ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Chapter 1: Interest Rates and Related Contracts

1.5 Market Conventions

Interest Rate Models

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1.5 Market Conventions

- Day-count conventions
- Accrued interest
- Clean price and dirty price

Measuring Time

By convention we measure the time in units of years.

But what is the time $\delta(t, T)$ in units of years between the calendar dates $t = 4$ January 2000 and $T = 4$ July 2002?

The market evaluates year fraction $\delta(t, T)$ between calendar dates t and T in different ways: day-count conventions

Day-Count Conventions

Here are some of the most popular day-count conventions:

- actual/365: the year has 365 days

$$\delta(t, T) = \frac{\text{actual number of days between } t \text{ and } T}{365}.$$

- actual/360: as above but the year counts 360 days
- 30/360: months count 30 and years 360 days. I.e. for calendar dates $t = d_1/m_1/y_1$ and $T = d_2/m_2/y_2$:

$$\delta(t, T) = \frac{\min(d_2, 30) + (30 - d_1)^+}{360} + \frac{m_2 - m_1 - 1}{12} + y_2 - y_1.$$

Example

The time between calendar dates $t = 4$ January 2000 and $T = 4$ July 2002 amounts to:

- In the 30/360 convention:

$$\delta(t, T) = \frac{4 + (30 - 4)}{360} + \frac{7 - 1 - 1}{12} + 2002 - 2000 = 2.5$$

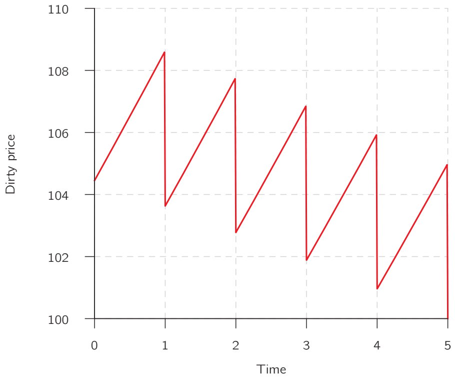
- In the actual/365 convention: $\delta(t, T) = 2.4986$

Dirty Price

Recall the coupon bond ex-dividend price formula

$$p(t) = \sum_{i=1}^{n} P(t, T_i) c_i 1_{\{t<T_i\}} + P(t, T_n) N.$$

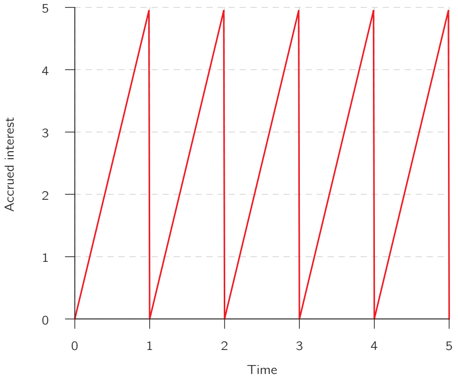
Systematic discontinuities of price trajectory at $t = T_i$



Accrued Interest

The accrued interest on the coupon c_i at $t \in [T_{i-1}, T_i)$ is defined by

$$AI(i;t) = c_i \frac{t - T_{i-1}}{T_i - T_{i-1}}.$$



Clean Price

The clean price (quoted) of coupon bond at $t \in [T_{i-1}, T_i)$ is

$$p_{clean}(t) = p(t) - AI(i; t).$$

The dirty price (to pay) is

$$p(t) = p_{clean}(t) + AI(i; t).$$