ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Chapter 1: Interest Rates and Related Contracts

1.3 Coupon Bonds and Interest Rate Swaps

Interest Rate Models

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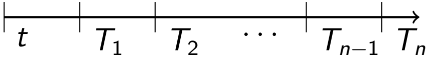
1.3 Coupon Bonds and Interest Rate Swaps

- Coupon bonds
- Floating rate notes
- Interest rate swaps

Fixed Coupon Bonds

A (fixed) coupon bond is specified by

- coupon dates $T_1 < \cdots < T_n$ (T_n=maturity)
- fixed coupons $c_1, \ldots, c_n$
- a principal value N



The ex-dividend price at time $t \leq T_n$ is

$$p(t) = \sum_{i=1}^{n} P(t, T_i) c_i 1_{\{t < T_i\}} + P(t, T_n) N.$$

Floating Rate Notes

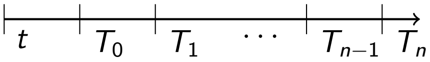
A floating rate note is specified by

- reset/settlement dates $T_0 < \cdots < T_n$ (T_0=first reset date, T_n=maturity)
- a principal value N
- floating coupon payments at $T_1 \ldots T_n$

$$C_i = (T_i - T_{i-1}) L(T_{i-1}, T_i) N$$

The price at time $t \leq T_0$ (replicate cash flows by buying N T_0-bonds) is

$$p(t) = NP(t, T_0).$$



Pricing by Replicating the Cash Flows

At $t \leq T_0$: buy N T_0-bonds

- Initial cost $= NP(t, T_0)$

For $i = 1, \dots, n$:

- at T_{i-1}: invest N in $\frac{N}{P(T_{i-1},T_i)}$ T_i-bonds
- at T_i: receive $\frac{N}{P(T_{i-1},T_i)} = \underbrace{N\left(\frac{1}{P(T_{i-1}, T_i)} - 1\right)}_{=C_i} + N$
    - Pay off C_i
    - Keep and reinvest (or pay off if $i = n$) N

Absence of arbitrage implies: a self-financing strategy yielding the same cash flows as a traded security with price $p(t)$ must have initial (t) cost equal to $p(t)$.

Floating Rate Note in Words

Paying

- the floating coupons C_i at coupon dates $T_1 \ldots T_n$ and
- the principal N at maturity T_n

is equivalent to paying the principal N at reset date T_0.

Interest Rate Swaps

Exchange of fixed and floating coupon payments.

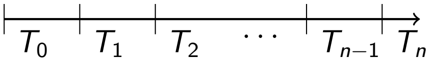
A payer interest rate swap settled in arrears is specified by:

- reset/settlement dates $T_0 < T_1 < \cdots < T_n$ (T_0=first reset date, T_n=maturity, $T_n - T_0$=length)
- a fixed rate K
- a notional N
- for simplicity assume: $T_i - T_{i-1} \equiv \delta$

Swap Cash Flows

At $T_1 \dots T_n$, the holder of contract

- pays fixed $\delta K N$,
- receives floating $\delta L(T_{i-1}, T_i) N$.



Swap Value

The value of a payer interest rate swap at $t \leq T_0$ is

$$V_p(t) = N\left(P(t, T_0) - P(t, T_n) - \delta K \sum_{i=1}^{n} P(t, T_i)\right)$$
$$= N \sum_{i=1}^{n} P(t, T_i)\delta\left(F(t, T_{i-1}, T_i) - K\right).$$

The value of a receiver interest rate swap at $t \leq T_0$ is

$$V_r(t) = -V_p(t).$$

Swap Rate

The fixed rate K which makes

$$V_r(t) = -V_p(t) = 0$$

is the forward (spot) swap rate prevailing at $t < T_0$ ($t = T_0$)

$$R_{swap}(t) = \frac{P(t, T_0) - P(t, T_n)}{\delta \sum_{i=1}^{n} P(t, T_i)}.$$

Exercise: an equivalent alternative expression is

$$R_{swap}(t) = \sum_{i=1}^{n} w_i(t) F(t, T_{i-1}, T_i) \text{ with weights } w_i(t) = \frac{P(t, T_i)}{\sum_{j=1}^{n} P(t, T_j)}.$$

Relation to Coupon Bonds

Lemma 1

A coupon bond has par value at T_0 if and only if its coupon rates equal the corresponding swap rate:

$$1 = \sum_{i=1}^{n} P(T_0, T_i) \underbrace{\delta R_{swap}(T_0)}_{coupon\ c_i} + P(T_0, T_n)$$

Proof.

Exercise. □

Market Quotes for Swap Rates: Example

Lenghts \ Forwards	Swap Rate	1W	1MO	3MO	6MO	1YR	2YR	3YR	4YR	5YR	10YR
1Yr	0.44	0.45	0.47	0.53	0.62	0.81	1.34	1.91	2.40	2.69	3.64
2Yr	0.62	0.63	0.66	0.73	0.84	1.08	1.62	2.15	2.55	2.82	3.64
3Yr	0.86	0.87	0.90	0.98	1.10	1.35	1.88	2.33	2.68	2.93	3.61
4Yr	1.12	1.12	1.16	1.24	1.36	1.60	2.07	2.48	2.79	3.03	3.62
5Yr	1.37	1.37	1.40	1.48	1.59	1.81	2.24	2.61	2.90	3.13	3.63
6Yr	1.58	1.58	1.61	1.68	1.78	1.99	2.38	2.73	3.00	3.20	3.56
7Yr	1.76	1.76	1.79	1.85	1.95	2.14	2.51	2.83	3.08	3.26	3.51
8Yr	1.92	1.92	1.95	2.01	2.10	2.28	2.62	2.92	3.14	3.29	3.48
9Yr	2.06	2.07	2.09	2.15	2.23	2.40	2.72	2.99	3.18	3.32	3.45
10Yr	2.19	2.19	2.22	2.27	2.35	2.51	2.80	3.04	3.22	3.36	3.43

EUR forward swap rates [in %] from 30 Aug 2013. Times to first reset date: spot (first column) to 10 years forward. Swap lengths: from 1 to 10 years. Euro zone swaps pay annual coupons ($\delta = 1$).

Interest Rate Swap Markets

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- Interest rate swap markets are over the counter.
- But swap contracts exist in standardized form, e.g. by the ISDA (International Swaps and Derivatives Association, Inc.).
- Swap markets are extremely liquid, the outstanding notional very large.
- Maturities from 1 to 30 years are standard, swap rate quotes available up to 60 years.
- Gives market participants, such as life insurers, opportunity to create synthetically long-dated investments.