ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

# Chapter 4: Interest Rate Derivatives

## 4.3 Swaptions

**Interest Rate Models**

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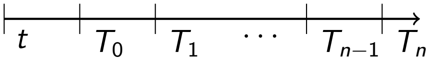
# 4.3 Swaptions

- Payer and receiver swaptions
- Moneyness
- Callable bonds
- Black's price formula
- Bachelier's price formula
- Swaption quotes in terms of implied volatilities

# Swaptions

A payer (receiver) swaption with strike rate $K$ gives the holder right to enter a payer (receiver) swap with fixed rate $K$ at swaption expiry date.

Usually the swaption expiry date equals first reset date $T_0$ of underlying swap.



# Swaption Payoff

The payer swaption payoff at expiry date $T_0$ is

$$V_p(T_0)^+ = N\left(\sum_{i=1}^{n} P(T_0, T_i)\delta(F(T_0, T_{i-1}, T_i) - K)\right)^+$$

This payoff cannot be decomposed into more elementary payoffs.

The dependence between different forward rates will enter valuation procedure.

# Moneyness

An equivalent expression for the payer swaption payoff at expiry date $T_0$ is

$$N \sum_{i=1}^{n} P(T_0, T_i)\delta \left(R_{swap}(T_0) - K\right)^{+} .$$

A payer (receiver) swaption is said to be

- at-the-money (ATM) if $K = R_{swap}(t)$
- in-the-money (ITM) if $K < R_{swap}(t)$ (receiver: $K > R_{swap}(t)$)
- out-of-the-money (OTM) if $K > R_{swap}(t)$ (receiver: $K < R_{swap}(t)$)

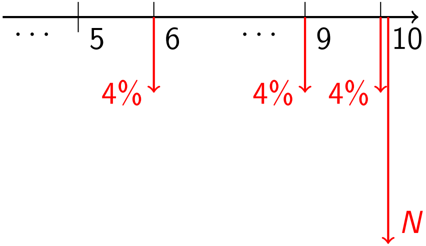
The $x \times y$-swaption has expiry in $x$ years and underlying swap length is $y$ years.

# Application: Callable Bond

Swaptions can be used to synthetically create callable bonds.

Example: company issued 10-year bond with 4% annual coupon and principal $N$.

- It wants to add the right to call (prepay) the bond at par after 5 years.
- But it cannot change original bond.




Time-5 value of bond

$$p(5) = N\left(\sum_{k=6}^{10} P(5,k) \times 4\% + P(5,10)\right)$$

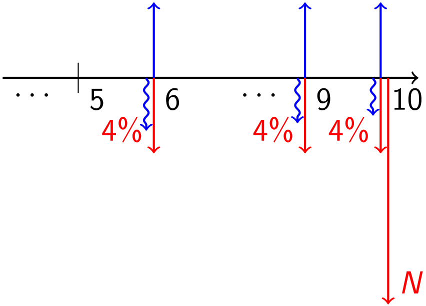
# Application: Callable Bond

Solution: buy a $5 \times 5$ receiver swaption with strike rate 4%:

- The fixed leg of the swap cancels the fixed coupon payments.
- The exchange of notionals between $t = 5$ and $T = 10$ is equivalent to paying floating.

Time-5 value of swaption

$$V_r(5)^+ = (p(5) - N)^+$$



# Black's Formula for Swaptions

Black's formula for payer and receiver swaptions price is

$$Swpt_p(t) = N\delta \sum_{i=1}^{n} P(t, T_i) \left(R_{swap}(t)\Phi(d_1) - K\Phi(d_2)\right)$$

$$Swpt_r(t) = N\delta \sum_{i=1}^{n} P(t, T_i) \left(K\Phi(-d_2) - R_{swap}(t)\Phi(-d_1)\right)$$

where $\Phi$ is the standard normal cumulative distribution function and

$$d_{1,2} = \frac{\log\left[\frac{R_{swap}(t)}{K}\right] \pm \frac{1}{2}\sigma^2(T_0 - t)}{\sigma\sqrt{T_0 - t}}.$$

$\sigma$: Black (or relative) volatility

# Bachelier's Formula for Swaptions

Bachelier's formula for payer and receiver swaptions price is

$$Swpt_p(t) = N\delta \sum_{i=1}^{n} P(t, T_i)\, \sigma \sqrt{T_0 - t}\, (D\Phi(D) + \phi(D))$$

$$Swpt_r(t) = N\delta \sum_{i=1}^{n} P(t, T_i)\, \sigma \sqrt{T_0 - t}\, (-D\Phi(-D) + \phi(-D))$$

where $\Phi$ is the standard normal cumulative distribution function, $\phi = \Phi'$, and

$$D = \frac{R_{swap}(t) - K}{\sigma \sqrt{T_0 - t}}.$$

$\sigma$: normal (basis point, absolute) volatility

# Swaption Quotes

Swaption prices are quoted in terms of Black or normal implied volatilities.

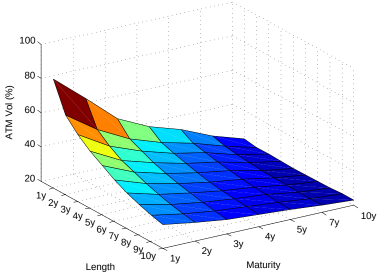
The accrual period $\delta = T_i - T_{i-1}$ for underlying swap can differ from prevailing $\delta$ for caps within the same market region.

E.g. euro zone: caps are written on semiannual LIBOR ($\delta = 1/2$), while swaps pay annual coupons ($\delta = 1$).

# Example of Swaption Quotes - Black ATM volatilities

Black's implied volatilities (in %) of EUR ATM swaptions on August 30, 2013. Maturities are 1,2,3,4,5,7,10 years, swaps lengths from 1 to 10 years:

| Maturities \ Lenghts | 1 YR | 2 YR | 3 YR | 4 YR | 5 YR | 6 YR | 7 YR | 8 YR | 9 YR | 10 YR |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 YR | 83.56 | 66.6 | 59.1 | 53 | 48.82 | 44.02 | 40.42 | 37.61 | 35.34 | 33.53 |
| 2 YR | 67.95 | 53.7 | 47.89 | 43.7 | 40.8 | 37.86 | 35.48 | 33.53 | 31.92 | 30.65 |
| 3 YR | 52.12 | 44.2 | 40.85 | 38.01 | 35.72 | 33.72 | 32.03 | 30.62 | 29.48 | 28.53 |
| 4 YR | 43.26 | 38.03 | 35.61 | 33.59 | 31.86 | 30.52 | 29.38 | 28.51 | 27.78 | 27.16 |
| 5 YR | 37.37 | 33.45 | 31.75 | 30.29 | 29.03 | 28.11 | 27.41 | 26.87 | 26.46 | 26.09 |
| 7 YR | 29.64 | 27.44 | 26.44 | 25.6 | 24.99 | 24.63 | 24.37 | 24.24 | 24.23 | 24.17 |
| 10 YR | 23.76 | 22.86 | 22.58 | 22.39 | 22.27 | 22.39 | 22.52 | 22.69 | 22.9 | 23.01 |

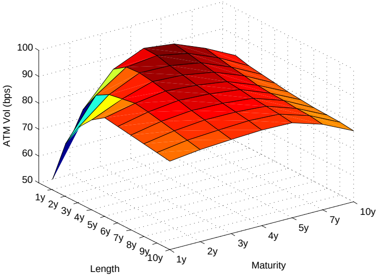



An interest rate model for swaptions valuation must fit given volatility surface.

# Example of Swaption Quotes - Normal ATM volatilities

Normal implied volatilities (in bps) of EUR ATM swaptions on August 30, 2013. Maturities are 1,2,3,4,5,7,10 years, swaps lenghts from 1 to 10 years:

| Maturities \ Lenghts | 1 YR | 2 YR | 3 YR | 4 YR | 5 YR | 6 YR | 7 YR | 8 YR | 9 YR | 10 YR |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 YR | 53.7 | 69.76 | 78.5 | 83.79 | 87.32 | 86.59 | 85.79 | 85.02 | 84.22 | 83.48 |
| 2 YR | 76.91 | 85.01 | 87.9 | 88.9 | 89.8 | 88.84 | 87.83 | 86.85 | 85.86 | 84.9 |
| 3 YR | 89.26 | 92.79 | 92.98 | 92.21 | 91.36 | 90.24 | 89.08 | 87.95 | 86.8 | 85.7 |
| 4 YR | 94.31 | 94.1 | 92.89 | 91.68 | 90.43 | 89.6 | 88.74 | 87.95 | 87.15 | 86.25 |
| 5 YR | 92.94 | 91.83 | 90.82 | 89.81 | 88.79 | 88.23 | 87.64 | 87.15 | 86.64 | 85.95 |
| 7 YR | 88.22 | 87.2 | 86.29 | 85.4 | 84.49 | 84.08 | 83.66 | 83.36 | 83.04 | 82.47 |
| 10 YR | 82.41 | 81.13 | 80.35 | 79.6 | 78.84 | 78.46 | 78.07 | 77.79 | 77.52 | 76.97 |




An interest rate model for swaptions valuation must fit given volatility surface.