Exercise 08 for 02612 Constrained Optimization

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1 Test problems

Look in the OSQP paper uploaded on Learn and make random QP and LP test problems that you can use to test your algorithms. Below you see example code for generating a random convex QP.

```
function [H,g,bl,A,bu,l,u] = RandomQP(n,alpha,density)
% RandomQP  Generates data for a random convex QP
%
%
%   min     0.5 x' H x + g' x
%    x
%   s.t.    bl <= A' x <= bu
%            l <=    x <= u
%
% Syntax: [H,g,bl,A,bu,l,u] = RandomQP(n,alpha,density)
%
%   Inputs:
%       n       number of variabels
%       alpha   regularization factor. alpha > 0
%       density density of sparse matrix. 0 < density < 1
%
%   Outputs: QP data

m = 10*n;
A = sprandn(n,m,density);
bl = -rand(m,1);
bu = rand(m,1);
M = sprandn(n,n,density);
H = M*M' + alpha*eye(n,n);
g = randn(n,1);
l = -ones(n,1);
u = ones(n,1);
```

1.1 Convex QP

$$\min_x \quad \phi = \frac{1}{2}x'Hx + g'x \tag{1a}$$

$$s.t. \quad b_l \leq A'x \leq b_u \tag{1b}$$

$$l \leq x \leq u \tag{1c}$$

- Implement a primal-dual interior point algorithm that can solve the above QP.
- Implement a primal active-set algorithm that can solve the above QP.
- Verify the algorithms using a number of test problems.

1.2 LP

$$\min_{x} \quad \phi = g'x \tag{2a}$$

$$s.t. \quad b_l \leq A'x \leq b_u \tag{2b}$$

$$l \leq x \leq u \tag{2c}$$

1.2.1 Solve the LP using convex QP algorithms

set $H = \varepsilon I$ for ε small i.e. $\varepsilon = \sqrt{\epsilon}$ where ϵ is the machine precision and solve the above LP using the convex QP algorithms you developed.

1.2.2 Solve the LP using tailored LP algorithms

- Implement a primal-dual interior-point algorithm tailored for the above LP.
- Implement a primal active-set (revised simplex) algorithm tailored for the above LP
- Verify the algorithms using a number of test problems. Compare the solution time to the QP algorithms adapated to the LP.

2 Extra exercise - Quadratic Programs

You should try to implement interior point and active set algorithms for convex quadratic programs with different structure. You also test these algorithm for problems you construct or find. Go through the procedure of writing up the algorithm in paper (write the problem, form the Lagrangian, write the first order conditions, do the steps in the primal-dual IP algorithm with the linear algebra specialized to the case you have, implement the algorithm in Matlab, test the algorithm).

2.1 General convex QP

$$\begin{aligned} \min_x \quad & \phi = \frac{1}{2}x'Hx + g'x && \text{(3a)} \\ s.t. \quad & A'x = b && \text{(3b)} \\ & C'x \geq d && \text{(3c)} \end{aligned}$$

2.2 Inequality constrained Convex QP

$$\begin{aligned} \min_x \quad & \phi = \frac{1}{2}x'Hx + g'x && \text{(4a)} \\ s.t. \quad & C'x \geq d && \text{(4b)} \end{aligned}$$

2.3 Box constrained Convex QP

$$\begin{aligned} \min_x \quad & \phi = \frac{1}{2}x'Hx + g'x && \text{(5a)} \\ s.t. \quad & l \leq x \leq u && \text{(5b)} \end{aligned}$$

2.4 Equality and box constrained Convex QP

$$\begin{aligned} \min_x \quad & \phi = \frac{1}{2}x'Hx + g'x && \text{(6a)} \\ s.t. \quad & A'x = b && \text{(6b)} \\ s.t. \quad & l \leq x \leq u && \text{(6c)} \end{aligned}$$

2.5 Inequality and box constrained Convex QP

$$\begin{aligned} \min_x \quad & \phi = \frac{1}{2}x'Hx + g'x && \text{(7a)} \\ s.t. \quad & b_l \leq C'x \leq b_u && \text{(7b)} \\ s.t. \quad & l \leq x \leq u && \text{(7c)} \end{aligned}$$

3 Extra exercise - Linear Programs

You should try to implement interior point and active set (revised simplex) algorithms for linear programs with different structure. You also test these algorithm for problems you construct or find. Go through the procedure of writing up the algorithm in paper (write the problem, form the Lagrangian, write the first order conditions, do the steps in the primal-dual IP algorithm with the linear algebra specialized to the case you have, implement the algorithm in Matlab, test the algorithm). Do NOT convert the LP to an LP in standard form.

3.1 General LP

$$\min_x \quad \phi = g'x \tag{8a}$$
$$s.t. \quad A'x = b \tag{8b}$$
$$C'x \geq d \tag{8c}$$

3.2 Extra Exercise - Inequality constrained LP

$$\min_x \quad \phi = g'x \tag{9a}$$
$$s.t. \quad C'x \geq d \tag{9b}$$

3.3 Box constrained LP

$$\min_x \quad \phi = g'x \tag{10a}$$
$$s.t. \quad l \leq x \leq u \tag{10b}$$

Can you in addition come up with a very simple and fast procedure for solving this problem?

3.4 Equality and box constrained LP

$$\min_x \quad \phi = g'x \tag{11a}$$
$$s.t. \quad A'x = b \tag{11b}$$
$$s.t. \quad l \leq x \leq u \tag{11c}$$

3.5 Inequality and box constrained LP

$$\min_x \quad \phi = g'x \tag{12a}$$
$$s.t. \quad b_l \leq C'x \leq b_u \tag{12b}$$
$$s.t. \quad l \leq x \leq u \tag{12c}$$