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A Review and a Comparison of Dugoff and Modified Dugoff Formula with Magic Formula

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***Abstract*— The most effective mathematical model to characterize tire behavior is the Magic formula. Current study presents a brief review of existing tire models namely Dugoff and modified Dugoff models and its comparison with Magic formula tire model. The mathematical equations represents these models were understood and the tire behavior is predicted for various normal load conditions and a comparative study is made. The longitudinal and lateral forces were plotted against slip ratio and slip angle respectively to compare with the results obtained from the Magic Formula under the same normal load conditions. This work also highlights the disadvantages of the older version of the Dugoff Model [1] when compared with the Magic Formula [2]. Furthermore, there was a new modification introduced in the Dugoff formula to make it comparable with the Magic formula at lower slip angles/slip ratios thereby reducing the error that was vividly evident in the older version of the Dugoff Model.**

Keywords*— *Magic formula, Dugoff model, MATLAB, Modified Dugoff model.

I. Introduction

The performance of a vehicle not only depends on engine, transmission, but also the interaction between the tire and road surface because it is the interface between driving environment and vehicle. The forces generated between tire and road plays an important role for safety of vehicle. Most of the vehicle active safety systems like anti-lock braking system (ABS), traction control system (TCS), electronic stability program (ESP), regulate the forces generated between tire and road for safety and comfort. And also these forces govern the different states of motion of the vehicle; tire and road interaction gives the input to the driver maneuverability operation.

Since the friction force is the major force generated between tire and road, it is always important to measure the friction force generated. It is very difficult to measure the friction forces by using expensive sensors and practical limitations on a driving vehicle. Due to these problems it is very important to generate a model for measuring tire road interaction forces so as to reduce the time and cost. Most of the research has been carried out to find the models which represent the actual tire road interaction [3] [4]. Among these models the Magic Formula is most popular model developed by Prof. Hans B. Pacejka [2]. In this model the coefficient of friction is composite trigonometric function of slip. Magic formula is the only model which gives more accurate results in nonlinear zone and is considered to be the standard, to compare the other tire models to study tire behavior

In this work, comparison has been done between the Magic formula and Dugoff model, and graphs plotted for longitudinal force vs. slip ratio, lateral force vs. slip angle by developing code in MATLAB. A modified version of the Dugoff model has been introduced after considerations from other literature [7].

II. Methodology

A. Magic Formula

The Magic Formula is an empirical method used for characterizing tire behavior as well as for vehicle handling simulations.

The Magic Formula in its basic form is used to fit experimental tire test data for characterizing the relationship between the cornering force and slip angle, self-aligning torque and slip angle, or traction/braking effort and longitudinal slip/skid. The MATLAB code for the Magic formula has been developed to compare and validate the Dugoff and modified Dugoff Formulae. The Magic Formula describes how much traction is available at the tire i.e., how much power from the power plant has reached the tire as traction. The most important region of the curve is considered as the linear region after which, the non-linear region, is where the vehicle starts to spin. Therefore the study should be limited to the linear region and the peak value is the maximum value of traction available at the tire.

The Magic Formula can be expressed by the formulae mentioned in (1), (2) and (3) [6]. The values of the constants have not been varied and have been taken from literature for a load of 4000N, 6000N and 8000N [6]. The formula (1) describes the variation of cornering force/tractive force on the tire as a function of slip angle/slip ratio.

$$y(x) = D\sin\{ C\ \tan^{-1}[Bx - E(Bx - \tan^{-1} Bx)]\} \quad (1)$$

$$Y(X) = y(x) + S_v \quad (2)$$

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$$x = X + S_h \quad (3)$$

Where,
Y(X) denotes cornering force, self-aligning torque, or traction/braking effort
X denotes slip angle or longitudinal slip/skid
B= stiffness factor
C= shape factor
D= peak factor
E= curvature factor
S_h= horizontal shift
S_v= vertical shift

B. *Dugoff Model*

In this research work, we have considered the tire model "Dugoff model" to calculate longitudinal and lateral forces under pure longitudinal slip and pure side slip conditions. These works have been often done by a lot of researchers [7]. Dugoff tire model can be described by:

$$F_x = \frac{C_a S}{1-S} f(z) \quad (4)$$

$$F_y = \frac{C_b \tan(\alpha)}{1-S} f(z) \quad (5)$$

$$z = \frac{\mu Fz\,(1-S)}{2\sqrt{(C_a S)^2 + (C_b \tan(\alpha))^2}} \quad (6)$$

$$f(z) = \begin{cases} z(2-z), & z < 1 \\ 1, & z \geq 1 \end{cases} \quad (7)$$

$$V_s = u\sqrt{S^2 + tan^2\alpha} \quad (8)$$

$$\mu = \mu_0(1 - A_s V_s) \quad (9)$$

Where,

F_x = longitudinal forces of tires (N)

F_y = lateral forces of tires (N)

F_z =vertical force of tires (N)

C_a = longitudinal stiffness of tires (N/rad.)

C_b = lateral stiffness of tires (N/rad.)
α = side slip angle of tires (rad.)
S = longitudinal slip ratio of tires
μ = maximum friction coefficient.
u = velocity component in the wheel plane (m/s)

A_s = friction reduction factor (m/s)

The constants and parameters considered during the programming of the model are given in Table I and Table II.

TABLE I. CONSTANTS USED FOR LONGITUDINAL FORCE VS. SLIP RATIO:

Constant	Value
u	15 m/s
α	0.035 rad
C_a	80000 N/rad
μ_0	0.83
C_b	26000 N/rad
As	0.0115 m/s

TABLE II. CONSTANTS USED FOR LATERAL FORCE VS. SLIP ANGLE (α):

Constant	Value
u	15 m/s
s	0
C_a	80000 N/rad
μ_0	0.83
C_b	26000 N/rad
As	0.0115 m/s

Using this model, we have done the analysis using MATLAB. The code was formed to calculate the longitudinal and lateral forces at a constant normal load of 4000N, 6000N and 8000N and a similar code with the Magic Formula was run to compare both the studies.

C. *Modified Dugoff Model*

This research work focuses on introducing a new and modified version of the Dugoff formula. The correction factors are introduced in (4) and (5) for the longitudinal and lateral forces as given by (10) and (11). The correction factors are given in (14) and (15). The same normal loads of 4000N, 6000N and 8000N is considered for the purpose of comparative study. The correction factors have been taken from literature [7] and have been subjected to trial and error for further refinement of the results in MATLAB.
The modified formula can be described by the following equations:

$$F_x = \frac{C_a\, S}{1-S} \,.\, f(z).\, g_x \quad (10)$$

$$F_y = \frac{C_b \tan(\alpha)}{1-S} .\, f(z).\, g_y \quad (11)$$

$$z = \frac{\mu Fz\,(1-S)}{2\sqrt{(C_a S)^2 + (C_b \tan(\alpha))^2}} \quad (12)$$

$$f(z) = \begin{cases} z(2-z), & z < 1 \\ 1, & z \geq 1 \end{cases} \quad (13)$$

$$g_x = (1.15 - 0.75\mu)S^2 - (1.63 - 0.75\mu)S + 1.5 \quad (14)$$

$$g_y = (\mu - 1.6)\tan(\alpha) + 1.5 \quad (15)$$

$$V_s = u\sqrt{S^2 + tan^2\alpha} \quad (16)$$

$$\mu = \mu_0(1 - A_s V_s) \quad (17)$$

Where,
F_x = longitudinal forces of tires (N)
F_y = lateral forces of tires (N)
F_z =vertical force of tires (N)

C_a = longitudinal stiffness of tires (N/rad.)

C_b = lateral stiffness of tires (N/rad.)
α = side slip angle of tires (rad.)
S = longitudinal slip ratio of tires
μ = maximum friction coefficient
u = velocity component in the wheel plane (m/s)

A_s = friction reduction factor (m/s)
g =correction factor

The constants considered during the simulation of the model in MATLAB were kept the same as previously defined for the Dugoff formula for maintaining consistency and for fair comparison between the old and the new models.

III. RESULTS

The Magic Formula is considered as the most effective means of comparing and validating the other tire models. The Dugoff tire model (represented by (4) to (9)) was simulated on MATLAB and the plots for the longitudinal and lateral force was compared with those obtained from the Magic Formula (represented by (1) to (3)). The comparison was as follows shown in Fig. 1 and 2.
The solid line represents the variation in Magic Formula whereas the dotted line shows the Dugoff and the Modified Dugoff Model through Fig. 1 to Fig. 4.

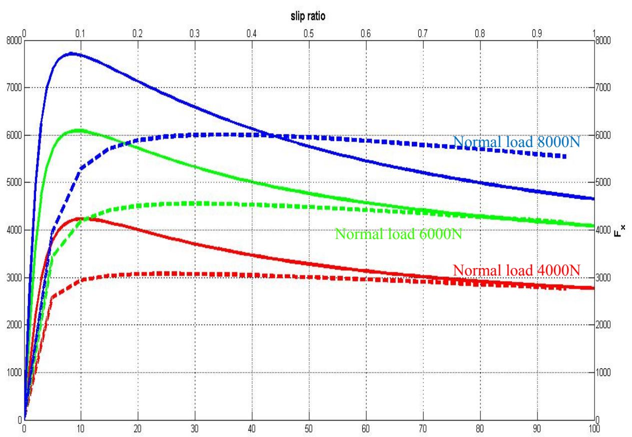

Fig. 1. Comparison of Dugoff Model with Magic Formula for Longitudinal Force (Fx) vs. slip ratio (S).

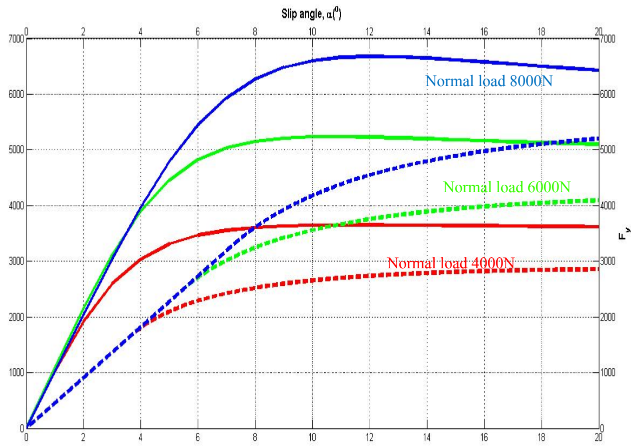

Fig. 2. Comparison of Dugoff Model with Magic Formula for Lateral Force (Fy) with slip angle (α).

As evident from the graphs in Fig. 1 and Fig. 2, the Dugoff Model has a large variation from the Magic Formula throughout the range, thereby giving inaccurate results for the same normal loads. The Dugoff model fails to incorporate the peak of the longitudinal force and the linear region as that in the magic formula. In case of lateral force too, the Dugoff model is unable to incorporate the entire region of the linear path and hence does not give accurate results for longitudinal/lateral force as the slip/slip angle changes.
To counter this, a modification in the existing model (represented by (10 to (17)) was done and then compared with the Magic formula (represented by (1) to (3)). The comparison showed the following results in Fig. 3 and Fig. 4:
As evident from the graph (Fig. 3), the modified Dugoff Formula is comparable to the Magic formula. The modified Dugoff Model is in tune with the Magic Formula at lower slip ratios where the curve is linear. The peak value is comparable in both the models unlike the older model, where it was not comparable throughout the range of the curve. The consistency in linearity at lower slip ratios for higher normal loads is obtained at the cost of getting a higher force value in the non-linear region i.e., when the vehicle starts to skid.

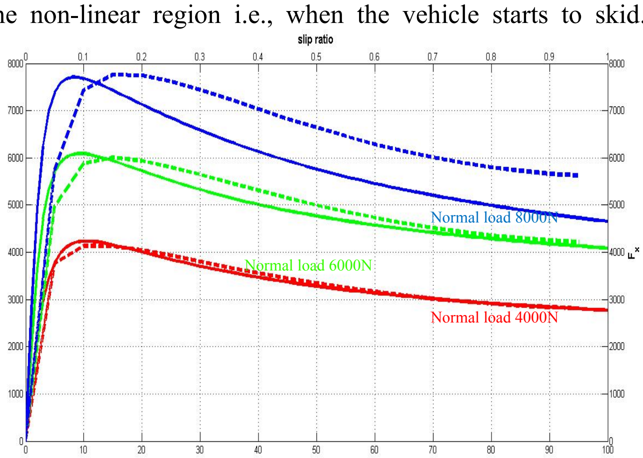

Fig. 3. Comparison of modified Dugoff Model with Magic Formula for Longitudinal Force (Fx) vs. slip ratio (S).

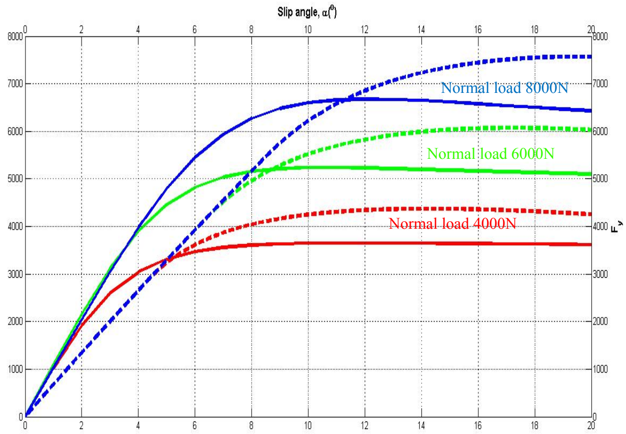

Fig. 4. Comparison of Modified Dugoff Model with Magic Formula for Lateral Force (F_y) vs. slip angle (α).

As evident from the graph (Fig. 4), the results are comparable with the Magic Formula in the sense that the modified Dugoff formula is comparable in the linear region of the graph when the value of slip angle is less. This is achieved at the cost of obtaining a higher value of force for larger slip angles or in a region where the vehicle starts to skid (i.e., in the non-linear zone). The Magic Formula, considered as the standard of measurement, gives minutely larger values of force in case of longitudinal force/lateral force when the values of slip angles is less. The modified Dugoff Model, thus is able to compare itself to the Magic Formula at lower values of slip angles at a cost of giving higher values of force for higher slip angles, quite unlike the older version of the Dugoff Model where the force varied from the Magic Formula by a large magnitude, throughout the region of the graph.

IV. CONCLUSION

In this paper, the Dugoff model was compared with the standard Magic formula and it could clearly be concluded that there was a large variation in results when comparing longitudinal force with slip ratio as well as lateral force with slip. From literature, the modified Dugoff model was taken and then some minor changes in the formula were made and then the comparison was done with magic formula. The changes in the formula were brought about by trial and error method by constantly varying the values for a better result. As per the results obtained, it can be said with confidence that even though the problem persists for higher slip angle/slip ratio where the force obtained is higher in magnitude as otherwise suggested by the Magic Formula, the results are quite comparable at lower slip angles/slip ratios i.e., in the linear zone of the graph. The modified Dugoff Model is also able to incorporate the peak represented by the Magic Formula in case of longitudinal force, unlike the older version of the Dugoff Model. It can also be inferred that although the results are better as compared to the older Dugoff Model, the variation from the Magic Formula increases as the normal load increases. Therefore the Modified Dugoff Model is best suited for lower normal loads whereas its variation with the curve obtained via the Magic Formula is almost negligible. This proves that the new modified Dugoff model, does decrease the variation from the Magic Formula at lower slip angles/slip ratios unlike the earlier version of the Dugoff Model.

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