

MOSEK ApS

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# Introduction

The **MOSEK** Optimization Suite 10.0.46 is a powerful software package capable of solving large-scale optimization problems of the following kind:

- linear,
- conic:
  - conic quadratic (also known as second-order cone),
  - involving the exponential cone,
  - involving the power cone,
  - semidefinite.
- convex quadratic and quadratically constrained,
- integer.

In order to obtain an overview of features in the MOSEK Optimization Suite consult the product introduction guide.

The most widespread class of optimization problems is *linear optimization problems*, where all relations are linear. The tremendous success of both applications and theory of linear optimization can be ascribed to the following factors:

- The required data are simple, i.e. just matrices and vectors.
- Convexity is guaranteed since the problem is convex by construction.
- Linear functions are trivially differentiable.
- There exist very efficient algorithms and software for solving linear problems.
- Duality properties for linear optimization are nice and simple.

Even if the linear optimization model is only an approximation to the true problem at hand, the advantages of linear optimization may outweigh the disadvantages. In some cases, however, the problem formulation is inherently nonlinear and a linear approximation is either intractable or inadequate. *Conic optimization* has proved to be a very expressive and powerful way to introduce nonlinearities, while preserving all the nice properties of linear optimization listed above.

The fundamental expression in linear optimization is a linear expression of the form

$$Ax - b \ge 0$$
.

In conic optimization this is replaced with a wider class of constraints

$$Ax - b \in \mathcal{K}$$

where  $\mathcal{K}$  is a *convex cone*. For example in 3 dimensions  $\mathcal{K}$  may correspond to an ice cream cone. The conic optimizer in **MOSEK** supports a number of different types of cones  $\mathcal{K}$ , which allows a surprisingly large number of nonlinear relations to be modeled, as described in the **MOSEK** Modeling Cookbook, while preserving the nice algorithmic and theoretical properties of linear optimization.

# 1.1 Why the Command Line Tools?

The **MOSEK** capabilities can be accessed from the command line without the need to use any programming language. The user can input optimization problems using files in a variety of *formats*, or via the AMPL language shell.

The Command Line Tools provides access to:

- Linear Optimization (LO)
- Conic Quadratic (Second-Order Cone) Optimization (CQO, SOCO)
- Power Cone Optimization
- Conic Exponential Optimization (CEO)
- Convex Quadratic and Quadratically Constrained Optimization (QO, QCQO)
- Semidefinite Optimization (SDO)
- Mixed-Integer Optimization (MIO)

as well as to additional utilities for:

- problem analysis,
- sensitivity analysis,
- infeasibility diagnostics.

# **Contact Information**

Phone	$+45\ 7174\ 9373$	
Website	mosek.com	
Email		
	sales@mosek.com	Sales, pricing, and licensing
	support@mosek.com	Technical support, questions and bug reports
	info@mosek.com	Everything else.
Mailing Address		
	MOSEK ApS	
	Fruebjergvej 3	
	Symbion Science Park, Box 16	
	2100 Copenhagen O	
	Denmark	

You can get in touch with  $\mathbf{MOSEK}$  using popular social media as well:

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In particular  $\mathbf{Twitter}$  is used for news, updates and release announcements.

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zlib.h -- interface of the 'zlib' general purpose compression library version 1.2.7, May 2nd, 2012

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Jean-loup Gailly Mark Adler

jloup@gzip.org madler@alumni.caltech.edu

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BLASFEO -- BLAS For Embedded Optimization.

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### one TBB

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# Installation

In this section we discuss how to install and setup the MOSEK Command Line Tools.

Important: Before running this MOSEK interface please make sure that you:

- Installed **MOSEK** correctly. Some operating systems require extra steps. See the Installation guide for instructions and common troubleshooting tips.
- Set up a license. See the Licensing guide for instructions.

### Locating files in the MOSEK Optimization Suite

The relevant files of the Command Line Tools are organized as reported in Table 4.1.

Table 4.1: Relevant files for the Command Line Tools.

Relative Path	Description	Label
<pre><mskhome>/mosek/10.0/tools/platform/<platform>/bin</platform></mskhome></pre>	Binaries	<bindir></bindir>
<pre><mskhome>/mosek/10.0/tools/platform/<platform>/bin/mosek</platform></mskhome></pre>	Mosek executable	
<mskhome>/mosek/10.0/tools/examples/data</mskhome>	Examples	<exdir></exdir>

#### where

- <MSKHOME> is the folder in which the MOSEK Optimization Suite has been installed,
- <PLATFORM> is the actual platform among those supported by MOSEK, i.e. win32x86, win64x86, linux64x86 or osx64x86.

### Setting up paths

The executable file is ready for use. It may be convenient to add the directory  ${\tt SINDIR}$  to the environment variable PATH, and then  ${\tt MOSEK}$  can simply be used by typing

#### mosek

in the command line.

# 4.1 Testing the installation

To test that Command Line Tools has been installed correctly go to the examples directory <EXDIR> and run MOSEK on any of the input files, for example lo1.mps:

```
mosek lo1.mps
```

Is should produce output similar to:

```
MOSEK Version 8.0.0.53 (Build date: 2017-1-12 22:21:45)
Copyright (c) MOSEK ApS, Denmark. WWW: mosek.com
Platform: Linux/64-X86
Open file 'lo1.mps'
Reading started.
[....]
Optimizer started.
Interior-point optimizer started.
[....]
Interior-point solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: 8.3333333280e+01
                                                Viol. con: 1e-08
                                                                     var: 0e+00
                                  nrm: 5e+01
                                                Viol. con: 2e-10
                                                                     var: 5e-09
 Dual.
          obj: 8.3333333242e+01
                                  nrm: 4e+00
Basic solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: 8.333333333e+01 nrm: 5e+01
                                                Viol. con: 7e-15
                                                                     var: 0e+00
 Dual.
          obj: 8.3333333245e+01 nrm: 4e+00
                                                Viol. con: 2e-10
                                                                     var: 5e-09
[....]
Open file 'lo1.sol'
Start writing.
done writing. Time: 0.00
Open file 'lo1.bas'
Start writing.
done writing. Time: 0.00
Return code - 0 [MSK_RES_OK]
```

# The Command Line Tool

## 5.1 Introduction

The MOSEK command line tool is used to solve optimization problems from the operating system command line. It is invoked as follows

```
mosek [options] [filename]
```

where both [options] and [filename] are optional arguments:

- [options] consists of command line arguments that modify the behavior of MOSEK. They are listed in Sec. 5.5. In particular, options can be used to set optimizer parameters.
- [filename] is a file describing the optimization problem. The MOSEK command line accepts files in any of the *supported file formats* or in the AMPL .nl format.

If no arguments are given, MOSEK will display a splash screen and exit.

```
user@host:~$ mosek/8/tools/platform/linux64x86/bin/mosek

MOSEK Version 8.0.0.32(BETA) (Build date: 2016-7-12 10:29:26)
Copyright (c) MOSEK ApS, Denmark. WWW: mosek.com
Platform: Linux/64-X86

*** No input file specified. No optimization is performed.

Return code - 0 [MSK_RES_OK]
```

### 5.2 Files

The MOSEK command line tool communicates with the user via files and prints some execution logs and solution summary to the terminal.

#### Input files

Optimization problems are read from files. See Sec. 12 for details.

### File format conversion

To convert between two file formats supported by  $\mathbf{MOSEK}$  use the option -x together with -out to specify the target file name. The target file type must support the problem type of the source file, otherwise the conversion will be partial. For instance in case a MPS file must be converted in a more readable OPF format, the following line can be used

```
mosek -x -out lo1.opf lo1.mps
```

With the -x option the solver will not actually solve the problem.

#### **Output files**

Solutions are written to files:

- .bas basic solution,
- .sol interior point solution,
- .itg integer solution (the only available solution for mixed-integer problems).

For linear problems both the basic and interior point solution may be present. Infeasibility certificates are stored in the same files. See Sec. 12.8 for details.

## 5.3 Example

To solve a problem stored in file, say lol.mps, write:

```
mosek lo1.mps
```

The solver will

- read lo1.mps from disk,
- solve the problem and display the solution log and
- store the relevant solution files if any solution exists; file content explained in Sec. 12.8.

```
MOSEK Version 8.0.0.34(BETA) (Build date: 2016-8-24 00:51:13)
Copyright (c) MOSEK ApS, Denmark. WWW: mosek.com
Platform: Linux/64-X86
Open file '/home/andrea/mosek/8/tools/examples/data/lo1.mps'
Reading started.
Using 'obj' as objective vector
Read 13 number of A nonzeros in 0.00 seconds.
Using 'rhs' as rhs vector
Using 'bound' as bound vector
Reading terminated. Time: 0.00
Read summary
                   : LO (linear optimization problem)
 Type
 Objective sense : max
 Scalar variables: 4
 Matrix variables : 0
 Constraints
                 : 3
 Cones
                   : 0
 Time
                   : 0.0
Problem
 Name
                         : 101
```

```
Objective sense
                       : max
                       : LO (linear optimization problem)
 Type
 Constraints
                       : 3
 Cones
                       : 0
 Scalar variables
                       : 0
 Matrix variables
 Integer variables
                       : 0
Optimizer started.
Interior-point optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Eliminator started.
Freed constraints in eliminator: 0
Eliminator terminated.
Eliminator - tries
                                  : 1
                                                     time
                                                                          : 0.00
Lin. dep. - tries
                                  : 1
                                                                           : 0.00
                                                     time
Lin. dep. - number
                                  : 0
Presolve terminated. Time: 0.00
Optimizer - threads
                                 : 2
Optimizer - solved problem
                                : the primal
Optimizer - Constraints
                                 : 3
Optimizer - Cones
                                 : 0
Optimizer - Scalar variables
                                 : 6
                                                                           : 0
                                                     conic
Optimizer - Semi-definite variables: 0
                                                     scalarized
                                                                          : 0
Factor - setup time
                                                                          : 0.00
                                : 0.00
                                                     dense det. time
Factor
         - ML order time
                                : 0.00
                                                     GP order time
                                                                          : 0.00
Factor
        - nonzeros before factor : 6
                                                     after factor
                                                                          : 6
Factor
        - dense dim.
                                  : 0
                                                     flops
                                                                          : 1.
⊶06e+02
ITE PFEAS DFEAS
                             PRSTATUS
                                       POBJ
                                                        DOBJ
                    GFEAS
                                                                          MU
→ TIME
0 8.0e+00 3.2e+00 3.5e+00 1.00e+00
                                       1.000000000e+01
                                                        0.00000000e+00
                                                                          1.0e+00
1 4.2e+00 2.5e+00 4.7e-01 0.00e+00
                                       3.093970927e+01
                                                        2.766058702e+01
                                                                          2.6e+00
→ 0.01
2 4.2e-01 2.5e-01 4.6e-02 -1.82e-02 6.511676243e+01
                                                        6.308843559e+01
                                                                          2.6e-01
→ 0.01
3 3.6e-02 2.1e-02 3.9e-03 5.84e-01
                                       8.096141239e+01
                                                       8.061962333e+01
                                                                          2.2e-02
→ 0.01
  1.5e-05 9.1e-06 1.7e-06 9.43e-01
                                       8.333280389e+01
                                                       8.333241803e+01
                                                                          9.2e-06
→ 0.01
5 1.5e-09 9.1e-10 1.7e-10 1.00e+00 8.333333328e+01
                                                       8.333333324e+01
                                                                          9.2e-10
→ 0.01
Basis identification started.
Primal basis identification phase started.
ITER
         TIME
         0.00
Primal basis identification phase terminated. Time: 0.00
Dual basis identification phase started.
ITER
         TIME
         0.00
Dual basis identification phase terminated. Time: 0.00
Basis identification terminated. Time: 0.00
Interior-point optimizer terminated. Time: 0.01.
```

```
Optimizer terminated. Time: 0.02
Interior-point solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: 8.3333333280e+01
                                  nrm: 5e+01
                                                Viol. con: 1e-08
                                                                    var: 0e+00
          obj: 8.3333333242e+01
 Dual.
                                  nrm: 4e+00
                                                Viol. con: 2e-10
                                                                    var: 5e-09
Basic solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: 8.333333333e+01 nrm: 5e+01
                                                Viol. con: 7e-15
                                                                    var: 0e+00
          obj: 8.3333333245e+01 nrm: 4e+00
                                                Viol. con: 2e-10
                                                                    var: 5e-09
Optimizer summary
 Optimizer
                                                   time: 0.02
    Interior-point
                                                   time: 0.01
                           - iterations : 5
     Basis identification -
                                                   time: 0.00
                          - iterations : 0
       Primal
                                                   time: 0.00
       Dual
                          - iterations : 0
                                                  time: 0.00
       Clean primal
                         - iterations : 0
                                                  time: 0.00
       Clean dual
                          - iterations : 0
                                                   time: 0.00
                                                   time: 0.00
   Simplex
     Primal simplex
                          - iterations : 0
                                                   time: 0.00
     Dual simplex
                           - iterations : 0
                                                   time: 0.00
   Mixed integer
                           - relaxations: 0
                                                   time: 0.00
Open file '/home/andrea/mosek/8/tools/examples/data/lo1.sol'
Start writing.
done writing. Time: 0.00
Open file '/home/andrea/mosek/8/tools/examples/data/lo1.bas'
Start writing.
done writing. Time: 0.00
Return code - 0 [MSK_RES_OK]
```

## 5.4 Solver Parameters

MOSEK comes with a large number of parameters that allows the user to tune the behavior of the optimizer. The typical settings which can be changed with solver parameters include:

- choice of the optimizer for linear problems,
- choice of primal/dual solver,
- turning presolve on/off,
- turning heuristics in the mixed-integer optimizer on/off,
- level of multi-threading,
- ullet feasibility tolerances,
- solver termination criteria,
- behaviour of the license manager,

and more. All parameters have default settings which will be suitable for most typical users. Each parameter is identified by a unique string name and it can accept either integers or symbolic names, floating point values or symbolic strings. Please refer to Sec. 11.2 for the complete list of available solver parameters.

### 5.4.1 Setting from command line

Setting solver parameters is possible using the command line option -d. If multiple parameters must be specified, option -d must be repeated for each one. For example, the next command will switch off presolve, set a feasibility tolerance and solve the problem from lol.opf:

```
mosek -d MSK_IPAR_PRESOLVE_USE MSK_OFF -d MSK_DPAR_INTPNT_TOL_PFEAS 1.0e-8 lo1.opf
```

## 5.4.2 Using the Parameter File

Solver parameters can also be set using a parameter file, for example:

The syntax of the parameter file must obey a few simple rules:

- The file must begin with BEGIN MOSEK and end with END MOSEK.
- Empty lines and lines starting from a % sign are ignored.
- Each line contains a valid MOSEK parameter name followed by its value.

The parameter file can have any name. Assuming it has been called mosek.par, it can be used using the -p option as follows:

```
mosek -p mosek.par afiro.mps
```

Command-line parameters override those from the parameter file in case of repetition. For instance

```
mosek -p mosek.par -d MSK_DPAR_INTPNT_TOL_PFEAS 1.0e-8 afiro.mps
```

will set  $MSK_DPAR_INTPNT_TOL_PFEAS$  to  $10^{-8}$  using the value provided on the command line.

# 5.5 Command Line Arguments

The following list shows the available command-line arguments for MOSEK:

-anapro

Analyze the problem data.

-anasoli <name>

Analyze the initial solution name e.g. -anasoli bas.

-anasolo <name>

Analyze the final solution name e.g. -anasolo itg.

-a

MOSEK is started in AMPL mode.

-basi <name>

Input basic solution file name.

-baso <name>

Output basic solution file name.

-d <name> <value>

Define the value value for the MOSEK parameter name.

```
-dbgmem <name>
     Name of memory debug file.
-f
     Complete license information is printed.
-h, -?
     Help.
```

-inti <name>

Input integer solution file name.

-into <name>

Output integer solution file name.

-itri <name>

Input interior point solution file name.

-itro <name>

Output interior point solution file name.

-info <name>

Infeasible subproblem output file name.

-infrepo <name>

Feasibility reparation output file.

-jsoli <name>

Input JSON format solution file name.

-jsolo <name>

Output JSON format solution file name.

-1,-L <dir>

dir is the directory where the MOSEK license file mosek.lic is located.

-max

The problem is maximized.

-min

The problem is minimized.

-n

Ignore errors in subsequent parameter settings.

-optserv <url>

Use an OptServer specified by an URL of the form http://host:port.

-out <name>

Write the task to a data file named name. See Sec. 12.

-p <name>, -pari <name>

Name of the input parameter file.

-paro <name>

Name of the output parameter file.

-primalrepair

Repair a primal infeasible problem. See Sec. 10.2.

If the option is present, the program returns -1 if an error occurred, otherwise 0.

-removeitg

Removes all integer constraints after reading the problem.

-rout <name>

If the option is present, the program writes the return code to file name.

-q <name>

Name of an optional log file.

-sen <file>

Perform sensitivity analysis based on file.

As little information as possible is send to the terminal.

-toconic

Translate to conic form after reading.

MOSEK version is printed and no optimization is performed.

If this options is on, then MOSEK will wait for a license.

-x

Do not run the optimizer. Useful for converting between file formats.

List all possible solver parameters with default value, lower bound and upper bound (if applicable).

# 5.6 The license system

MOSEK is a commercial product that always needs a valid license to work. MOSEK uses a third party license manager to implement license checking. The number of license tokens provided determines the number of optimizations that can be run simultaneously.

By default a license token remains checked out for the whole execution of the command line tool. If the license is not unlimited, then the number of tokens determines the maximal number of processes that can run simultaneously. In this case setting the license wait flag with the parameter <code>MSK\_IPAR\_LICENSE\_WAIT</code> will force <code>MOSEK</code> to wait until a license token becomes available instead of returning with an error.

# The MOSEK-bundled AMPL shell

AMPL is a modeling language for specifying linear and nonlinear optimization models in a natural way. AMPL also makes it easy to solve the problem and e.g. display the solution or part of it. We will not discuss the specifics of the AMPL language here but instead refer the reader to [FGK03], http://ampl.com/BOOK/download.html and the AMPL website http://www.ampl.com.

AMPL cannot solve optimization problems by itself but requires a link to an optimizer. The **MOSEK** distribution includes an AMPL link which makes it possible to use **MOSEK** as an optimizer within AMPL. The link can be used from any AMPL shell.

#### Note:

- To use **MOSEK** from AMPL you need to set up the system path to the **MOSEK** command line tool.
- It is possible to specify problems in AMPL that cannot be solved by **MOSEK**. The optimization problem must be a smooth convex optimization problem as discussed in Sec. 8.

We refer to your AMPL documentation for how to specify optimization problems in AMPL and attach **MOSEK** as a solver.

# 6.1 An example

In many instances, you can successfully apply **MOSEK** simply by specifying the model and data, setting the solver option to **MOSEK**, and typing solve.

Consider a simple linear optimization problem formulated as an AMPL model in Listing 6.1.

Listing 6.1: An example of an optimization problem in AMPL language.

We can specify the input data using an input file again following the AMPL syntax, as in Listing 6.2.

Listing 6.2: An example of data for an optimization problem using AMPL language.

```
param: FOOD:
                             cost f_min f_max :=
 "Quarter Pounder w/ Cheese"
                             1.84
 "McLean Deluxe w/ Cheese"
                             2.19
 "Big Mac"
                            1.84
 "Filet-O-Fish"
                            1.44
 "McGrilled Chicken"
                           2.29
 "Fries, small"
                             .77
 "Sausage McMuffin"
                           1.29
 "1% Lowfat Milk"
                            .60
 "Orange Juice"
                             .72
param: NUTR: n_min n_max :=
       Cal
              2000
       Carbo
               350
                     375
       Protein
               55
               100
       VitA
               100
       VitC
       Calc
              100
       Iron
               100
                      . ;
param amt (tr):
                           Cal Carbo Protein VitA
                                                    VitC Calc Iron :=
 "Quarter Pounder w/ Cheese"
                           510
                                  34
                                        28
                                              15
                                                    6
                                                          30
                                                               20
 "McLean Deluxe w/ Cheese"
                           370
                                  35
                                        24
                                              15
                                                    10
                                                          20
                                                               20
 "Big Mac"
                           500
                                  42
                                        25
                                               6
                                                    2
                                                          25
                                                               20
 "Filet-O-Fish"
                           370
                                  38
                                              2
                                                    0
                                       14
                                                          15
                                                               10
 "McGrilled Chicken"
                         400
                                  42
                                        31
                                              8 15
                                                          15
 "Fries, small"
                           220
                                  26
                                        3
                                               0
                                                          0
                                                               2
                                                   15
 "Sausage McMuffin"
                           345
                                  27
                                        15
                                              4
                                                    0
                                                          20
                                                               15
                                                    4
 "1% Lowfat Milk"
                           110
                                  12
                                        9
                                              10
                                                          30
                                                                0
                                                                2;
 "Orange Juice"
                            80
                                  20
                                               2
                                                    120
                                                          2
```

In AMPL type in the commands:

```
ampl: model diet.mod;
ampl: data diet.dat;
ampl: option solver mosek;
ampl: solve;
```

The resulting output is:

MOSEK finished.

Problem status - PRIMAL\_AND\_DUAL\_FEASIBLE

Solution status - OPTIMAL Primal objective - 14.8557377 Dual objective - 14.8557377

Objective = Total\_Cost

# 6.2 Retrieving solutions

### 6.2.1 Status codes

The AMPL parameter solve\_result\_num is used to indicate the outcome of the optimization process. It is used as follows

```
ampl: display solve_result_num
```

Please refer to table Table 6.1 for possible values of this parameter.

Table 6.1: Interpretation of solve\_result\_num.

Value	Message
0	the solution is optimal.
100	suboptimal primal solution.
101	superoptimal (dual feasible) solution.
150	the solution is near optimal.
200	primal infeasible problem.
300	dual infeasible problem.
400	too many iterations.
500	solution status is unknown.
501	ill-posed problem, solution status is unknown.
> 501	Mapped MOSEK response code. See note below.

MOSEK response codes are mapped to AMPL return codes greater than 501. In order to get the actual response code the base value 501 must be subtracted. For example: the AMPL return code 502corresponds to MOSEK response code 1.

### 6.2.2 Which solution is returned

MOSEK can produce three types of solutions: basic, interior point and integer. The solution returned to AMPL is determined according to the following rules:

- For problems containing integer variables only the integer solution is available and it is returned.
- For nonlinear problems only the interior point solution is available and it is returned.
- For linear problems, if both basic and interior point solution are available, then the basic solution is returned. Otherwise the only available solution is returned.

## 6.3 Optimizer options

## 6.3.1 The MOSEK parameter database

The **MOSEK** optimizer can be controller using solver parameters, as described in Sec. 5.4. These parameters can be modified within AMPL as shown in the example below:

```
ampl: model diet.mod;
ampl: data diet.dat;
ampl: option solver mosek;
ampl: option mosek_options
ampl? 'msk_ipar_optimizer = msk_optimizer_primal_simplex \
ampl? msk_ipar_sim_max_iterations = 100000';
ampl: solve;
```

In the example above a string called mosek\_options is created which contains the parameter settings. Each parameter setting has the format

```
parameter_name = value
```

where parameter\_name is a valid MOSEK parameter name. See Sec. 11.2 for a description of all valid MOSEK parameters.

An alternative way of specifying the parameters is

```
ampl: option mosek_options
ampl? 'msk_ipar_optimizer = msk_optimizer_primal_simplex'
ampl? 'msk_ipar_sim_max_iterations = 1000000';
```

New parameters can also be appended to an existing option string as shown below.

```
ampl: option mosek_options $mosek_options
ampl? ' msk_ipar_sim_print_freq = 0 msk_ipar_sim_max_iterations = 1000';
```

The expression \$mosek\_options expands to the current value of the option. Line two in the example appends an additional value msk\_ipar\_sim\_max\_iterations to the option string.

## 6.3.2 Options

MOSEK recognizes the following AMPL options.

#### outlev

Controls the amount of printed output. 0 means no printed output and a higher value means progressively more output. An example of setting outlev is as follows:

```
ampl: option mosek_options 'outlev=2';
```

### wantsol

Controls the solution information generated when run in standal one mode (called without the argument  ${\tt -AMPL}).$  It should be constructed as the sum of

1	to write a .sol file
2	to print the primal variable values
4	to print the dual variable values
8	to suppress printing the solution message

We refer the reader to the AMPL manual [FGK03] for more details.

## 6.3.3 Passing variable names to MOSEK

AMPL assigns meaningful names to all the constraints and variables. Since **MOSEK** uses item names in error and log messages, it may be useful to pass the AMPL names to **MOSEK**. This can be achieved with the command:

```
ampl: option auxfiles rc;
ampl: solve;
```

### 6.4 Hot-start

Frequently, a sequence of optimization problems is solved where each problem differs only slightly from the previous problem. In that case it may be advantageous to use the previous optimal solution to warm-start the optimizer. Such a facility is available in **MOSEK** only when the simplex optimizer is used

The warm-start facility exploits the AMPL variable suffix sstatus to communicate the optimal basis back to AMPL, and AMPL uses this facility to communicate an initial basis to MOSEK. The following example demonstrates this feature.

```
ampl: model diet.mod;
ampl: data diet.dat;
ampl: option solver mosek;
ampl: option mosek_options
ampl? 'msk_ipar_optimizer = msk_optimizer_primal_simplex outlev=2';
ampl: solve;
ampl: display Buy.sstatus;
ampl: solve;
```

The resulting output is:

```
Accepted: msk_ipar_optimizer
                                              = MSK_OPTIMIZER_PRIMAL_SIMPLEX
Accepted: outlev
Computer
           - Platform
                                     : Linux/64-X86
                                    : Intel-P4
Computer
          - CPU type
MOSEK
           - task name
MOSEK
           - objective sense
                                    : min
MOSEK
           - problem type
                                    : LO (linear optimization problem)
MOSEK
           - constraints
                                    : 7
                                                         variables
                                                                                : 9
           - integer variables
MOSEK
                                    : 0
Optimizer started.
Simplex optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Presolve - Stk. size (kb) : 0
Eliminator - tries
                                     : 0
                                                                                : 0.00
                                                         time
Eliminator - elim's
                                     : 0
Lin. dep. - tries
                                                                                 : 0.00
                                     : 1
                                                         time
Lin. dep. - number
Presolve terminated. Time: 0.00
Primal simplex optimizer started.
Primal simplex optimizer setup started.
Primal simplex optimizer setup terminated.
Optimizer - solved problem
                                    : the primal
Optimizer - constraints
                                    : 7
                                                                                 : 9
                                                         variables
Optimizer - hotstart
                                     : no
         DEGITER(%) PFEAS
                                              POBJ
                                                                     DOBJ
                                  DFEAS
      TTMF:
                TOTTIME
                                                                      (continues on next page)
```

```
0
         0.00
                     1.40e+03
                                NA
                                             1.2586666667e+01
                                                                   NA
     0.00
               0.01
\hookrightarrow
3
         0.00
                     0.00e+00
                                 NΑ
                                             1.4855737705e+01
                                                                   NA
                                                                                    11
     0.00
               0.01
Primal simplex optimizer terminated.
Simplex optimizer terminated. Time: 0.00.
Optimizer terminated. Time: 0.01
Return code - 0 [MSK_RES_OK]
MOSEK finished.
Problem status : PRIMAL_AND_DUAL_FEASIBLE
Solution status : OPTIMAL
Primal objective : 14.8557377
Dual objective : 14.8557377
Objective = Total_Cost
Buy.sstatus [*] :=
'Quarter Pounder w/ Cheese' bas
'McLean Deluxe w/ Cheese' low
'Big Mac' low
Filet-O-Fish low
'McGrilled Chicken' low
'Fries, small' bas
'Sausage McMuffin' low
'1% Lowfat Milk' bas
'Orange Juice' low
Accepted: msk_ipar_optimizer
                                             = MSK_OPTIMIZER_PRIMAL_SIMPLEX
Accepted: outlev
                                             = 2
Basic solution
Problem status : UNKNOWN
Solution status : UNKNOWN
Primal - objective: 1.4855737705e+01
                                    eq. infeas.: 3.97e+03 max bound infeas.: 2.
→00e+03
Dual - objective: 0.00000000000e+00 eq. infeas.: 7.14e-01 max bound infeas.: 0.
→00e+00
                                  : Linux/64-X86
Computer - Platform
Computer - CPU type
                                   : Intel-P4
MOSEK
          - task name
          - objective sense
                                   : min
MOSEK
MOSEK

    problem type

                                   : LO (linear optimization problem)
MOSEK
          - constraints
                                   : 7
                                                       variables
MOSEK
          - integer variables
                                   : 0
Optimizer started.
Simplex optimizer started.
Presolve started.
Presolve - Stk. size (kb) : 0
Eliminator - tries
                                   : 0
                                                       time
                                                                              : 0.00
Eliminator - elim's
                                   : 0
Lin. dep. - tries
                                   : 0
                                                       time
                                                                              : 0.00
Lin. dep. - number
                                   : 0
Presolve terminated. Time: 0.00
Primal simplex optimizer started.
Primal simplex optimizer setup started.
Primal simplex optimizer setup terminated.
Optimizer - solved problem
                                   : the primal
```

```
Optimizer
          - constraints
                                     : 7
                                                         variables
Optimizer
          - hotstart
                                     : yes
Optimizer
           - Num. basic
                                     : 7
                                                         Basis rank
                                                                                  : 7
Optimizer - Valid bas. fac.
                                     : no
          DEGITER(%) PFEAS
ITER
                                               POB.I
                                                                      DOB.J
                                   DFEAS
                                                                                       Ш
      TIME
                TOTTIME
0
          0.00
                      0.00e+00
                                               1.4855737705e+01
                                                                      NA
                                                                                       Ш
      0.00
                0.01
0
                                               1.4855737705e+01
          0.00
                      0.00e+00
                                   NΑ
                                                                      NΑ
      0.00
                0.01
Primal simplex optimizer terminated.
Simplex optimizer terminated. Time: 0.00.
Optimizer terminated. Time: 0.01
Return code - 0 [MSK_RES_OK]
MOSEK finished.
Problem status
                  : PRIMAL_AND_DUAL_FEASIBLE
Solution status
                 : OPTIMAL
Primal objective
                 : 14.8557377
Dual objective
                  : 14.8557377
Objective = Total_Cost
```

Please note that the second solve takes fewer iterations since the previous optimal basis is reused.

## 6.5 Infeasibility report

For linear optimization problems without any integer constrained variables **MOSEK** can generate an infeasibility report automatically. The report provides important information about the infeasibility. The generation of the infeasibility report is turned on using the parameter setting

```
option auxfiles rc;
option mosek_options 'msk_ipar_infeas_report_auto=msk_on';
```

For further details about infeasibility report see Sec. 10.2.

# 6.6 Sensitivity analysis

**MOSEK** can calculate sensitivity information for the objective and constraints. To enable sensitivity information set the option:

```
sensitivity = 1
```

Results are returned in variable/constraint suffixes as follows:

- .down Smallest value of objective coefficient/right hand side before the optimal basis changes.
- .up Largest value of objective coefficient/right hand side before the optimal basis changes.
- .current Current value of objective coefficient/right hand side.

For ranged constraints sensitivity information is returned only for the lower bound. The example below returns sensitivity information on the diet model.

```
ampl: model diet.mod;
ampl: data diet.dat;
ampl: option solver mosek;
ampl: option mosek_options 'sensitivity=1';
```

```
ampl: solve;
#display sensitivity information and current solution.
ampl: display _var.down,_var.current,_var.up,_var;
#display sensitivity information and optimal dual values.
ampl: display _con.down,_con.current,_con.up,_con;
```

The resulting output is:

```
Return code - 0 [MSK_RES_OK]
MOSEK finished.
Problem status
                 : PRIMAL_AND_DUAL_FEASIBLE
Solution status : OPTIMAL
Primal objective : 14.8557377
Dual objective
                 : 14.8557377
suffix up OUT;
suffix down OUT;
suffix current OUT;
Objective = Total_Cost
  _var.down _var.current
                                _var.up
                                               _var
    1.37385
                1.84
                                  1.86075
                                              4.38525
2
   1.8677
                 2.19
                            Infinity
3
   1.82085
                 1.84
                                              0
                            Infinity
4
  1.35466
                 1.44
                                              0
                            Infinity
5
  1.57633
                 2.29
                                              0
                            Infinity
6
   0.094
                 0.77
                                   0.794851
                                              6.14754
7
   1.22759
                 1.29
                                              0
                            Infinity
8
   0.57559
                 0.6
                                   0.910769
                                              3.42213
9
   0.657279
                 0.72
                            Infinity
ampl: display _con.down,_con.current,_con.up,_con;
      _con.down
                _con.current _con.up
                                                          :=
    -Infinity
                        2000
                                  3965.37
2
          297.6
                        350
                                   375
                                             0.0277049
3
                         55
                                   172.029
   -Infinity
                                             0
4
           63.0531
                         100
                                   195.388
                                             0.0267541
5
   -Infinity
                         100
                                   132.213
6
   -Infinity
                         100
                                   234.221
                                             0
7
           17.6923
                         100
                                   142.821
                                             0.0248361
```

# 6.7 Using the command line version of the AMPL interface

AMPL can generate a data file containing the optimization problem and all relevant information which can then be read and solved by the  $\mathbf{MOSEK}$  command line tool.

When the problem has been loaded into AMPL, the commands

```
ampl: option auxfiles rc;
ampl: write bprob;
```

will make AMPL write the appropriate data files, i.e.

```
prob.nl
prob.col
prob.row
```

Then the problem can be solved using the command line version of  $\mathbf{MOSEK}$  as follows

```
mosek prob.nl outlev=10 -a
```

The option -a indicates that MOSEK is invoked in AMPL mode. When MOSEK is invoked in AMPL mode the standard MOSEK command line options should appear after the -a option except for the file name which should be the first argument. As the above example demonstrates MOSEK accepts command line options following the AMPL convention. To see which command line arguments MOSEK accepts in AMPL mode write:

```
mosek -= -a
```

For linear, quadratic and quadratically constrained problems a text file representation of the problem can be obtained by performing one of the following conversions:

```
mosek prob.nl -a -x -out prob.mps
mosek prob.nl -a -x -out prob.opf
mosek prob.nl -a -x -out prob.lp
```

# **Debugging Tutorials**

This collection of tutorials contains basic techniques for debugging optimization problems using tools available in MOSEK: optimizer log, solution summary, infeasibility report, command-line tools. It is intended as a first line of technical help for issues such as: Why do I get solution status *unknown* and how can I fix it? Why is my model infeasible while it shouldn't be? Should I change some parameters? Can the model solve faster? etc.

### The major steps when debugging a model are always:

- Consult the log output.
- Run the optimization and analyze the log output, see Sec. 7.1. In particular:
  - check if the problem setup (number of constraints/variables etc.) matches your expectation.
  - check solution summary and solution status.
- Dump the problem to disk if necessary to continue analysis.
  - use a human-readable text format, preferably \*.ptf if you want to check the problem structure by hand. Assign names to variables and constraints to make them easier to identify.
  - use the MOSEK native format \*.task.gz when submitting a bug report or support question.
- Fix problem setup, improve the model, locate infeasibility or adjust parameters, depending on the diagnosis.

See the following sections for details.

# 7.1 Understanding optimizer log

The optimizer produces a log which splits roughly into four sections:

- 1. summary of the input data,
- 2. presolve and other pre-optimize problem setup stages,
- 3. actual optimizer iterations,
- 4. solution summary.

In this tutorial we show how to analyze the most important parts of the log when initially debugging a model: input data (1) and solution summary (4). For the iterations log (3) see Sec. 9.3.4 or Sec. 9.4.4.

### 7.1.1 Input data

If **MOSEK** behaves very far from expectations it may be due to errors in problem setup. The log file will begin with a summary of the structure of the problem, which looks for instance like:

```
Problem
 Name
  Objective sense
                          : minimize
  Type
                          : CONIC (conic optimization problem)
 Constraints
                          : 234
                          : 5348
  Affine conic cons.
 Disjunctive cons.
                          : 0
 Cones
                          : 20693
 Scalar variables
 Matrix variables
                          : 0
  Integer variables
                          : 0
```

This can be consulted to eliminate simple errors: wrong objective sense, wrong number of variables etc. Note that some modeling tools can introduce additional variables and constraints to the model and perturb the model even further (such as by dualizing). In most **MOSEK** APIs the problem dimensions should match exactly what the user specified.

If this is not sufficient a bit more information can be obtained by dumping the problem to a file (see Sec. 7) and using the anapro option of any of the command line tools. This will produce a longer summary similar to:

```
** Variables
scalar: 20414
                   integer: 0
                                      matrix: 0
low: 2082
                   up: 5014
                                      ranged: 0
                                                         free: 12892
                                                                             fixed: 426
** Constraints
all: 20413
low: 10028
                                                                             fixed: 10385
                   up: 0
                                      ranged: 0
                                                         free: 0
** Affine conic constraints (ACC)
QUAD: 1
                   dims: 2865: 1
RQUAD: 2507
                   dims: 3: 2507
** Problem data (numerics)
|c|
                   nnz: 10028
                                      min=2.09e-05
                                                         max=1.00e+00
| A |
                   nnz: 597023
                                      min=1.17e-10
                                                         max=1.00e+00
                   fin: 2508
                                      min=-3.60e+09
                                                         max=2.75e+05
blx
bux
                   fin: 5440
                                      min=0.00e+00
                                                         \max=2.94e+08
blc
                   fin: 20413
                                      min=-7.61e+05
                                                         \max = 7.61e + 05
buc
                   fin: 10385
                                      min=-5.00e-01
                                                         max=0.00e+00
|F|
                   nnz: 612301
                                      min=8.29e-06
                                                         max=9.31e+01
|g|
                   nnz: 1203
                                      min=5.00e-03
                                                         max=1.00e+00
```

Again, this can be used to detect simple errors, such as:

- Wrong type of conic constraint was used or it has wrong dimension.
- The bounds for variables or constraints are incorrect or incomplete.
- The model is otherwise incomplete.
- Suspicious values of coefficients.
- For various data sizes the model does not scale as expected.

Finally saving the problem in a human-friendly text format such as LP or PTF (see Sec. 7) and analyzing it by hand can reveal if the model is correct.

### Warnings and errors

At this stage the user can encounter warnings which should not be ignored, unless they are well-understood. They can also serve as hints as to numerical issues with the problem data. A typical warning of this kind is

```
MOSEK warning 53: A numerically large upper bound value 2.9e+08 is specified for 

⇒variable 'absh[107]' (2613).
```

Warnings do not stop the problem setup. If, on the other hand, an error occurs then the model will become invalid. The user should make sure to test for errors/exceptions from all API calls that set up the problem and validate the data.

## 7.1.2 Solution summary

The last item in the log is the solution summary.

### Continuous problem

### **Optimal solution**

A typical solution summary for a continuous (linear, conic, quadratic) problem looks like:

It contains the following elements:

- Problem and solution status.
- A summary of the primal solution: objective value, infinity norm of the solution vector and maximal violations of variables and constraints of different types. The violation of a linear constraint such as  $a^T x \leq b$  is  $\max(a^T x b, 0)$ . The violation of a conic constraint is the distance to the cone.
- The same for the dual solution.

The features of the solution summary which characterize a very good and accurate solution and a well-posed model are:

- Status: The solution status is OPTIMAL.
- **Duality gap:** The primal and dual objective values are (almost) identical, which proves the solution is (almost) optimal.
- Norms: Ideally the norms of the solution and the objective values should not be too large. This of course depends on the input data, but a huge solution norm can be an indicator of issues with the scaling, conditioning and/or well-posedness of the model. It may also indicate that the problem is borderline between feasibility and infeasibility and sensitive to small perturbations in this respect.
- Violations: The violations are close to zero, which proves the solution is (almost) feasible. Observe that due to rounding errors it can be expected that the violations are proportional to the norm (nrm:) of the solution. It is rarely the case that violations are exactly zero.

#### Solution status UNKNOWN

A typical example with solution status UNKNOWN due to numerical problems will look like:

Note that:

- The primal and dual objective are very different.
- The dual solution has very large norm.
- There are considerable violations so the solution is likely far from feasible.

Follow the hints in Sec. 7.2 to resolve the issue.

### Solution status UNKNOWN with a potentially useful solution

Solution status UNKNOWN does not necessarily mean that the solution is completely useless. It only means that the solver was unable to make any more progress due to numerical difficulties, and it was not able to reach the accuracy required by the termination criteria (see Sec. 9.3.2). Consider for instance:

```
Problem status
               : UNKNOWN
Solution status : UNKNOWN
Primal.
        obj: 3.4531019648e+04
                                  nrm: 1e+05
                                                 Viol.
                                                        con: 7e-02
                                                                      var: 0e+00
→acc: 0e+00
Dual.
         obj: 3.4529720645e+04
                                  nrm: 8e+03
                                                 Viol.
                                                       con: 1e-04
                                                                      var: 2e-04
→acc: 0e+00
```

Such a solution may still be useful, and it is always up to the user to decide. It may be a good enough approximation of the optimal point. For example, the large constraint violation may be due to the fact that one constraint contained a huge coefficient.

#### Infeasibility certificate

A primal infeasibility certificate is stored in the dual variables:

```
Problem status : PRIMAL_INFEASIBLE
Solution status : PRIMAL_INFEASIBLE_CER
Dual. obj: 2.9238975853e+02 nrm: 6e+02 Viol. con: 0e+00 var: 1e-11 u

acc: 0e+00
```

It is a Farkas-type certificate as described in Sec. 8.2.2. In particular, for a good certificate:

- The dual objective is positive for a minimization problem, negative for a maximization problem. Ideally it is well bounded away from zero.
- The norm is not too big and the violations are small (as for a solution).

If the model was not expected to be infeasible, the likely cause is an error in the problem formulation. Use the hints in Sec. 7.1.1 and Sec. 7.3 to locate the issue.

Just like a solution, the infeasibility certificate can be of better or worse quality. The infeasibility certificate above is very solid. However, there can be less clear-cut cases, such as for example:

```
Problem status : PRIMAL_INFEASIBLE
Solution status : PRIMAL_INFEASIBLE_CER
Dual. obj: 1.6378689238e-06 nrm: 6e+05 Viol. con: 7e-03 var: 2e-04 u

acc: 0e+00
```

This infeasibility certificate is more dubious because the dual objective is positive, but barely so in comparison with the large violations. It also has rather large norm. This is more likely an indication that the problem is borderline between feasibility and infeasibility or simply ill-posed and sensitive to tiny variations in input data. See Sec. 7.3 and Sec. 7.2.

The same remarks apply to dual infeasibility (i.e. unboundedness) certificates. Here the primal objective should be negative a minimization problem and positive for a maximization problem.

### 7.1.3 Mixed-integer problem

### Optimal integer solution

For a mixed-integer problem there is no dual solution and a typical optimal solution report will look as follows:

The interpretation of all elements is as for a continuous problem. The additional field itg denotes the maximum violation of an integer variable from being an exact integer.

### Feasible integer solution

If the solver found an integer solution but did not prove optimality, for instance because of a time limit, the solution status will be PRIMAL\_FEASIBLE:

```
Problem status : PRIMAL_FEASIBLE
Solution status : PRIMAL_FEASIBLE
Primal. obj: 6.0114607792e+06 nrm: 1e+03 Viol. con: 2e-13 var: 2e-13 u
itg: 4e-15
```

In this case it is valuable to go back to the optimizer summary to see how good the best solution is:

In this case the best integer solution found has objective value 6.011460779193e+06, the best proved lower bound is 6.007896089225e+06 and so the solution is guaranteed to be within 0.06% from optimum. The same data can be obtained as information items through an API. See also Sec. 9.4 for more details.

### Infeasible problem

If the problem is declared infeasible the summary is simply

```
Problem status : PRIMAL_INFEASIBLE
Solution status : UNKNOWN
Primal. obj: 0.00000000000e+00 nrm: 0e+00 Viol. con: 0e+00 var: 0e+00 ⊔
→itg: 0e+00
```

If infeasibility was not expected, consult Sec. 7.3.

## 7.2 Addressing numerical issues

The suggestions in this section should help diagnose and solve issues with numerical instability, in particular UNKNOWN solution status or solutions with large violations. Since numerically stable models tend to solve faster, following these hints can also dramatically shorten solution times.

We always recommend that issues of this kind are addressed by reformulating or rescaling the model, since it is the modeler who has the best insight into the structure of the problem and can fix the cause of the issue.

## 7.2.1 Formulating problems

### **Scaling**

Make sure that all the data in the problem are of comparable orders of magnitude. This applies especially to the linear constraint matrix. Use Sec. 7.1.1 if necessary. For example a report such as

means that the ratio of largest to smallest elements in A is  $10^{11}$ . In this case the user should rescale or reformulate the model to avoid such spread which makes it difficult for **MOSEK** to scale the problem internally. In many cases it may be possible to change the units, i.e. express the model in terms of rescaled variables (for instance work with millions of dollars instead of dollars, etc.).

Similarly, if the objective contains very different coefficients, say

maximize 
$$10^{10}x + y$$

then it is likely to lead to inaccuracies. The objective will be dominated by the contribution from x and y will become insignificant.

### Removing huge bounds

**Never** use a very large number as replacement for  $\infty$ . Instead define the variable or constraint as unbounded from below/above. Similarly, avoid artificial huge bounds if you expect they will not become tight in the optimal solution.

### **Avoiding linear dependencies**

As much as possible try to avoid linear dependencies and near-linear dependencies in the model. See Example 7.3.

### **Avoiding ill-posedness**

Avoid continuous models which are ill-posed: the solution space is degenerate, for example consists of a single point (technically, the Slater condition is not satisfied). In general, this refers to problems which are borderline between feasible and infeasible. See Example 7.1.

## Scaling the expected solution

Try to formulate the problem in such a way that the expected solution (both primal and dual) is not very large. Consult the solution summary Sec. 7.1.2 to check the objective values or solution norms.

#### 7.2.2 Further suggestions

Here are other simple suggestions that can help locate the cause of the issues. They can also be used as hints for how to tune the optimizer if fixing the root causes of the issue is not possible.

- Remove the objective and solve the feasibility problem. This can reveal issues with the objective.
- Change the objective or change the objective sense from minimization to maximization (if applicable). If the two objective values are almost identical, this may indicate that the feasible set is very small, possibly degenerate.
- Perturb the data, for instance bounds, very slightly, and compare the results.
- $\bullet$  For linear problems: solve the problem using a different optimizer by setting the parameter  $MSK\_IPAR\_OPTIMIZER$  and compare the results.
- Force the optimizer to solve the primal/dual versions of the problem by setting the parameter <code>MSK\_IPAR\_INTPNT\_SOLVE\_FORM</code> or <code>MSK\_IPAR\_SIM\_SOLVE\_FORM</code>. **MOSEK** has a heuristic to decide whether to dualize, but for some problems the guess is wrong an explicit choice may give better results.
- Solve the problem without presolve or some of its parts by setting the parameter MSK\_IPAR\_PRESOLVE\_USE, see Sec. 9.1.
- Use different numbers of threads (MSK\_IPAR\_NUM\_THREADS) and compare the results. Very different results indicate numerical issues resulting from round-off errors.

If the problem was dumped to a file, experimenting with various parameters is facilitated with the **MOSEK** Command Line Tool or **MOSEK** Python Console Sec. 7.4.

## 7.2.3 Typical pitfalls

Example 7.1 (Ill-posedness). A toy example of this situation is the feasibility problem

$$(x-1)^2 < 1, (x+1)^2 < 1$$

whose only solution is x=0 and moreover replacing any 1 on the right hand side by  $1-\varepsilon$  makes the problem infeasible and replacing it by  $1+\varepsilon$  yields a problem whose solution set is an interval (fully-dimensional). This is an example of ill-posedness.

**Example 7.2** (Huge solution). If the norm of the expected solution is very large it may lead to numerical issues or infeasibility. For example the problem

$$(10^{-4}, x, 10^3) \in \mathcal{Q}_x^3$$

may be declared infeasible because the expected solution must satisfy  $x \ge 5 \cdot 10^9$ .

**Example 7.3** (Near linear dependency). Consider the following problem:

If we add the equalities together we obtain:

$$0 = \varepsilon$$

which is infeasible for any  $\varepsilon \neq 0$ . Here infeasibility is caused by a linear dependency in the constraint matrix coupled with a precision error represented by the  $\varepsilon$ . Indeed if a problem contains linear dependencies then the problem is either infeasible or contains redundant constraints. In the above case any of the equality constraints can be removed while not changing the set of feasible solutions. To summarize linear dependencies in the constraints can give rise to infeasible problems and therefore it is better to avoid them.

**Example 7.4** (Presolving very tight bounds). Next consider the problem

Now the **MOSEK** presolve will, for the sake of efficiency, fix variables (and constraints) that have tight bounds where tightness is controlled by the parameter  $MSK\_DPAR\_PRESOLVE\_TOL\_X$ . Since the bounds

$$-10^{-9} < x_1 < 10^{-9}$$

are tight, presolve will set  $x_1 = 0$ . It easy to see that this implies  $x_4 = 0$ , which leads to the incorrect conclusion that the problem is infeasible. However a tiny change of the value  $10^{-9}$  makes the problem feasible. In general it is recommended to avoid ill-posed problems, but if that is not possible then one solution is to reduce parameters such as  $MSK_DPAR_PRESOLVE_TOL_X$  to say  $10^{-10}$ . This will at least make sure that presolve does not make the undesired reduction.

## 7.3 Debugging infeasibility

When solving an optimization problem one typically expects to get an optimal solution, but in some cases, either by design, or (most frequently) due to an error in the formulation, the problem may become infeasible (have no solution at all).

This section

- describes the intuitions behind infeasibility,
- helps to debug (unexpectedly) infeasible problems using the command line tool and by inspecting infeasibility reports and problem data by hand,
- gives some hints for how to modify the formulation to identify the reasons for infeasibility.

An infeasibility certificate is only available for continuous problems, however the hints in Sec. 7.3.4 apply to a large extent also to mixed-integer problems.

## 7.3.1 Numerical issues

Infeasible problem status may be just an artifact of numerical issues appearing when the problem is badly-scaled, barely feasible or otherwise ill-conditioned so that it is unstable under small perturbations of the data or round-off errors. This may be visible in the solution summary if the infeasibility certificate has poor quality. See Sec. 7.1.2 for how to diagnose that and Sec. 7.2 for possible hints. Sec. 7.2.3 contains examples of situations which may lead to infeasibility for numerical reasons.

We refer to Sec. 7.2 for further information on dealing with those sort of issues. For the rest of this section we concentrate on the case when the solution summary leaves little doubt that the problem solved by the optimizer actually is infeasible.

## 7.3.2 Locating primal infeasibility

As an example of a primal infeasible problem consider minimizing the cost of transportation between a number of production plants and stores: Each plant produces a fixed number of goods, and each store has a fixed demand that must be met. Supply, demand and cost of transportation per unit are given in Fig. 7.1.

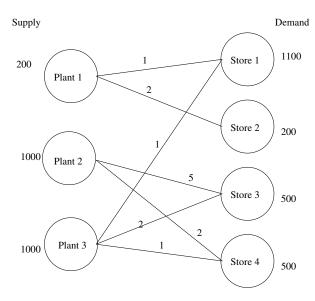


Fig. 7.1: Supply, demand and cost of transportation.

The problem represented in Fig. 7.1 is infeasible, since the total demand

$$2300 = 1100 + 200 + 500 + 500$$

exceeds the total supply

$$2200 = 200 + 1000 + 1000$$

If we denote the number of transported goods from plant i to store j by  $x_{ij}$ , the problem can be formulated as the LP:

minimize 
$$x_{11} + 2x_{12} + 5x_{23} + 2x_{24} + x_{31} + 2x_{33} + x_{34}$$
 subject to  $x_{0}$ :  $x_{11} + x_{12}$   $x_{23} + x_{24}$   $x_{24} + x_{31} + x_{24}$   $x_{23} + x_{34}$   $x_{24} + x_{31} + x_{32} + x_{34}$   $x_{24} + x_{31} + x_{32} + x_{34} + x_{3$ 

Solving problem (7.1) using **MOSEK** will result in an infeasibility status. The infeasibility certificate is contained in the dual variables an can be accessed from an API. The variables and constraints with nonzero solution values form an infeasible subproblem, which frequently is very small. See Sec. 8.1.2 or Sec. 8.2.2 for detailed specifications of infeasibility certificates.

A short infeasibility report can also be printed to the log stream. It can be turned on by setting the parameter <code>MSK\_IPAR\_INFEAS\_REPORT\_AUTO</code> to <code>MSK\_ON</code>. This causes <code>MOSEK</code> to print a report on variables and constraints which are involved in infeasibility in the above sense, i.e. have nonzero values in the certificate. The parameter <code>MSK\_IPAR\_INFEAS\_REPORT\_LEVEL</code> controls the amount of information presented in the infeasibility report. The default value is 1. For the above example the report is

#### MOSEK PRIMAL INFEASIBILITY REPORT.

Problem status: The problem is primal infeasible

The following constraints are involved in the primal infeasibility.

Index	Name	Lower bound	Upper bound	Dual lower	Dual upper
0	<b>s</b> 0	NONE	2.000000e+002	0.000000e+000	1.000000e+000
2	s2	NONE	1.000000e+003	0.000000e+000	1.000000e+000
3	d1	1.100000e+003	1.100000e+003	1.000000e+000	0.000000e+000
4	d2	2.000000e+002	2.000000e+002	1.000000e+000	0.000000e+000

The following bound constraints are involved in the infeasibility.

Index	Name	Lower bound	Upper bound	Dual lower	Dual upper
8	x33	0.000000e+000	NONE	1.000000e+000	0.000000e+000
10	x34	0.000000e+000	NONE	1.000000e+000	0.000000e+000

The infeasibility report is divided into two sections corresponding to constraints and variables. It is a selection of those lines from the problem solution which are important in understanding primal infeasibility. In this case the constraints s0, s2, d1, d2 and variables x33, x34 are of importance because of nonzero dual values. The columns Dual lower and Dual upper contain the values of dual variables  $s_l^c$ ,  $s_u^c$ ,  $s_u^s$  and  $s_u^s$  in the primal infeasibility certificate (see Sec. 8.1.2).

In our example the certificate means that an appropriate linear combination of constraints s0, s1 with coefficient  $s_u^c = 1$ , constraints d1 and d2 with coefficient  $s_u^c - s_l^c = 0 - 1 = -1$  and lower bounds on x33 and x34 with coefficient  $-s_l^x = -1$  gives a contradiction. Indeed, the combination of the four involved constraints is  $x_{33} + x_{34} \le -100$  (as indicated in the introduction, the difference between supply and demand).

It is also possible to extract the infeasible subproblem with the command-line tool. For an infeasible problem called infeas.lp the command:

```
mosek -d MSK_IPAR_INFEAS_REPORT_AUTO MSK_ON infeas.lp -info rinfeas.lp
```

will produce the file rinfeas.bas.inf.lp which contains the infeasible subproblem. Because of its size it may be easier to work with than the original problem file.

Returning to the transportation example, we discover that removing the fifth constraint  $x_{12} = 200$  makes the problem feasible. Almost all undesired infeasibilities should be fixable at the modeling stage.

### 7.3.3 Locating dual infeasibility

A problem may also be  $dual\ infeasible$ . In this case the primal problem is usually unbounded, meaning that feasible solutions exists such that the objective tends towards infinity. For example, consider the problem

```
maximize 200y_1 + 1000y_2 + 1000y_3 + 1100y_4 + 200y_5 + 500y_6 + 500y_7 subject to y_1 + y_4 \le 1, \ y_1 + y_5 \le 2, \ y_2 + y_6 \le 5, \ y_2 + y_7 \le 2, \\ y_3 + y_4 \le 1, \ y_3 + y_6 \le 2, \ y_3 + y_7 \le 1 \\ y_1, y_2, y_3 \le 0
```

which is dual to (7.1) (and therefore is dual infeasible). The dual infeasibility report may look as follows:

```
MOSEK DUAL INFEASIBILITY REPORT.

Problem status: The problem is dual infeasible

The following constraints are involved in the infeasibility.

Index Name Activity Objective Lower bound Upperubound
```

(continues on next page)

(continued from previous page)

				(continu	ed from previous page)
5	x33	-1.000000e+00		NONE	2.
<b>→</b> 00000	0e+00				
6	x34	-1.00000e+00		NONE	1.
<b>→</b> 00000	0e+00				
The fol	lowing varia	bles are involved in the	he infeasibility	7.	
Index	Name	Activity	Objective	Lower boun	d Upper⊔
⇔bound					
0	y1	-1.000000e+00	2.000000e+02	NONE	0.
<b>→</b> 00000	0e+00				
2	у3	-1.00000e+00	1.000000e+03	NONE	0.
<b>→</b> 00000	0e+00				
3	y4	1.000000e+00	1.100000e+03	NONE	NONE
4	y5	1.000000e+00	2.000000e+02	NONE	NONE
Probl Solut	ion status :	DUAL_INFEASIBLE DUAL_INFEASIBLE_CER	1e+00 Viol	con: 0e+00	var: 0e+00
		DUAL_INFEASIBLE_CER 0000000000e+02 nrm:	1e+00 Viol.	con: 0e+00	var: 0e+0

In the report we see that the variables y1, y3, y4, y5 and two constraints contribute to infeasibility with non-zero values in the Activity column. Therefore

$$(y_1,\ldots,y_7)=(-1,0,-1,1,1,0,0)$$

is the dual infeasibility certificate as in Sec. 8.1.2. This just means, that along the ray

$$(0,0,0,0,0,0,0) + t(y_1,\ldots,y_7) = (-t,0,-t,t,t,0,0), t > 0,$$

which belongs to the feasible set, the objective value 100t can be arbitrarily large, i.e. the problem is unbounded.

In the example problem we could

- Add a lower bound on y3. This will directly invalidate the certificate of dual infeasibility.
- Increase the objective coefficient of y3. Changing the coefficients sufficiently will invalidate the inequality  $c^T y^* > 0$  and thus the certificate.

## 7.3.4 Suggestions

#### Primal infeasibility

When trying to understand what causes the unexpected primal infeasible status use the following hints:

- Remove the objective function. This does not change the infeasibility status but simplifies the problem, eliminating any possibility of issues related to the objective function.
- Remove cones, semidefinite variables and integer constraints. Solve only the linear part of the problem. Typical simple modeling errors will lead to infeasibility already at this stage.
- Consider whether your problem has some obvious necessary conditions for feasibility and examine if these are satisfied, e.g. total supply should be greater than or equal to total demand.
- Verify that coefficients and bounds are reasonably sized in your problem.
- See if there are any obvious contradictions, for instance a variable is bounded both in the variables and constraints section, and the bounds are contradictory.
- Consider replacing suspicious equality constraints by inequalities. For instance, instead of  $x_{12} = 200$  see what happens for  $x_{12} \ge 200$  or  $x_{12} \le 200$ .

- Relax bounds of the suspicious constraints or variables.
- For integer problems, remove integrality constraints on some/all variables and see if the problem solves.
- Form an **elastic model**: allow to violate constraints at a cost. Introduce slack variables and add them to the objective as penalty. For instance, suppose we have a constraint

which might be causing infeasibility. Then create a new variable y and form the problem which contains:

Solving this problem will reveal by how much the constraint needs to be relaxed in order to become feasible. This is equivalent to inspecting the infeasibility certificate but may be more intuitive.

- If you think you have a feasible solution or its part, fix all or some of the variables to those values.
   Presolve will propagate them through the model and potentially reveal more localized sources of infeasibility.
- Dump the problem in PTF or LP format and verify that the problem that was passed to the optimizer corresponds to the problem expressed in the high-level modeling language, if any such was used.

#### **Dual infeasibility**

When trying to understand what causes the unexpected dual infeasible status use the following hints:

- Verify that the objective coefficients are reasonably sized.
- Check if no bounds and constraints are missing, for example if all variables that should be nonnegative have been declared as such etc.
- Strengthen bounds of the suspicious constraints or variables.
- Form an series of models with decreasing bounds on the objective, that is, instead of objective

minimize 
$$c^T x$$

solve the problem with an additional constraint such as

$$c^T x = -10^5$$

and inspect the solution to figure out the mechanism behind arbitrarily decreasing objective values. This is equivalent to inspecting the infeasibility certificate but may be more intuitive.

Dump the problem in PTF or LP format and verify that the problem that was passed to the
optimizer corresponds to the problem expressed in the high-level modeling language, if any such
was used.

Please note that modifying the problem to invalidate the reported certificate does *not* imply that the problem becomes feasible — the reason for infeasibility may simply *move*, resulting a problem that is still infeasible, but for a different reason. More often, the reported certificate can be used to give a hint about errors or inconsistencies in the model that produced the problem.

## 7.4 Python Console

The **MOSEK** Python Console is an alternative to the **MOSEK** Command Line Tool. It can be used for interactive loading, solving and debugging optimization problems stored in files, for example **MOSEK** task files. It facilitates debugging techniques described in Sec. 7.

#### 7.4.1 Usage

The tool requires Python 3. The **MOSEK** interface for Python must be installed following the installation instructions for Python API or Python Fusion API. The easiest option is

#### pip install Mosek

The Python Console is contained in the file mosekconsole.py in the folder with MOSEK binaries. It can be copied to an arbitrary location. The file is also available for download here (mosekconsole.py). To run the console in interactive mode use

```
python mosekconsole.py
```

To run the console in batch mode provide a semicolon-separated list of commands as the second argument of the script, for example:

```
python mosekconsole.py "read data.task.gz; solve form=dual; writesol data"
```

The script is written using the **MOSEK** Python API and can be extended by the user if more specific functionality is required. We refer to the documentation of the Python API.

#### 7.4.2 Examples

To read a problem from data.task.gz, solve it, and write solutions to data.sol, data.bas or data.itg:

```
read data.task.gz; solve; writesol data
```

To convert between file formats:

```
read data.task.gz; write data.mps
```

To set a parameter before solving:

```
read data.task.gz; param INTPNT_CO_TOL_DFEAS 1e-9; solve"
```

To list parameter values related to the mixed-integer optimizer in the task file:

```
read data.task.gz; param MIO
```

To print a summary of problem structure:

```
read data.task.gz; anapro
```

To solve a problem forcing the dual and switching off presolve:

```
read data.task.gz; solve form=dual presolve=no
```

To write an infeasible subproblem to a file for debugging purposes:

```
read data.task.gz; solve; infsub; write inf.opf
```

## 7.4.3 Full list of commands

Below is a brief description of all the available commands. Detailed information about a specific command  ${\sf cmd}$  and its options can be obtained with

help cmd

Table 7.1: List of commands of the MOSEK Python Console.

Command	Description
help [command]	Print list of commands or info about a specific command
log filename	Save the session to a file
intro	Print MOSEK splashscreen
testlic	Test the license system
read filename	Load problem from file
reread	Reload last problem file
solve	Solve current problem
[options]	
write filename	Write current problem to file
param [name	Set a parameter or get parameter values
[value]]	
paramdef	Set all parameters to default values
paramdiff	Show parameters with non-default values
info [name]	Get an information item
anapro	Analyze problem data
hist	Plot a histogram of problem data
histsol	Plot a histogram of the solutions
spy	Plot the sparsity pattern of the data matrices
truncate	Truncate small coefficients down to 0
epsilon	
resobj [fac]	Rescale objective by a factor
anasol	Analyze solutions
removeitg	Remove integrality constraints
removecones	Remove all cones and leave just the linear part
infsub	Replace current problem with its infeasible subproblem
writesol	Write solution(s) to file(s) with given basename
basename	
delsol	Remove all solutions from the task
optserver	Use an OptServer to optimize
[url]	
exit	Leave

# Chapter 8

# Problem Formulation and Solutions

In this chapter we will discuss the following topics:

- The formal, mathematical formulations of the problem types that MOSEK can solve and their duals.
- The solution information produced by MOSEK.
- The infeasibility certificate produced by MOSEK if the problem is infeasible.

For the underlying mathematical concepts, derivations and proofs see the Modeling Cookbook or any book on convex optimization. This chapter explains how the related data is organized specifically within the **MOSEK** API.

## 8.1 Linear Optimization

MOSEK accepts linear optimization problems of the form

where

- $\bullet$  m is the number of constraints.
- $\bullet$  *n* is the number of decision variables.
- $x \in \mathbb{R}^n$  is a vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear part of the objective function.
- $c^f \in \mathbb{R}$  is a constant term in the objective
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.

Lower and upper bounds can be infinite, or in other words the corresponding bound may be omitted. A primal solution (x) is (primal) feasible if it satisfies all constraints in (8.1). If (8.1) has at least one primal feasible solution, then (8.1) is said to be (primal) feasible. In case (8.1) does not have a feasible solution, the problem is said to be (primal) infeasible

## 8.1.1 Duality for Linear Optimization

Corresponding to the primal problem (8.1), there is a dual problem

maximize 
$$(l^{c})^{T} s_{l}^{c} - (u^{c})^{T} s_{u}^{c} + (l^{x})^{T} s_{l}^{x} - (u^{x})^{T} s_{u}^{x} + c^{f}$$

$$A^{T} y + s_{l}^{x} - s_{u}^{x} = c,$$
subject to 
$$-y + s_{l}^{c} - s_{u}^{c} = 0,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \geq 0,$$

$$(8.2)$$

where

- $s_l^c$  are the dual variables for lower bounds of constraints,
- $s_u^c$  are the dual variables for upper bounds of constraints,
- $s_l^x$  are the dual variables for lower bounds of variables,
- $s_u^x$  are the dual variables for upper bounds of variables.

If a bound in the primal problem is plus or minus infinity, the corresponding dual variable is fixed at 0, and we use the convention that the product of the bound value and the corresponding dual variable is 0. This is equivalent to removing the corresponding dual variable from the dual problem. For example:

$$l_j^x = -\infty \quad \Rightarrow \quad (s_l^x)_j = 0 \text{ and } l_j^x \cdot (s_l^x)_j = 0.$$

A solution

$$(y, s_l^c, s_u^c, s_l^x, s_u^x)$$

to the dual problem is feasible if it satisfies all the constraints in (8.2). If (8.2) has at least one feasible solution, then (8.2) is (dual) feasible, otherwise the problem is (dual) infeasible.

A solution

$$(x^*, y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)$$

is denoted a *primal-dual feasible solution*, if  $(x^*)$  is a solution to the primal problem (8.1) and  $(y^*, (s_l^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*$  is a solution to the corresponding dual problem (8.2). We also define an auxiliary vector

$$(x^c)^* := Ax^*$$

containing the activities of linear constraints.

For a primal-dual feasible solution we define the *duality gap* as the difference between the primal and the dual objective value,

$$c^{T}x^{*} + c^{f} - \left\{ (l^{c})^{T}(s_{l}^{c})^{*} - (u^{c})^{T}(s_{u}^{c})^{*} + (l^{x})^{T}(s_{l}^{x})^{*} - (u^{x})^{T}(s_{u}^{x})^{*} + c^{f} \right\}$$

$$= \sum_{i=0}^{m-1} \left[ (s_{l}^{c})_{i}^{*}((x_{i}^{c})^{*} - l_{i}^{c}) + (s_{u}^{c})_{i}^{*}(u_{i}^{c} - (x_{i}^{c})^{*}) \right]$$

$$+ \sum_{j=0}^{m-1} \left[ (s_{l}^{x})_{j}^{*}(x_{j} - l_{j}^{x}) + (s_{u}^{x})_{j}^{*}(u_{j}^{x} - x_{j}^{*}) \right] \geq 0$$

$$(8.3)$$

where the first relation can be obtained by transposing and multiplying the dual constraints (8.2) by  $x^*$  and  $(x^c)^*$  respectively, and the second relation comes from the fact that each term in each sum is nonnegative. It follows that the primal objective will always be greater than or equal to the dual objective.

It is well-known that a linear optimization problem has an optimal solution if and only if there exist feasible primal-dual solution so that the duality gap is zero, or, equivalently, that the *complementarity conditions* 

$$\begin{array}{rclcrcl} (s_l^c)_i^*((x_i^c)^*-l_i^c) & = & 0, & i=0,\dots,m-1, \\ (s_u^c)_i^*(u_i^c-(x_i^c)^*) & = & 0, & i=0,\dots,m-1, \\ (s_l^x)_j^*(x_j^*-l_j^x) & = & 0, & j=0,\dots,n-1, \\ (s_u^x)_j^*(u_j^x-x_j^*) & = & 0, & j=0,\dots,n-1, \end{array}$$

are satisfied.

If (8.1) has an optimal solution and **MOSEK** solves the problem successfully, both the primal and dual solution are reported, including a status indicating the exact state of the solution.

#### 8.1.2 Infeasibility for Linear Optimization

#### **Primal Infeasible Problems**

If the problem (8.1) is infeasible (has no feasible solution), **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the modified dual problem

such that the objective value is strictly positive, i.e. a solution

$$(y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)$$

to (8.4) so that

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* > 0.$$

Such a solution implies that (8.4) is unbounded, and that (8.1) is infeasible.

#### **Dual Infeasible Problems**

If the problem (8.2) is infeasible (has no feasible solution), **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the modified primal problem

minimize 
$$c^T x$$
  
subject to  $\hat{l}^c \leq Ax \leq \hat{u}^c$ ,  $\hat{l}^x \leq x \leq \hat{u}^x$ , (8.5)

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

and

$$\hat{l}_{j}^{x} = \left\{ \begin{array}{ll} 0 & \text{if } l_{j}^{x} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{j}^{x} := \left\{ \begin{array}{ll} 0 & \text{if } u_{j}^{x} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

such that

$$c^T x < 0.$$

Such a solution implies that (8.5) is unbounded, and that (8.2) is infeasible.

In case that both the primal problem (8.1) and the dual problem (8.2) are infeasible, **MOSEK** will report only one of the two possible certificates — which one is not defined (**MOSEK** returns the first certificate found).

#### 8.1.3 Minimalization vs. Maximalization

When the objective sense of problem (8.1) is maximization, i.e.

the objective sense of the dual problem changes to minimization, and the domain of all dual variables changes sign in comparison to (8.2). The dual problem thus takes the form

This means that the duality gap, defined in (8.3) as the primal minus the dual objective value, becomes nonpositive. It follows that the dual objective will always be greater than or equal to the primal objective. The primal infeasibility certificate will be reported by **MOSEK** as a solution to the system

$$A^{T}y + s_{l}^{x} - s_{u}^{x} = 0,$$

$$-y + s_{l}^{c} - s_{u}^{c} = 0,$$

$$s_{l}^{c}, s_{u}^{x}, s_{l}^{x}, s_{u}^{x} \leq 0,$$
(8.6)

such that the objective value is strictly negative

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* < 0.$$

Similarly, the certificate of dual infeasibility is an x satisfying the requirements of (8.5) such that  $c^T x > 0$ .

## 8.2 Conic Optimization

Conic optimization is an extension of linear optimization (see Sec. 8.1) allowing conic domains to be specified for affine expressions. A conic optimization problem to be solved by **MOSEK** can be written as

minimize 
$$c^T x + c^f$$
  
subject to  $l^c \le Ax \le u^c$ ,  
 $l^x \le x \le u^x$ ,  
 $Fx + g \in \mathcal{D}$ , (8.7)

where

- $\bullet$  m is the number of constraints.
- n is the number of decision variables.
- $x \in \mathbb{R}^n$  is a vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear part of the objective function.
- $c^f \in \mathbb{R}$  is a constant term in the objective
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.

is the same as in Sec. 8.1 and moreover:

- $F \in \mathbb{R}^{k \times n}$  is the affine conic constraint matrix.,
- $g \in \mathbb{R}^k$  is the affine conic constraint constant term vector.,
- $\mathcal{D}$  is a Cartesian product of conic domains, namely  $\mathcal{D} = \mathcal{D}_1 \times \cdots \times \mathcal{D}_p$ , where p is the number of individual affine conic constraints (ACCs), and each domain is one from Sec. 11.5.

The total dimension of the domain  $\mathcal{D}$  must be equal to k, the number of rows in F and g. Lower and upper bounds can be infinite, or in other words the corresponding bound may be omitted.

**MOSEK** supports also the cone of positive semidefinite matrices. In order not to obscure this section with additional notation, that extension is discussed in Sec. 8.3.

## 8.2.1 Duality for Conic Optimization

Corresponding to the primal problem (8.7), there is a dual problem

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x - g^T \dot{y} + c^f$$
 subject to 
$$A^T y + s_l^x - s_u^x + F^T \dot{y} = c, \\ -y + s_l^c - s_u^c = 0, \\ s_l^c, s_u^c, s_l^x, s_u^x \ge 0, \\ \dot{y} \in \mathcal{D}^*, \end{cases}$$
(8.8)

where

- $s_l^c$  are the dual variables for lower bounds of constraints,
- $s_u^c$  are the dual variables for upper bounds of constraints,
- $s_l^x$  are the dual variables for lower bounds of variables,
- $s_u^x$  are the dual variables for upper bounds of variables,
- $\dot{y}$  are the dual variables for affine conic constraints,
- the dual domain  $\mathcal{D}^* = \mathcal{D}_1^* \times \cdots \times \mathcal{D}_p^*$  is a Cartesian product of cones dual to  $\mathcal{D}_i$ .

One can check that the dual problem of the dual problem is identical to the original primal problem. If a bound in the primal problem is plus or minus infinity, the corresponding dual variable is fixed at 0, and we use the convention that the product of the bound value and the corresponding dual variable is 0. This is equivalent to removing the corresponding dual variable  $(s_l^x)_j$  from the dual problem. For example:

$$l_j^x = -\infty \quad \Rightarrow \quad (s_l^x)_j = 0 \text{ and } l_j^x \cdot (s_l^x)_j = 0.$$

A solution

$$(y, s_l^c, s_u^c, s_l^x, s_u^x, \dot{y})$$

to the dual problem is feasible if it satisfies all the constraints in (8.8). If (8.8) has at least one feasible solution, then (8.8) is (dual) feasible, otherwise the problem is (dual) infeasible.

A solution

$$(x^*, y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*, (\dot{y})^*)$$

is denoted a *primal-dual feasible solution*, if  $(x^*)$  is a solution to the primal problem (8.7) and  $(y^*, (s_l^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*$  is a solution to the corresponding dual problem (8.8). We also define an auxiliary vector

$$(x^c)^* := Ax^*$$

containing the activities of linear constraints.

For a primal-dual feasible solution we define the  $duality\ gap$  as the difference between the primal and the dual objective value,

$$c^{T}x^{*} + c^{f} - \left\{ (l^{c})^{T}(s_{l}^{c})^{*} - (u^{c})^{T}(s_{u}^{c})^{*} + (l^{x})^{T}(s_{l}^{x})^{*} - (u^{x})^{T}(s_{u}^{x})^{*} - g^{T}(\dot{y})^{*} + c^{f} \right\}$$

$$= \sum_{i=0}^{m-1} \left[ (s_{l}^{c})_{i}^{*}((x_{i}^{c})^{*} - l_{i}^{c}) + (s_{u}^{c})_{i}^{*}(u_{i}^{c} - (x_{i}^{c})^{*}) \right]$$

$$+ \sum_{j=0}^{m-1} \left[ (s_{l}^{x})_{j}^{*}(x_{j} - l_{j}^{x}) + (s_{u}^{x})_{j}^{*}(u_{j}^{x} - x_{j}^{*}) \right]$$

$$+ ((\dot{y})^{*})^{T}(Fx^{*} + g) \ge 0$$

$$(8.9)$$

where the first relation can be obtained by transposing and multiplying the dual constraints (8.2) by  $x^*$  and  $(x^c)^*$  respectively, and the second relation comes from the fact that each term in each sum is nonnegative. It follows that the primal objective will always be greater than or equal to the dual objective.

It is well-known that, under some non-degeneracy assumptions that exclude ill-posed cases, a conic optimization problem has an optimal solution if and only if there exist feasible primal-dual solution so that the duality gap is zero, or, equivalently, that the *complementarity conditions* 

$$\begin{aligned}
(s_l^c)_i^* & ((x_i^c)^* - l_i^c) &= 0, & i = 0, \dots, m - 1, \\
(s_u^c)_i^* & (u_i^c - (x_i^c)^*) &= 0, & i = 0, \dots, m - 1, \\
& (s_l^c)_j^* & (x_j^* - l_j^c) &= 0, & j = 0, \dots, n - 1, \\
& (s_u^c)_j^* & (u_i^c - x_j^*) &= 0, & j = 0, \dots, n - 1, \\
& ((\dot{y})^*)^T & (Fx^* + g) &= 0,
\end{aligned} (8.10)$$

are satisfied.

If (8.7) has an optimal solution and **MOSEK** solves the problem successfully, both the primal and dual solution are reported, including a status indicating the exact state of the solution.

## 8.2.2 Infeasibility for Conic Optimization

#### **Primal Infeasible Problems**

If the problem (8.7) is infeasible (has no feasible solution), **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the modified dual problem

maximize 
$$(l^{c})^{T}s_{l}^{c} - (u^{c})^{T}s_{u}^{c} + (l^{x})^{T}s_{l}^{x} - (u^{x})^{T}s_{u}^{x} - g^{T}\dot{y}$$
 subject to 
$$A^{T}y + s_{l}^{x} - s_{u}^{x} + F^{T}\dot{y} = 0,$$
 
$$-y + s_{l}^{c} - s_{u}^{c} = 0,$$
 
$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \geq 0,$$
 
$$\dot{y} \in \mathcal{D}^{*},$$
 (8.11)

such that the objective value is strictly positive, i.e. a solution

$$(y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*, (\dot{y})^*)$$

to (8.11) so that

$$(l^c)^T(s^c_l)^* - (u^c)^T(s^c_u)^* + (l^x)^T(s^x_l)^* - (u^x)^T(s^x_u)^* - g^T\dot{y} > 0.$$

Such a solution implies that (8.11) is unbounded, and that (8.7) is infeasible.

#### **Dual Infeasible Problems**

If the problem (8.8) is infeasible (has no feasible solution), **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the modified primal problem

minimize 
$$c^T x$$
  
subject to  $\hat{l}^c \leq Ax \leq \hat{u}^c$ ,  
 $\hat{l}^x \leq x \leq \hat{u}^x$ ,  
 $Fx \in \mathcal{D}$  (8.12)

where

$$\hat{l}_{i}^{c} = \left\{ \begin{array}{ll} 0 & \text{if } l_{i}^{c} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{i}^{c} := \left\{ \begin{array}{ll} 0 & \text{if } u_{i}^{c} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$
 (8.13)

and

$$\hat{l}_{j}^{x} = \left\{ \begin{array}{cc} 0 & \text{if } l_{j}^{x} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{j}^{x} := \left\{ \begin{array}{cc} 0 & \text{if } u_{j}^{x} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$
(8.14)

such that

$$c^T x < 0.$$

Such a solution implies that (8.12) is unbounded, and that (8.8) is infeasible.

In case that both the primal problem (8.7) and the dual problem (8.8) are infeasible, **MOSEK** will report only one of the two possible certificates — which one is not defined (**MOSEK** returns the first certificate found).

#### 8.2.3 Minimalization vs. Maximalization

When the objective sense of problem (8.7) is maximization, i.e.

the objective sense of the dual problem changes to minimization, and the domain of all dual variables changes sign in comparison to (8.2). The dual problem thus takes the form

$$\begin{array}{ll} \text{minimize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x - g^T \dot{y} + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x + F^T \dot{y} = c, \\ & -y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \leq 0, \\ & -\dot{y} \in \mathcal{D}^* \\ \end{array}$$

This means that the duality gap, defined in (8.9) as the primal minus the dual objective value, becomes nonpositive. It follows that the dual objective will always be greater than or equal to the primal objective. The primal infeasibility certificate will be reported by **MOSEK** as a solution to the system

$$\begin{split} A^T y + s_l^x - s_u^x + F^T \dot{y} &= 0, \\ -y + s_l^c - s_u^c &= 0, \\ s_l^c, s_u^c, s_l^x, s_u^x &\leq 0, \\ -\dot{y} &\in \mathcal{D}^* \end{split} \tag{8.15}$$

such that the objective value is strictly negative

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* - g^T \dot{y} < 0.$$

Similarly, the certificate of dual infeasibility is an x satisfying the requirements of (8.12) such that  $c^T x > 0$ .

## 8.3 Semidefinite Optimization

Semidefinite optimization is an extension of conic optimization (see Sec. 8.2) allowing positive semidefinite matrix variables to be used in addition to the usual scalar variables. All the other parts of the input are defined exactly as in Sec. 8.2, and the discussion from that section applies verbatim to all properties of problems with semidefinite variables. We only briefly indicate how the corresponding formulae should be modified with semidefinite terms.

A semidefinite optimization problem can be written as

$$\begin{array}{llll} \text{minimize} & & c^Tx + \langle \overline{C}, \overline{X} \rangle + c^f \\ \text{subject to} & l^c & \leq & Ax + \langle \overline{A}, \overline{X} \rangle & \leq & u^c, \\ & l^x & \leq & x & \leq & u^x, \\ & & & Fx + \langle \overline{F}, \overline{X} \rangle + g & \in & \mathcal{D}, \\ & & & \overline{X}_j & \in & \mathcal{S}_+^{r_j}, j = 1, \dots, s \end{array}$$

where

- *m* is the number of constraints.
- $\bullet$  *n* is the number of decision variables.
- $x \in \mathbb{R}^n$  is a vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear part of the objective function.
- $c^f \in \mathbb{R}$  is a constant term in the objective
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $F \in \mathbb{R}^{k \times n}$  is the affine conic constraint matrix.,
- $g \in \mathbb{R}^k$  is the affine conic constraint constant term vector.,
- $\mathcal{D}$  is a Cartesian product of conic domains, namely  $\mathcal{D} = \mathcal{D}_1 \times \cdots \times \mathcal{D}_p$ , where p is the number of individual affine conic constraints (ACCs), and each domain is one from Sec. 11.5.

is the same as in Sec. 8.2 and moreover:

- there are s symmetric positive semidefinite variables, the j-th of which is  $\overline{X}_j \in \mathcal{S}_+^{r_j}$  of dimension  $r_j$ ,
- $\overline{C} = (\overline{C}_j)_{j=1,\dots,s}$  is a collection of symmetric coefficient matrices in the objective, with  $\overline{C}_j \in \mathcal{S}^{r_j}$ , and we interpret the notation  $\langle \overline{C}, \overline{X} \rangle$  as a shorthand for

$$\langle \overline{C}, \overline{X} \rangle := \sum_{j=1}^{s} \langle \overline{C}_j, \overline{X}_j \rangle.$$

•  $\overline{A} = (\overline{A}_{ij})_{i=1,\dots,m,j=1,\dots,s}$  is a collection of symmetric coefficient matrices in the constraints, with  $\overline{A}_{ij} \in \mathcal{S}^{r_j}$ , and we interpret the notation  $\langle \overline{A}, \overline{X} \rangle$  as a shorthand for the vector

$$\langle \overline{A}, \overline{X} \rangle := \left( \sum_{j=1}^{s} \langle \overline{A}_{ij}, \overline{X}_{j} \rangle \right)_{i=1,\dots,m}$$

•  $\overline{F} = (\overline{F}_{ij})_{i=1,\dots,k,j=1,\dots,s}$  is a collection of symmetric coefficient matrices in the affine conic constraints, with  $\overline{F}_{ij} \in \mathcal{S}^{r_j}$ , and we interpret the notation  $\langle \overline{F}, \overline{X} \rangle$  as a shorthand for the vector

$$\langle \overline{F}, \overline{X} \rangle := \left( \sum_{j=1}^{s} \langle \overline{F}_{ij}, \overline{X}_{j} \rangle \right)_{i=1,\dots,k}.$$

In each case the matrix inner product between symmetric matrices of the same dimension r is defined as

$$\langle U, V \rangle := \sum_{i=1}^{r} \sum_{j=1}^{r} U_{ij} V_{ij}.$$

To summarize, above the formulation extends that from Sec. 8.2 by the possibility of including semidefinite terms in the objective, constraints and affine conic constraints.

#### **Duality**

The definition of the dual problem (8.8) becomes:

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x - g^T \dot{y} + c^f$$
 subject to 
$$A^T y + s_l^x - s_u^x + F^T \dot{y} = c, \\ -y + s_l^c - s_u^c = 0, \\ \overline{C}_j - \sum_{i=1}^m y_i \overline{A}_{ij} - \sum_{i=1}^k \dot{y}_i \overline{F}_{ij} = S_j, \qquad j = 1, \dots, s, \\ s_l^c, s_u^c, s_l^x, s_u^x \ge 0, \\ \dot{y} \in \mathcal{D}^*, \\ \overline{S}_j \in \mathcal{S}_+^{r_j}, \qquad j = 1, \dots, s. \end{cases}$$
(8.16)

Complementarity conditions (8.10) include the additional relation:

$$\langle \overline{X}_j, \overline{S}_j \rangle = 0 \quad j = 1, \dots, s.$$
 (8.17)

#### Infeasibility

A certificate of primal infeasibility (8.11) is now a feasible solution to:

maximize 
$$(l^{c})^{T} s_{l}^{c} - (u^{c})^{T} s_{u}^{c} + (l^{x})^{T} s_{l}^{x} - (u^{x})^{T} s_{u}^{x} - g^{T} \dot{y}$$
 subject to
$$A^{T} y + s_{l}^{x} - s_{u}^{x} + F^{T} \dot{y} = 0,$$

$$-y + s_{l}^{c} - s_{u}^{c} = 0,$$

$$-\sum_{i=1}^{m} y_{i} \overline{A}_{ij} - \sum_{i=1}^{k} \dot{y}_{i} \overline{F}_{ij} = S_{j}, \qquad j = 1, \dots, s,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \geq 0,$$

$$\dot{y} \in \mathcal{D}^{*},$$

$$\overline{S}_{j} \in \mathcal{S}_{+}^{rj}, \qquad j = 1, \dots, s.$$

$$(8.18)$$

such that the objective value is strictly positive.

Similarly, a dual infeasibility certificate (8.12) is a feasible solution to

minimize 
$$c^{T}x + \langle \overline{C}, \overline{X} \rangle$$
subject to 
$$\hat{l}^{c} \leq Ax + \langle \overline{A}, \overline{X} \rangle \leq \hat{u}^{c},$$

$$\hat{l}^{x} \leq x \leq \hat{u}^{x},$$

$$Fx + \langle \overline{F}, \overline{X} \rangle \in \mathcal{D},$$

$$\overline{X}_{j} \in \mathcal{S}_{+}^{r_{j}}, j = 1, \dots, s$$

$$(8.19)$$

where the modified bounds are as in (8.13) and (8.14) and the objective value is strictly negative.

## 8.4 Quadratic and Quadratically Constrained Optimization

A convex quadratic and quadratically constrained optimization problem has the form

minimize 
$$\frac{1}{2}x^{T}Q^{o}x + c^{T}x + c^{f}$$
subject to 
$$l_{k}^{c} \leq \frac{1}{2}x^{T}Q^{k}x + \sum_{j=0}^{n-1}a_{kj}x_{j} \leq u_{k}^{c}, \quad k = 0, \dots, m-1,$$

$$l_{j}^{x} \leq x_{j} \leq u_{j}^{x}, \quad j = 0, \dots, n-1,$$

$$(8.20)$$

where all variables and bounds have the same meaning as for linear problems (see Sec. 8.1) and  $Q^o$  and all  $Q^k$  are symmetric matrices. Moreover, for convexity,  $Q^o$  must be a positive semidefinite matrix and  $Q^k$  must satisfy

$$\begin{array}{rcl} -\infty < l_k^c & \Rightarrow & Q^k \text{ is negative semidefinite,} \\ u_k^c < \infty & \Rightarrow & Q^k \text{ is positive semidefinite,} \\ -\infty < l_k^c \leq u_k^c < \infty & \Rightarrow & Q^k = 0. \end{array}$$

The convexity requirement is very important and MOSEK checks whether it is fulfilled.

#### 8.4.1 A Recommendation

Any convex quadratic optimization problem can be reformulated as a conic quadratic optimization problem, see Modeling Cookbook and [And13]. In fact **MOSEK** does such conversion internally as a part of the solution process for the following reasons:

- the conic optimizer is numerically more robust than the one for quadratic problems.
- the conic optimizer is usually faster because quadratic cones are simpler than quadratic functions, even though the conic reformulation usually has more constraints and variables than the original quadratic formulation.
- it is easy to dualize the conic formulation if deemed worthwhile potentially leading to (huge) computational savings.

However, instead of relying on the automatic reformulation we recommend to formulate the problem as a conic problem from scratch because:

- it saves the computational overhead of the reformulation including the convexity check. A conic problem is convex by construction and hence no convexity check is needed for conic problems.
- usually the modeler can do a better reformulation than the automatic method because the modeler can exploit the knowledge of the problem at hand.

To summarize we recommend to formulate quadratic problems and in particular quadratically constrained problems directly in conic form.

## 8.4.2 Duality for Quadratic and Quadratically Constrained Optimization

The dual problem corresponding to the quadratic and quadratically constrained optimization problem (8.20) is given by

$$\begin{array}{ll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + \frac{1}{2} x^T \left\{ \sum_{k=0}^{m-1} y_k Q^k - Q^o \right\} x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x + \left\{ \sum_{k=0}^{m-1} y_k Q^k - Q^o \right\} x = c, \\ & - y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_u^x, s_u^x \geq 0. \end{array}$$

The dual problem is related to the dual problem for linear optimization (see Sec. 8.1.1), but depends on the variable x which in general can not be eliminated. In the solutions reported by **MOSEK**, the value of x is the same for the primal problem (8.20) and the dual problem (8.21).

## 8.4.3 Infeasibility for Quadratic Optimization

In case **MOSEK** finds a problem to be infeasible it reports a certificate of infeasibility. We write them out explicitly for quadratic problems, that is when  $Q^k = 0$  for all k and quadratic terms appear only in the objective  $Q^o$ . In this case the constraints both in the primal and dual problem are linear, and **MOSEK** produces for them the same infeasibility certificate as for linear problems.

The certificate of primal infeasibility is a solution to the problem (8.4) such that the objective value is strictly positive.

The certificate of dual infeasibility is a solution to the problem (8.5) together with an additional constraint

$$Q^o x = 0$$

such that the objective value is strictly negative.

Below is an outline of the different problem types for quick reference.

#### Continuous problem formulations

• Linear optimization (LO)

• Conic optimization (CO)

Conic optimization extends linear optimization with affine conic constraints (ACC):

where  $\mathcal{D}$  is a product of domains from Sec. 11.5.

• Semidefinite optimization (SDO)

A conic optimization problem can be further extended with semidefinite variables:

where  $\mathcal{D}$  is a product of domains from Sec. 11.5 and  $\mathcal{S}_+$  is a product of PSD cones meaning that  $\overline{X}$  is a sequence of PSD matrix variables.

• Quadratic and quadratically constrained optimization (QO, QCQO)

A quadratic problem or quadratically constrained problem has the form

$$\begin{array}{lll} \text{minimize} & & \frac{1}{2}x^TQ^ox + c^Tx + c^f \\ \text{subject to} & l^c & \leq & \frac{1}{2}x^TQ^cx + Ax & \leq & u^c, \\ & l^x & \leq & x & \leq & u^x. \end{array}$$

#### Mixed-integer extensions

Coninuous problems can be extended with constraints requiring the mixed-integer optimizer. We outline them briefly here. The continuous part of a mixed-integer problem is formulated according to one of the continuous types above, however only the primal information and solution fields are relevant, there are no dual values and no infeasibility certificates.

• Integer variables. Specifies that a subset of variables take integer values, that is

$$x_I \in \mathbb{Z}$$

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for some index set I. Available for problems of type LO, CO, QO and QCQO.

# Chapter 9

# **Optimizers**

The most essential part of MOSEK are the optimizers:

- primal simplex (linear problems),
- dual simplex (linear problems),
- interior-point (linear, quadratic and conic problems),
- mixed-integer (problems with integer variables).

The structure of a successful optimization process is roughly:

#### • Presolve

- 1. Elimination: Reduce the size of the problem.
- 2. Dualizer: Choose whether to solve the primal or the dual form of the problem.
- 3. Scaling: Scale the problem for better numerical stability.

#### • Optimization

- 1. Optimize: Solve the problem using selected method.
- 2. Terminate: Stop the optimization when specific termination criteria have been met.
- 3. Report: Return the solution or an infeasibility certificate.

The preprocessing stage is transparent to the user, but useful to know about for tuning purposes. The purpose of the preprocessing steps is to make the actual optimization more efficient and robust. We discuss the details of the above steps in the following sections.

## 9.1 Presolve

Before an optimizer actually performs the optimization the problem is preprocessed using the so-called presolve. The purpose of the presolve is to

- 1. remove redundant constraints,
- 2. eliminate fixed variables,
- 3. remove linear dependencies,
- 4. substitute out (implied) free variables, and
- 5. reduce the size of the optimization problem in general.

After the presolved problem has been optimized the solution is automatically postsolved so that the returned solution is valid for the original problem. Hence, the presolve is completely transparent. For further details about the presolve phase, please see [AA95] and [AGMeszarosX96].

It is possible to fine-tune the behavior of the presolve or to turn it off entirely. If presolve consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This is done by setting the parameter MSK\_IPAR\_PRESOLVE\_USE to MSK\_PRESOLVE\_MODE\_OFF.

In the following we describe in more detail the presolve applied to continuous, i.e., linear and conic optimization problems, see Sec. 9.2 and Sec. 9.3. The mixed-integer optimizer, Sec. 9.4, applies similar techniques. The two most time-consuming steps of the presolve for continuous optimization problems are

- the eliminator, and
- the linear dependency check.

Therefore, in some cases it is worthwhile to disable one or both of these.

#### Numerical issues in the presolve

During the presolve the problem is reformulated so that it hopefully solves faster. However, in rare cases the presolved problem may be harder to solve then the original problem. The presolve may also be infeasible although the original problem is not. If it is suspected that presolved problem is much harder to solve than the original, we suggest to first turn the eliminator off by setting the parameter  $MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES$  to 0. If that does not help, then trying to turn entire presolve off may help.

Since all computations are done in finite precision, the presolve employs some tolerances when concluding a variable is fixed or a constraint is redundant. If it happens that  $\mathbf{MOSEK}$  incorrectly concludes a problem is primal or dual infeasible, then it is worthwhile to try to reduce the parameters  $MSK\_DPAR\_PRESOLVE\_TOL\_X$  and  $MSK\_DPAR\_PRESOLVE\_TOL\_S$ . However, if reducing the parameters actually helps then this should be taken as an indication that the problem is badly formulated.

#### Eliminator

The purpose of the eliminator is to eliminate free and implied free variables from the problem using substitution. For instance, given the constraints

$$\begin{array}{rcl} y & = & \sum_{j} x_{j}, \\ y, x & \geq & 0, \end{array}$$

y is an implied free variable that can be substituted out of the problem, if deemed worthwhile. If the eliminator consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This can be done by setting the parameter  $MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES$  to 0. In rare cases the eliminator may cause that the problem becomes much hard to solve.

#### Linear dependency checker

The purpose of the linear dependency check is to remove linear dependencies among the linear equalities. For instance, the three linear equalities

$$\begin{array}{rcl} x_1 + x_2 + x_3 & = & 1, \\ x_1 + 0.5x_2 & = & 0.5, \\ 0.5x_2 + x_3 & = & 0.5. \end{array}$$

contain exactly one linear dependency. This implies that one of the constraints can be dropped without changing the set of feasible solutions. Removing linear dependencies is in general a good idea since it reduces the size of the problem. Moreover, the linear dependencies are likely to introduce numerical problems in the optimization phase. It is best practice to build models without linear dependencies, but that is not always easy for the user to control. If the linear dependencies are removed at the modeling stage, the linear dependency check can safely be disabled by setting the parameter  $MSK\_IPAR\_PRESOLVE\_LINDEP\_USE$  to  $MSK\_OFF$ .

#### **Dualizer**

All linear, conic, and convex optimization problems have an equivalent dual problem associated with them. **MOSEK** has built-in heuristics to determine if it is more efficient to solve the primal or dual problem. The form (primal or dual) is displayed in the **MOSEK** log and available as an information item from the solver. Should the internal heuristics not choose the most efficient form of the problem it may be worthwhile to set the dualizer manually by setting the parameters:

- MSK\_IPAR\_INTPNT\_SOLVE\_FORM: In case of the interior-point optimizer.
- MSK\_IPAR\_SIM\_SOLVE\_FORM: In case of the simplex optimizer.

Note that currently only linear and conic (but not semidefinite) problems may be automatically dualized.

#### **Scaling**

Problems containing data with large and/or small coefficients, say 1.0e + 9 or 1.0e - 7, are often hard to solve. Significant digits may be truncated in calculations with finite precision, which can result in the optimizer relying on inaccurate data. Since computers work in finite precision, extreme coefficients should be avoided. In general, data around the same *order of magnitude* is preferred, and we will refer to a problem, satisfying this loose property, as being *well-scaled*. If the problem is not well scaled, **MOSEK** will try to scale (multiply) constraints and variables by suitable constants. **MOSEK** solves the scaled problem to improve the numerical properties.

The scaling process is transparent, i.e. the solution to the original problem is reported. It is important to be aware that the optimizer terminates when the termination criterion is met on the scaled problem, therefore significant primal or dual infeasibilities may occur after unscaling for badly scaled problems. The best solution of this issue is to reformulate the problem, making it better scaled.

By default **MOSEK** heuristically chooses a suitable scaling. The scaling for interior-point and simplex optimizers can be controlled with the parameters <code>MSK\_IPAR\_INTPNT\_SCALING</code> and <code>MSK\_IPAR\_SIM\_SCALING</code> respectively.

## 9.2 Linear Optimization

#### 9.2.1 Optimizer Selection

Two different types of optimizers are available for linear problems: The default is an interior-point method, and the alternative is the simplex method (primal or dual). The optimizer can be selected using the parameter  $MSK\_IPAR\_OPTIMIZER$ .

### The Interior-point or the Simplex Optimizer?

Given a linear optimization problem, which optimizer is the best: the simplex or the interior-point optimizer? It is impossible to provide a general answer to this question. However, the interior-point optimizer behaves more predictably: it tends to use between 20 and 100 iterations, almost independently of problem size, but cannot perform warm-start. On the other hand the simplex method can take advantage of an initial solution, but is less predictable from cold-start. The interior-point optimizer is used by default.

#### The Primal or the Dual Simplex Variant?

MOSEK provides both a primal and a dual simplex optimizer. Predicting which simplex optimizer is faster is impossible, however, in recent years the dual optimizer has seen several algorithmic and computational improvements, which, in our experience, make it faster on average than the primal version. Still, it depends much on the problem structure and size. Setting the MSK\_IPAR\_OPTIMIZER parameter to MSK\_OPTIMIZER\_FREE\_SIMPLEX instructs MOSEK to choose one of the simplex variants automatically.

To summarize, if you want to know which optimizer is faster for a given problem type, it is best to try all the options.

## 9.2.2 The Interior-point Optimizer

The purpose of this section is to provide information about the algorithm employed in the **MOSEK** interior-point optimizer for linear problems and about its termination criteria.

#### The homogeneous primal-dual problem

In order to keep the discussion simple it is assumed that  $\mathbf{MOSEK}$  solves linear optimization problems of standard form

$$\begin{array}{lll} \text{minimize} & c^T x \\ \text{subject to} & Ax & = & b, \\ & x > 0. \end{array} \tag{9.1}$$

This is in fact what happens inside MOSEK; for efficiency reasons MOSEK converts the problem to standard form before solving, then converts it back to the input form when reporting the solution.

Since it is not known beforehand whether problem (9.1) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason why **MOSEK** solves the so-called homogeneous model

$$\begin{array}{rcl}
Ax - b\tau & = & 0, \\
A^{T}y + s - c\tau & = & 0, \\
-c^{T}x + b^{T}y - \kappa & = & 0, \\
x, s, \tau, \kappa & \geq & 0,
\end{array} \tag{9.2}$$

where y and s correspond to the dual variables in (9.1), and  $\tau$  and  $\kappa$  are two additional scalar variables. Note that the homogeneous model (9.2) always has solution since

$$(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)$$

is a solution, although not a very interesting one. Any solution

$$(x^*, y^*, s^*, \tau^*, \kappa^*)$$

to the homogeneous model (9.2) satisfies

$$x_i^* s_i^* = 0$$
 and  $\tau^* \kappa^* = 0$ .

Moreover, there is always a solution that has the property  $\tau^* + \kappa^* > 0$ .

First, assume that  $\tau^* > 0$ . It follows that

$$\begin{array}{rcl} A\frac{x^*}{\tau^*} & = & b, \\ A^T\frac{y^*}{\tau^*} + \frac{s^*}{\tau^*} & = & c, \\ -c^T\frac{x^*}{\tau^*} + b^T\frac{y^*}{\tau^*} & = & 0, \\ x^*, s^*, \tau^*, \kappa^* & \geq & 0. \end{array}$$

This shows that  $\frac{x^*}{\tau^*}$  is a primal optimal solution and  $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$  is a dual optimal solution; this is reported as the optimal interior-point solution since

$$(x, y, s) = \left\{ \frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau^*} \right\}$$

is a primal-dual optimal solution (see Sec. 8.1 for the mathematical background on duality and optimality).

On other hand, if  $\kappa^* > 0$  then

$$\begin{array}{rcl} Ax^* & = & 0, \\ A^Ty^* + s^* & = & 0, \\ -c^Tx^* + b^Ty^* & = & \kappa^*, \\ x^*, s^*, \tau^*, \kappa^* & \geq & 0. \end{array}$$

This implies that at least one of

$$c^T x^* < 0 (9.3)$$

or

$$b^T y^* > 0 (9.4)$$

is satisfied. If (9.3) is satisfied then  $x^*$  is a certificate of dual infeasibility, whereas if (9.4) is satisfied then  $y^*$  is a certificate of primal infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm [And09].

#### **Interior-point Termination Criterion**

For efficiency reasons it is not practical to solve the homogeneous model exactly. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In the k-th iteration of the interior-point algorithm a trial solution

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

to homogeneous model is generated, where

$$x^k, s^k, \tau^k, \kappa^k > 0.$$

#### Optimal case

Whenever the trial solution satisfies the criterion

$$\left\| A \frac{x^{k}}{\tau^{k}} - b \right\|_{\infty} \leq \epsilon_{p} (1 + \|b\|_{\infty}), 
\left\| A^{T} \frac{y^{k}}{\tau^{k}} + \frac{s^{k}}{\tau^{k}} - c \right\|_{\infty} \leq \epsilon_{d} (1 + \|c\|_{\infty}), \text{ and} 
\min \left( \frac{(x^{k})^{T} s^{k}}{(\tau^{k})^{2}}, \left| \frac{c^{T} x^{k}}{\tau^{k}} - \frac{b^{T} y^{k}}{\tau^{k}} \right| \right) \leq \epsilon_{g} \max \left( 1, \frac{\min \left( \left| c^{T} x^{k} \right|, \left| b^{T} y^{k} \right| \right)}{\tau^{k}} \right),$$
(9.5)

the interior-point optimizer is terminated and

$$\frac{(x^k, y^k, s^k)}{\tau^k}$$

is reported as the primal-dual optimal solution. The interpretation of (9.5) is that the optimizer is terminated if

- $\frac{x^k}{\tau^k}$  is approximately primal feasible,
- $\bullet \ \left\{ \frac{y^k}{\tau^k}, \frac{s^k}{\tau^k} \right\}$  is approximately dual feasible, and
- the duality gap is almost zero.

#### **Dual infeasibility certificate**

On the other hand, if the trial solution satisfies

$$-\epsilon_i c^T x^k > \frac{\|c\|_{\infty}}{\max\left(1, \|b\|_{\infty}\right)} \|Ax^k\|_{\infty}$$

then the problem is declared dual infeasible and  $x^k$  is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows: First assume that  $\|Ax^k\|_{\infty} = 0$ ; then  $x^k$  is an exact certificate of dual infeasibility. Next assume that this is not the case, i.e.

$$||Ax^k||_{\infty} > 0,$$

and define

$$\bar{x} := \epsilon_i \frac{\max\left(1, \|b\|_{\infty}\right)}{\|Ax^k\|_{\infty} \|c\|_{\infty}} x^k.$$

It is easy to verify that

$$||A\bar{x}||_{\infty} = \epsilon_i \frac{\max(1, ||b||_{\infty})}{||c||_{\infty}} \text{ and } -c^T \bar{x} > 1,$$

which shows  $\bar{x}$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation. A smaller value means a better approximation.

#### Primal infeasibility certificate

Finally, if

$$\epsilon_i b^T y^k > \frac{\|b\|_{\infty}}{\max\left(1, \|c\|_{\infty}\right)} \left\|A^T y^k + s^k\right\|_{\infty}$$

then  $y^k$  is reported as a certificate of primal infeasibility.

#### Adjusting optimality criteria

It is possible to adjust the tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$  using parameters; see table for details.

Table 9.1: Parameters employed in termination criterion

ToleranceParameter	name
$\varepsilon_p$	MSK_DPAR_INTPNT_TOL_PFEAS
$arepsilon_d$	MSK_DPAR_INTPNT_TOL_DFEAS
$arepsilon_g$	MSK_DPAR_INTPNT_TOL_REL_GAP
$arepsilon_i$	MSK_DPAR_INTPNT_TOL_INFEAS

The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (9.5) reveals that the quality of the solution depends on  $||b||_{\infty}$  and  $||c||_{\infty}$ ; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by **MOSEK** will converge toward optimality and primal and dual feasibility at the same rate [And09]. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances,  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$ , have to be relaxed together to achieve an effect.

The basis identification discussed in Sec. 9.2.2 requires an optimal solution to work well; hence basis identification should be turned off if the termination criterion is relaxed.

To conclude the discussion in this section, relaxing the termination criterion is usually not worthwhile.

#### **Basis Identification**

An interior-point optimizer does not return an optimal basic solution unless the problem has a unique primal and dual optimal solution. Therefore, the interior-point optimizer has an optimal post-processing step that computes an optimal basic solution starting from the optimal interior-point solution. More information about the basis identification procedure may be found in [AY96]. In the following we provide an overall idea of the procedure.

There are some cases in which a basic solution could be more valuable:

- a basic solution is often more accurate than an interior-point solution,
- a basic solution can be used to warm-start the simplex algorithm in case of reoptimization,
- a basic solution is in general more sparse, i.e. more variables are fixed to zero. This is particularly appealing when solving continuous relaxations of mixed integer problems, as well as in all applications in which sparser solutions are preferred.

To illustrate how the basis identification routine works, we use the following trivial example:

$$\begin{array}{lll} \text{minimize} & x+y \\ \text{subject to} & x+y & = & 1, \\ & x,y \geq 0. \end{array}$$

It is easy to see that all feasible solutions are also optimal. In particular, there are two basic solutions, namely

$$(x_1^*, y_1^*) = (1, 0),$$
  
 $(x_2^*, y_2^*) = (0, 1).$ 

The interior point algorithm will actually converge to the center of the optimal set, i.e. to  $(x^*, y^*) = (1/2, 1/2)$  (to see this in **MOSEK** deactivate *Presolve*).

In practice, when the algorithm gets close to the optimal solution, it is possible to construct in polynomial time an initial basis for the simplex algorithm from the current interior point solution. This basis is used to warm-start the simplex algorithm that will provide the optimal basic solution. In most cases the constructed basis is optimal, or very few iterations are required by the simplex algorithm to make it optimal and hence the final *clean-up* phase be short. However, for some cases of ill-conditioned problems the additional simplex clean up phase may take of lot a time.

By default **MOSEK** performs a basis identification. However, if a basic solution is not needed, the basis identification procedure can be turned off. The parameters

- MSK\_IPAR\_INTPNT\_BASIS,
- MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER, and
- MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR

control when basis identification is performed.

The type of simplex algorithm to be used (primal/dual) can be tuned with the parameter  $MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER$ , and the maximum number of iterations can be set with  $MSK\_IPAR\_BI\_MAX\_ITERATIONS$ .

Finally, it should be mentioned that there is no guarantee on which basic solution will be returned.

#### The Interior-point Log

Below is a typical log output from the interior-point optimizer:

```
Optimizer
          - threads
                                    : 1
                                    : the dual
Optimizer
          - solved problem
Optimizer
          - Constraints
                                    : 2
Optimizer
          - Cones
Optimizer
          - Scalar variables
                                    : 6
                                                                                : 0
                                                         conic
                                                                                : 0
          - Semi-definite variables: 0
Optimizer
                                                         scalarized
Factor
           - setup time
                                    : 0.00
                                                         dense det. time
                                                                                : 0.00
Factor
           - ML order time
                                    : 0.00
                                                         GP order time
                                                                                : 0.00
Factor
           - nonzeros before factor : 3
                                                         after factor
                                                                                : 3
                                                                                : 7.
Factor
           - dense dim.
                                                         flops
→00e+001
ITE PFEAS
             DFEAS
                      GFEAS
                               PRSTATUS
                                                             DOBJ
                                          POBJ
                                                                               MU
→ TIME
Ω
   1.0e+000 8.6e+000 6.1e+000 1.00e+000
                                          0.00000000e+000
                                                            -2.208000000e+003 1.
→0e+000 0.00
   1.1e+000 2.5e+000 1.6e-001 0.00e+000
                                          -7.901380925e+003 -7.394611417e+003 2.
→5e+000 0.00
   1.4e-001 3.4e-001 2.1e-002 8.36e-001
                                          -8.113031650e+003 -8.055866001e+003 3.3e-
→001 0.00
   2.4e-002 5.8e-002 3.6e-003 1.27e+000
                                          -7.777530698e+003 -7.766471080e+003 5.7e-
-002 0.01
   1.3e-004 3.2e-004 2.0e-005 1.08e+000 -7.668323435e+003 -7.668207177e+003 3.2e-
→004 0.01
```

(continues on next page)

```
5 1.3e-008 3.2e-008 2.0e-009 1.00e+000 -7.668000027e+003 -7.668000015e+003 3.2e-

-008 0.01

6 1.3e-012 3.2e-012 2.0e-013 1.00e+000 -7.667999994e+003 -7.667999994e+003 3.2e-

-012 0.01
```

The first line displays the number of threads used by the optimizer and the second line indicates if the optimizer chose to solve the primal or dual problem (see MSK\_IPAR\_INTPNT\_SOLVE\_FORM). The next lines display the problem dimensions as seen by the optimizer, and the Factor... lines show various statistics. This is followed by the iteration log.

Using the same notation as in Sec. 9.2.2 the columns of the iteration log have the following meaning:

- ITE: Iteration index k.
- PFEAS:  $||Ax^k b\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- DFEAS:  $\|A^Ty^k + s^k c\tau^k\|_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- GFEAS:  $|-c^Tx^k+b^Ty^k-\kappa^k|$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- PRSTATUS: This number converges to 1 if the problem has an optimal solution whereas it converges to −1 if that is not the case.
- POBJ:  $c^T x^k / \tau^k$ . An estimate for the primal objective value.
- DOBJ:  $b^T y^k / \tau^k$ . An estimate for the dual objective value.
- $\bullet$  MU:  $\frac{(x^k)^Ts^k+\tau^k\kappa^k}{n+1}$  . The numbers in this column should always converge to zero.
- TIME: Time spent since the optimization started.

### 9.2.3 The Simplex Optimizer

An alternative to the interior-point optimizer is the simplex optimizer. The simplex optimizer uses a different method that allows exploiting an initial guess for the optimal solution to reduce the solution time. Depending on the problem it may be faster or slower to use an initial guess; see Sec. 9.2.1 for a discussion. **MOSEK** provides both a primal and a dual variant of the simplex optimizer.

#### **Simplex Termination Criterion**

The simplex optimizer terminates when it finds an optimal basic solution or an infeasibility certificate. A basic solution is optimal when it is primal and dual feasible; see Sec. 8.1 for a definition of the primal and dual problem. Due to the fact that computations are performed in finite precision **MOSEK** allows violations of primal and dual feasibility within certain tolerances. The user can control the allowed primal and dual tolerances with the parameters <code>MSK\_DPAR\_BASIS\_TOL\_X</code> and <code>MSK\_DPAR\_BASIS\_TOL\_S</code>.

Setting the parameter MSK\_IPAR\_OPTIMIZER to MSK\_OPTIMIZER\_FREE\_SIMPLEX instructs MOSEK to select automatically between the primal and the dual simplex optimizers. Hence, MOSEK tries to choose the best optimizer for the given problem and the available solution. The same parameter can also be used to force one of the variants.

#### Starting From an Existing Solution

When using the simplex optimizer it may be possible to reuse an existing solution and thereby reduce the solution time significantly. When a simplex optimizer starts from an existing solution it is said to perform a *warm-start*. If the user is solving a sequence of optimization problems by solving the problem, making modifications, and solving again, **MOSEK** will warm-start automatically.

By default **MOSEK** uses presolve when performing a warm-start. If the optimizer only needs very few iterations to find the optimal solution it may be better to turn off the presolve.

#### **Numerical Difficulties in the Simplex Optimizers**

Though MOSEK is designed to minimize numerical instability, completely avoiding it is impossible when working in finite precision. MOSEK treats a "numerically unexpected behavior" event inside the optimizer as a *set-back*. The user can define how many set-backs the optimizer accepts; if that number is exceeded, the optimization will be aborted. Set-backs are a way to escape long sequences where the optimizer tries to recover from an unstable situation.

Examples of set-backs are: repeated singularities when factorizing the basis matrix, repeated loss of feasibility, degeneracy problems (no progress in objective) and other events indicating numerical difficulties. If the simplex optimizer encounters a lot of set-backs the problem is usually badly scaled; in such a situation try to reformulate it into a better scaled problem. Then, if a lot of set-backs still occur, trying one or more of the following suggestions may be worthwhile:

- Raise tolerances for allowed primal or dual feasibility: increase the value of
  - MSK\_DPAR\_BASIS\_TOL\_X, and
  - MSK\_DPAR\_BASIS\_TOL\_S.
- Raise or lower pivot tolerance: Change the MSK\_DPAR\_SIMPLEX\_ABS\_TOL\_PIV parameter.
- Switch optimizer: Try another optimizer.
- Switch off crash: Set both MSK\_IPAR\_SIM\_PRIMAL\_CRASH and MSK\_IPAR\_SIM\_DUAL\_CRASH to 0.
- Experiment with other pricing strategies: Try different values for the parameters
  - MSK\_IPAR\_SIM\_PRIMAL\_SELECTION and
  - MSK\_IPAR\_SIM\_DUAL\_SELECTION.
- If you are using warm-starts, in rare cases switching off this feature may improve stability. This is controlled by the  $MSK\_IPAR\_SIM\_HOTSTART$  parameter.
- Increase maximum number of set-backs allowed controlled by MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS.
- If the problem repeatedly becomes infeasible try switching off the special degeneracy handling. See the parameter  $MSK\_IPAR\_SIM\_DEGEN$  for details.

#### The Simplex Log

Below is a typical log output from the simplex optimizer:

-	nizer – so nizer – Co	lved problem	: the p:	rimal		
-			: 1424	conic		: 0
-		alar variables		Conic		. 0
Uptim	nizer - ho		: no			
ITER	DEGIT	ER(%) PFEAS	DFEAS	POBJ	DOBJ	Ц
$\hookrightarrow$	TIME	TOTTIME				
0	0.00	1.43e+05	NA	6.5584140832e+03	NA	П
$\hookrightarrow$	0.00	0.02				
1000	1.10	0.00e+00	NA	1.4588289726e+04	NA	П
$\hookrightarrow$	0.13	0.14				
2000	0.75	0.00e+00	NA	7.3705564855e+03	NA	П
$\hookrightarrow$	0.21	0.22				_

(continues on next page)

(continued from previous page)

3000	0.67		0.00e+00	NA	6.0509727712e+03	NA	Ц
$\hookrightarrow$	0.29	0.31					
4000	0.52		0.00e+00	NA	5.5771203906e+03	NA	ш
$\hookrightarrow$	0.38	0.39					
4533	0.49		0.00e+00	NA	5.5018458883e+03	NA	Ц
$\hookrightarrow$	0.42	0.44					

The first lines summarize the problem the optimizer is solving. This is followed by the iteration log, with the following meaning:

- ITER: Number of iterations.
- DEGITER(%): Ratio of degenerate iterations.
- PFEAS: Primal feasibility measure reported by the simplex optimizer. The numbers should be 0 if the problem is primal feasible (when the primal variant is used).
- DFEAS: Dual feasibility measure reported by the simplex optimizer. The number should be 0 if the problem is dual feasible (when the dual variant is used).
- POBJ: An estimate for the primal objective value (when the primal variant is used).
- DOBJ: An estimate for the dual objective value (when the dual variant is used).
- TIME: Time spent since this instance of the simplex optimizer was invoked (in seconds).
- TOTTIME: Time spent since optimization started (in seconds).

## 9.3 Conic Optimization - Interior-point optimizer

For conic optimization problems only an interior-point type optimizer is available. The same optimizer is used for quadratic optimization problems which are internally reformulated to conic form.

#### 9.3.1 The homogeneous primal-dual problem

The interior-point optimizer is an implementation of the so-called homogeneous and self-dual algorithm. For a detailed description of the algorithm, please see [ART03]. In order to keep our discussion simple we will assume that **MOSEK** solves a conic optimization problem of the form:

minimize 
$$c^T x$$
  
subject to  $Ax = b$ , (9.6)

where K is a convex cone. The corresponding dual problem is

$$\begin{array}{lll} \text{maximize} & b^T y \\ \text{subject to} & A^T y + s & = & c, \\ & s \in \mathcal{K}^* \end{array} \tag{9.7}$$

where  $\mathcal{K}^*$  is the dual cone of  $\mathcal{K}$ . See Sec. 8.2 for definitions.

Since it is not known beforehand whether problem (9.6) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason that **MOSEK** solves the so-called homogeneous model

$$Ax - b\tau = 0,$$

$$A^{T}y + s - c\tau = 0,$$

$$-c^{T}x + b^{T}y - \kappa = 0,$$

$$x \in \mathcal{K},$$

$$s \in \mathcal{K}^{*},$$

$$\tau, \kappa \geq 0,$$

$$(9.8)$$

where y and s correspond to the dual variables in (9.6), and  $\tau$  and  $\kappa$  are two additional scalar variables. Note that the homogeneous model (9.8) always has a solution since

$$(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)$$

is a solution, although not a very interesting one. Any solution

$$(x^*, y^*, s^*, \tau^*, \kappa^*)$$

to the homogeneous model (9.8) satisfies

$$(x^*)^T s^* + \tau^* \kappa^* = 0$$

i.e. complementarity. Observe that  $x^* \in \mathcal{K}$  and  $s^* \in \mathcal{K}^*$  implies

$$(x^*)^T s^* > 0$$

and therefore

$$\tau^* \kappa^* = 0.$$

since  $\tau^*, \kappa^* \geq 0$ . Hence, at least one of  $\tau^*$  and  $\kappa^*$  is zero.

First, assume that  $\tau^* > 0$  and hence  $\kappa^* = 0$ . It follows that

$$\begin{array}{rcl} A \frac{x^*}{\tau^*} & = & b, \\ A^T \frac{y^*}{\tau^*} + \frac{s^*}{\tau^*} & = & c, \\ -c^T \frac{x^*}{\tau^*} + b^T \frac{y^*}{\tau^*} & = & 0, \\ x^*/\tau^* & \in & \mathcal{K}, \\ s^*/\tau^* & \in & \mathcal{K}^*. \end{array}$$

This shows that  $\frac{x^*}{\tau^*}$  is a primal optimal solution and  $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$  is a dual optimal solution; this is reported as the optimal interior-point solution since

$$(x, y, s) = \left(\frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau^*}\right)$$

is a primal-dual optimal solution.

On other hand, if  $\kappa^* > 0$  then

$$\begin{array}{rcl} Ax^* & = & 0, \\ A^Ty^* + s^* & = & 0, \\ -c^Tx^* + b^Ty^* & = & \kappa^*, \\ x^* & \in & \mathcal{K}, \\ s^* & \in & \mathcal{K}^*. \end{array}$$

This implies that at least one of

$$c^T x^* < 0 (9.9)$$

or

$$b^T y^* > 0 (9.10)$$

holds. If (9.9) is satisfied, then  $x^*$  is a certificate of dual infeasibility, whereas if (9.10) holds then  $y^*$  is a certificate of primal infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm [And09].

## 9.3.2 Interior-point Termination Criterion

Since computations are performed in finite precision, and for efficiency reasons, it is not possible to solve the homogeneous model exactly in general. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In every iteration k of the interior-point algorithm a trial solution

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

to the homogeneous model is generated, where

$$x^k \in \mathcal{K}, s^k \in \mathcal{K}^*, \tau^k, \kappa^k > 0.$$

Therefore, it is possible to compute the values:

$$\begin{array}{lll} \rho_p^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A \frac{x^k}{\tau^k} - b \right\|_{\infty} \leq \rho \varepsilon_p (1 + \|b\|_{\infty}) \right\}, \\ \rho_d^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A^T \frac{y^k}{\tau^k} + \frac{s^k}{\tau^k} - c \right\|_{\infty} \leq \rho \varepsilon_d (1 + \|c\|_{\infty}) \right\}, \\ \rho_g^k &=& \arg\min_{\rho} \left\{ \rho \mid \left( \frac{(x^k)^T s^k}{(\tau^k)^2}, |\frac{c^T x^k}{\tau^k} - \frac{b^T y^k}{\tau^k}| \right) \leq \rho \varepsilon_g \max \left( 1, \frac{\min(|c^T x^k|, |b^T y^k|)}{\tau^k} \right) \right\}, \\ \rho_{pi}^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A^T y^k + s^k \right\|_{\infty} \leq \rho \varepsilon_i b^T y^k, \, b^T y^k > 0 \right\} \text{ and } \\ \rho_{di}^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A x^k \right\|_{\infty} \leq -\rho \varepsilon_i c^T x^k, \, c^T x^k < 0 \right\}. \end{array}$$

Note  $\varepsilon_p, \varepsilon_d, \varepsilon_q$  and  $\varepsilon_i$  are nonnegative user specified tolerances.

#### **Optimal Case**

Observe  $\rho_p^k$  measures how far  $x^k/\tau^k$  is from being a good approximate primal feasible solution. Indeed if  $\rho_p^k \leq 1$ , then

$$\left\| A \frac{x^k}{\tau^k} - b \right\|_{\infty} \le \varepsilon_p (1 + \|b\|_{\infty}). \tag{9.11}$$

This shows the violations in the primal equality constraints for the solution  $x^k/\tau^k$  is small compared to the size of b given  $\varepsilon_p$  is small.

Similarly, if  $\rho_d^k \leq 1$ , then  $(y^k, s^k)/\tau^k$  is an approximate dual feasible solution. If in addition  $\rho_g \leq 1$ , then the solution  $(x^k, y^k, s^k)/\tau^k$  is approximate optimal because the associated primal and dual objective values are almost identical.

In other words if  $\max(\rho_n^k, \rho_d^k, \rho_q^k) \leq 1$ , then

$$\frac{(x^k, y^k, s^k)}{\tau^k}$$

is an approximate optimal solution.

#### **Dual Infeasibility Certificate**

Next assume that  $\rho_{di}^k \leq 1$  and hence

$$||Ax^k||_{\infty} \le -\varepsilon_i c^T x^k$$
 and  $-c^T x^k > 0$ 

holds. Now in this case the problem is declared dual infeasible and  $x^k$  is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows. Let

$$\bar{x} := \frac{x^k}{-c^T x^k}$$

and it is easy to verify that

$$||A\bar{x}||_{\infty} \leq \varepsilon_i$$
 and  $c^T\bar{x} = -1$ 

which shows  $\bar{x}$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation.

#### **Primal Infeasiblity Certificate**

Next assume that  $\rho_{pi}^k \leq 1$  and hence

$$||A^T y^k + s^k||_{\infty} \le \varepsilon_i b^T y^k$$
 and  $b^T y^k > 0$ 

holds. Now in this case the problem is declared primal infeasible and  $(y^k, s^k)$  is reported as a certificate of primal infeasibility. The motivation for this stopping criterion is as follows. Let

$$\bar{y} := \frac{y^k}{b^T y^k}$$
 and  $\bar{s} := \frac{s^k}{b^T y^k}$ 

and it is easy to verify that

$$||A^T \bar{y} + \bar{s}||_{\infty} \leq \varepsilon_i \text{ and } b^T \bar{y} = 1$$

which shows  $(y^k, s^k)$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation.

## 9.3.3 Adjusting optimality criteria

It is possible to adjust the tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$  using parameters; see the next table for details. Note that although this section discusses the conic optimizer, if the problem was originally input as a quadratic or quadratically constrained optimization problem then the parameter names that apply are those from the third column (with infix Q0 instead of C0).

Table 9.2: Parameters employed in termination criterion

The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (9.11) reveals that the quality of the solution depends on  $\|b\|_{\infty}$  and  $\|c\|_{\infty}$ ; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by **MOSEK** will converge toward optimality and primal and dual feasibility at the same rate [And09]. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances,  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$ , have to be relaxed together to achieve an effect.

If the optimizer terminates without locating a solution that satisfies the termination criteria, for example because of a stall or other numerical issues, then it will check if the solution found up to that point satisfies the same criteria with all tolerances multiplied by the value of  $MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL$ . If this is the case, the solution is still declared as optimal.

To conclude the discussion in this section, relaxing the termination criterion is usually not worthwhile.

#### 9.3.4 The Interior-point Log

Below is a typical log output from the interior-point optimizer:

Optimizer	- threads	:	20			
Optimizer	- solved problem	:	the primal			
Optimizer	- Constraints	:	1			
Optimizer	- Cones	:	2			
Optimizer	- Scalar variables	:	6	conic	:	6
Optimizer	- Semi-definite variables	3:	0	scalarized	:	0
Factor	- setup time	:	0.00	dense det. time	:	0.00
Factor	- ML order time	:	0.00	GP order time	:	0.00

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					,	,
Factor	- nonzeros	s before f	actor : 1	aft	er factor	: 1
Factor	- dense di	im.	: 0	flo	ps	: 1.
⊶70e+01						
ITE PFEAS	DFEAS	GFEAS	PRSTATUS	POBJ	DOBJ	MU 🔟
→ TIME						_
0 1.0e+00	2.9e-01	3.4e+00	0.00e+00	2.414213562e+00	0.00000000e+00	1.0e+00
→ 0.01						_
1 2.7e-01	7.9e-02	2.2e+00	8.83e-01	6.969257574e-01	-9.685901771e-03	2.7e-01
→ 0.01						
2 6.5e-02	1.9e-02	1.2e+00	1.16e+00	7.606090061e-01	6.046141322e-01	6.5e-02
→ 0.01						
3 1.7e-03	5.0e-04	2.2e-01	1.12e+00	7.084385672e-01	7.045122560e-01	1.7e-03
→ 0.01	0.00 01		11120 00			27.0 00
4 1.4e-08	4 2e-09	4.9e-08	1.00e+00	7.071067941e-01	7.071067599e-01	1.4e-08
→ 0.01	1.20-00	1.50-00	1.000.00	1.0110010410-01	7.0710070000-01	1.10-00
→ 0.01						

The first line displays the number of threads used by the optimizer and the second line indicates if the optimizer chose to solve the primal or dual problem (see <code>MSK\_IPAR\_INTPNT\_SOLVE\_FORM</code>). The next lines display the problem dimensions as seen by the optimizer, and the <code>Factor...</code> lines show various statistics. This is followed by the iteration log.

Using the same notation as in Sec. 9.3.1 the columns of the iteration log have the following meaning:

- ITE: Iteration index k.
- PFEAS:  $||Ax^k b\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- DFEAS:  $\|A^Ty^k + s^k c\tau^k\|_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- GFEAS:  $|-c^Tx^k+b^Ty^k-\kappa^k|$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- PRSTATUS: This number converges to 1 if the problem has an optimal solution whereas it converges to -1 if that is not the case.
- POBJ:  $c^T x^k / \tau^k$ . An estimate for the primal objective value.
- DOBJ:  $b^T y^k / \tau^k$ . An estimate for the dual objective value.
- MU:  $\frac{(x^k)^T s^k + \tau^k \kappa^k}{n+1}$ . The numbers in this column should always converge to zero.
- TIME: Time spent since the optimization started (in seconds).

# 9.4 The Optimizer for Mixed-Integer Problems

Solving optimization problems where one or more of the variables are constrained to be integer valued is called Mixed-Integer Optimization (MIO). For an introduction to model building with integer variables, the reader is recommended to consult the **MOSEK** Modeling Cookbook, and for further reading we highlight textbooks such as [Wol98] or [CCornuejolsZ14].

MOSEK can perform mixed-integer

- linear (MILO),
- quadratic (MIQO) and quadratically constrained (MIQCQO), and
- conic (MICO)

optimization, except for mixed-integer semidefinite problems.

By default the mixed-integer optimizer is run-to-run deterministic. This means that if a problem is solved twice on the same computer with identical parameter settings and no time limit, then the obtained solutions will be identical. The mixed-integer optimizer is parallelized, i.e., it can exploit multiple cores during the optimization.

In practice, a predominant special case of integer variables are binary variables, taking values in  $\{0,1\}$ . Mixed- or pure binary problems are important subclasses of mixed-integer optimization where all integer variables are of this type. In the general setting however, an integer variable may have arbitrary lower and upper bounds.

## 9.4.1 Branch-and-Bound

In order to succeed in solving mixed-integer problems, it can be useful to have a basic understanding of the underlying solution algorithms. The most important concept in this regard is arguably the so-called Branch-and-Bound algorithm, employed also by **MOSEK**. In order to comprehend Branch-and-Bound, the concept of a *relaxation* is important.

Consider for example a mixed-integer linear optimization problem of minimization type

$$z^* = \text{minimize} \quad c^T x$$
subject to  $Ax = b$ 

$$x \ge 0$$

$$x_j \in \mathbb{Z}, \qquad \forall j \in \mathcal{J}.$$

$$(9.12)$$

It has the continuous relaxation

$$\underline{z} = \text{minimize} \quad c^T x$$
subject to  $Ax = b$ 
 $x > 0$ , (9.13)

obtained simply by ignoring the integrality restrictions. The first step in Branch-and-Bound is to solve this so-called root relaxation, which is a continuous optimization problem. Since (9.13) is less constrained than (9.12), one certainly gets

$$\underline{z} \leq z^*$$
,

and z is therefore called the *objective bound*: it bounds the optimal objective value from below.

After the solution of the root relaxation, in the most likely outcome there will be one or more integer constrained variables with fractional values, i.e., violating the integrality constraints. Branch-and-Bound now takes such a variable,  $x_j = f_j \in \mathbb{R} \setminus \mathbb{Z}$  with  $j \in \mathcal{J}$ , say, and creates two branches leading to relaxations with the additional constraint  $x_j \leq \lfloor f_j \rfloor$  or  $x_j \geq \lceil f_j \rceil$ , respectively. The intuitive idea here is to push the variable away from the fractional value, closer towards integrality. If the variable was binary, say, branching would lead to fixing its value to 0 in one branch, and to 1 in the other.

The Branch-and-Bound process continues in this way and successively solves relaxations and creates branches to refined relaxations. Whenever a relaxation solution  $\hat{x}$  does not violate any integrality constraints, it is feasible to (9.12) and is called an *integer feasible solution*. Clearly, its solution value  $\bar{z} := c^T \hat{x}$  is an upper bound on the optimal objective value,

$$z^* < \bar{z}$$

Since refining a relaxation by adding constraints to it can only increase its solution value, the objective bound  $\underline{z}$ , now defined as the minimum over all solution values of so far solved relaxations, can only increase during the algorithm. If as upper bound  $\bar{z}$  one records the solution value of the best integer feasible solution encountered so far, the so-called *incumbent solution*,  $\bar{z}$  can only decrease during the algorithm. Since at any time we also have

$$\underline{z} \le z^* \le \bar{z}$$
,

objective bound and incumbent solution value are encapsulating the optimal objective value, eventually converging to it.

The Branch-and-Bound scheme can be depicted by means of a tree, where branches and relaxations correspond to edges and nodes. Figure Fig. 9.1 shows an example of such a tree. The strength of Branch-and-Bound is its ability to prune nodes in this tree, meaning that no new child nodes will be created. Pruning can occur in several cases:

- A relaxation leads to an integer feasible solution  $\hat{x}$ . In this case we may update the incumbent and its solution value  $\bar{z}$ , but no new branches need to be created.
- A relaxation is infeasible. The subtree rooted at this node cannot contain any feasible relaxation, so it can be discarded.
- A relaxation has a solution value that exceeds  $\bar{z}$ . The subtree rooted at this node cannot contain any integer feasible solution with a solution value better than the incumbent we already have, so it can be discarded.

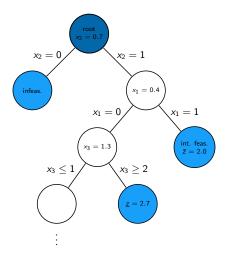


Fig. 9.1: An examplary Branch-and-Bound tree. Pruned nodes are shown in light blue.

Having objective bound and incumbent solution value is a quite fundamental property of Branch-and-Bound, and helps to asses solution quality and control termination of the algorithm, as we detail in the next section. Note that the above explanation is coined for minimization problems, but the Branch-and-bound scheme has a straightforward extension to maximization problems.

#### 9.4.2 Solution quality and termination criteria

The issue of terminating the mixed-integer optimizer is rather delicate. Recalling the Branch-and-Bound scheme from the previous section, one may see that mixed-integer optimization is generally much harder than continuous optimization; in fact, solving continuous sub-problems is just one component of a mixed-integer optimizer. Despite the ability to prune nodes in the tree, the computational effort required to solve mixed-integer problems grows exponentially with the size of the problem in a worst-case scenario (solving mixed-integer problems is NP-hard). For instance, a problem with n binary variables, may require the solution of  $2^n$  relaxations. The value of  $2^n$  is huge even for moderate values of n. In practice it is often advisable to accept near-optimal or appproximate solutions in order to counteract this complexity burden. The user has numerous possibilities of influencing optimizer termination with various parameters, in particular related to solution quality, and the most important ones are highlighted here.

#### Solution quality in terms of optimality

In order to assess the quality of any incumbent solution in terms of its objective value, one may check the *optimality gap*, defined as

$$\epsilon = |(\text{incumbent solution value}) - (\text{objective bound})| = |\bar{z} - \underline{z}|.$$

It measures how much the objectives of the incumbent and the optimal solution can deviate in the worst case. Often it is more meaningful to look at the relative optimality gap

$$\epsilon_{\rm rel} = \frac{|\bar{z} - \underline{z}|}{\max(\delta_1, |\bar{z}|)}.$$

This is essentially the above absolute optimality gap normalized against the magnitude of the incumbent solution value; the purpose of the (small) constant  $\delta_1$  is to avoid overweighing incumbent solution values that are very close to zero. The relative optimality gap can thus be interpreted as answering the question: "Within what fraction of the optimal solution is the incumbent solution in the worst case?"

Absolute and relative optimality gaps provide useful means to define termination criteria for the mixed-integer optimizer in MOSEK. The idea is to terminate the optimization process as soon as the quality of the incumbent solution, measured in absolute or relative gap, is good enough. In fact, whenever an incumbent solution is located, the criterion

$$\bar{z} - z \leq \max(\delta_2, \delta_3 \max(\delta_1, |\bar{z}|))$$

is checked. If satisfied, i.e., if either absolute or relative optimality gap are below the thresholds  $\delta_2$  or  $\delta_3$ , respectively, the optimizer terminates and reports the incumbent as an optimal solution. The optimality gaps can always be retrieved through the information items  $MSK\_DINF\_MIO\_OBJ\_ABS\_GAP$  and  $MSK\_DINF\_MIO\_OBJ\_REL\_GAP$ .

The tolerances discussed above can be adjusted using suitable parameters, see Table 9.3. By default, the optimality parameters  $\delta_2$  and  $\delta_3$  are quite small, i.e., restrictive. These default values for the absolute and relative gap amount to solving any instance to (almost) optimality: the incumbent is required to be within at most a tiny percentage of the optimal solution. As anticipated, this is not tractable in most practical situations, and one should resort to finding near-optimal solutions quickly rather than insisting on finding the optimal one. It may happen, for example, that an optimal or close-to-optimal solution is found very early by the optimizer, but it does not terminate because the objective bound  $\underline{z}$  is of poor quality. Instead, the vast majority of computational time is spent on trying to improve  $\underline{z}$ : a typical situation that practioneers would want to avoid. The concept of optimality gaps is fundamental for controlling solution quality when resorting to near-optimal solutions.

#### MIO performance tweaks: termination criteria

One of the first things to do in order to cut down excessive solution time is to increase the relative gap tolerance MSK\_DPAR\_MIO\_TOL\_REL\_GAP to some non-default value, so as to not insist on finding optimal solutions. Typical values could be 0.01,0.05 or 0.1, guaranteeing that the delivered solutions lie within 1%,5% or 10% of the optimum. Increasing the tolerance will lead to less computational time spent by the optimizer.

#### Solution quality in terms of feasibility

For an optimizer relying on floating-point arithmetic like the mixed-integer optimizer in **MOSEK**, it may be hard to achieve exact integrality of the solution values of integer variables in most cases, and it makes sense to numerically relax this constraint. Any candidate solution  $\hat{x}$  is accepted as integer feasible if the criterion

$$\min(\hat{x_j} - \lfloor \hat{x_j} \rfloor, \lceil \hat{x_j} \rceil - \hat{x_j}) \le \delta_4 \quad \forall j \in \mathcal{J}$$

is satisfied, meaning that  $\hat{x_j}$  is at most  $\delta_4$  away from the nearest integer. As above,  $\delta_4$  can be adjusted using a parameter, see Table 9.3, and impacts the quality of the acieved solution in terms of integer feasibility. By influencing what solution may be accepted as imcumbent, it can also have an impact on the termination of the optimizer.

#### MIO performance tweaks: feasibility criteria

Whether increasing the integer feasibility tolerance  $MSK\_DPAR\_MIO\_TOL\_ABS\_RELAX\_INT$  leads to less solution time is highly problem dependent. Intuitively, the optimizer is more flexible in finding new incumbent soutions so as to improve  $\bar{z}$ . But this effect has do be examined with care on indivuidual instances: it may worsen solution quality with no effect at all on the solution time. It may in some cases even lead to contrary effects on the solution time.

Table 9.3: Tolerances for the mixed-integer optimizer.

Tolerance	Parameter name	Default value
$\delta_1$	MSK_DPAR_MIO_REL_GAP_CONST	1.0e-10
$\delta_2$	MSK_DPAR_MIO_TOL_ABS_GAP	0.0
$\delta_3$	MSK_DPAR_MIO_TOL_REL_GAP	1.0e-4
$\delta_4$	MSK_DPAR_MIO_TOL_ABS_RELAX_INT	1.0e-5

### Further controlling optimizer termination

There are more ways to limit the computational effort employed by the mixed-integer optimizer by simply limiting the number of explored branches, solved relaxations or updates of the incumbent solution. When any of the imposed limits is hit, the optimizer terminates and the incumbent solution may be retrieved. See Table 9.4 for a list of corresponding parameters. In contrast to the parameters discussed in Sec. 9.4.2, interfering with these does not maintain any guarantees in terms of solution quality.

Table 9.4: Other parameters affecting the integer optimizer termination criterion.

Parameter name	Explanation
MSK_IPAR_MIO_MAX_NUM_BRANCHES	Maximum number of branches allowed.
MSK_IPAR_MIO_MAX_NUM_RELAXS	Maximum number of relaxations allowed.
MSK_IPAR_MIO_MAX_NUM_SOLUTIONS	Maximum number of feasible integer solutions allowed.

# 9.4.3 Additional components of the mixed-integer Optimizer

The Branch-and-Bound scheme from Sec. 9.4.1 is only the basic skeleton of the mixed-integer optimizer in **MOSEK**, and several components are built on top of that in order to enhance its functionality and increase its speed. A mixed-integer optimizer is sometimes referred to as a "giant bag of tricks", and it would be impossible to describe all of these tricks here. Yet, some of the additional components are worth mentioning to the user. They can be influenced by various user parameters, and although the default values of these parameters are optimized to work well on average mixed-integer problems, it may pay off to adjust them for an individual problem, or a specific problem class.

#### **Presolve**

Similar to the case of continuous problems, see Sec. 9.1, the mixed-integer optimizer applies various presolve reductions before the actual solution process is initiated. Just as in the continuous case, the use of presolve can be controlled with the parameter  $MSK\_IPAR\_PRESOLVE\_USE$ .

#### **Primal Heuristics**

Solving relaxations in the Branch-and-bound tree to an integer feasible solution  $\hat{x}$  is not the only way to find new incumbent solutions. There is a variety of procedures that, given a mixed-integer problem in a generic form like (9.12), attempt to produce integer feasible solutions in an ad-hoc way. These procedures are called Primal Heuristics, and several of them are implemented in **MOSEK**. For example, whenever a relaxation leads to a fractional solution, one may round the solution values of the integer variables, in various ways, and hope that the outcome is still feasible to the remaining constraints. Primal heuristics are mostly employed while processing the root node, but play a role throughout the whole solution process. The goal of a primal heuristic is to improve the incumbent solution and thus the bound  $\bar{z}$ , and this can of course affect the quality of the solution that is returned after termination of the optimizer. The user parameters affecting primal heuristics are listed in Table 9.5.

# MIO performance tweaks: primal heuristics

- If the mixed-integer optimizer struggles to improve the incumbent solution  $\bar{z}$ , see Sec. 9.4.4, it can be helpful to intensify the use of primal heuristics.
  - Set parameters related to primal heuristics to more aggressive values than the default ones, so
    that more effort is spent in this component. A List of the respective parameters can be found in

Table 9.5. In particular, if the optimizer has difficulties finding any integer feasible solution at all, indicated by NA in the column BEST\_INT\_OBJ in the mixed-integer log, one may try to activate a construction heuristic like the Feasibility Pump with MSK\_IPAR\_MIO\_FEASPUMP\_LEVEL.

- Specify a good initial solution: In many cases a good feasible solution is either known or easily computed using problem-specific knowledge that the optimizer does not have. If so, it is usually worthwhile to use this as a starting point for the mixed-integer optimizer.
- For feasibility problems, i.e., problems having a constant objective, the goal is to find a single integer feasible solution, and this can be hard by itself on some instances. Try setting the objective to something meaningful anyway, even if the underlying application does not require this. After all, the feasible set is not changed, but the optimizer might benefit from being able to pursue a concrete goal.
- In rare cases it may also happen that the optimizer spends an excessive amount of time on primal heuristics without drawing any benefit from it, and one may try to limit their use with the respective parameters.

Parameter name Explanation

MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL Primal heuristics aggressivity level.

MSK\_IPAR\_MIO\_RINS\_MAX\_NODES Maximum number of nodes allowed in the RINS heuristic.

MSK\_IPAR\_MIO\_FEASPUMP\_LEVEL Way of using the Feasibility Pump heuristic.

Table 9.5: Parameters affecting primal heuristics

#### **Cutting Planes**

Cutting planes (cuts) are simply constraints that are valid for a mixed-integer problem, for example in the form (9.12), meaning they do not remove any integer feasible solutions from the feasible set. Therefore they are also called valid inequalities. They do not have to be valid for the relaxation (9.13) though, and of interest and potentially useful are those cuts that do remove solutions from the feasible set of the relaxation. The latter is a superset of the feasible region of the mixed-integer problem, and the rationale behind cuts is thus to bring the integer problem and its relaxation closer together in terms of their feasible sets.

As an example, take the constraints

$$2x_1 + 3x_2 + x_3 \le 4$$
,  $x_1, x_2 \in \{0, 1\}$ ,  $x_3 \ge 0$ . (9.14)

One may realize that there cannot be a feasible solution in which both binary variables take on a value of 1. So certainly

$$x_1 + x_2 \le 1 \tag{9.15}$$

is a valid inequality. In fact, there is no integer solution satisfying (9.14), but violating (9.15). The latter does cut off a portion of the feasible region of the continuous relaxation of (9.14) though, obtained by replacing  $x_1, x_2 \in \{0, 1\}$  with  $x_1, x_2 \in [0, 1]$ . For example, the fractional point  $(x_1, x_2, x_3) = (0.5, 1, 0)$  is feasible to the relaxation, but violates the cut (9.15).

There are many classes of general-purpose cuttting planes that may be generated for a mixed-integer problem in a generic form like (9.12), and **MOSEK**'s mixed-integer optimizer supports several of them. For instance, the above is an example of a so-called clique cut. The most effort on generating cutting planes is spent after the solution of the root relaxation, but cuts can also be generated later on in the Branch-and-Bound tree. Cuts aim at improving the objective bound  $\underline{z}$  and can thus have significant impact on the solution time. The user parameters affecting cut generation can be seen in Table 9.6.

# MIO performance tweaks: cutting planes

- If the mixed-integer optimizer struggles to improve the objective bound  $\underline{z}$ , see Sec. 9.4.4, it can be helpful to intensify the use of cutting planes.
  - Some types of cutting planes are not activated by default, but doing so may help to improve the objective bound.

- The parameters  $MSK\_DPAR\_MIO\_TOL\_REL\_DUAL\_BOUND\_IMPROVEMENT$  and  $MSK\_IPAR\_MIO\_CUT\_SELECTION\_LEVEL$  determine how aggressively cuts will be generated and selected.
- If some valid inequalities can be deduced from problem-specific knowledge that the optimizer does not have, it may be helpful to add these to the problem formulation as constraints.
   This has to be done with care, since there is a tradeoff between the benefit obtained from an improved objective boud, and the amount of additional constraints that make the relaxations larger.
- In rare cases it may also be observed that the optimizer spends an excessive amount of time on cutting planes, see Sec. 9.4.4, and one may limit their use with <code>MSK\_IPAR\_MIO\_MAX\_NUM\_ROOT\_CUT\_ROUNDS</code>, or by disabling a certain type of cutting planes.

Table 9.6: Parameters affecting cutting planes

Parameter name	Explanation
MSK_IPAR_MIO_CUT_CLIQUE	Should clique cuts be enabled?
MSK_IPAR_MIO_CUT_CMIR	Should mixed-integer rounding cuts be enabled?
MSK_IPAR_MIO_CUT_GMI	Should GMI cuts be enabled?
MSK_IPAR_MIO_CUT_IMPLIED_BOUND	Should implied bound cuts be enabled?
MSK_IPAR_MIO_CUT_KNAPSACK_COVER	Should knapsack cover cuts be enabled?
MSK_IPAR_MIO_CUT_LIPRO	Should lift-and-project cuts be enabled?
MSK_IPAR_MIO_CUT_SELECTION_LEVEL	Cut selection aggressivity level.
MSK_IPAR_MIO_MAX_NUM_ROOT_CUT_ROUN	Maximum number of root cut rounds.
MSK_DPAR_MIO_TOL_REL_DUAL_BOUND_IM	PMWHIMIN required objective bound improvement during
	root cut generation.

# 9.4.4 The Mixed-Integer Log

Below is a typical log output from the mixed-integer optimizer:

Pres	solved probl	.em: 1176 va:	riables,	1344 constraints, 496	88 non-zeros		
Pres	solved probl	.em: 328 gen	eral inte	ger, 392 binary, 456	continuous		
Cli	que table si	.ze: 55					
BRAI	NCHES RELAXS	ACT_NDS	DEPTH	BEST_INT_OBJ	BEST_RELAX_OBJ	REL_GA	P(
<b>→</b> %2	) TIME						
0	0	1	0	8.3888091139e+07	NA	NA	ш
$\hookrightarrow$	0.0						
0	1	1	0	8.3888091139e+07	2.5492512136e+07	69.61	ш
$\hookrightarrow$	0.1						
0	1	1	0	3.1273162420e+07	2.5492512136e+07	18.48	ш
$\hookrightarrow$	0.1						
0	1	1	0	2.6047699632e+07	2.5492512136e+07	2.13	ш
<b>↔</b>	0.2						
	generation						
0	1	1	0	2.6047699632e+07	2.5492512136e+07	2.13	Ш
<b>↔</b>	0.2		^	0.0047000000.07	0.550000004707	4 70	
0	2	1	0	2.6047699632e+07	2.5589986247e+07	1.76	Ш
<b>⇔</b>	0.2	±	T:	٥٢			
	•	terminated.			0. 5000744004 .07	4 00	
0	4	1	0	2.5990071367e+07	2.5662741991e+07	1.26	Ш
0	0.3 8	1	0	0 5071000767	0 56607410010107	1.19	
"	0.5	1	U	2.5971002767e+07	2.5662741991e+07	1.19	Ш
0	11	1	0	2.5925040617e+07	2.5662741991e+07	1.01	
	0.5	1	U	2.09200400176+07	2.00021419910+01	1.01	П
$\hookrightarrow$	0.5						Ų

(continues on next page)

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					(continued from	previous p	age)
0	12	1	0	2.5915504014e+07	2.5662741991e+07	0.98	П
$\hookrightarrow$	0.5						
2	23	1	0	2.5915504014e+07	2.5662741991e+07	0.98	Ш
$\hookrightarrow$	0.6						
14	35	1	0	2.5915504014e+07	2.5662741991e+07	0.98	ш
$\hookrightarrow$	0.6						
[	]						
Object	cive of b	est integer	solutio	n : 2.578282162804e+07			
Best o	bjective	bound		: 2.569877601306e+07			
	•		ive	: Not employed			
		_		: Not employed			
	•	generated		- ·			
		•					
		IR cuts		: 137			
Numb	per of cl	ique cuts		: 3			
	of bran	-		: 29252			
		xations sol	ved				
		rior point					
		lex iterati					
	-	solving the					
		imizing the					
		_		d. Time: 6.46			
IIIVea	THERET	obermirger c	CIMITHALE	u. 11me. 0.40			

The first lines contain a summary of the problem after mixed-integer presolve has been applied. This is followed by the iteration log, reflecting the progress made during the Branch-and-bound process. The columns have the following meanings:

- BRANCHES: Number of branches / nodes generated.
- RELAXS: Number of relaxations solved.
- ACT\_NDS: Number of active / non-processed nodes.
- DEPTH: Depth of the last solved node.
- BEST\_INT\_OBJ: The incumbent solution / best integer objective value,  $\bar{z}$ .
- BEST\_RELAX\_OBJ: The objective bound,  $\underline{z}$ .
- REL\_GAP(%): Relative optimality gap,  $100\% \cdot \epsilon_{\rm rel}$
- TIME: Time (in seconds) from the start of optimization.

The beginning and the end of the root cut generation is highlighted as well, and the number of log lines in between reflects to the computational effort spent here.

Finally there is a summary of the optimization process, containing also information on the type of generated cuts and the total number of iterations needed to solve all occurring continuous relaxations.

When the solution time for a mixed-integer problem has to be cut down, it can sometimes be useful to examine the log in order to understand where time is spent and what might be improved. In particular, it might happen that the values in either of the column BEST\_INT\_OBJ or BEST\_RELAX\_OBJ stall over a long period of log lines, an indication that the optimizer has a hard time improving either the incumbent solution, i.e.,  $\bar{z}$ , or the objective bound  $\underline{z}$ , see also Sec. 9.4.3 and Sec. 9.4.3.

# 9.4.5 Mixed-Integer Nonlinear Optimization

Due to the involved non-linearities, MI(QC)QO or MICO problems are on average harder than MILO problems of comparable size. Yet, the Branch-and-Bound scheme can be applied to these problem classes in a straightforward manner. The relaxations have to be solved as conic problems with the interior point algorithm in that case, see Sec. 9.3, opposed to MILO where it is often beneficial to solve relaxations with the dual simplex method, see Sec. 9.2.3. There is another solution approach for these types of problems implemented in **MOSEK**, namely the Outer-Approximation algorithm, making use of dynamically refined linear approximations of the non-linearities.

### MICO performance tweaks: choice of algorithm

Whether conic Branch-and-Bound or Outer-Approximation is applied to a mixed-integer conic problem can be set with  $MSK\_IPAR\_MIO\_CONIC\_OUTER\_APPROXIMATION$ . The best value for this option is highly problem dependent.

#### MI(QC)QO

**MOSEK** is specialized in solving linear and conic optimization problems, both with or without mixed-integer variables. Just like for continuous problems, mixed-integer quadratic problems are converted internally to conic form, see Sec. 8.4.1

Contrary to the continuous case,  $\mathbf{MOSEK}$  can solve certain mixed-integer quadratic problems where one or more of the involved matrices are not positive semidefinite, so-called non-convex  $\mathbf{MI}(\mathbf{QC})\mathbf{QO}$  problems. These are automatically reformulated to an equivalent convex  $\mathbf{MI}(\mathbf{QC})\mathbf{QO}$  problem, provided that such a reformulation is possible on the given instance (otherwiese  $\mathbf{MOSEK}$  will reject the problem and issue an error message). The concept of reformulations can also affect the solution times of  $\mathbf{MI}(\mathbf{QC})\mathbf{QO}$  problems.

## MI(QC)QO performance tweaks: applying a reformulation method

There are several reformulation methods for MI(QC)QO problems, available through the parameter  $MSK\_IPAR\_MIO\_QCQO\_REFORMULATION\_METHOD$ . The chosen method can have significant impact on the mixed-integer optimizer's speed on such problems, both convex and non-convex. The best value for this option is highly problem dependent.

# 9.4.6 Disjunctive constraints

Problems with disjunctive constraints (DJC) are typically reformulated to mixed-integer problems, and even if this is not the case they are solved with an algorithm that is based on the mixed-integer optimizer. In **MOSEK**, these problems thus fall into the realm of MIO. In particular, **MOSEK** automatically attempts to replace any DJC by so called big-M constraints, potentially after transforming it to several, less complicated DJCs. As an example, take the DJC

$$[z=0] \vee [z=1, x_1 + x_2 \ge 1000],$$

where  $z \in \{0, 1\}$  and  $x_1, x_2 \in [0, 750]$ . This is an example of a DJC formulation of a so-called indicator constraint. A big-M reformulation is given by

$$x_1 + x_2 \ge 1000 - M \cdot (1 - z),$$

where M>0 is a large constant. The practical difficulty of these constructs is that M should always be sufficiently large, but ideally not larger. Too large values for M can be harmful for the mixed-integer optimizer. During presolve, and taking into account the bounds of the involved variables,  $\mathbf{MOSEK}$  automatically reformulates DJCs to big-M constraints if the required M values do not exceed the parameter  $MSK\_DPAR\_MIO\_DJC\_MAX\_BIGM$ . From a performance point-of-view, all DJCs would ideally be linearized to big-Ms after presolve without changing this parameter's default value of 1.0e6. Whether or not this is the case can be seen by retrieving the information item  $MSK\_IINF\_MIO\_PRESOLVED\_NUMDJC$ , or by a line in the mixed-integer optimizer's log as in the example below. Both state the number of remaining disjunctions after presolve.

	Presolved problem: 305 variables, 204 constraints, 708 non-zeros  Presolved problem: 0 general integer, 100 binary, 205 continuous							
	Presolved problem: 0 general integer, 100 binary, 205 continuous  Presolved problem: 100 disjunctions							
	-		Junctions					
_	le table size		DEDTH	DECT INT OD I	DECT DELAY OD I	DEL CAD(		
	HES RELAXS	ACT_NDS	DEPIH	BEST_INT_OBJ	BEST_RELAX_OBJ	REL_GAP(		
<b>→</b> %)	TIME	4	^	27.4	0.0000000000000000000000000000000000000	27.4		
0	1	1	0	NA	0.000000000e+00	NA 🔟		
$\hookrightarrow$	0.0							
0	1	1	0	5.0574653969e+05	0.000000000e+00	100.00 🔟		
$\hookrightarrow$	0.0							
[	]							

## DJC performance tweaks: managing variable bounds

- Always specify the tightest known bounds on the variables of any problem with DJCs, even if they seem trivial from the user-perspective. The mixed-integer optimizer can only benefit from these when reformulating DJCs and thus gain performance; even if bounds don't help with reformulations, it is very unlikely that they hurt the optimizer.
- Increasing MSK\_DPAR\_MIO\_DJC\_MAX\_BIGM can lead to more DJC reformulations and thus increase optimizer speed, but it may in turn hurt numerical solution quality and has to be examined with care. The other way round, on numerically challenging instances with DJCs, decreasing MSK\_DPAR\_MIO\_DJC\_MAX\_BIGM may lead to numerically more robust solutions.

# 9.4.7 Randomization

A mixed-integer optimizer is usually prone to performance variability, meaning that a small change in either

- problem data, or
- computer hardware, or
- algorithmic parameters

can lead to significant changes in solution time, due to different solution paths in the Branch-and-Bound tree. In extreme cases the exact same problem can vary from being solvable in less than a second to seemingly unsolvable in any reasonable amount of time on a different computer.

One practical implication of this is that one should ideally verify whether a seemingly beneficial set of parameters, established experimentally on a single problem, is still beneficial (on average) on a larger set of problems from the same problem class. This protects against making parameter changes that had positive effects only due to random effects on that single problem.

In the absence of a large set of test problems, one may also change the random seed of the optimizer to a series of different values in order to hedge against drawing such wrong conclusions regarding parameters. The random seed, accessible through  $MSK\_IPAR\_MIO\_SEED$ , impacts for example random tie-breaking in many of the mixed-integer optimizer's components. Changing the random seed can be combined with a permutation of the problem data to further incite randomness, accessible through the parameter  $MSK\_IPAR\_MIO\_DATA\_PERMUTATION\_METHOD$ .

# 9.4.8 Further performance tweaks

In addition to what was mentioned previously, there may be other ways to speed up the solution of a given mixed-integer problem. For example, there are further user parameters affecting some algorithmic settings in the mixed-integer optimizer. As mentioned above, default parameter values are optimized to work well on average, but on individual problems they may be adjusted.

### MIO performance tweaks: miscellaneous

- When relaxations in the Branch-and-Bound tree are linear optimization problems (e.g., in MILO or when solving MICO problems with the Outer-Approximation method), it is usually best to employ the dual simplex method for their solution. In rare cases the primal simplex method may actually be the better choice, and this can be set with the parameter MSK\_IPAR\_MIO\_NODE\_OPTIMIZER.
- Some problems are numerically more challenging than others, for example if the ratio between the smallest and the largest involved coefficients is large, say  $\geq 1e9$ . An indication of numerical issues are, for example, large violations in the final solution, observable in the solution summery of the log output, see Sec. 7.1.3. Similarly, a problem that is known to be feasible by the user may be declared infeasible by the optimizer. In such cases it is usually best to try to rescale the model. Otherwise, the mixed-integer optimizer can be instructed to be more cautios regarding numerics with the parameter  $MSK\_IPAR\_MIO\_NUMERICAL\_EMPHASIS\_LEVEL$ . This may in turn be at the cost of solution speed though.
- Improve the formulation: A MIO problem may be impossible to solve in one form and quite easy in another form. However, it is beyond the scope of this manual to discuss good formulations for mixed-integer problems. For discussions on this topic see for example [Wol98].

# Chapter 10

# Additional features

In this section we describe additional features and tools which enable more detailed analysis of optimization problems with  $\mathbf{MOSEK}$ .

# 10.1 Problem Analyzer

The problem analyzer prints a survey of the structure of the problem, with information about linear constraints and objective, quadratic constraints, conic constraints and variables.

In the initial stages of model formulation the problem analyzer may be used as a quick way of verifying that the model has been built or imported correctly. In later stages it can help revealing special structures within the model that may be used to tune the optimizer's performance or to identify the causes of numerical difficulties.

The problem analyzer is run from the command line using the <code>-anapro</code> argument and produces output similar to the following (this is the problem analyzer's survey of the <code>aflow30a</code> problem from the MIPLIB 2003 collection.)

	Analyzir	ng the problem			
*** Structural	report Dimensions				
	Constraints 479	Variables 842	Matrix var. O	Cones O	
	Constraint ar	nd bound types			
	Free	Lower	Upper	Ranged	Fixed
Constraints:	0	0	421	0	58
Variables:	0	0	0	842	0
	Integer const	traint types			
	Binary	General			
	421	0			
*** Data repor	rt				
	Nonzeros	Min	Max		
lcjl:	421	1.1e+01	5.0e+02		
Aij :		1.0e+00	1.0e+02		
	# finite	Min	Max		
blci :	58	1.0e+00	1.0e+01		
buci :		0.0e+00	1.0e+01		
blxj :		0.0e+00	0.0e+00		
buxj :		1.0e+00	1.0e+02		
					(continues on next page)

(continues on next page)

#### \*\*\* Done analyzing the problem

The survey is divided into a structural and numerical report. The content should be self-explanatory.

# 10.2 Automatic Repair of Infeasible Problems

**MOSEK** provides an automatic repair tool for infeasible linear problems which we cover in this section. Note that most infeasible models are so due to bugs which can (and should) be more reliably fixed manually, using the knowledge of the model structure. We discuss this approach in Sec. 7.3.

# 10.2.1 Automatic repair

The main idea can be described as follows. Consider the linear optimization problem with m constraints and n variables

which is assumed to be infeasible.

One way of making the problem feasible is to reduce the lower bounds and increase the upper bounds. If the change is sufficiently large the problem becomes feasible. Now an obvious idea is to compute the optimal relaxation by solving an optimization problem. The problem

minimize 
$$p(v_l^c, v_u^c, v_l^x, v_u^x)$$
  
subject to  $l^c - v_l^c \le Ax \le u^c + v_u^c,$   
 $l^x - v_l^x \le x \le u^x + v_u^x,$   
 $v_l^c, v_u^c, v_l^x, v_u^x \ge 0$  (10.1)

does exactly that. The additional variables  $(v_l^c)_i$ ,  $(v_u^c)_i$ ,  $(v_l^x)_j$  and  $(v_u^c)_j$  are elasticity variables because they allow a constraint to be violated and hence add some elasticity to the problem. For instance, the elasticity variable  $(v_l^c)_i$  controls how much the lower bound  $(l^c)_i$  should be relaxed to make the problem feasible. Finally, the so-called penalty function

$$p(v_l^c, v_u^c, v_l^x, v_u^x)$$

is chosen so it penalizes changes to bounds. Given the weights

- $w_l^c \in \mathbb{R}^m$  (associated with  $l^c$ ),
- $w_u^c \in \mathbb{R}^m$  (associated with  $u^c$ ),
- $w_l^x \in \mathbb{R}^n$  (associated with  $l^x$ ),
- $w_u^x \in \mathbb{R}^n$  (associated with  $u^x$ ),

a natural choice is

$$p(v_l^c, v_u^c, v_l^x, v_u^x) = (w_l^c)^T v_l^c + (w_u^c)^T v_u^c + (w_l^x)^T v_l^x + (w_u^x)^T v_u^x.$$

Hence, the penalty function p() is a weighted sum of the elasticity variables and therefore the problem (10.1) keeps the amount of relaxation at a minimum. Please observe that

- the problem (10.1) is always feasible.
- a negative weight implies problem (10.1) is unbounded. For this reason if the value of a weight is negative **MOSEK** fixes the associated elasticity variable to zero. Clearly, if one or more of the weights are negative, it may imply that it is not possible to repair the problem.

A simple choice of weights is to set them all to 1, but of course that does not take into account that constraints may have different importance.

#### **Caveats**

Observe if the infeasible problem

minimize 
$$x + z$$
  
subject to  $x = -1$ ,  
 $x > 0$ 

is repaired then it will become unbounded. Hence, a repaired problem may not have an optimal solution. Another and more important caveat is that only a minimal repair is performed i.e. the repair that barely makes the problem feasible. Hence, the repaired problem is barely feasible and that sometimes makes the repaired problem hard to solve.

# Using the automatic repair tool

In this subsection we consider an infeasible linear optimization example:

The problem (10.2) is contained in a file:

Listing 10.1: Problem (10.2) in LP format.

```
minimize
obj: - 10 x1 - 9 x2
st
c1: + 7e-01 x1 + x2 <= 630
c2: + 5e-01 x1 + 8.333333333e-01 x2 <= 600
c3: + x1 + 6.6666667e-01 x2 <= 708
c4: + 1e-01 x1 + 2.5e-01 x2 <= 135
bounds
x2 >= 650
end
```

Given the assumption that all weights are 1 the command

```
mosek -primalrepair -d MSK_IPAR_LOG_FEAS_REPAIR 3 feasrepair.lp
```

will form the repaired problem and solve it. The parameter MSK\_IPAR\_LOG\_FEAS\_REPAIR controls the amount of log output from the repair. A value of 2 causes the optimal repair to printed out. The output from running the above command is:

```
MOSEK Version 9.0.0.25(ALPHA) (Build date: 2017-11-7 16:11:50)
Copyright (c) MOSEK ApS, Denmark. WWW: mosek.com
Platform: Linux/64-X86

Open file 'feasrepair.lp'
Reading started.
Reading terminated. Time: 0.00

Read summary
Type : LO (linear optimization problem)
Objective sense : min
Scalar variables : 2
Matrix variables : 0
```

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```
Constraints : 4
 Cones
                 : 0
 Time
                  : 0.0
Problem
 Name
 Objective sense
                      : min
 Type
                      : LO (linear optimization problem)
 Constraints
                      : 4
 Cones
                       : 0
 Cones
Scalar variables
                       : 2
 Matrix variables
                      : 0
 Integer variables
                      : 0
Primal feasibility repair started.
Optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Eliminator started.
Freed constraints in eliminator : 2
Eliminator terminated.
Eliminator - tries
                                  : 1
                                                                          : 0.00
                                                     time
Lin. dep. - tries
                                  : 1
                                                     time
                                                                           : 0.00
Lin. dep. - number
                                  : 0
Presolve terminated. Time: 0.00
Problem
 Name
 Objective sense
                      : min
                      : LO (linear optimization problem)
 Constraints
                      : 8
                       : 0
 Cones
 Scalar variables : 14
Matrix variables : 0
 Integer variables
Optimizer - threads
                                 : 20
Optimizer - solved problem
                                : the primal
Optimizer - Constraints
                                 : 2
Optimizer - Cones
                                 : 0
Optimizer - Scalar variables : 5
                                                    conic
                                                                          : 0
Optimizer - Semi-definite variables: 0
                                                    scalarized
                                                                          : 0
                                                  dense det. time
GP order time
Factor - setup time : 0.00
Factor - ML order time : 0.00
                                                                         : 0.00
                                                                         : 0.00
Factor - nonzeros before factor : 3
                                                   after factor
                                                                         : 3
Factor
         - dense dim.
                                  : 0
                                                     flops
                                                                          : 5.
⊶00e+01
ITE PFEAS DFEAS
                    GFEAS
                             PRSTATUS
                                      POBJ
                                                         DOBJ
                                                                          MU
→ TIME
0 2.7e+01 1.0e+00 4.0e+00 1.00e+00
                                       3.00000000e+00
                                                         0.00000000e+00
                                                                          1.0e+00
→ 0.00
1 2.5e+01 9.1e-01 1.4e+00 0.00e+00
                                      8.711262850e+00
                                                         1.115287830e+01
                                                                          2.4e+00<sub>L</sub>
→ 0.00
2 2.4e+00 8.8e-02 1.4e-01 -7.33e-01 4.062505701e+01
                                                         4.422203730e+01
                                                                          2.3e-01
→ 0.00
3 9.4e-02 3.4e-03 5.5e-03 1.33e+00 4.250700434e+01
                                                         4.258548510e+01
                                                                          9.1e-03
→ 0.00
                                                                 (continues on next page)
```

```
(continued from previous page)
                                      4.249996599e+01
  2.0e-05 7.2e-07 1.1e-06 1.02e+00
                                                         4.249998669e+01
                                                                           1.9e-06
→ 0.00
 2.0e-09 7.2e-11 1.1e-10 1.00e+00 4.250000000e+01 4.250000000e+01
                                                                           1.9e-10
Basis identification started.
Basis identification terminated. Time: 0.00
Optimizer terminated. Time: 0.01
Basic solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: 4.2500000000e+01 nrm: 6e+02
                                                Viol. con: 1e-13
                                                                    var: 0e+00
          obj: 4.2499999999e+01 nrm: 2e+00
                                               Viol. con: 0e+00
                                                                    var: 9e-11
 Dual.
Optimal objective value of the penalty problem: 4.250000000000e+01
Repairing bounds.
Increasing the upper bound 1.35e+02 on constraint 'c4' (3) with 2.25e+01.
Decreasing the lower bound 6.50e+02 on variable 'x2' (4) with 2.00e+01.
Primal feasibility repair terminated.
Optimizer started.
Optimizer terminated. Time: 0.00
Interior-point solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: -5.6700000000e+03 nrm: 6e+02
                                                Viol. con: 0e+00
                                                                    var: 0e+00
          obj: -5.6700000000e+03 nrm: 1e+01
                                                Viol. con: 0e+00
                                                                    var: 0e+00
 Dual.
Basic solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: -5.6700000000e+03 nrm: 6e+02
                                                Viol. con: 0e+00
                                                                    var: 0e+00
         obj: -5.6700000000e+03 nrm: 1e+01
                                                Viol. con: 0e+00
                                                                    var: 0e+00
Optimizer summary
 Optimizer
                                                  time: 0.00
   Interior-point
                           - iterations : 0
                                                  time: 0.00
     Basis identification -
                                                  time: 0.00
                                                  time: 0.00
       Primal
                          - iterations : 0
                                                  time: 0.00
       Dual
                          - iterations : 0
       Clean primal
                          - iterations : 0
                                                  time: 0.00
       Clean dual
                          - iterations : 0
                                                  time: 0.00
   Simplex
                                                  time: 0.00
     Primal simplex
                          - iterations : 0
                                                  time: 0.00
     Dual simplex
                          - iterations : 0
                                                  time: 0.00
   Mixed integer
                          - relaxations: 0
                                                   time: 0.00
```

In this case the optimal repair it is to increase the upper bound on constraint c4 by 22.5 and decrease the lower bound on variable x2 by 20.

# 10.3 Sensitivity Analysis

Given an optimization problem it is often useful to obtain information about how the optimal objective value changes when the problem parameters are perturbed. E.g, assume that a bound represents the capacity of a machine. Now, it may be possible to expand the capacity for a certain cost and hence it is worthwhile knowing what the value of additional capacity is. This is precisely the type of questions the sensitivity analysis deals with.

Analyzing how the optimal objective value changes when the problem data is changed is called sensitivity analysis.

#### References

The book [Chvatal83] discusses the classical sensitivity analysis in Chapter 10 whereas the book [RTV97] presents a modern introduction to sensitivity analysis. Finally, it is recommended to read the short paper [Wal00] to avoid some of the pitfalls associated with sensitivity analysis.

Warning: Currently, sensitivity analysis is only available for continuous linear optimization problems. Moreover, MOSEK can only deal with perturbations of bounds and objective function coefficients.

# 10.3.1 Sensitivity Analysis for Linear Problems

# The Optimal Objective Value Function

Assume that we are given the problem

$$z(l^{c}, u^{c}, l^{x}, u^{x}, c) = \underset{\text{subject to}}{\text{minimize}} c^{T}x$$

$$l^{c} \leq Ax \leq u^{c},$$

$$l^{x} \leq x \leq u^{x},$$

$$(10.3)$$

and we want to know how the optimal objective value changes as  $l_i^c$  is perturbed. To answer this question we define the perturbed problem for  $l_i^c$  as follows

$$\begin{array}{lcl} f_{l_i^c}(\beta) & = & \text{minimize} & & c^T x \\ & & \text{subject to} & l^c + \beta e_i & \leq & Ax & \leq u^c, \\ & & l^x & \leq & x \leq & u^x, \end{array}$$

where  $e_i$  is the *i*-th column of the identity matrix. The function

$$f_{l_{\cdot}^{c}}(\beta) \tag{10.4}$$

shows the optimal objective value as a function of  $\beta$ . Please note that a change in  $\beta$  corresponds to a perturbation in  $l_i^c$  and hence (10.4) shows the optimal objective value as a function of varying  $l_i^c$  with the other bounds fixed.

It is possible to prove that the function (10.4) is a piecewise linear and convex function, i.e. its graph may look like in Fig. 10.1 and Fig. 10.2.

Clearly, if the function  $f_{l_i^c}(\beta)$  does not change much when  $\beta$  is changed, then we can conclude that the optimal objective value is insensitive to changes in  $l_i^c$ . Therefore, we are interested in the rate of change in  $f_{l_i^c}(\beta)$  for small changes in  $\beta$ —specifically the gradient

$$f_{l_i^c}'(0),$$

which is called the *shadow price* related to  $l_i^c$ . The shadow price specifies how the objective value changes for small changes of  $\beta$  around zero. Moreover, we are interested in the *linearity interval* 

$$\beta \in [\beta_1, \beta_2]$$

for which

$$f'_{l_i^c}(\beta) = f'_{l_i^c}(0).$$

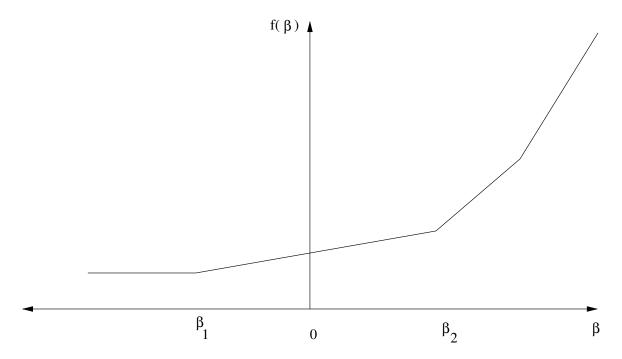


Fig. 10.1:  $\beta=0$  is in the interior of linearity interval.

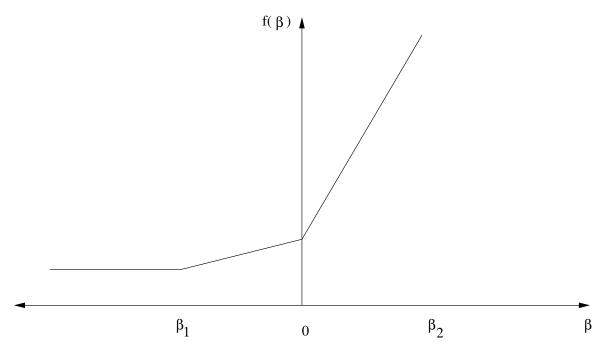


Fig. 10.2:  $\beta=0$  is a breakpoint.

Since  $f_{l_i^c}$  is not a smooth function  $f'_{l_i^c}$  may not be defined at 0, as illustrated in Fig. 10.2. In this case we can define a left and a right shadow price and a left and a right linearity interval.

The function  $f_{l_i^c}$  considered only changes in  $l_i^c$ . We can define similar functions for the remaining parameters of the z defined in (10.3) as well:

$$f_{l_i^c}(\beta) = z(l^c + \beta e_i, u^c, l^x, u^x, c), \quad i = 1, \dots, m,$$

$$f_{u_i^c}(\beta) = z(l^c, u^c + \beta e_i, l^x, u^x, c), \quad i = 1, \dots, m,$$

$$f_{l_j^x}(\beta) = z(l^c, u^c, l^x + \beta e_j, u^x, c), \quad j = 1, \dots, n,$$

$$f_{u_j^x}(\beta) = z(l^c, u^c, l^x, u^x + \beta e_j, c), \quad j = 1, \dots, n,$$

$$f_{c_j}(\beta) = z(l^c, u^c, l^x, u^x, c + \beta e_j), \quad j = 1, \dots, n.$$

Given these definitions it should be clear how linearity intervals and shadow prices are defined for the parameters  $u_i^c$  etc.

## **Equality Constraints**

In **MOSEK** a constraint can be specified as either an equality constraint or a ranged constraint. If some constraint  $e_i^c$  is an equality constraint, we define the optimal value function for this constraint as

$$f_{e_i^c}(\beta) = z(l^c + \beta e_i, u^c + \beta e_i, l^x, u^x, c)$$

Thus for an equality constraint the upper and the lower bounds (which are equal) are perturbed simultaneously. Therefore, **MOSEK** will handle sensitivity analysis differently for a ranged constraint with  $l_i^c = u_i^c$  and for an equality constraint.

### The Basis Type Sensitivity Analysis

The classical sensitivity analysis discussed in most textbooks about linear optimization, e.g. [Chvatal83], is based on an optimal basis. This method may produce misleading results [RTV97] but is computationally cheap. This is the type of sensitivity analysis implemented in **MOSEK**.

We will now briefly discuss the basis type sensitivity analysis. Given an optimal basic solution which provides a partition of variables into basic and non-basic variables, the basis type sensitivity analysis computes the linearity interval  $[\beta_1, \beta_2]$  so that the basis remains optimal for the perturbed problem. A shadow price associated with the linearity interval is also computed. However, it is well-known that an optimal basic solution may not be unique and therefore the result depends on the optimal basic solution employed in the sensitivity analysis. If the optimal objective value function has a breakpoint for  $\beta = 0$  then the basis type sensitivity method will only provide a subset of either the left or the right linearity interval.

In summary, the basis type sensitivity analysis is computationally cheap but does not provide complete information. Hence, the results of the basis type sensitivity analysis should be used with care.

#### **Example: Sensitivity Analysis**

As an example we will use the following transportation problem. Consider the problem of minimizing the transportation cost between a number of production plants and stores. Each plant supplies a number of goods and each store has a given demand that must be met. Supply, demand and cost of transportation per unit are shown in Fig. 10.3.

If we denote the number of transported goods from location i to location j by  $x_{ij}$ , problem can be formulated as the linear optimization problem of minimizing

$$1x_{11} + 2x_{12} + 5x_{23} + 2x_{24} + 1x_{31} + 2x_{33} + 1x_{34}$$

subject to

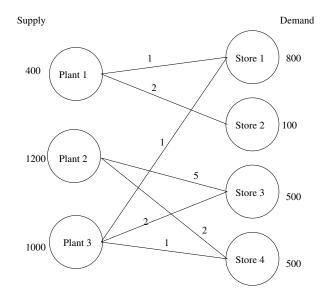


Fig. 10.3: Supply, demand and cost of transportation.

The sensitivity parameters are shown in Table 10.1 and Table 10.2.

Table 10.1: Ranges and shadow prices related to bounds on constraints and variables.

Con.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
1	-300.00	0.00	3.00	3.00
2	-700.00	$+\infty$	0.00	0.00
3	-500.00	0.00	3.00	3.00
4	-0.00	500.00	4.00	4.00
5	-0.00	300.00	5.00	5.00
6	-0.00	700.00	5.00	5.00
7	-500.00	700.00	2.00	2.00
'	000.00	100.00	2.00	2.00
Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$var.$ $x_{11}$	$\beta_1$ $-\infty$	$\beta_2$ 300.00	$\sigma_1$ 0.00	$\frac{\sigma_2}{0.00}$
Var. $x_{11}$ $x_{12}$	$\beta_1$ $-\infty$ $-\infty$	$\beta_2$ $300.00$ $100.00$	$\sigma_1$ 0.00 0.00	$\frac{\sigma_2}{0.00}$
Var. $x_{11}$ $x_{12}$ $x_{23}$	$\beta_1$ $-\infty$ $-\infty$ $-\infty$	$\beta_2$ 300.00 100.00 0.00	$\sigma_1$ 0.00 0.00 0.00	$\sigma_2$ 0.00 0.00 0.00
Var. $x_{11}$ $x_{12}$ $x_{23}$ $x_{24}$	$\beta_1$ $-\infty$ $-\infty$ $-\infty$ $-\infty$	$\beta_2$ 300.00 100.00 0.00 500.00	$\sigma_1$ 0.00 0.00 0.00 0.00	$\sigma_2$ 0.00 0.00 0.00 0.00

Table 10.2: Ranges and shadow prices related to the objective coefficients.

Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$c_1$	$-\infty$	3.00	300.00	300.00
$c_2$	$-\infty$	$\infty$	100.00	100.00
$c_3$	-2.00	$\infty$	0.00	0.00
$c_4$	$-\infty$	2.00	500.00	500.00
$c_5$	-3.00	$\infty$	500.00	500.00
$c_6$	$-\infty$	2.00	500.00	500.00
$c_7$	-2.00	$\infty$	0.00	0.00

Examining the results from the sensitivity analysis we see that for constraint number 1 we have  $\sigma_1 = 3$  and  $\beta_1 = -300$ ,  $\beta_2 = 0$ .

If the upper bound on constraint 1 is decreased by

$$\beta \in [0, 300]$$

then the optimal objective value will increase by the value

$$\sigma_1 \beta = 3\beta$$
.

# 10.3.2 Sensitivity Analysis with MOSEK

A sensitivity analysis can be performed with the **MOSEK** command line tool specifying the option -sen, e.g.

```
mosek myproblem.mps -sen sensitivity.ssp
```

where sensitivity.ssp is a file in the format described in the next section. The ssp file describes which parts of the problem the sensitivity analysis should be performed on, see Sec. 10.3.2.

By default results are written to a file named myproblem.sen. If necessary, this file name can be changed by setting the MSK\_SPAR\_SENSITIVITY\_RES\_FILE\_NAME parameter.

# Sensitivity Analysis Specification File

**MOSEK** employs an MPS-like file format to specify on which model parameters the sensitivity analysis should be performed. The format of the sensitivity specification file is shown in Listing 10.2, where capitalized names are keywords, and names in brackets are names of the constraints and variables to be included in the analysis.

Listing 10.2: Sensitivity analysis file specification.

```
BOUNDS CONSTRAINTS
U|L|LU [cname1]
U|L|LU [cname2]-[cname3]
BOUNDS VARIABLES
U|L|LU [vname1]
U|L|LU [vname2]-[vname3]
OBJECTIVE VARIABLES
[vname1]
[vname2]-[vname3]
```

The sensitivity specification file has three sections, i.e.

- BOUNDS CONSTRAINTS: Specifies on which bounds on constraints the sensitivity analysis should be performed.
- BOUNDS VARIABLES: Specifies on which bounds on variables the sensitivity analysis should be performed.
- OBJECTIVE VARIABLES: Specifies on which objective coefficients the sensitivity analysis should be performed.

A line in the body of a section must begin with a whitespace. In the BOUNDS sections one of the keys L, U, and LU must appear next. These keys specify whether the sensitivity analysis is performed on the lower bound, on the upper bound, or on both the lower and the upper bound respectively. Next, a single constraint (variable) or range of constraints (variables) is specified.

Recall from Sec. 10.3.1 that equality constraints are handled in a special way. Sensitivity analysis of an equality constraint can be specified with either L, U, or LU, all indicating the same, namely that upper and lower bounds (which are equal) are perturbed simultaneously.

As an example consider

```
BOUNDS CONSTRAINTS
L "cons1"
U "cons2"
LU "cons3"-"cons6"
```

which requests that sensitivity analysis is performed on the lower bound of the constraint named cons1, on the upper bound of the constraint named cons2, and on both lower and upper bound on the constraints named cons3 to cons6.

It is allowed to use indexes instead of names, for instance

```
BOUNDS CONSTRAINTS
L "cons1"
U 2
LU 3 - 6
```

The character \* indicates that the line contains a comment and is ignored.

#### **Example: Sensitivity Analysis from Command Line**

As an example consider problem (10.5): the sensitivity file shown below (included in the distribution among the examples).

Listing 10.3: Sensitivity file for problem (10.5).

```
* Comment 1
BOUNDS CONSTRAINTS
U "c1"
                * Analyze upper bound for constraints named c1
U 2
                 * Analyze upper bound for constraints with index 2
U 3-5
                 * Analyze upper bound for constraint with index in interval [3:5]
VARIABLES CONSTRAINTS
L 2-4
                 * This section specifies which bounds on variables should be analyzed.
\hookrightarrow
L "x11"
OBJECTIVE CONSTRAINTS
"x11"
                 * This section specifies which objective coeficients should be I
\rightarrowanalysed.
```

The command

```
mosek transport.lp -sen sensitivity.ssp
```

produces the output file as follow

Listing 10.4: Results of sensitivity analysis

BOUNDS CONSTRAINTS					
INDEX NAME	BOUND	LEFTRANGE	RIGHTRANGE	LEFTPRICE	11
⇔RIGHTPRICE					_
0 c1	UP	-6.574875e-18	5.000000e+02	1.000000e+00	ш
→1.000000e+00					
2 c3	UP	-6.574875e-18	5.000000e+02	1.000000e+00	ш
→1.000000e+00					
3 c4	FIX	-5.000000e+02	6.574875e-18	2.000000e+00	ш
→2.000000e+00					_
4 c5	FIX	-1.000000e+02	6.574875e-18	3.000000e+00	11
→3.000000e+00					_
5 c6	FIX	-5.000000e+02	6.574875e-18	3.000000e+00	11
→3.000000e+00					

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					1 . 0 . /
BOUNDS VARIABLES					
INDEX NAME	BOUND	LEFTRANGE	RIGHTRANGE	LEFTPRICE	⊔
→RIGHTPRICE					
2 x23	LO	-6.574875e-18	5.000000e+02	2.000000e+00	ш
→2.000000e+00					ū
3 x24	LO	-inf	5.000000e+02	0.000000e+00	1.1
→0.000000e+00	20	ada da da	0.00000000000	0.0000000000000000000000000000000000000	
4 x31	LO	-inf	5.000000e+02	0 0000000+00	
→0.000000e+00	LU	-1111	3.000000e+02	0.00000e100	Ц
	T 0		0.000000.00	0.000000.00	
0 x11	LO	-inf	3.000000e+02	0.000000e+00	Ц
⊶0.00000e+00					
OBJECTIVE VARIABLES					
INDEX NAME		LEFTRANGE	RIGHTRANGE	LEFTPRICE	⊔
→RIGHTPRICE					
0 x11		-inf	1.000000e+00	3.000000e+02	11
→3.000000e+02					
2 x23		-2.000000e+00	+inf	0.000000e+00	1.1
→0.000000e+00		2.00000000000			П
→0.00000e:00					

# **Controlling Log Output**

Setting the parameter  $MSK\_IPAR\_LOG\_SENSITIVITY$  to 1 or 0 (default) controls whether or not the results from sensitivity calculations are printed to the message stream.

The parameter  $MSK\_IPAR\_LOG\_SENSITIVITY\_OPT$  controls the amount of debug information on internal calculations from the sensitivity analysis.

# Chapter 11

# **API** Reference

- Optimizer parameters:
  - Double, Integer, String
  - Full list
  - Browse by topic
- Optimizer response codes
- Constants
- List of supported domains

# 11.1 Parameters grouped by topic

# **Analysis**

- MSK\_DPAR\_ANA\_SOL\_INFEAS\_TOL
- MSK\_IPAR\_ANA\_SOL\_BASIS
- MSK\_IPAR\_ANA\_SOL\_PRINT\_VIOLATED
- MSK\_IPAR\_LOG\_ANA\_PRO

# **Basis identification**

- MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV
- MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER
- MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER
- MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR
- MSK\_IPAR\_BI\_MAX\_ITERATIONS
- MSK\_IPAR\_INTPNT\_BASIS
- MSK\_IPAR\_LOG\_BI
- MSK\_IPAR\_LOG\_BI\_FREQ

## Conic interior-point method

- MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP

# Data check

- MSK\_DPAR\_DATA\_SYM\_MAT\_TOL
- MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_HUGE
- MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_LARGE
- MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE
- MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE
- MSK\_DPAR\_DATA\_TOL\_BOUND\_INF
- MSK\_DPAR\_DATA\_TOL\_BOUND\_WRN
- MSK\_DPAR\_DATA\_TOL\_C\_HUGE
- MSK\_DPAR\_DATA\_TOL\_CJ\_LARGE
- MSK\_DPAR\_DATA\_TOL\_QIJ
- MSK\_DPAR\_DATA\_TOL\_X
- MSK\_DPAR\_SEMIDEFINITE\_TOL\_APPROX
- MSK\_IPAR\_CHECK\_CONVEXITY
- MSK\_IPAR\_LOG\_CHECK\_CONVEXITY

# Data input/output

- MSK\_IPAR\_INFEAS\_REPORT\_AUTO
- MSK\_IPAR\_LOG\_FILE
- MSK\_IPAR\_OPF\_WRITE\_HEADER
- MSK\_IPAR\_OPF\_WRITE\_HINTS
- MSK\_IPAR\_OPF\_WRITE\_LINE\_LENGTH
- MSK\_IPAR\_OPF\_WRITE\_PARAMETERS
- MSK\_IPAR\_OPF\_WRITE\_PROBLEM
- MSK\_IPAR\_OPF\_WRITE\_SOL\_BAS
- MSK\_IPAR\_OPF\_WRITE\_SOL\_ITG
- MSK\_IPAR\_OPF\_WRITE\_SOL\_ITR
- MSK\_IPAR\_OPF\_WRITE\_SOLUTIONS

- MSK\_IPAR\_PARAM\_READ\_CASE\_NAME
- MSK\_IPAR\_PARAM\_READ\_IGN\_ERROR
- MSK\_IPAR\_PTF\_WRITE\_PARAMETERS
- MSK\_IPAR\_PTF\_WRITE\_SOLUTIONS
- MSK\_IPAR\_PTF\_WRITE\_TRANSFORM
- MSK\_IPAR\_READ\_DEBUG
- MSK\_IPAR\_READ\_KEEP\_FREE\_CON
- MSK\_IPAR\_READ\_MPS\_FORMAT
- MSK\_IPAR\_READ\_MPS\_WIDTH
- MSK\_IPAR\_READ\_TASK\_IGNORE\_PARAM
- MSK\_IPAR\_SOL\_READ\_NAME\_WIDTH
- MSK\_IPAR\_SOL\_READ\_WIDTH
- MSK\_IPAR\_WRITE\_BAS\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_BAS\_HEAD
- MSK\_IPAR\_WRITE\_BAS\_VARIABLES
- MSK\_IPAR\_WRITE\_COMPRESSION
- MSK\_IPAR\_WRITE\_DATA\_PARAM
- MSK\_IPAR\_WRITE\_FREE\_CON
- MSK\_IPAR\_WRITE\_GENERIC\_NAMES
- MSK\_IPAR\_WRITE\_GENERIC\_NAMES\_IO
- MSK\_IPAR\_WRITE\_IGNORE\_INCOMPATIBLE\_ITEMS
- MSK\_IPAR\_WRITE\_INT\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_INT\_HEAD
- MSK\_IPAR\_WRITE\_INT\_VARIABLES
- MSK\_IPAR\_WRITE\_JSON\_INDENTATION
- MSK\_IPAR\_WRITE\_LP\_FULL\_OBJ
- MSK\_IPAR\_WRITE\_LP\_LINE\_WIDTH
- MSK\_IPAR\_WRITE\_MPS\_FORMAT
- MSK\_IPAR\_WRITE\_MPS\_INT
- MSK\_IPAR\_WRITE\_SOL\_BARVARIABLES
- MSK\_IPAR\_WRITE\_SOL\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_SOL\_HEAD
- MSK\_IPAR\_WRITE\_SOL\_IGNORE\_INVALID\_NAMES
- MSK\_IPAR\_WRITE\_SOL\_VARIABLES
- MSK\_IPAR\_WRITE\_TASK\_INC\_SOL
- MSK\_IPAR\_WRITE\_XML\_MODE

- MSK\_SPAR\_BAS\_SOL\_FILE\_NAME
- MSK\_SPAR\_DATA\_FILE\_NAME
- MSK\_SPAR\_DEBUG\_FILE\_NAME
- MSK\_SPAR\_INT\_SOL\_FILE\_NAME
- MSK\_SPAR\_ITR\_SOL\_FILE\_NAME
- MSK\_SPAR\_MIO\_DEBUG\_STRING
- MSK\_SPAR\_PARAM\_COMMENT\_SIGN
- MSK\_SPAR\_PARAM\_READ\_FILE\_NAME
- MSK\_SPAR\_PARAM\_WRITE\_FILE\_NAME
- MSK\_SPAR\_READ\_MPS\_BOU\_NAME
- MSK\_SPAR\_READ\_MPS\_OBJ\_NAME
- MSK\_SPAR\_READ\_MPS\_RAN\_NAME
- MSK\_SPAR\_READ\_MPS\_RHS\_NAME
- MSK\_SPAR\_SENSITIVITY\_FILE\_NAME
- MSK\_SPAR\_SENSITIVITY\_RES\_FILE\_NAME
- MSK\_SPAR\_SOL\_FILTER\_XC\_LOW
- MSK\_SPAR\_SOL\_FILTER\_XC\_UPR
- MSK\_SPAR\_SOL\_FILTER\_XX\_LOW
- MSK\_SPAR\_SOL\_FILTER\_XX\_UPR
- MSK\_SPAR\_STAT\_KEY
- MSK\_SPAR\_STAT\_NAME
- MSK\_SPAR\_WRITE\_LP\_GEN\_VAR\_NAME

#### **Debugging**

• MSK\_IPAR\_AUTO\_SORT\_A\_BEFORE\_OPT

# **Dual simplex**

- MSK\_IPAR\_SIM\_DUAL\_CRASH
- MSK\_IPAR\_SIM\_DUAL\_RESTRICT\_SELECTION
- MSK\_IPAR\_SIM\_DUAL\_SELECTION

# Infeasibility report

- MSK\_IPAR\_INFEAS\_GENERIC\_NAMES
- MSK\_IPAR\_INFEAS\_REPORT\_LEVEL
- MSK\_IPAR\_LOG\_INFEAS\_ANA

#### Interior-point method

- MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL
- MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_QO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_QO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_TOL\_DSAFE
- MSK\_DPAR\_INTPNT\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_TOL\_PATH
- MSK\_DPAR\_INTPNT\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_TOL\_PSAFE
- MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_TOL\_REL\_STEP
- MSK\_DPAR\_INTPNT\_TOL\_STEP\_SIZE
- MSK\_DPAR\_QCQO\_REFORMULATE\_REL\_DROP\_TOL
- MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER
- MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR
- MSK\_IPAR\_INTPNT\_BASIS
- MSK\_IPAR\_INTPNT\_DIFF\_STEP
- MSK\_IPAR\_INTPNT\_HOTSTART
- MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS
- MSK\_IPAR\_INTPNT\_MAX\_NUM\_COR
- MSK\_IPAR\_INTPNT\_MAX\_NUM\_REFINEMENT\_STEPS
- MSK\_IPAR\_INTPNT\_OFF\_COL\_TRH
- MSK\_IPAR\_INTPNT\_ORDER\_GP\_NUM\_SEEDS
- MSK\_IPAR\_INTPNT\_ORDER\_METHOD

- MSK\_IPAR\_INTPNT\_PURIFY
- MSK\_IPAR\_INTPNT\_REGULARIZATION\_USE
- MSK\_IPAR\_INTPNT\_SCALING
- MSK\_IPAR\_INTPNT\_SOLVE\_FORM
- MSK\_IPAR\_INTPNT\_STARTING\_POINT
- MSK\_IPAR\_LOG\_INTPNT

#### License manager

- MSK\_IPAR\_CACHE\_LICENSE
- MSK\_IPAR\_LICENSE\_DEBUG
- MSK\_IPAR\_LICENSE\_PAUSE\_TIME
- MSK\_IPAR\_LICENSE\_SUPPRESS\_EXPIRE\_WRNS
- MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN
- MSK\_IPAR\_LICENSE\_WAIT

# Logging

- MSK\_IPAR\_LOG
- MSK\_IPAR\_LOG\_ANA\_PRO
- MSK\_IPAR\_LOG\_BI
- MSK\_IPAR\_LOG\_BI\_FREQ
- MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT
- MSK\_IPAR\_LOG\_EXPAND
- MSK\_IPAR\_LOG\_FEAS\_REPAIR
- MSK\_IPAR\_LOG\_FILE
- MSK\_IPAR\_LOG\_INCLUDE\_SUMMARY
- MSK\_IPAR\_LOG\_INFEAS\_ANA
- MSK\_IPAR\_LOG\_INTPNT
- MSK\_IPAR\_LOG\_LOCAL\_INFO
- MSK\_IPAR\_LOG\_MIO
- $\bullet \ \textit{MSK\_IPAR\_LOG\_MIO\_FREQ}$
- MSK\_IPAR\_LOG\_ORDER
- MSK\_IPAR\_LOG\_PRESOLVE
- MSK\_IPAR\_LOG\_RESPONSE
- MSK\_IPAR\_LOG\_SENSITIVITY
- MSK\_IPAR\_LOG\_SENSITIVITY\_OPT
- MSK\_IPAR\_LOG\_SIM
- MSK\_IPAR\_LOG\_SIM\_FREQ
- MSK\_IPAR\_LOG\_STORAGE

#### Mixed-integer optimization

- MSK\_DPAR\_MIO\_DJC\_MAX\_BIGM
- MSK\_DPAR\_MIO\_MAX\_TIME
- MSK\_DPAR\_MIO\_REL\_GAP\_CONST
- MSK\_DPAR\_MIO\_TOL\_ABS\_GAP
- MSK\_DPAR\_MIO\_TOL\_ABS\_RELAX\_INT
- MSK\_DPAR\_MIO\_TOL\_FEAS
- MSK\_DPAR\_MIO\_TOL\_REL\_DUAL\_BOUND\_IMPROVEMENT
- MSK\_DPAR\_MIO\_TOL\_REL\_GAP
- MSK\_IPAR\_LOG\_MIO
- MSK\_IPAR\_LOG\_MIO\_FREQ
- MSK\_IPAR\_MIO\_BRANCH\_DIR
- MSK\_IPAR\_MIO\_CONIC\_OUTER\_APPROXIMATION
- MSK\_IPAR\_MIO\_CONSTRUCT\_SOL
- MSK\_IPAR\_MIO\_CUT\_CLIQUE
- MSK\_IPAR\_MIO\_CUT\_CMIR
- MSK\_IPAR\_MIO\_CUT\_GMI
- MSK\_IPAR\_MIO\_CUT\_IMPLIED\_BOUND
- MSK\_IPAR\_MIO\_CUT\_KNAPSACK\_COVER
- MSK\_IPAR\_MIO\_CUT\_LIPRO
- MSK\_IPAR\_MIO\_CUT\_SELECTION\_LEVEL
- MSK\_IPAR\_MIO\_DATA\_PERMUTATION\_METHOD
- MSK\_IPAR\_MIO\_FEASPUMP\_LEVEL
- MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL
- MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES
- MSK\_IPAR\_MIO\_MAX\_NUM\_RELAXS
- MSK\_IPAR\_MIO\_MAX\_NUM\_ROOT\_CUT\_ROUNDS
- MSK\_IPAR\_MIO\_MAX\_NUM\_SOLUTIONS
- MSK\_IPAR\_MIO\_MEMORY\_EMPHASIS\_LEVEL
- MSK\_IPAR\_MIO\_NODE\_OPTIMIZER
- MSK\_IPAR\_MIO\_NODE\_SELECTION
- MSK\_IPAR\_MIO\_NUMERICAL\_EMPHASIS\_LEVEL
- MSK\_IPAR\_MIO\_PERSPECTIVE\_REFORMULATE
- MSK\_IPAR\_MIO\_PROBING\_LEVEL
- MSK\_IPAR\_MIO\_PROPAGATE\_OBJECTIVE\_CONSTRAINT
- MSK\_IPAR\_MIO\_QCQO\_REFORMULATION\_METHOD

- MSK\_IPAR\_MIO\_RINS\_MAX\_NODES
- MSK\_IPAR\_MIO\_ROOT\_OPTIMIZER
- MSK\_IPAR\_MIO\_ROOT\_REPEAT\_PRESOLVE\_LEVEL
- MSK\_IPAR\_MIO\_SEED
- MSK\_IPAR\_MIO\_SYMMETRY\_LEVEL
- MSK\_IPAR\_MIO\_VB\_DETECTION\_LEVEL

# **Output** information

- MSK\_IPAR\_INFEAS\_REPORT\_LEVEL
- MSK\_IPAR\_LICENSE\_SUPPRESS\_EXPIRE\_WRNS
- MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN
- MSK\_IPAR\_LOG
- MSK\_IPAR\_LOG\_BI
- MSK\_IPAR\_LOG\_BI\_FREQ
- MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT
- MSK\_IPAR\_LOG\_EXPAND
- MSK\_IPAR\_LOG\_FEAS\_REPAIR
- MSK\_IPAR\_LOG\_FILE
- MSK\_IPAR\_LOG\_INCLUDE\_SUMMARY
- MSK\_IPAR\_LOG\_INFEAS\_ANA
- MSK\_IPAR\_LOG\_INTPNT
- MSK\_IPAR\_LOG\_LOCAL\_INFO
- MSK\_IPAR\_LOG\_MIO
- MSK\_IPAR\_LOG\_MIO\_FREQ
- MSK\_IPAR\_LOG\_ORDER
- MSK\_IPAR\_LOG\_RESPONSE
- MSK\_IPAR\_LOG\_SENSITIVITY
- MSK\_IPAR\_LOG\_SENSITIVITY\_OPT
- MSK\_IPAR\_LOG\_SIM
- MSK\_IPAR\_LOG\_SIM\_FREQ
- MSK\_IPAR\_LOG\_SIM\_MINOR
- MSK\_IPAR\_LOG\_STORAGE
- MSK\_IPAR\_MAX\_NUM\_WARNINGS

## Overall solver

- MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER
- MSK\_IPAR\_INFEAS\_PREFER\_PRIMAL
- MSK\_IPAR\_LICENSE\_WAIT
- MSK\_IPAR\_MIO\_MODE
- MSK\_IPAR\_OPTIMIZER
- MSK\_IPAR\_PRESOLVE\_LEVEL
- MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_REDUCTIONS
- MSK\_IPAR\_PRESOLVE\_USE
- MSK\_IPAR\_PRIMAL\_REPAIR\_OPTIMIZER
- MSK\_IPAR\_SENSITIVITY\_ALL
- MSK\_IPAR\_SENSITIVITY\_OPTIMIZER
- MSK\_IPAR\_SENSITIVITY\_TYPE
- MSK\_IPAR\_SOLUTION\_CALLBACK

### Overall system

- MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO
- MSK\_IPAR\_LICENSE\_WAIT
- MSK\_IPAR\_LOG\_STORAGE
- MSK\_IPAR\_MT\_SPINCOUNT
- MSK\_IPAR\_NUM\_THREADS
- MSK\_IPAR\_REMOVE\_UNUSED\_SOLUTIONS
- MSK\_IPAR\_TIMING\_LEVEL
- MSK\_SPAR\_REMOTE\_OPTSERVER\_HOST
- MSK\_SPAR\_REMOTE\_TLS\_CERT
- MSK\_SPAR\_REMOTE\_TLS\_CERT\_PATH

### **Presolve**

- MSK\_DPAR\_PRESOLVE\_TOL\_ABS\_LINDEP
- MSK\_DPAR\_PRESOLVE\_TOL\_AIJ
- MSK\_DPAR\_PRESOLVE\_TOL\_PRIMAL\_INFEAS\_PERTURBATION
- MSK\_DPAR\_PRESOLVE\_TOL\_REL\_LINDEP
- MSK\_DPAR\_PRESOLVE\_TOL\_S
- MSK\_DPAR\_PRESOLVE\_TOL\_X
- MSK\_IPAR\_MIO\_PRESOLVE\_AGGREGATOR\_USE
- MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_FILL

- MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES
- MSK\_IPAR\_PRESOLVE\_LEVEL
- MSK\_IPAR\_PRESOLVE\_LINDEP\_ABS\_WORK\_TRH
- MSK\_IPAR\_PRESOLVE\_LINDEP\_REL\_WORK\_TRH
- MSK\_IPAR\_PRESOLVE\_LINDEP\_USE
- MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_PASS
- MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_REDUCTIONS
- MSK\_IPAR\_PRESOLVE\_USE

# **Primal simplex**

- MSK\_IPAR\_SIM\_PRIMAL\_CRASH
- MSK\_IPAR\_SIM\_PRIMAL\_RESTRICT\_SELECTION
- MSK\_IPAR\_SIM\_PRIMAL\_SELECTION

# **Progress callback**

• MSK\_IPAR\_SOLUTION\_CALLBACK

# Simplex optimizer

- MSK\_DPAR\_BASIS\_REL\_TOL\_S
- MSK\_DPAR\_BASIS\_TOL\_S
- MSK\_DPAR\_BASIS\_TOL\_X
- MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV
- MSK\_DPAR\_SIMPLEX\_ABS\_TOL\_PIV
- MSK\_IPAR\_BASIS\_SOLVE\_USE\_PLUS\_ONE
- MSK\_IPAR\_LOG\_SIM
- MSK\_IPAR\_LOG\_SIM\_FREQ
- MSK\_IPAR\_LOG\_SIM\_MINOR
- MSK\_IPAR\_SENSITIVITY\_OPTIMIZER
- $\bullet \ \textit{MSK\_IPAR\_SIM\_BASIS\_FACTOR\_USE}$
- MSK\_IPAR\_SIM\_DEGEN
- MSK\_IPAR\_SIM\_DETECT\_PWL
- MSK\_IPAR\_SIM\_DUAL\_PHASEONE\_METHOD
- MSK\_IPAR\_SIM\_EXPLOIT\_DUPVEC
- MSK\_IPAR\_SIM\_HOTSTART
- MSK\_IPAR\_SIM\_HOTSTART\_LU
- MSK\_IPAR\_SIM\_MAX\_ITERATIONS

- MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS
- MSK\_IPAR\_SIM\_NON\_SINGULAR
- MSK\_IPAR\_SIM\_PRIMAL\_PHASEONE\_METHOD
- MSK\_IPAR\_SIM\_REFACTOR\_FREQ
- MSK\_IPAR\_SIM\_REFORMULATION
- MSK\_IPAR\_SIM\_SAVE\_LU
- MSK\_IPAR\_SIM\_SCALING
- MSK\_IPAR\_SIM\_SCALING\_METHOD
- MSK\_IPAR\_SIM\_SEED
- MSK\_IPAR\_SIM\_SOLVE\_FORM
- MSK\_IPAR\_SIM\_STABILITY\_PRIORITY
- MSK\_IPAR\_SIM\_SWITCH\_OPTIMIZER

# Solution input/output

- MSK\_IPAR\_INFEAS\_REPORT\_AUTO
- MSK\_IPAR\_SOL\_FILTER\_KEEP\_BASIC
- MSK\_IPAR\_SOL\_FILTER\_KEEP\_RANGED
- MSK\_IPAR\_SOL\_READ\_NAME\_WIDTH
- MSK\_IPAR\_SOL\_READ\_WIDTH
- MSK\_IPAR\_WRITE\_BAS\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_BAS\_HEAD
- MSK\_IPAR\_WRITE\_BAS\_VARIABLES
- MSK\_IPAR\_WRITE\_INT\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_INT\_HEAD
- MSK\_IPAR\_WRITE\_INT\_VARIABLES
- MSK\_IPAR\_WRITE\_SOL\_BARVARIABLES
- MSK\_IPAR\_WRITE\_SOL\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_SOL\_HEAD
- MSK\_IPAR\_WRITE\_SOL\_IGNORE\_INVALID\_NAMES
- MSK\_IPAR\_WRITE\_SOL\_VARIABLES
- MSK\_SPAR\_BAS\_SOL\_FILE\_NAME
- MSK\_SPAR\_INT\_SOL\_FILE\_NAME
- MSK\_SPAR\_ITR\_SOL\_FILE\_NAME
- MSK\_SPAR\_SOL\_FILTER\_XC\_LOW
- MSK\_SPAR\_SOL\_FILTER\_XC\_UPR
- MSK\_SPAR\_SOL\_FILTER\_XX\_LOW
- MSK\_SPAR\_SOL\_FILTER\_XX\_UPR

#### Termination criteria

- MSK\_DPAR\_BASIS\_REL\_TOL\_S
- MSK\_DPAR\_BASIS\_TOL\_S
- MSK\_DPAR\_BASIS\_TOL\_X
- MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_QO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_QO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP
- MSK\_DPAR\_LOWER\_OBJ\_CUT
- MSK\_DPAR\_LOWER\_OBJ\_CUT\_FINITE\_TRH
- MSK\_DPAR\_MIO\_MAX\_TIME
- MSK\_DPAR\_MIO\_REL\_GAP\_CONST
- MSK\_DPAR\_MIO\_TOL\_REL\_GAP
- MSK\_DPAR\_OPTIMIZER\_MAX\_TIME
- MSK\_DPAR\_UPPER\_OBJ\_CUT
- MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH
- MSK\_IPAR\_BI\_MAX\_ITERATIONS
- MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS
- MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES
- MSK\_IPAR\_MIO\_MAX\_NUM\_ROOT\_CUT\_ROUNDS
- MSK\_IPAR\_MIO\_MAX\_NUM\_SOLUTIONS
- MSK\_IPAR\_SIM\_MAX\_ITERATIONS

#### Other

- MSK\_IPAR\_COMPRESS\_STATFILE
- MSK\_IPAR\_NG
- MSK\_IPAR\_REMOTE\_USE\_COMPRESSION

# 11.2 Parameters (alphabetical list sorted by type)

- Double parameters
- Integer parameters
- String parameters

# 11.2.1 Double parameters

# MSK\_DPAR\_ANA\_SOL\_INFEAS\_TOL

If a constraint violates its bound with an amount larger than this value, the constraint name, index and violation will be printed by the solution analyzer.

Default 1e-6

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_ANA\_SOL\_INFEAS\_TOL 1e-6 file

Groups Analysis

#### MSK\_DPAR\_BASIS\_REL\_TOL\_S

Maximum relative dual bound violation allowed in an optimal basic solution.

**Default** 1.0e-12

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_BASIS\_REL\_TOL\_S 1.0e-12 file

Groups Simplex optimizer, Termination criteria

# MSK\_DPAR\_BASIS\_TOL\_S

Maximum absolute dual bound violation in an optimal basic solution.

Default 1.0e-6

Accepted [1.0e-9; +inf]

Example mosek -d MSK\_DPAR\_BASIS\_TOL\_S 1.0e-6 file

Groups Simplex optimizer, Termination criteria

#### MSK\_DPAR\_BASIS\_TOL\_X

Maximum absolute primal bound violation allowed in an optimal basic solution.

Default 1.0e-6

Accepted [1.0e-9; +inf]

Example mosek -d MSK\_DPAR\_BASIS\_TOL\_X 1.0e-6 file

Groups Simplex optimizer, Termination criteria

# MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL

This parameter controls when the full convexity check declares a problem to be non-convex. Increasing this tolerance relaxes the criteria for declaring the problem non-convex.

A problem is declared non-convex if negative (positive) pivot elements are detected in the Cholesky factor of a matrix which is required to be PSD (NSD). This parameter controls how much this non-negativity requirement may be violated.

If  $d_i$  is the pivot element for column i, then the matrix Q is considered to not be PSD if:

$$d_i \leq -|Q_{ii}|$$
 check convexity rel tol

Default 1e-10

Accepted [0; +inf]

Example mosek -d MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL 1e-10 file

Groups Interior-point method

#### MSK\_DPAR\_DATA\_SYM\_MAT\_TOL

Absolute zero tolerance for elements in in symmetric matrices. If any value in a symmetric matrix is smaller than this parameter in absolute terms **MOSEK** will treat the values as zero and generate a warning.

**Default** 1.0e-12

**Accepted** [1.0e-16; 1.0e-6]

Example mosek -d MSK\_DPAR\_DATA\_SYM\_MAT\_TOL 1.0e-12 file

Groups Data check

#### MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_HUGE

An element in a symmetric matrix which is larger than this value in absolute size causes an error.

**Default** 1.0e20

Accepted [0.0; +inf]

 $\mathbf{Example} \ \mathtt{mosek} \ \mathtt{-d} \ \mathtt{MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_HUGE} \ 1.0e20 \ \mathtt{file}$ 

Groups Data check

#### MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_LARGE

An element in a symmetric matrix which is larger than this value in absolute size causes a warning message to be printed.

**Default** 1.0e10

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_LARGE 1.0e10 file

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE

An element in A which is larger than this value in absolute size causes an error.

**Default** 1.0e20

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE 1.0e20 file

Groups Data check

# MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE

An element in A which is larger than this value in absolute size causes a warning message to be printed.

**Default** 1.0e10

**Accepted** [0.0; +inf]

Example mosek -d MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE 1.0e10 file

Groups Data check

# MSK\_DPAR\_DATA\_TOL\_BOUND\_INF

Any bound which in absolute value is greater than this parameter is considered infinite.

Default 1.0e16

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_DATA\_TOL\_BOUND\_INF 1.0e16 file

Groups Data check

### MSK\_DPAR\_DATA\_TOL\_BOUND\_WRN

If a bound value is larger than this value in absolute size, then a warning message is issued.

Default 1.0e8

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_DATA\_TOL\_BOUND\_WRN 1.0e8 file

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_C\_HUGE

An element in c which is larger than the value of this parameter in absolute terms is considered to be huge and generates an error.

Default 1.0e16

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_DATA\_TOL\_C\_HUGE 1.0e16 file

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_CJ\_LARGE

An element in c which is larger than this value in absolute terms causes a warning message to be printed.

Default 1.0e8

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_DATA\_TOL\_CJ\_LARGE 1.0e8 file

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_QIJ

Absolute zero tolerance for elements in Q matrices.

**Default** 1.0e-16

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_DATA\_TOL\_QIJ 1.0e-16 file

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_X

Zero tolerance for constraints and variables i.e. if the distance between the lower and upper bound is less than this value, then the lower and upper bound is considered identical.

**Default** 1.0e-8

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_DATA\_TOL\_X 1.0e-8 file

 ${\bf Groups}\ {\it Data\ check}$ 

## MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS

Dual feasibility tolerance used by the interior-point optimizer for conic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS 1.0e-8 file

See also MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

Groups Interior-point method, Termination criteria, Conic interior-point method

#### MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS

Infeasibility tolerance used by the interior-point optimizer for conic problems. Controls when the interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-12

**Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS 1.0e-12 file

Groups Interior-point method, Termination criteria, Conic interior-point method

#### MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED

Relative complementarity gap tolerance used by the interior-point optimizer for conic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED 1.0e-8 file

Groups Interior-point method, Termination criteria, Conic interior-point method

#### MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

Optimality tolerance used by the interior-point optimizer for conic problems. If **MOSEK** cannot compute a solution that has the prescribed accuracy then it will check if the solution found satisfies the termination criteria with all tolerances multiplied by the value of this parameter. If yes, then the solution is also declared optimal.

Default 1000

Accepted [1.0; +inf]

Example mosek -d MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL 1000 file

Groups Interior-point method, Termination criteria, Conic interior-point method

#### MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS

Primal feasibility tolerance used by the interior-point optimizer for conic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS 1.0e-8 file

See also MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

Groups Interior-point method, Termination criteria, Conic interior-point method

#### MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP

Relative gap termination tolerance used by the interior-point optimizer for conic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP 1.0e-8 file

See also MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

Groups Interior-point method, Termination criteria, Conic interior-point method

# MSK\_DPAR\_INTPNT\_QO\_TOL\_DFEAS

Dual feasibility tolerance used by the interior-point optimizer for quadratic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_QO\_TOL\_DFEAS 1.0e-8 file

See also MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

Groups Interior-point method, Termination criteria

#### MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS

Infeasibility tolerance used by the interior-point optimizer for quadratic problems. Controls when the interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-12

**Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS 1.0e-12 file

Groups Interior-point method, Termination criteria

# MSK\_DPAR\_INTPNT\_QO\_TOL\_MU\_RED

Relative complementarity gap tolerance used by the interior-point optimizer for quadratic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_QO\_TOL\_MU\_RED 1.0e-8 file

Groups Interior-point method, Termination criteria

# MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

Optimality tolerance used by the interior-point optimizer for quadratic problems. If **MOSEK** cannot compute a solution that has the prescribed accuracy then it will check if the solution found satisfies the termination criteria with all tolerances multiplied by the value of this parameter. If yes, then the solution is also declared optimal.

Default 1000

Accepted [1.0; +inf]

Example mosek -d MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL 1000 file

Groups Interior-point method, Termination criteria

# MSK\_DPAR\_INTPNT\_QO\_TOL\_PFEAS

Primal feasibility tolerance used by the interior-point optimizer for quadratic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_QO\_TOL\_PFEAS 1.0e-8 file

See also MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

Groups Interior-point method, Termination criteria

#### MSK\_DPAR\_INTPNT\_QO\_TOL\_REL\_GAP

Relative gap termination tolerance used by the interior-point optimizer for quadratic problems.

Default 1.0e-8

Accepted [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_QO\_TOL\_REL\_GAP 1.0e-8 file

See also MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

Groups Interior-point method, Termination criteria

#### MSK\_DPAR\_INTPNT\_TOL\_DFEAS

Dual feasibility tolerance used by the interior-point optimizer for linear problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_TOL\_DFEAS 1.0e-8 file

Groups Interior-point method, Termination criteria

# MSK\_DPAR\_INTPNT\_TOL\_DSAFE

Controls the initial dual starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it might be worthwhile to increase this value.

**Default** 1.0

Accepted [1.0e-4; +inf]

Example mosek -d MSK\_DPAR\_INTPNT\_TOL\_DSAFE 1.0 file

Groups Interior-point method

# MSK\_DPAR\_INTPNT\_TOL\_INFEAS

Infeasibility tolerance used by the interior-point optimizer for linear problems. Controls when the interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-10

Accepted [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_TOL\_INFEAS 1.0e-10 file

## Groups Interior-point method, Termination criteria

### MSK\_DPAR\_INTPNT\_TOL\_MU\_RED

Relative complementarity gap tolerance used by the interior-point optimizer for linear problems.

**Default** 1.0e-16 **Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_TOL\_MU\_RED 1.0e-16 file

Groups Interior-point method, Termination criteria

### MSK\_DPAR\_INTPNT\_TOL\_PATH

Controls how close the interior-point optimizer follows the central path. A large value of this parameter means the central path is followed very closely. On numerically unstable problems it may be worthwhile to increase this parameter.

Default 1.0e-8

**Accepted** [0.0; 0.9999]

Example mosek -d MSK\_DPAR\_INTPNT\_TOL\_PATH 1.0e-8 file

Groups Interior-point method

#### MSK\_DPAR\_INTPNT\_TOL\_PFEAS

Primal feasibility tolerance used by the interior-point optimizer for linear problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_INTPNT\_TOL\_PFEAS 1.0e-8 file

Groups Interior-point method, Termination criteria

## MSK\_DPAR\_INTPNT\_TOL\_PSAFE

Controls the initial primal starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it may be worthwhile to increase this value.

Default 1.0

Accepted [1.0e-4; +inf]

Example mosek -d MSK\_DPAR\_INTPNT\_TOL\_PSAFE 1.0 file

Groups Interior-point method

## MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP

Relative gap termination tolerance used by the interior-point optimizer for linear problems.

Default 1.0e-8

Accepted [1.0e-14; +inf]

Example mosek -d MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP 1.0e-8 file

Groups Termination criteria, Interior-point method

### MSK\_DPAR\_INTPNT\_TOL\_REL\_STEP

Relative step size to the boundary for linear and quadratic optimization problems.

**Default** 0.9999

**Accepted** [1.0e-4; 0.999999]

Example mosek -d MSK\_DPAR\_INTPNT\_TOL\_REL\_STEP 0.9999 file

 ${\bf Groups} \ \ Interior\mbox{-}point \ method$ 

### MSK\_DPAR\_INTPNT\_TOL\_STEP\_SIZE

Minimal step size tolerance. If the step size falls below the value of this parameter, then the interior-point optimizer assumes that it is stalled. In other words the interior-point optimizer does not make any progress and therefore it is better to stop.

Default 1.0e-6

```
Accepted [0.0; 1.0]
```

Example mosek -d MSK\_DPAR\_INTPNT\_TOL\_STEP\_SIZE 1.0e-6 file

Groups Interior-point method

### MSK\_DPAR\_LOWER\_OBJ\_CUT

If either a primal or dual feasible solution is found proving that the optimal objective value is outside the interval [ MSK\_DPAR\_LOWER\_OBJ\_CUT, MSK\_DPAR\_UPPER\_OBJ\_CUT ], then MOSEK is terminated.

**Default** -1.0e30

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_DPAR\_LOWER\_OBJ\_CUT -1.0e30 file

See also MSK\_DPAR\_LOWER\_OBJ\_CUT\_FINITE\_TRH

Groups Termination criteria

### MSK\_DPAR\_LOWER\_OBJ\_CUT\_FINITE\_TRH

If the lower objective cut is less than the value of this parameter value, then the lower objective cut i.e.  $MSK\_DPAR\_LOWER\_OBJ\_CUT$  is treated as  $-\infty$ .

**Default** -0.5e30

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_DPAR\_LOWER\_OBJ\_CUT\_FINITE\_TRH -0.5e30 file

Groups Termination criteria

### MSK\_DPAR\_MIO\_DJC\_MAX\_BIGM

Maximum allowed big-M value when reformulating disjunctive constraints to linear constraints. Higher values make it more likely that a disjunction is reformulated to linear constraints, but also increase the risk of numerical problems.

Default 1.0e6

Accepted [0; +inf]

Example mosek -d MSK\_DPAR\_MIO\_DJC\_MAX\_BIGM 1.0e6 file

 ${\bf Groups}\ \textit{Mixed-integer optimization}$ 

# MSK\_DPAR\_MIO\_MAX\_TIME

This parameter limits the maximum time spent by the mixed-integer optimizer. A negative number means infinity.

**Default** -1.0

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_DPAR\_MIO\_MAX\_TIME -1.0 file

 ${\bf Groups}\ {\it Mixed-integer}\ optimization,\ Termination\ criteria$ 

# MSK\_DPAR\_MIO\_REL\_GAP\_CONST

This value is used to compute the relative gap for the solution to an integer optimization problem.

**Default** 1.0e-10

Accepted [1.0e-15; +inf]

Example mosek -d MSK\_DPAR\_MIO\_REL\_GAP\_CONST 1.0e-10 file

Groups Mixed-integer optimization, Termination criteria

### MSK\_DPAR\_MIO\_TOL\_ABS\_GAP

Absolute optimality tolerance employed by the mixed-integer optimizer.

Default 0.0

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_MIO\_TOL\_ABS\_GAP 0.0 file

**Groups** Mixed-integer optimization

### MSK\_DPAR\_MIO\_TOL\_ABS\_RELAX\_INT

Absolute integer feasibility tolerance. If the distance to the nearest integer is less than this tolerance then an integer constraint is assumed to be satisfied.

Default 1.0e-5

Accepted [1e-9; +inf]

Example mosek -d MSK\_DPAR\_MIO\_TOL\_ABS\_RELAX\_INT 1.0e-5 file

**Groups** Mixed-integer optimization

#### MSK\_DPAR\_MIO\_TOL\_FEAS

Feasibility tolerance for mixed integer solver.

Default 1.0e-6

**Accepted** [1e-9; 1e-3]

Example mosek -d MSK\_DPAR\_MIO\_TOL\_FEAS 1.0e-6 file

**Groups** Mixed-integer optimization

### MSK\_DPAR\_MIO\_TOL\_REL\_DUAL\_BOUND\_IMPROVEMENT

If the relative improvement of the dual bound is smaller than this value, the solver will terminate the root cut generation. A value of 0.0 means that the value is selected automatically.

**Default** 0.0

**Accepted** [0.0; 1.0]

Example mosek -d MSK\_DPAR\_MIO\_TOL\_REL\_DUAL\_BOUND\_IMPROVEMENT 0.0 file

**Groups** Mixed-integer optimization

#### MSK\_DPAR\_MIO\_TOL\_REL\_GAP

Relative optimality tolerance employed by the mixed-integer optimizer.

Default 1.0e-4

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_MIO\_TOL\_REL\_GAP 1.0e-4 file

Groups Mixed-integer optimization, Termination criteria

### MSK\_DPAR\_OPTIMIZER\_MAX\_TIME

Maximum amount of time the optimizer is allowed to spent on the optimization. A negative number means infinity.

Default -1.0

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_DPAR\_OPTIMIZER\_MAX\_TIME -1.0 file

Groups Termination criteria

#### MSK\_DPAR\_PRESOLVE\_TOL\_ABS\_LINDEP

Absolute tolerance employed by the linear dependency checker.

Default 1.0e-6

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_PRESOLVE\_TOL\_ABS\_LINDEP 1.0e-6 file

Groups Presolve

# MSK\_DPAR\_PRESOLVE\_TOL\_AIJ

Absolute zero tolerance employed for  $a_{ij}$  in the presolve.

**Default** 1.0e-12

Accepted [1.0e-15; +inf]

Example mosek -d MSK\_DPAR\_PRESOLVE\_TOL\_AIJ 1.0e-12 file

Groups Presolve

### MSK\_DPAR\_PRESOLVE\_TOL\_PRIMAL\_INFEAS\_PERTURBATION

The presolve is allowed to perturbe a bound on a constraint or variable by this amount if it removes an infeasibility.

Default 1.0e-6

Accepted [0.0; +inf]

 $\mathbf{Example} \ \mathtt{mosek} \ \mathtt{-d} \ \mathtt{MSK\_DPAR\_PRESOLVE\_TOL\_PRIMAL\_INFEAS\_PERTURBATION} \ \mathtt{1}.$ 

0e-6 file

Groups Presolve

### MSK\_DPAR\_PRESOLVE\_TOL\_REL\_LINDEP

Relative tolerance employed by the linear dependency checker.

**Default** 1.0e-10

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_PRESOLVE\_TOL\_REL\_LINDEP 1.0e-10 file

Groups Presolve

## MSK\_DPAR\_PRESOLVE\_TOL\_S

Absolute zero tolerance employed for  $s_i$  in the presolve.

Default 1.0e-8

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_PRESOLVE\_TOL\_S 1.0e-8 file

Groups Presolve

# MSK\_DPAR\_PRESOLVE\_TOL\_X

Absolute zero tolerance employed for  $x_i$  in the presolve.

Default 1.0e-8

Accepted [0.0; +inf]

Example mosek -d MSK\_DPAR\_PRESOLVE\_TOL\_X 1.0e-8 file

Groups Presolve

### MSK\_DPAR\_QCQO\_REFORMULATE\_REL\_DROP\_TOL

This parameter determines when columns are dropped in incomplete Cholesky factorization during reformulation of quadratic problems.

Default 1e-15

**Accepted**  $[0; +\inf]$ 

Example mosek -d MSK\_DPAR\_QCQO\_REFORMULATE\_REL\_DROP\_TOL 1e-15 file

 ${\bf Groups} \ \, \textit{Interior-point method} \, \,$ 

# MSK\_DPAR\_SEMIDEFINITE\_TOL\_APPROX

Tolerance to define a matrix to be positive semidefinite.

**Default** 1.0e-10

Accepted [1.0e-15; +inf]

Example mosek -d MSK\_DPAR\_SEMIDEFINITE\_TOL\_APPROX 1.0e-10 file

Groups Data check

## MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV

Relative pivot tolerance employed when computing the LU factorization of the basis in the simplex optimizers and in the basis identification procedure. A value closer to 1.0 generally improves numerical stability but typically also implies an increase in the computational work.

Default 0.01

**Accepted** [1.0e-6; 0.999999]

Example mosek -d MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV 0.01 file

Groups Basis identification, Simplex optimizer

#### MSK\_DPAR\_SIMPLEX\_ABS\_TOL\_PIV

Absolute pivot tolerance employed by the simplex optimizers.

Default 1.0e-7

Accepted [1.0e-12; +inf]

Example mosek -d MSK\_DPAR\_SIMPLEX\_ABS\_TOL\_PIV 1.0e-7 file

Groups Simplex optimizer

## MSK\_DPAR\_UPPER\_OBJ\_CUT

If either a primal or dual feasible solution is found proving that the optimal objective value is outside the interval [ MSK\_DPAR\_LOWER\_OBJ\_CUT, MSK\_DPAR\_UPPER\_OBJ\_CUT ], then MOSEK is terminated.

**Default** 1.0e30

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_DPAR\_UPPER\_OBJ\_CUT 1.0e30 file

See also MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH

Groups Termination criteria

## MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH

If the upper objective cut is greater than the value of this parameter, then the upper objective cut  $MSK\_DPAR\_UPPER\_OBJ\_CUT$  is treated as  $\infty$ .

Default 0.5e30

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH 0.5e30 file

Groups Termination criteria

# 11.2.2 Integer parameters

### MSK\_IPAR\_ANA\_SOL\_BASIS

Controls whether the basis matrix is analyzed in solution analyzer.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_ANA\_SOL\_BASIS MSK\_ON file

Groups Analysis

### MSK\_IPAR\_ANA\_SOL\_PRINT\_VIOLATED

A parameter of the problem analyzer. Controls whether a list of violated constraints is printed. All constraints violated by more than the value set by the parameter  $MSK\_DPAR\_ANA\_SOL\_INFEAS\_TOL$  will be printed.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_ANA\_SOL\_PRINT\_VIOLATED MSK\_OFF file

Groups Analysis

### MSK\_IPAR\_AUTO\_SORT\_A\_BEFORE\_OPT

Controls whether the elements in each column of A are sorted before an optimization is performed. This is not required but makes the optimization more deterministic.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_AUTO\_SORT\_A\_BEFORE\_OPT MSK\_OFF file

Groups Debugging

### MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO

Controls whether the solution information items are automatically updated after an optimization is performed.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO MSK\_OFF file

Groups Overall system

## MSK\_IPAR\_BASIS\_SOLVE\_USE\_PLUS\_ONE

If a slack variable is in the basis, then the corresponding column in the basis is a unit vector with -1 in the right position. However, if this parameter is set to  $MSK_ON$ , -1 is replaced by 1.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_BASIS\_SOLVE\_USE\_PLUS\_ONE MSK\_OFF file

Groups Simplex optimizer

### MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER

Controls which simplex optimizer is used in the clean-up phase. Anything else than  $MSK\_OPTIMIZER\_PRIMAL\_SIMPLEX$  or  $MSK\_OPTIMIZER\_DUAL\_SIMPLEX$  is equivalent to  $MSK\_OPTIMIZER\_FREE\_SIMPLEX$ .

Default FREE

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT

Example mosek -d MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER MSK\_OPTIMIZER\_FREE file

**Groups** Basis identification, Overall solver

### MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER

If the parameter  $MSK\_IPAR\_INTPNT\_BASIS$  has the value  $MSK\_BI\_NO\_ERROR$  and the interior-point optimizer has terminated due to maximum number of iterations, then basis identification is performed if this parameter has the value  $MSK\_ON$ .

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER MSK\_OFF file

**Groups** Interior-point method, Basis identification

### MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR

If the parameter  $MSK\_IPAR\_INTPNT\_BASIS$  has the value  $MSK\_BI\_NO\_ERROR$  and the interior-point optimizer has terminated due to a numerical problem, then basis identification is performed if this parameter has the value  $MSK\_ON$ .

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR MSK\_OFF file

**Groups** Interior-point method, Basis identification

#### MSK\_IPAR\_BI\_MAX\_ITERATIONS

Controls the maximum number of simplex iterations allowed to optimize a basis after the basis identification.

**Default** 1000000

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_BI\_MAX\_ITERATIONS 1000000 file

Groups Basis identification, Termination criteria

#### MSK\_IPAR\_CACHE\_LICENSE

Specifies if the license is kept checked out for the lifetime of the **MOSEK** environment/model/process  $(MSK_-ON)$  or returned to the server immediately after the optimization  $(MSK_-OFF)$ .

Check-in and check-out of licenses have an overhead. Frequent communication with the license server should be avoided.

```
Default ON
```

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_CACHE\_LICENSE MSK\_ON file

Groups License manager

#### MSK\_IPAR\_CHECK\_CONVEXITY

Specify the level of convexity check on quadratic problems.

Default FULL

Accepted NONE, SIMPLE, FULL

Example mosek -d MSK\_IPAR\_CHECK\_CONVEXITY MSK\_CHECK\_CONVEXITY\_FULL file

Groups Data check

### MSK\_IPAR\_COMPRESS\_STATFILE

Control compression of stat files.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_COMPRESS\_STATFILE MSK\_ON file

### MSK\_IPAR\_INFEAS\_GENERIC\_NAMES

Controls whether generic names are used when an infeasible subproblem is created.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_INFEAS\_GENERIC\_NAMES MSK\_OFF file

Groups Infeasibility report

### MSK\_IPAR\_INFEAS\_PREFER\_PRIMAL

If both certificates of primal and dual infeasibility are supplied then only the primal is used when this option is turned on.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_INFEAS\_PREFER\_PRIMAL MSK\_ON file

Groups Overall solver

#### MSK\_IPAR\_INFEAS\_REPORT\_AUTO

Controls whether an infeasibility report is automatically produced after the optimization if the problem is primal or dual infeasible.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_INFEAS\_REPORT\_AUTO MSK\_OFF file

Groups Data input/output, Solution input/output

### MSK\_IPAR\_INFEAS\_REPORT\_LEVEL

Controls the amount of information presented in an infeasibility report. Higher values imply more information.

Default 1

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_INFEAS\_REPORT\_LEVEL 1 file

Groups Infeasibility report, Output information

#### MSK\_IPAR\_INTPNT\_BASIS

Controls whether the interior-point optimizer also computes an optimal basis.

Default ALWAYS

Accepted NEVER, ALWAYS, NO\_ERROR, IF\_FEASIBLE, RESERVERED

```
Example mosek -d MSK_IPAR_INTPNT_BASIS MSK_BI_ALWAYS file
```

See also MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER, MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR, MSK\_IPAR\_BI\_MAX\_ITERATIONS, MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER

Groups Interior-point method, Basis identification

#### MSK\_IPAR\_INTPNT\_DIFF\_STEP

Controls whether different step sizes are allowed in the primal and dual space.

## Default ON

# Accepted

- ON: Different step sizes are allowed.
- OFF: Different step sizes are not allowed.

Example mosek -d MSK\_IPAR\_INTPNT\_DIFF\_STEP MSK\_ON file

Groups Interior-point method

### MSK\_IPAR\_INTPNT\_HOTSTART

Currently not in use.

Default NONE

Accepted NONE, PRIMAL, DUAL, PRIMAL\_DUAL

Example mosek -d MSK\_IPAR\_INTPNT\_HOTSTART MSK\_INTPNT\_HOTSTART\_NONE file

Groups Interior-point method

#### MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS

Controls the maximum number of iterations allowed in the interior-point optimizer.

Default 400

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS 400 file

Groups Interior-point method, Termination criteria

### MSK\_IPAR\_INTPNT\_MAX\_NUM\_COR

Controls the maximum number of correctors allowed by the multiple corrector procedure. A negative value means that **MOSEK** is making the choice.

#### Default -1

Accepted  $[-1; +\inf]$ 

Example mosek -d MSK\_IPAR\_INTPNT\_MAX\_NUM\_COR -1 file

**Groups** Interior-point method

### MSK\_IPAR\_INTPNT\_MAX\_NUM\_REFINEMENT\_STEPS

Maximum number of steps to be used by the iterative refinement of the search direction. A negative value implies that the optimizer chooses the maximum number of iterative refinement steps.

#### Default -1

Accepted [-inf; +inf]

Example mosek -d MSK\_IPAR\_INTPNT\_MAX\_NUM\_REFINEMENT\_STEPS -1 file

**Groups** Interior-point method

## MSK\_IPAR\_INTPNT\_OFF\_COL\_TRH

Controls how many offending columns are detected in the Jacobian of the constraint matrix.

0	no detection
1	aggressive detection
> 1	higher values mean less aggressive detection

### Default 40

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_INTPNT\_OFF\_COL\_TRH 40 file

#### Groups Interior-point method

### MSK\_IPAR\_INTPNT\_ORDER\_GP\_NUM\_SEEDS

The GP ordering is dependent on a random seed. Therefore, trying several random seeds may lead to a better ordering. This parameter controls the number of random seeds tried.

A value of 0 means that MOSEK makes the choice.

Default 0

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_INTPNT\_ORDER\_GP\_NUM\_SEEDS 0 file

Groups Interior-point method

### MSK\_IPAR\_INTPNT\_ORDER\_METHOD

Controls the ordering strategy used by the interior-point optimizer when factorizing the Newton equation system.

Default FREE

Accepted FREE, APPMINLOC, EXPERIMENTAL, TRY\_GRAPHPAR, FORCE\_GRAPHPAR, NONE

Example mosek -d MSK\_IPAR\_INTPNT\_ORDER\_METHOD MSK\_ORDER\_METHOD\_FREE file

Groups Interior-point method

### MSK\_IPAR\_INTPNT\_PURIFY

Currently not in use.

Default NONE

Accepted NONE, PRIMAL, DUAL, PRIMAL\_DUAL, AUTO

Example mosek -d MSK\_IPAR\_INTPNT\_PURIFY MSK\_PURIFY\_NONE file

Groups Interior-point method

## MSK\_IPAR\_INTPNT\_REGULARIZATION\_USE

Controls whether regularization is allowed.

Default ON

Accepted ON, OFF

 $\mathbf{Example} \ \mathtt{mosek} \ \mathtt{-d} \ \mathtt{MSK\_IPAR\_INTPNT\_REGULARIZATION\_USE} \ \mathtt{MSK\_ON} \ \mathtt{file}$ 

 ${\bf Groups} \ \, \textit{Interior-point method} \, \,$ 

# MSK\_IPAR\_INTPNT\_SCALING

Controls how the problem is scaled before the interior-point optimizer is used.

Default FREE

Accepted FREE, NONE

Example mosek -d MSK\_IPAR\_INTPNT\_SCALING MSK\_SCALING\_FREE file

Groups Interior-point method

## MSK\_IPAR\_INTPNT\_SOLVE\_FORM

Controls whether the primal or the dual problem is solved.

Default FREE

Accepted FREE, PRIMAL, DUAL

Example mosek -d MSK\_IPAR\_INTPNT\_SOLVE\_FORM MSK\_SOLVE\_FREE file

Groups Interior-point method

# MSK\_IPAR\_INTPNT\_STARTING\_POINT

Starting point used by the interior-point optimizer.

Default FREE

Accepted FREE, GUESS, CONSTANT, SATISFY\_BOUNDS

```
Example mosek -d MSK_IPAR_INTPNT_STARTING_POINT MSK_STARTING_POINT_FREE file
```

Groups Interior-point method

# MSK\_IPAR\_LICENSE\_DEBUG

This option is used to turn on debugging of the license manager.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_LICENSE\_DEBUG MSK\_OFF file

Groups License manager

## MSK\_IPAR\_LICENSE\_PAUSE\_TIME

If  $MSK\_IPAR\_LICENSE\_WAIT$  is  $MSK\_ON$  and no license is available, then  $\mathbf{MOSEK}$  sleeps a number of milliseconds between each check of whether a license has become free.

Default 100

Accepted [0; 1000000]

Example mosek -d MSK\_IPAR\_LICENSE\_PAUSE\_TIME 100 file

Groups License manager

### MSK\_IPAR\_LICENSE\_SUPPRESS\_EXPIRE\_WRNS

Controls whether license features expire warnings are suppressed.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_LICENSE\_SUPPRESS\_EXPIRE\_WRNS MSK\_OFF file

Groups License manager, Output information

#### MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN

If a license feature expires in a numbers of days less than the value of this parameter then a warning will be issued.

Default 7

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN 7 file

Groups License manager, Output information

### MSK\_IPAR\_LICENSE\_WAIT

If all licenses are in use **MOSEK** returns with an error code. However, by turning on this parameter **MOSEK** will wait for an available license.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_LICENSE\_WAIT MSK\_OFF file

**Groups** Overall solver, Overall system, License manager

#### MSK\_IPAR\_LOG

Controls the amount of log information. The value 0 implies that all log information is suppressed. A higher level implies that more information is logged.

Please note that if a task is employed to solve a sequence of optimization problems the value of this parameter is reduced by the value of  $MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT$  for the second and any subsequent optimizations.

Default 10

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG 10 file

See also MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT

Groups Output information, Logging

#### MSK\_IPAR\_LOG\_ANA\_PRO

Controls amount of output from the problem analyzer.

Default 1

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_ANA\_PRO 1 file

Groups Analysis, Logging

### MSK\_IPAR\_LOG\_BI

Controls the amount of output printed by the basis identification procedure. A higher level implies that more information is logged.

Default 1

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_BI 1 file

**Groups** Basis identification, Output information, Logging

### MSK\_IPAR\_LOG\_BI\_FREQ

Controls how frequently the optimizer outputs information about the basis identification and how frequent the user-defined callback function is called.

Default 2500

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_BI\_FREQ 2500 file

**Groups** Basis identification, Output information, Logging

#### MSK\_IPAR\_LOG\_CHECK\_CONVEXITY

Controls logging in convexity check on quadratic problems. Set to a positive value to turn logging on. If a quadratic coefficient matrix is found to violate the requirement of PSD (NSD) then a list of negative (positive) pivot elements is printed. The absolute value of the pivot elements is also shown.

Default 0

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_CHECK\_CONVEXITY 0 file

Groups Data check

### MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT

If a task is employed to solve a sequence of optimization problems, then the value of the log levels is reduced by the value of this parameter. E.g  $MSK\_IPAR\_LOG$  and  $MSK\_IPAR\_LOG\_SIM$  are reduced by the value of this parameter for the second and any subsequent optimizations.

Default 1

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT 1 file

See also MSK\_IPAR\_LOG, MSK\_IPAR\_LOG\_INTPNT, MSK\_IPAR\_LOG\_MIO, MSK\_IPAR\_LOG\_SIM

Groups Output information, Logging

## MSK\_IPAR\_LOG\_EXPAND

Controls the amount of logging when a data item such as the maximum number constrains is expanded.

**Default** 0

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_EXPAND 0 file

Groups Output information, Logging

#### MSK\_IPAR\_LOG\_FEAS\_REPAIR

Controls the amount of output printed when performing feasibility repair. A value higher than one means extensive logging.

Default 1

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_FEAS\_REPAIR 1 file

Groups Output information, Logging

### MSK\_IPAR\_LOG\_FILE

If turned on, then some log info is printed when a file is written or read.

Default 1

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_FILE 1 file

Groups Data input/output, Output information, Logging

### MSK\_IPAR\_LOG\_INCLUDE\_SUMMARY

Not relevant for this API.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_LOG\_INCLUDE\_SUMMARY MSK\_OFF file

Groups Output information, Logging

### MSK\_IPAR\_LOG\_INFEAS\_ANA

Controls amount of output printed by the infeasibility analyzer procedures. A higher level implies that more information is logged.

Default 1

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_INFEAS\_ANA 1 file

Groups Infeasibility report, Output information, Logging

### MSK\_IPAR\_LOG\_INTPNT

Controls amount of output printed by the interior-point optimizer. A higher level implies that more information is logged.

Default 1

**Accepted**  $[0; +\inf]$ 

Example mosek -d MSK\_IPAR\_LOG\_INTPNT 1 file

Groups Interior-point method, Output information, Logging

### MSK\_IPAR\_LOG\_LOCAL\_INFO

Controls whether local identifying information like environment variables, filenames, IP addresses etc. are printed to the log.

Note that this will only affect some functions. Some functions that specifically emit system information will not be affected.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_LOG\_LOCAL\_INFO MSK\_ON file

Groups Output information, Logging

## MSK\_IPAR\_LOG\_MIO

Controls the log level for the mixed-integer optimizer. A higher level implies that more information is logged.

 ${\bf Default}\ 4$ 

Accepted [0; +inf]

```
Example mosek -d MSK_IPAR_LOG_MIO 4 file
```

Groups Mixed-integer optimization, Output information, Logging

#### MSK\_IPAR\_LOG\_MIO\_FREQ

Controls how frequent the mixed-integer optimizer prints the log line. It will print line every time  $MSK\_IPAR\_LOG\_MIO\_FREQ$  relaxations have been solved.

Default 10

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_IPAR\_LOG\_MIO\_FREQ 10 file

Groups Mixed-integer optimization, Output information, Logging

#### MSK\_IPAR\_LOG\_ORDER

If turned on, then factor lines are added to the log.

Default 1

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_ORDER 1 file

Groups Output information, Logging

### MSK\_IPAR\_LOG\_PRESOLVE

Controls amount of output printed by the presolve procedure. A higher level implies that more information is logged.

Default 1

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_PRESOLVE 1 file

Groups Logging

### MSK\_IPAR\_LOG\_RESPONSE

Controls amount of output printed when response codes are reported. A higher level implies that more information is logged.

Default 0

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_RESPONSE O file

Groups Output information, Logging

### MSK\_IPAR\_LOG\_SENSITIVITY

Controls the amount of logging during the sensitivity analysis.

- 0. Means no logging information is produced.
- 1. Timing information is printed.
- 2. Sensitivity results are printed.

Default 1

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_SENSITIVITY 1 file

Groups Output information, Logging

#### MSK\_IPAR\_LOG\_SENSITIVITY\_OPT

Controls the amount of logging from the optimizers employed during the sensitivity analysis. 0 means no logging information is produced.

Default 0

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_SENSITIVITY\_OPT 0 file

Groups Output information, Logging

#### MSK\_IPAR\_LOG\_SIM

Controls amount of output printed by the simplex optimizer. A higher level implies that more information is logged.

Default 4

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_SIM 4 file

Groups Simplex optimizer, Output information, Logging

### MSK\_IPAR\_LOG\_SIM\_FREQ

Controls how frequent the simplex optimizer outputs information about the optimization and how frequent the user-defined callback function is called.

Default 1000

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_SIM\_FREQ 1000 file

Groups Simplex optimizer, Output information, Logging

#### MSK\_IPAR\_LOG\_SIM\_MINOR

Currently not in use.

Default 1

**Accepted**  $[0; +\inf]$ 

Example mosek -d MSK\_IPAR\_LOG\_SIM\_MINOR 1 file

**Groups** Simplex optimizer, Output information

#### MSK\_IPAR\_LOG\_STORAGE

When turned on, MOSEK prints messages regarding the storage usage and allocation.

Default 0

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_LOG\_STORAGE 0 file

Groups Output information, Overall system, Logging

#### MSK\_IPAR\_MAX\_NUM\_WARNINGS

Each warning is shown a limited number of times controlled by this parameter. A negative value is identical to infinite number of times.

Default 10

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_IPAR\_MAX\_NUM\_WARNINGS 10 file

Groups Output information

### MSK\_IPAR\_MIO\_BRANCH\_DIR

Controls whether the mixed-integer optimizer is branching up or down by default.

Default FREE

Accepted FREE, UP, DOWN, NEAR, FAR, ROOT\_LP, GUIDED, PSEUDOCOST

Example mosek -d MSK\_IPAR\_MIO\_BRANCH\_DIR MSK\_BRANCH\_DIR\_FREE file

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_CONIC\_OUTER\_APPROXIMATION

If this option is turned on outer approximation is used when solving relaxations of conic problems; otherwise interior point is used.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_MIO\_CONIC\_OUTER\_APPROXIMATION MSK\_OFF file

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_CONSTRUCT\_SOL

If set to  $MSK_ON$  and all integer variables have been given a value for which a feasible mixed integer solution exists, then MOSEK generates an initial solution to the mixed integer problem by fixing all integer values and solving the remaining problem.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_MIO\_CONSTRUCT\_SOL MSK\_OFF file

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_CUT\_CLIQUE

Controls whether clique cuts should be generated.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_MIO\_CUT\_CLIQUE MSK\_ON file

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_CUT\_CMIR

Controls whether mixed integer rounding cuts should be generated.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_MIO\_CUT\_CMIR MSK\_ON file

**Groups** Mixed-integer optimization

#### MSK\_IPAR\_MIO\_CUT\_GMI

Controls whether GMI cuts should be generated.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_MIO\_CUT\_GMI MSK\_ON file

 ${\bf Groups}\ \textit{Mixed-integer optimization}$ 

### MSK\_IPAR\_MIO\_CUT\_IMPLIED\_BOUND

Controls whether implied bound cuts should be generated.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_MIO\_CUT\_IMPLIED\_BOUND MSK\_ON file

**Groups** Mixed-integer optimization

## MSK\_IPAR\_MIO\_CUT\_KNAPSACK\_COVER

Controls whether knapsack cover cuts should be generated.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_MIO\_CUT\_KNAPSACK\_COVER MSK\_OFF file

**Groups** Mixed-integer optimization

## MSK\_IPAR\_MIO\_CUT\_LIPRO

Controls whether lift-and-project cuts should be generated.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_MIO\_CUT\_LIPRO MSK\_OFF file

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_CUT\_SELECTION\_LEVEL

Controls how aggressively generated cuts are selected to be included in the relaxation.

- $\bullet$  -1. The optimizer chooses the level of cut selection
- 0. Generated cuts less likely to be added to the relaxation
- 1. Cuts are more aggressively selected to be included in the relaxation

Default -1

Accepted [-1; +1]

Example mosek -d MSK\_IPAR\_MIO\_CUT\_SELECTION\_LEVEL -1 file

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_DATA\_PERMUTATION\_METHOD

Controls what problem data permutation method is appplied to mixed-integer problems.

Default NONE

Accepted NONE, CYCLIC\_SHIFT, RANDOM

Example mosek -d MSK\_IPAR\_MIO\_DATA\_PERMUTATION\_METHOD MSK\_MIO\_DATA\_PERMUTATION\_METHOD\_NONE file

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_FEASPUMP\_LEVEL

Controls the way the Feasibility Pump heuristic is employed by the mixed-integer optimizer.

- $\bullet$  -1. The optimizer chooses how the Feasibility Pump is used
- 0. The Feasibility Pump is disabled
- 1. The Feasibility Pump is enabled with an effort to improve solution quality
- 2. The Feasibility Pump is enabled with an effort to reach feasibility early

Default -1

Accepted [-1; 2]

Example mosek -d MSK\_IPAR\_MIO\_FEASPUMP\_LEVEL -1 file

**Groups** Mixed-integer optimization

## MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL

Controls the heuristic employed by the mixed-integer optimizer to locate an initial good integer feasible solution. A value of zero means the heuristic is not used at all. A larger value than 0 means that a gradually more sophisticated heuristic is used which is computationally more expensive. A negative value implies that the optimizer chooses the heuristic. Normally a value around 3 to 5 should be optimal.

Default -1

**Accepted** [-inf; +inf]

Example mosek -d MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL -1 file

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES

Maximum number of branches allowed during the branch and bound search. A negative value means infinite.

Default -1

Accepted  $[-\inf; +\inf]$ 

 $\mathbf{Example} \ \mathtt{mosek} \ \mathtt{-d} \ \mathtt{MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES} \ \mathtt{-1} \ \mathtt{file}$ 

Groups Mixed-integer optimization, Termination criteria

## MSK\_IPAR\_MIO\_MAX\_NUM\_RELAXS

Maximum number of relaxations allowed during the branch and bound search. A negative value means infinite.

Default -1

Accepted  $[-\inf; +\inf]$ 

```
Example mosek -d MSK_IPAR_MIO_MAX_NUM_RELAXS -1 file
```

**Groups** Mixed-integer optimization

#### MSK\_IPAR\_MIO\_MAX\_NUM\_ROOT\_CUT\_ROUNDS

Maximum number of cut separation rounds at the root node.

Default 100

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_MIO\_MAX\_NUM\_ROOT\_CUT\_ROUNDS 100 file

Groups Mixed-integer optimization, Termination criteria

### MSK\_IPAR\_MIO\_MAX\_NUM\_SOLUTIONS

The mixed-integer optimizer can be terminated after a certain number of different feasible solutions has been located. If this parameter has the value n > 0, then the mixed-integer optimizer will be terminated when n feasible solutions have been located.

Default -1

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_IPAR\_MIO\_MAX\_NUM\_SOLUTIONS -1 file

Groups Mixed-integer optimization, Termination criteria

### MSK\_IPAR\_MIO\_MEMORY\_EMPHASIS\_LEVEL

Controls how much emphasis is put on reducing memory usage. Being more conservative about memory usage may come at the cost of decreased solution speed.

- 0. The optimizer chooses
- 1. More emphasis is put on reducing memory usage and less on speed

Default 0

Accepted [0; +1]

Example mosek -d MSK\_IPAR\_MIO\_MEMORY\_EMPHASIS\_LEVEL 0 file

 ${\bf Groups} \ \textit{Mixed-integer optimization}$ 

## MSK\_IPAR\_MIO\_MODE

Controls whether the optimizer includes the integer restrictions and disjunctive constraints when solving a (mixed) integer optimization problem.

Default SATISFIED

Accepted IGNORED, SATISFIED

Example mosek -d MSK\_IPAR\_MIO\_MODE MSK\_MIO\_MODE\_SATISFIED file

Groups Overall solver

# MSK\_IPAR\_MIO\_NODE\_OPTIMIZER

Controls which optimizer is employed at the non-root nodes in the mixed-integer optimizer.

Default FREE

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT

Example mosek -d MSK\_IPAR\_MIO\_NODE\_OPTIMIZER MSK\_OPTIMIZER\_FREE file

Groups Mixed-integer optimization

## MSK\_IPAR\_MIO\_NODE\_SELECTION

Controls the node selection strategy employed by the mixed-integer optimizer.

Default FREE

Accepted FREE, FIRST, BEST, PSEUDO

Example mosek -d MSK\_IPAR\_MIO\_NODE\_SELECTION MSK\_MIO\_NODE\_SELECTION\_FREE file

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_NUMERICAL\_EMPHASIS\_LEVEL

Controls how much emphasis is put on reducing numerical problems possibly at the expense of solution speed.

- 0. The optimizer chooses
- 1. More emphasis is put on reducing numerical problems
- 2. Even more emphasis

### Default 0

Accepted [0; +2]

Example mosek -d MSK\_IPAR\_MIO\_NUMERICAL\_EMPHASIS\_LEVEL 0 file

**Groups** Mixed-integer optimization

#### MSK\_IPAR\_MIO\_PERSPECTIVE\_REFORMULATE

Enables or disables perspective reformulation in presolve.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_MIO\_PERSPECTIVE\_REFORMULATE MSK\_ON file

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_PRESOLVE\_AGGREGATOR\_USE

Controls if the aggregator should be used.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_MIO\_PRESOLVE\_AGGREGATOR\_USE MSK\_ON file

Groups Presolve

#### MSK\_IPAR\_MIO\_PROBING\_LEVEL

Controls the amount of probing employed by the mixed-integer optimizer in presolve.

- $\bullet$  -1. The optimizer chooses the level of probing employed
- 0. Probing is disabled
- 1. A low amount of probing is employed
- 2. A medium amount of probing is employed
- 3. A high amount of probing is employed

## Default -1

Accepted [-1; 3]

Example mosek -d MSK\_IPAR\_MIO\_PROBING\_LEVEL -1 file

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_PROPAGATE\_OBJECTIVE\_CONSTRAINT

Use objective domain propagation.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_MIO\_PROPAGATE\_OBJECTIVE\_CONSTRAINT MSK\_OFF

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_QCQO\_REFORMULATION\_METHOD

Controls what reformulation method is applied to mixed-integer quadratic problems.

Default FREE

Accepted FREE, NONE, LINEARIZATION, EIGEN\_VAL\_METHOD, DIAG\_SDP, RELAX\_SDP

Example mosek -d MSK\_IPAR\_MIO\_QCQO\_REFORMULATION\_METHOD MSK\_MIO\_QCQO\_REFORMULATION\_METHOD\_FREE file

#### **Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_RINS\_MAX\_NODES

Controls the maximum number of nodes allowed in each call to the RINS heuristic. The default value of -1 means that the value is determined automatically. A value of zero turns off the heuristic.

#### Default -1

Accepted  $[-1; +\inf]$ 

Example mosek -d MSK\_IPAR\_MIO\_RINS\_MAX\_NODES -1 file

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_ROOT\_OPTIMIZER

Controls which optimizer is employed at the root node in the mixed-integer optimizer.

#### Default FREE

```
Accepted FREE, INTPNT, CONIC, PRIMAL_SIMPLEX, DUAL_SIMPLEX, FREE_SIMPLEX, MIXED INT
```

Example mosek -d MSK\_IPAR\_MIO\_ROOT\_OPTIMIZER MSK\_OPTIMIZER\_FREE file Groups Mixed-integer optimization

### MSK\_IPAR\_MIO\_ROOT\_REPEAT\_PRESOLVE\_LEVEL

Controls whether presolve can be repeated at root node.

- $\bullet$  -1. The optimizer chooses whether presolve is repeated
- 0. Never repeat presolve
- 1. Always repeat presolve

### Default -1

Accepted [-1; 1]

Example mosek -d MSK\_IPAR\_MIO\_ROOT\_REPEAT\_PRESOLVE\_LEVEL -1 file

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_SEED

Sets the random seed used for randomization in the mixed integer optimizer. Selecting a different seed can change the path the optimizer takes to the optimal solution.

## Default 42

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_MIO\_SEED 42 file

 ${\bf Groups}\ {\it Mixed-integer}\ optimization$ 

## MSK\_IPAR\_MIO\_SYMMETRY\_LEVEL

Controls the amount of symmetry detection and handling employed by the mixed-integer optimizer in presolve.

- -1. The optimizer chooses the level of symmetry detection and handling employed
- 0. Symmetry detection and handling is disabled
- 1. A low amount of symmetry detection and handling is employed
- 2. A medium amount of symmetry detection and handling is employed
- 3. A high amount of symmetry detection and handling is employed
- 4. An extremely high amount of symmetry detection and handling is employed

### Default -1

Accepted [-1; 4]

Example mosek -d MSK\_IPAR\_MIO\_SYMMETRY\_LEVEL -1 file

**Groups** Mixed-integer optimization

### MSK\_IPAR\_MIO\_VB\_DETECTION\_LEVEL

Controls how much effort is put into detecting variable bounds.

- $\bullet$  -1. The optimizer chooses
- 0. No variable bounds are detected
- 1. Only detect variable bounds that are directly represented in the problem
- 2. Detect variable bounds in probing

#### Default -1

Accepted [-1; +2]

Example mosek -d MSK\_IPAR\_MIO\_VB\_DETECTION\_LEVEL -1 file

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MT\_SPINCOUNT

Set the number of iterations to spin before sleeping.

#### Default 0

**Accepted** [0; 1000000000]

Example mosek -d MSK\_IPAR\_MT\_SPINCOUNT O file

Groups Overall system

### MSK\_IPAR\_NG

Not in use.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_NG MSK\_OFF file

### MSK\_IPAR\_NUM\_THREADS

Controls the number of threads employed by the optimizer. If set to 0 the number of threads used will be equal to the number of cores detected on the machine.

#### Default 0

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_NUM\_THREADS 0 file

Groups Overall system

## MSK\_IPAR\_OPF\_WRITE\_HEADER

Write a text header with date and MOSEK version in an OPF file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_OPF\_WRITE\_HEADER MSK\_ON file

Groups Data input/output

### MSK\_IPAR\_OPF\_WRITE\_HINTS

Write a hint section with problem dimensions in the beginning of an OPF file.

Default ON

Accepted ON, OFF

 $\mathbf{Example} \ \mathtt{mosek} \ \mathtt{-d} \ \mathtt{MSK\_IPAR\_OPF\_WRITE\_HINTS} \ \mathtt{MSK\_ON} \ \mathtt{file}$ 

Groups Data input/output

# MSK\_IPAR\_OPF\_WRITE\_LINE\_LENGTH

Aim to keep lines in OPF files not much longer than this.

Default 80

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_OPF\_WRITE\_LINE\_LENGTH 80 file

### Groups Data input/output

### MSK\_IPAR\_OPF\_WRITE\_PARAMETERS

Write a parameter section in an OPF file.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_OPF\_WRITE\_PARAMETERS MSK\_OFF file

Groups Data input/output

## MSK\_IPAR\_OPF\_WRITE\_PROBLEM

Write objective, constraints, bounds etc. to an OPF file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_OPF\_WRITE\_PROBLEM MSK\_ON file

Groups Data input/output

### MSK\_IPAR\_OPF\_WRITE\_SOL\_BAS

If  $MSK\_IPAR\_OPF\_WRITE\_SOLUTIONS$  is  $MSK\_ON$  and a basic solution is defined, include the basic solution in OPF files.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_OPF\_WRITE\_SOL\_BAS MSK\_ON file

Groups Data input/output

### MSK\_IPAR\_OPF\_WRITE\_SOL\_ITG

If  $MSK\_IPAR\_OPF\_WRITE\_SOLUTIONS$  is  $MSK\_ON$  and an integer solution is defined, write the integer solution in OPF files.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_OPF\_WRITE\_SOL\_ITG MSK\_ON file

Groups Data input/output

### MSK\_IPAR\_OPF\_WRITE\_SOL\_ITR

If  $MSK\_IPAR\_OPF\_WRITE\_SOLUTIONS$  is  $MSK\_ON$  and an interior solution is defined, write the interior solution in OPF files.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_OPF\_WRITE\_SOL\_ITR MSK\_ON file

Groups Data input/output

## MSK\_IPAR\_OPF\_WRITE\_SOLUTIONS

Enable inclusion of solutions in the OPF files.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_OPF\_WRITE\_SOLUTIONS MSK\_OFF file

Groups Data input/output

### MSK\_IPAR\_OPTIMIZER

The parameter controls which optimizer is used to optimize the task.

Default FREE

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT

Example mosek -d MSK\_IPAR\_OPTIMIZER MSK\_OPTIMIZER\_FREE file

## Groups Overall solver

### MSK\_IPAR\_PARAM\_READ\_CASE\_NAME

If turned on, then names in the parameter file are case sensitive.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_PARAM\_READ\_CASE\_NAME MSK\_ON file

Groups Data input/output

## MSK\_IPAR\_PARAM\_READ\_IGN\_ERROR

If turned on, then errors in parameter settings is ignored.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_PARAM\_READ\_IGN\_ERROR MSK\_OFF file

Groups Data input/output

### MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_FILL

Controls the maximum amount of fill-in that can be created by one pivot in the elimination phase of the presolve. A negative value means the parameter value is selected automatically.

Default -1

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_FILL -1 file

Groups Presolve

## MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES

Control the maximum number of times the eliminator is tried. A negative value implies MOSEK decides.

Default -1

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES -1 file

Groups Presolve

## MSK\_IPAR\_PRESOLVE\_LEVEL

Currently not used.

Default -1

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_IPAR\_PRESOLVE\_LEVEL -1 file

Groups Overall solver, Presolve

### MSK\_IPAR\_PRESOLVE\_LINDEP\_ABS\_WORK\_TRH

Controls linear dependency check in presolve. The linear dependency check is potentially computationally expensive.

Default 100

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_IPAR\_PRESOLVE\_LINDEP\_ABS\_WORK\_TRH 100 file

Groups Presolve

### MSK\_IPAR\_PRESOLVE\_LINDEP\_REL\_WORK\_TRH

Controls linear dependency check in presolve. The linear dependency check is potentially computationally expensive.

 $\textbf{Default} \ 100$ 

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_IPAR\_PRESOLVE\_LINDEP\_REL\_WORK\_TRH 100 file

### Groups Presolve

### MSK\_IPAR\_PRESOLVE\_LINDEP\_USE

Controls whether the linear constraints are checked for linear dependencies.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_PRESOLVE\_LINDEP\_USE MSK\_ON file

Groups Presolve

### MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_PASS

Control the maximum number of times presolve passes over the problem. A negative value implies  $\mathbf{MOSEK}$  decides.

Default -1

Accepted [-inf; +inf]

Example mosek -d MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_PASS -1 file

Groups Presolve

### MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_REDUCTIONS

Controls the maximum number of reductions performed by the presolve. The value of the parameter is normally only changed in connection with debugging. A negative value implies that an infinite number of reductions are allowed.

Default -1

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_REDUCTIONS -1 file

Groups Overall solver, Presolve

#### MSK\_IPAR\_PRESOLVE\_USE

Controls whether the presolve is applied to a problem before it is optimized.

Default FREE

Accepted OFF, ON, FREE

Example mosek -d MSK\_IPAR\_PRESOLVE\_USE MSK\_PRESOLVE\_MODE\_FREE file

Groups Overall solver, Presolve

# MSK\_IPAR\_PRIMAL\_REPAIR\_OPTIMIZER

Controls which optimizer that is used to find the optimal repair.

Default FREE

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX,

Example mosek -d MSK\_IPAR\_PRIMAL\_REPAIR\_OPTIMIZER MSK\_OPTIMIZER\_FREE file

Groups Overall solver

# MSK\_IPAR\_PTF\_WRITE\_PARAMETERS

If  $MSK\_IPAR\_PTF\_WRITE\_PARAMETERS$  is  $MSK\_ON$ , the parameters section is written.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_PTF\_WRITE\_PARAMETERS MSK\_OFF file

Groups Data input/output

#### MSK\_IPAR\_PTF\_WRITE\_SOLUTIONS

If  $MSK\_IPAR\_PTF\_WRITE\_SOLUTIONS$  is  $MSK\_ON$ , the solution section is written if any solutions are available, otherwise solution section is not written even if solutions are available.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_PTF\_WRITE\_SOLUTIONS MSK\_OFF file

Groups Data input/output

### MSK\_IPAR\_PTF\_WRITE\_TRANSFORM

If  $MSK\_IPAR\_PTF\_WRITE\_TRANSFORM$  is  $MSK\_ON$ , constraint blocks with identifiable conic slacks are transformed into conic constraints and the slacks are eliminated.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_PTF\_WRITE\_TRANSFORM MSK\_ON file

Groups Data input/output

### MSK\_IPAR\_READ\_DEBUG

Turns on additional debugging information when reading files.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_READ\_DEBUG MSK\_OFF file

Groups Data input/output

## MSK\_IPAR\_READ\_KEEP\_FREE\_CON

Controls whether the free constraints are included in the problem.

 ${\bf Default} \ \textit{OFF}$ 

# Accepted

- ON: The free constraints are kept.
- OFF: The free constraints are discarded.

Example mosek -d MSK\_IPAR\_READ\_KEEP\_FREE\_CON MSK\_OFF file

Groups Data input/output

### MSK\_IPAR\_READ\_MPS\_FORMAT

Controls how strictly the MPS file reader interprets the MPS format.

 ${\bf Default}\ \textit{FREE}$ 

Accepted STRICT, RELAXED, FREE, CPLEX

Example mosek -d MSK\_IPAR\_READ\_MPS\_FORMAT MSK\_MPS\_FORMAT\_FREE file

Groups Data input/output

### MSK\_IPAR\_READ\_MPS\_WIDTH

Controls the maximal number of characters allowed in one line of the MPS file.

Default 1024

Accepted [80; +inf]

 $\mathbf{Example} \ \mathtt{mosek} \ \mathtt{-d} \ \mathtt{MSK\_IPAR\_READ\_MPS\_WIDTH} \ \mathtt{1024} \ \mathtt{file}$ 

Groups Data input/output

# MSK\_IPAR\_READ\_TASK\_IGNORE\_PARAM

Controls whether  $\mathbf{MOSEK}$  should ignore the parameter setting defined in the task file and use the default parameter setting instead.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_READ\_TASK\_IGNORE\_PARAM MSK\_OFF file

Groups Data input/output

# MSK\_IPAR\_REMOTE\_USE\_COMPRESSION

Use compression when sending data to an optimization server.

Default ZSTD

```
Accepted NONE, FREE, GZIP, ZSTD
```

Example mosek -d MSK\_IPAR\_REMOTE\_USE\_COMPRESSION MSK\_COMPRESS\_ZSTD file

#### MSK\_IPAR\_REMOVE\_UNUSED\_SOLUTIONS

Removes unused solutions before the optimization is performed.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_REMOVE\_UNUSED\_SOLUTIONS MSK\_OFF file

Groups Overall system

### MSK\_IPAR\_SENSITIVITY\_ALL

Not applicable.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_SENSITIVITY\_ALL MSK\_OFF file

Groups Overall solver

### MSK\_IPAR\_SENSITIVITY\_OPTIMIZER

Controls which optimizer is used for optimal partition sensitivity analysis.

Default FREE\_SIMPLEX

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT

Example mosek -d MSK\_IPAR\_SENSITIVITY\_OPTIMIZER MSK\_OPTIMIZER\_FREE\_SIMPLEX file

Groups Overall solver, Simplex optimizer

### MSK\_IPAR\_SENSITIVITY\_TYPE

Controls which type of sensitivity analysis is to be performed.

Default BASIS

Accepted BASIS

Example mosek -d MSK\_IPAR\_SENSITIVITY\_TYPE MSK\_SENSITIVITY\_TYPE\_BASIS file

Groups Overall solver

### MSK\_IPAR\_SIM\_BASIS\_FACTOR\_USE

Controls whether an LU factorization of the basis is used in a hot-start. Forcing a refactorization sometimes improves the stability of the simplex optimizers, but in most cases there is a performance penalty.

Default ON

Accepted ON, OFF

 $\mathbf{Example} \ \mathtt{mosek} \ \mathtt{-d} \ \mathtt{MSK\_IPAR\_SIM\_BASIS\_FACTOR\_USE} \ \mathtt{MSK\_ON} \ \mathtt{file}$ 

Groups Simplex optimizer

## MSK\_IPAR\_SIM\_DEGEN

Controls how aggressively degeneration is handled.

Default FREE

Accepted NONE, FREE, AGGRESSIVE, MODERATE, MINIMUM

Example mosek -d MSK\_IPAR\_SIM\_DEGEN MSK\_SIM\_DEGEN\_FREE file

Groups Simplex optimizer

## MSK\_IPAR\_SIM\_DETECT\_PWL

Not in use.

Default ON

# Accepted

- $\bullet$  ON: PWL are detected.
- OFF: PWL are not detected.

Example mosek -d MSK\_IPAR\_SIM\_DETECT\_PWL MSK\_ON file

Groups Simplex optimizer

#### MSK\_IPAR\_SIM\_DUAL\_CRASH

Controls whether crashing is performed in the dual simplex optimizer. If this parameter is set to x, then a crash will be performed if a basis consists of more than  $(100 - x) \mod f_v$  entries, where  $f_v$  is the number of fixed variables.

Default 90

**Accepted**  $[0; +\inf]$ 

Example mosek -d MSK\_IPAR\_SIM\_DUAL\_CRASH 90 file

Groups Dual simplex

### MSK\_IPAR\_SIM\_DUAL\_PHASEONE\_METHOD

An experimental feature.

Default 0

**Accepted** [0; 10]

Example mosek -d MSK\_IPAR\_SIM\_DUAL\_PHASEONE\_METHOD 0 file

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_DUAL\_RESTRICT\_SELECTION

The dual simplex optimizer can use a so-called restricted selection/pricing strategy to choose the outgoing variable. Hence, if restricted selection is applied, then the dual simplex optimizer first choose a subset of all the potential outgoing variables. Next, for some time it will choose the outgoing variable only among the subset. From time to time the subset is redefined. A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

**Default** 50

**Accepted** [0; 100]

Example mosek -d MSK\_IPAR\_SIM\_DUAL\_RESTRICT\_SELECTION 50 file

Groups Dual simplex

### MSK\_IPAR\_SIM\_DUAL\_SELECTION

Controls the choice of the incoming variable, known as the selection strategy, in the dual simplex optimizer.

Default FREE

Accepted FREE, FULL, ASE, DEVEX, SE, PARTIAL

Example mosek -d MSK\_IPAR\_SIM\_DUAL\_SELECTION MSK\_SIM\_SELECTION\_FREE file

Groups Dual simplex

## MSK\_IPAR\_SIM\_EXPLOIT\_DUPVEC

Controls if the simplex optimizers are allowed to exploit duplicated columns.

Default OFF

Accepted ON, OFF, FREE

Example mosek -d MSK\_IPAR\_SIM\_EXPLOIT\_DUPVEC MSK\_SIM\_EXPLOIT\_DUPVEC\_OFF file

Groups Simplex optimizer

# MSK\_IPAR\_SIM\_HOTSTART

Controls the type of hot-start that the simplex optimizer perform.

Default FREE

```
Accepted NONE, FREE, STATUS_KEYS
```

Example mosek -d MSK\_IPAR\_SIM\_HOTSTART MSK\_SIM\_HOTSTART\_FREE file

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_HOTSTART\_LU

Determines if the simplex optimizer should exploit the initial factorization.

# Default ON

## Accepted

- $\bullet$  ON: Factorization is reused if possible.
- $\bullet$  *OFF*: Factorization is recomputed.

Example mosek -d MSK\_IPAR\_SIM\_HOTSTART\_LU MSK\_ON file

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_MAX\_ITERATIONS

Maximum number of iterations that can be used by a simplex optimizer.

**Default** 10000000

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_SIM\_MAX\_ITERATIONS 10000000 file

Groups Simplex optimizer, Termination criteria

### MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS

Controls how many set-backs are allowed within a simplex optimizer. A set-back is an event where the optimizer moves in the wrong direction. This is impossible in theory but may happen due to numerical problems.

Default 250

**Accepted**  $[0; +\inf]$ 

Example mosek -d MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS 250 file

Groups Simplex optimizer

## MSK\_IPAR\_SIM\_NON\_SINGULAR

Controls if the simplex optimizer ensures a non-singular basis, if possible.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_SIM\_NON\_SINGULAR MSK\_ON file

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_PRIMAL\_CRASH

Controls whether crashing is performed in the primal simplex optimizer. In general, if a basis consists of more than (100-this parameter value)% fixed variables, then a crash will be performed.

Default 90

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_SIM\_PRIMAL\_CRASH 90 file

Groups Primal simplex

### MSK\_IPAR\_SIM\_PRIMAL\_PHASEONE\_METHOD

An experimental feature.

Default 0

**Accepted** [0; 10]

Example mosek -d MSK\_IPAR\_SIM\_PRIMAL\_PHASEONE\_METHOD 0 file

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_PRIMAL\_RESTRICT\_SELECTION

The primal simplex optimizer can use a so-called restricted selection/pricing strategy to choose the outgoing variable. Hence, if restricted selection is applied, then the primal simplex optimizer first choose a subset of all the potential incoming variables. Next, for some time it will choose the incoming variable only among the subset. From time to time the subset is redefined. A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

Default 50
Accepted [0; 100]
Example mosek -d MSK\_IPAR\_SIM\_PRIMAL\_RESTRICT\_SELECTION 50 file
Groups Primal simplex

### MSK\_IPAR\_SIM\_PRIMAL\_SELECTION

Controls the choice of the incoming variable, known as the selection strategy, in the primal simplex optimizer.

Default FREE
Accepted FREE, FULL, ASE, DEVEX, SE, PARTIAL
Example mosek -d MSK\_IPAR\_SIM\_PRIMAL\_SELECTION MSK\_SIM\_SELECTION\_FREE file
Groups Primal simplex

#### MSK\_IPAR\_SIM\_REFACTOR\_FREQ

Controls how frequent the basis is refactorized. The value 0 means that the optimizer determines the best point of refactorization. It is strongly recommended NOT to change this parameter.

 $\begin{tabular}{ll} \textbf{Default 0} \\ \textbf{Accepted [0; +inf]} \\ \textbf{Example mosek -d MSK_IPAR_SIM_REFACTOR\_FREQ 0 file} \\ \textbf{Groups } \textit{Simplex optimizer} \\ \end{tabular}$ 

## MSK\_IPAR\_SIM\_REFORMULATION

Default OFF

Controls if the simplex optimizers are allowed to reformulate the problem.

Accepted ON, OFF, FREE, AGGRESSIVE

Example mosek -d MSK\_IPAR\_SIM\_REFORMULATION MSK\_SIM\_REFORMULATION\_OFF
file

Groups Simplex optimizer

#### MSK\_IPAR\_SIM\_SAVE\_LU

Controls if the LU factorization stored should be replaced with the LU factorization corresponding to the initial basis.

Default OFF
Accepted ON, OFF
Example mosek -d MSK\_IPAR\_SIM\_SAVE\_LU MSK\_OFF file
Groups Simplex optimizer

### MSK\_IPAR\_SIM\_SCALING

Controls how much effort is used in scaling the problem before a simplex optimizer is used.

Default FREE
Accepted FREE, NONE
Example mosek -d MSK\_IPAR\_SIM\_SCALING MSK\_SCALING\_FREE file
Groups Simplex optimizer

### MSK\_IPAR\_SIM\_SCALING\_METHOD

Controls how the problem is scaled before a simplex optimizer is used.

```
Default POW2
```

Accepted POW2, FREE

Example mosek -d MSK\_IPAR\_SIM\_SCALING\_METHOD MSK\_SCALING\_METHOD\_POW2 file

Groups Simplex optimizer

#### MSK\_IPAR\_SIM\_SEED

Sets the random seed used for randomization in the simplex optimizers.

Default 23456

**Accepted** [0; 32749]

Example mosek -d MSK\_IPAR\_SIM\_SEED 23456 file

Groups Simplex optimizer

## MSK\_IPAR\_SIM\_SOLVE\_FORM

Controls whether the primal or the dual problem is solved by the primal-/dual-simplex optimizer.

Default FREE

Accepted FREE, PRIMAL, DUAL

Example mosek -d MSK\_IPAR\_SIM\_SOLVE\_FORM MSK\_SOLVE\_FREE file

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_STABILITY\_PRIORITY

Controls how high priority the numerical stability should be given.

**Default** 50

**Accepted** [0; 100]

Example mosek -d MSK\_IPAR\_SIM\_STABILITY\_PRIORITY 50 file

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_SWITCH\_OPTIMIZER

The simplex optimizer sometimes chooses to solve the dual problem instead of the primal problem. This implies that if you have chosen to use the dual simplex optimizer and the problem is dualized, then it actually makes sense to use the primal simplex optimizer instead. If this parameter is on and the problem is dualized and furthermore the simplex optimizer is chosen to be the primal (dual) one, then it is switched to the dual (primal).

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_SIM\_SWITCH\_OPTIMIZER MSK\_OFF file

Groups Simplex optimizer

# MSK\_IPAR\_SOL\_FILTER\_KEEP\_BASIC

If turned on, then basic and super basic constraints and variables are written to the solution file independent of the filter setting.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_SOL\_FILTER\_KEEP\_BASIC MSK\_OFF file

Groups Solution input/output

### MSK\_IPAR\_SOL\_FILTER\_KEEP\_RANGED

If turned on, then ranged constraints and variables are written to the solution file independent of the filter setting.

 $\mathbf{Default} \ \textit{OFF}$ 

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_SOL\_FILTER\_KEEP\_RANGED MSK\_OFF file

Groups Solution input/output

#### MSK\_IPAR\_SOL\_READ\_NAME\_WIDTH

When a solution is read by **MOSEK** and some constraint, variable or cone names contain blanks, then a maximum name width much be specified. A negative value implies that no name contain blanks.

Default -1

Accepted  $[-\inf; +\inf]$ 

Example mosek -d MSK\_IPAR\_SOL\_READ\_NAME\_WIDTH -1 file

Groups Data input/output, Solution input/output

#### MSK\_IPAR\_SOL\_READ\_WIDTH

Controls the maximal acceptable width of line in the solutions when read by MOSEK.

Default 1024

Accepted [80; +inf]

Example mosek -d MSK\_IPAR\_SOL\_READ\_WIDTH 1024 file

Groups Data input/output, Solution input/output

### MSK\_IPAR\_SOLUTION\_CALLBACK

Indicates whether solution callbacks will be performed during the optimization.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_SOLUTION\_CALLBACK MSK\_OFF file

Groups Progress callback, Overall solver

#### MSK\_IPAR\_TIMING\_LEVEL

Controls the amount of timing performed inside **MOSEK**.

Default 1

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_TIMING\_LEVEL 1 file

Groups Overall system

### MSK\_IPAR\_WRITE\_BAS\_CONSTRAINTS

Controls whether the constraint section is written to the basic solution file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_BAS\_CONSTRAINTS MSK\_ON file

Groups Data input/output, Solution input/output

## MSK\_IPAR\_WRITE\_BAS\_HEAD

Controls whether the header section is written to the basic solution file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_BAS\_HEAD MSK\_ON file

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_BAS\_VARIABLES

Controls whether the variables section is written to the basic solution file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_BAS\_VARIABLES MSK\_ON file

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_COMPRESSION

Controls whether the data file is compressed while it is written. 0 means no compression while higher values mean more compression.

Default 9

Accepted [0; +inf]

Example mosek -d MSK\_IPAR\_WRITE\_COMPRESSION 9 file

Groups Data input/output

#### MSK\_IPAR\_WRITE\_DATA\_PARAM

If this option is turned on the parameter settings are written to the data file as parameters.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_DATA\_PARAM MSK\_OFF file

Groups Data input/output

### MSK\_IPAR\_WRITE\_FREE\_CON

Controls whether the free constraints are written to the data file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_FREE\_CON MSK\_ON file

Groups Data input/output

### MSK\_IPAR\_WRITE\_GENERIC\_NAMES

Controls whether generic names should be used instead of user-defined names when writing to the data file.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_GENERIC\_NAMES MSK\_OFF file

Groups Data input/output

### MSK\_IPAR\_WRITE\_GENERIC\_NAMES\_IO

Index origin used in generic names.

Default 1

**Accepted**  $[0; +\inf]$ 

Example mosek -d MSK\_IPAR\_WRITE\_GENERIC\_NAMES\_IO 1 file

Groups Data input/output

# MSK\_IPAR\_WRITE\_IGNORE\_INCOMPATIBLE\_ITEMS

Controls if the writer ignores incompatible problem items when writing files.

Default OFF

# Accepted

- ON: Ignore items that cannot be written to the current output file format.
- *OFF*: Produce an error if the problem contains items that cannot the written to the current output file format.

Example mosek -d MSK\_IPAR\_WRITE\_IGNORE\_INCOMPATIBLE\_ITEMS MSK\_OFF file

Groups Data input/output

### MSK\_IPAR\_WRITE\_INT\_CONSTRAINTS

Controls whether the constraint section is written to the integer solution file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_INT\_CONSTRAINTS MSK\_ON file

## Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_INT\_HEAD

Controls whether the header section is written to the integer solution file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_INT\_HEAD MSK\_ON file

Groups Data input/output, Solution input/output

## MSK\_IPAR\_WRITE\_INT\_VARIABLES

Controls whether the variables section is written to the integer solution file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_INT\_VARIABLES MSK\_ON file

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_JSON\_INDENTATION

When set, the JSON task and solution files are written with indentation for better readability.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_JSON\_INDENTATION MSK\_OFF file

Groups Data input/output

### MSK\_IPAR\_WRITE\_LP\_FULL\_OBJ

Write all variables, including the ones with 0-coefficients, in the objective.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_LP\_FULL\_OBJ MSK\_ON file

Groups Data input/output

### MSK\_IPAR\_WRITE\_LP\_LINE\_WIDTH

Maximum width of line in an LP file written by MOSEK.

Default 80

Accepted [40; +inf]

Example mosek -d MSK\_IPAR\_WRITE\_LP\_LINE\_WIDTH 80 file

 ${\bf Groups}\ \ {\it Data\ input/output}$ 

#### MSK\_IPAR\_WRITE\_MPS\_FORMAT

Controls in which format the MPS is written.

Default FREE

Accepted STRICT, RELAXED, FREE, CPLEX

Example mosek -d MSK\_IPAR\_WRITE\_MPS\_FORMAT MSK\_MPS\_FORMAT\_FREE file

Groups Data input/output

### MSK\_IPAR\_WRITE\_MPS\_INT

Controls if marker records are written to the MPS file to indicate whether variables are integer restricted.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_MPS\_INT MSK\_ON file

Groups Data input/output

#### MSK\_IPAR\_WRITE\_SOL\_BARVARIABLES

Controls whether the symmetric matrix variables section is written to the solution file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_SOL\_BARVARIABLES MSK\_ON file

Groups Data input/output, Solution input/output

#### MSK\_IPAR\_WRITE\_SOL\_CONSTRAINTS

Controls whether the constraint section is written to the solution file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_SOL\_CONSTRAINTS MSK\_ON file

Groups Data input/output, Solution input/output

#### MSK\_IPAR\_WRITE\_SOL\_HEAD

Controls whether the header section is written to the solution file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_SOL\_HEAD MSK\_ON file

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_SOL\_IGNORE\_INVALID\_NAMES

Even if the names are invalid MPS names, then they are employed when writing the solution file.

Default OFF

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_SOL\_IGNORE\_INVALID\_NAMES MSK\_OFF file

Groups Data input/output, Solution input/output

## MSK\_IPAR\_WRITE\_SOL\_VARIABLES

Controls whether the variables section is written to the solution file.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_SOL\_VARIABLES MSK\_ON file

Groups Data input/output, Solution input/output

#### MSK\_IPAR\_WRITE\_TASK\_INC\_SOL

Controls whether the solutions are stored in the task file too.

Default ON

Accepted ON, OFF

Example mosek -d MSK\_IPAR\_WRITE\_TASK\_INC\_SOL MSK\_ON file

Groups Data input/output

## MSK\_IPAR\_WRITE\_XML\_MODE

Controls if linear coefficients should be written by row or column when writing in the XML file format.

Default ROW

Accepted ROW, COL

Example mosek -d MSK\_IPAR\_WRITE\_XML\_MODE MSK\_WRITE\_XML\_MODE\_ROW file

Groups Data input/output

# 11.2.3 String parameters

#### MSK\_SPAR\_BAS\_SOL\_FILE\_NAME

Name of the bas solution file.

Accepted Any valid file name.

Example mosek -d MSK\_SPAR\_BAS\_SOL\_FILE\_NAME somevalue file

Groups Data input/output, Solution input/output

#### MSK\_SPAR\_DATA\_FILE\_NAME

Data are read and written to this file.

Accepted Any valid file name.

Example mosek -d MSK\_SPAR\_DATA\_FILE\_NAME somevalue file

Groups Data input/output

## MSK\_SPAR\_DEBUG\_FILE\_NAME

MOSEK debug file.

Accepted Any valid file name.

Example mosek -d MSK\_SPAR\_DEBUG\_FILE\_NAME somevalue file

Groups Data input/output

### MSK\_SPAR\_INT\_SOL\_FILE\_NAME

Name of the int solution file.

**Accepted** Any valid file name.

Example mosek -d MSK\_SPAR\_INT\_SOL\_FILE\_NAME somevalue file

Groups Data input/output, Solution input/output

### MSK\_SPAR\_ITR\_SOL\_FILE\_NAME

Name of the itr solution file.

Accepted Any valid file name.

Example mosek -d MSK\_SPAR\_ITR\_SOL\_FILE\_NAME somevalue file

Groups Data input/output, Solution input/output

## MSK\_SPAR\_MIO\_DEBUG\_STRING

For internal debugging purposes.

**Accepted** Any valid string.

Example mosek -d MSK\_SPAR\_MIO\_DEBUG\_STRING somevalue file

Groups Data input/output

# MSK\_SPAR\_PARAM\_COMMENT\_SIGN

Only the first character in this string is used. It is considered as a start of comment sign in the **MOSEK** parameter file. Spaces are ignored in the string.

# Default

%%

Accepted Any valid string.

Example mosek -d MSK\_SPAR\_PARAM\_COMMENT\_SIGN %% file

Groups Data input/output

## MSK\_SPAR\_PARAM\_READ\_FILE\_NAME

Modifications to the parameter database is read from this file.

**Accepted** Any valid file name.

Example mosek -d MSK\_SPAR\_PARAM\_READ\_FILE\_NAME somevalue file

Groups Data input/output

#### MSK\_SPAR\_PARAM\_WRITE\_FILE\_NAME

The parameter database is written to this file.

Accepted Any valid file name.

Example mosek -d MSK\_SPAR\_PARAM\_WRITE\_FILE\_NAME somevalue file

Groups Data input/output

### MSK\_SPAR\_READ\_MPS\_BOU\_NAME

Name of the BOUNDS vector used. An empty name means that the first BOUNDS vector is used.

**Accepted** Any valid MPS name.

Example mosek -d MSK\_SPAR\_READ\_MPS\_BOU\_NAME somevalue file

Groups Data input/output

#### MSK\_SPAR\_READ\_MPS\_OBJ\_NAME

Name of the free constraint used as objective function. An empty name means that the first constraint is used as objective function.

Accepted Any valid MPS name.

Example mosek -d MSK\_SPAR\_READ\_MPS\_OBJ\_NAME somevalue file

Groups Data input/output

### MSK\_SPAR\_READ\_MPS\_RAN\_NAME

Name of the RANGE vector used. An empty name means that the first RANGE vector is used.

**Accepted** Any valid MPS name.

Example mosek -d MSK\_SPAR\_READ\_MPS\_RAN\_NAME somevalue file

Groups Data input/output

### MSK\_SPAR\_READ\_MPS\_RHS\_NAME

Name of the RHS used. An empty name means that the first RHS vector is used.

Accepted Any valid MPS name.

Example mosek -d MSK\_SPAR\_READ\_MPS\_RHS\_NAME somevalue file

Groups Data input/output

## MSK\_SPAR\_REMOTE\_OPTSERVER\_HOST

URL of the remote optimization server in the format (http|https)://server:port. If set, all subsequent calls to any MOSEK function that involves synchronous optimization will be sent to the specified OptServer instead of being executed locally. Passing empty string deactivates this redirection.

Accepted Any valid URL.

Example mosek -d MSK\_SPAR\_REMOTE\_OPTSERVER\_HOST somevalue file

Groups Overall system

### MSK\_SPAR\_REMOTE\_TLS\_CERT

List of known server certificates in PEM format.

**Accepted** PEM files separated by new-lines.

Example mosek -d MSK\_SPAR\_REMOTE\_TLS\_CERT somevalue file

Groups Overall system

#### MSK\_SPAR\_REMOTE\_TLS\_CERT\_PATH

Path to known server certificates in PEM format.

Accepted Any valid path.

Example mosek -d MSK\_SPAR\_REMOTE\_TLS\_CERT\_PATH somevalue file

Groups Overall system

#### MSK\_SPAR\_SENSITIVITY\_FILE\_NAME

Not applicable.

Accepted Any valid string.

Example mosek -d MSK\_SPAR\_SENSITIVITY\_FILE\_NAME somevalue file

Groups Data input/output

## MSK\_SPAR\_SENSITIVITY\_RES\_FILE\_NAME

Not applicable.

Accepted Any valid string.

Example mosek -d MSK\_SPAR\_SENSITIVITY\_RES\_FILE\_NAME somevalue file

Groups Data input/output

#### MSK\_SPAR\_SOL\_FILTER\_XC\_LOW

A filter used to determine which constraints should be listed in the solution file. A value of 0.5 means that all constraints having xc[i]>0.5 should be listed, whereas +0.5 means that all constraints having xc[i]>=blc[i]+0.5 should be listed. An empty filter means that no filter is applied.

Accepted Any valid filter.

Example mosek -d MSK\_SPAR\_SOL\_FILTER\_XC\_LOW somevalue file

Groups Data input/output, Solution input/output

### MSK\_SPAR\_SOL\_FILTER\_XC\_UPR

A filter used to determine which constraints should be listed in the solution file. A value of 0.5 means that all constraints having xc[i]<0.5 should be listed, whereas -0.5 means all constraints having xc[i]<-buc[i]-0.5 should be listed. An empty filter means that no filter is applied.

Accepted Any valid filter.

Example mosek -d MSK\_SPAR\_SOL\_FILTER\_XC\_UPR somevalue file

 ${\bf Groups}\ \textit{Data input/output}, \textit{Solution input/output}$ 

#### MSK\_SPAR\_SOL\_FILTER\_XX\_LOW

A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having xx[j] >= 0.5 should be listed, whereas "+0.5" means that all constraints having xx[j] >= blx[j] + 0.5 should be listed. An empty filter means no filter is applied.

Accepted Any valid filter.

Example mosek -d MSK\_SPAR\_SOL\_FILTER\_XX\_LOW somevalue file

Groups Data input/output, Solution input/output

### MSK\_SPAR\_SOL\_FILTER\_XX\_UPR

A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having xx[j]<0.5 should be printed, whereas "-0.5" means all constraints having xx[j]<=bux[j]-0.5 should be listed. An empty filter means no filter is applied.

Accepted Any valid file name.

Example mosek -d MSK\_SPAR\_SOL\_FILTER\_XX\_UPR somevalue file

Groups Data input/output, Solution input/output

### MSK\_SPAR\_STAT\_KEY

Key used when writing the summary file.

Accepted Any valid string.

Example mosek -d MSK\_SPAR\_STAT\_KEY somevalue file

Groups Data input/output

## MSK\_SPAR\_STAT\_NAME

Name used when writing the statistics file.

**Accepted** Any valid XML string.

Example mosek -d MSK\_SPAR\_STAT\_NAME somevalue file Groups Data input/output

### MSK\_SPAR\_WRITE\_LP\_GEN\_VAR\_NAME

Sometimes when an LP file is written additional variables must be inserted. They will have the prefix denoted by this parameter.

Default xmskgen

Accepted Any valid string.

Example mosek -d MSK\_SPAR\_WRITE\_LP\_GEN\_VAR\_NAME xmskgen file

Groups Data input/output

# 11.3 Response codes

Response codes include:

- Termination codes
- Warnings
- Errors

The numerical code (in brackets) identifies the response in error messages and in the log output.

### 11.3.1 Termination

MSK\_RES\_OK (0)

No error occurred.

MSK\_RES\_TRM\_MAX\_ITERATIONS (100000)

The optimizer terminated at the maximum number of iterations.

MSK\_RES\_TRM\_MAX\_TIME (100001)

The optimizer terminated at the maximum amount of time.

MSK\_RES\_TRM\_OBJECTIVE\_RANGE (100002)

The optimizer terminated with an objective value outside the objective range.

MSK\_RES\_TRM\_MIO\_NUM\_RELAXS (100008)

The mixed-integer optimizer terminated as the maximum number of relaxations was reached.

MSK\_RES\_TRM\_MIO\_NUM\_BRANCHES (100009)

The mixed-integer optimizer terminated as the maximum number of branches was reached.

MSK\_RES\_TRM\_NUM\_MAX\_NUM\_INT\_SOLUTIONS (100015)

The mixed-integer optimizer terminated as the maximum number of feasible solutions was reached.  $MSK_RES_TRM_STALL$  (100006)

The optimizer is terminated due to slow progress.

Stalling means that numerical problems prevent the optimizer from making reasonable progress and that it makes no sense to continue. In many cases this happens if the problem is badly scaled or otherwise ill-conditioned. There is no guarantee that the solution will be feasible or optimal. However, often stalling happens near the optimum, and the returned solution may be of good quality. Therefore, it is recommended to check the status of the solution. If the solution status is optimal the solution is most likely good enough for most practical purposes.

Please note that if a linear optimization problem is solved using the interior-point optimizer with basis identification turned on, the returned basic solution likely to have high accuracy, even though the optimizer stalled.

Some common causes of stalling are a) badly scaled models, b) near feasible or near infeasible problems.

MSK\_RES\_TRM\_USER\_CALLBACK (100007)

The optimizer terminated due to the return of the user-defined callback function.

MSK\_RES\_TRM\_MAX\_NUM\_SETBACKS (100020)

The optimizer terminated as the maximum number of set-backs was reached. This indicates serious numerical problems and a possibly badly formulated problem.

#### MSK\_RES\_TRM\_NUMERICAL\_PROBLEM (100025)

The optimizer terminated due to numerical problems.

MSK\_RES\_TRM\_LOST\_RACE (100027)

Lost a race.

MSK\_RES\_TRM\_INTERNAL (100030)

The optimizer terminated due to some internal reason. Please contact MOSEK support.

MSK\_RES\_TRM\_INTERNAL\_STOP (100031)

The optimizer terminated for internal reasons. Please contact MOSEK support.

# 11.3.2 Warnings

#### MSK\_RES\_WRN\_OPEN\_PARAM\_FILE (50)

The parameter file could not be opened.

MSK\_RES\_WRN\_LARGE\_BOUND (51)

A numerically large bound value is specified.

MSK\_RES\_WRN\_LARGE\_LO\_BOUND (52)

A numerically large lower bound value is specified.

MSK\_RES\_WRN\_LARGE\_UP\_BOUND (53)

A numerically large upper bound value is specified.

MSK\_RES\_WRN\_LARGE\_CON\_FX (54)

An equality constraint is fixed to a numerically large value. This can cause numerical problems.

MSK\_RES\_WRN\_LARGE\_CJ (57)

A numerically large value is specified for one  $c_i$ .

MSK\_RES\_WRN\_LARGE\_AIJ (62)

A numerically large value is specified for an  $a_{i,j}$  element in A. The parameter  $MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE$  controls when an  $a_{i,j}$  is considered large.

MSK\_RES\_WRN\_ZERO\_AIJ (63)

One or more zero elements are specified in A.

MSK\_RES\_WRN\_NAME\_MAX\_LEN (65)

A name is longer than the buffer that is supposed to hold it.

MSK\_RES\_WRN\_SPAR\_MAX\_LEN (66)

A value for a string parameter is longer than the buffer that is supposed to hold it.

MSK\_RES\_WRN\_MPS\_SPLIT\_RHS\_VECTOR (70)

An RHS vector is split into several nonadjacent parts in an MPS file.

MSK\_RES\_WRN\_MPS\_SPLIT\_RAN\_VECTOR (71)

A RANGE vector is split into several nonadjacent parts in an MPS file.

MSK\_RES\_WRN\_MPS\_SPLIT\_BOU\_VECTOR (72)

A BOUNDS vector is split into several nonadjacent parts in an MPS file.

MSK\_RES\_WRN\_LP\_OLD\_QUAD\_FORMAT (80)

Missing '/2' after quadratic expressions in bound or objective.

MSK\_RES\_WRN\_LP\_DROP\_VARIABLE (85)

Ignored a variable because the variable was not previously defined. Usually this implies that a variable appears in the bound section but not in the objective or the constraints.

MSK\_RES\_WRN\_NZ\_IN\_UPR\_TRI (200)

Non-zero elements specified in the upper triangle of a matrix were ignored.

MSK\_RES\_WRN\_DROPPED\_NZ\_QOBJ (201)

One or more non-zero elements were dropped in the Q matrix in the objective.

MSK\_RES\_WRN\_IGNORE\_INTEGER (250)

Ignored integer constraints.

MSK\_RES\_WRN\_NO\_GLOBAL\_OPTIMIZER (251)

No global optimizer is available.

MSK\_RES\_WRN\_MIO\_INFEASIBLE\_FINAL (270)

The final mixed-integer problem with all the integer variables fixed at their optimal values is infeasible.

MSK\_RES\_WRN\_SOL\_FILTER (300)

Invalid solution filter is specified.

MSK\_RES\_WRN\_UNDEF\_SOL\_FILE\_NAME (350)

Undefined name occurred in a solution.

# MSK\_RES\_WRN\_SOL\_FILE\_IGNORED\_CON (351)

One or more lines in the constraint section were ignored when reading a solution file.

### MSK\_RES\_WRN\_SOL\_FILE\_IGNORED\_VAR (352)

One or more lines in the variable section were ignored when reading a solution file.

# MSK\_RES\_WRN\_TOO\_FEW\_BASIS\_VARS (400)

An incomplete basis has been specified. Too few basis variables are specified.

# MSK\_RES\_WRN\_TOO\_MANY\_BASIS\_VARS (405)

A basis with too many variables has been specified.

#### MSK\_RES\_WRN\_LICENSE\_EXPIRE (500)

The license expires.

### MSK\_RES\_WRN\_LICENSE\_SERVER (501)

The license server is not responding.

### MSK\_RES\_WRN\_EMPTY\_NAME (502)

A variable or constraint name is empty. The output file may be invalid.

# MSK\_RES\_WRN\_USING\_GENERIC\_NAMES (503)

Generic names are used because a name is not valid. For instance when writing an LP file the names must not contain blanks or start with a digit.

### MSK\_RES\_WRN\_INVALID\_MPS\_NAME (504)

A name e.g. a row name is not a valid MPS name.

### MSK\_RES\_WRN\_INVALID\_MPS\_OBJ\_NAME (505)

The objective name is not a valid MPS name.

# MSK\_RES\_WRN\_LICENSE\_FEATURE\_EXPIRE (509)

The license expires.

### MSK\_RES\_WRN\_PARAM\_NAME\_DOU (510)

The parameter name is not recognized as a double parameter.

### MSK\_RES\_WRN\_PARAM\_NAME\_INT (511)

The parameter name is not recognized as a integer parameter.

### MSK\_RES\_WRN\_PARAM\_NAME\_STR (512)

The parameter name is not recognized as a string parameter.

## MSK\_RES\_WRN\_PARAM\_STR\_VALUE (515)

The string is not recognized as a symbolic value for the parameter.

# MSK\_RES\_WRN\_PARAM\_IGNORED\_CMIO (516)

A parameter was ignored by the conic mixed integer optimizer.

# MSK\_RES\_WRN\_ZEROS\_IN\_SPARSE\_ROW (705)

One or more (near) zero elements are specified in a sparse row of a matrix. Since, it is redundant to specify zero elements then it may indicate an error.

# MSK\_RES\_WRN\_ZEROS\_IN\_SPARSE\_COL (710)

One or more (near) zero elements are specified in a sparse column of a matrix. It is redundant to specify zero elements. Hence, it may indicate an error.

# MSK\_RES\_WRN\_INCOMPLETE\_LINEAR\_DEPENDENCY\_CHECK (800)

The linear dependency check(s) is incomplete. Normally this is not an important warning unless the optimization problem has been formulated with linear dependencies. Linear dependencies may prevent **MOSEK** from solving the problem.

## MSK\_RES\_WRN\_ELIMINATOR\_SPACE (801)

The eliminator is skipped at least once due to lack of space.

# MSK\_RES\_WRN\_PRESOLVE\_OUTOFSPACE (802)

The presolve is incomplete due to lack of space.

# MSK\_RES\_WRN\_PRESOLVE\_PRIMAL\_PERTUBATIONS (803)

The presolve perturbed the bounds of the primal problem. This is an indication that the problem is nearly infeasible.

# MSK\_RES\_WRN\_WRITE\_CHANGED\_NAMES (830)

Some names were changed because they were invalid for the output file format.

## MSK\_RES\_WRN\_WRITE\_DISCARDED\_CFIX (831)

The fixed objective term could not be converted to a variable and was discarded in the output file.

## MSK\_RES\_WRN\_DUPLICATE\_CONSTRAINT\_NAMES (850)

Two constraint names are identical.

# MSK\_RES\_WRN\_DUPLICATE\_VARIABLE\_NAMES (851)

Two variable names are identical.

#### MSK\_RES\_WRN\_DUPLICATE\_BARVARIABLE\_NAMES (852)

Two barvariable names are identical.

MSK\_RES\_WRN\_DUPLICATE\_CONE\_NAMES (853)

Two cone names are identical.

MSK\_RES\_WRN\_WRITE\_LP\_INVALID\_VAR\_NAMES (854)

LP file will be written with generic variable names.

MSK\_RES\_WRN\_WRITE\_LP\_DUPLICATE\_VAR\_NAMES (855)

LP file will be written with generic variable names.

MSK\_RES\_WRN\_WRITE\_LP\_INVALID\_CON\_NAMES (856)

LP file will be written with generic constraint names.

MSK\_RES\_WRN\_WRITE\_LP\_DUPLICATE\_CON\_NAMES (857)

LP file will be written with generic constraint names.

MSK\_RES\_WRN\_ANA\_LARGE\_BOUNDS (900)

This warning is issued by the problem analyzer, if one or more constraint or variable bounds are very large. One should consider omitting these bounds entirely by setting them to +inf or -inf.

MSK\_RES\_WRN\_ANA\_C\_ZERO (901)

This warning is issued by the problem analyzer, if the coefficients in the linear part of the objective are all zero.

MSK\_RES\_WRN\_ANA\_EMPTY\_COLS (902)

This warning is issued by the problem analyzer, if columns, in which all coefficients are zero, are found.

MSK\_RES\_WRN\_ANA\_CLOSE\_BOUNDS (903)

This warning is issued by problem analyzer, if ranged constraints or variables with very close upper and lower bounds are detected. One should consider treating such constraints as equalities and such variables as constants.

MSK\_RES\_WRN\_ANA\_ALMOST\_INT\_BOUNDS (904)

This warning is issued by the problem analyzer if a constraint is bound nearly integral.

MSK\_RES\_WRN\_NO\_INFEASIBILITY\_REPORT\_WHEN\_MATRIX\_VARIABLES (930)

An infeasibility report is not available when the problem contains matrix variables.

MSK\_RES\_WRN\_NO\_DUALIZER (950)

No automatic dualizer is available for the specified problem. The primal problem is solved.

MSK\_RES\_WRN\_SYM\_MAT\_LARGE (960)

A numerically large value is specified for an  $e_{i,j}$  element in E. The parameter  $MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_LARGE$  controls when an  $e_{i,j}$  is considered large.

MSK\_RES\_WRN\_MODIFIED\_DOUBLE\_PARAMETER (970)

A double parameter related to solver tolerances has a non-default value.

MSK\_RES\_WRN\_LARGE\_FIJ (980)

A numerically large value is specified for an  $f_{i,j}$  element in F. The parameter  $MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE$  controls when an  $f_{i,j}$  is considered large.

## 11.3.3 Errors

MSK\_RES\_ERR\_LICENSE (1000)

Invalid license.

MSK\_RES\_ERR\_LICENSE\_EXPIRED (1001)

The license has expired.

MSK\_RES\_ERR\_LICENSE\_VERSION (1002)

The license is valid for another version of MOSEK.

MSK\_RES\_ERR\_LICENSE\_OLD\_SERVER\_VERSION (1003)

The version of the FlexLM license server is too old. You should upgrade the license server to one matching this version of **MOSEK**. It will support this and all older versions of **MOSEK**.

This error can appear if the client was updated to a new version which includes an upgrade of the licensing module, making it incompatible with a much older license server.

MSK\_RES\_ERR\_SIZE\_LICENSE (1005)

The problem is bigger than the license.

MSK\_RES\_ERR\_PROB\_LICENSE (1006)

The software is not licensed to solve the problem.

#### MSK\_RES\_ERR\_FILE\_LICENSE (1007)

Invalid license file.

### MSK\_RES\_ERR\_MISSING\_LICENSE\_FILE (1008)

MOSEK cannot find license file or a token server. See the MOSEK licensing manual for details.

# MSK\_RES\_ERR\_SIZE\_LICENSE\_CON (1010)

The problem has too many constraints to be solved with the available license.

## MSK\_RES\_ERR\_SIZE\_LICENSE\_VAR (1011)

The problem has too many variables to be solved with the available license.

#### MSK\_RES\_ERR\_SIZE\_LICENSE\_INTVAR (1012)

The problem contains too many integer variables to be solved with the available license.

### MSK\_RES\_ERR\_OPTIMIZER\_LICENSE (1013)

The optimizer required is not licensed.

#### MSK\_RES\_ERR\_FLEXLM (1014)

The FLEXIm license manager reported an error.

# MSK\_RES\_ERR\_LICENSE\_SERVER (1015)

The license server is not responding.

# MSK\_RES\_ERR\_LICENSE\_MAX (1016)

Maximum number of licenses is reached.

## MSK\_RES\_ERR\_LICENSE\_MOSEKLM\_DAEMON (1017)

The MOSEKLM license manager daemon is not up and running.

### MSK\_RES\_ERR\_LICENSE\_FEATURE (1018)

A requested feature is not available in the license file(s). Most likely due to an incorrect license system setup.

### MSK\_RES\_ERR\_PLATFORM\_NOT\_LICENSED (1019)

A requested license feature is not available for the required platform.

## MSK\_RES\_ERR\_LICENSE\_CANNOT\_ALLOCATE (1020)

The license system cannot allocate the memory required.

### MSK\_RES\_ERR\_LICENSE\_CANNOT\_CONNECT (1021)

MOSEK cannot connect to the license server. Most likely the license server is not up and running. MSK\_RES\_ERR\_LICENSE\_INVALID\_HOSTID (1025)

The host ID specified in the license file does not match the host ID of the computer.

# MSK\_RES\_ERR\_LICENSE\_SERVER\_VERSION (1026)

The version specified in the checkout request is greater than the highest version number the daemon supports.

# MSK\_RES\_ERR\_LICENSE\_NO\_SERVER\_SUPPORT (1027)

The license server does not support the requested feature. Possible reasons for this error include:

- The feature has expired.
- The feature's start date is later than today's date.
- The version requested is higher than feature's the highest supported version.
- A corrupted license file.

Try restarting the license and inspect the license server debug file, usually called lmgrd.log.

# MSK\_RES\_ERR\_LICENSE\_NO\_SERVER\_LINE (1028)

There is no SERVER line in the license file. All non-zero license count features need at least one SERVER line.

## MSK\_RES\_ERR\_OLDER\_DLL (1035)

The dynamic link library is older than the specified version.

# MSK\_RES\_ERR\_NEWER\_DLL (1036)

The dynamic link library is newer than the specified version.

## MSK\_RES\_ERR\_LINK\_FILE\_DLL (1040)

A file cannot be linked to a stream in the DLL version.

### MSK\_RES\_ERR\_THREAD\_MUTEX\_INIT (1045)

Could not initialize a mutex.

# MSK\_RES\_ERR\_THREAD\_MUTEX\_LOCK (1046)

Could not lock a mutex.

# MSK\_RES\_ERR\_THREAD\_MUTEX\_UNLOCK (1047)

Could not unlock a mutex.

### MSK\_RES\_ERR\_THREAD\_CREATE (1048)

Could not create a thread. This error may occur if a large number of environments are created and not deleted again. In any case it is a good practice to minimize the number of environments created.

# MSK\_RES\_ERR\_THREAD\_COND\_INIT (1049)

Could not initialize a condition.

MSK\_RES\_ERR\_UNKNOWN (1050)

Unknown error.

MSK\_RES\_ERR\_SPACE (1051)

Out of space.

MSK\_RES\_ERR\_FILE\_OPEN (1052)

Error while opening a file.

MSK\_RES\_ERR\_FILE\_READ (1053)

File read error.

MSK\_RES\_ERR\_FILE\_WRITE (1054)

File write error.

MSK\_RES\_ERR\_DATA\_FILE\_EXT (1055)

The data file format cannot be determined from the file name.

MSK\_RES\_ERR\_INVALID\_FILE\_NAME (1056)

An invalid file name has been specified.

MSK\_RES\_ERR\_INVALID\_SOL\_FILE\_NAME (1057)

An invalid file name has been specified.

MSK\_RES\_ERR\_END\_OF\_FILE (1059)

End of file reached.

MSK\_RES\_ERR\_NULL\_ENV (1060)

env is a NULL pointer.

MSK\_RES\_ERR\_NULL\_TASK (1061)

task is a NULL pointer.

MSK\_RES\_ERR\_INVALID\_STREAM (1062)

An invalid stream is referenced.

MSK\_RES\_ERR\_NO\_INIT\_ENV (1063)

env is not initialized.

MSK\_RES\_ERR\_INVALID\_TASK (1064)

The task is invalid.

MSK\_RES\_ERR\_NULL\_POINTER (1065)

An argument to a function is unexpectedly a NULL pointer.

MSK\_RES\_ERR\_LIVING\_TASKS (1066)

All tasks associated with an environment must be deleted before the environment is deleted. There are still some undeleted tasks.

MSK\_RES\_ERR\_BLANK\_NAME (1070)

An all blank name has been specified.

MSK\_RES\_ERR\_DUP\_NAME (1071)

The same name was used multiple times for the same problem item type.

MSK\_RES\_ERR\_FORMAT\_STRING (1072)

The name format string is invalid.

MSK\_RES\_ERR\_SPARSITY\_SPECIFICATION (1073)

The sparsity included an index that was out of bounds of the shape.

MSK\_RES\_ERR\_MISMATCHING\_DIMENSION (1074)

Mismatching dimensions specified in arguments

MSK\_RES\_ERR\_INVALID\_OBJ\_NAME (1075)

An invalid objective name is specified.

MSK\_RES\_ERR\_INVALID\_CON\_NAME (1076)

An invalid constraint name is used.

MSK\_RES\_ERR\_INVALID\_VAR\_NAME (1077)

An invalid variable name is used.

MSK\_RES\_ERR\_INVALID\_CONE\_NAME (1078)

An invalid cone name is used.

#### MSK\_RES\_ERR\_INVALID\_BARVAR\_NAME (1079)

An invalid symmetric matrix variable name is used.

MSK\_RES\_ERR\_SPACE\_LEAKING (1080)

MOSEK is leaking memory. This can be due to either an incorrect use of MOSEK or a bug.

MSK\_RES\_ERR\_SPACE\_NO\_INFO (1081)

No available information about the space usage.

MSK\_RES\_ERR\_DIMENSION\_SPECIFICATION (1082)

Invalid dimension specification

MSK\_RES\_ERR\_AXIS\_NAME\_SPECIFICATION (1083)

Invalid axis names specification

MSK\_RES\_ERR\_READ\_FORMAT (1090)

The specified format cannot be read.

MSK\_RES\_ERR\_MPS\_FILE (1100)

An error occurred while reading an MPS file.

MSK\_RES\_ERR\_MPS\_INV\_FIELD (1101)

A field in the MPS file is invalid. Probably it is too wide.

MSK\_RES\_ERR\_MPS\_INV\_MARKER (1102)

An invalid marker has been specified in the MPS file.

MSK\_RES\_ERR\_MPS\_NULL\_CON\_NAME (1103)

An empty constraint name is used in an MPS file.

MSK\_RES\_ERR\_MPS\_NULL\_VAR\_NAME (1104)

An empty variable name is used in an MPS file.

MSK\_RES\_ERR\_MPS\_UNDEF\_CON\_NAME (1105)

An undefined constraint name occurred in an MPS file.

MSK\_RES\_ERR\_MPS\_UNDEF\_VAR\_NAME (1106)

An undefined variable name occurred in an MPS file.

MSK\_RES\_ERR\_MPS\_INVALID\_CON\_KEY (1107)

An invalid constraint key occurred in an MPS file.

MSK\_RES\_ERR\_MPS\_INVALID\_BOUND\_KEY (1108)

An invalid bound key occurred in an MPS file.

MSK\_RES\_ERR\_MPS\_INVALID\_SEC\_NAME (1109)

An invalid section name occurred in an MPS file.

MSK\_RES\_ERR\_MPS\_NO\_OBJECTIVE (1110)

No objective is defined in an MPS file.

MSK\_RES\_ERR\_MPS\_SPLITTED\_VAR (1111)

All elements in a column of the A matrix must be specified consecutively. Hence, it is illegal to specify non-zero elements in A for variable 1, then for variable 2 and then variable 1 again.

MSK\_RES\_ERR\_MPS\_MUL\_CON\_NAME (1112)

A constraint name was specified multiple times in the ROWS section.

MSK\_RES\_ERR\_MPS\_MUL\_QSEC (1113)

Multiple QSECTIONs are specified for a constraint in the MPS data file.

MSK\_RES\_ERR\_MPS\_MUL\_QOBJ (1114)

The Q term in the objective is specified multiple times in the MPS data file.

MSK\_RES\_ERR\_MPS\_INV\_SEC\_ORDER (1115)

The sections in the MPS data file are not in the correct order.

MSK\_RES\_ERR\_MPS\_MUL\_CSEC (1116)

Multiple CSECTIONs are given the same name.

MSK\_RES\_ERR\_MPS\_CONE\_TYPE (1117)

Invalid cone type specified in a CSECTION.

MSK\_RES\_ERR\_MPS\_CONE\_OVERLAP (1118)

A variable is specified to be a member of several cones.

MSK\_RES\_ERR\_MPS\_CONE\_REPEAT (1119)

A variable is repeated within the CSECTION.

MSK\_RES\_ERR\_MPS\_NON\_SYMMETRIC\_Q (1120)

A non symmetric matrix has been speciefied.

MSK\_RES\_ERR\_MPS\_DUPLICATE\_Q\_ELEMENT (1121)

Duplicate elements is specified in a Q matrix.

MSK\_RES\_ERR\_MPS\_INVALID\_OBJSENSE (1122)

An invalid objective sense is specified.

MSK\_RES\_ERR\_MPS\_TAB\_IN\_FIELD2 (1125)

A tab char occurred in field 2.

MSK\_RES\_ERR\_MPS\_TAB\_IN\_FIELD3 (1126)

A tab char occurred in field 3.

MSK\_RES\_ERR\_MPS\_TAB\_IN\_FIELD5 (1127)

A tab char occurred in field 5.

MSK\_RES\_ERR\_MPS\_INVALID\_OBJ\_NAME (1128)

An invalid objective name is specified.

MSK\_RES\_ERR\_MPS\_INVALID\_KEY (1129)

An invalid indicator key occurred in an MPS file.

MSK\_RES\_ERR\_MPS\_INVALID\_INDICATOR\_CONSTRAINT (1130)

An invalid indicator constraint is used. It must not be a ranged constraint.

MSK\_RES\_ERR\_MPS\_INVALID\_INDICATOR\_VARIABLE (1131)

An invalid indicator variable is specified. It must be a binary variable.

MSK\_RES\_ERR\_MPS\_INVALID\_INDICATOR\_VALUE (1132)

An invalid indicator value is specified. It must be either 0 or 1.

MSK\_RES\_ERR\_MPS\_INVALID\_INDICATOR\_QUADRATIC\_CONSTRAINT (1133)

A quadratic constraint can be be an indicator constraint.

MSK\_RES\_ERR\_OPF\_SYNTAX (1134)

Syntax error in an OPF file

MSK\_RES\_ERR\_OPF\_PREMATURE\_EOF (1136)

Premature end of file in an OPF file.

MSK\_RES\_ERR\_OPF\_MISMATCHED\_TAG (1137)

Mismatched end-tag in OPF file

MSK\_RES\_ERR\_OPF\_DUPLICATE\_BOUND (1138)

Either upper or lower bound was specified twice in OPF file

MSK\_RES\_ERR\_OPF\_DUPLICATE\_CONSTRAINT\_NAME (1139)

Duplicate constraint name in OPF File

MSK\_RES\_ERR\_OPF\_INVALID\_CONE\_TYPE (1140)

Invalid cone type in OPF File

MSK\_RES\_ERR\_OPF\_INCORRECT\_TAG\_PARAM (1141)

Invalid number of parameters in start-tag in OPF File

MSK\_RES\_ERR\_OPF\_INVALID\_TAG (1142)

Invalid start-tag in OPF File

MSK\_RES\_ERR\_OPF\_DUPLICATE\_CONE\_ENTRY (1143)

Same variable appears in multiple cones in OPF File

MSK\_RES\_ERR\_OPF\_TOO\_LARGE (1144)

The problem is too large to be correctly loaded

MSK\_RES\_ERR\_OPF\_DUAL\_INTEGER\_SOLUTION (1146)

Dual solution values are not allowed in OPF File

MSK\_RES\_ERR\_LP\_INCOMPATIBLE (1150)

The problem cannot be written to an LP formatted file.

MSK\_RES\_ERR\_LP\_EMPTY (1151)

The problem cannot be written to an LP formatted file.

MSK\_RES\_ERR\_LP\_DUP\_SLACK\_NAME (1152)

The name of the slack variable added to a ranged constraint already exists.

MSK\_RES\_ERR\_WRITE\_MPS\_INVALID\_NAME (1153)

An invalid name is created while writing an MPS file. Usually this will make the MPS file unreadable.

MSK\_RES\_ERR\_LP\_INVALID\_VAR\_NAME (1154)

A variable name is invalid when used in an LP formatted file.

MSK\_RES\_ERR\_LP\_FREE\_CONSTRAINT (1155)

Free constraints cannot be written in LP file format.

MSK\_RES\_ERR\_WRITE\_OPF\_INVALID\_VAR\_NAME (1156)

Empty variable names cannot be written to OPF files.

MSK\_RES\_ERR\_LP\_FILE\_FORMAT (1157)

Syntax error in an LP file.

MSK\_RES\_ERR\_WRITE\_LP\_FORMAT (1158)

Problem cannot be written as an LP file.

MSK\_RES\_ERR\_READ\_LP\_MISSING\_END\_TAG (1159)

Syntax error in LP file. Possibly missing End tag.

MSK\_RES\_ERR\_LP\_INDICATOR\_VAR (1160)

An indicator variable was not declared binary

MSK\_RES\_ERR\_WRITE\_LP\_NON\_UNIQUE\_NAME (1161)

An auto-generated name is not unique.

MSK\_RES\_ERR\_READ\_LP\_NONEXISTING\_NAME (1162)

A variable never occurred in objective or constraints.

MSK\_RES\_ERR\_LP\_WRITE\_CONIC\_PROBLEM (1163)

The problem contains cones that cannot be written to an LP formatted file.

MSK\_RES\_ERR\_LP\_WRITE\_GECO\_PROBLEM (1164)

The problem contains general convex terms that cannot be written to an LP formatted file.

MSK\_RES\_ERR\_WRITING\_FILE (1166)

An error occurred while writing file

MSK\_RES\_ERR\_INVALID\_NAME\_IN\_SOL\_FILE (1170)

An invalid name occurred in a solution file.

MSK\_RES\_ERR\_LP\_INVALID\_CON\_NAME (1171)

A constraint name is invalid when used in an LP formatted file.

MSK\_RES\_ERR\_JSON\_SYNTAX (1175)

Syntax error in an JSON data

MSK\_RES\_ERR\_JSON\_STRING (1176)

Error in JSON string.

MSK\_RES\_ERR\_JSON\_NUMBER\_OVERFLOW (1177)

Invalid number entry - wrong type or value overflow.

MSK\_RES\_ERR\_JSON\_FORMAT (1178)

Error in an JSON Task file

MSK\_RES\_ERR\_JSON\_DATA (1179)

Inconsistent data in JSON Task file

MSK\_RES\_ERR\_JSON\_MISSING\_DATA (1180)

Missing data section in JSON task file.

MSK\_RES\_ERR\_PTF\_INCOMPATIBILITY (1181)
Incompatible item

MSK\_RES\_ERR\_PTF\_UNDEFINED\_ITEM (1182)

Undefined symbol referenced

MSK\_RES\_ERR\_PTF\_INCONSISTENCY (1183)

Inconsistent size of item

MSK\_RES\_ERR\_PTF\_FORMAT (1184)

Syntax error in an PTF file

MSK\_RES\_ERR\_ARGUMENT\_LENNEQ (1197)

Incorrect length of arguments.

MSK\_RES\_ERR\_ARGUMENT\_TYPE (1198)

Incorrect argument type.

MSK\_RES\_ERR\_NUM\_ARGUMENTS (1199)

Incorrect number of function arguments.

MSK\_RES\_ERR\_IN\_ARGUMENT (1200)

A function argument is incorrect.

MSK\_RES\_ERR\_ARGUMENT\_DIMENSION (1201)

A function argument is of incorrect dimension.

MSK\_RES\_ERR\_SHAPE\_IS\_TOO\_LARGE (1202)

The size of the n-dimensional shape is too large.

MSK\_RES\_ERR\_INDEX\_IS\_TOO\_SMALL (1203)

An index in an argument is too small.

MSK\_RES\_ERR\_INDEX\_IS\_TOO\_LARGE (1204)

An index in an argument is too large.

#### MSK\_RES\_ERR\_INDEX\_IS\_NOT\_UNIQUE (1205)

An index in an argument is is unique.

MSK\_RES\_ERR\_PARAM\_NAME (1206)

The parameter name is not correct.

MSK\_RES\_ERR\_PARAM\_NAME\_DOU (1207)

The parameter name is not correct for a double parameter.

MSK\_RES\_ERR\_PARAM\_NAME\_INT (1208)

The parameter name is not correct for an integer parameter.

MSK\_RES\_ERR\_PARAM\_NAME\_STR (1209)

The parameter name is not correct for a string parameter.

MSK\_RES\_ERR\_PARAM\_INDEX (1210)

Parameter index is out of range.

MSK\_RES\_ERR\_PARAM\_IS\_TOO\_LARGE (1215)

The parameter value is too large.

MSK\_RES\_ERR\_PARAM\_IS\_TOO\_SMALL (1216)

The parameter value is too small.

MSK\_RES\_ERR\_PARAM\_VALUE\_STR (1217)

The parameter value string is incorrect.

MSK\_RES\_ERR\_PARAM\_TYPE (1218)

The parameter type is invalid.

MSK\_RES\_ERR\_INF\_DOU\_INDEX (1219)

A double information index is out of range for the specified type.

MSK\_RES\_ERR\_INF\_INT\_INDEX (1220)

An integer information index is out of range for the specified type.

MSK\_RES\_ERR\_INDEX\_ARR\_IS\_TOO\_SMALL (1221)

An index in an array argument is too small.

MSK\_RES\_ERR\_INDEX\_ARR\_IS\_TOO\_LARGE (1222)

An index in an array argument is too large.

MSK\_RES\_ERR\_INF\_LINT\_INDEX (1225)

A long integer information index is out of range for the specified type.

MSK\_RES\_ERR\_ARG\_IS\_TOO\_SMALL (1226)

The value of a argument is too small.

MSK\_RES\_ERR\_ARG\_IS\_TOO\_LARGE (1227)

The value of a argument is too large.

MSK\_RES\_ERR\_INVALID\_WHICHSOL (1228) whichsol is invalid.

MSK\_RES\_ERR\_INF\_DOU\_NAME (1230)

A double information name is invalid.

MSK\_RES\_ERR\_INF\_INT\_NAME (1231)

An integer information name is invalid.

MSK\_RES\_ERR\_INF\_TYPE (1232)

The information type is invalid.

MSK\_RES\_ERR\_INF\_LINT\_NAME (1234)

A long integer information name is invalid.

MSK\_RES\_ERR\_INDEX (1235)

An index is out of range.

MSK\_RES\_ERR\_WHICHSOL (1236)

The solution defined by whichsol does not exists.

MSK\_RES\_ERR\_SOLITEM (1237)

The solution item number solitem is invalid. Please note that MSK\_SOL\_ITEM\_SNX is invalid for the basic solution.

MSK\_RES\_ERR\_WHICHITEM\_NOT\_ALLOWED (1238)

whichitem is unacceptable.

MSK\_RES\_ERR\_MAXNUMCON (1240)

The maximum number of constraints specified is smaller than the number of constraints in the task.

# MSK\_RES\_ERR\_MAXNUMVAR (1241)

The maximum number of variables specified is smaller than the number of variables in the task.

#### MSK\_RES\_ERR\_MAXNUMBARVAR (1242)

The maximum number of semidefinite variables specified is smaller than the number of semidefinite variables in the task.

### MSK\_RES\_ERR\_MAXNUMQNZ (1243)

The maximum number of non-zeros specified for the Q matrices is smaller than the number of non-zeros in the current Q matrices.

# MSK\_RES\_ERR\_TOO\_SMALL\_MAX\_NUM\_NZ (1245)

The maximum number of non-zeros specified is too small.

### MSK\_RES\_ERR\_INVALID\_IDX (1246)

A specified index is invalid.

# MSK\_RES\_ERR\_INVALID\_MAX\_NUM (1247)

A specified index is invalid.

#### MSK\_RES\_ERR\_UNALLOWED\_WHICHSOL (1248)

The value od whichsol is not allowed.

### MSK\_RES\_ERR\_NUMCONLIM (1250)

Maximum number of constraints limit is exceeded.

# MSK\_RES\_ERR\_NUMVARLIM (1251)

Maximum number of variables limit is exceeded.

#### MSK\_RES\_ERR\_TOO\_SMALL\_MAXNUMANZ (1252)

The maximum number of non-zeros specified for A is smaller than the number of non-zeros in the current A.

### MSK\_RES\_ERR\_INV\_APTRE (1253)

aptre[j] is strictly smaller than aptrb[j] for some j.

### MSK\_RES\_ERR\_MUL\_A\_ELEMENT (1254)

An element in A is defined multiple times.

# MSK\_RES\_ERR\_INV\_BK (1255)

Invalid bound key.

### MSK\_RES\_ERR\_INV\_BKC (1256)

Invalid bound key is specified for a constraint.

# MSK\_RES\_ERR\_INV\_BKX (1257)

An invalid bound key is specified for a variable.

# MSK\_RES\_ERR\_INV\_VAR\_TYPE (1258)

An invalid variable type is specified for a variable.

## MSK\_RES\_ERR\_SOLVER\_PROBTYPE (1259)

Problem type does not match the chosen optimizer.

### MSK\_RES\_ERR\_OBJECTIVE\_RANGE (1260)

Empty objective range.

# MSK\_RES\_ERR\_BASIS (1266)

An invalid basis is specified. Either too many or too few basis variables are specified.

# MSK\_RES\_ERR\_INV\_SKC (1267)

Invalid value in skc.

#### MSK\_RES\_ERR\_INV\_SKX (1268)

Invalid value in skx.

# MSK\_RES\_ERR\_INV\_SKN (1274)

Invalid value in skn.

# MSK\_RES\_ERR\_INV\_SK\_STR (1269)

Invalid status key string encountered.

# MSK\_RES\_ERR\_INV\_SK (1270)

Invalid status key code.

#### MSK\_RES\_ERR\_INV\_CONE\_TYPE\_STR (1271)

Invalid cone type string encountered.

# MSK\_RES\_ERR\_INV\_CONE\_TYPE (1272)

Invalid cone type code is encountered.

# MSK\_RES\_ERR\_INVALID\_SURPLUS (1275)

Invalid surplus.

# MSK\_RES\_ERR\_INV\_NAME\_ITEM (1280)

An invalid name item code is used.

MSK\_RES\_ERR\_PRO\_ITEM (1281)

An invalid problem is used.

MSK\_RES\_ERR\_INVALID\_FORMAT\_TYPE (1283)

Invalid format type.

MSK\_RES\_ERR\_FIRSTI (1285)

Invalid firsti.

MSK\_RES\_ERR\_LASTI (1286)

Invalid lasti.

MSK\_RES\_ERR\_FIRSTJ (1287)

Invalid firstj.

MSK\_RES\_ERR\_LASTJ (1288)

Invalid lastj.

MSK\_RES\_ERR\_MAX\_LEN\_IS\_TOO\_SMALL (1289)

A maximum length that is too small has been specified.

MSK\_RES\_ERR\_NONLINEAR\_EQUALITY (1290)

The model contains a nonlinear equality which defines a nonconvex set.

MSK\_RES\_ERR\_NONCONVEX (1291)

The optimization problem is nonconvex.

MSK\_RES\_ERR\_NONLINEAR\_RANGED (1292)

Nonlinear constraints with finite lower and upper bound always define a nonconvex feasible set.

MSK\_RES\_ERR\_CON\_Q\_NOT\_PSD (1293)

The quadratic constraint matrix is not positive semidefinite as expected for a constraint with finite upper bound. This results in a nonconvex problem. The parameter  $MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL$  can be used to relax the convexity check.

MSK\_RES\_ERR\_CON\_Q\_NOT\_NSD (1294)

The quadratic constraint matrix is not negative semidefinite as expected for a constraint with finite lower bound. This results in a nonconvex problem. The parameter  $MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL$  can be used to relax the convexity check.

MSK\_RES\_ERR\_OBJ\_Q\_NOT\_PSD (1295)

The quadratic coefficient matrix in the objective is not positive semidefinite as expected for a minimization problem. The parameter  $MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL$  can be used to relax the convexity check.

MSK\_RES\_ERR\_OBJ\_Q\_NOT\_NSD (1296)

The quadratic coefficient matrix in the objective is not negative semidefinite as expected for a maximization problem. The parameter  $MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL$  can be used to relax the convexity check.

MSK\_RES\_ERR\_ARGUMENT\_PERM\_ARRAY (1299)

An invalid permutation array is specified.

MSK\_RES\_ERR\_CONE\_INDEX (1300)

An index of a non-existing cone has been specified.

MSK\_RES\_ERR\_CONE\_SIZE (1301)

A cone with incorrect number of members is specified.

MSK\_RES\_ERR\_CONE\_OVERLAP (1302)

One or more of the variables in the cone to be added is already member of another cone. Now assume the variable is  $x_j$  then add a new variable say  $x_k$  and the constraint

$$x_j = x_k$$

and then let  $x_k$  be member of the cone to be appended.

MSK\_RES\_ERR\_CONE\_REP\_VAR (1303)

A variable is included multiple times in the cone.

MSK\_RES\_ERR\_MAXNUMCONE (1304)

The value specified for maxnumcone is too small.

MSK\_RES\_ERR\_CONE\_TYPE (1305)

Invalid cone type specified.

MSK\_RES\_ERR\_CONE\_TYPE\_STR (1306)

Invalid cone type specified.

MSK\_RES\_ERR\_CONE\_OVERLAP\_APPEND (1307)

The cone to be appended has one variable which is already member of another cone.

#### MSK\_RES\_ERR\_REMOVE\_CONE\_VARIABLE (1310)

A variable cannot be removed because it will make a cone invalid.

MSK\_RES\_ERR\_APPENDING\_TOO\_BIG\_CONE (1311)

Trying to append a too big cone.

MSK\_RES\_ERR\_CONE\_PARAMETER (1320)

An invalid cone parameter.

MSK\_RES\_ERR\_SOL\_FILE\_INVALID\_NUMBER (1350)

An invalid number is specified in a solution file.

MSK\_RES\_ERR\_HUGE\_C (1375)

A huge value in absolute size is specified for one  $c_j$ .

MSK\_RES\_ERR\_HUGE\_AIJ (1380)

A numerically huge value is specified for an  $a_{i,j}$  element in A. The parameter  $MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE$  controls when an  $a_{i,j}$  is considered huge.

MSK\_RES\_ERR\_DUPLICATE\_AIJ (1385)

An element in the A matrix is specified twice.

MSK\_RES\_ERR\_LOWER\_BOUND\_IS\_A\_NAN (1390)

The lower bound specified is not a number (nan).

MSK\_RES\_ERR\_UPPER\_BOUND\_IS\_A\_NAN (1391)

The upper bound specified is not a number (nan).

MSK\_RES\_ERR\_INFINITE\_BOUND (1400)

A numerically huge bound value is specified.

MSK\_RES\_ERR\_INV\_QOBJ\_SUBI (1401)

Invalid value in qosubi.

MSK\_RES\_ERR\_INV\_QOBJ\_SUBJ (1402)

Invalid value in qosubj.

MSK\_RES\_ERR\_INV\_QOBJ\_VAL (1403)

Invalid value in goval.

MSK\_RES\_ERR\_INV\_QCON\_SUBK (1404)

Invalid value in qcsubk.

MSK\_RES\_ERR\_INV\_QCON\_SUBI (1405)

Invalid value in qcsubi.

MSK\_RES\_ERR\_INV\_QCON\_SUBJ (1406)

Invalid value in qcsubj.

MSK\_RES\_ERR\_INV\_QCON\_VAL (1407)

Invalid value in qcval.

MSK\_RES\_ERR\_QCON\_SUBI\_TOO\_SMALL (1408)

Invalid value in qcsubi.

MSK\_RES\_ERR\_QCON\_SUBI\_TOO\_LARGE (1409)

Invalid value in qcsubi.

MSK\_RES\_ERR\_QOBJ\_UPPER\_TRIANGLE (1415)

An element in the upper triangle of  $Q^o$  is specified. Only elements in the lower triangle should be specified.

MSK\_RES\_ERR\_QCON\_UPPER\_TRIANGLE (1417)

An element in the upper triangle of a  $Q^k$  is specified. Only elements in the lower triangle should be specified.

MSK\_RES\_ERR\_FIXED\_BOUND\_VALUES (1420)

A fixed constraint/variable has been specified using the bound keys but the numerical value of the lower and upper bound is different.

MSK\_RES\_ERR\_TOO\_SMALL\_A\_TRUNCATION\_VALUE (1421)

A too small value for the A trucation value is specified.

MSK\_RES\_ERR\_INVALID\_OBJECTIVE\_SENSE (1445)

An invalid objective sense is specified.

MSK\_RES\_ERR\_UNDEFINED\_OBJECTIVE\_SENSE (1446)

The objective sense has not been specified before the optimization.

MSK\_RES\_ERR\_Y\_IS\_UNDEFINED (1449)

The solution item y is undefined.

MSK\_RES\_ERR\_NAN\_IN\_DOUBLE\_DATA (1450)

An invalid floating point value was used in some double data.

### MSK\_RES\_ERR\_INF\_IN\_DOUBLE\_DATA (1451)

An infinite floating point value was used in some double data.

MSK\_RES\_ERR\_NAN\_IN\_BLC (1461)

 $l^c$  contains an invalid floating point value, i.e. a NaN.

MSK\_RES\_ERR\_NAN\_IN\_BUC (1462)

 $u^c$  contains an invalid floating point value, i.e. a NaN.

MSK\_RES\_ERR\_INVALID\_CFIX (1469)

An invalid fixed term in the objective is speficied.

MSK\_RES\_ERR\_NAN\_IN\_C (1470)

c contains an invalid floating point value, i.e. a NaN.

MSK\_RES\_ERR\_NAN\_IN\_BLX (1471)

 $l^x$  contains an invalid floating point value, i.e. a NaN.

MSK\_RES\_ERR\_NAN\_IN\_BUX (1472)

 $u^x$  contains an invalid floating point value, i.e. a NaN.

MSK\_RES\_ERR\_INVALID\_AIJ (1473)

 $a_{i,j}$  contains an invalid floating point value, i.e. a NaN or an infinite value.

MSK\_RES\_ERR\_INVALID\_CJ (1474)

 $c_j$  contains an invalid floating point value, i.e. a NaN or an infinite value.

MSK\_RES\_ERR\_SYM\_MAT\_INVALID (1480)

A symmetric matrix contains an invalid floating point value, i.e. a NaN or an infinite value.

MSK\_RES\_ERR\_SYM\_MAT\_HUGE (1482)

A symmetric matrix contains a huge value in absolute size. The parameter  $MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_HUGE$  controls when an  $e_{i,j}$  is considered huge.

MSK\_RES\_ERR\_INV\_PROBLEM (1500)

Invalid problem type. Probably a nonconvex problem has been specified.

MSK\_RES\_ERR\_MIXED\_CONIC\_AND\_NL (1501)

The problem contains nonlinear terms conic constraints. The requested operation cannot be applied to this type of problem.

MSK\_RES\_ERR\_GLOBAL\_INV\_CONIC\_PROBLEM (1503)

The global optimizer can only be applied to problems without semidefinite variables.

MSK\_RES\_ERR\_INV\_OPTIMIZER (1550)

An invalid optimizer has been chosen for the problem.

MSK\_RES\_ERR\_MIO\_NO\_OPTIMIZER (1551)

No optimizer is available for the current class of integer optimization problems.

MSK\_RES\_ERR\_NO\_OPTIMIZER\_VAR\_TYPE (1552)

No optimizer is available for this class of optimization problems.

MSK\_RES\_ERR\_FINAL\_SOLUTION (1560)

An error occurred during the solution finalization.

MSK\_RES\_ERR\_FIRST (1570)

Invalid first.

MSK\_RES\_ERR\_LAST (1571)

Invalid index last. A given index was out of expected range.

MSK\_RES\_ERR\_SLICE\_SIZE (1572)

Invalid slice size specified.

MSK\_RES\_ERR\_NEGATIVE\_SURPLUS (1573)

Negative surplus.

MSK\_RES\_ERR\_NEGATIVE\_APPEND (1578)

Cannot append a negative number.

MSK\_RES\_ERR\_POSTSOLVE (1580)

An error occurred during the postsolve. Please contact **MOSEK** support.

MSK\_RES\_ERR\_OVERFLOW (1590)

A computation produced an overflow i.e. a very large number.

MSK\_RES\_ERR\_NO\_BASIS\_SOL (1600)

No basic solution is defined.

MSK\_RES\_ERR\_BASIS\_FACTOR (1610)

The factorization of the basis is invalid.

MSK\_RES\_ERR\_BASIS\_SINGULAR (1615)

The basis is singular and hence cannot be factored.

#### MSK\_RES\_ERR\_FACTOR (1650)

An error occurred while factorizing a matrix.

MSK\_RES\_ERR\_FEASREPAIR\_CANNOT\_RELAX (1700)

An optimization problem cannot be relaxed.

MSK\_RES\_ERR\_FEASREPAIR\_SOLVING\_RELAXED (1701)

The relaxed problem could not be solved to optimality. Please consult the log file for further details.

MSK\_RES\_ERR\_FEASREPAIR\_INCONSISTENT\_BOUND (1702)

The upper bound is less than the lower bound for a variable or a constraint. Please correct this before running the feasibility repair.

MSK\_RES\_ERR\_REPAIR\_INVALID\_PROBLEM (1710)

The feasibility repair does not support the specified problem type.

MSK\_RES\_ERR\_REPAIR\_OPTIMIZATION\_FAILED (1711)

Computation the optimal relaxation failed. The cause may have been numerical problems.

MSK\_RES\_ERR\_NAME\_MAX\_LEN (1750)

A name is longer than the buffer that is supposed to hold it.

MSK\_RES\_ERR\_NAME\_IS\_NULL (1760)

The name buffer is a NULL pointer.

MSK\_RES\_ERR\_INVALID\_COMPRESSION (1800)

Invalid compression type.

MSK\_RES\_ERR\_INVALID\_IOMODE (1801)

Invalid io mode.

MSK\_RES\_ERR\_NO\_PRIMAL\_INFEAS\_CER (2000)

A certificate of primal infeasibility is not available.

MSK\_RES\_ERR\_NO\_DUAL\_INFEAS\_CER (2001)

A certificate of infeasibility is not available.

MSK\_RES\_ERR\_NO\_SOLUTION\_IN\_CALLBACK (2500)

The required solution is not available.

MSK\_RES\_ERR\_INV\_MARKI (2501)

Invalid value in marki.

MSK\_RES\_ERR\_INV\_MARKJ (2502)

Invalid value in markj.

MSK\_RES\_ERR\_INV\_NUMI (2503)

Invalid numi.

MSK\_RES\_ERR\_INV\_NUMJ (2504)

Invalid numj.

MSK\_RES\_ERR\_TASK\_INCOMPATIBLE (2560)

The Task file is incompatible with this platform. This results from reading a file on a 32 bit platform generated on a 64 bit platform.

MSK\_RES\_ERR\_TASK\_INVALID (2561)

The Task file is invalid.

MSK\_RES\_ERR\_TASK\_WRITE (2562)

Failed to write the task file.

MSK\_RES\_ERR\_LU\_MAX\_NUM\_TRIES (2800)

Could not compute the LU factors of the matrix within the maximum number of allowed tries.

MSK\_RES\_ERR\_INVALID\_UTF8 (2900)

An invalid UTF8 string is encountered.

MSK\_RES\_ERR\_INVALID\_WCHAR (2901)

An invalid wchar string is encountered.

MSK\_RES\_ERR\_NO\_DUAL\_FOR\_ITG\_SOL (2950)

No dual information is available for the integer solution.

MSK\_RES\_ERR\_NO\_SNX\_FOR\_BAS\_SOL (2953)

 $s_n^x$  is not available for the basis solution.

MSK\_RES\_ERR\_INTERNAL (3000)

An internal error occurred. Please report this problem.

MSK\_RES\_ERR\_API\_ARRAY\_TOO\_SMALL (3001)

An input array was too short.

MSK\_RES\_ERR\_API\_CB\_CONNECT (3002)

Failed to connect a callback object.

### MSK\_RES\_ERR\_API\_FATAL\_ERROR (3005)

An internal error occurred in the API. Please report this problem.

# MSK\_RES\_ERR\_API\_INTERNAL (3999)

An internal fatal error occurred in an interface function.

## MSK\_RES\_ERR\_SEN\_FORMAT (3050)

Syntax error in sensitivity analysis file.

# MSK\_RES\_ERR\_SEN\_UNDEF\_NAME (3051)

An undefined name was encountered in the sensitivity analysis file.

### MSK\_RES\_ERR\_SEN\_INDEX\_RANGE (3052)

Index out of range in the sensitivity analysis file.

### MSK\_RES\_ERR\_SEN\_BOUND\_INVALID\_UP (3053)

Analysis of upper bound requested for an index, where no upper bound exists.

### MSK\_RES\_ERR\_SEN\_BOUND\_INVALID\_LO (3054)

Analysis of lower bound requested for an index, where no lower bound exists.

# MSK\_RES\_ERR\_SEN\_INDEX\_INVALID (3055)

Invalid range given in the sensitivity file.

## MSK\_RES\_ERR\_SEN\_INVALID\_REGEXP (3056)

Syntax error in regexp or regexp longer than 1024.

## MSK\_RES\_ERR\_SEN\_SOLUTION\_STATUS (3057)

No optimal solution found to the original problem given for sensitivity analysis.

### MSK\_RES\_ERR\_SEN\_NUMERICAL (3058)

Numerical difficulties encountered performing the sensitivity analysis.

### MSK\_RES\_ERR\_SEN\_UNHANDLED\_PROBLEM\_TYPE (3080)

Sensitivity analysis cannot be performed for the specified problem. Sensitivity analysis is only possible for linear problems.

# MSK\_RES\_ERR\_UNB\_STEP\_SIZE (3100)

A step size in an optimizer was unexpectedly unbounded. For instance, if the step-size becomes unbounded in phase 1 of the simplex algorithm then an error occurs. Normally this will happen only if the problem is badly formulated. Please contact **MOSEK** support if this error occurs.

## MSK\_RES\_ERR\_IDENTICAL\_TASKS (3101)

Some tasks related to this function call were identical. Unique tasks were expected.

# MSK\_RES\_ERR\_AD\_INVALID\_CODELIST (3102)

The code list data was invalid.

## MSK\_RES\_ERR\_INTERNAL\_TEST\_FAILED (3500)

An internal unit test function failed.

# MSK\_RES\_ERR\_XML\_INVALID\_PROBLEM\_TYPE (3600)

The problem type is not supported by the XML format.

# MSK\_RES\_ERR\_INVALID\_AMPL\_STUB (3700)

Invalid AMPL stub.

# MSK\_RES\_ERR\_INT64\_TO\_INT32\_CAST (3800)

A 64 bit integer could not be cast to a 32 bit integer.

### MSK\_RES\_ERR\_SIZE\_LICENSE\_NUMCORES (3900)

The computer contains more cpu cores than the license allows for.

## MSK\_RES\_ERR\_INFEAS\_UNDEFINED (3910)

The requested value is not defined for this solution type.

# MSK\_RES\_ERR\_NO\_BARX\_FOR\_SOLUTION (3915)

There is no  $\overline{X}$  available for the solution specified. In particular note there are no  $\overline{X}$  defined for the basic and integer solutions.

# MSK\_RES\_ERR\_NO\_BARS\_FOR\_SOLUTION (3916)

There is no  $\bar{s}$  available for the solution specified. In particular note there are no  $\bar{s}$  defined for the basic and integer solutions.

# MSK\_RES\_ERR\_BAR\_VAR\_DIM (3920)

The dimension of a symmetric matrix variable has to be greater than 0.

# MSK\_RES\_ERR\_SYM\_MAT\_INVALID\_ROW\_INDEX (3940)

A row index specified for sparse symmetric matrix is invalid.

## MSK\_RES\_ERR\_SYM\_MAT\_INVALID\_COL\_INDEX (3941)

A column index specified for sparse symmetric matrix is invalid.

MSK\_RES\_ERR\_SYM\_MAT\_NOT\_LOWER\_TRINGULAR (3942)

Only the lower triangular part of sparse symmetric matrix should be specified.

MSK\_RES\_ERR\_SYM\_MAT\_INVALID\_VALUE (3943)

The numerical value specified in a sparse symmetric matrix is not a floating point value.

MSK\_RES\_ERR\_SYM\_MAT\_DUPLICATE (3944)

A value in a symmetric matric as been specified more than once.

MSK\_RES\_ERR\_INVALID\_SYM\_MAT\_DIM (3950)

A sparse symmetric matrix of invalid dimension is specified.

MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_SYM\_MAT (4000)

The file format does not support a problem with symmetric matrix variables.

MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_CFIX (4001)

The file format does not support a problem with nonzero fixed term in c.

MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_RANGED\_CONSTRAINTS (4002)

The file format does not support a problem with ranged constraints.

MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_FREE\_CONSTRAINTS (4003)

The file format does not support a problem with free constraints.

MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_CONES (4005)

The file format does not support a problem with conic constraints.

MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_QUADRATIC\_TERMS (4006)

The file format does not support a problem with quadratic terms.

MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_NONLINEAR (4010)

The file format does not support a problem with nonlinear terms.

MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_DISJUNCTIVE\_CONSTRAINTS (4011)

The file format does not support a problem with disjunctive constraints.

MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_AFFINE\_CONIC\_CONSTRAINTS (4012)

The file format does not support a problem with affine conic constraints.

MSK\_RES\_ERR\_DUPLICATE\_CONSTRAINT\_NAMES (4500)

Two constraint names are identical.

MSK\_RES\_ERR\_DUPLICATE\_VARIABLE\_NAMES (4501)

Two variable names are identical.

MSK\_RES\_ERR\_DUPLICATE\_BARVARIABLE\_NAMES (4502)

Two barvariable names are identical.

MSK\_RES\_ERR\_DUPLICATE\_CONE\_NAMES (4503)

Two cone names are identical.

MSK\_RES\_ERR\_DUPLICATE\_DOMAIN\_NAMES (4504)

Two domain names are identical.

MSK\_RES\_ERR\_DUPLICATE\_DJC\_NAMES (4505)

Two disjunctive constraint names are identical.

MSK\_RES\_ERR\_NON\_UNIQUE\_ARRAY (5000)

An array does not contain unique elements.

MSK\_RES\_ERR\_ARGUMENT\_IS\_TOO\_SMALL (5004)

The value of a function argument is too small.

MSK\_RES\_ERR\_ARGUMENT\_IS\_TOO\_LARGE (5005)

The value of a function argument is too large.

MSK\_RES\_ERR\_MIO\_INTERNAL (5010)

A fatal error occurred in the mixed integer optimizer. Please contact MOSEK support.

MSK\_RES\_ERR\_INVALID\_PROBLEM\_TYPE (6000)

An invalid problem type.

MSK\_RES\_ERR\_UNHANDLED\_SOLUTION\_STATUS (6010)

Unhandled solution status.

MSK\_RES\_ERR\_UPPER\_TRIANGLE (6020)

An element in the upper triangle of a lower triangular matrix is specified.

MSK\_RES\_ERR\_LAU\_SINGULAR\_MATRIX (7000)

A matrix is singular.

MSK\_RES\_ERR\_LAU\_NOT\_POSITIVE\_DEFINITE (7001)

A matrix is not positive definite.

MSK\_RES\_ERR\_LAU\_INVALID\_LOWER\_TRIANGULAR\_MATRIX (7002)

An invalid lower triangular matrix.

MSK\_RES\_ERR\_LAU\_UNKNOWN (7005)

An unknown error.

MSK\_RES\_ERR\_LAU\_ARG\_M (7010)

Invalid argument m.

MSK\_RES\_ERR\_LAU\_ARG\_N (7011)

Invalid argument n.

MSK\_RES\_ERR\_LAU\_ARG\_K (7012)

Invalid argument k.

MSK\_RES\_ERR\_LAU\_ARG\_TRANSA (7015)

Invalid argument transa.

MSK\_RES\_ERR\_LAU\_ARG\_TRANSB (7016)

Invalid argument transb.

MSK\_RES\_ERR\_LAU\_ARG\_UPLO (7017)

Invalid argument uplo.

MSK\_RES\_ERR\_LAU\_ARG\_TRANS (7018)

Invalid argument trans.

MSK\_RES\_ERR\_LAU\_INVALID\_SPARSE\_SYMMETRIC\_MATRIX (7019)

An invalid sparse symmetric matrix is specified. Note only the lower triangular part with no duplicates is specified.

MSK\_RES\_ERR\_CBF\_PARSE (7100)

An error occurred while parsing an CBF file.

MSK\_RES\_ERR\_CBF\_OBJ\_SENSE (7101)

An invalid objective sense is specified.

MSK\_RES\_ERR\_CBF\_NO\_VARIABLES (7102)

No variables are specified.

MSK\_RES\_ERR\_CBF\_TOO\_MANY\_CONSTRAINTS (7103)

Too many constraints specified.

MSK\_RES\_ERR\_CBF\_TOO\_MANY\_VARIABLES (7104)

Too many variables specified.

MSK\_RES\_ERR\_CBF\_NO\_VERSION\_SPECIFIED (7105)

No version specified.

MSK\_RES\_ERR\_CBF\_SYNTAX (7106)

Invalid syntax.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_OBJ (7107)

Duplicate OBJ keyword.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_CON (7108)

Duplicate CON keyword.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_VAR (7110)

Duplicate VAR keyword.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_INT (7111)

Duplicate INT keyword.

MSK\_RES\_ERR\_CBF\_INVALID\_VAR\_TYPE (7112)

Invalid variable type.

MSK\_RES\_ERR\_CBF\_INVALID\_CON\_TYPE (7113)

Invalid constraint type.

MSK\_RES\_ERR\_CBF\_INVALID\_DOMAIN\_DIMENSION (7114)

Invalid domain dimension.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_OBJACOORD (7115)

 $\label{eq:Duplicate index in OBJCOORD.}$ 

MSK\_RES\_ERR\_CBF\_DUPLICATE\_BCOORD (7116)

Duplicate index in BCOORD.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_ACOORD (7117)

Duplicate index in ACOORD.

MSK\_RES\_ERR\_CBF\_TOO\_FEW\_VARIABLES (7118)

Too few variables defined.

MSK\_RES\_ERR\_CBF\_TOO\_FEW\_CONSTRAINTS (7119)

Too few constraints defined.

MSK\_RES\_ERR\_CBF\_TOO\_FEW\_INTS (7120)

Too few ints are specified.

MSK\_RES\_ERR\_CBF\_TOO\_MANY\_INTS (7121)

Too many ints are specified.

MSK\_RES\_ERR\_CBF\_INVALID\_INT\_INDEX (7122)

Invalid INT index.

MSK\_RES\_ERR\_CBF\_UNSUPPORTED (7123)

Unsupported feature is present.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_PSDVAR (7124)

Duplicate PSDVAR keyword.

MSK\_RES\_ERR\_CBF\_INVALID\_PSDVAR\_DIMENSION (7125)

Invalid PSDVAR dimension.

MSK\_RES\_ERR\_CBF\_TOO\_FEW\_PSDVAR (7126)

Too few variables defined.

MSK\_RES\_ERR\_CBF\_INVALID\_EXP\_DIMENSION (7127)

Invalid dimension of a exponential cone.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_POW\_CONES (7130)

Multiple POWCONES specified.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_POW\_STAR\_CONES (7131)

Multiple POW\*CONES specified.

MSK\_RES\_ERR\_CBF\_INVALID\_POWER (7132)

Invalid power specified.

MSK\_RES\_ERR\_CBF\_POWER\_CONE\_IS\_TOO\_LONG (7133)

Power cone is too long.

MSK\_RES\_ERR\_CBF\_INVALID\_POWER\_CONE\_INDEX (7134)

Invalid power cone index.

MSK\_RES\_ERR\_CBF\_INVALID\_POWER\_STAR\_CONE\_INDEX (7135)

Invalid power star cone index.

MSK\_RES\_ERR\_CBF\_UNHANDLED\_POWER\_CONE\_TYPE (7136)

An unhandled power cone type.

MSK\_RES\_ERR\_CBF\_UNHANDLED\_POWER\_STAR\_CONE\_TYPE (7137)

An unhandled power star cone type.

MSK\_RES\_ERR\_CBF\_POWER\_CONE\_MISMATCH (7138)

The power cone does not match with it definition.

MSK\_RES\_ERR\_CBF\_POWER\_STAR\_CONE\_MISMATCH (7139)

The power star cone does not match with it definition.

MSK\_RES\_ERR\_CBF\_INVALID\_NUMBER\_OF\_CONES (7140)

Invalid number of cones.

MSK\_RES\_ERR\_CBF\_INVALID\_DIMENSION\_OF\_CONES (7141)

Invalid number of cones.

MSK\_RES\_ERR\_CBF\_INVALID\_NUM\_PSDCON (7200)

Invalid number of PSDCON.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_PSDCON (7201)

Duplicate CON keyword.

MSK\_RES\_ERR\_CBF\_INVALID\_DIMENSION\_OF\_PSDCON (7202)

Invalid PSDCON dimension.

MSK\_RES\_ERR\_CBF\_INVALID\_PSDCON\_INDEX (7203)

Invalid PSDCON index.

MSK\_RES\_ERR\_CBF\_INVALID\_PSDCON\_VARIABLE\_INDEX (7204)

Invalid PSDCON index.

MSK\_RES\_ERR\_CBF\_INVALID\_PSDCON\_BLOCK\_INDEX (7205)

Invalid PSDCON index.

MSK\_RES\_ERR\_CBF\_UNSUPPORTED\_CHANGE (7210)

The CHANGE section is not supported.

MSK\_RES\_ERR\_MIO\_INVALID\_ROOT\_OPTIMIZER (7700)

An invalid root optimizer was selected for the problem type.

MSK\_RES\_ERR\_MIO\_INVALID\_NODE\_OPTIMIZER (7701)

An invalid node optimizer was selected for the problem type.

### MSK\_RES\_ERR\_MPS\_WRITE\_CPLEX\_INVALID\_CONE\_TYPE (7750)

An invalid cone type occurs when writing a CPLEX formatted MPS file.

MSK\_RES\_ERR\_TOCONIC\_CONSTR\_Q\_NOT\_PSD (7800)

The matrix defining the quadratric part of constraint is not positive semidefinite.

MSK\_RES\_ERR\_TOCONIC\_CONSTRAINT\_FX (7801)

The quadratic constraint is an equality, thus not convex.

MSK\_RES\_ERR\_TOCONIC\_CONSTRAINT\_RA (7802)

The quadratic constraint has finite lower and upper bound, and therefore it is not convex.

MSK\_RES\_ERR\_TOCONIC\_CONSTR\_NOT\_CONIC (7803)

The constraint is not conic representable.

MSK\_RES\_ERR\_TOCONIC\_OBJECTIVE\_NOT\_PSD (7804)

The matrix defining the quadratric part of the objective function is not positive semidefinite.

MSK\_RES\_ERR\_SERVER\_CONNECT (8000)

Failed to connect to remote solver server. The server string or the port string were invalid, or the server did not accept connection.

MSK\_RES\_ERR\_SERVER\_PROTOCOL (8001)

Unexpected message or data from solver server.

MSK\_RES\_ERR\_SERVER\_STATUS (8002)

Server returned non-ok HTTP status code

MSK\_RES\_ERR\_SERVER\_TOKEN (8003)

The job ID specified is incorrect or invalid

MSK\_RES\_ERR\_SERVER\_ADDRESS (8004)

Invalid address string

MSK\_RES\_ERR\_SERVER\_CERTIFICATE (8005)

Invalid TLS certificate format or path

MSK\_RES\_ERR\_SERVER\_TLS\_CLIENT (8006)

Failed to create TLS cleint

MSK\_RES\_ERR\_SERVER\_ACCESS\_TOKEN (8007)

Invalid access token

MSK\_RES\_ERR\_SERVER\_PROBLEM\_SIZE (8008)

The size of the problem exceeds the dimensions permitted by the instance of the OptServer where it was run.

MSK\_RES\_ERR\_DUPLICATE\_FIJ (20100)

An element in the F matrix is specified twice.

MSK\_RES\_ERR\_INVALID\_FIJ (20101)

 $f_{i,j}$  contains an invalid floating point value, i.e. a NaN or an infinite value.

MSK\_RES\_ERR\_HUGE\_FIJ (20102)

A numerically huge value is specified for an  $f_{i,j}$  element in F. The parameter  $MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE$  controls when an  $f_{i,j}$  is considered huge.

MSK\_RES\_ERR\_INVALID\_G (20103)

g contains an invalid floating point value, i.e. a NaN or an infinite value.

MSK\_RES\_ERR\_INVALID\_B (20150)

b contains an invalid floating point value, i.e. a NaN or an infinite value.

MSK\_RES\_ERR\_DOMAIN\_INVALID\_INDEX (20400)

A domain index is invalid.

MSK\_RES\_ERR\_DOMAIN\_DIMENSION (20401)

A domain dimension is invalid.

MSK\_RES\_ERR\_DOMAIN\_DIMENSION\_PSD (20402)

A PSD domain dimension is invalid.

MSK\_RES\_ERR\_NOT\_POWER\_DOMAIN (20403)

The function is only applicable to primal and dual power cone domains.

MSK\_RES\_ERR\_DOMAIN\_POWER\_INVALID\_ALPHA (20404)

Alpha contains an invalid floating point value, i.e. a NaN or an infinite value.

MSK\_RES\_ERR\_DOMAIN\_POWER\_NEGATIVE\_ALPHA (20405)

Alpha contains a negative value or zero.

MSK\_RES\_ERR\_DOMAIN\_POWER\_NLEFT (20406)

The value of  $n_{\text{left}}$  is not in [1, n-1] where n is the dimension.

#### MSK\_RES\_ERR\_AFE\_INVALID\_INDEX (20500)

An affine expression index is invalid.

MSK\_RES\_ERR\_ACC\_INVALID\_INDEX (20600)

A affine conic constraint index is invalid.

MSK\_RES\_ERR\_ACC\_INVALID\_ENTRY\_INDEX (20601)

The index of an element in an affine conic constraint is invalid.

MSK\_RES\_ERR\_ACC\_AFE\_DOMAIN\_MISMATCH (20602)

There is a mismatch between between the number of affine expressions and total dimension of the domain(s).

MSK\_RES\_ERR\_DJC\_INVALID\_INDEX (20700)

A disjunctive constraint index is invalid.

MSK\_RES\_ERR\_DJC\_UNSUPPORTED\_DOMAIN\_TYPE (20701)

An unsupported domain type has been used in a disjunctive constraint.

MSK\_RES\_ERR\_DJC\_AFE\_DOMAIN\_MISMATCH (20702)

There is a mismatch between the number of affine expressions and total dimension of the domain(s).

MSK\_RES\_ERR\_DJC\_INVALID\_TERM\_SIZE (20703)

A termize is invalid.

MSK\_RES\_ERR\_DJC\_DOMAIN\_TERMSIZE\_MISMATCH (20704)

There is a mismatch between the number of domains and the term sizes.

MSK\_RES\_ERR\_DJC\_TOTAL\_NUM\_TERMS\_MISMATCH (20705)

There total number of terms in all domains does not match.

MSK\_RES\_ERR\_UNDEF\_SOLUTION (22000)

MOSEK has the following solution types:

- an interior-point solution,
- a basic solution,
- and an integer solution.

Each optimizer may set one or more of these solutions; e.g by default a successful optimization with the interior-point optimizer defines the interior-point solution and, for linear problems, also the basic solution. This error occurs when asking for a solution or for information about a solution that is not defined.

MSK\_RES\_ERR\_NO\_DOTY (22010)

No doty is available

# 11.4 Constants

# 11.4.1 Basis identification

MSK\_BI\_NEVER

Never do basis identification.

MSK\_BI\_ALWAYS

Basis identification is always performed even if the interior-point optimizer terminates abnormally.  ${\tt MSK\_BI\_NO\_ERROR}$ 

Basis identification is performed if the interior-point optimizer terminates without an error.

MSK\_BI\_IF\_FEASIBLE

Basis identification is not performed if the interior-point optimizer terminates with a problem status saying that the problem is primal or dual infeasible.

MSK\_BI\_RESERVERED

Not currently in use.

# 11.4.2 Bound keys

MSK\_BK\_LO

The constraint or variable has a finite lower bound and an infinite upper bound.

MSK\_BK\_UP

The constraint or variable has an infinite lower bound and an finite upper bound.

MSK\_BK\_FX

The constraint or variable is fixed.

MSK\_BK\_FR

The constraint or variable is free.

MSK\_BK\_RA

The constraint or variable is ranged.

#### 11.4.3 Mark

MSK\_MARK\_LO

The lower bound is selected for sensitivity analysis.

MSK\_MARK\_UP

The upper bound is selected for sensitivity analysis.

# 11.4.4 Degeneracy strategies

MSK\_SIM\_DEGEN\_NONE

The simplex optimizer should use no degeneration strategy.

MSK\_SIM\_DEGEN\_FREE

The simplex optimizer chooses the degeneration strategy.

MSK\_SIM\_DEGEN\_AGGRESSIVE

The simplex optimizer should use an aggressive degeneration strategy.

MSK\_SIM\_DEGEN\_MODERATE

The simplex optimizer should use a moderate degeneration strategy.

MSK\_SIM\_DEGEN\_MINIMUM

The simplex optimizer should use a minimum degeneration strategy.

# 11.4.5 Transposed matrix.

MSK\_TRANSPOSE\_NO

No transpose is applied.

MSK\_TRANSPOSE\_YES

A transpose is applied.

# 11.4.6 Triangular part of a symmetric matrix.

MSK\_UPLO\_LO

Lower part.

MSK\_UPLO\_UP

Upper part.

# 11.4.7 Problem reformulation.

MSK\_SIM\_REFORMULATION\_ON

Allow the simplex optimizer to reformulate the problem.

MSK\_SIM\_REFORMULATION\_OFF

Disallow the simplex optimizer to reformulate the problem.

MSK\_SIM\_REFORMULATION\_FREE

The simplex optimizer can choose freely.

MSK\_SIM\_REFORMULATION\_AGGRESSIVE

The simplex optimizer should use an aggressive reformulation strategy.

# 11.4.8 Exploit duplicate columns.

# MSK\_SIM\_EXPLOIT\_DUPVEC\_ON

Allow the simplex optimizer to exploit duplicated columns.

## MSK\_SIM\_EXPLOIT\_DUPVEC\_OFF

Disallow the simplex optimizer to exploit duplicated columns.

## MSK\_SIM\_EXPLOIT\_DUPVEC\_FREE

The simplex optimizer can choose freely.

# 11.4.9 Hot-start type employed by the simplex optimizer

## MSK\_SIM\_HOTSTART\_NONE

The simplex optimizer performs a coldstart.

### MSK\_SIM\_HOTSTART\_FREE

The simplex optimize chooses the hot-start type.

### MSK\_SIM\_HOTSTART\_STATUS\_KEYS

Only the status keys of the constraints and variables are used to choose the type of hot-start.

# 11.4.10 Hot-start type employed by the interior-point optimizers.

### MSK\_INTPNT\_HOTSTART\_NONE

The interior-point optimizer performs a coldstart.

### MSK\_INTPNT\_HOTSTART\_PRIMAL

The interior-point optimizer exploits the primal solution only.

#### MSK\_INTPNT\_HOTSTART\_DUAL

The interior-point optimizer exploits the dual solution only.

## MSK\_INTPNT\_HOTSTART\_PRIMAL\_DUAL

The interior-point optimizer exploits both the primal and dual solution.

# 11.4.11 Solution purification employed optimizer.

#### MSK\_PURIFY\_NONE

The optimizer performs no solution purification.

# MSK\_PURIFY\_PRIMAL

The optimizer purifies the primal solution.

#### MSK\_PURIFY\_DUAL

The optimizer purifies the dual solution.

## MSK\_PURIFY\_PRIMAL\_DUAL

The optimizer purifies both the primal and dual solution.

# MSK\_PURIFY\_AUTO

TBD

# 11.4.12 Progress callback codes

# MSK\_CALLBACK\_BEGIN\_BI

The basis identification procedure has been started.

#### MSK\_CALLBACK\_BEGIN\_CONIC

The callback function is called when the conic optimizer is started.

#### MSK\_CALLBACK\_BEGIN\_DUAL\_BI

The callback function is called from within the basis identification procedure when the dual phase is started.

#### MSK\_CALLBACK\_BEGIN\_DUAL\_SENSITIVITY

Dual sensitivity analysis is started.

# MSK\_CALLBACK\_BEGIN\_DUAL\_SETUP\_BI

The callback function is called when the dual BI phase is started.

# MSK\_CALLBACK\_BEGIN\_DUAL\_SIMPLEX

The callback function is called when the dual simplex optimizer started.

#### MSK\_CALLBACK\_BEGIN\_DUAL\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure when the dual simplex clean-up phase is started.

### MSK\_CALLBACK\_BEGIN\_INFEAS\_ANA

The callback function is called when the infeasibility analyzer is started.

### MSK\_CALLBACK\_BEGIN\_INTPNT

The callback function is called when the interior-point optimizer is started.

#### MSK\_CALLBACK\_BEGIN\_LICENSE\_WAIT

Begin waiting for license.

### MSK\_CALLBACK\_BEGIN\_MIO

The callback function is called when the mixed-integer optimizer is started.

#### MSK\_CALLBACK\_BEGIN\_OPTIMIZER

The callback function is called when the optimizer is started.

# MSK\_CALLBACK\_BEGIN\_PRESOLVE

The callback function is called when the presolve is started.

## MSK\_CALLBACK\_BEGIN\_PRIMAL\_BI

The callback function is called from within the basis identification procedure when the primal phase is started.

## MSK\_CALLBACK\_BEGIN\_PRIMAL\_REPAIR

Begin primal feasibility repair.

### MSK\_CALLBACK\_BEGIN\_PRIMAL\_SENSITIVITY

Primal sensitivity analysis is started.

### MSK\_CALLBACK\_BEGIN\_PRIMAL\_SETUP\_BI

The callback function is called when the primal BI setup is started.

# MSK\_CALLBACK\_BEGIN\_PRIMAL\_SIMPLEX

The callback function is called when the primal simplex optimizer is started.

#### MSK\_CALLBACK\_BEGIN\_PRIMAL\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure when the primal simplex clean-up phase is started.

## MSK\_CALLBACK\_BEGIN\_QCQO\_REFORMULATE

Begin QCQO reformulation.

# MSK\_CALLBACK\_BEGIN\_READ

MOSEK has started reading a problem file.

## MSK\_CALLBACK\_BEGIN\_ROOT\_CUTGEN

The callback function is called when root cut generation is started.

### MSK\_CALLBACK\_BEGIN\_SIMPLEX

The callback function is called when the simplex optimizer is started.

# MSK\_CALLBACK\_BEGIN\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure when the simplex clean-up phase is started.

## MSK\_CALLBACK\_BEGIN\_SOLVE\_ROOT\_RELAX

The callback function is called when solution of root relaxation is started.

# MSK\_CALLBACK\_BEGIN\_TO\_CONIC

Begin conic reformulation.

## MSK\_CALLBACK\_BEGIN\_WRITE

MOSEK has started writing a problem file.

#### MSK\_CALLBACK\_CONIC

The callback function is called from within the conic optimizer after the information database has been updated.

#### MSK\_CALLBACK\_DUAL\_SIMPLEX

The callback function is called from within the dual simplex optimizer.

## MSK\_CALLBACK\_END\_BI

The callback function is called when the basis identification procedure is terminated.

# MSK\_CALLBACK\_END\_CONIC

The callback function is called when the conic optimizer is terminated.

#### MSK\_CALLBACK\_END\_DUAL\_BI

The callback function is called from within the basis identification procedure when the dual phase is terminated.

#### MSK\_CALLBACK\_END\_DUAL\_SENSITIVITY

Dual sensitivity analysis is terminated.

### MSK\_CALLBACK\_END\_DUAL\_SETUP\_BI

The callback function is called when the dual BI phase is terminated.

### MSK\_CALLBACK\_END\_DUAL\_SIMPLEX

The callback function is called when the dual simplex optimizer is terminated.

#### MSK\_CALLBACK\_END\_DUAL\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure when the dual clean-up phase is terminated.

## MSK\_CALLBACK\_END\_INFEAS\_ANA

The callback function is called when the infeasibility analyzer is terminated.

#### MSK\_CALLBACK\_END\_INTPNT

The callback function is called when the interior-point optimizer is terminated.

# MSK\_CALLBACK\_END\_LICENSE\_WAIT

End waiting for license.

## MSK\_CALLBACK\_END\_MIO

The callback function is called when the mixed-integer optimizer is terminated.

### MSK\_CALLBACK\_END\_OPTIMIZER

The callback function is called when the optimizer is terminated.

#### MSK\_CALLBACK\_END\_PRESOLVE

The callback function is called when the presolve is completed.

### MSK\_CALLBACK\_END\_PRIMAL\_BI

The callback function is called from within the basis identification procedure when the primal phase is terminated.

# MSK\_CALLBACK\_END\_PRIMAL\_REPAIR

End primal feasibility repair.

#### MSK\_CALLBACK\_END\_PRIMAL\_SENSITIVITY

Primal sensitivity analysis is terminated.

# MSK\_CALLBACK\_END\_PRIMAL\_SETUP\_BI

The callback function is called when the primal BI setup is terminated.

#### MSK\_CALLBACK\_END\_PRIMAL\_SIMPLEX

The callback function is called when the primal simplex optimizer is terminated.

# MSK\_CALLBACK\_END\_PRIMAL\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure when the primal clean-up phase is terminated.

# MSK\_CALLBACK\_END\_QCQO\_REFORMULATE

End QCQO reformulation.

# MSK\_CALLBACK\_END\_READ

**MOSEK** has finished reading a problem file.

# MSK\_CALLBACK\_END\_ROOT\_CUTGEN

The callback function is called when root cut generation is terminated.

#### MSK\_CALLBACK\_END\_SIMPLEX

The callback function is called when the simplex optimizer is terminated.

## MSK\_CALLBACK\_END\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure when the simplex clean-up phase is terminated.

# MSK\_CALLBACK\_END\_SOLVE\_ROOT\_RELAX

The callback function is called when solution of root relaxation is terminated.

## MSK\_CALLBACK\_END\_TO\_CONIC

End conic reformulation.

# MSK\_CALLBACK\_END\_WRITE

 $\mathbf{MOSEK}$  has finished writing a problem file.

#### MSK\_CALLBACK\_IM\_BI

The callback function is called from within the basis identification procedure at an intermediate point.

### MSK\_CALLBACK\_IM\_CONIC

The callback function is called at an intermediate stage within the conic optimizer where the information database has not been updated.

#### MSK\_CALLBACK\_IM\_DUAL\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the dual phase.

### MSK\_CALLBACK\_IM\_DUAL\_SENSIVITY

The callback function is called at an intermediate stage of the dual sensitivity analysis.

### MSK\_CALLBACK\_IM\_DUAL\_SIMPLEX

The callback function is called at an intermediate point in the dual simplex optimizer.

#### MSK\_CALLBACK\_IM\_INTPNT

The callback function is called at an intermediate stage within the interior-point optimizer where the information database has not been updated.

#### MSK\_CALLBACK\_IM\_LICENSE\_WAIT

**MOSEK** is waiting for a license.

#### MSK\_CALLBACK\_IM\_LU

The callback function is called from within the LU factorization procedure at an intermediate point.

### MSK\_CALLBACK\_IM\_MIO

The callback function is called at an intermediate point in the mixed-integer optimizer.

#### MSK\_CALLBACK\_IM\_MIO\_DUAL\_SIMPLEX

The callback function is called at an intermediate point in the mixed-integer optimizer while running the dual simplex optimizer.

#### MSK\_CALLBACK\_IM\_MIO\_INTPNT

The callback function is called at an intermediate point in the mixed-integer optimizer while running the interior-point optimizer.

#### MSK\_CALLBACK\_IM\_MIO\_PRIMAL\_SIMPLEX

The callback function is called at an intermediate point in the mixed-integer optimizer while running the primal simplex optimizer.

### MSK\_CALLBACK\_IM\_ORDER

The callback function is called from within the matrix ordering procedure at an intermediate point.

#### MSK\_CALLBACK\_IM\_PRESOLVE

The callback function is called from within the presolve procedure at an intermediate stage.

## MSK\_CALLBACK\_IM\_PRIMAL\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the primal phase.

### MSK\_CALLBACK\_IM\_PRIMAL\_SENSIVITY

The callback function is called at an intermediate stage of the primal sensitivity analysis.

# MSK\_CALLBACK\_IM\_PRIMAL\_SIMPLEX

The callback function is called at an intermediate point in the primal simplex optimizer.

# MSK\_CALLBACK\_IM\_QO\_REFORMULATE

The callback function is called at an intermediate stage of the conic quadratic reformulation.

#### MSK\_CALLBACK\_IM\_READ

Intermediate stage in reading.

## MSK\_CALLBACK\_IM\_ROOT\_CUTGEN

The callback is called from within root cut generation at an intermediate stage.

# MSK\_CALLBACK\_IM\_SIMPLEX

The callback function is called from within the simplex optimizer at an intermediate point.

#### MSK\_CALLBACK\_IM\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the simplex clean-up phase. The frequency of the callbacks is controlled by the  $MSK\_IPAR\_LOG\_SIM\_FREQ$  parameter.

# MSK\_CALLBACK\_INTPNT

The callback function is called from within the interior-point optimizer after the information database has been updated.

## MSK\_CALLBACK\_NEW\_INT\_MIO

The callback function is called after a new integer solution has been located by the mixed-integer optimizer.

# MSK\_CALLBACK\_PRIMAL\_SIMPLEX

The callback function is called from within the primal simplex optimizer.

# MSK\_CALLBACK\_READ\_OPF

The callback function is called from the OPF reader.

#### MSK\_CALLBACK\_READ\_OPF\_SECTION

A chunk of Q non-zeros has been read from a problem file.

# MSK\_CALLBACK\_SOLVING\_REMOTE

The callback function is called while the task is being solved on a remote server.

## MSK\_CALLBACK\_UPDATE\_DUAL\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the dual phase.

#### MSK\_CALLBACK\_UPDATE\_DUAL\_SIMPLEX

The callback function is called in the dual simplex optimizer.

## MSK\_CALLBACK\_UPDATE\_DUAL\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the dual simplex clean-up phase. The frequency of the callbacks is controlled by the  $MSK\_IPAR\_LOG\_SIM\_FREQ$  parameter.

# MSK\_CALLBACK\_UPDATE\_PRESOLVE

The callback function is called from within the presolve procedure.

## MSK\_CALLBACK\_UPDATE\_PRIMAL\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the primal phase.

## MSK\_CALLBACK\_UPDATE\_PRIMAL\_SIMPLEX

The callback function is called in the primal simplex optimizer.

### MSK\_CALLBACK\_UPDATE\_PRIMAL\_SIMPLEX\_BI

The callback function is called from within the basis identification procedure at an intermediate point in the primal simplex clean-up phase. The frequency of the callbacks is controlled by the  $MSK\_IPAR\_LOG\_SIM\_FREQ$  parameter.

# MSK\_CALLBACK\_UPDATE\_SIMPLEX

The callback function is called from simplex optimizer.

#### MSK\_CALLBACK\_WRITE\_OPF

The callback function is called from the OPF writer.

# 11.4.13 Types of convexity checks.

# MSK\_CHECK\_CONVEXITY\_NONE

No convexity check.

### MSK\_CHECK\_CONVEXITY\_SIMPLE

Perform simple and fast convexity check.

### MSK\_CHECK\_CONVEXITY\_FULL

Perform a full convexity check.

# 11.4.14 Compression types

## MSK\_COMPRESS\_NONE

No compression is used.

# MSK\_COMPRESS\_FREE

The type of compression used is chosen automatically.

## MSK\_COMPRESS\_GZIP

The type of compression used is gzip compatible.

### MSK\_COMPRESS\_ZSTD

The type of compression used is zstd compatible.

# 11.4.15 Cone types

MSK\_CT\_QUAD

The cone is a quadratic cone.

MSK\_CT\_RQUAD

The cone is a rotated quadratic cone.

MSK\_CT\_PEXP

A primal exponential cone.

MSK\_CT\_DEXP

A dual exponential cone.

MSK\_CT\_PPOW

A primal power cone.

MSK\_CT\_DPOW

A dual power cone.

MSK\_CT\_ZERO

The zero cone.

# 11.4.16 Cone types

MSK\_DOMAIN\_R

R.

MSK\_DOMAIN\_RZERO

The zero vector.

MSK\_DOMAIN\_RPLUS

The positive orthant.

MSK\_DOMAIN\_RMINUS

The negative orthant.

MSK\_DOMAIN\_QUADRATIC\_CONE

The quadratic cone.

MSK\_DOMAIN\_RQUADRATIC\_CONE

The rotated quadratic cone.

MSK\_DOMAIN\_PRIMAL\_EXP\_CONE

The primal exponential cone.

MSK\_DOMAIN\_DUAL\_EXP\_CONE

The dual exponential cone.

MSK\_DOMAIN\_PRIMAL\_POWER\_CONE

The primal power cone.

MSK\_DOMAIN\_DUAL\_POWER\_CONE

The dual power cone.

MSK\_DOMAIN\_PRIMAL\_GEO\_MEAN\_CONE

The primal geometric mean cone.

MSK\_DOMAIN\_DUAL\_GEO\_MEAN\_CONE

The dual geometric mean cone.

MSK\_DOMAIN\_SVEC\_PSD\_CONE

The vectorized positive semidefinite cone.

# 11.4.17 Name types

MSK\_NAME\_TYPE\_GEN

General names. However, no duplicate and blank names are allowed.

MSK\_NAME\_TYPE\_MPS

MPS type names.

MSK\_NAME\_TYPE\_LP

LP type names.

# 11.4.18 Cone types

#### MSK\_SYMMAT\_TYPE\_SPARSE

Sparse symmetric matrix.

# 11.4.19 Data format types

# MSK\_DATA\_FORMAT\_EXTENSION

The file extension is used to determine the data file format.

#### MSK\_DATA\_FORMAT\_MPS

The data file is MPS formatted.

MSK\_DATA\_FORMAT\_LP

The data file is LP formatted.

MSK\_DATA\_FORMAT\_OP

The data file is an optimization problem formatted file.

MSK\_DATA\_FORMAT\_FREE\_MPS

The data a free MPS formatted file.

MSK\_DATA\_FORMAT\_TASK

Generic task dump file.

MSK\_DATA\_FORMAT\_PTF

(P)retty (T)ext (F)format.

MSK\_DATA\_FORMAT\_CB

Conic benchmark format,

MSK\_DATA\_FORMAT\_JSON\_TASK

JSON based task format.

# 11.4.20 Data format types

### MSK\_SOL\_FORMAT\_EXTENSION

The file extension is used to determine the data file format.

MSK\_SOL\_FORMAT\_B

Simple binary format

MSK\_SOL\_FORMAT\_TASK

Tar based format.

MSK\_SOL\_FORMAT\_JSON\_TASK

JSON based format.

# 11.4.21 Double information items

### MSK\_DINF\_ANA\_PRO\_SCALARIZED\_CONSTRAINT\_MATRIX\_DENSITY

Density percentage of the scalarized constraint matrix.

MSK\_DINF\_BI\_CLEAN\_DUAL\_TIME

Time spent within the dual clean-up optimizer of the basis identification procedure since its invocation.

### MSK\_DINF\_BI\_CLEAN\_PRIMAL\_TIME

Time spent within the primal clean-up optimizer of the basis identification procedure since its invocation.

### MSK\_DINF\_BI\_CLEAN\_TIME

Time spent within the clean-up phase of the basis identification procedure since its invocation.

MSK\_DINF\_BI\_DUAL\_TIME

Time spent within the dual phase basis identification procedure since its invocation.

# MSK\_DINF\_BI\_PRIMAL\_TIME

Time spent within the primal phase of the basis identification procedure since its invocation.

# MSK\_DINF\_BI\_TIME

Time spent within the basis identification procedure since its invocation.

#### MSK\_DINF\_INTPNT\_DUAL\_FEAS

Dual feasibility measure reported by the interior-point optimizer. (For the interior-point optimizer this measure is not directly related to the original problem because a homogeneous model is employed.)

#### MSK\_DINF\_INTPNT\_DUAL\_OBJ

Dual objective value reported by the interior-point optimizer.

# MSK\_DINF\_INTPNT\_FACTOR\_NUM\_FLOPS

An estimate of the number of flops used in the factorization.

## MSK\_DINF\_INTPNT\_OPT\_STATUS

A measure of optimality of the solution. It should converge to +1 if the problem has a primal-dual optimal solution, and converge to -1 if the problem is (strictly) primal or dual infeasible. If the measure converges to another constant, or fails to settle, the problem is usually ill-posed.

#### MSK\_DINF\_INTPNT\_ORDER\_TIME

Order time (in seconds).

## MSK\_DINF\_INTPNT\_PRIMAL\_FEAS

Primal feasibility measure reported by the interior-point optimizer. (For the interior-point optimizer this measure is not directly related to the original problem because a homogeneous model is employed).

# MSK\_DINF\_INTPNT\_PRIMAL\_OBJ

Primal objective value reported by the interior-point optimizer.

#### MSK\_DINF\_INTPNT\_TIME

Time spent within the interior-point optimizer since its invocation.

## MSK\_DINF\_MIO\_CLIQUE\_SEPARATION\_TIME

Separation time for clique cuts.

### MSK\_DINF\_MIO\_CMIR\_SEPARATION\_TIME

Separation time for CMIR cuts.

### MSK\_DINF\_MIO\_CONSTRUCT\_SOLUTION\_OBJ

If **MOSEK** has successfully constructed an integer feasible solution, then this item contains the optimal objective value corresponding to the feasible solution.

# MSK\_DINF\_MIO\_DUAL\_BOUND\_AFTER\_PRESOLVE

Value of the dual bound after presolve but before cut generation.

## MSK\_DINF\_MIO\_GMI\_SEPARATION\_TIME

Separation time for GMI cuts.

## MSK\_DINF\_MIO\_IMPLIED\_BOUND\_TIME

Separation time for implied bound cuts.

# MSK\_DINF\_MIO\_INITIAL\_FEASIBLE\_SOLUTION\_OBJ

If the user provided solution was found to be feasible this information item contains it's objective value.

# MSK\_DINF\_MIO\_KNAPSACK\_COVER\_SEPARATION\_TIME

Separation time for knapsack cover.

# MSK\_DINF\_MIO\_LIPRO\_SEPARATION\_TIME

Separation time for lift-and-project cuts.

#### MSK\_DINF\_MIO\_OBJ\_ABS\_GAP

Given the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the absolute gap defined by

|(objective value of feasible solution) – (objective bound)|.

Otherwise it has the value -1.0.

### MSK\_DINF\_MIO\_OBJ\_BOUND

The best known bound on the objective function. This value is undefined until at least one relaxation has been solved: To see if this is the case check that  $MSK\_IINF\_MIO\_NUM\_RELAX$  is strictly positive.

# MSK\_DINF\_MIO\_OBJ\_INT

The primal objective value corresponding to the best integer feasible solution. Please note that at least one integer feasible solution must have been located i.e. check  $MSK\_IINF\_MIO\_NUM\_INT\_SOLUTIONS$ .

## MSK\_DINF\_MIO\_OBJ\_REL\_GAP

Given that the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the relative gap defined by

 $\frac{|(\text{objective value of feasible solution}) - (\text{objective bound})|}{\max(\delta, |(\text{objective value of feasible solution})|)}.$ 

where  $\delta$  is given by the parameter  $MSK\_DPAR\_MIO\_REL\_GAP\_CONST$ . Otherwise it has the value -1.0.

MSK\_DINF\_MIO\_PROBING\_TIME

Total time for probing.

MSK\_DINF\_MIO\_ROOT\_CUTGEN\_TIME

Total time for cut generation.

MSK\_DINF\_MIO\_ROOT\_OPTIMIZER\_TIME

Time spent in the continuous optimizer while processing the root node relaxation.

MSK\_DINF\_MIO\_ROOT\_PRESOLVE\_TIME

Time spent presolving the problem at the root node.

MSK\_DINF\_MIO\_ROOT\_TIME

Time spent processing the root node.

MSK\_DINF\_MIO\_TIME

Time spent in the mixed-integer optimizer.

MSK\_DINF\_MIO\_USER\_OBJ\_CUT

If the objective cut is used, then this information item has the value of the cut.

MSK\_DINF\_OPTIMIZER\_TIME

Total time spent in the optimizer since it was invoked.

MSK\_DINF\_PRESOLVE\_ELI\_TIME

Total time spent in the eliminator since the presolve was invoked.

MSK\_DINF\_PRESOLVE\_LINDEP\_TIME

Total time spent in the linear dependency checker since the presolve was invoked.

MSK\_DINF\_PRESOLVE\_TIME

Total time (in seconds) spent in the presolve since it was invoked.

MSK\_DINF\_PRESOLVE\_TOTAL\_PRIMAL\_PERTURBATION

Total perturbation of the bounds of the primal problem.

MSK\_DINF\_PRIMAL\_REPAIR\_PENALTY\_OBJ

The optimal objective value of the penalty function.

MSK\_DINF\_QCQO\_REFORMULATE\_MAX\_PERTURBATION

Maximum absolute diagonal perturbation occurring during the QCQO reformulation.

MSK\_DINF\_QCQO\_REFORMULATE\_TIME

Time spent with conic quadratic reformulation.

MSK\_DINF\_QCQO\_REFORMULATE\_WORST\_CHOLESKY\_COLUMN\_SCALING

Worst Cholesky column scaling.

MSK\_DINF\_QCQO\_REFORMULATE\_WORST\_CHOLESKY\_DIAG\_SCALING

Worst Cholesky diagonal scaling.

MSK\_DINF\_READ\_DATA\_TIME

Time spent reading the data file.

MSK\_DINF\_REMOTE\_TIME

The total real time in seconds spent when optimizing on a server by the process performing the optimization on the server

MSK\_DINF\_SIM\_DUAL\_TIME

Time spent in the dual simplex optimizer since invoking it.

MSK\_DINF\_SIM\_FEAS

Feasibility measure reported by the simplex optimizer.

MSK\_DINF\_SIM\_OBJ

Objective value reported by the simplex optimizer.

MSK\_DINF\_SIM\_PRIMAL\_TIME

Time spent in the primal simplex optimizer since invoking it.

MSK\_DINF\_SIM\_TIME

Time spent in the simplex optimizer since invoking it.

MSK\_DINF\_SOL\_BAS\_DUAL\_OBJ

Dual objective value of the basic solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

MSK\_DINF\_SOL\_BAS\_DVIOLCON

Maximal dual bound violation for  $x^c$  in the basic solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

MSK\_DINF\_SOL\_BAS\_DVIOLVAR

Maximal dual bound violation for  $x^x$  in the basic solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

#### MSK\_DINF\_SOL\_BAS\_NRM\_BARX

Infinity norm of  $\overline{X}$  in the basic solution.

# MSK\_DINF\_SOL\_BAS\_NRM\_SLC

Infinity norm of  $s_l^c$  in the basic solution.

## MSK\_DINF\_SOL\_BAS\_NRM\_SLX

Infinity norm of  $s_l^x$  in the basic solution.

## MSK\_DINF\_SOL\_BAS\_NRM\_SUC

Infinity norm of  $s_u^c$  in the basic solution.

### MSK\_DINF\_SOL\_BAS\_NRM\_SUX

Infinity norm of  $s_u^X$  in the basic solution.

# MSK\_DINF\_SOL\_BAS\_NRM\_XC

Infinity norm of  $x^c$  in the basic solution.

# MSK\_DINF\_SOL\_BAS\_NRM\_XX

Infinity norm of  $x^x$  in the basic solution.

#### MSK\_DINF\_SOL\_BAS\_NRM\_Y

Infinity norm of y in the basic solution.

## MSK\_DINF\_SOL\_BAS\_PRIMAL\_OBJ

Primal objective value of the basic solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set

## MSK\_DINF\_SOL\_BAS\_PVIOLCON

Maximal primal bound violation for  $x^c$  in the basic solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

## MSK\_DINF\_SOL\_BAS\_PVIOLVAR

Maximal primal bound violation for  $x^x$  in the basic solution. Updated in  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

### MSK\_DINF\_SOL\_ITG\_NRM\_BARX

Infinity norm of  $\overline{X}$  in the integer solution.

# MSK\_DINF\_SOL\_ITG\_NRM\_XC

Infinity norm of  $x^c$  in the integer solution.

# MSK\_DINF\_SOL\_ITG\_NRM\_XX

Infinity norm of  $x^x$  in the integer solution.

## MSK\_DINF\_SOL\_ITG\_PRIMAL\_OBJ

Primal objective value of the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

# MSK\_DINF\_SOL\_ITG\_PVIOLACC

Maximal primal violation for affine conic constraints in the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

# MSK\_DINF\_SOL\_ITG\_PVIOLBARVAR

Maximal primal bound violation for  $\overline{X}$  in the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

# MSK\_DINF\_SOL\_ITG\_PVIOLCON

Maximal primal bound violation for  $x^c$  in the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

## MSK\_DINF\_SOL\_ITG\_PVIOLCONES

Maximal primal violation for primal conic constraints in the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

## MSK\_DINF\_SOL\_ITG\_PVIOLDJC

Maximal primal violation for disjunctive constraints in the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

## MSK\_DINF\_SOL\_ITG\_PVIOLITG

Maximal violation for the integer constraints in the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

# MSK\_DINF\_SOL\_ITG\_PVIOLVAR

Maximal primal bound violation for  $x^x$  in the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

# MSK\_DINF\_SOL\_ITR\_DUAL\_OBJ

Dual objective value of the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

### MSK\_DINF\_SOL\_ITR\_DVIOLACC

Maximal dual violation for the affine conic constraints in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

### MSK\_DINF\_SOL\_ITR\_DVIOLBARVAR

Maximal dual bound violation for  $\overline{X}$  in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

#### MSK\_DINF\_SOL\_ITR\_DVIOLCON

Maximal dual bound violation for  $x^c$  in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

#### MSK\_DINF\_SOL\_ITR\_DVIOLCONES

Maximal dual violation for conic constraints in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

# ${\tt MSK\_DINF\_SOL\_ITR\_DVIOLVAR}$

Maximal dual bound violation for  $x^x$  in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set.

#### MSK\_DINF\_SOL\_ITR\_NRM\_BARS

Infinity norm of  $\overline{S}$  in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_NRM\_BARX

Infinity norm of  $\overline{X}$  in the interior-point solution.

## MSK\_DINF\_SOL\_ITR\_NRM\_SLC

Infinity norm of  $s_I^c$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_SLX

Infinity norm of  $s_I^x$  in the interior-point solution.

#### MSK\_DINF\_SOL\_ITR\_NRM\_SNX

Infinity norm of  $s_n^x$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_SUC

Infinity norm of  $s_u^c$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_SUX

Infinity norm of  $s_u^X$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_XC

Infinity norm of  $x^c$  in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_NRM\_XX

Infinity norm of  $x^x$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_Y

Infinity norm of y in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_PRIMAL\_OBJ

Primal objective value of the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

## MSK\_DINF\_SOL\_ITR\_PVIOLACC

Maximal primal violation for affine conic constraints in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

# ${\tt MSK\_DINF\_SOL\_ITR\_PVIOLBARVAR}$

Maximal primal bound violation for  $\overline{X}$  in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

### MSK\_DINF\_SOL\_ITR\_PVIOLCON

Maximal primal bound violation for  $x^c$  in the interior-point solution. Updated if  $\texttt{MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO}$  is set .

#### MSK\_DINF\_SOL\_ITR\_PVIOLCONES

Maximal primal violation for conic constraints in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set .

# MSK\_DINF\_SOL\_ITR\_PVIOLVAR

Maximal primal bound violation for  $x^x$  in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set.

# MSK\_DINF\_TO\_CONIC\_TIME

Time spent in the last to conic reformulation.

# MSK\_DINF\_WRITE\_DATA\_TIME

Time spent writing the data file.

## 11.4.22 License feature

MSK FEATURE PTS

Base system.

MSK\_FEATURE\_PTON

Conic extension.

# 11.4.23 Long integer information items.

#### MSK\_LIINF\_ANA\_PRO\_SCALARIZED\_CONSTRAINT\_MATRIX\_NUM\_COLUMNS

Number of columns in the scalarized constraint matrix.

MSK\_LIINF\_ANA\_PRO\_SCALARIZED\_CONSTRAINT\_MATRIX\_NUM\_NZ

Number of non-zero entries in the scalarized constraint matrix.

MSK\_LIINF\_ANA\_PRO\_SCALARIZED\_CONSTRAINT\_MATRIX\_NUM\_ROWS

Number of rows in the scalarized constraint matrix.

MSK\_LIINF\_BI\_CLEAN\_DUAL\_DEG\_ITER

Number of dual degenerate clean iterations performed in the basis identification.

MSK\_LIINF\_BI\_CLEAN\_DUAL\_ITER

Number of dual clean iterations performed in the basis identification.

MSK\_LIINF\_BI\_CLEAN\_PRIMAL\_DEG\_ITER

Number of primal degenerate clean iterations performed in the basis identification.

MSK\_LIINF\_BI\_CLEAN\_PRIMAL\_ITER

Number of primal clean iterations performed in the basis identification.

MSK\_LIINF\_BI\_DUAL\_ITER

Number of dual pivots performed in the basis identification.

MSK\_LIINF\_BI\_PRIMAL\_ITER

Number of primal pivots performed in the basis identification.

MSK\_LIINF\_INTPNT\_FACTOR\_NUM\_NZ

Number of non-zeros in factorization.

MSK\_LIINF\_MIO\_ANZ

Number of non-zero entries in the constraint matrix of the problem to be solved by the mixed-integer optimizer.

MSK\_LIINF\_MIO\_INTPNT\_ITER

Number of interior-point iterations performed by the mixed-integer optimizer.

MSK\_LIINF\_MIO\_NUM\_DUAL\_ILLPOSED\_CER

Number of dual illposed certificates encountered by the mixed-integer optimizer.

MSK\_LIINF\_MIO\_NUM\_PRIM\_ILLPOSED\_CER

Number of primal illposed certificates encountered by the mixed-integer optimizer.

MSK\_LIINF\_MIO\_PRESOLVED\_ANZ

Number of non-zero entries in the constraint matrix of the problem after the mixed-integer optimizer's presolve.

MSK\_LIINF\_MIO\_SIMPLEX\_ITER

Number of simplex iterations performed by the mixed-integer optimizer.

MSK\_LIINF\_RD\_NUMACC

Number of affince conic constraints.

MSK\_LIINF\_RD\_NUMANZ

Number of non-zeros in A that is read.

MSK\_LIINF\_RD\_NUMDJC

Number of disjuncive constraints.

MSK\_LIINF\_RD\_NUMQNZ

Number of Q non-zeros.

MSK\_LIINF\_SIMPLEX\_ITER

Number of iterations performed by the simplex optimizer.

# 11.4.24 Integer information items.

MSK\_IINF\_ANA\_PRO\_NUM\_CON

Number of constraints in the problem.

MSK\_IINF\_ANA\_PRO\_NUM\_CON\_EQ

Number of equality constraints.

MSK\_IINF\_ANA\_PRO\_NUM\_CON\_FR

Number of unbounded constraints.

MSK\_IINF\_ANA\_PRO\_NUM\_CON\_LO

Number of constraints with a lower bound and an infinite upper bound.

MSK\_IINF\_ANA\_PRO\_NUM\_CON\_RA

Number of constraints with finite lower and upper bounds.

MSK\_IINF\_ANA\_PRO\_NUM\_CON\_UP

Number of constraints with an upper bound and an infinite lower bound.

MSK\_IINF\_ANA\_PRO\_NUM\_VAR

Number of variables in the problem.

MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_BIN

Number of binary (0-1) variables.

MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_CONT

Number of continuous variables.

MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_EQ

Number of fixed variables.

MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_FR

Number of free variables.

MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_INT

Number of general integer variables.

MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_LO

Number of variables with a lower bound and an infinite upper bound.

MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_RA

Number of variables with finite lower and upper bounds.

MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_UP

Number of variables with an upper bound and an infinite lower bound.

MSK\_IINF\_INTPNT\_FACTOR\_DIM\_DENSE

Dimension of the dense sub system in factorization.

MSK\_IINF\_INTPNT\_ITER

Number of interior-point iterations since invoking the interior-point optimizer.

MSK\_IINF\_INTPNT\_NUM\_THREADS

Number of threads that the interior-point optimizer is using.

MSK\_IINF\_INTPNT\_SOLVE\_DUAL

Non-zero if the interior-point optimizer is solving the dual problem.

 ${\tt MSK\_IINF\_MIO\_ABSGAP\_SATISFIED}$ 

Non-zero if absolute gap is within tolerances.

MSK\_IINF\_MIO\_CLIQUE\_TABLE\_SIZE

Size of the clique table.

MSK\_IINF\_MIO\_CONSTRUCT\_SOLUTION

This item informs if MOSEK constructed an initial integer feasible solution.

- -1: tried, but failed,
- $\bullet\,$  0: no partial solution supplied by the user,
- 1: constructed feasible solution.

#### MSK\_IINF\_MIO\_INITIAL\_FEASIBLE\_SOLUTION

This item informs if MOSEK found the solution provided by the user to be feasible

- 0: solution provided by the user was not found to be feasible for the current problem,
- 1: user provided solution was found to be feasible.

# MSK\_IINF\_MIO\_NODE\_DEPTH

Depth of the last node solved.

# MSK\_IINF\_MIO\_NUM\_ACTIVE\_NODES

Number of active branch and bound nodes.

### MSK\_IINF\_MIO\_NUM\_BRANCH

Number of branches performed during the optimization.

# MSK\_IINF\_MIO\_NUM\_CLIQUE\_CUTS

Number of clique cuts.

# MSK\_IINF\_MIO\_NUM\_CMIR\_CUTS

Number of Complemented Mixed Integer Rounding (CMIR) cuts.

### MSK\_IINF\_MIO\_NUM\_GOMORY\_CUTS

Number of Gomory cuts.

### MSK\_IINF\_MIO\_NUM\_IMPLIED\_BOUND\_CUTS

Number of implied bound cuts.

### MSK\_IINF\_MIO\_NUM\_INT\_SOLUTIONS

Number of integer feasible solutions that have been found.

# MSK\_IINF\_MIO\_NUM\_KNAPSACK\_COVER\_CUTS

Number of clique cuts.

### MSK\_IINF\_MIO\_NUM\_LIPRO\_CUTS

Number of lift-and-project cuts.

## MSK\_IINF\_MIO\_NUM\_RELAX

Number of relaxations solved during the optimization.

### MSK\_IINF\_MIO\_NUM\_REPEATED\_PRESOLVE

Number of times presolve was repeated at root.

### MSK\_IINF\_MIO\_NUMBIN

Number of binary variables in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_NUMBINCONEVAR

Number of binary cone variables in the problem to be solved by the mixed-integer optimizer.

#### MSK\_IINF\_MIO\_NUMCON

Number of constraints in the problem to be solved by the mixed-integer optimizer.

#### MSK\_IINF\_MIO\_NUMCONE

Number of cones in the problem to be solved by the mixed-integer optimizer.

#### MSK\_IINF\_MIO\_NUMCONEVAR

Number of cone variables in the problem to be solved by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_NUMCONT

Number of continuous variables in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_NUMCONTCONEVAR

Number of continuous cone variables in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_NUMDEXPCONES

Number of dual exponential cones in the problem to be solved by the mixed-integer optimizer.

#### MSK\_IINF\_MIO\_NUMDJC

Number of disjunctive constraints in the problem to be solved by the mixed-integer optimizer.

## MSK\_IINF\_MIO\_NUMDPOWCONES

Number of dual power cones in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_NUMINT

Number of integer variables in the problem to be solved by the mixed-integer optimizer.

#### MSK\_IINF\_MIO\_NUMINTCONEVAR

Number of integer cone variables in the problem to be solved by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_NUMPEXPCONES

Number of primal exponential cones in the problem to be solved by the mixed-integer optimizer.

## MSK\_IINF\_MIO\_NUMPPOWCONES

Number of primal power cones in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_NUMQCONES

Number of quadratic cones in the problem to be solved by the mixed-integer optimizer.

#### MSK\_IINF\_MIO\_NUMRQCONES

Number of rotated quadratic cones in the problem to be solved by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_NUMVAR

Number of variables in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_OBJ\_BOUND\_DEFINED

Non-zero if a valid objective bound has been found, otherwise zero.

# MSK\_IINF\_MIO\_PRESOLVED\_NUMBIN

Number of binary variables in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMBINCONEVAR

Number of binary cone variables in the problem after the mixed-integer optimizer's presolve.

# MSK\_IINF\_MIO\_PRESOLVED\_NUMCON

Number of constraints in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMCONE

Number of cones in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMCONEVAR

Number of cone variables in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMCONT

Number of continuous variables in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMCONTCONEVAR

Number of continuous cone variables in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMDEXPCONES

Number of dual exponential cones in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMDJC

Number of disjunctive constraints in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMDPOWCONES

Number of dual power cones in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMINT

Number of integer variables in the problem after the mixed-integer optimizer's presolve.

## MSK\_IINF\_MIO\_PRESOLVED\_NUMINTCONEVAR

Number of integer cone variables in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMPEXPCONES

Number of primal exponential cones in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMPPOWCONES

Number of primal power cones in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMQCONES

Number of quadratic cones in the problem after the mixed-integer optimizer's presolve.

## MSK\_IINF\_MIO\_PRESOLVED\_NUMRQCONES

Number of rotated quadratic cones in the problem after the mixed-integer optimizer's presolve.

# ${\tt MSK\_IINF\_MIO\_PRESOLVED\_NUMVAR}$

Number of variables in the problem after the mixed-integer optimizer's presolve.

# MSK\_IINF\_MIO\_RELGAP\_SATISFIED

Non-zero if relative gap is within tolerances.

# MSK\_IINF\_MIO\_TOTAL\_NUM\_CUTS

Total number of cuts generated by the mixed-integer optimizer.

#### MSK\_IINF\_MIO\_USER\_OBJ\_CUT

If it is non-zero, then the objective cut is used.

## MSK\_IINF\_OPT\_NUMCON

Number of constraints in the problem solved when the optimizer is called.

# MSK\_IINF\_OPT\_NUMVAR

Number of variables in the problem solved when the optimizer is called

### MSK\_IINF\_OPTIMIZE\_RESPONSE

The response code returned by optimize.

# MSK\_IINF\_PRESOLVE\_NUM\_PRIMAL\_PERTURBATIONS

Number perturbations to the bounds of the primal problem.

## MSK\_IINF\_PURIFY\_DUAL\_SUCCESS

Is nonzero if the dual solution is purified.

## MSK\_IINF\_PURIFY\_PRIMAL\_SUCCESS

Is nonzero if the primal solution is purified.

## MSK\_IINF\_RD\_NUMBARVAR

Number of symmetric variables read.

#### MSK\_IINF\_RD\_NUMCON

Number of constraints read.

# MSK\_IINF\_RD\_NUMCONE

Number of conic constraints read.

#### MSK\_IINF\_RD\_NUMINTVAR

Number of integer-constrained variables read.

#### MSK\_IINF\_RD\_NUMQ

Number of nonempty Q matrices read.

#### MSK\_IINF\_RD\_NUMVAR

Number of variables read.

#### MSK\_IINF\_RD\_PROTYPE

Problem type.

#### MSK\_IINF\_SIM\_DUAL\_DEG\_ITER

The number of dual degenerate iterations.

#### MSK\_IINF\_SIM\_DUAL\_HOTSTART

If 1 then the dual simplex algorithm is solving from an advanced basis.

## MSK\_IINF\_SIM\_DUAL\_HOTSTART\_LU

If 1 then a valid basis factorization of full rank was located and used by the dual simplex algorithm.

#### MSK\_IINF\_SIM\_DUAL\_INF\_ITER

The number of iterations taken with dual infeasibility.

#### MSK\_IINF\_SIM\_DUAL\_ITER

Number of dual simplex iterations during the last optimization.

#### MSK\_IINF\_SIM\_NUMCON

Number of constraints in the problem solved by the simplex optimizer.

## MSK\_IINF\_SIM\_NUMVAR

Number of variables in the problem solved by the simplex optimizer.

## MSK\_IINF\_SIM\_PRIMAL\_DEG\_ITER

The number of primal degenerate iterations.

# MSK\_IINF\_SIM\_PRIMAL\_HOTSTART

If 1 then the primal simplex algorithm is solving from an advanced basis.

#### MSK\_IINF\_SIM\_PRIMAL\_HOTSTART\_LU

If 1 then a valid basis factorization of full rank was located and used by the primal simplex algorithm.

### MSK\_IINF\_SIM\_PRIMAL\_INF\_ITER

The number of iterations taken with primal infeasibility.

# MSK\_IINF\_SIM\_PRIMAL\_ITER

Number of primal simplex iterations during the last optimization.

# MSK\_IINF\_SIM\_SOLVE\_DUAL

Is non-zero if dual problem is solved.

#### MSK\_IINF\_SOL\_BAS\_PROSTA

Problem status of the basic solution. Updated after each optimization.

# MSK\_IINF\_SOL\_BAS\_SOLSTA

Solution status of the basic solution. Updated after each optimization.

# MSK\_IINF\_SOL\_ITG\_PROSTA

Problem status of the integer solution. Updated after each optimization.

#### MSK\_IINF\_SOL\_ITG\_SOLSTA

Solution status of the integer solution. Updated after each optimization.

## MSK\_IINF\_SOL\_ITR\_PROSTA

Problem status of the interior-point solution. Updated after each optimization.

# MSK\_IINF\_SOL\_ITR\_SOLSTA

Solution status of the interior-point solution. Updated after each optimization.

# MSK\_IINF\_STO\_NUM\_A\_REALLOC

Number of times the storage for storing A has been changed. A large value may indicates that memory fragmentation may occur.

# 11.4.25 Information item types

MSK\_INF\_DOU\_TYPE

Is a double information type.

MSK\_INF\_INT\_TYPE

Is an integer.

MSK\_INF\_LINT\_TYPE

Is a long integer.

# 11.4.26 Input/output modes

MSK\_IOMODE\_READ

The file is read-only.

MSK\_IOMODE\_WRITE

The file is write-only. If the file exists then it is truncated when it is opened. Otherwise it is created when it is opened.

MSK\_IOMODE\_READWRITE

The file is to read and write.

# 11.4.27 Specifies the branching direction.

MSK\_BRANCH\_DIR\_FREE

The mixed-integer optimizer decides which branch to choose.

MSK\_BRANCH\_DIR\_UP

The mixed-integer optimizer always chooses the up branch first.

MSK\_BRANCH\_DIR\_DOWN

The mixed-integer optimizer always chooses the down branch first.

MSK\_BRANCH\_DIR\_NEAR

Branch in direction nearest to selected fractional variable.

MSK\_BRANCH\_DIR\_FAR

Branch in direction farthest from selected fractional variable.

MSK\_BRANCH\_DIR\_ROOT\_LP

Chose direction based on root lp value of selected variable.

MSK\_BRANCH\_DIR\_GUIDED

Branch in direction of current incumbent.

MSK\_BRANCH\_DIR\_PSEUDOCOST

Branch based on the pseudocost of the variable.

# 11.4.28 Specifies the reformulation method for mixed-integer quadratic problems.

MSK\_MIO\_QCQO\_REFORMULATION\_METHOD\_FREE

The mixed-integer optimizer decides which reformulation method to apply.

MSK\_MIO\_QCQO\_REFORMULATION\_METHOD\_NONE

No reformulation method is applied.

MSK\_MIO\_QCQO\_REFORMULATION\_METHOD\_LINEARIZATION

A reformulation via linearization is applied.

MSK\_MIO\_QCQO\_REFORMULATION\_METHOD\_EIGEN\_VAL\_METHOD

The eigenvalue method is applied.

MSK\_MIO\_QCQO\_REFORMULATION\_METHOD\_DIAG\_SDP

A perturbation of matrix diagonals via the solution of SDPs is applied.

MSK\_MIO\_QCQO\_REFORMULATION\_METHOD\_RELAX\_SDP

A Reformulation based on the solution of an SDP-relaxation of the problem is applied.

# 11.4.29 Specifies the problem data permutation method for mixed-integer problems.

#### MSK\_MIO\_DATA\_PERMUTATION\_METHOD\_NONE

No problem data permutation is applied.

MSK\_MIO\_DATA\_PERMUTATION\_METHOD\_CYCLIC\_SHIFT

A random cyclic shift is applied to permute the problem data.

MSK\_MIO\_DATA\_PERMUTATION\_METHOD\_RANDOM

A random permutation is applied to the problem data.

# 11.4.30 Continuous mixed-integer solution type

MSK\_MIO\_CONT\_SOL\_NONE

No interior-point or basic solution are reported when the mixed-integer optimizer is used.

MSK\_MIO\_CONT\_SOL\_ROOT

The reported interior-point and basic solutions are a solution to the root node problem when mixed-integer optimizer is used.

MSK\_MIO\_CONT\_SOL\_ITG

The reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. A solution is only reported in case the problem has a primal feasible solution.

MSK\_MIO\_CONT\_SOL\_ITG\_REL

In case the problem is primal feasible then the reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. If the problem is primal infeasible, then the solution to the root node problem is reported.

# 11.4.31 Integer restrictions

MSK\_MIO\_MODE\_IGNORED

The integer constraints are ignored and the problem is solved as a continuous problem.

MSK\_MIO\_MODE\_SATISFIED

Integer restrictions should be satisfied.

# 11.4.32 Mixed-integer node selection types

MSK\_MIO\_NODE\_SELECTION\_FREE

The optimizer decides the node selection strategy.

MSK\_MIO\_NODE\_SELECTION\_FIRST

The optimizer employs a depth first node selection strategy.

MSK\_MIO\_NODE\_SELECTION\_BEST

The optimizer employs a best bound node selection strategy.

MSK\_MIO\_NODE\_SELECTION\_PSEUDO

The optimizer employs selects the node based on a pseudo cost estimate.

# 11.4.33 MPS file format type

MSK\_MPS\_FORMAT\_STRICT

It is assumed that the input file satisfies the MPS format strictly.

MSK\_MPS\_FORMAT\_RELAXED

It is assumed that the input file satisfies a slightly relaxed version of the MPS format.

MSK\_MPS\_FORMAT\_FREE

It is assumed that the input file satisfies the free MPS format. This implies that spaces are not allowed in names. Otherwise the format is free.

MSK\_MPS\_FORMAT\_CPLEX

The CPLEX compatible version of the MPS format is employed.

# 11.4.34 Objective sense types

MSK\_OBJECTIVE\_SENSE\_MINIMIZE

The problem should be minimized.

MSK\_OBJECTIVE\_SENSE\_MAXIMIZE

The problem should be maximized.

# 11.4.35 On/off

MSK\_ON

Switch the option on.

MSK\_OFF

Switch the option off.

# 11.4.36 Optimizer types

MSK\_OPTIMIZER\_CONIC

The optimizer for problems having conic constraints.

MSK\_OPTIMIZER\_DUAL\_SIMPLEX

The dual simplex optimizer is used.

MSK\_OPTIMIZER\_FREE

The optimizer is chosen automatically.

MSK\_OPTIMIZER\_FREE\_SIMPLEX

One of the simplex optimizers is used.

MSK\_OPTIMIZER\_INTPNT

The interior-point optimizer is used.

MSK\_OPTIMIZER\_MIXED\_INT

The mixed-integer optimizer.

MSK\_OPTIMIZER\_PRIMAL\_SIMPLEX

The primal simplex optimizer is used.

# 11.4.37 Ordering strategies

MSK\_ORDER\_METHOD\_FREE

The ordering method is chosen automatically.

MSK\_ORDER\_METHOD\_APPMINLOC

Approximate minimum local fill-in ordering is employed.

MSK\_ORDER\_METHOD\_EXPERIMENTAL

This option should not be used.

MSK\_ORDER\_METHOD\_TRY\_GRAPHPAR

Always try the graph partitioning based ordering.

 ${\tt MSK\_ORDER\_METHOD\_FORCE\_GRAPHPAR}$ 

Always use the graph partitioning based ordering even if it is worse than the approximate minimum local fill ordering.

MSK\_ORDER\_METHOD\_NONE

No ordering is used.

# 11.4.38 Presolve method.

MSK\_PRESOLVE\_MODE\_OFF

The problem is not presolved before it is optimized.

MSK\_PRESOLVE\_MODE\_ON

The problem is presolved before it is optimized.

MSK\_PRESOLVE\_MODE\_FREE

It is decided automatically whether to presolve before the problem is optimized.

# 11.4.39 Parameter type

MSK\_PAR\_INVALID\_TYPE

Not a valid parameter.

MSK\_PAR\_DOU\_TYPE

Is a double parameter.

MSK\_PAR\_INT\_TYPE

Is an integer parameter.

MSK\_PAR\_STR\_TYPE

Is a string parameter.

#### 11.4.40 Problem data items

MSK\_PI\_VAR

Item is a variable.

MSK\_PI\_CON

Item is a constraint.

MSK\_PI\_CONE

Item is a cone.

# 11.4.41 Problem types

MSK\_PROBTYPE\_LO

The problem is a linear optimization problem.

MSK\_PROBTYPE\_QO

The problem is a quadratic optimization problem.

MSK\_PROBTYPE\_QCQO

The problem is a quadratically constrained optimization problem.

MSK\_PROBTYPE\_CONIC

A conic optimization.

MSK\_PROBTYPE\_MIXED

General nonlinear constraints and conic constraints. This combination can not be solved by MOSEK.

# 11.4.42 Problem status keys

MSK\_PRO\_STA\_UNKNOWN

Unknown problem status.

MSK\_PRO\_STA\_PRIM\_AND\_DUAL\_FEAS

The problem is primal and dual feasible.

MSK\_PRO\_STA\_PRIM\_FEAS

The problem is primal feasible.

MSK\_PRO\_STA\_DUAL\_FEAS

The problem is dual feasible.

MSK\_PRO\_STA\_PRIM\_INFEAS

The problem is primal infeasible.

MSK\_PRO\_STA\_DUAL\_INFEAS

The problem is dual infeasible.

 ${\tt MSK\_PRO\_STA\_PRIM\_AND\_DUAL\_INFEAS}$ 

The problem is primal and dual infeasible.

MSK\_PRO\_STA\_ILL\_POSED

The problem is ill-posed. For example, it may be primal and dual feasible but have a positive duality gap.

MSK\_PRO\_STA\_PRIM\_INFEAS\_OR\_UNBOUNDED

The problem is either primal infeasible or unbounded. This may occur for mixed-integer problems.

# 11.4.43 XML writer output mode

MSK\_WRITE\_XML\_MODE\_ROW

Write in row order.

MSK\_WRITE\_XML\_MODE\_COL

Write in column order.

# 11.4.44 Response code type

MSK\_RESPONSE\_OK

The response code is OK.

MSK\_RESPONSE\_WRN

The response code is a warning.

MSK\_RESPONSE\_TRM

The response code is an optimizer termination status.

MSK\_RESPONSE\_ERR

The response code is an error.

MSK\_RESPONSE\_UNK

The response code does not belong to any class.

# 11.4.45 Scaling type

MSK\_SCALING\_FREE

The optimizer chooses the scaling heuristic.

MSK\_SCALING\_NONE

No scaling is performed.

# 11.4.46 Scaling method

MSK\_SCALING\_METHOD\_POW2

Scales only with power of 2 leaving the mantissa untouched.

MSK\_SCALING\_METHOD\_FREE

The optimizer chooses the scaling heuristic.

# 11.4.47 Sensitivity types

MSK\_SENSITIVITY\_TYPE\_BASIS

Basis sensitivity analysis is performed.

# 11.4.48 Simplex selection strategy

MSK\_SIM\_SELECTION\_FREE

The optimizer chooses the pricing strategy.

MSK\_SIM\_SELECTION\_FULL

The optimizer uses full pricing.

 ${\tt MSK\_SIM\_SELECTION\_ASE}$ 

The optimizer uses approximate steepest-edge pricing.

MSK\_SIM\_SELECTION\_DEVEX

The optimizer uses devex steepest-edge pricing (or if it is not available an approximate steep-edge selection).

MSK\_SIM\_SELECTION\_SE

The optimizer uses steepest-edge selection (or if it is not available an approximate steep-edge selection).

MSK\_SIM\_SELECTION\_PARTIAL

The optimizer uses a partial selection approach. The approach is usually beneficial if the number of variables is much larger than the number of constraints.

# 11.4.49 Solution items

MSK\_SOL\_ITEM\_XC

Solution for the constraints.

MSK\_SOL\_ITEM\_XX

Variable solution.

MSK\_SOL\_ITEM\_Y

Lagrange multipliers for equations.

MSK\_SOL\_ITEM\_SLC

Lagrange multipliers for lower bounds on the constraints.

MSK\_SOL\_ITEM\_SUC

Lagrange multipliers for upper bounds on the constraints.

MSK\_SOL\_ITEM\_SLX

Lagrange multipliers for lower bounds on the variables.

MSK\_SOL\_ITEM\_SUX

Lagrange multipliers for upper bounds on the variables.

MSK\_SOL\_ITEM\_SNX

Lagrange multipliers corresponding to the conic constraints on the variables.

# 11.4.50 Solution status keys

MSK\_SOL\_STA\_UNKNOWN

Status of the solution is unknown.

MSK\_SOL\_STA\_OPTIMAL

The solution is optimal.

MSK\_SOL\_STA\_PRIM\_FEAS

The solution is primal feasible.

MSK\_SOL\_STA\_DUAL\_FEAS

The solution is dual feasible.

MSK\_SOL\_STA\_PRIM\_AND\_DUAL\_FEAS

The solution is both primal and dual feasible.

MSK\_SOL\_STA\_PRIM\_INFEAS\_CER

The solution is a certificate of primal infeasibility.

MSK\_SOL\_STA\_DUAL\_INFEAS\_CER

The solution is a certificate of dual infeasibility.

MSK\_SOL\_STA\_PRIM\_ILLPOSED\_CER

The solution is a certificate that the primal problem is illposed.

MSK\_SOL\_STA\_DUAL\_ILLPOSED\_CER

The solution is a certificate that the dual problem is illposed.

MSK\_SOL\_STA\_INTEGER\_OPTIMAL

The primal solution is integer optimal.

# 11.4.51 Solution types

MSK\_SOL\_BAS

The basic solution.

MSK\_SOL\_ITR

The interior solution.

MSK\_SOL\_ITG

The integer solution.

# 11.4.52 Solve primal or dual form

#### MSK\_SOLVE\_FREE

The optimizer is free to solve either the primal or the dual problem.

#### MSK\_SOLVE\_PRIMAL

The optimizer should solve the primal problem.

## MSK\_SOLVE\_DUAL

The optimizer should solve the dual problem.

# 11.4.53 Status keys

#### MSK\_SK\_UNK

The status for the constraint or variable is unknown.

#### MSK\_SK\_BAS

The constraint or variable is in the basis.

#### MSK\_SK\_SUPBAS

The constraint or variable is super basic.

#### MSK\_SK\_LOW

The constraint or variable is at its lower bound.

#### MSK\_SK\_UPR

The constraint or variable is at its upper bound.

#### MSK SK FIX

The constraint or variable is fixed.

# MSK\_SK\_INF

The constraint or variable is infeasible in the bounds.

# 11.4.54 Starting point types

#### MSK\_STARTING\_POINT\_FREE

The starting point is chosen automatically.

# MSK\_STARTING\_POINT\_GUESS

The optimizer guesses a starting point.

#### MSK\_STARTING\_POINT\_CONSTANT

The optimizer constructs a starting point by assigning a constant value to all primal and dual variables. This starting point is normally robust.

#### MSK\_STARTING\_POINT\_SATISFY\_BOUNDS

The starting point is chosen to satisfy all the simple bounds on nonlinear variables. If this starting point is employed, then more care than usual should employed when choosing the bounds on the nonlinear variables. In particular very tight bounds should be avoided.

# 11.4.55 Stream types

## MSK\_STREAM\_LOG

Log stream. Contains the aggregated contents of all other streams. This means that a message written to any other stream will also be written to this stream.

#### MSK\_STREAM\_MSG

Message stream. Log information relating to performance and progress of the optimization is written to this stream.

#### MSK\_STREAM\_ERR

Error stream. Error messages are written to this stream.

# MSK\_STREAM\_WRN

Warning stream. Warning messages are written to this stream.

# 11.4.56 Integer values

MSK\_MAX\_STR\_LEN

Maximum string length allowed in MOSEK.

MSK\_LICENSE\_BUFFER\_LENGTH

The length of a license key buffer.

# 11.4.57 Variable types

MSK\_VAR\_TYPE\_CONT

Is a continuous variable.

MSK\_VAR\_TYPE\_INT

Is an integer variable.

# 11.5 Supported domains

This section lists the domains supported by MOSEK.

# 11.5.1 Linear domains

Each linear domain is determined by the dimension n.

- : the **zero domain**, consisting of the origin  $0^n \in \mathbb{R}^n$ .
- : the nonnegative orthant domain  $\mathbb{R}^n_{>0}$ .
- : the nonpositive orthant domain  $\mathbb{R}^n_{<0}$ .
- : the **free domain**, consisting of the whole  $\mathbb{R}^n$ .

Membership in a linear domain is equivalent to imposing the corresponding set of n linear constraints, for instance  $Fx + g \in 0^n$  is equivalent to Fx + g = 0 and so on. The free domain imposes no restriction.

# 11.5.2 Quadratic cone domains

The quadratic domains are determined by the dimension n.

• : the quadratic cone domain is the subset of  $\mathbb{R}^n$  defined as

$$Q^n = \left\{ x \in \mathbb{R}^n : x_1 \ge \sqrt{x_2^2 + \dots + x_n^2} \right\}.$$

ullet : the **rotated quadratic cone domain** is the subset of  $\mathbb{R}^n$  defined as

$$Q_r^n = \{x \in \mathbb{R}^n : 2x_1x_2 \ge x_3^2 + \dots + x_n^2, x_1, x_2 \ge 0\}.$$

# 11.5.3 Exponential cone domains

ullet: the **primal exponential cone domain** is the subset of  $\mathbb{R}^3$  defined as

$$K_{\text{exp}} = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 \ge x_2 \exp(x_3/x_2), x_1, x_2 \ge 0\}.$$

• : the dual exponential cone domain is the subset of  $\mathbb{R}^3$  defined as

$$K_{\text{exp}}^* = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 \le -x_3 \exp(x_2/x_3 - 1), x_1 \ge 0, x_3 \le 0\}.$$

# 11.5.4 Power cone domains

A power cone domain is determined by the dimension n and a sequence of  $1 \le n_l < n$  positive real numbers (weights)  $\alpha_1, \ldots, \alpha_{n_l}$ .

• : the **primal power cone domain** is the subset of  $\mathbb{R}^n$  defined as

$$\mathcal{P}_n^{(\alpha_1, \dots, \alpha_{n_l})} = \left\{ x \in \mathbb{R}^n : \prod_{i=1}^{n_l} x_i^{\beta_i} \ge \sqrt{x_{n_l+1}^2 + \dots + x_n^2}, \ x_1, \dots, x_{n_l} \ge 0 \right\}.$$

where  $\beta_i$  are the weights normalized to add up to 1, ie.  $\beta_i = \alpha_i/(\sum_j \alpha_j)$  for  $i = 1, ..., n_l$ . The name  $n_l$  reads as "n left", the length of the product on the left-hand side of the definition.

• : the dual power cone domain is the subset of  $\mathbb{R}^n$  defined as

$$\left(\mathcal{P}_{n}^{(\alpha_{1},\dots,\alpha_{n_{l}})}\right)^{*} = \left\{x \in \mathbb{R}^{n} : \prod_{i=1}^{n_{l}} \left(\frac{x_{i}}{\beta_{i}}\right)^{\beta_{i}} \geq \sqrt{x_{n_{l}+1}^{2} + \dots + x_{n}^{2}}, \ x_{1},\dots,x_{n_{l}} \geq 0\right\}.$$

where  $\beta_i$  are the weights normalized to add up to 1, ie.  $\beta_i = \alpha_i/(\sum_j \alpha_j)$  for  $i = 1, ..., n_l$ . The name  $n_l$  reads as "n left", the length of the product on the left-hand side of the definition.

• **Remark:** in MOSEK 9 power cones were available only in the special case with  $n_l = 2$  and weights  $(\alpha, 1 - \alpha)$  for some  $0 < \alpha < 1$  specified as cone parameter.

# 11.5.5 Geometric mean cone domains

A geometric mean cone domain is determined by the dimension n.

• : the **primal geometric mean cone domain** is the subset of  $\mathbb{R}^n$  defined as

$$\mathcal{GM}^n = \left\{ x \in \mathbb{R}^n : \left( \prod_{i=1}^{n-1} x_i \right)^{1/(n-1)} \ge |x_n|, \ x_1, \dots, x_{n-1} \ge 0 \right\}.$$

It is a special case of the primal power cone domain with  $n_l = n-1$  and weights  $\alpha = (1, \dots, 1)$ .

ullet : the dual geometric mean cone domain is the subset of  $\mathbb{R}^n$  defined as

$$(\mathcal{GM}^n)^* = \left\{ x \in \mathbb{R}^n : (n-1) \left( \prod_{i=1}^{n-1} x_i \right)^{1/(n-1)} \ge |x_n|, x_1, \dots, x_{n-1} \ge 0 \right\}.$$

It is a special case of the dual power cone domain with  $n_l = n - 1$  and weights  $\alpha = (1, \dots, 1)$ .

# 11.5.6 Vectorized semidefinite domain

• : the vectorized PSD cone domain is determined by the dimension n, which must be of the form n = d(d+1)/2. Then the domain is defined as

$$\mathcal{S}_{+}^{d,\text{vec}} = \left\{ (x_1, \dots, x_{d(d+1)/2}) \in \mathbb{R}^n : \text{sMat}(x) \in \mathcal{S}_{+}^d \right\},\,$$

where

$$sMat(x) = \begin{bmatrix} x_1 & x_2/\sqrt{2} & \cdots & x_d/\sqrt{2} \\ x_2/\sqrt{2} & x_{d+1} & \cdots & x_{2d-1}/\sqrt{2} \\ \cdots & \cdots & \cdots & \cdots \\ x_d/\sqrt{2} & x_{2d-1}/\sqrt{2} & \cdots & x_{d(d+1)/2} \end{bmatrix},$$

or equivalently

$$\mathcal{S}_{+}^{d, \text{vec}} = \left\{ \text{sVec}(X) : X \in \mathcal{S}_{+}^{d} \right\},$$

where

$$sVec(X) = (X_{11}, \sqrt{2}X_{21}, \dots, \sqrt{2}X_{d1}, X_{22}, \sqrt{2}X_{32}, \dots, X_{dd}).$$

In other words, the domain consists of vectorizations of the lower-triangular part of a positive semidefinite matrix, with the non-diagonal elements additionally rescaled.

# Chapter 12

# Supported File Formats

**MOSEK** supports a range of problem and solution formats listed in Table 12.1 and Table 12.2. The most important are:

- the **Task format**, **MOSEK**'s native binary format which supports all features that **MOSEK** supports. It is the closest possible representation of the internal data in a task and it is ideal for submitting problem data support questions.
- the **PTF** format, **MOSEK**'s human-readable format that supports all linear, conic and mixed-integer features. It is ideal for debugging. It is not an exact copy of all the data in the task, but it contains all information required to reconstruct it, presented in a readable fashion.
- MPS, LP, CBF formats are industry standards, each supporting some limited set of features, and potentially requiring some degree of reformulation during read/write.

#### **Problem formats**

Table 12.1: List of supported file formats for optimization problems.

Format Type	Ext.	Binary/Text	LP	QCQO	ACC	SDP	DJC	Sol	Param
LP	lp	plain text	X	X					
MPS	mps	plain text	X	X					
PTF	ptf	plain text	X		X	X	X	X	
CBF	cbf	plain text	X		X	X			
Task format	task	binary	X	X	X	X	X	X	X
Jtask format	jtask	text/JSON	X	X	X	X	X	X	X
<i>OPF</i> (deprecated for conic problems)	opf	plain text	X	X				X	X

The columns of the table indicate if the specified file format supports:

- $\bullet~{\rm LP}$  linear problems,
- QCQO quadratic objective or constraints,
- ACC affine conic constraints,
- SDP semidefinite cone/variables,
- DJC disjunctive constraints,
- Sol solutions,
- Param optimizer parameters.

## **Solution formats**

Table 12.2: List of supported solution formats.

Format Type	Ext.	Binary/Text	Description
SOL	sol	plain text	Interior Solution
	bas	plain text	Basic Solution
	int	plain text	Integer
Jsol format	jsol	text/JSON	All solutions

# Compression

MOSEK supports GZIP and Zstandard compression. Problem files with extension <code>.gz</code> (for GZIP) and <code>.zst</code> (for Zstandard) are assumed to be compressed when read, and are automatically compressed when written. For example, a file called

problem.mps.zst

will be considered as a Zstandard compressed MPS file.

# 12.1 The LP File Format

MOSEK supports the LP file format with some extensions. The LP format is not a completely well-defined standard and hence different optimization packages may interpret the same LP file in slightly different ways. MOSEK tries to emulate as closely as possible CPLEX's behavior, but tries to stay backward compatible.

The LP file format can specify problems of the form

$$\begin{array}{lll} \text{minimize/maximize} & & c^Tx + \frac{1}{2}q^o(x) \\ \text{subject to} & l^c & \leq & Ax + \frac{1}{2}q(x) & \leq & u^c, \\ l^x & \leq & x & \leq & u^x, \\ & & & x_{\mathcal{I}} \text{ integer}, \end{array}$$

where

- $x \in \mathbb{R}^n$  is the vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear term in the objective.
- $q^o :\in \mathbb{R}^n \to \mathbb{R}$  is the quadratic term in the objective where

$$q^o(x) = x^T Q^o x$$

and it is assumed that

$$Q^o = (Q^o)^T.$$

- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $q: \mathbb{R}^n \to \mathbb{R}$  is a vector of quadratic functions. Hence,

$$q_i(x) = x^T Q^i x$$

where it is assumed that

$$Q^i = (Q^i)^T.$$

•  $\mathcal{J} \subseteq \{1, 2, \dots, n\}$  is an index set of the integer constrained variables.

#### 12.1.1 File Sections

An LP formatted file contains a number of sections specifying the objective, constraints, variable bounds, and variable types. The section keywords may be any mix of upper and lower case letters.

## **Objective Function**

The first section beginning with one of the keywords

max
maximum
maximize
min
minimum
minimize

defines the objective sense and the objective function, i.e.

$$c^T x + \frac{1}{2} x^T Q^o x.$$

The objective may be given a name by writing

#### myname:

before the expressions. If no name is given, then the objective is named obj.

The objective function contains linear and quadratic terms. The linear terms are written as

```
4 x1 + x2 - 0.1 x3
```

and so forth. The quadratic terms are written in square brackets ([ ]/2) and are either squared or multiplied as in the examples

x1^2

and

x1 \* x2

There may be zero or more pairs of brackets containing quadratic expressions. An example of an objective section is

```
minimize
myobj: 4 x1 + x2 - 0.1 x3 + [ x1^2 + 2.1 x1 * x2 ]/2
```

Please note that the quadratic expressions are multiplied with  $\frac{1}{2}$ , so that the above expression means

minimize 
$$4x_1 + x_2 - 0.1 \cdot x_3 + \frac{1}{2}(x_1^2 + 2.1 \cdot x_1 \cdot x_2)$$

If the same variable occurs more than once in the linear part, the coefficients are added, so that  $4 \times 1 + 2 \times 1$  is equivalent to  $6 \times 1$ . In the quadratic expressions  $\times 1 \times 2$  is equivalent to  $\times 2 \times 1$  and, as in the linear part, if the same variables multiplied or squared occur several times their coefficients are added.

# Constraints

The second section beginning with one of the keywords

```
subj to
subject to
s.t.
st
```

defines the linear constraint matrix A and the quadratic matrices  $Q^{i}$ .

A constraint contains a name (optional), expressions adhering to the same rules as in the objective and a bound:

```
subject to con1: x1 + x2 + [ x3^2 ]/2 <= 5.1
```

The bound type (here <=) may be any of <, <=, =, >, >= (< and <= mean the same), and the bound may be any number.

In the standard LP format it is not possible to define more than one bound per line, but **MOSEK** supports defining ranged constraints by using double-colon (::) instead of a single-colon (:) after the constraint name, i.e.

$$-5 \le x_1 + x_2 \le 5 \tag{12.1}$$

may be written as

```
con:: -5 < x_1 + x_2 < 5
```

By default MOSEK writes ranged constraints this way.

If the files must adhere to the LP standard, ranged constraints must either be split into upper bounded and lower bounded constraints or be written as an equality with a slack variable. For example the expression (12.1) may be written as

$$x_1 + x_2 - sl_1 = 0, -5 \le sl_1 \le 5.$$

#### **Bounds**

Bounds on the variables can be specified in the bound section beginning with one of the keywords

```
bound bounds
```

The bounds section is optional but should, if present, follow the **subject to** section. All variables listed in the bounds section must occur in either the objective or a constraint.

The default lower and upper bounds are 0 and  $+\infty$ . A variable may be declared free with the keyword free, which means that the lower bound is  $-\infty$  and the upper bound is  $+\infty$ . Furthermore it may be assigned a finite lower and upper bound. The bound definitions for a given variable may be written in one or two lines, and bounds can be any number or  $\pm\infty$  (written as +inf/-inf/+infinity/-infinity) as in the example

```
bounds

x1 free

x2 <= 5

0.1 <= x2

x3 = 42

2 <= x4 < +inf
```

## Variable Types

The final two sections are optional and must begin with one of the keywords

```
bin
binaries
binary
```

and

```
gen
general
```

Under general all integer variables are listed, and under binary all binary (integer variables with bounds 0 and 1) are listed:

```
general
x1 x2
binary
x3 x4
```

Again, all variables listed in the binary or general sections must occur in either the objective or a constraint.

# **Terminating Section**

Finally, an LP formatted file must be terminated with the keyword

end

# 12.1.2 LP File Examples

# Linear example 1o1.1p

```
\ File: lo1.lp
maximize
obj: 3 x1 + x2 + 5 x3 + x4
subject to
c1: 3 x1 + x2 + 2 x3 = 30
c2: 2 x1 + x2 + 3 x3 + x4 >= 15
c3: 2 x2 + 3 x4 <= 25
bounds
0 <= x1 <= +infinity
0 <= x2 <= 10
0 <= x3 <= +infinity
0 <= x4 <= +infinity
end</pre>
```

# Mixed integer example milo1.lp

```
maximize
obj: x1 + 6.4e-01 x2
subject to
c1: 5e+01 x1 + 3.1e+01 x2 <= 2.5e+02
c2: 3e+00 x1 - 2e+00 x2 >= -4e+00
bounds
0 <= x1 <= +infinity
0 <= x2 <= +infinity
general
x1 x2
end</pre>
```

# 12.1.3 LP Format peculiarities

#### Comments

Anything on a line after a \ is ignored and is treated as a comment.

#### **Names**

A name for an objective, a constraint or a variable may contain the letters a-z, A-Z, the digits 0-9 and the characters

The first character in a name must not be a number, a period or the letter **e** or **E**. Keywords must not be used as names.

**MOSEK** accepts any character as valid for names, except \0. A name that is not allowed in LP file will be changed and a warning will be issued.

The algorithm for making names LP valid works as follows: The name is interpreted as an utf-8 string. For a Unicode character c:

- If c==\_ (underscore), the output is \_\_ (two underscores).
- If c is a valid LP name character, the output is just c.
- If c is another character in the ASCII range, the output is <code>\_XX</code>, where <code>XX</code> is the hexadecimal code for the character.
- If c is a character in the range 127-65535, the output is \_uXXXX, where XXXX is the hexadecimal code for the character.
- If c is a character above 65535, the output is \_UXXXXXXXX, where XXXXXXXX is the hexadecimal code for the character.

Invalid  $\mathtt{utf-8}$  substrings are escaped as  $\mathtt{XX'}$ , and if a name starts with a period, e or E, that character is escaped as  $\mathtt{XX}$ .

## Variable Bounds

Specifying several upper or lower bounds on one variable is possible but **MOSEK** uses only the tightest bounds. If a variable is fixed (with =), then it is considered the tightest bound.

# 12.2 The MPS File Format

**MOSEK** supports the standard MPS format with some extensions. For a detailed description of the MPS format see the book by Nazareth [Naz87].

## 12.2.1 MPS File Structure

The version of the MPS format supported by **MOSEK** allows specification of an optimization problem of the form

maximize/minimize 
$$c^T x + q_0(x)$$
  
 $l^c \le Ax + q(x) \le u^c,$   
 $l^x \le x \le u^x,$   
 $x \in \mathcal{K},$   
 $x_{\mathcal{J}}$  integer, (12.2)

where

- $x \in \mathbb{R}^n$  is the vector of decision variables.
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.

- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $q: \mathbb{R}^n \to \mathbb{R}$  is a vector of quadratic functions. Hence,

$$q_i(x) = \frac{1}{2}x^T Q^i x$$

where it is assumed that  $Q^i = (Q^i)^T$ . Please note the explicit  $\frac{1}{2}$  in the quadratic term and that  $Q^i$  is required to be symmetric. The same applies to  $q_0$ .

- $\mathcal{K}$  is a convex cone.
- $\mathcal{J} \subseteq \{1, 2, \dots, n\}$  is an index set of the integer-constrained variables.
- ullet c is the vector of objective coefficients.

An MPS file with one row and one column can be illustrated like this:

```
3
                                           4
                                                      5
                                                                 6
         1
                    2
*2345678901234567890123456789012345678901234567890
NAME
               [name]
OBJSENSE
    [objsense]
OBJNAME
               [objname]
ROWS
   [cname1]
COLUMNS
    [vname1]
               [cname1]
                          [value1]
                                           [cname2]
                                                      [value2]
RHS
               [cname1]
                          [value1]
                                           [cname2]
                                                      [value2]
    [name]
RANGES
    [name]
               [cname1]
                          [value1]
                                           [cname2]
                                                      [value2]
QSECTION
               [cname1]
               [vname2]
                          [value1]
                                           [vname3]
                                                      [value2]
    [vname1]
QMATRIX
               [vname2]
                          [value1]
    [vname1]
QUADOBJ
               [vname2]
                          [value1]
    [vname1]
QCMATRIX
               [cname1]
               [vname2]
                          [value1]
    [vname1]
BOUNDS
?? [name]
               [vname1]
                          [value1]
CSECTION
               [kname1]
                          [value1]
                                           [ktype]
    [vname1]
ENDATA
```

Here the names in capitals are keywords of the MPS format and names in brackets are custom defined names or values. A couple of notes on the structure:

• Fields: All items surrounded by brackets appear in *fields*. The fields named "valueN" are numerical values. Hence, they must have the format

```
[+|-]XXXXXXX.XXXXXX[[e|E][+|-]XXX]
```

where

```
X = [0|1|2|3|4|5|6|7|8|9].
```

• Sections: The MPS file consists of several sections where the names in capitals indicate the beginning of a new section. For example, COLUMNS denotes the beginning of the columns section.

- Comments: Lines starting with an \* are comment lines and are ignored by MOSEK.
- Keys: The question marks represent keys to be specified later.
- Extensions: The sections QSECTION and CSECTION are specific MOSEK extensions of the MPS format. The sections QMATRIX, QUADOBJ and QCMATRIX are included for sake of compatibility with other vendors extensions to the MPS format.
- The standard MPS format is a fixed format, i.e. everything in the MPS file must be within certain fixed positions. **MOSEK** also supports a *free format*. See Sec. 12.2.5 for details.

# Linear example lo1.mps

A concrete example of a MPS file is presented below:

```
* File: lo1.mps
NAME
               lo1
OBJSENSE
    MAX
ROWS
N obj
E c1
G c2
L c3
COLUMNS
    x1
               obj
                          3
    x1
               c1
                          3
                          2
               c2
    x1
    x2
               obj
                          1
    x2
               c1
                          1
    x2
               c2
                          1
    x2
               сЗ
                          2
                          5
    хЗ
               obj
    xЗ
                          2
               c1
               c2
                          3
    xЗ
                          1
    x4
               obj
    x4
               c2
                          1
               сЗ
                          3
    x4
RHS
                          30
               c1
    rhs
    rhs
               c2
                          15
    rhs
               сЗ
                          25
RANGES
BOUNDS
UP bound
               x2
                          10
ENDATA
```

Subsequently each individual section in the MPS format is discussed.

## NAME (optional)

In this section a name ([name]) is assigned to the problem.

## OBJSENSE (optional)

This is an optional section that can be used to specify the sense of the objective function. The OBJSENSE section contains one line at most which can be one of the following:

MIN
MINIMIZE
MAX
MAXIMIZE

It should be obvious what the implication is of each of these four lines.

## OBJNAME (optional)

This is an optional section that can be used to specify the name of the row that is used as objective function. objname should be a valid row name.

#### ROWS

A record in the ROWS section has the form

#### ? [cname1]

where the requirements for the fields are as follows:

Field	Starting Position	Max Width	required	Description	
?	2	1	Yes	Constraint key	i
[cname1]	5	8	Yes	Constraint name	

Hence, in this section each constraint is assigned a unique name denoted by [cname1]. Please note that [cname1] starts in position 5 and the field can be at most 8 characters wide. An initial key ? must be present to specify the type of the constraint. The key can have values E, G, L, or N with the following interpretation:

Constraint type	$l_i^c$	$u_i^c$
E (equal)	finite	$=l_i^c$
G (greater)	finite	$\infty$
L (lower)	$-\infty$	finite
N (none)	$-\infty$	$\infty$

In the MPS format the objective vector is not specified explicitly, but one of the constraints having the key  $\mathbb N$  will be used as the objective vector c. In general, if multiple  $\mathbb N$  type constraints are specified, then the first will be used as the objective vector c, unless something else was specified in the section OBJNAME.

# COLUMNS

In this section the elements of A are specified using one or more records having the form:

[vname1]	[cname1] [value1]	name1]	[cname2]	[value2]		
----------	-------------------	--------	----------	----------	--	--

where the requirements for each field are as follows:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

Hence, a record specifies one or two elements  $a_{ij}$  of A using the principle that [vname1] and [cname1] determines j and i respectively. Please note that [cname1] must be a constraint name specified in the

ROWS section. Finally, [value1] denotes the numerical value of  $a_{ij}$ . Another optional element is specified by [cname2], and [value2] for the variable specified by [vname1]. Some important comments are:

- All elements belonging to one variable must be grouped together.
- Zero elements of A should not be specified.
- At least one element for each variable should be specified.

### RHS (optional)

A record in this section has the format

ue1] [cname2] [va	[cname1]	[name]	[cname2]
-------------------	----------	--------	----------

where the requirements for each field are as follows:

Field	Starting Position	Max Width	required	Description
[name]	5	8	Yes	Name of the RHS vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The interpretation of a record is that [name] is the name of the RHS vector to be specified. In general, several vectors can be specified. [cname1] denotes a constraint name previously specified in the ROWS section. Now, assume that this name has been assigned to the i-h constraint and  $v_1$  denotes the value specified by [value1], then the interpretation of  $v_1$  is:

Constraint	$l_i^c$	$u_i^c$
E	$v_1$	$v_1$
G	$v_1$	
L		$v_1$
N		

An optional second element is specified by [cname2] and [value2] and is interpreted in the same way. Please note that it is not necessary to specify zero elements, because elements are assumed to be zero.

## RANGES (optional)

A record in this section has the form

[value1] [cname2] [valu
-------------------------

where the requirements for each fields are as follows:

Field	Starting Position	Max Width	required	Description
[name]	5	8	Yes	Name of the RANGE vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The records in this section are used to modify the bound vectors for the constraints, i.e. the values in  $l^c$  and  $u^c$ . A record has the following interpretation: [name] is the name of the RANGE vector and [cname1] is a valid constraint name. Assume that [cname1] is assigned to the *i*-th constraint and let  $v_1$  be the value specified by [value1], then a record has the interpretation:

Constraint type	Sign of $v_1$	$l_i^c$	$u_i^c$
E	_	$u_i^c + v_1$	
E	+		$l_i^c + v_1$
G	- or +		$l_i^c +  v_1 $
L	- or +	$u_i^c -  v_1 $	
N			

Another constraint bound can optionally be modified using [cname2] and [value2] the same way.

# QSECTION (optional)

Within the QSECTION the label [cname1] must be a constraint name previously specified in the ROWS section. The label [cname1] denotes the constraint to which the quadratic terms belong. A record in the QSECTION has the form

	]
--	---

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value
[vname3]	40	8	No	Variable name
[value2]	50	12	No	Numerical value

A record specifies one or two elements in the lower triangular part of the  $Q^i$  matrix where [cname1] specifies the i. Hence, if the names [vname1] and [vname2] have been assigned to the k-th and j-th variable, then  $Q^i_{kj}$  is assigned the value given by [value1] An optional second element is specified in the same way by the fields [vname1], [vname3], and [value2].

The example

$$\begin{array}{ll} \text{minimize} & -x_2 + \frac{1}{2}(2x_1^2 - 2x_1x_3 + 0.2x_2^2 + 2x_3^2) \\ \text{subject to} & x_1 + x_2 + x_3 & \geq & 1, \\ & x \geq 0 & \end{array}$$

has the following MPS file representation

		4	
		qo1.mps	
NAM	ΙE	qo1	
ROW	IS		
	obj		
	c1		
	UMNS		
COL			4 0
	x1	c1	1.0
	x2	obj	-1.0
	x2	c1	1.0
	xЗ	c1	1.0
RHS	;		
	rhs	c1	1.0
QSE	CTION		
•	x1	x1	2.0
	x1	х3	-1.0
	x2	x2	0.2
	x3	x3	2.0
END		AO	2.0
END	HIA		

Regarding the QSECTIONs please note that:

• Only one QSECTION is allowed for each constraint.

- The QSECTIONs can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QSECTION must already be specified in the COLUMNS section.
- ullet All entries specified in a QSECTION are assumed to belong to the lower triangular part of the quadratic term of Q .

# QMATRIX/QUADOBJ (optional)

The QMATRIX and QUADOBJ sections allow to define the quadratic term of the objective function. They differ in how the quadratic term of the objective function is stored:

- QMATRIX stores all the nonzeros coefficients, without taking advantage of the symmetry of the Q
  matrix
- QUADOBJ stores the upper diagonal nonzero elements of the Q matrix.

A record in both sections has the form:

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value

A record specifies one elements of the Q matrix in the objective function. Hence, if the names [vname1] and [vname2] have been assigned to the k-th and j-th variable, then  $Q_{kj}$  is assigned the value given by [value1]. Note that a line must appear for each off-diagonal coefficient if using a QMATRIX section, while only one entry is required in a QUADOBJ section. The quadratic part of the objective function will be evaluated as  $1/2x^TQx$ .

The example

minimize 
$$-x_2 + \frac{1}{2}(2x_1^2 - 2x_1x_3 + 0.2x_2^2 + 2x_3^2)$$
 subject to 
$$x_1 + x_2 + x_3 \geq 1,$$
 
$$x > 0$$

has the following MPS file representation using QMATRIX

* F:	ile:	qo1_matrix.mp	s
NAMI		qo1_qma	trix
ROWS			
	obj		
	c1		
COLU	UMNS		
	x1	c1	1.0
	x2	obj	-1.0
	x2	c1	1.0
	xЗ	c1	1.0
RHS			
	rhs	c1	1.0
QMA.	TRIX		
	x1	x1	2.0
	x1	x3	-1.0
	x3	x1	-1.0
	x2	x2	0.2
	xЗ	x3	2.0
END	ATA		

or the following using QUADOBJ

```
* File: qo1_quadobj.mps
NAME
               qo1_quadobj
ROWS
   obj
N
G c1
COLUMNS
    x1
               с1
                          1.0
    x2
               obj
                          -1.0
    x2
               с1
                          1.0
    хЗ
               с1
                          1.0
RHS
                          1.0
    rhs
               c1
QUADOBJ
                          2.0
    x1
               x1
               xЗ
                          -1.0
    x1
               x2
                          0.2
    x2
    x3
               x3
                          2.0
ENDATA
```

Please also note that:

- A QMATRIX/QUADOBJ section can appear in an arbitrary order after the COLUMNS section.
- ullet All variable names occurring in the QMATRIX/QUADOBJ section must already be specified in the COLUMNS section.

## QCMATRIX (optional)

A QCMATRIX section allows to specify the quadratic part of a given constraint. Within the QCMATRIX the label [cname1] must be a constraint name previously specified in the ROWS section. The label [cname1] denotes the constraint to which the quadratic term belongs. A record in the QSECTION has the form

|--|

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value

A record specifies an entry of the  $Q^i$  matrix where [cname1] specifies the i. Hence, if the names [vname1] and [vname2] have been assigned to the k-th and j-th variable, then  $Q^i_{kj}$  is assigned the value given by [value1]. Moreover, the quadratic term is represented as  $1/2x^TQx$ .

The example

$$\begin{array}{lll} \text{minimize} & x_2 \\ \text{subject to} & x_1 + x_2 + x_3 & \geq & 1, \\ & \frac{1}{2} (-2x_1x_3 + 0.2x_2^2 + 2x_3^2) & \leq & 10, \\ & x \geq 0 & \end{array}$$

has the following MPS file representation

```
* File: qo1.mps
NAME qo1
ROWS
N obj
G c1
L q1
COLUMNS
```

(continues on next page)

(continued from previous page)

				(	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
x1	c1	1.0			
x2	obj	-1.0			
x2	c1	1.0			
x3	c1	1.0			
RHS					
rhs	c1	1.0			
rhs	q1	10.0			
QCMATRIX	q1				
x1	x1	2.0			
x1	x3	-1.0			
x3	x1	-1.0			
x2	x2	0.2			
x3	x3	2.0			
ENDATA					

Regarding the QCMATRIXs please note that:

- Only one QCMATRIX is allowed for each constraint.
- The QCMATRIXs can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QSECTION must already be specified in the COLUMNS section.
- QCMATRIX does not exploit the symmetry of Q: an off-diagonal entry (i,j) should appear twice.

## BOUNDS (optional)

In the BOUNDS section changes to the default bounds vectors  $l^x$  and  $u^x$  are specified. The default bounds vectors are  $l^x=0$  and  $u^x=\infty$ . Moreover, it is possible to specify several sets of bound vectors. A record in this section has the form

|--|--|

where the requirements for each field are:

Field	Starting Position	Max Width	Required	Description
??	2	2	Yes	Bound key
[name]	5	8	Yes	Name of the BOUNDS vector
[vname1]	15	8	Yes	Variable name
[value1]	25	12	No	Numerical value

Hence, a record in the BOUNDS section has the following interpretation: [name] is the name of the bound vector and [vname1] is the name of the variable for which the bounds are modified by the record. ?? and [value1] are used to modify the bound vectors according to the following table:

??	$l_j^x$	$u_j^x$	Made integer (added to $\mathcal{J}$ )
FR	$-\infty$	$\infty$	No
FX	$v_1$	$v_1$	No
LO	$v_1$	unchanged	No
MI	$-\infty$	unchanged	No
PL	unchanged	$\infty$	No
UP	unchanged	$v_1$	No
BV	0	1	Yes
LI	$\lceil v_1 \rceil$	unchanged	Yes
UI	unchanged	$\lfloor v_1 \rfloor$	Yes

Here  $v_1$  is the value specified by [value1].

#### CSECTION (optional)

The purpose of the CSECTION is to specify the conic constraint

$$x \in \mathcal{K}$$

in (12.2). It is assumed that K satisfies the following requirements. Let

$$x^t \in \mathbb{R}^{n^t}, \quad t = 1, \dots, k$$

be vectors comprised of parts of the decision variables x so that each decision variable is a member of exactly **one** vector  $x^t$ , for example

$$x^1 = \begin{bmatrix} x_1 \\ x_4 \\ x_7 \end{bmatrix}$$
 and  $x^2 = \begin{bmatrix} x_6 \\ x_5 \\ x_3 \\ x_2 \end{bmatrix}$ .

Next define

$$\mathcal{K} := \{ x \in \mathbb{R}^n : x^t \in \mathcal{K}_t, \quad t = 1, \dots, k \}$$

where  $\mathcal{K}_t$  must have one of the following forms:

•  $\mathbb{R}$  set:

$$\mathcal{K}_t = \mathbb{R}^{n^t}$$
.

• Zero cone:

$$\mathcal{K}_t = \{0\} \subseteq \mathbb{R}^{n^t}. \tag{12.3}$$

• Quadratic cone:

$$\mathcal{K}_{t} = \left\{ x \in \mathbb{R}^{n^{t}} : x_{1} \ge \sqrt{\sum_{j=2}^{n^{t}} x_{j}^{2}} \right\}.$$
 (12.4)

• Rotated quadratic cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : 2x_1 x_2 \ge \sum_{j=3}^{n^t} x_j^2, \quad x_1, x_2 \ge 0 \right\}.$$
 (12.5)

• Primal exponential cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^3 : x_1 \ge x_2 \exp(x_3/x_2), \quad x_1, x_2 \ge 0 \right\}. \tag{12.6}$$

• Primal power cone (with parameter  $0 < \alpha < 1$ ):

$$\mathcal{K}_{t} = \left\{ x \in \mathbb{R}^{n^{t}} : x_{1}^{\alpha} x_{2}^{1-\alpha} \ge \sqrt{\sum_{j=3}^{n^{t}} x_{j}^{2}}, \quad x_{1}, x_{2} \ge 0 \right\}.$$
 (12.7)

• Dual exponential cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^3 : x_1 \ge -x_3 e^{-1} \exp(x_2/x_3), \quad x_3 \le 0, x_1 \ge 0 \right\}. \tag{12.8}$$

• Dual power cone (with parameter  $0 < \alpha < 1$ ):

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : \left(\frac{x_1}{\alpha}\right)^{\alpha} \left(\frac{x_2}{1-\alpha}\right)^{1-\alpha} \ge \sqrt{\sum_{j=3}^{n^t} x_j^2}, \quad x_1, x_2 \ge 0 \right\}.$$
 (12.9)

In general, membership in the  $\mathbb{R}$  set is not specified. If a variable is not a member of any other cone then it is assumed to be a member of the  $\mathbb{R}$  cone.

Next, let us study an example. Assume that the power cone

$$x_4^{1/3} x_5^{2/3} \ge |x_8|$$

and the rotated quadratic cone

$$2x_3x_7 \ge x_1^2 + x_0^2, \quad x_3, x_7 \ge 0,$$

should be specified in the MPS file. One CSECTION is required for each cone and they are specified as follows:

*	1	2	3	4	5	6
*23456	789012	_	_	4567890123456	-	-
CSECTI	ON	konea	3e-1	PPOW		
x4						
x5						
x8						
CSECTI	ON	koneb	0.0	RQUAD		
x7						
x3						
x1						
x0						

In general, a CSECTION header has the format

|--|

where the requirements for each field are as follows:

Field	Starting Position	Max Width	Required	Description
[kname1]	15	8	Yes	Name of the cone
[value1]	25	12	No	Cone parameter
[ktype]	40		Yes	Type of the cone.

The possible cone type keys are:

[ktype]	Members	[value1]	Interpretation.
ZERO	$\geq 0$	unused	Zero cone (12.3).
QUAD	$\geq 1$	unused	Quadratic cone (12.4).
RQUAD	$\geq 2$	unused	Rotated quadratic cone (12.5).
PEXP	3	unused	Primal exponential cone (12.6).
PPOW	$\geq 2$	α	Primal power cone (12.7).
DEXP	3	unused	Dual exponential cone (12.8).
DPOW	$\geq 2$	α	Dual power cone (12.9).

A record in the CSECTION has the format

# [vname1]

where the requirements for each field are

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	A valid variable name

A variable must occur in at most one CSECTION.

#### **ENDATA**

This keyword denotes the end of the MPS file.

# 12.2.2 Integer Variables

Using special bound keys in the BOUNDS section it is possible to specify that some or all of the variables should be integer-constrained i.e. be members of  $\mathcal{J}$ . However, an alternative method is available. This method is available only for backward compatibility and we recommend that it is not used. This method requires that markers are placed in the COLUMNS section as in the example:

```
COLUMNS
x1
                                                  0.7
           obj
                      -10.0
                                       c1
x1
           c2
                      0.5
                                       сЗ
                                                  1.0
x1
           c4
                      0.1
* Start of integer-constrained variables.
MARK000
                                       'INTORG'
           'MARKER'
x2
           obj
                      -9.0
                                                  1.0
                                       c1
x2
                      0.8333333333
                                                  0.6666667
           c2
                                       с3
x2
           c4
                      0.25
xЗ
                      1.0
                                                  2.0
           obj
                                       c6
MARKO01
           'MARKER'
                                       'INTEND'
* End of integer-constrained variables.
```

Please note that special marker lines are used to indicate the start and the end of the integer variables. Furthermore be aware of the following

- All variables between the markers are assigned a default lower bound of 0 and a default upper bound of 1. This may not be what is intended. If it is not intended, the correct bounds should be defined in the BOUNDS section of the MPS formatted file.
- MOSEK ignores field 1, i.e. MARKO001 and MARKO01, however, other optimization systems require
  them.
- Field 2, i.e. MARKER, must be specified including the single quotes. This implies that no row can be assigned the name MARKER.
- Field 3 is ignored and should be left blank.
- Field 4, i.e. INTORG and INTEND, must be specified.
- It is possible to specify several such integer marker sections within the COLUMNS section.

#### 12.2.3 General Limitations

• An MPS file should be an ASCII file.

# 12.2.4 Interpretation of the MPS Format

Several issues related to the MPS format are not well-defined by the industry standard. However, **MOSEK** uses the following interpretation:

- If a matrix element in the COLUMNS section is specified multiple times, then the multiple entries are added together.
- If a matrix element in a QSECTION section is specified multiple times, then the multiple entries are added together.

# 12.2.5 The Free MPS Format

**MOSEK** supports a free format variation of the MPS format. The free format is similar to the MPS file format but less restrictive, e.g. it allows longer names. However, a name must not contain any blanks.

Moreover, by default a line in the MPS file must not contain more than 1024 characters. By modifying the parameter  $MSK\_IPAR\_READ\_MPS\_WIDTH$  an arbitrary large line width will be accepted.

The free MPS format is default. To change to the strict and other formats use the parameter  $MSK\_IPAR\_READ\_MPS\_FORMAT$ .

**Warning:** This file format is to a large extent deprecated. While it can still be used for linear and quadratic problems, for conic problems the Sec. 12.5 is recommended.

# 12.3 The OPF Format

The Optimization Problem Format (OPF) is an alternative to LP and MPS files for specifying optimization problems. It is row-oriented, inspired by the CPLEX LP format.

Apart from containing objective, constraints, bounds etc. it may contain complete or partial solutions, comments and extra information relevant for solving the problem. It is designed to be easily read and modified by hand and to be forward compatible with possible future extensions.

#### Intended use

The OPF file format is meant to replace several other files:

- The LP file format: Any problem that can be written as an LP file can be written as an OPF file too; furthermore it naturally accommodates ranged constraints and variables as well as arbitrary characters in names, fixed expressions in the objective, empty constraints, and conic constraints.
- Parameter files: It is possible to specify integer, double and string parameters along with the problem (or in a separate OPF file).
- Solution files: It is possible to store a full or a partial solution in an OPF file and later reload it.

## 12.3.1 The File Format

The format uses tags to structure data. A simple example with the basic sections may look like this:

```
[comment]
This is a comment. You may write almost anything here...
[/comment]
# This is a single-line comment.

[objective min 'myobj']
x + 3 y + x^2 + 3 y^2 + z + 1
[/objective]

[constraints]
[con 'con01'] 4 <= x + y [/con]
[/constraints]

[bounds]
[b] -10 <= x,y <= 10 [/b]

[cone quad] x,y,z [/cone]
[/bounds]</pre>
```

A scope is opened by a tag of the form [tag] and closed by a tag of the form [/tag]. An opening tag may accept a list of unnamed and named arguments, for examples:

```
[tag value] tag with one unnamed argument [/tag]
[tag arg=value] tag with one named argument [/tag]
```

Unnamed arguments are identified by their order, while named arguments may appear in any order, but never before an unnamed argument. The value can be a quoted, single-quoted or double-quoted text string, i.e.

```
[tag 'value'] single-quoted value [/tag]
[tag arg='value'] single-quoted value [/tag]
[tag "value"] double-quoted value [/tag]
[tag arg="value"] double-quoted value [/tag]
```

### 12.3.2 Sections

The recognized tags are

## [comment]

A comment section. This can contain *almost* any text: Between single quotes (') or double quotes (") any text may appear. Outside quotes the markup characters ([ and ]) must be prefixed by backslashes. Both single and double quotes may appear alone or inside a pair of quotes if it is prefixed by a backslash.

## [objective]

The objective function: This accepts one or two parameters, where the first one (in the above example min) is either min or max (regardless of case) and defines the objective sense, and the second one (above myobj), if present, is the objective name. The section may contain linear and quadratic expressions.

If several objectives are specified, all but the last are ignored.

# [constraints]

This does not directly contain any data, but may contain subsections con defining a linear constraint.

#### [con]

Defines a single constraint; if an argument is present ([con NAME]) this is used as the name of the constraint, otherwise it is given a null-name. The section contains a constraint definition written as linear and quadratic expressions with a lower bound, an upper bound, with both or with an equality. Examples:

Constraint names are unique. If a constraint is specified which has the same name as a previously defined constraint, the new constraint replaces the existing one.

#### [bounds]

This does not directly contain any data, but may contain subsections **b** (linear bounds on variables) and cone (cones).

[b]

Bound definition on one or several variables separated by comma (,). An upper or lower bound on a variable replaces any earlier defined bound on that variable. If only one bound (upper or lower) is given only this bound is replaced. This means that upper and lower bounds can be specified separately. So the OPF bound definition:

[b] 
$$x,y \ge -10$$
 [/b]  
[b]  $x,y \le 10$  [/b]

results in the bound  $-10 \le x, y \le 10$ .

#### [cone]

Specifies a cone. A cone is defined as a sequence of variables which belong to a single unique cone. The supported cone types are:

• quad: a quadratic cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1^2 \ge \sum_{i=2}^n x_i^2, \quad x_1 \ge 0.$$

• rquad: a rotated quadratic cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$2x_1x_2 \ge \sum_{i=3}^n x_i^2, \quad x_1, x_2 \ge 0.$$

• pexp: primal exponential cone of 3 variables  $x_1, x_2, x_3$  defines a constraint of the form

$$x_1 \ge x_2 \exp(x_3/x_2), \quad x_1, x_2 \ge 0.$$

• ppow with parameter  $0 < \alpha < 1$ : primal power cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1^{\alpha} x_2^{1-\alpha} \ge \sqrt{\sum_{j=3}^n x_j^2}, \quad x_1, x_2 \ge 0.$$

• dexp: dual exponential cone of 3 variables  $x_1, x_2, x_3$  defines a constraint of the form

$$x_1 \ge -x_3 e^{-1} \exp(x_2/x_3), \quad x_3 \le 0, x_1 \ge 0.$$

• dpow with parameter  $0 < \alpha < 1$ : dual power cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$\left(\frac{x_1}{\alpha}\right)^{\alpha} \left(\frac{x_2}{1-\alpha}\right)^{1-\alpha} \ge \sqrt{\sum_{j=3}^{n^t} x_j^2}, \quad x_1, x_2 \ge 0.$$

• zero: zero cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1 = \dots = x_n = 0$$

A [bounds]-section example:

By default all variables are free.

#### [variables]

This defines an ordering of variables as they should appear in the problem. This is simply a space-separated list of variable names.

#### [integer]

This contains a space-separated list of variables and defines the constraint that the listed variables must be integer-valued.

#### [hints]

This may contain only non-essential data; for example estimates of the number of variables, constraints and non-zeros. Placed before all other sections containing data this may reduce the time spent reading the file.

In the hints section, any subsection which is not recognized by MOSEK is simply ignored. In this section a hint is defined as follows:

```
[hint ITEM] value [/hint]
```

The hints recognized by **MOSEK** are:

- number of variables),
- numcon (number of linear/quadratic constraints),
- numanz (number of linear non-zeros in constraints),
- numqnz (number of quadratic non-zeros in constraints).

#### [solutions]

This section can contain a set of full or partial solutions to a problem. Each solution must be specified using a [solution]-section, i.e.

```
[solutions]
[solution]...[/solution] #solution 1
[solution]...[/solution] #solution 2
#other solutions....
[solution]...[/solution] #solution n
[/solutions]
```

The syntax of a [solution]-section is the following:

```
[solution SOLTYPE status=STATUS]...[/solution]
```

where SOLTYPE is one of the strings

- interior, a non-basic solution,
- basic, a basic solution,

- integer, an integer solution, and STATUS is one of the strings
- UNKNOWN,
- OPTIMAL,
- INTEGER\_OPTIMAL,
- PRIM\_FEAS,
- DUAL\_FEAS,
- PRIM\_AND\_DUAL\_FEAS,
- NEAR\_OPTIMAL,
- NEAR\_PRIM\_FEAS,
- NEAR\_DUAL\_FEAS,
- NEAR\_PRIM\_AND\_DUAL\_FEAS,
- PRIM\_INFEAS\_CER,
- DUAL\_INFEAS\_CER,
- NEAR\_PRIM\_INFEAS\_CER,
- NEAR\_DUAL\_INFEAS\_CER,
- NEAR\_INTEGER\_OPTIMAL.

Most of these values are irrelevant for input solutions; when constructing a solution for simplex hot-start or an initial solution for a mixed integer problem the safe setting is UNKNOWN.

A [solution]-section contains [con] and [var] sections. Each [con] and [var] section defines solution information for a single variable or constraint, specified as list of KEYWORD/value pairs, in any order, written as

# KEYWORD=value

Allowed keywords are as follows:

- sk. The status of the item, where the value is one of the following strings:
  - LOW, the item is on its lower bound.
  - UPR, the item is on its upper bound.
  - FIX, it is a fixed item.
  - BAS, the item is in the basis.
  - SUPBAS, the item is super basic.
  - UNK, the status is unknown.
  - INF, the item is outside its bounds (infeasible).
- 1vl Defines the level of the item.
- s1 Defines the level of the dual variable associated with its lower bound.
- su Defines the level of the dual variable associated with its upper bound.
- sn Defines the level of the variable associated with its cone.
- y Defines the level of the corresponding dual variable (for constraints only).

A [var] section should always contain the items sk, lvl, sl and su. Items sl and su are not required for integer solutions.

A [con] section should always contain sk, lvl, sl, su and y.

An example of a solution section

• [vendor] This contains solver/vendor specific data. It accepts one argument, which is a vendor ID – for MOSEK the ID is simply mosek – and the section contains the subsection parameters defining solver parameters. When reading a vendor section, any unknown vendor can be safely ignored. This is described later.

Comments using the # may appear anywhere in the file. Between the # and the following line-break any text may be written, including markup characters.

## **12.3.3** Numbers

Numbers, when used for parameter values or coefficients, are written in the usual way by the printf function. That is, they may be prefixed by a sign (+ or -) and may contain an integer part, decimal part and an exponent. The decimal point is always . (a dot). Some examples are

```
1
1.0
.0
1.
1e10
1e+10
1e-10
```

Some invalid examples are

```
e10  # invalid, must contain either integer or decimal part
.  # invalid
.e10  # invalid
```

More formally, the following standard regular expression describes numbers as used:

```
[+|-]?([0-9]+[.][0-9]*|[.][0-9]+)([eE][+|-]?[0-9]+)?
```

#### 12.3.4 Names

Variable names, constraint names and objective name may contain arbitrary characters, which in some cases must be enclosed by quotes (single or double) that in turn must be preceded by a backslash. Unquoted names must begin with a letter (a-z or A-Z) and contain only the following characters: the letters a-z and A-Z, the digits 0-9, braces ({ and }) and underscore (\_).

Some examples of legal names:

```
an_unquoted_name
another_name{123}
'single quoted name'
"double quoted name"
"name with \\"quote\\" in it"
"name with []s in it"
```

# 12.3.5 Parameters Section

In the vendor section solver parameters are defined inside the parameters subsection. Each parameter is written as

```
[p PARAMETER_NAME] value [/p]
```

where PARAMETER\_NAME is replaced by a **MOSEK** parameter name, usually of the form MSK\_IPAR\_. . . . , MSK\_DPAR\_. . . . , and the value is replaced by the value of that parameter; both integer values and named values may be used. Some simple examples are

# 12.3.6 Writing OPF Files from MOSEK

To write an OPF file then make sure the file extension is .opf.

Then modify the following parameters to define what the file should contain:

MSK_IPAR_OPF_WRITE_SOL_BAS	Include basic solution, if defined.	
MSK_IPAR_OPF_WRITE_SOL_ITG	Include integer solution, if defined.	
MSK_IPAR_OPF_WRITE_SOL_ITR	Include interior solution, if defined.	
MSK_IPAR_OPF_WRITE_SOLUTION	SInclude solutions if they are defined. If this is off, no solutions are	
	included.	
MSK_IPAR_OPF_WRITE_HEADER	Include a small header with comments.	
MSK_IPAR_OPF_WRITE_PROBLEM	Include the problem itself — objective, constraints and bounds.	
MSK_IPAR_OPF_WRITE_PARAMETER\$nclude all parameter settings.		
MSK_IPAR_OPF_WRITE_HINTS Include hints about the size of the problem.		

# 12.3.7 Examples

This section contains a set of small examples written in OPF and describing how to formulate linear, quadratic and conic problems.

# Linear Example 101.opf

Consider the example:

having the bounds

$$\begin{array}{rcl}
0 & \leq & x_0 & \leq & \infty, \\
0 & \leq & x_1 & \leq & 10, \\
0 & \leq & x_2 & \leq & \infty, \\
0 & \leq & x_3 & \leq & \infty.
\end{array}$$

In the OPF format the example is displayed as shown in Listing 12.1.

Listing 12.1: Example of an OPF file for a linear problem.

```
[comment]
  The lo1 example in OPF format
[/comment]
[hints]
  [hint NUMVAR] 4 [/hint]
  [hint NUMCON] 3 [/hint]
  [hint NUMANZ] 9 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3 x4
[/variables]
[objective maximize 'obj']
   3 x1 + x2 + 5 x3 + x4
[/objective]
[constraints]
  [con 'c1'] 3 \times 1 + \times 2 + 2 \times 3 = 30 [/con] [con 'c2'] 2 \times 1 + \times 2 + 3 \times 3 + \times 4 >= 15 [/con]
  [con 'c3'] 2 x2 + 3 x4 <= 25 [/con]
[/constraints]
[bounds]
  [b] 0 \le * [/b]
  [b] 0 \le x2 \le 10 [/b]
[/bounds]
```

### Quadratic Example qo1.opf

An example of a quadratic optimization problem is

minimize 
$$x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2$$
 subject to 
$$1 \le x_1 + x_2 + x_3,$$
 
$$x > 0$$

This can be formulated in opf as shown below.

Listing 12.2: Example of an OPF file for a quadratic problem.

```
[comment]
  The qo1 example in OPF format
[/comment]

[hints]
  [hint NUMVAR] 3 [/hint]
  [hint NUMCON] 1 [/hint]
  [hint NUMANZ] 3 [/hint]
  [hint NUMQNZ] 4 [/hint]
[/hints]

[variables disallow_new_variables]
  x1 x2 x3
[/variables]
```

(continued from previous page)

```
[objective minimize 'obj']
  # The quadratic terms are often written with a factor of 1/2 as here,
  # but this is not required.

- x2 + 0.5 ( 2.0 x1 ^ 2 - 2.0 x3 * x1 + 0.2 x2 ^ 2 + 2.0 x3 ^ 2 )
[/objective]

[constraints]
  [con 'c1'] 1.0 <= x1 + x2 + x3 [/con]
[/constraints]

[bounds]
  [b] 0 <= * [/b]
[/bounds]</pre>
```

### Conic Quadratic Example cqo1.opf

Consider the example:

minimize 
$$x_3 + x_4 + x_5$$
  
subject to  $x_0 + x_1 + 2x_2 = 1$ ,  
 $x_0, x_1, x_2 \ge 0$ ,  
 $x_3 \ge \sqrt{x_0^2 + x_1^2}$ ,  
 $2x_4x_5 \ge x_2^2$ .

Please note that the type of the cones is defined by the parameter to [cone ...]; the content of the cone-section is the names of variables that belong to the cone. The resulting OPF file is in Listing 12.3.

Listing 12.3: Example of an OPF file for a conic quadratic problem.

```
[comment]
 The cqo1 example in OPF format.
[/comment]
[hints]
  [hint NUMVAR] 6 [/hint]
  [hint NUMCON] 1 [/hint]
 [hint NUMANZ] 3 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3 x4 x5 x6
[/variables]
[objective minimize 'obj']
  x4 + x5 + x6
[/objective]
[constraints]
  [con 'c1'] x1 + x2 + 2e+00 x3 = 1e+00 [/con]
[/constraints]
[bounds]
 # We let all variables default to the positive orthant
 [b] 0 \le * [/b]
 # ...and change those that differ from the default
```

(continued from previous page)

```
[b] x4,x5,x6 free [/b]

# Define quadratic cone: x4 >= sqrt( x1^2 + x2^2 )
[cone quad 'k1'] x4, x1, x2 [/cone]

# Define rotated quadratic cone: 2 x5 x6 >= x3^2
[cone rquad 'k2'] x5, x6, x3 [/cone]
[/bounds]
```

## Mixed Integer Example milo1.opf

Consider the mixed integer problem:

This can be implemented in OPF with the file in Listing 12.4.

Listing 12.4: Example of an OPF file for a mixed-integer linear problem.

```
[comment]
 The milo1 example in OPF format
[/comment]
[hints]
  [hint NUMVAR] 2 [/hint]
  [hint NUMCON] 2 [/hint]
 [hint NUMANZ] 4 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2
[/variables]
[objective maximize 'obj']
  x1 + 6.4e-1 x2
[/objective]
[constraints]
  [con 'c1'] 5e+1 x1 + 3.1e+1 x2 \le 2.5e+2 [/con]
  [con 'c2'] -4 \le 3 x1 - 2 x2 [/con]
[/constraints]
[bounds]
 [b] 0 \ll * [/b]
[/bounds]
[integer]
 x1 x2
[/integer]
```

# 12.4 The CBF Format

This document constitutes the technical reference manual of the *Conic Benchmark Format* with file extension: .cbf or .CBF. It unifies linear, second-order cone (also known as conic quadratic), exponential cone, power cone and semidefinite optimization with mixed-integer variables. The format has been designed with benchmark libraries in mind, and therefore focuses on compact and easily parsable representations. The CBF format separates problem structure from the problem data.

# 12.4.1 How Instances Are Specified

This section defines the spectrum of conic optimization problems that can be formulated in terms of the keywords of the CBF format.

In the CBF format, conic optimization problems are considered in the following form:

$$\min / \max \quad g^{obj} 
g_i \in \mathcal{K}_i, \quad i \in \mathcal{I}, 
\text{s.t.} \quad G_i \in \mathcal{K}_i, \quad i \in \mathcal{I}^{PSD}, 
x_j \in \mathcal{K}_j, \quad j \in \mathcal{J}, 
\overline{X}_j \in \mathcal{K}_j, \quad j \in \mathcal{J}^{PSD}.$$
(12.10)

- Variables are either scalar variables,  $x_j$  for  $j \in \mathcal{J}$ , or matrix variables,  $\overline{X}_j$  for  $j \in \mathcal{J}^{PSD}$ . Scalar variables can also be declared as integer.
- Constraints are affine expressions of the variables, either scalar-valued  $g_i$  for  $i \in \mathcal{I}$ , or matrix-valued  $G_i$  for  $i \in \mathcal{I}^{PSD}$

$$g_i = \sum_{j \in \mathcal{J}^{PSD}} \langle F_{ij}, X_j \rangle + \sum_{j \in \mathcal{J}} a_{ij} x_j + b_i,$$
$$G_i = \sum_{j \in \mathcal{J}} x_j H_{ij} + D_i.$$

• The **objective function** is a scalar-valued affine expression of the variables, either to be minimized or maximized. We refer to this expression as  $q^{obj}$ 

$$g^{obj} = \sum_{j \in \mathcal{J}^{PSD}} \langle F_j^{obj}, X_j \rangle + \sum_{j \in \mathcal{J}} a_j^{obj} x_j + b^{obj}.$$

As of version 4 of the format, CBF files can represent the following non-parametric cones  $\mathcal{K}$ :

• Free domain - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n\}$$
, for  $n \ge 1$ .

• Positive orthant - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_i \ge 0 \text{ for } j = 1, \dots, n\}, \text{ for } n \ge 1.$$

• Negative orthant - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_i < 0 \text{ for } j = 1, \dots, n\}, \text{ for } n > 1.$$

• Fixpoint zero - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_i = 0 \text{ for } j = 1, \dots, n\}, \text{ for } n \ge 1.$$

• Quadratic cone - A cone in the second-order cone family defined by

$$\left\{ \left(\begin{array}{c} p \\ x \end{array}\right) \in \mathbb{R} \times \mathbb{R}^{n-1}, \ p^2 \ge x^T x, \ p \ge 0 \right\}, \ \text{for } n \ge 2.$$

• Rotated quadratic cone - A cone in the second-order cone family defined by

$$\left\{ \begin{pmatrix} p \\ q \\ x \end{pmatrix} \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{n-2}, \ 2pq \ge x^T x, \ p \ge 0, \ q \ge 0 \right\}, \text{ for } n \ge 3.$$

• Exponential cone - A cone in the exponential cone family defined by

$$\operatorname{cl}(S_1) = S_1 \cup S_2$$

where,

$$S_1 = \left\{ \begin{pmatrix} t \\ s \\ r \end{pmatrix} \in \mathbb{R}^3, \ t \ge se^{\frac{r}{s}}, \ s \ge 0 \right\}.$$

and,

$$S_2 = \left\{ \begin{pmatrix} t \\ s \\ r \end{pmatrix} \in \mathbb{R}^3, \ t \ge 0, \ r \le 0, \ s = 0 \right\}.$$

• Dual Exponential cone - A cone in the exponential cone family defined by

$$\operatorname{cl}(S_1) = S_1 \cup S_2$$

where,

$$S_1 = \left\{ \begin{pmatrix} t \\ s \\ r \end{pmatrix} \in \mathbb{R}^3, \ et \ge (-r)e^{\frac{s}{r}}, \ -r \ge 0 \right\}.$$

and,

$$S_2 = \left\{ \begin{pmatrix} t \\ s \\ r \end{pmatrix} \in \mathbb{R}^3, \ et \ge 0, \ s \ge 0, \ r = 0 \right\}.$$

• Radial geometric mean cone - A cone in the power cone family defined by

$$\left\{ \left( \begin{array}{c} p \\ x \end{array} \right) \in \mathbb{R}^k_+ \times \mathbb{R}^1, \ \left( \prod_{j=1}^k p_j \right)^{\frac{1}{k}} \ge |x| \right\}, \text{ for } n = k+1 \ge 2.$$

• Dual radial geometric mean cone - A cone in the power cone family defined by

$$\left\{ \left( \begin{array}{c} p \\ x \end{array} \right) \in \mathbb{R}^k_+ \times \mathbb{R}^1, \ \left( \prod_{j=1}^k k p_j \right)^{\frac{1}{k}} \ge |x| \right\}, \ \text{for } n = k+1 \ge 2.$$

and, the following parametric cones:

• Radial power cone - A cone in the power cone family defined by

$$\left\{ \left( \begin{array}{c} p \\ x \end{array} \right) \in \mathbb{R}_+^k \times \mathbb{R}^{n-k}, \ \left( \prod_{j=1}^k p_j^{\alpha_j} \right)^{\frac{1}{\sigma}} \ge \|x\|_2 \right\}, \text{ for } n \ge k \ge 1.$$

where, 
$$\sigma = \sum_{j=1}^{k} \alpha_j$$
 and  $\alpha = \mathbb{R}_{++}^k$ .

• Dual radial power cone - A cone in the power cone family defined by

$$\left\{ \left( \begin{array}{c} p \\ x \end{array} \right) \in \mathbb{R}_+^k \times \mathbb{R}^{n-k}, \ \left( \prod_{j=1}^k \left( \frac{\sigma p_j}{\alpha_j} \right)^{\alpha_j} \right)^{\frac{1}{\sigma}} \ge \|x\|_2 \right\}, \text{ for } n \ge k \ge 1.$$

where, 
$$\sigma = \sum_{j=1}^k \alpha_j$$
 and  $\alpha = \mathbb{R}_{++}^k$ .

# 12.4.2 The Structure of CBF Files

This section defines how information is written in the CBF format, without being specific about the type of information being communicated.

All information items belong to exactly one of the three groups of information. These information groups, and the order they must appear in, are:

- 1. File format.
- 2. Problem structure.
- 3. Problem data.

The first group, file format, provides information on how to interpret the file. The second group, problem structure, provides the information needed to deduce the type and size of the problem instance. Finally, the third group, problem data, specifies the coefficients and constants of the problem instance.

# Information items

The format is composed as a list of information items. The first line of an information item is the KEYWORD, revealing the type of information provided. The second line - of some keywords only - is the HEADER, typically revealing the size of information that follows. The remaining lines are the BODY holding the actual information to be specified.

The KEYWORD determines how each line in the HEADER and BODY is structured. Moreover, the number of lines in the BODY follows either from the KEYWORD, the HEADER, or from another information item required to precede it.

### File encoding and line width restrictions

The format is based on the US-ASCII printable character set with two extensions as listed below. Note, by definition, that none of these extensions can be misinterpreted as printable US-ASCII characters:

- A line feed marks the end of a line, carriage returns are ignored.
- Comment-lines may contain unicode characters in UTF-8 encoding.

The line width is restricted to 512 bytes, with 3 bytes reserved for the potential carriage return, line feed and null-terminator.

Integers and floating point numbers must follow the ISO C decimal string representation in the standard C locale. The format does not impose restrictions on the magnitude of, or number of significant digits in numeric data, but the use of 64-bit integers and 64-bit IEEE 754 floating point numbers should be sufficient to avoid loss of precision.

### Comment-line and whitespace rules

The format allows single-line comments respecting the following rule:

• Lines having first byte equal to '#' (US-ASCII 35) are comments, and should be ignored. Comments are only allowed between information items.

Given that a line is not a comment-line, whitespace characters should be handled according to the following rules:

- Leading and trailing whitespace characters should be ignored.
  - The seperator between multiple pieces of information on one line, is either one or more whitespace characters.
- Lines containing only whitespace characters are empty, and should be ignored. Empty lines are only allowed between information items.

## 12.4.3 Problem Specification

### The problem structure

The problem structure defines the objective sense, whether it is minimization and maximization. It also defines the index sets,  $\mathcal{J}$ ,  $\mathcal{J}^{PSD}$ ,  $\mathcal{I}$  and  $\mathcal{I}^{PSD}$ , which are all numbered from zero,  $\{0, 1, \ldots\}$ , and empty until explicitly constructed.

• Scalar variables are constructed in vectors restricted to a conic domain, such as  $(x_0, x_1) \in \mathbb{R}^2_+$ ,  $(x_2, x_3, x_4) \in \mathcal{Q}^3$ , etc. In terms of the Cartesian product, this generalizes to

$$x \in \mathcal{K}_1^{n_1} \times \mathcal{K}_2^{n_2} \times \dots \times \mathcal{K}_k^{n_k}$$

which in the CBF format becomes:

```
VAR
n k
K1 n1
K2 n2
...
Kk nk
```

where  $\sum_{i} n_{i} = n$  is the total number of scalar variables. The list of supported cones is found in Table 12.3. Integrality of scalar variables can be specified afterwards.

• **PSD variables** are constructed one-by-one. That is,  $X_j \succeq \mathbf{0}^{n_j \times n_j}$  for  $j \in \mathcal{J}^{PSD}$ , constructs a matrix-valued variable of size  $n_j \times n_j$  restricted to be symmetric positive semidefinite. In the CBF format, this list of constructions becomes:



where N is the total number of PSD variables.

• Scalar constraints are constructed in vectors restricted to a conic domain, such as  $(g_0, g_1) \in \mathbb{R}^2_+$ ,  $(g_2, g_3, g_4) \in \mathcal{Q}^3$ , etc. In terms of the Cartesian product, this generalizes to

$$g \in \mathcal{K}_1^{m_1} \times \mathcal{K}_2^{m_2} \times \dots \times \mathcal{K}_k^{m_k}$$

which in the CBF format becomes:

```
CON
m k
K1 m1
K2 m2
...
Kk mk
```

where  $\sum_{i} m_{i} = m$  is the total number of scalar constraints. The list of supported cones is found in Table 12.3.

• **PSD constraints** are constructed one-by-one. That is,  $G_i \succeq \mathbf{0}^{m_i \times m_i}$  for  $i \in \mathcal{I}^{PSD}$ , constructs a matrix-valued affine expressions of size  $m_i \times m_i$  restricted to be symmetric positive semidefinite. In the CBF format, this list of constructions becomes

```
PSDCON
M
m1
m2
..
mM
```

where M is the total number of PSD constraints.

With the objective sense, variables (with integer indications) and constraints, the definitions of the many affine expressions follow in problem data.

#### Problem data

The problem data defines the coefficients and constants of the affine expressions of the problem instance. These are considered zero until explicitly defined, implying that instances with no keywords from this information group are, in fact, valid. Duplicating or conflicting information is a failure to comply with the standard. Consequently, two coefficients written to the same position in a matrix (or to transposed positions in a symmetric matrix) is an error.

The affine expressions of the objective,  $g^{obj}$ , of the scalar constraints,  $g_i$ , and of the PSD constraints,  $G_i$ , are defined separately. The following notation uses the standard trace inner product for matrices,  $\langle X, Y \rangle = \sum_{i,j} X_{ij} Y_{ij}$ .

 $\bullet\,$  The affine expression of the objective is defined as

$$g^{obj} = \sum_{j \in \mathcal{J}^{PSD}} \langle F_j^{obj}, X_j \rangle + \sum_{j \in \mathcal{J}} a_j^{obj} x_j + b^{obj},$$

in terms of the symmetric matrices,  $F_j^{obj}$ , and scalars,  $a_j^{obj}$  and  $b^{obj}$ .

• The affine expressions of the scalar constraints are defined, for  $i \in \mathcal{I}$ , as

$$g_i = \sum_{j \in \mathcal{J}^{PSD}} \langle F_{ij}, X_j \rangle + \sum_{j \in \mathcal{J}} a_{ij} x_j + b_i,$$

in terms of the symmetric matrices,  $F_{ij}$ , and scalars,  $a_{ij}$  and  $b_i$ .

• The affine expressions of the PSD constraints are defined, for  $i \in \mathcal{I}^{PSD}$ , as

$$G_i = \sum_{j \in \mathcal{J}} x_j H_{ij} + D_i,$$

in terms of the symmetric matrices,  $H_{ij}$  and  $D_i$ .

### List of cones

The format uses an explicit syntax for symmetric positive semidefinite cones as shown above. For scalar variables and constraints, constructed in vectors, the supported conic domains and their sizes are given as follows.

Table 12.3: Cones available in the CBF format

Name	CBF keyword	Cone family	Cone size
Free domain	F	linear	$n \ge 1$
Positive orthant	L+	linear	$n \ge 1$
Negative orthant	L-	linear	$n \ge 1$
Fixpoint zero	L=	linear	$n \ge 1$
Quadratic cone	Q	second-order	$n \ge 1$
Rotated quadratic cone	QR	second-order	$n \ge 2$
Exponential cone	EXP	exponential	n=3
Dual exponential cone	EXP*	exponential	n=3
Radial geometric mean cone	GMEANABS	power	$n = k + 1 \ge 2$
Dual radial geometric mean cone	GMEANABS*	power	$n = k + 1 \ge 2$
Radial power cone (parametric)	POW	power	$n \ge k \ge 1$
Dual radial power cone (parametric)	POW*	power	$n \ge k \ge 1$

# 12.4.4 File Format Keywords

### **VER**

Description: The version of the Conic Benchmark Format used to write the file.

**HEADER:** None

BODY: One line formatted as:

INT

This is the version number.

Must appear exactly once in a file, as the first keyword.

### **POWCONES**

Description: Define a lookup table for power cone domains.

**HEADER**: One line formatted as:

INT INT

This is the number of cones to be specified and the combined length of their dense parameter vectors.

BODY: A list of chunks each specifying the dense parameter vector of a power cone. CHUNKHEADER: One line formatted as:

INT

This is the parameter vector length.

CHUNKBODY: A list of lines formatted as:

REAL

This is the parameter vector values. The number of lines should match the number stated in the chunk header.

The cone specified at index k (with 0-based indexing) is registered under the CBF name @k:POW.

### **POW\*CONES**

Description: Define a lookup table for dual power cone domains.

**HEADER**: One line formatted as:

INT INT

This is the number of cones to be specified and the combined length of their dense parameter vectors.

BODY: A list of chunks each specifying the dense parameter vector of a dual power cone. CHUNKHEADER: One line formatted as:

INT

This is the parameter vector length.

CHUNKBODY: A list of lines formatted as:

REAL

This is the parameter vector values. The number of lines should match the number stated in the chunk header.

The cone specified at index k (with 0-based indexing) is registered under the CBF name @k:POW\*.

## **OBJSENSE**

Description: Define the objective sense.

**HEADER:** None

BODY: One line formatted as:

STR

having MIN indicates minimize, and MAX indicates maximize. Upper-case letters are required. Must appear exactly once in a file.

#### **PSDVAR**

Description: Construct the PSD variables.

**HEADER**: One line formatted as:

INT

This is the number of PSD variables in the problem.

BODY: A list of lines formatted as:

INT

This indicates the number of rows (equal to the number of columns) in the matrix-valued PSD variable. The number of lines should match the number stated in the header.

#### **VAR**

Description: Construct the scalar variables.

**HEADER**: One line formatted as:

INT INT

This is the number of scalar variables, followed by the number of conic domains they are restricted to.

BODY: A list of lines formatted as:

STR INT

This indicates the cone name (see Table 12.3), and the number of scalar variables restricted to this cone. These numbers should add up to the number of scalar variables stated first in the header. The number of lines should match the second number stated in the header.

#### INT

Description: Declare integer requirements on a selected subset of scalar variables.

**HEADER**: one line formatted as:

INT

This is the number of integer scalar variables in the problem.

BODY: a list of lines formatted as:

INT

This indicates the scalar variable index  $j \in \mathcal{J}$ . The number of lines should match the number stated in the header.

Can only be used after the keyword VAR.

## **PSDCON**

Description: Construct the PSD constraints.

**HEADER**: One line formatted as:

INT

This is the number of PSD constraints in the problem.

BODY: A list of lines formatted as:

INT

This indicates the number of rows (equal to the number of columns) in the matrix-valued affine expression of the PSD constraint. The number of lines should match the number stated in the header. Can only be used after these keywords: PSDVAR, VAR.

CON

Description: Construct the scalar constraints.

**HEADER**: One line formatted as:

INT INT

This is the number of scalar constraints, followed by the number of conic domains they restrict to. BODY: A list of lines formatted as:

STR INT

This indicates the cone name (see Table 12.3), and the number of affine expressions restricted to this cone. These numbers should add up to the number of scalar constraints stated first in the header. The number of lines should match the second number stated in the header.

Can only be used after these keywords: PSDVAR, VAR

### **OBJFCOORD**

Description: Input sparse coordinates (quadruplets) to define the symmetric matrices  $F_j^{obj}$ , as used in the objective.

**HEADER:** One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT INT REAL

This indicates the PSD variable index  $j \in \mathcal{J}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

### **OBJACOORD**

Description: Input sparse coordinates (pairs) to define the scalars,  $a_j^{obj}$ , as used in the objective. HEADER: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT REAL

This indicates the scalar variable index  $j \in \mathcal{J}$  and the coefficient value. The number of lines should match the number stated in the header.

### **OBJBCOORD**

Description: Input the scalar,  $b^{obj}$ , as used in the objective.

HEADER: None.

BODY: One line formatted as:

REAL

This indicates the coefficient value.

### **FCOORD**

Description: Input sparse coordinates (quintuplets) to define the symmetric matrices,  $F_{ij}$ , as used in the scalar constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT INT INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$ , the PSD variable index  $j \in \mathcal{J}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

### **ACOORD**

Description: Input sparse coordinates (triplets) to define the scalars,  $a_{ij}$ , as used in the scalar constraints. HEADER: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$ , the scalar variable index  $j \in \mathcal{J}$  and the coefficient value. The number of lines should match the number stated in the header.

#### **BCOORD**

Description: Input sparse coordinates (pairs) to define the scalars,  $b_i$ , as used in the scalar constraints. HEADER: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$  and the coefficient value. The number of lines should match the number stated in the header.

# **HCOORD**

Description: Input sparse coordinates (quintuplets) to define the symmetric matrices,  $H_{ij}$ , as used in the PSD constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as

# INT INT INT INT REAL

This indicates the PSD constraint index  $i \in \mathcal{I}^{PSD}$ , the scalar variable index  $j \in \mathcal{J}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

## **DCOORD**

Description: Input sparse coordinates (quadruplets) to define the symmetric matrices,  $D_i$ , as used in the PSD constraints.

**HEADER**: One line formatted as

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT INT REAL

This indicates the PSD constraint index  $i \in \mathcal{I}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

# 12.4.5 CBF Format Examples

## Minimal Working Example

The conic optimization problem (12.11), has three variables in a quadratic cone - first one is integer - and an affine expression in domain 0 (equality constraint).

minimize 
$$5.1 x_0$$
  
subject to  $6.2 x_1 + 7.3 x_2 - 8.4 \in \{0\}$   
 $x \in \mathcal{Q}^3, x_0 \in \mathbb{Z}.$  (12.11)

Its formulation in the Conic Benchmark Format begins with the version of the CBF format used, to safeguard against later revisions.

```
VER 4
```

Next follows the problem structure, consisting of the objective sense, the number and domain of variables, the indices of integer variables, and the number and domain of scalar-valued affine expressions (i.e., the equality constraint).

```
OBJSENSE
MIN

VAR
3 1
Q 3

INT
1
0

CON
1 1
L= 1
```

Finally follows the problem data, consisting of the coefficients of the objective, the coefficients of the constraints, and the constant terms of the constraints. All data is specified on a sparse coordinate form.

```
OBJACOORD

1
0 5.1

ACOORD
2
0 1 6.2
0 2 7.3

BCOORD
1
0 -8.4
```

This concludes the example.

### Mixing Linear, Second-order and Semidefinite Cones

The conic optimization problem (12.12), has a semidefinite cone, a quadratic cone over unordered subindices, and two equality constraints.

The equality constraints are easily rewritten to the conic form,  $(g_0, g_1) \in \{0\}^2$ , by moving constants such that the right-hand-side becomes zero. The quadratic cone does not fit under the VAR keyword in this variable permutation. Instead, it takes a scalar constraint  $(g_2, g_3, g_4) = (x_1, x_0, x_2) \in \mathcal{Q}^3$ , with scalar variables constructed as  $(x_0, x_1, x_2) \in \mathbb{R}^3$ . Its formulation in the CBF format is reported in the following list

```
# File written using this version of the Conic Benchmark Format:
      | Version 4.
VER
4
# The sense of the objective is:
      | Minimize.
OBJSENSE
MIN
# One PSD variable of this size:
      | Three times three.
PSDVAR
1
3
# Three scalar variables in this one conic domain:
#
      | Three are free.
VAR
3 1
F 3
# Five scalar constraints with affine expressions in two conic domains:
      | Two are fixed to zero.
      | Three are in conic quadratic domain.
CON
5 2
L=2
# Five coordinates in F^{obj}_j coefficients:
      | F^{obj}[0][0,0] = 2.0
      | F^{obj}[0][1,0] = 1.0
      | and more...
OBJFCOORD
```

 $({\rm continued\ from\ previous\ page})$ 

```
0 0 0 2.0
0 1 0 1.0
0 1 1 2.0
0 2 1 1.0
0 2 2 2.0
# One coordinate in a{obj}_{j} coefficients:
      | a^{obj}[1] = 1.0
OBJACOORD
1 1.0
# Nine coordinates in F_ij coefficients:
     | F[0,0][0,0] = 1.0
      | F[0,0][1,1] = 1.0
      | and more...
FCOORD
0 0 0 0 1.0
0 0 1 1 1.0
0 0 2 2 1.0
1 0 0 0 1.0
1 0 1 0 1.0
1 0 2 0 1.0
1 0 1 1 1.0
1 0 2 1 1.0
1 0 2 2 1.0
# Six coordinates in a_ij coefficients:
      | a[0,1] = 1.0
      | a[1,0] = 1.0
      | and more...
ACOORD
0 1 1.0
1 0 1.0
1 2 1.0
2 1 1.0
3 0 1.0
4 2 1.0
# Two coordinates in b_i coefficients:
    | b[0] = -1.0
      | b[1] = -0.5
BCOORD
0 -1.0
1 -0.5
```

### Mixing Semidefinite Variables and Linear Matrix Inequalities

The standard forms in semidefinite optimization are usually based either on semidefinite variables or linear matrix inequalities. In the CBF format, both forms are supported and can even be mixed as shown.

minimize 
$$\left\langle \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, X_1 \right\rangle + x_1 + x_2 + 1$$
  
subject to  $\left\langle \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, X_1 \right\rangle - x_1 - x_2 \qquad \geq 0.0,$   
 $x_1 \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix} + x_2 \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \succeq \mathbf{0},$   
 $X_1 \succeq \mathbf{0}.$  (12.13)

Its formulation in the CBF format is written in what follows

```
# File written using this version of the Conic Benchmark Format:
      | Version 4.
VER
4
# The sense of the objective is:
# | Minimize.
OBJSENSE
MIN
# One PSD variable of this size:
     | Two times two.
PSDVAR
1
2
# Two scalar variables in this one conic domain:
      | Two are free.
VAR
2 1
# One PSD constraint of this size:
# | Two times two.
PSDCON
2
# One scalar constraint with an affine expression in this one conic domain:
     | One is greater than or equal to zero.
CON
1 1
L+ 1
# Two coordinates in F^{obj}_j coefficients:
      | F^{obj}[0][0,0] = 1.0
      | F^{obj}[0][1,1] = 1.0
OBJFCOORD
0 0 0 1.0
0 1 1 1.0
# Two coordinates in a^{obj}_j coefficients:
```

 $({\rm continued\ from\ previous\ page})$ 

```
| a^{obj}[0] = 1.0
     | a^{obj}[1] = 1.0
OBJACOORD
0 1.0
1 1.0
# One coordinate in b^{obj} coefficient:
# | b^{obj} = 1.0
OBJBCOORD
1.0
# One coordinate in F_ij coefficients:
     | F[0,0][1,0] = 1.0
FCOORD
0 0 1 0 1.0
# Two coordinates in a_ij coefficients:
\# | a[0,0] = -1.0
     | a[0,1] = -1.0
ACOORD
0 0 -1.0
0 1 -1.0
# Four coordinates in H_ij coefficients:
# | H[0,0][1,0] = 1.0
    | H[0,0][1,1] = 3.0
     | and more...
HCOORD
0 0 1 0 1.0
0 0 1 1 3.0
0 1 0 0 3.0
0 1 1 0 1.0
# Two coordinates in D_i coefficients:
    | D[0][0,0] = -1.0
     | D[0][1,1] = -1.0
DCOORD
0 0 0 -1.0
0 1 1 -1.0
```

### The exponential cone

The conic optimization problem (12.14), has one equality constraint, one quadratic cone constraint and an exponential cone constraint.

minimize 
$$x_0 - x_3$$
  
subject to  $x_0 + 2x_1 - x_2 \in \{0\}$   
 $(5.0, x_0, x_1) \in \mathcal{Q}^3$   
 $(x_2, 1.0, x_3) \in EXP.$  (12.14)

The nonlinear conic constraints enforce  $\sqrt{x_0^2 + x_1^2} \le 0.5$  and  $x_3 \le \log(x_2)$ .

```
# File written using this version of the Conic Benchmark Format:
       | Version 3.
VER
3
# The sense of the objective is:
       | Minimize.
OBJSENSE
MIN
# Four scalar variables in this one conic domain:
       | Four are free.
VAR
4 1
F 4
# Seven scalar constraints with affine expressions in three conic domains:
       | One is fixed to zero.
       | Three are in conic quadratic domain.
#
       | Three are in exponential cone domain.
CON
7 3
L= 1
03
EXP 3
# Two coordinates in a^{obj}_j coefficients:
       | a^{obj}[0] = 1.0
       | a^{obj}[3] = -1.0
OBJACOORD
0 1.0
3 -1.0
# Seven coordinates in a_ij coefficients:
       | a[0,0] = 1.0
       | a[0,1] = 2.0
       | and more...
ACOORD
0 0 1.0
0 1 2.0
0 2 -1.0
2 0 1.0
3 1 1.0
4 2 1.0
6 3 1.0
```

(continued from previous page)

#### Parametric cones

The problem (12.15), has three variables in a power cone with parameter  $\alpha_1 = (1, 1)$  and two power cone constraints each with parameter  $\alpha_0 = (8, 1)$ .

minimize 
$$x_3$$
  
subject to  $(1.0, x_1, x_1 + x_2) \in POW_{\alpha_0}$   
 $(1.0, x_2, x_1 + x_2) \in POW_{\alpha_0}$   
 $x \in POW_{\alpha_1}$ . (12.15)

The nonlinear conic constraints enforce  $x_3 \leq x_1 x_2$  and  $x_1 + x_2 \leq \min(x_1^{\frac{1}{9}}, x_2^{\frac{1}{9}})$ .

```
# File written using this version of the Conic Benchmark Format:
#
       | Version 3.
VER.
3
# Two power cone domains defined in a total of four parameters:
       | @0:POW (specification 0) has two parameters:
#
       | alpha[0] = 8.0.
#
       | alpha[1] = 1.0.
#
       | @1:POW (specification 1) has two parameters:
       | alpha[0] = 1.0.
       | alpha[1] = 1.0.
POWCONES
2 4
2
8.0
1.0
2
1.0
1.0
# The sense of the objective is:
       | Maximize.
OBJSENSE
MAX
# Three scalar variable in this one conic domain:
       | Three are in power cone domain (specification 1).
VAR
3 1
@1:POW 3
# Six scalar constraints with affine expressions in two conic domains:
#
       | Three are in power cone domain (specification 0).
#
       | Three are in power cone domain (specification 0).
```

(continued from previous page)

```
CON
6 2
@0:POW 3
@0:POW 3
# One coordinate in a^{obj}_j coefficients:
       | a^{obj}[2] = 1.0
OBJACOORD
2 1.0
# Six coordinates in a_ij coefficients:
       | a[1,0] = 1.0
       | a[2,0] = 1.0
       | and more...
ACOORD
1 0 1.0
2 0 1.0
2 1 1.0
4 1 1.0
5 0 1.0
5 1 1.0
# Two coordinates in b_i coefficients:
       | b[0] = 1.0
       | b[3] = 1.0
BCOORD
0 1.0
3 1.0
```

# 12.5 The PTF Format

The PTF format is a human-readable, natural text format supporting all of **MOSEK** optimization problems in conic form, possibly with integer variables and disjunctive constraints.

## 12.5.1 The overall format

The format is indentation based, where each section is started by a head line and followed by a section body with deeper indentation that the head line. For example:

```
Header line
Body line 1
Body line 1
Body line 1
```

Section can also be nested:

```
Header line A
Body line in A
Header line A.1
Body line in A.1
Body line in A.1
Body line in A.1
```

The indentation of blank lines is ignored, so a subsection can contain a blank line with no indentation. The character # defines a line comment and anything between the # character and the end of the line is ignored.

In a PTF file, the first section must be a Task section. The order of the remaining section is arbitrary, and sections may occur multiple times or not at all.

MOSEK will ignore any top-level section it does not recognize.

#### **Names**

In the description of the format we use following definitions for name strings:

```
NAME: PLAIN_NAME | QUOTED_NAME
PLAIN_NAME: [a-zA-Z_] [a-zA-ZO-9_-.!|]
QUOTED_NAME: "'" ( [^'\\r\n] | "\\" ( [\\rn] | "x" [0-9a-fA-F] [0-9a-fA-F] ) )* "'"
```

#### **Expressions**

An expression is a sum of terms. A term is either a linear term (a coefficient and a variable name, where the coefficient can be left out if it is 1.0), or a matrix inner product.

An expression:

```
EXPR: EMPTY | [+-]? TERM ( [+-] TERM )*
TERM: LINEAR_TERM | MATRIX_TERM
```

A linear term

```
LINEAR_TERM: FLOAT? NAME
```

A matrix term

```
MATRIX_TERM: "<" FLOAT? NAME ( [+-] FLOAT? NAME)* ";" NAME ">"
```

Here the right-hand name is the name of a (semidefinite) matrix variable, and the left-hand side is a sum of symmetric matrixes. The actual matrixes are defined in a separate section.

Expressions can span multiple lines by giving subsequent lines a deeper indentation.

For example following two section are equivalent:

```
# Everything on one line:
x1 + x2 + x3 + x4

# Split into multiple lines:
x1
+ x2
+ x3
+ x4
```

## 12.5.2 Task section

The first section of the file must be a Task. The text in this section is not used and may contain comments, or meta-information from the writer or about the content.

Format:

```
Task NAME
Anything goes here...
```

NAME is a the task name.

## 12.5.3 Objective section

The Objective section defines the objective name, sense and function. The format:

```
"Objective" NAME?
( "Minimize" | "Maximize" ) EXPR
```

For example:

```
Objective 'obj'
Minimize x1 + 0.2 x2 + < M1 ; X1 >
```

#### 12.5.4 Constraints section

The constraints section defines a series of constraints. A constraint defines a term  $A \cdot x + b \in K$ . For linear constraints A is just one row, while for conic constraints it can be multiple rows. If a constraint spans multiple rows these can either be written inline separated by semi-colons, or each expression in a separete sub-section.

Simple linear constraints:

```
"Constraints"
NAME? "[" [-+] (FLOAT | "Inf") (";" [-+] (FLOAT | "Inf") )? "]" EXPR
```

If the brackets contain two values, they are used as upper and lower bounds. It they contain one value the constraint is an equality.

For example:

```
Constraints
'c1' [0;10] x1 + x2 + x3
[0] x1 + x2 + x3
```

Constraint blocks put the expression either in a subsection or inline. The cone type (domain) is written in the brackets, and MOSEK currently supports following types:

- SOC(N) Second order cone of dimension N
- RSOC(N) Rotated second order cone of dimension N
- PSD(N) Symmetric positive semidefinite cone of dimension N. This contains N\*(N+1)/2 elements.
- PEXP Primal exponential cone of dimension 3
- DEXP Dual exponential cone of dimension 3
- PPOW(N,P) Primal power cone of dimension N with parameter P
- $\bullet$  DPOW(N,P) Dual power cone of dimension N with parameter P
- ZERO(N) The zero-cone of dimension N.

```
"Constraints"
NAME? "[" DOMAIN "]" EXPR_LIST
```

For example:

```
Constraints

'K1' [SOC(3)] x1 + x2 ; x2 + x3 ; x3 + x1

'K2' [RSOC(3)]

x1 + x2

x2 + x3

x3 + x1
```

## 12.5.5 Variables section

Any variable used in an expression must be defined in a variable section. The variable section defines each variable domain.

```
"Variables"

NAME "[" [-+] (FLOAT | "Inf") (";" [-+] (FLOAT | "Inf") )? "]"

NAME "[" DOMAIN "]" NAMES

For example, a linear variable
```

```
Variables x1 [0;Inf]
```

As with constraints, members of a conic domain can be listed either inline or in a subsection:

```
Variables
k1 [SOC(3)] x1; x2; x3
k2 [RSOC(3)]
x1
x2
x3
```

# 12.5.6 Integer section

This section contains a list of variables that are integral. For example:

```
Integer x1 x2 x3
```

## 12.5.7 SymmetricMatrixes section

This section defines the symmetric matrixes used for matrix coefficients in matrix inner product terms. The section lists named matrixes, each with a size and a number of non-zeros. Only non-zeros in the lower triangular part should be defined.

```
"SymmetricMatrixes"

NAME "SYMMAT" "(" INT ")" ( "(" INT "," INT "," FLOAT ")" )*

...
```

For example:

```
SymmetricMatrixes
M1 SYMMAT(3) (0,0,1.0) (1,1,2.0) (2,1,0.5)
M2 SYMMAT(3)
(0,0,1.0)
(1,1,2.0)
(2,1,0.5)
```

## 12.5.8 Solutions section

Each subsection defines a solution. A solution defines for each constraint and for each variable exactly one primal value and either one (for conic domains) or two (for linear domains) dual values. The values follow the same logic as in the **MOSEK** C API. A primal and a dual solution status defines the meaning of the values primal and dual (solution, certificate, unknown, etc.)

The format is this:

```
"Solutions"
  "Solution" WHICHSOL
      "ProblemStatus" PROSTA PROSTA?
  "SolutionStatus" SOLSTA SOLSTA?
  "Objective" FLOAT FLOAT
  "Variables"
      # Linear variable status: level, slx, sux
     NAME "[" STATUS "]" FLOAT (FLOAT FLOAT)?
      # Conic variable status: level, snx
      NAME.
         "[" STATUS "]" FLOAT FLOAT?
         . . .
   "Constraints"
      # Linear variable status: level, slx, sux
      NAME "[" STATUS "]" FLOAT (FLOAT FLOAT)?
      # Conic variable status: level, snx
      NAME
         "[" STATUS "]" FLOAT FLOAT?
```

Following values for WHICHSOL are supported:

- interior Interior solution, the result of an interior-point solver.
- basic Basic solution, as produced by a simplex solver.
- integer Integer solution, the solution to a mixed-integer problem. This does not define a dual solution.

Following values for PROSTA are supported:

- unknown The problem status is unknown
- feasible The problem has been proven feasible
- infeasible The problem has been proven infeasible
- illposed The problem has been proved to be ill posed
- infeasible\_or\_unbounded The problem is infeasible or unbounded

Following values for SOLSTA are supported:

- unknown The solution status is unknown
- feasible The solution is feasible
- optimal The solution is optimal
- infeas\_cert The solution is a certificate of infeasibility
- illposed\_cert The solution is a certificate of illposedness

Following values for STATUS are supported:

- unknown The value is unknown
- super\_basic The value is super basic

- at\_lower The value is basic and at its lower bound
- at\_upper The value is basic and at its upper bound
- fixed The value is basic fixed
- infinite The value is at infinity

# 12.5.9 Examples

## Linear example lo1.ptf

```
Task ''
   # Written by MOSEK v10.0.13
   # problemtype: Linear Problem
   # number of linear variables: 4
   # number of linear constraints: 3
    # number of old-style A nonzeros: 9
Objective obj
   Maximize + 3 x1 + x2 + 5 x3 + x4
Constraints
   c1 [3e+1] + 3 x1 + x2 + 2 x3
    c2 [1.5e+1;+inf] + 2 x1 + x2 + 3 x3 + x4
    c3 [-inf; 2.5e+1] + 2 x2 + 3 x4
Variables
   x1 [0;+inf]
   x2 [0;1e+1]
   x3 [0;+inf]
   x4 [0;+inf]
```

### Conic example cqo1.ptf

```
Task ''
   # Written by MOSEK v10.0.17
   # problemtype: Conic Problem
   # number of linear variables: 6
   # number of linear constraints: 1
   # number of old-style cones: 0
   # number of positive semidefinite variables: 0
   # number of positive semidefinite matrixes: 0
   # number of affine conic constraints: 2
   # number of disjunctive constraints: 0
   # number scalar affine expressions/nonzeros : 6/6
    # number of old-style A nonzeros: 3
Objective obj
   Minimize + x4 + x5 + x6
Constraints
    c1 [1] + x1 + x2 + 2 x3
   k1 [QUAD(3)]
        0ac1: + x4
        0ac2: + x1
        @ac3: + x2
   k2 [RQUAD(3)]
        0ac4: + x5
        0ac5: + x6
        @ac6: + x3
Variables
```

(continued from previous page)

```
x4
x1 [0;+inf]
x2 [0;+inf]
x5
x6
x3 [0;+inf]
```

# Disjunctive example djc1.ptf

```
Task djc1
Objective ''
    Minimize + 2 'x[0]' + 'x[1]' + 3 'x[2]' + 'x[3]'
    @c0 [-10;+inf] + 'x[0]' + 'x[1]' + 'x[2]' + 'x[3]'
    @DO [OR]
        [AND]
            [NEGATIVE(1)]
                 + 'x[0]' - 2 'x[1]' + 1
            [ZERO(2)]
                 + 'x[2]'
                  + 'x[3]'
        [AND]
            [NEGATIVE(1)]
                 + 'x[2]' - 3 'x[3]' + 2
            [ZERO(2)]
                 + 'x[0]'
                 + 'x[1]'
    @D1 [OR]
        [ZERO(1)]
             + 'x[0]' - 2.5
        [ZERO(1)]
             + 'x[1]' - 2.5
        [ZERO(1)]
             + 'x[2]' - 2.5
        [ZERO(1)]
             + 'x[3]' - 2.5
Variables
    'x[0]'
    'x[1]'
    'x[2]'
    'x[3]'
```

# 12.6 The Task Format

The Task format is  $\mathbf{MOSEK}$ 's native binary format. It contains a complete image of a  $\mathbf{MOSEK}$  task, i.e.

- Problem data: Linear, conic, semidefinite and quadratic data
- Problem item names: Variable names, constraints names, cone names etc.
- Parameter settings
- Solutions

There are a few things to be aware of:

- Status of a solution read from a file will always be unknown.
- Parameter settings in a task file *always override* any parameters set on the command line or in a parameter file.

The format is based on the TAR (USTar) file format. This means that the individual pieces of data in a .task file can be examined by unpacking it as a TAR file. Please note that the inverse may not work: Creating a file using TAR will most probably not create a valid **MOSEK** Task file since the order of the entries is important.

# 12.7 The JSON Format

**MOSEK** provides the possibility to read/write problems and solutions in JSON format. The official JSON website <a href="http://www.json.org">http://www.json.org</a> provides plenty of information along with the format definition. JSON is an industry standard for data exchange and JSON files can be easily written and read in most programming languages using dedicated libraries.

MOSEK uses two JSON-based formats:

• JTASK, for storing problem instances together with solutions and parameters. The JTASK format contains the same information as a native MOSEK task task format, that is a very close representation of the internal data storage in the task object.

You can write a JTASK file specifying the extension .jtask. When the parameter  $MSK\_IPAR\_WRITE\_JSON\_INDENTATION$  is set the JTASK file will be indented to slightly improve readability.

• **JSOL**, for storing solutions and information items.

You can write a JSOL solution file using the option -jsolo. When the parameter MSK\_IPAR\_WRITE\_JSON\_INDENTATION is set the JSOL file will be indented to slightly improve readability.

You can read a JSOL solution into an existing task file using the option -jsoli. Only the Task/solutions section of the data will be taken into consideration.

## 12.7.1 JTASK Specification

The JTASK is a dictionary containing the following sections. All sections are optional and can be omitted if irrelevant for the problem.

- \$schema: JSON schema.
- Task/name: The name of the task (string).
- Task/INFO: Information about problem data dimensions and similar. These are treated as hints when reading the file.
  - number of variables (int32).
  - numcon: number of constraints (int32).
  - numcone: number of cones (int32, deprecated).
  - number of symmetric matrix variables (int32).
  - numanz: number of nonzeros in A (int64).
  - numsymmat: number of matrices in the symmetric matrix storage E (int64).
  - numafe: number of affine expressions in AFE storage (int64).
  - numfnz: number of nonzeros in F (int64).
  - numacc: number of affine conic constraints (ACCs) (int64).
  - numdjc: number of disjunctive constraints (DJCs) (int64).
  - number of domains (int64).
  - mosekver: MOSEK version (list(int32)).

- Task/data: Numerical and structural data of the problem.
  - var: Information about variables. All fields present must have the same length as bk. All or none of bk, bl, and bu must appear.
    - \* name: Variable names (list(string)).
    - \* bk: Bound keys (list(string)).
    - \* bl: Lower bounds (list(double)).
    - \* bu: Upper bounds (list(double)).
    - \* type: Variable types (list(string)).
  - con: Information about linear constraints. All fields present must have the same length as
     bk. All or none of bk, bl, and bu must appear.
    - \* name: Constraint names (list(string)).
    - \* bk: Bound keys (list(string)).
    - \* bl: Lower bounds (list(double)).
    - \* bu: Upper bounds (list(double)).
  - barvar: Information about symmetric matrix variables. All fields present must have the same length as dim.
    - \* name: Barvar names (list(string)).
    - \* dim: Dimensions (list(int32)).
  - objective: Information about the objective.
    - \* name: Objective name (string).
    - \* sense: Objective sense (string).
    - \* c: The linear part c of the objective as a sparse vector. Both arrays must have the same length.
      - · subj: indices of nonzeros (list(int32)).
      - · val: values of nonzeros (list(double)).
    - \* cfix: Constant term in the objective (double).
    - \* Q: The quadratic part  $Q^o$  of the objective as a sparse matrix, only lower-triangular part included. All arrays must have the same length.
      - · subi: row indices of nonzeros (list(int32)).
      - · subj: column indices of nonzeros (list(int32)).
      - · val: values of nonzeros (list(double)).
    - \* barc: The semidefinite part  $\overline{C}$  of the objective (list). Each element of the list is a list describing one entry  $\overline{C}_j$  using three fields:
      - · index j (int32).
      - · weights of the matrices from the storage E forming  $C_j$  (list(double)).
      - · indices of the matrices from the storage E forming  $\overline{C}_i$  (list(int64)).
  - A: The linear constraint matrix A as a sparse matrix. All arrays must have the same length.
    - \* subi: row indices of nonzeros (list(int32)).
    - \* subj: column indices of nonzeros (list(int32)).
    - \* val: values of nonzeros (list(double)).
  - bara: The semidefinite part  $\overline{A}$  of the constraints (list). Each element of the list is a list describing one entry  $\overline{A}_{ij}$  using four fields:
    - \* index i (int32).
    - \* index j (int32).
    - \* weights of the matrices from the storage E forming  $\overline{A}_{ij}$  (list(double)).
    - \* indices of the matrices from the storage E forming  $\overline{A}_{ij}$  (list(int64)).
  - AFE: The affine expression storage.
    - \* numafe: number of rows in the storage (int64).
    - \* F: The matrix F as a sparse matrix. All arrays must have the same length.

- · subi: row indices of nonzeros (list(int64)).
- · subj: column indices of nonzeros (list(int32)).
- · val: values of nonzeros (list(double)).
- $\ast$  g: The vector g of constant terms as a sparse vector. Both arrays must have the same length.
  - · subi: indices of nonzeros (list(int64)).
  - · val: values of nonzeros (list(double)).
- \* barf: The semidefinite part  $\overline{F}$  of the expressions in AFE storage (list). Each element of the list is a list describing one entry  $\overline{F}_{ij}$  using four fields:
  - · index i (int64).
  - · index j (int32).
  - · weights of the matrices from the storage E forming  $\overline{F}_{ij}$  (list(double)).
  - · indices of the matrices from the storage E forming  $\overline{F}_{ij}$  (list(int64)).
- domains: Information about domains. All fields present must have the same length as type.
  - \* name: Domain names (list(string)).
  - \* type: Description of the type of each domain (list). Each element of the list is a list describing one domain using at least one field:
    - · domain type (string).
    - · (except pexp, dexp) dimension (int64).
    - · (only ppow, dpow) weights (list(double)).
- ACC: Information about affine conic constraints (ACC). All fields present must have the same length as domain.
  - \* name: ACC names (list(string)).
  - \* domain: Domains (list(int64)).
  - \* afeidx: AFE indices, grouped by ACC (list(list(int64))).
  - \* b: constant vectors b, grouped by ACC (list(list(double))).
- DJC: Information about disjunctive constraints (DJC). All fields present must have the same length as termsize.
  - \* name: DJC names (list(string)).
  - \* termsize: Term sizes, grouped by DJC (list(list(int64))).
  - \* domain: Domains, grouped by DJC (list(list(int64))).
  - \* afeidx: AFE indices, grouped by DJC (list(list(int64))).
  - \* b: constant vectors b, grouped by DJC (list(list(double))).
- MatrixStore: The symmetric matrix storage E (list). Each element of the list is a list describing one entry E using four fields in sparse matrix format, lower-triangular part only:
  - \* dimension (int32).
  - \* row indices of nonzeros (list(int32)).
  - \* column indices of nonzeros (list(int32)).
  - \* values of nonzeros (list(double)).
- Q: The quadratic part  $Q^c$  of the constraints (list). Each element of the list is a list describing one entry  $Q_i^c$  using four fields in sparse matrix format, lower-triangular part only:
  - \* the row index i (int32).
  - \* row indices of nonzeros (list(int32)).
  - \* column indices of nonzeros (list(int32)).
  - \* values of nonzeros (list(double)).
- qcone (deprecated). The description of cones. All fields present must have the same length as type.
  - \* name: Cone names (list(string)).
  - \* type: Cone types (list(string)).
  - \* par: Additional cone parameters (list(double)).

- \* members: Members, grouped by cone (list(list(int32))).
- Task/solutions: Solutions. This section can contain up to three subsections called:
  - interior
  - basic
  - integer

corresponding to the three solution types in MOSEK. Each of these sections has the same structure:

- prosta: problem status (string).
- solsta: solution status (string).
- xx, xc, y, slc, suc, slx, sux, snx: one for each component of the solution of the same name (list(double)).
- skx, skc, skn: status keys (list(string)).
- doty: the dual  $\dot{y}$  solution, grouped by ACC (list(list(double))).
- barx, bars: the primal/dual semidefinite solution, grouped by matrix variable
   (list(list(double))).
- Task/parameters: Parameters.
  - iparam: Integer parameters (dictionary). A dictionary with entries of the form name:value, where name is a shortened parameter name (without leading MSK\_IPAR\_) and value is either an integer or string if the parameter takes values from an enum.
  - dparam: Double parameters (dictionary). A dictionary with entries of the form name:value, where name is a shortened parameter name (without leading MSK\_DPAR\_) and value is a double.
  - sparam: String parameters (dictionary). A dictionary with entries of the form name:value, where name is a shortened parameter name (without leading MSK\_SPAR\_) and value is a string. Note that this section is allowed but MOSEK ignores it both when writing and reading JTASK files.

## 12.7.2 JSOL Specification

The JSOL is a dictionary containing the following sections. All sections are optional and can be omitted if irrelevant for the problem.

- \$schema: JSON schema.
- Task/name: The name of the task (string).
- Task/solutions: Solutions. This section can contain up to three subsections called:
  - interior
  - basic
  - integer

corresponding to the three solution types in MOSEK. Each of these section has the same structure:

- prosta: problem status (string).
- solsta: solution status (string).
- xx, xc, y, slc, suc, slx, sux, snx: one for each component of the solution of the same name (list(double)).
- skx, skc, skn: status keys (list(string)).
- doty: the dual  $\dot{y}$  solution, grouped by ACC (list(list(double))).
- barx, bars: the primal/dual semidefinite solution, grouped by matrix variable (list(list(double))).

- Task/information: Information items from the optimizer.
  - int32: int32 information items (dictionary). A dictionary with entries of the form name: value.
  - int64: int64 information items (dictionary). A dictionary with entries of the form name:
     value.
  - double: double information items (dictionary). A dictionary with entries of the form name:
     value.

# 12.7.3 A jtask example

Listing 12.5: A formatted jtask file for a simple portfolio optimization problem.

```
"$schema": "http://mosek.com/json/schema#",
  "Task/name": "Markowitz portfolio with market impact",
  "Task/INFO":{"numvar":7, "numcon":1, "numcone":0, "numbarvar":0, "numanz":6, "numsymmat
-":0, "numafe":13, "numfnz":12, "numacc":4, "numdjc":0, "numdom":3, "mosekver":[10,0,0,3]},
  "Task/data":{
      "var":{
         "name":["1.0","x[0]","x[1]","x[2]","t[0]","t[1]","t[2]"],
         "bk":["fx","lo","lo","lo","fr","fr","fr"],
         "bl": [1,0.0,0.0,0.0,-1e+30,-1e+30,-1e+30],
         "bu": [1,1e+30,1e+30,1e+30,1e+30,1e+30],
         "type":["cont","cont","cont","cont","cont","cont"]
     },
      "con":{
         "name":["budget[]"],
         "bk":["fx"],
         "bl":[1],
         "bu":[1]
     },
     "objective":{
         "sense": "max".
         "name": "obj",
         "c":{
            "subj": [1,2,3],
            "val": [0.1073,0.0737,0.0627]
         },
         "cfix":0.0
     },
     "A":{
         "subi":[0,0,0,0,0,0],
         "subj":[1,2,3,4,5,6],
         "val": [1,1,1,0.01,0.01,0.01]
     },
      "AFE":{
         "numafe":13,
         "F":{
            "subi":[1,1,1,2,2,3,4,6,7,9,10,12],
            "subj": [1,2,3,2,3,3,4,1,5,2,6,3],
            "val":[0.166673333200005,0.0232190712557243,0.0012599496030238,0.
\hookrightarrow 102863378954911, -0.00222873156550421, 0.0338148677744977, 1, 1, 1, 1, 1, 1]
         },
         "g":{
            "subi": [0,5,8,11],
```

```
"val": [0.035,1,1,1]
         }
      },
      "domains": {
         "type":[["r",0],
                  ["quad",4],
                  ["ppow",3,[0.66666666666666666,0.3333333333333333]]]]
      },
      "ACC":{
         "name":["risk[]","tz[0]","tz[1]","tz[2]"],
         "domain":[1,2,2,2],
         "afeidx":[[0,1,2,3],
                    [4,5,6],
                    [7,8,9],
                    [10,11,12]
      }
   },
   "Task/solutions":{
      "interior":{
         "prosta": "unknown",
         "solsta": "unknown",
         "skx":["fix", "supbas", "supbas", "supbas", "supbas", "supbas"],
         "skc":["fix"],
         "xx":[1,0.10331580274282556,0.11673185566457132,0.7724326587076371,0.
\hookrightarrow 033208600335718846,0.03988270849469869,0.6788769587942524],
         "xc":[1],
         "slx":[0.0,-5.585840467641202e-10,-8.945844685006369e-10,-7.815248786428623e-
\hookrightarrow11,0.0,0.0,0.0],
         "sux":[0.0,0.0,0.0,0.0,0.0,0.0,0.0],
         "snx":[0.0,0.0,0.0,0.0,0.0,0.0,0.0],
         "slc":[0.0],
         "suc":[-0.046725814048521205],
         "y":[0.046725814048521205],
         "doty":[[-0.6062603164682975,0.3620818321879349,0.17817754087278295,0.
→4524390346223723].
                  [-4.6725842015519993e-4,-7.708781121860897e-6,2.24800624747081e-4],
                  [-4.6725842015519993e-4, -9.268264309496919e-6, 2.390390600079771e-4],
                  [-4.6725842015519993e-4,-1.5854982159992136e-4,6.159249331148646e-4]]
      }
   },
   "Task/parameters":{
      "iparam":{
         "LICENSE_DEBUG": "ON",
         "MIO_SEED":422
      },
      "dparam":{
         "MIO_MAX_TIME":100
      },
      "sparam":{
   }
}
```

# 12.8 The Solution File Format

**MOSEK** can output solutions to a text file:

- basis solution file (extension .bas) if the problem is optimized using the simplex optimizer or basis identification is performed,
- interior solution file (extension .sol) if a problem is optimized using the interior-point optimizer and no basis identification is required,
- integer solution file (extension .int) if the problem is solved with the mixed-integer optimizer.

All solution files have the format:

```
NAME
                     : <problem name>
PROBLEM STATUS
                     : <status of the problem>
SOLUTION STATUS
                     : <status of the solution>
OBJECTIVE NAME
                     : <name of the objective function>
PRIMAL OBJECTIVE
                     : <primal objective value corresponding to the solution>
DUAL OBJECTIVE
                     : <dual objective value corresponding to the solution>
CONSTRAINTS
INDEX NAME
                AT ACTIVITY
                                LOWER LIMIT
                                               UPPER LIMIT
                                                             DUAL LOWER
                                                                           DUAL UPPER
                ?? <a value>
       <name>
                                <a value>
                                               <a value>
                                                              <a value>
                                                                           <a value>
AFFINE CONIC CONSTRAINTS
INDEX NAME
                Ι
                            ACTIVITY
                                        DUAT.
       <name>
                <a value>
                            <a value>
                                        <a value>
VARIABLES
INDEX NAME
                AT ACTIVITY
                                LOWER LIMIT
                                               UPPER LIMIT
                                                             DUAL LOWER
                                                                           DUAL UPPER
→ [CONIC DUAL]
       <name>
                ?? <a value>
                                <a value>
                                               <a value>
                                                              <a value>
                                                                           <a value>
→[<a value>]
SYMMETRIC MATRIX VARIABLES
                                                          DUAT.
INDEX NAME
                                            PR.TMAT.
                Τ
                             J.
       <name>
                 <a value>
                             <a value>
                                            <a value>
                                                          <a value>
```

The fields ?, ?? and <> will be filled with problem and solution specific information as described below. The solution contains sections corresponding to parts of the input. Empty sections may be omitted and fields in [] are optional, depending on what type of problem is solved. The notation below follows the MOSEK naming convention for parts of the solution as defined in the problem specifications in Sec. 8.

• HEADER In this section, first the name of the problem is listed and afterwards the problem and solution status are shown. Next the primal and dual objective values are displayed.

### • CONSTRAINTS

- INDEX: A sequential index assigned to the constraint by MOSEK
- NAME: The name of the constraint assigned by the user or autogenerated.
- AT: The status key bkc of the constraint as in Table 12.4.
- ACTIVITY: the activity xc of the constraint expression.
- LOWER LIMIT: the lower bound blc of the constraint.
- UPPER LIMIT: the upper bound buc of the constraint.
- DUAL LOWER: the dual multiplier slc corresponding to the lower limit on the constraint.
- DUAL UPPER: the dual multiplier suc corresponding to the upper limit on the constraint.

### • AFFINE CONIC CONSTRAINTS

- INDEX: A sequential index assigned to the affine expressions by MOSEK
- NAME: The name of the affine conic constraint assigned by the user or autogenerated.
- I: The sequential index of the affine expression in the affine conic constraint.
- ACTIVITY: the activity of the I-th affine expression in the affine conic constraint.
- DUAL: the dual multiplier doty for the I-th entry in the affine conic constraint.

#### VARIABLES

- INDEX: A sequential index assigned to the variable by MOSEK
- NAME: The name of the variable assigned by the user or autogenerated.
- AT: The status key bkx of the variable as in Table 12.4.
- ACTIVITY: the value xx of the variable.
- LOWER LIMIT: the lower bound blx of the variable.
- UPPER LIMIT: the upper bound bux of the variable.
- DUAL LOWER: the dual multiplier slx corresponding to the lower limit on the variable.
- DUAL UPPER: the dual multiplier sux corresponding to the upper limit on the variable.
- CONIC DUAL: the dual multiplier skx corresponding to a conic variable (deprecated).

#### • SYMMETRIC MATRIX VARIABLES

- INDEX: A sequential index assigned to each symmetric matrix entry by MOSEK
- NAME: The name of the symmetric matrix variable assigned by the user or autogenerated.
- I: The row index in the symmetric matrix variable.
- J: The column index in the symmetric matrix variable.
- PRIMAL: the value of barx for the (I, J)-th entry in the symmetric matrix variable.
- DUAL: the dual multiplier bars for the (I, J)-th entry in the symmetric matrix variable.

Table 12.4: Status keys.

Status key	Interpretation
UN	Unknown status
BS	Is basic
SB	Is superbasic
LL	Is at the lower limit (bound)
UL	Is at the upper limit (bound)
EQ	Lower limit is identical to upper limit
**	Is infeasible i.e. the lower limit is greater than the upper limit.

## Example.

Below is an example of a solution file.

Listing 12.6: An example of .sol file.

NAME :

PROBLEM STATUS : PRIMAL\_AND\_DUAL\_FEASIBLE

SOLUTION STATUS : OPTIMAL OBJECTIVE NAME : OBJ

PRIMAL OBJECTIVE : 0.70571049347734 DUAL OBJECTIVE : 0.70571048919757

CONSTRAINTS

INDEX NAME AT ACTIVITY LOWER LIMIT UPPER LIMIT 

→ DUAL LOWER

DUAL UPPER

DUAL UPPER

(continued from previous page)

					(COIII)	mued from previou	s page)	
	CONIC CONSTRAIN							
INDEX	NAME	Ι	ACTIVITY			DUAL		
0	A1	0	1.000000009656			0.54475821296644		
1	A1	1	0.5000000152223		0.32190455246225			
2	A2	0	0.25439922724695			0.4552417870329		
3	A2	1	0.17988741850378			-0.32190455246178		
4	A2	2	0.17988741850378		-0.32190455246178			
VARIAB	T EC							
TNDEX	NAME	ΛТ ΛС	TIVITY		LOWER LIMIT	UPPER LIN	итт	
TNDEX	DUAL LOWER	AI AC	DUAL UPPER.		TOWLIL TILITI	OI LEIK PH	U	
0	X1	SB 0 1	254399227246		NONE	NONE		
	0	DD 0.2	0		NOIVL	IVOIVL		
1	X2	SR 0	[79887418503	78	NONE	NONE		
	0	DD 0	0		1101111	1101111		
2	Х3	SB 0.	179887418503	78	NONE	NONE	ш	
∠ ⇔	0	25 0	0	. •		1,0111		
	-		-					
SYMMET	RIC MATRIX VARIA	BLES						
INDEX	NAME	I	J	PRIMAL		DUAL		
0	BARX1	0	0	0.21725	0.21725733689874		1.1333372337141	
1	BARX1	1	0	-0.2599	-0.25997257078534 0			
<b>→</b> 6780	9544651396							
2	BARX1	2	0	0.21725	733648507	-0.		
<b>→</b> 3219	045527104							
3	BARX1	1	1	0.31108	610088839	1.133337233	32693	
4	BARX1	2	1	-0.2599	7257078534	0.		
<b>→</b> 6780	9544651435							
5	BARX1	2	2	0.21725	733689874	1.133337233	37145	
6	BARX2	0	0	0 4.8362272828127e-10		0.		
<b>→</b> 5447	5821339698							
7	BARX2	1	0	0		0		
8	BARX2	1	1	4.83622	72828127e-10	0.		
<b>→</b> 5447	5821339698							

## Chapter 13

# List of examples

List of examples shipped in the distribution of Command Line Tools:

Table 13.1: List of distributed examples

File	Description
25fv47.mps	A large linear problem from the Netlib library
ampl1.res	Interfacing MOSEK from AMPL
ampl2.res	Interfacing MOSEK from AMPL
ampl3.res	Interfacing MOSEK from AMPL
cqo1.jtask	A simple conic quadratic problem
cqo1.mps	A simple conic quadratic problem
cqo1.ptf	A simple conic quadratic problem
diet.dat	Data for the diet example diet.mod
diet.mod	A diet balancing AMPL example
dinfeas.lp	A simple dual infeasible linear problem
djc1.jtask	A simple problem with disjunctive constraints (DJC)
djc1.ptf	A simple problem with disjunctive constraints (DJC)
feasrepair.lp	An example demonstrating repair of infeasible problems
infeas.lp	A simple primal infeasible problem
lo1.jtask	A simple linear problem
lo1.lp	A simple linear problem
lo1.mps	A simple linear problem
lo1.ptf	A simple linear problem
milo1.jtask	A simple mixed-integer linear problem
milo1.lp	A simple mixed-integer linear problem
milo1.ptf	A simple mixed-integer linear problem
qo1.jtask	A simple quadratic problem
qo1.mps	A simple quadratic problem
qo1.opf	A simple quadratic problem
sdo1.cbf	A simple semidefinite problem with one matrix variable and a quadratic cone
sdo1.jtask	A simple semidefinite problem with one matrix variable and a quadratic cone
sdo1.ptf	A simple semidefinite problem with one matrix variable and a quadratic cone
sensitivity.	Sensitivity analysis specification for transport.lp
ssp	
transport.lp	A linear problem in the sensitivity analysis example

Additional examples can be found on the  $\mathbf{MOSEK}$  website and in other  $\mathbf{MOSEK}$  publications.

## Chapter 14

## Interface changes

The section shows interface-specific changes to the **MOSEK** Command Line Tools in version 10.0 compared to version 9. See the release notes for general changes and new features of the **MOSEK** Optimization Suite.

### 14.1 Important changes compared to version 9

- Parameters. Users who set parameters to tune the performance and numerical properties of the solver (termination criteria, tolerances, solving primal or dual, presolve etc.) are recommended to reevaluate such tuning. It may be that other, or default, parameter settings will be more beneficial in the current version. The hints in Sec. 7 may be useful for some cases.
- Multithreading. In the interior-point optimizer it is possible to set the number of threads with <code>MSK\_IPAR\_NUM\_THREADS</code> before each optimization, and not just once per process. The parameter <code>MSK\_IPAR\_INTPNT\_MULTI\_THREAD</code> is no longer relevant and was removed.

## 14.2 Changes compared to version 9

## 14.3 Parameters compared to version 9

#### Added

- MSK\_DPAR\_MIO\_DJC\_MAX\_BIGM
- MSK\_DPAR\_PRESOLVE\_TOL\_PRIMAL\_INFEAS\_PERTURBATION
- MSK\_IPAR\_MIO\_CONSTRUCT\_SOL
- MSK\_IPAR\_MIO\_CUT\_LIPRO
- MSK\_IPAR\_MIO\_DATA\_PERMUTATION\_METHOD
- MSK\_IPAR\_MIO\_MEMORY\_EMPHASIS\_LEVEL
- MSK\_IPAR\_MIO\_NUMERICAL\_EMPHASIS\_LEVEL
- MSK\_IPAR\_MIO\_PRESOLVE\_AGGREGATOR\_USE
- MSK\_IPAR\_MIO\_QCQO\_REFORMULATION\_METHOD
- MSK\_IPAR\_MIO\_SYMMETRY\_LEVEL
- MSK\_IPAR\_NG
- MSK\_IPAR\_PTF\_WRITE\_PARAMETERS
- MSK\_IPAR\_PTF\_WRITE\_SOLUTIONS

- MSK\_IPAR\_REMOTE\_USE\_COMPRESSION
- MSK\_IPAR\_SIM\_DETECT\_PWL
- MSK\_IPAR\_WRITE\_JSON\_INDENTATION
- MSK\_SPAR\_REMOTE\_OPTSERVER\_HOST
- MSK\_SPAR\_REMOTE\_TLS\_CERT
- MSK\_SPAR\_REMOTE\_TLS\_CERT\_PATH

#### Removed

- MSK\_IPAR\_INTPNT\_MULTI\_THREAD
- MSK\_IPAR\_READ\_LP\_DROP\_NEW\_VARS\_IN\_BOU
- MSK\_IPAR\_READ\_LP\_QUOTED\_NAMES
- MSK\_IPAR\_WRITE\_LP\_QUOTED\_NAMES
- MSK\_IPAR\_WRITE\_LP\_STRICT\_FORMAT
- MSK\_IPAR\_WRITE\_LP\_TERMS\_PER\_LINE
- MSK\_IPAR\_WRITE\_PRECISION
- MSK\_SPAR\_REMOTE\_ACCESS\_TOKEN
- MSK\_SPAR\_STAT\_FILE\_NAME

### 14.4 Constants compared to version 9

#### **Added**

- MSK\_CALLBACK\_BEGIN\_SOLVE\_ROOT\_RELAX
- MSK\_CALLBACK\_END\_SOLVE\_ROOT\_RELAX
- MSK\_CALLBACK\_UPDATE\_SIMPLEX
- MSK\_DINF\_ANA\_PRO\_SCALARIZED\_CONSTRAINT\_MATRIX\_DENSITY
- MSK\_DINF\_MIO\_INITIAL\_FEASIBLE\_SOLUTION\_OBJ
- MSK\_DINF\_MIO\_LIPRO\_SEPARATION\_TIME
- MSK\_DINF\_MIO\_ROOT\_TIME
- MSK\_DINF\_PRESOLVE\_TOTAL\_PRIMAL\_PERTURBATION
- MSK\_DINF\_READ\_DATA\_TIME
- MSK\_DINF\_REMOTE\_TIME
- MSK\_DINF\_SOL\_ITG\_PVIOLACC
- MSK\_DINF\_SOL\_ITG\_PVIOLDJC
- MSK\_DINF\_SOL\_ITR\_DVIOLACC
- MSK\_DINF\_SOL\_ITR\_PVIOLACC
- MSK\_DINF\_WRITE\_DATA\_TIME

- MSK\_IINF\_MIO\_INITIAL\_FEASIBLE\_SOLUTION
- MSK\_IINF\_MIO\_NUM\_LIPRO\_CUTS
- MSK\_IINF\_MIO\_NUMDJC
- MSK\_IINF\_MIO\_PRESOLVED\_NUMDJC
- MSK\_IINF\_PRESOLVE\_NUM\_PRIMAL\_PERTURBATIONS
- MSK\_LIINF\_ANA\_PRO\_SCALARIZED\_CONSTRAINT\_MATRIX\_NUM\_COLUMNS
- MSK\_LIINF\_ANA\_PRO\_SCALARIZED\_CONSTRAINT\_MATRIX\_NUM\_NZ
- MSK\_LIINF\_ANA\_PRO\_SCALARIZED\_CONSTRAINT\_MATRIX\_NUM\_ROWS
- MSK\_LIINF\_MIO\_NUM\_DUAL\_ILLPOSED\_CER
- MSK\_LIINF\_MIO\_NUM\_PRIM\_ILLPOSED\_CER
- MSK\_LIINF\_RD\_NUMACC
- MSK\_LIINF\_RD\_NUMDJC
- MSK\_LIINF\_SIMPLEX\_ITER

#### Removed

- MSK\_CALLBACKCODE\_BEGIN\_FULL\_CONVEXITY\_CHECK
- MSK\_CALLBACKCODE\_END\_FULL\_CONVEXITY\_CHECK
- MSK\_CALLBACKCODE\_IM\_FULL\_CONVEXITY\_CHECK
- MSK\_DINFITEM\_RD\_TIME
- MSK\_SCALINGTYPE\_AGGRESSIVE
- MSK\_SCALINGTYPE\_MODERATE

### 14.5 Response Codes compared to version 9

#### Added

- MSK\_RES\_ERR\_ACC\_AFE\_DOMAIN\_MISMATCH
- MSK\_RES\_ERR\_ACC\_INVALID\_ENTRY\_INDEX
- MSK\_RES\_ERR\_ACC\_INVALID\_INDEX
- MSK\_RES\_ERR\_AFE\_INVALID\_INDEX
- MSK\_RES\_ERR\_ARGUMENT\_IS\_TOO\_SMALL
- MSK\_RES\_ERR\_AXIS\_NAME\_SPECIFICATION
- MSK\_RES\_ERR\_CBF\_DUPLICATE\_PSDCON
- MSK\_RES\_ERR\_CBF\_INVALID\_DIMENSION\_OF\_PSDCON
- MSK\_RES\_ERR\_CBF\_INVALID\_NUM\_PSDCON
- MSK\_RES\_ERR\_CBF\_INVALID\_PSDCON\_BLOCK\_INDEX
- MSK\_RES\_ERR\_CBF\_INVALID\_PSDCON\_INDEX

- MSK\_RES\_ERR\_CBF\_INVALID\_PSDCON\_VARIABLE\_INDEX
- MSK\_RES\_ERR\_CBF\_UNSUPPORTED\_CHANGE
- MSK\_RES\_ERR\_DIMENSION\_SPECIFICATION
- MSK\_RES\_ERR\_DJC\_AFE\_DOMAIN\_MISMATCH
- MSK\_RES\_ERR\_DJC\_DOMAIN\_TERMSIZE\_MISMATCH
- MSK\_RES\_ERR\_DJC\_INVALID\_INDEX
- MSK\_RES\_ERR\_DJC\_INVALID\_TERM\_SIZE
- MSK\_RES\_ERR\_DJC\_TOTAL\_NUM\_TERMS\_MISMATCH
- MSK\_RES\_ERR\_DJC\_UNSUPPORTED\_DOMAIN\_TYPE
- MSK\_RES\_ERR\_DOMAIN\_DIMENSION
- MSK\_RES\_ERR\_DOMAIN\_DIMENSION\_PSD
- MSK\_RES\_ERR\_DOMAIN\_INVALID\_INDEX
- MSK\_RES\_ERR\_DOMAIN\_POWER\_INVALID\_ALPHA
- MSK\_RES\_ERR\_DOMAIN\_POWER\_NEGATIVE\_ALPHA
- MSK\_RES\_ERR\_DOMAIN\_POWER\_NLEFT
- MSK\_RES\_ERR\_DUPLICATE\_DJC\_NAMES
- MSK\_RES\_ERR\_DUPLICATE\_DOMAIN\_NAMES
- MSK\_RES\_ERR\_DUPLICATE\_FIJ
- $\bullet \ \mathit{MSK\_RES\_ERR\_HUGE\_FIJ}$
- MSK\_RES\_ERR\_INDEX\_IS\_NOT\_UNIQUE
- MSK\_RES\_ERR\_INF\_IN\_DOUBLE\_DATA
- $\bullet \ \textit{MSK\_RES\_ERR\_INVALID\_B}$
- MSK\_RES\_ERR\_INVALID\_CFIX
- MSK\_RES\_ERR\_INVALID\_FIJ
- MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_AFFINE\_CONIC\_CONSTRAINTS
- MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_DISJUNCTIVE\_CONSTRAINTS
- MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_QUADRATIC\_TERMS
- MSK\_RES\_ERR\_INVALID\_G
- MSK\_RES\_ERR\_LICENSE\_OLD\_SERVER\_VERSION
- MSK\_RES\_ERR\_LP\_INDICATOR\_VAR
- MSK\_RES\_ERR\_MISMATCHING\_DIMENSION
- MSK\_RES\_ERR\_MPS\_INVALID\_BOUND\_KEY
- MSK\_RES\_ERR\_MPS\_INVALID\_CON\_KEY
- MSK\_RES\_ERR\_MPS\_INVALID\_INDICATOR\_CONSTRAINT
- MSK\_RES\_ERR\_MPS\_INVALID\_INDICATOR\_QUADRATIC\_CONSTRAINT
- MSK\_RES\_ERR\_MPS\_INVALID\_INDICATOR\_VALUE

- MSK\_RES\_ERR\_MPS\_INVALID\_INDICATOR\_VARIABLE
- MSK\_RES\_ERR\_MPS\_INVALID\_KEY
- MSK\_RES\_ERR\_MPS\_INVALID\_SEC\_NAME
- MSK\_RES\_ERR\_MPS\_WRITE\_CPLEX\_INVALID\_CONE\_TYPE
- MSK\_RES\_ERR\_NO\_DOTY
- MSK\_RES\_ERR\_NOT\_POWER\_DOMAIN
- MSK\_RES\_ERR\_OPF\_DUAL\_INTEGER\_SOLUTION
- MSK\_RES\_ERR\_OPF\_DUPLICATE\_BOUND
- MSK\_RES\_ERR\_OPF\_DUPLICATE\_CONE\_ENTRY
- MSK\_RES\_ERR\_OPF\_DUPLICATE\_CONSTRAINT\_NAME
- MSK\_RES\_ERR\_OPF\_INCORRECT\_TAG\_PARAM
- MSK\_RES\_ERR\_OPF\_INVALID\_CONE\_TYPE
- MSK\_RES\_ERR\_OPF\_INVALID\_TAG
- MSK\_RES\_ERR\_OPF\_MISMATCHED\_TAG
- MSK\_RES\_ERR\_OPF\_SYNTAX
- MSK\_RES\_ERR\_OPF\_TOO\_LARGE
- MSK\_RES\_ERR\_PTF\_INCOMPATIBILITY
- MSK\_RES\_ERR\_PTF\_INCONSISTENCY
- MSK\_RES\_ERR\_PTF\_UNDEFINED\_ITEM
- MSK\_RES\_ERR\_SERVER\_ACCESS\_TOKEN
- MSK\_RES\_ERR\_SERVER\_ADDRESS
- MSK\_RES\_ERR\_SERVER\_CERTIFICATE
- MSK\_RES\_ERR\_SERVER\_TLS\_CLIENT
- MSK\_RES\_ERR\_SPARSITY\_SPECIFICATION
- MSK\_RES\_ERR\_UNALLOWED\_WHICHSOL
- MSK\_RES\_TRM\_LOST\_RACE
- MSK\_RES\_WRN\_INVALID\_MPS\_NAME
- MSK\_RES\_WRN\_INVALID\_MPS\_OBJ\_NAME
- MSK\_RES\_WRN\_LARGE\_FIJ
- MSK\_RES\_WRN\_MODIFIED\_DOUBLE\_PARAMETER
- MSK\_RES\_WRN\_NO\_INFEASIBILITY\_REPORT\_WHEN\_MATRIX\_VARIABLES
- MSK\_RES\_WRN\_PRESOLVE\_PRIMAL\_PERTUBATIONS
- MSK\_RES\_WRN\_WRITE\_LP\_DUPLICATE\_CON\_NAMES
- MSK\_RES\_WRN\_WRITE\_LP\_DUPLICATE\_VAR\_NAMES
- MSK\_RES\_WRN\_WRITE\_LP\_INVALID\_CON\_NAMES
- MSK\_RES\_WRN\_WRITE\_LP\_INVALID\_VAR\_NAMES

### Removed

- MSK\_RES\_ERR\_LP\_FORMAT
- MSK\_RES\_ERR\_MPS\_INV\_BOUND\_KEY
- MSK\_RES\_ERR\_MPS\_INV\_CON\_KEY
- MSK\_RES\_ERR\_MPS\_INV\_SEC\_NAME
- MSK\_RES\_ERR\_OPF\_FORMAT
- MSK\_RES\_ERR\_OPF\_NEW\_VARIABLE
- MSK\_RES\_WRN\_EXP\_CONES\_WITH\_VARIABLES\_FIXED\_AT\_ZERO
- MSK\_RES\_WRN\_POW\_CONES\_WITH\_ROOT\_FIXED\_AT\_ZERO
- MSK\_RES\_WRN\_QUAD\_CONES\_WITH\_ROOT\_FIXED\_AT\_ZERO
- MSK\_RES\_WRN\_RQUAD\_CONES\_WITH\_ROOT\_FIXED\_AT\_ZERO

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