MANISA CELAL BAYAR UNIVERSITY – DEPARTMENT OF COMPUTER ENGINEERING
PROBLEM SET FOR NUMERICAL ANALYSIS FOR COMPUTER ENGINEERS

WEEK 02: DATA STRUCTURES IN PYTHON

1. What is the correct writing of the programming language that we used in this course?

( ) Phyton
( ) Pyhton
( ) Pthyon
( ) Python

2. What is the output of the code below?

```
my_name = "Bora Canbula"
print(my_name[2::-1])
```

( ) alu
( ) ula
( ) roB
( ) Bor

3. Which one is not a valid variable name?

( ) for_
( ) Manisa_Celal_Bayar_University
( ) IF
( ) not

4. What is the output of the code below?

```
for i in range(1, 5):
    print(f"{i:2d}{(i/2):4.2f}", end='')
```

```
( ) 010.50021.00031.50042.00
( )  10.50 21.00 31.50 42.00
( ) 1 0.5 2 1.0 3 1.5 4 2.0
( ) 100.5 201.0 301.5 402.0
```

5. Which one is the correct way to print Bora’s age?

```
profs = [
    {"name": "Yener", "age": 25},
    {"name": "Bora", "age": 37},
    {"name": "Ali", "age": 42}
]
```

( ) profs[“Bora”][“age”]
( ) profs[1][1]
( ) profs[1][“age”]
( ) profs.age[name=“Bora”]

6. What is the output of the code below?

```
x = set([int(i/2) for i in range(8)])
print(x)
```

( ) {0, 1, 2, 3, 4, 5, 6, 7}
( ) {0, 1, 2, 3}
( ) {0, 0, 1, 1, 2, 2, 3, 3}
( ) {0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4}

7. What is the output of the code below?

```
x = set(i for i in range(0, 4, 2))
y = set(i for i in range(1, 5, 2))
print(x^y)
```

( ) {0, 1, 2, 3}
( ) {}
( ) {0, 8}
( ) SyntaxError: invalid syntax

8. Which of the following sequences is immutable?

( ) List
( ) Set
( ) Dictionary
( ) String

9. What is the output of the code below?

```
print(int(2_999_999.999))
```

( ) 2
( ) 3000000
( ) ValueError: invalid literal
( ) 2999999

10. What is the output of the code below?

```
x = (1, 5, 1)
print(x, type(x))
```

( ) [1, 2, 3, 4] <class ‘list’>
( ) (1, 5, 1) <class ‘range’>
( ) (1, 5, 1) <class ‘tuple’>
( ) (1, 2, 3, 4) <class ‘set’>

WEEK 03: INTRODUCTION TO NUMPY

1. What is the correct way to create a NumPy array?

( ) np.list([1, 2, 3])
( ) np([1, 2, 3])
( ) np.array([1, 2, 3])
( ) np(array([1, 2, 3]))

2. Which of the following arrays is a 2-D array?

( ) [3, 5]
( ) [[3], [5]]
( ) [{1, 3}, {5, 7}]
( ) [2]

3. What is the correct way to print 5 from the array given below?

```
a = np.array([[1, 2], [3, 4], [5, 6]])
```

( ) print(a[3, 1])
( ) print(a[2, 0])
( ) print(a[1, 2])
( ) print(a[1, 3])

4. What is the correct way to print every other item from the array given below?

```
a = np.arange(5)
```

( ) print(a[1:3:5])
( ) print(a[::2])
( ) print(a[1:5])
( ) print(a[0:2:4]

5. What does the shape mean of a NumPy array?

( ) Number of columns
( ) Total number of items
( ) Number of items in each dimension
( ) Number of rows

6. What is the output of the code below?

```
n_1 = np.array([1, 2, 3])
n_2 = np.array([4, 5, 6])
n_3 = np.array([7, 8, 9])
print(np.array([n_1, n_2, n_3]).ndim)
```

Your answer:

7. What is the output of the code below?

```
n_1 = np.array([1, 2, 3])
n_2 = np.array([4, 5, 6])
n_3 = np.array([7, 8, 9])
print(np.array([n_1 + n_2 + n_3]).shape)
```

Your answer:

8. Which of the following is created with the code given below?

```
np.array([[1, 2, 3], [4, 5, 6]])
```

( ) 1-d array of shape 6 x 1
( ) 2-d array of shape 2 x 3
( ) 3-d array of shape 3 x 2
( ) 3-d array of shape 2 x 3

9. What is the output of the code below?

```
print(np.arange(10).reshape(2, -1))
```

10. What is the output of the code below?

```
Print(np.array([0.5, 1.5, 2.5]).dtype)
```

WEEK 04: BINARY REPRESENTATION OF NUMBERS

1. In binary system, which of the following digits are used to represent a number?
( ) 1 and 2
( ) 0 and 1
( ) 0, 1 and 2
( ) A and B

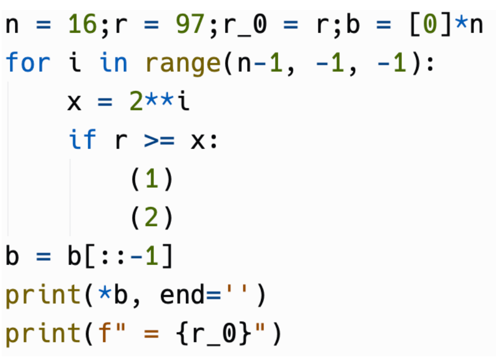
2. Which of the following codes gives a binary representation of 97?
( ) binary(97)
( ) (97).binary()
( ) f"{97:b}"
( ) to_binary(97)

3. What is the name of the NumPy method which converts a number to binary system?
( ) np.binary()
( ) np.bin()
( ) np.binary_representation()
( ) np.binary_repr()

4. The code given below produces this output:

```
> 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 1 = 97
```

Complete the code with appropriate statements for the lines given with (1) and (2).

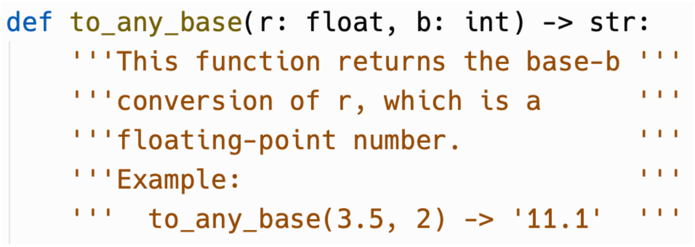


```python
n = 16;r = 97;r_0 = r;b = [0]*n
for i in range(n-1, -1, -1):
    x = 2**i
    if r >= x:
        (1)
        (2)
b = b[::-1]
print(*b, end='')
print(f" = {r_0}")
```

5. Modify the code given in question **4** to avoid fixing the number of digits (**n**).
Hint: use bit_length() method of integer object.

6. Use the codes given in the question **4** as a starting point and write Python codes which converts the decimal of a base-10 number into binary system.

7. Try to write a general function which converts a base-10 floating point number into any base including the decimal part.



```python
def to_any_base(r: float, b: int) -> str:
    '''This function returns the base-b '''
    '''conversion of r, which is a      '''
    '''floating-point number.          '''
    '''Example:                        '''
    '''   to_any_base(3.5, 2) -> '11.1'  '''
```

WEEK 05: IEEE 754 REPRESENTATION

1. Find the smallest and the largest value that you can represent with 16-bit IEEE 754 standard?

2. Find the 16 bit IEEE 754 representation of -5.875.

3. Calculate the error if we use 16 bit IEEE 754 representation to store the value 0.17 in memory.

4. Use a custom IEEE 754 representation as 1-bit for the sign of the number and (4-bit exponent) + (20-bit mantissa). Convert 0.17 into this representation and compare the result with the previous question.

5. Calculate the bias for the 8 bit exponent part.

WEEK 06: IEEE 754 PRECISIONS

1. The numbers used in the following equation are given in Half Precision IEEE 754 format, but in hexadecimal notation. Please find the result as a base-10 number.

67C8 + 3C00 =

2. Suppose that we want to save the value 0.1 in our PC. How many bits do we need for the mantissa part?

3. Which one is the correct representation of zero in IEEE 754 half precision?

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

or

0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0

Explain your answer.

4. You can find the current version of the file ieee754.py in the folder Week06 of GitHub repo of this course. List the weak points of this code that must be fixed.

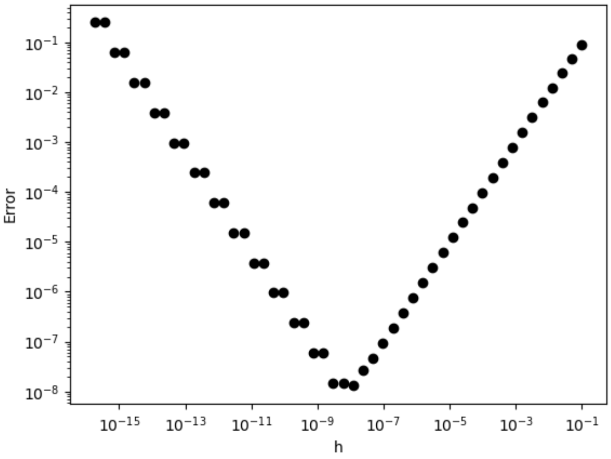
5. Using NumPy arrays to save the zeros and ones in ieee754.py, was it a correct choice or not? Explain your answer.

WEEK 11: DERIVATIVES WITH FINITE DIFFERENCE METHOD

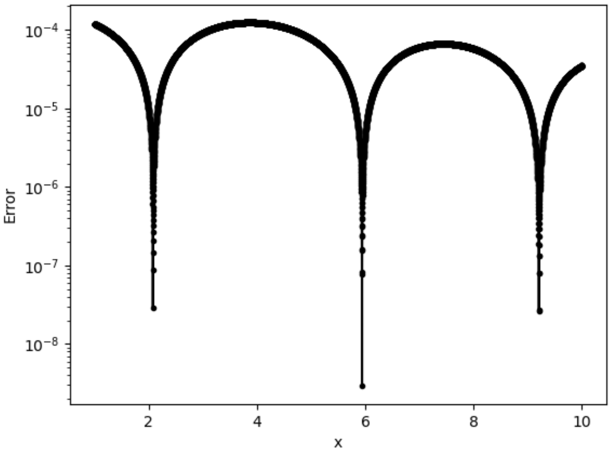
1. Derive the expression of Four Point Backward Difference Approximation.

2. To find a second derivative of a function, is it a good idea that use the same approximation two times or a combination of the approximations?

3. Find the relationship between the value of **h** and the error of the finite difference approximation. You can find an example below for the First Forward Approximation and f(x)=1/x at x=1.



4. Find the relationship between the **x** values and the resulting error of numerical derivative of the function sin(x) / x. You can find an example below.



WEEK 12: ROOT FINDING

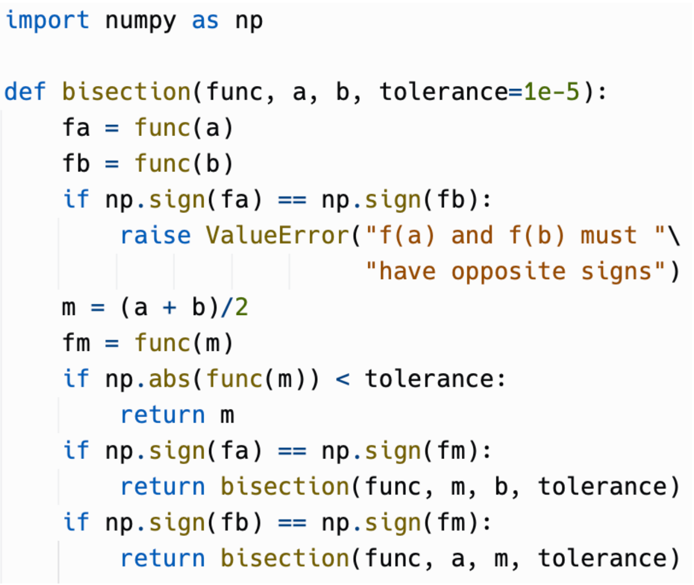
1. Fill the table with the intervals for bisection method for each iteration to find the root of the function given below:

$$f(x) = x^3 - 3x - 5$$

Intervals for Bisection Method		
Iteration	a	b
1	1	2

2. The function given below has multiple roots in interval [-5, 5]. Combine bisection method with a random number generator to improve the code given below to be able to find and return multiple roots.

$$f(x) = x^3 - 9x + 3$$



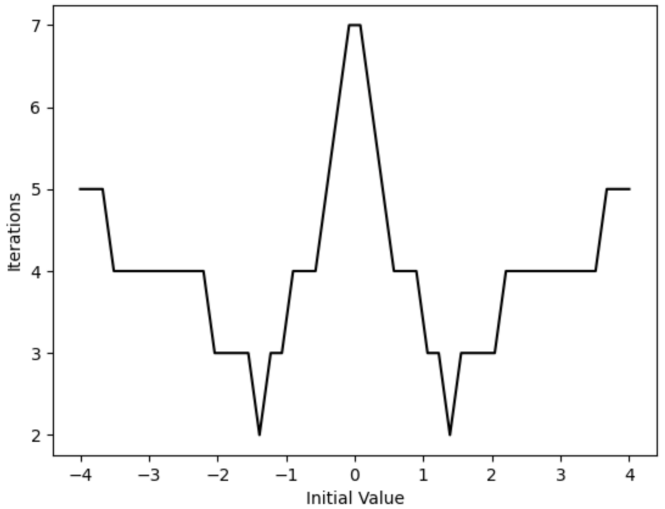
```python
import numpy as np

def bisection(func, a, b, tolerance=1e-5):
    fa = func(a)
    fb = func(b)
    if np.sign(fa) == np.sign(fb):
        raise ValueError("f(a) and f(b) must "\
                         "have opposite signs")
    m = (a + b)/2
    fm = func(m)
    if np.abs(func(m)) < tolerance:
        return m
    if np.sign(fa) == np.sign(fm):
        return bisection(func, m, b, tolerance)
    if np.sign(fb) == np.sign(fm):
        return bisection(func, a, m, tolerance)
```

3. Find the relationship between the initial value and the number of iterations which is needed to find the root of a function by using Newton-Raphson method.
Example:

$$f(x) = x^2 - 2$$

Tolerance is 10^{-5}.



4. Use the last two digits of your student id to build the number AB. Use this number in the function given below and find the root by using Newton-Raphson method:

$$f(x) = x - \sqrt[3]{BA}$$

WEEK 13: NUMERICAL INTEGRATION

1. Find the result of the integral given below by using midpoint rule and trapezoidal rule, then compare results.

$$\int_1^2 1/x$$

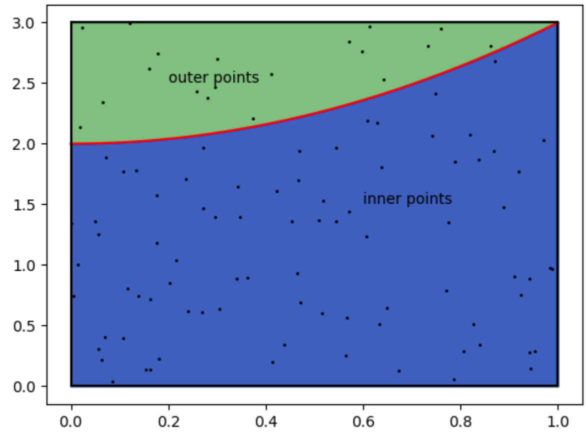
2. The y-coordinates of the points on a circle with the radius r can be given as:

$$y = \sqrt{r^2 - x^2}.$$

Use midpoint rule with 10 steps in the interval [0, r] to calculate the area of a circle with r=7. Calculate the error by comparing your numerical result with

$$\pi r^2.$$

3. Develop a numerical integration method which uses a well-known shape to cover the area that you want to integrate your function. As given in the example plot, you can use a rectangle which covers $x^2 + 2$ from 0 to 1. Generate random coordinates for large number of points in the covering area. Count the points which are below the function and find the ratio to the total number of points. This ratio gives you an estimation of the integral.


outer points
inner points