02612 Constrained Optimization
Exercise 07 Linear Programming

John Bagterp Jørgensen

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In this exercise we consider an LP in standard form

$$\min_x \quad c'x$$
$$Ax = b$$
$$x \geq 0$$

The Lagrange function for this probem is

$$L(x, \lambda, s) = c'x - \lambda'(Ax - b) - s'x$$

such that the first order necessary and sufficient optimality conditions are

$$\nabla_x L = c - A'\lambda - s = 0$$
$$Ax = b$$
$$x \geq 0 \quad \perp \quad s \geq 0$$

To test code for LP algorithms, we choose $A \in \mathbb{R}^{m \times n}$ randomly, and define

$$x_i = \begin{cases} \text{random positive number} & i = 1, 2, \ldots, m \\ 0 & i = m+1, m+2, \ldots, n \end{cases}$$

$$s_i = \begin{cases} \text{random positive number} & i = m+1, m+2, \ldots, n \\ 0 & i = 1, 2, \ldots, m \end{cases}$$

$$\lambda = \text{random vector}$$
$$c = A'\lambda + s$$
$$b = Ax$$

such that we know this combinations satisfies the first order optimality conditions.

Problem 1 - Revised Simplex Algorithm

- Describe the revised simplex algorithm for the standard LP and write a pseudo-code for it.

- Implement the revised simplex algorithm in Matlab for the standard LP

- Test your implementation of the revised simplex algorithm

Problem 2 - Interior-Point Algorithm

- Describe the primal-dual interior-point algorithm for the standard LP and write a pseudo-code for it.

- Implement the primal-dual interior-point algorithm in Matlab for the standard LP

- Test your implementation of the primal-dual interior-point algorithm

Problem 3 - Interior-Point Algorithm (Inequality form)

Consider a linear program in the form

$$\min_{x \in \mathbb{R}^n} \quad f(x) = g'x$$
$$s.t. \quad A'x \geq b$$

- Describe the primal-dual interior-point algorithm for this LP.

- Implement the primal-dual interior-point algorithm in Matlab for this problem

- Describe a method to generate random test problems that you can use to test your solver for this LP.

- Test your LP solver and compare it to `linprog`

- *Extra (hard problem)*: Implement a revised linear program algorithm tailored for this problem.

Problem 4 - Interior-Point Algorithm (General LP form)

Consider a linear program in the form

$$\min_{x \in \mathbb{R}^n} \quad f(x) = g'x$$
$$s.t. \quad x_l \leq x \leq x_u$$
$$b_l \leq A'x \leq b_u$$

- Describe the primal-dual interior-point algorithm for this LP.

- Implement the primal-dual interior-point algorithm in Matlab for this problem

- Describe a method to generate random test problems that you can use to test your solver for this LP.

- Test your LP solver and compare it to `linprog`

- *Extra (hard problem)*: Implement a revised linear program algorithm tailored for this problem.