ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Chapter 1: Interest Rates and Related Contracts

1.1 Interest Rates and Discount Bonds

Interest Rate Models

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1.1 Interest Rates and Discount Bonds

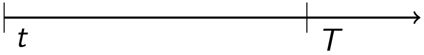
- Various notions of interest rates
- LIBOR, yield, short rate
- Money market account
- Zero-coupon bonds (discount bonds)

Interest

Interest refers to the rent paid by a borrower of money to a lender of money over a period of time.

Cash flows for the lender:

- at t: pay a loan of 1 to the borrower
- at T: receive 1 plus interest R



Annualized Interest Rates

Interest is expressed in annualized interest rates:

- Simple rate $L(t, T)$:

$$1 + R = 1 + (T - t)L(t, T)$$

- Continuously compounded rate (yield) $y(t, T)$:

$$1 + R = \mathrm{e}^{(T-t)y(t,T)}$$

Market Example: LIBOR

LIBOR (London Interbank Offered Rate):

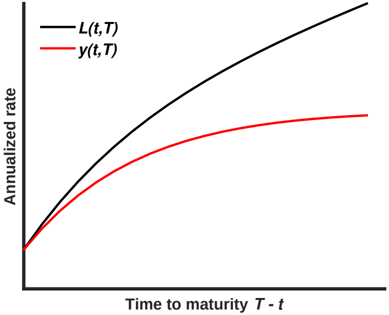
- Rate at which high-credit financial institutions can borrow on the London interbank money market
- Important reference rate for many interest rate contracts
- Quoted daily as simple rate $L(t, T)$ ...
- ... for various maturities $(T - t)$ ranging from 1 day to 1 year
- ... for various currencies

Term Structure and Yield Curve

Interest rates depend on lending period $[t, T]$. For varying maturities T we obtain the term structure of interest rates prevailing at t, e.g.

$$T \mapsto L(t, T).$$

The term structure of yields is also called the yield curve $T \mapsto y(t, T)$.

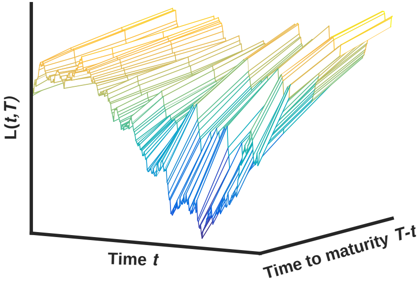


Term Structure Dynamics

For varying dates t we obtain a time series of term structures

$$(t, T) \mapsto L(t, T).$$

Modeling these time series is the aim of interest rate models.



Short Rate

Simple calculus shows

$$\lim_{T \to t} L(t, T) = \lim_{T \to t} y(t, T).$$

This common short end is called short rate $r(t)$.

It is the rate earned on a loan over the short period $[t, t + dt]$.

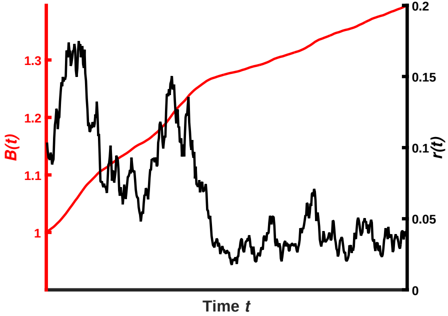
Money Market Account

Continuously reinvesting at the short rate gives the money market account

$$B(t+dt) = B(t)\,(1 + r(t)\,dt)\,.$$

With $B(0) = 1$ this is equivalent to

$$B(t) = \mathrm{e}^{\int_0^t r(s)\,ds}.$$



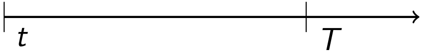
Zero-Coupon Bonds

A zero-coupon bond with maturity T is a contract that pays its holder 1 at T. Also called T-bond.

Its price at time t is denoted by $P(t, T)$.

It is the securitized (tradable) form of a loan:

- at t: lender buys a zero-coupon bond from and pays $P(t, T)$ to borrower
- at T: lender receives 1 from the borrower



Zero-Coupon Bonds and Interest Rates

Buying a T-bond at t and holding it until maturity generates a

- rate of return = simple rate $L(t, T)$:

$$P(t, T) = \frac{1}{1 + (T - t)L(t, T)}$$

- logarithmic rate of return = yield $y(t, T)$:

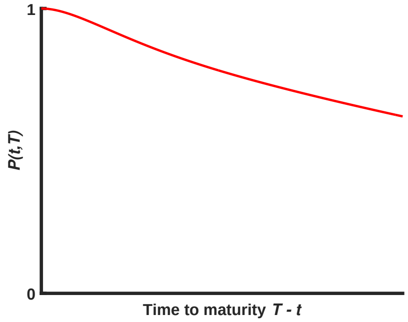
$$P(t, T) = \mathrm{e}^{-(T-t)y(t,T)}$$

Discount Curve

A zero-coupon bond is also called a discount bond: $P(t, T)$ is the price at time t of a cash flow of 1 at maturity T.

For varying maturities T we obtain the term structure of zero-coupon bond prices prevailing at t, also called the discount curve,

$$T \mapsto P(t, T).$$



Reality Check

In reality interest rates depend on:

- creditworthiness of the borrower
- liquidity needs of lender
- regulatory requirements
- market microstructure

In theory we assume, for this course:

- no credit risk: $P(T, T) = 1$
- there exists a frictionless market for all T-bonds