

Historical Periods: 
$$\{\hat{\boldsymbol{\xi}}_i: i \in \{0, 1, ..., E-1, E\}\}$$

Training Periods: 
$$\{\hat{\boldsymbol{\xi}}_i: i \in \{\tau, \ \tau+1, \ \dots, \tau+T_{TR}-1, \tau+T_{TR}\}\}$$

Validation Periods: 
$$\{\hat{\xi}_i : i \in \{\tau + T_{TR} + 1, \tau + T_{TR} + 2, ..., \tau + T_{TR} + T_V - 1, \tau + T_{TR} + T_V\}\}$$

Testing Periods:  $\{\hat{\xi}_i : i \in \{\tau + T_{TR} + T_V + 1, \tau + T_{TR} + T_V + 2, ..., \tau + T_{TR} + T_V - 1, \tau + T_{TR} + T_V + T_{TE}\}\}$