Fuzzy Logic & Its Applications

Unit 3

Syllabus

Introduction to Fuzzy Sets, Properties of Fuzzy Sets, Operations on Fuzzy Sets, Fuzzy Membership Functions, Fuzzy Relations with Operations and its Properties, Fuzzy Composition: Max-Min Composition, Max-Product Composition, Defuzzification Methods, Architecture of Mamdani Type Fuzzy Control System, Design of Fuzzy Controllers like Domestic Shower Controller, Washing Machine Controller, Water Purifier Controller, etc.

Self-learning Topics: Other Fuzzy Composition Operations, Fuzzy Inference System (FIS) & ANFIS.

Introduction

- The word “fuzzy” means “vaguness (ambiguity)”.
- Fuzziness occurs when the boundary of a piece of information is not clear-cut.
- Fuzzy sets - 1965 Lotfi Zadeh as an extension of classical notation set.
- Classical set theory allows the membership of the elements in the set in **binary terms**.
- Fuzzy set theory permits membership function valued in the interval [0,1].

Introduction

Example:

Words like young, tall, good or high are fuzzy.

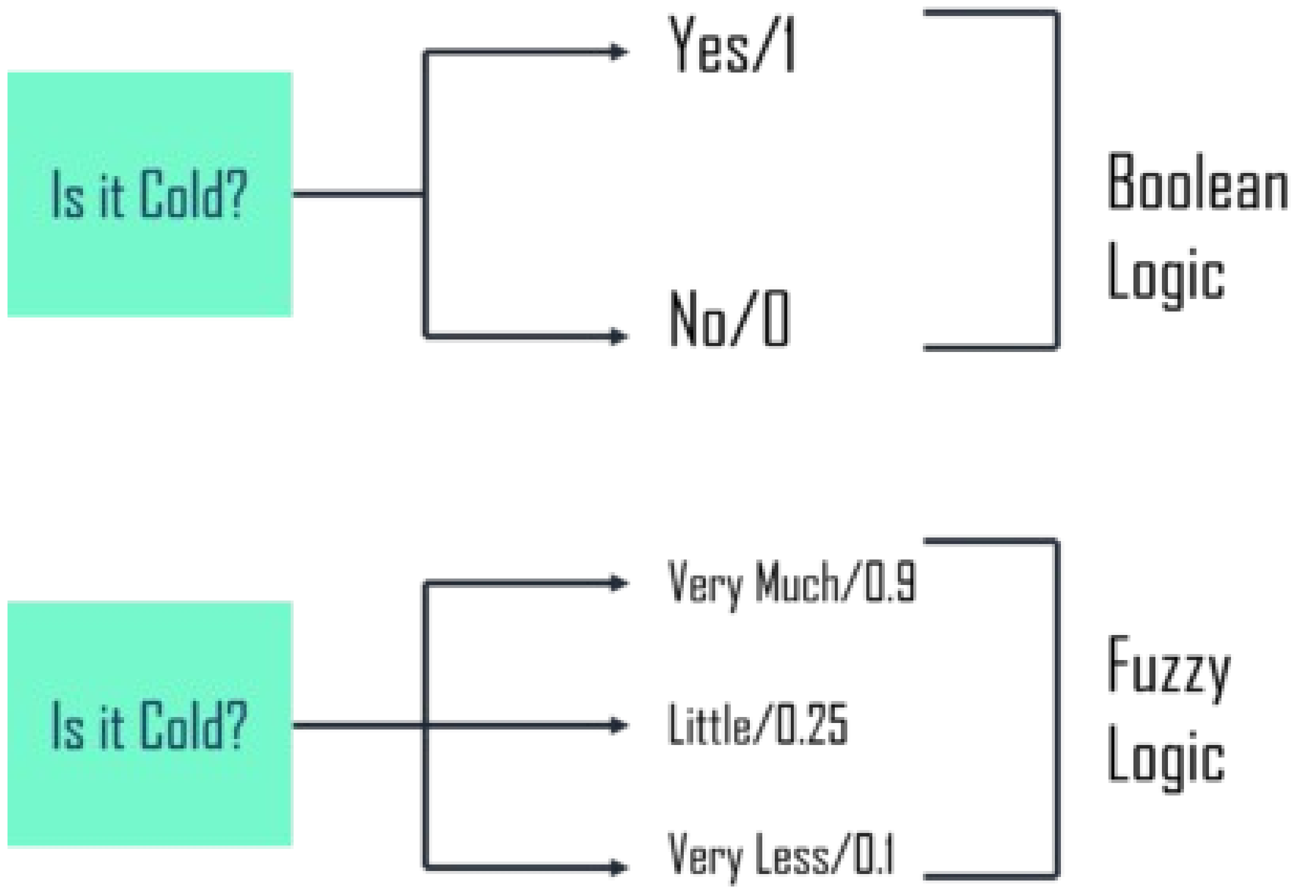
- There is no single quantitative value which defines the term young.
- For some people, age 25 is young, and for others, age 35 is young.
- The concept young has no clean boundary.
- Age 35 has some possibility of being young and usually depends on the context in which it is being considered.

Fuzzy set theory is an extension of classical set theory where elements have degree of membership.

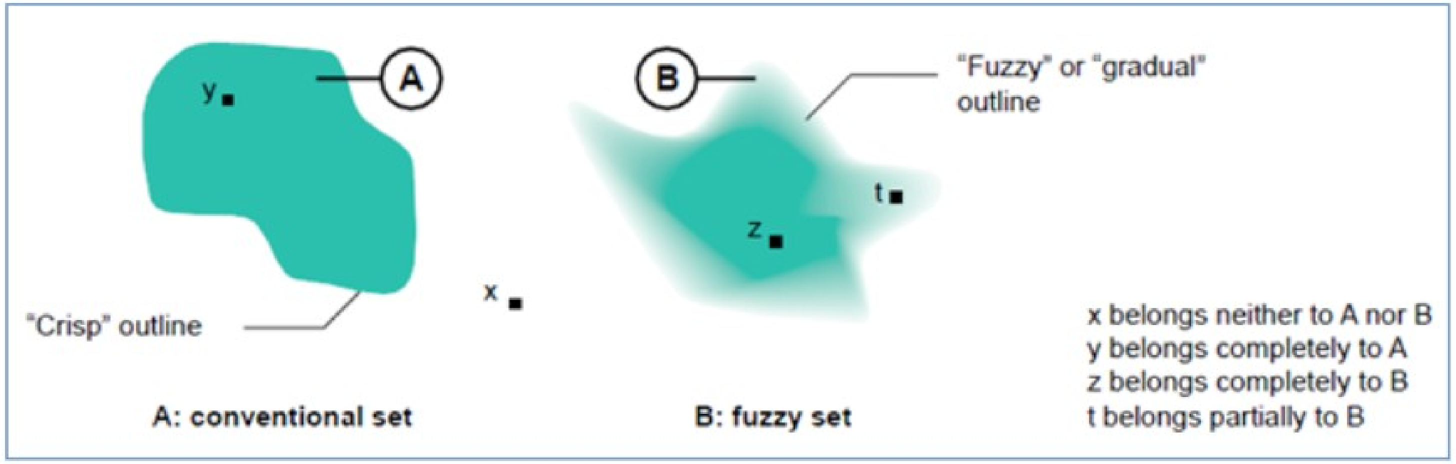
Application of Fuzzy Logic

- In automobile FL is used for gear selection and is based on the factor's engine load, road condition and style of driving
- In copy machine , FL is used to adjust drum voltage based on factor such as humidity, pictures, density and temperature
- In Medicine, FL is used for computer aided diagnosis based on symptoms and medical history
- In Chemical distillations, FL is used to control pH level and temperature variables
- In Dishwasher, FL is used to determine the washing strategy and power needed based on factor like number of dishes, level of food residue on the dishes .
- In aerospace, FL is used to manage altitude control for satellite and spacecraft based on environmental factors.
- In NLP , FL is used to determine the semantic relationship between concept represented by words and other linguistic variables
- In Environment control system such as air condition and heaters FL is used to determine the output based on factor current temperature and target temperature.
- In Business rule engine , FL used to streamline decision making according to predetermined criteria

Boolean Vs Fuzzy Logic



Classical Sets and Fuzzy Sets

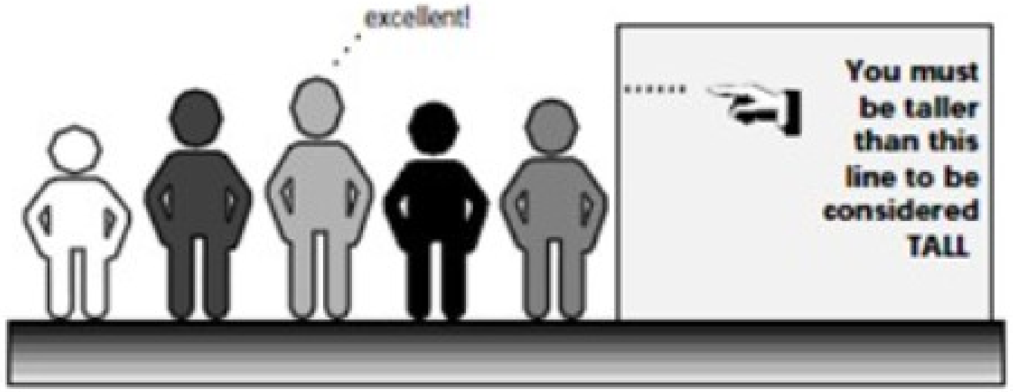


X (Universe of discourse)

A **classical set** is defined by ***crisp*** *boundaries*

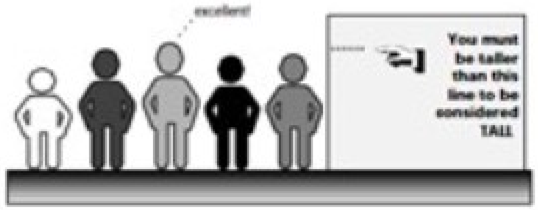
A fuzzy set is prescribed by **vague** or **ambiguous** properties; hence its boundaries are ambiguously specified

Fuzzy Sets

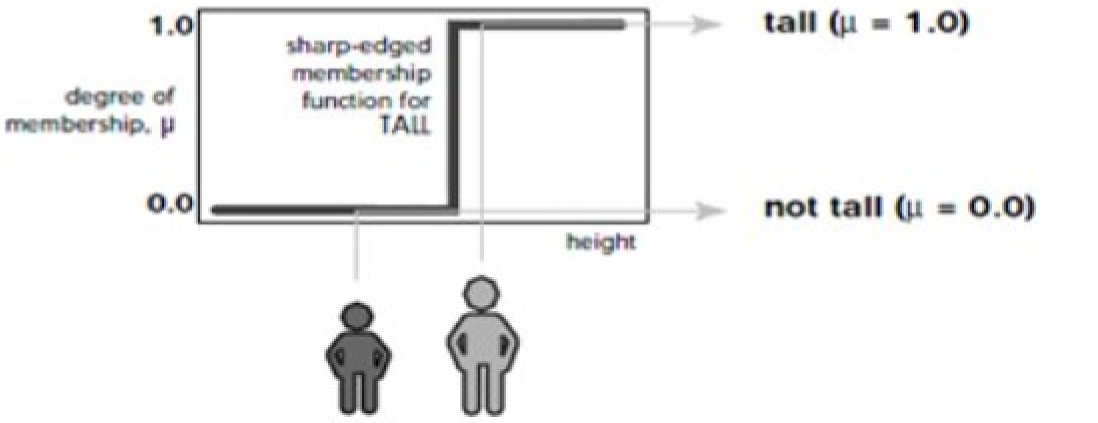


the set of tall people

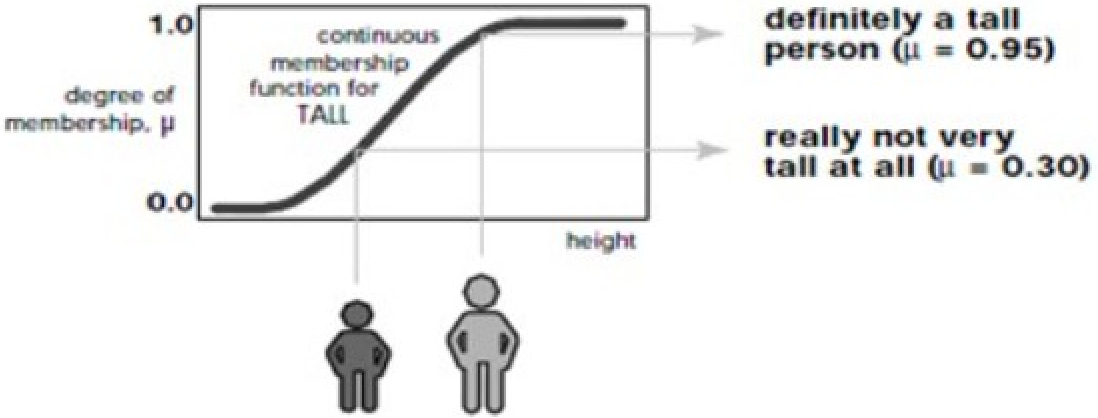
Fuzzy Sets



excellent!
You must be taller than this line to be considered TALL



1.0
0.0
degree of membership, µ
sharp-edged membership function for TALL
height
tall (µ = 1.0)
not tall (µ = 0.0)



1.0
0.0
degree of membership, µ
continuous membership function for TALL
height
definitely a tall person (µ = 0.95)
really not very tall at all (µ = 0.30)

Fuzzy Logic Membership Function

Fuzzy Sets

Elements of a fuzzy set are mapped to a universe of ***membership values*** *using a* function-theoretic form.

fuzzy sets are denoted by a set symbol with a tilde understrike;
$\underset{\sim}{A}$ would be the *fuzzy set A.*

This function maps elements of a fuzzy set $\underset{\sim}{A}$ to a real numbered value on the interval 0 to 1.
If an element in the universe, say *x, is a member of fuzzy set* $\underset{\sim}{A}$, then this mapping is given by

$$\mu_{\underset{\sim}{A}}(x) \in [0,1]$$

Fuzzy Set Operations

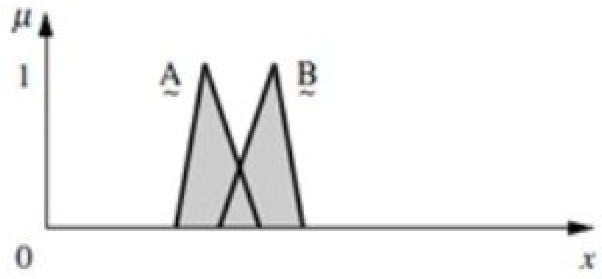
Three fuzzy sets A , B, and C on the universe X

For a given element *x of the* universe, the following **function-theoretic operations** for the set-theoretic operations of union, intersection, and complement are defined for aA, B, and C on X

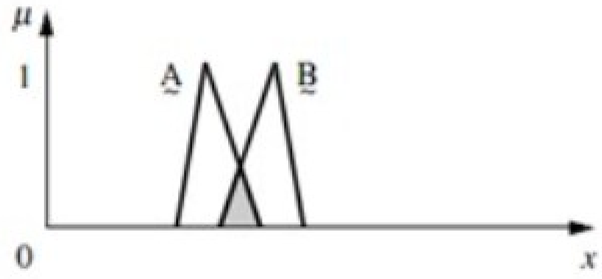
Union	$\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \vee \mu_{\underset{\sim}{B}}(x)$
Intersection	$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(x)$
Complement	$\mu_{\overline{\underset{\sim}{A}}}(x) = 1 - \mu_{\underset{\sim}{A}}(x)$

Standard fuzzy operations

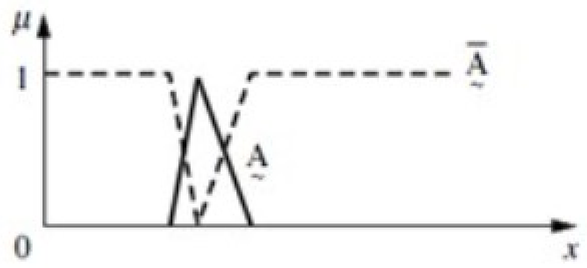
Fuzzy Set Operations



Union of fuzzy sets A~ and B~

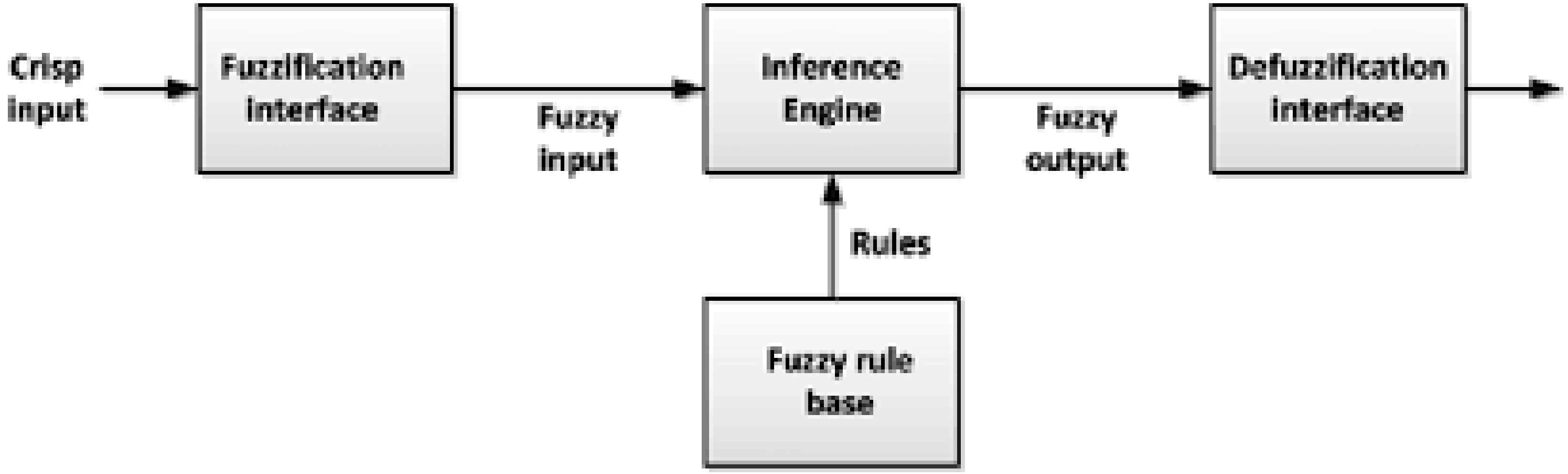


Intersection of fuzzy sets A~ and B~



Complement of fuzzy sets A~ and B~

Architecture of a Fuzzy Logic system



Architecture of Fuzzy Logic System

- Rule based : contain rules and membership function that control and regulate decision-making in FL .
- Fuzzification : convert the crisp input into fuzzy steps
  - Divide the input signal into
    - Large positive
    - Large negative
    - Medium positive
    - Medium negative
    - Small
- Inference engine :
  - Matched the fuzzy input and the rules
  - determine which rule to add and fire in combination for developing controlled action
- Defuzzification : transforms the signal converted into output

Advantage of FL system

- Structure of FL is easy to understand
- Used in commercial and practical purpose
- FL & AI Helps to control machine and consumer products
- Provide acceptable reasoning
- Helps to deal uncertainty in engineering
- Mostly robust since no precise input is required
- Can be modified to alter system performance
- Inexpensive sensors will help to reduce the overall cost of the system
- Provide effective solution to complex issues

Disadvantage of FL system

- Not always accurate
- Does not possess ML and NN pattern recognition
- Validation and verification of F.Knowledge system need extensive testing with hardware
- Setting exact rules and membership functions are very difficult
- Some FL are confused with probability .

CLASSICAL SET

- A set is a collection of distinct elements without ordering and without repetition.
- Defined by crisp boundaries
- No uncertainty about the set boundaries
- Fuzzy set is defined by its ambiguous boundaries

FUZZY SET THEORY

Examples

Satish is a rich man

That blue car is expensive

No well-defined demarcations between

- rich and poor
- very and little
- expensive and cheap
- old and young
- weak and strong

The classical set theory is not equipped to handle such vagueness as it does not allow an element to be a partial member

Fuzzy set theory is a generalization of the classical set theory

FUZZY SET THEORY

Fuzzy set theory was proposed by Zadeh in 1965

A fuzzy set is defined as a set of ordered pairs where the first element is the element and the second element is membership of that element

$$A = \{(x, \underbrace{\mu_A(x)}_{\text{membership function}}) \mid x \in U\}$$

U : universe of discourse.

$$\mu_A : U \to [0,1]$$

REPRESENTATION

$$A = \begin{Bmatrix} (1,0.1) & (2,0.3) & (3,0.8) & (4,1) \\ (5,0.9) & (6,0.5) & (7,0.2) & (8,0.1) \end{Bmatrix}$$

Alternative Representation:

$$A = 0.1/1 + 0.3/2 + 0.8/3 + 1.0/4 + 0.9/5 + 0.5/6 + 0.2/7 + 0.1/8$$

Properties of fuzzy sets

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$A \cup A = A$ $\qquad$ $A \cap A = A$

$A \cup X = X$ $\qquad$ $A \cap X = A$

$A \cup \varnothing = A$ $\qquad$ $A \cap \varnothing = \varnothing$

If $A \subseteq B \subseteq C$, then $A \subseteq C$

$A'' = A$

FUZZY SET OPERATIONS

Union

$$\mu_{A \cup B}(x) = \mu_A(x) \cup \mu_B(x)$$
$$= \max(\mu_A(x), \mu_B(x))$$

Intersection

$$\mu_{A \cap B}(x) = \mu_A(x) \cap \mu_B(x)$$
$$= \min(\mu_A(x), \mu_B(x))$$

Compliment

$$\mu_{A'}(x) = 1 - \mu_A(x)$$

FUZZY SET OPERATIONS

Comparison

Is $\bar{A} = \bar{B}$?

$\bar{A} = \bar{B}$ if $\mu_{\bar{A}}(x) = \mu_{\bar{B}}(x) \quad \forall x \in X$

Containment

Is $\bar{A} \subset \bar{B}$?

$\bar{A}$ is in $\bar{B}$ if $\mu_{\bar{A}}(x) < \mu_{\bar{B}}(x) \quad \forall x \in X$

Operations on Fuzzy Set operation with Example

$$\bar{A} = \{0.1/1 + 0.2/2 + 0.3/3)$$

$$\bar{B} = \{0.6/1 + 0.5/2 + 0.4/3 + 0.5/4\}$$

Fuzzy Membership Function

- Membership function $\mu_A(x)$ defines degree of membership of x in A
- It assumes value in the range [0,1]

E.g. $\mu_A(x) = 0.6$

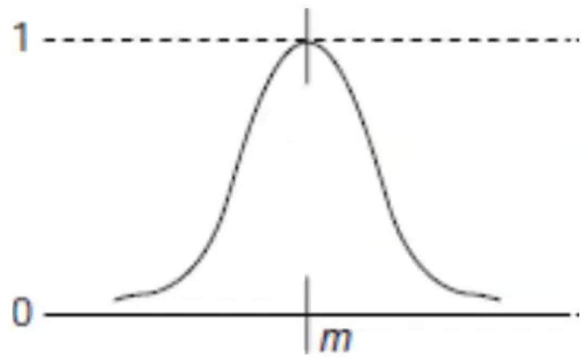
i.e. x belongs to Set A by 0.6 value

FUZZY MEMBERSHIP FUNCTION

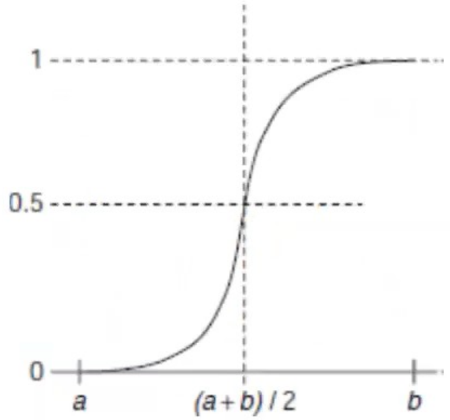
The most concise way to define a MF is to express it as a mathematical formula

$$\mu(x)=\begin{cases} 0, & \text{if} \quad x \le a \\ \dfrac{x-a}{m-a}, & \text{if} \quad a \le x \le m \\ \dfrac{b-x}{b-m}, & \text{if} \quad m \le x \le b \\ 0, & \text{if} \quad x \ge b \end{cases}$$

SOME POPULAR FUZZY MEMBERSHIP FUNCTIONS

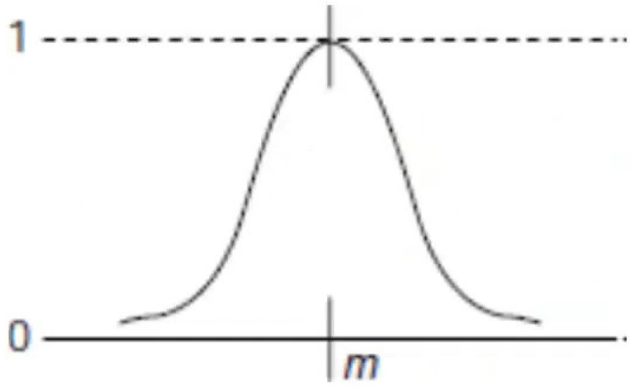


Gaussian function

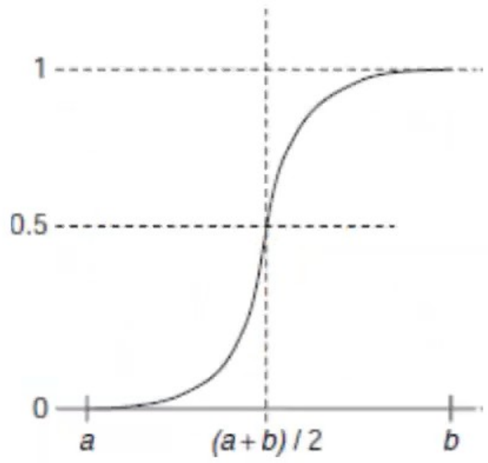


S-function

SOME POPULAR FUZZY MEMBERSHIP FUNCTIONS



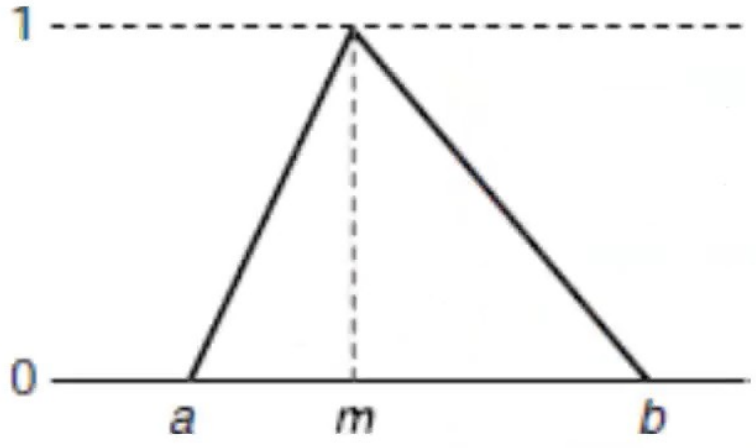
Gaussian function



S-function

SOME POPULAR FUZZY MEMBERSHIP FUNCTIONS

Triangular function- the most frequently used membership function



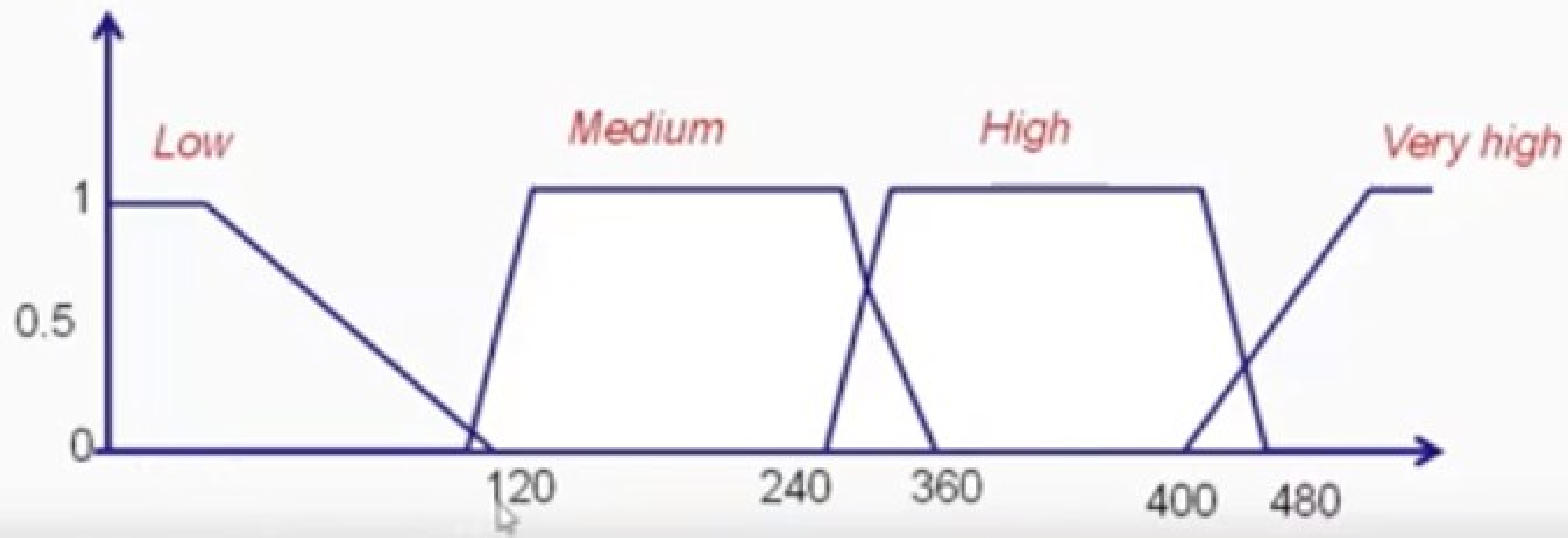
$$\mu(x)=\begin{cases} 0, & if \quad x \le a \\ \dfrac{x-a}{m-a}, & if \quad a \le x \le m \\ \dfrac{b-x}{b-m}, & if \quad m \le x \le b \\ 0, & if \quad x \ge b \end{cases}$$

Membership Functions

Consider a problem to identify the risk of cancer among a section of workforce employed in Lead industry based on no. of minutes a worker is exposed per day to the Lead processing unit.

So we identify 4 sets of risk of having cancer amongst the workers-

Low - [0 -120] ; Medium-[118- 360] ; High- [240-480] ;Very high=[400 or more]

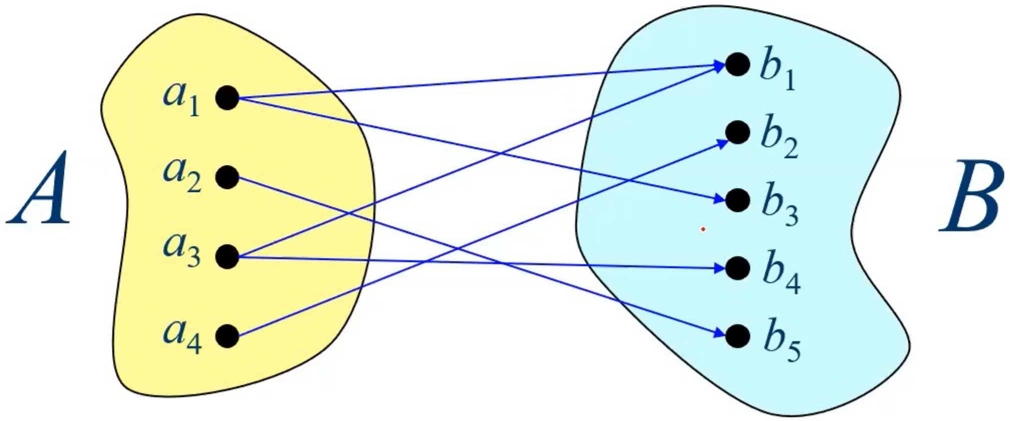


Fuzzy Relations with Operations and its Properties

THE REAL-LIFE RELATION

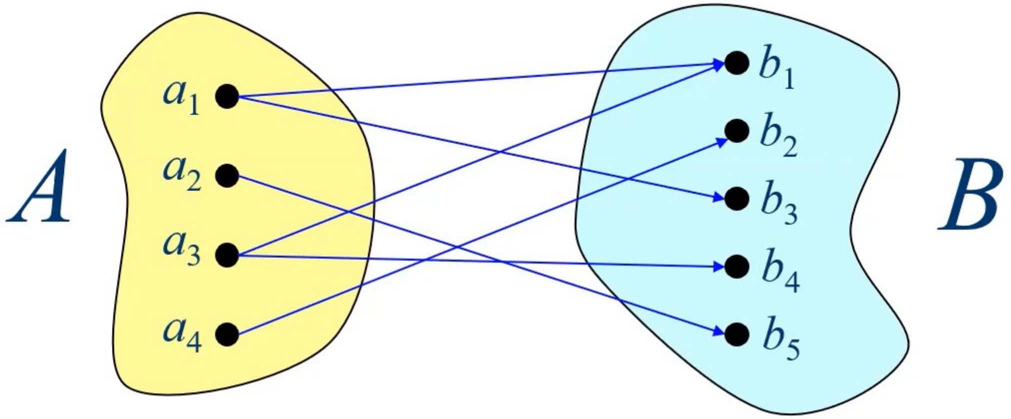
- x is close to y
  - x and y are numbers
- x depends on y
  - x and y are events
- x and y look alike
  - x and y are persons or objects

CRISP RELATION (R)



$$R \subseteq A \times B$$

CRISP RELATION (R)



$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$R \subseteq A \times B$$

Crisp Relations

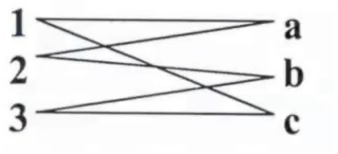
Example:

If X = {1,2,3}
  Y = {a,b,c}

R = { (1 a),(1 c),(2 a),(2 b),(3 b),(3 c) }

$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \end{array}$$

Using a diagram to represent the relation



FUZZY RELATIONS

Triples showing connection between two sets:

(a,b,#): a is related to b with degree #

Fuzzy relations are set themselves

Fuzzy relations can be expressed as matrices

FUZZY RELATIONS MATRICES

Example: relation between set of diseases and set of symptoms

$R_1(x, y)$	Running nose	High temp	shivering
Typhoid	0.1	0.9	0.8
Viral fever	0.2	0.9	0.7
Common cold	0.9	0.4	0.6

FUZZY RELATIONS

Let $A = B = C = \{0, 1, 2, 3\}$ and the relations R, S, and T defined as follows :

$R \subseteq A \times B$, $R = \{(a, b) \mid a + b \text{ is an even number}\}$

$S \subseteq A \times B$, $S = \{(a, b) \mid b = (a + 2) \text{ MOD } 3\}$

$T \subseteq B \times C$, $T = \{(b, c) \mid |b - c| = 1\}$

FUZZY RELATIONS

Let $A = B = C = \{0, 1, 2, 3\}$ and the relations R, S, and T defined as follows :

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$T \subseteq B \times C$, $T = \{(b, c) \mid \mid b - c \mid = 1\}$

These relations can be explicitly written as

$R = \{(0, 0), (0, 2), (1, 1), (1, 3), (2, 0), (2, 2), (3, 1), (3, 3)\}\}$

$S = \{(0, 2), (1, 0), (2, 1), (3, 2)\}$, and

$T = \{(0, 1), (1, 0), (1, 2), (2, 1), (2, 3), (3, 2)\}$.

FUZZY RELATIONS

$R = \{(0, 0), (0, 2), (1, 1), (1, 3), (2, 0), (2, 2), (3, 1), (3, 3)\}$

$S = \{(0, 2), (1, 0), (2, 1), (3, 2)\}$, and

$T = \{(0, 1), (1, 0), (1, 2), (2, 1), (2, 3), (3, 2)\}$.

$$T_R = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array} \quad T_S = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \quad T_T = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

OPERATIONS ON FUZZY RELATIONS

Union	$(R \cup S)\,(x, y) = max\,\{R\,(x, y), S\,(x, y)\}$
Intersection	$(R \cap S)\,(x, y) = min\,\{R\,(x, y), S\,(x, y)\}$
Complementation	$R'\,(x, y) = 1 - R\,(x, y)$

OPERATIONS ON FUZZY RELATIONS

$$T_R = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array} \quad T_S = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \quad T_T = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$T_{R\cup S} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array} \quad T_{R\cap S} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad T_{R'} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

OPERATIONS ON FUZZY RELATIONS

$$T_R = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array} \qquad T_S = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad T_T = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

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Fuzzy Composition

- Let R be a relation that relates, or maps, elements from universe X to universe Y, and let S be a relation that relates, or maps, elements from universe Y to universe Z.
- Can we find a relation, T, that relates the same elements in universe X that R contains to the same elements in universe Z that S contains?
- We can find such a relation using an operation known as *composition*.

Y

R	a	b	c	d
1	0.1	0.2	0.0	1.0
2	0.3	0.3	0.0	0.2
3	0.8	0.9	1.0	0.4

X

T?

Z

S	α	β	γ
a	0.9	0.0	0.3
b	0.2	1.0	0.8
c	0.8	0.0	0.7
d	0.4	0.2	0.3

Y

COMPOSITION

There are two common forms of the composition operation:

max–min composition

$$T = R \circ S,$$
$$\mu_T(x, z) = \bigvee_{y \in Y} (\mu_R(x, y) \wedge \mu_S(y, z)).$$

max–product composition.

$$T = R \circ S,$$
$$\mu_T(x, z) = \bigvee_{y \in Y} (\mu_R(x, y) \bullet \mu_S(y, z)).$$

EXAMPLE

$$\mu_{S\circ R}(x,y)=\max_v \min\left(\mu_R(x,v),\mu_S(v,y)\right)$$

R	a	b	c	d
1	0.1	0.2	0.0	1.0
2	0.3	0.3	0.0	0.2
3	0.8	0.9	1.0	0.4

S	α	β	γ
a	0.9	0.0	0.3
b	0.2	1.0	0.8
c	0.8	0.0	0.7
d	0.4	0.2	0.3

	0.1	0.2	0.0	1.0
min	0.9	0.2	0.8	0.4
max	0.1	0.2	0.0	0.4

$R\circ S$	α	β	γ
1	$\boxed{0.4}$	0.2	0.3
2	0.3	0.3	0.3
3	0.8	0.9	0.8

EXAMPLE

$$\mu_{R\circ S}(x,y) = \max_v \left(\mu_R(x,v)\mu_S(v,y)\right)$$

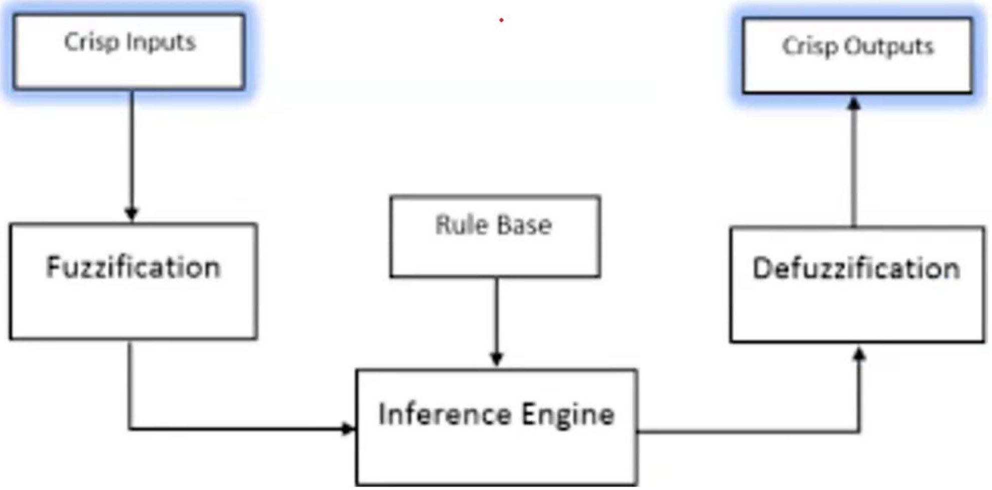
R	a	b	c	d
1	0.1	0.2	0.0	1.0
2	0.3	0.3	0.0	0.2
3	0.8	0.9	1.0	0.4

S	α	β	γ
a	0.9	0.0	0.3
b	0.2	1.0	0.8
c	0.8	0.0	0.7
d	0.4	0.2	0.3

	0.1	0.2	0.0	1.0
Product	0.9	0.2	0.8	0.4
max	.09	.04	0.0	0.4

R ° S	α	β	γ
	0.4	0.2	0.3
	0.27	0.3	0.24
	0.8	0.9	0.7

FUZZY INFERENCE PROCESS



FUZZY INFERENCE PROCESS

1. Fuzzification – input variables are assigned degrees of membership in various classes
2. Rule evaluation – inputs are applied to a set of if-then control rules

   E.g. if temperature is very hot then set fan speed to very high
3. Defuzzification – fuzzy outputs are combined into discrete values needed to drive the actual mechanism

FUZZY CONTROLLER

Design a fuzzy controller to determine the wash time of a domestic washing machine. Assume that input is dirt, grease on clothes. Use 3 descriptors for input variables and 5 descriptors for output variable. Device a set of rules for control action & defuzzification. Clearly indicate that if the clothes are soiled to a larger degree the wash time required will be more.

FUZZY CONTROLLER

Step 1: Identification of i/p and o/p using linguistic variables

Step 2: Assign membership function to the variables

Step 3: Build a rule base

Step 4: Generate a crisp controlled output

FUZZY CONTROLLER

Step 1: Identification of i/p and o/p using linguistic variables

Input 1: Dirt => { Small D, Medium D. Larae D }

Input 2: Grease => {Small G, Medium G, Large G }

Output: Wash time => {Very Short, Short, Medium, Long, Very Long} https://www.youtube.com/watch?v=qdeA6OO04ZI

Fuzzy Control system

Design a controller to determine the wash time of a domestic washing machine. Assume the input is dirt & grease on cloths. Use three descriptors for input variables and five descriptor for output variable. Derive the set of rules for controller action and defuzzification. The design should be supported by figure wherever possible. Show that if the cloths are solid to a larger degree the wash time will be more and vice versa.

Steps to solve

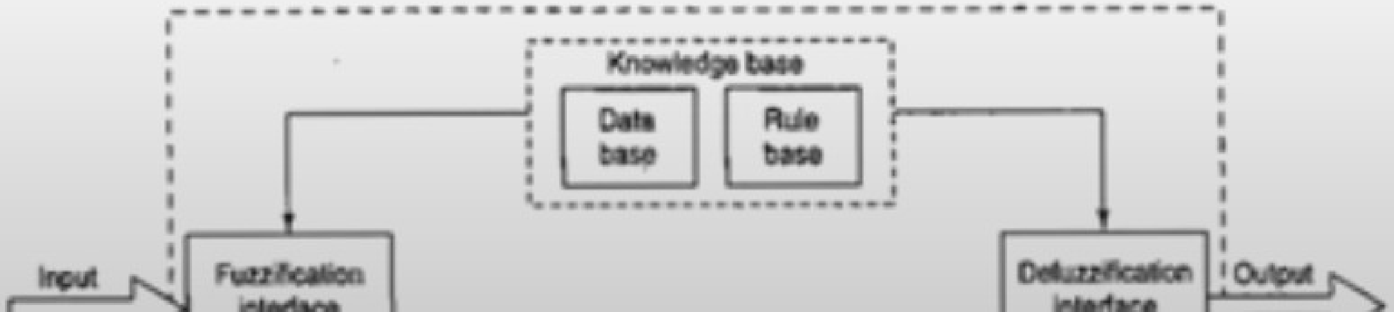
Step01: Identify input and output variables and decide descriptor for the same.

Step02: Define membership functions for each of input and output variables

Step03: Form a rule base

Step04: Rule Evaluation

Step05: Defuzzification



Step01: Identify input and output variables and decide descriptor for the same.

- Here inputs are “dirt” and “grease”. Assume they are in %
- Output is “wash time” measured in minute.

Descriptor for INPUT variable

Dirt

SD: Small dirt

MD: Medium dirt

LD: Large dirt

{**SD**, **MD**, **LD**}

Grease

NG: No Grease

MG: Medium Grease

LG: Large Grease

{**NG**, **MG**, **LD**}

Descriptor for OUTPUT variable

Wash Time

VS: Very Short

S: Short

M: Medium

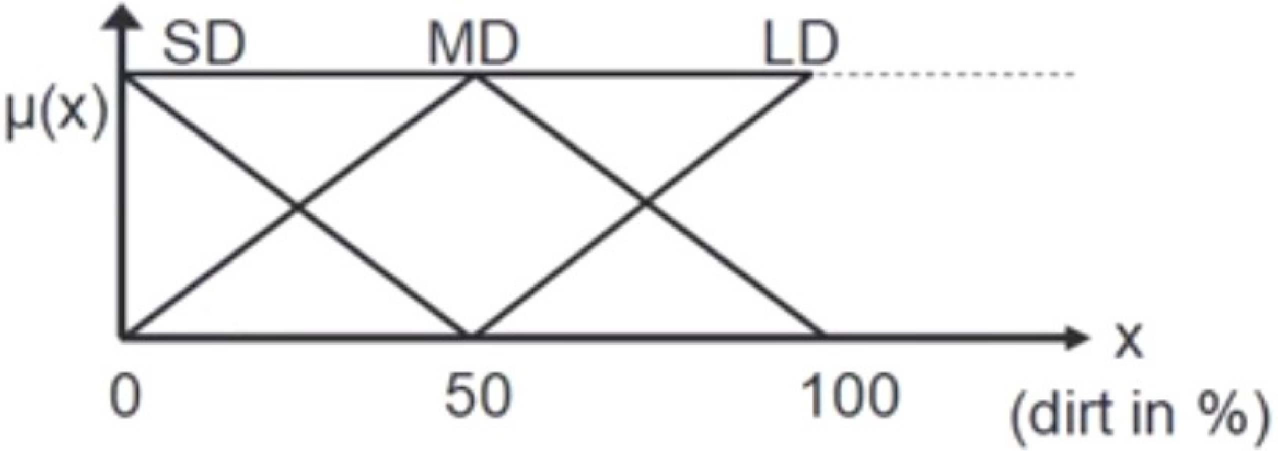
L: Large

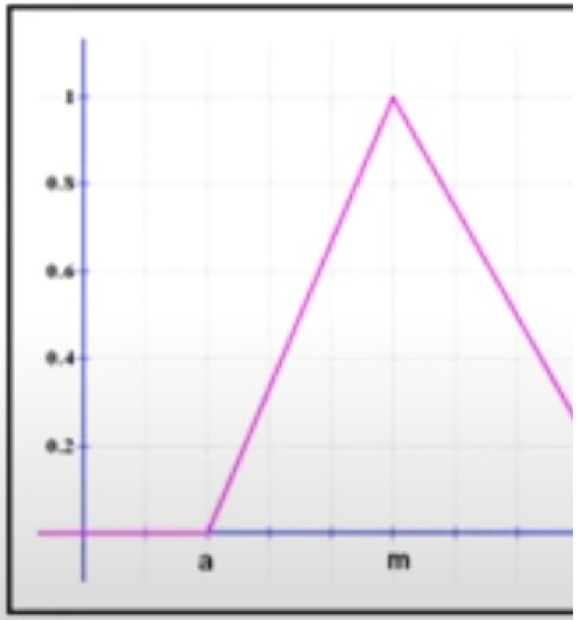
VL: Very Large

{**VS**, **S**, **M**, **L**, **VL**}

Step02: Define membership functions for each of inpu and output variables (We use triangular MF's)

(1) Membership function for dirt:



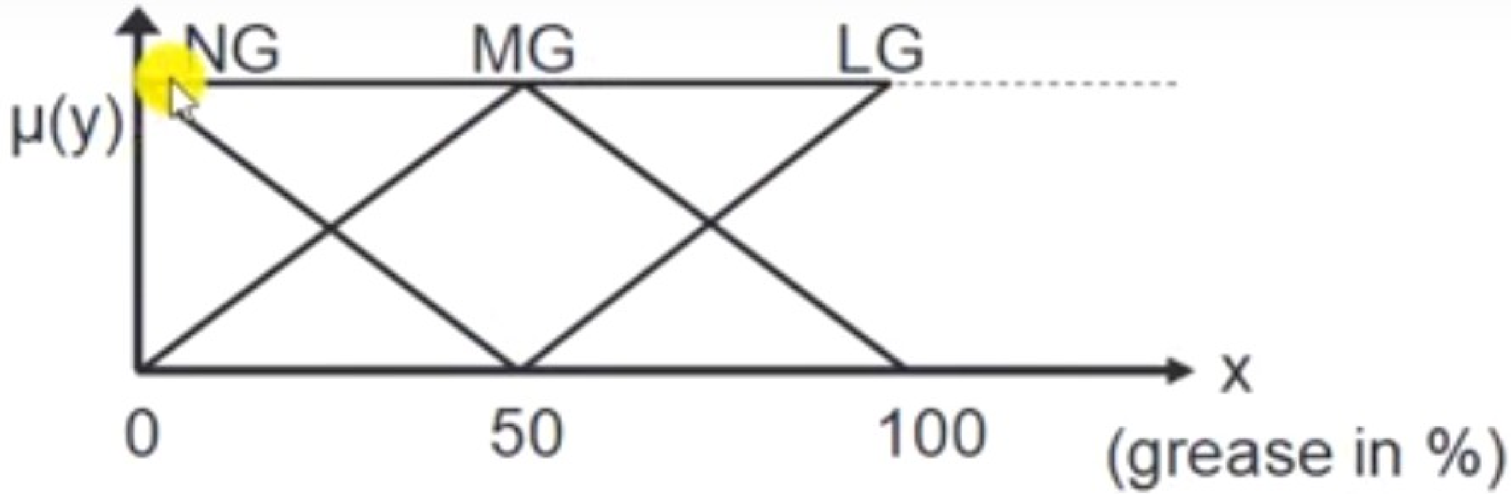


$$\mu_{SD}(x) = \frac{50-x}{50}, 0 \le x \le 50$$

$$\mu_{MD}(x) = \begin{cases} \frac{x}{50}, & 0 \le x \le 50 \\ \frac{100-x}{50}, & 50 \le x \le 100 \end{cases}$$

$$\mu_{LD}(x) = \frac{x-50}{50}, 50 \le x \le 100$$

(2) Membership function for grease:

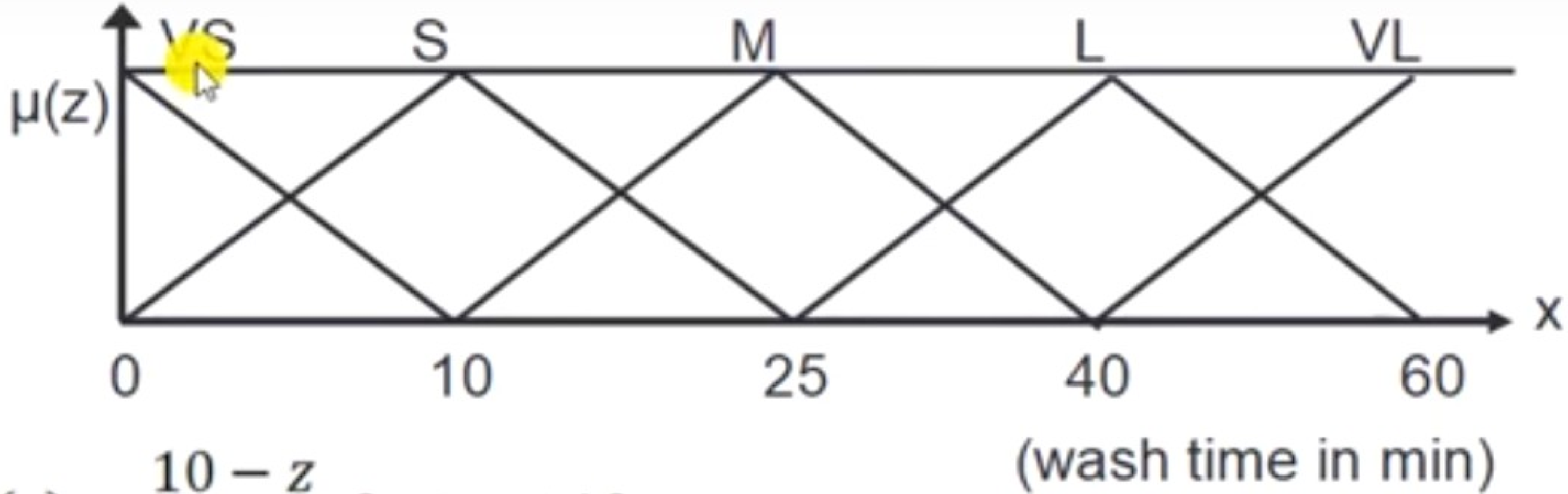


$$\mu_{NG}(y) = \frac{50 - y}{50}, 0 \leq y \leq 50$$

$$\mu_{MG}(y) = \begin{cases} \frac{y}{50}, & 0 \leq y \leq 50 \\ \frac{100 - y}{50}, & 50 \leq y \leq 100 \end{cases}$$

$$\mu_{LG}(y) = \frac{y - 50}{50}, 50 \leq y \leq 100$$

(3) Membership function for Wash time:



$$\mu_{VS}(z) = \frac{10 - z}{10}, 0 \le z \le 10$$

$$\mu_S(z) = \begin{cases} \frac{z}{10}, & 0 \le y \le 10 \\ \frac{25 - Z}{15}, & 10 \le z \le 25 \end{cases}$$

$$\mu_L(z) = \begin{cases} \frac{z - 25}{15}, & 25 \le z \le 40 \\ \frac{60 - z}{20}, & 40 \le z \le 60 \end{cases}$$

$$\mu_M(z) = \begin{cases} \frac{z - 10}{15}, & 10 \le Z \le 25 \\ \frac{40 - z}{\;}, & 25 \le z \le 40 \end{cases}$$

$$\mu_{VL}(z) = \frac{z - 40}{20}, 40 \le z \le 60$$

Step03: Form a rule base

x \ y	NG	MG	LG
SD	**VS**	**M**	**L**
MD	**S**	**M**	**L**
LD	**M**	**L**	**VL**

Step04: Rule Evaluation

Assume Dirt = 60%, Grease= 70%

Dirt=60% maps two MFs of dirt Grease=70% maps 2 MFs

$$\mu_{MD}(x) = \frac{100 - x}{50} \mid \mu_{LD}(x) = \frac{x - 50}{50}$$

$$\mu_{MG}(y) = \frac{100 - y}{50} \mid \mu_{LG}(y) = \frac{y - 50}{50}$$

Evaluate:

$$\mu_{MD}(60) = \frac{100 - 60}{50} = \frac{4}{5}$$

$$\mu_{MG}(70) = \frac{100 - 70}{50} = \frac{3}{5}$$

$$\mu_{LD}(60) = \frac{60 - 50}{50} = \frac{1}{5}$$

$$\mu_{LG}(70) = \frac{70 - 50}{50} = \frac{2}{5}$$

The above four equation leads to 4 rules need to evaluate:

1. Dirt is Medium and Grease is Medium
2. Dirt is Medium and Grease is Large
3. Dirt is Large and Grease is Medium
4. Dirt is Large and Grease is Large

SIS

Since the antecedent part of each of the above rule is connected by and operator we use min operator to evaluate strength of each rule.

Strength of Rule 1 DMGM

$$S1 = \min(\mu_{MD}(60), \mu_{MG}(70))$$
$$= \min\left(\frac{4}{5}, \frac{3}{5}\right)$$
$$= \frac{3}{5}$$

Strength of Rule 2 DMGL

$$S2 = \min(\mu_{MD}(60), \mu_{GL}(70))$$
$$= \min\left(\frac{4}{5}, \frac{2}{5}\right)$$
$$= \frac{2}{5}$$

Strength of Rule 3 DLGM

$$S3 = \min(\mu_{LD}(60), \mu_{MG}(70))$$
$$= \min\left(\frac{1}{5}, \frac{3}{5}\right)$$
$$= \frac{1}{5}$$

Strength of Rule 4 DLGL

$$S4 = \min(\mu_{LD}(60), \mu_{LG}(70))$$
$$= \min\left(\frac{1}{5}, \frac{2}{5}\right)$$
$$= \frac{1}{5}$$

Dirt \ Grease		MG	LG
	X	X	X
MD	X	M	L
LD	X	L	VL

MAX Membership Function

Dirt \ Grease		MG	LG
	X	X	X
MD	X	3/5	2/5
LD	X	1/5	1/5

Step05: Defuzzification

Since we use "Mean of Max" defuzzification technique

Maximum strength = Max(S1, S2, S3, S4)

= Max(3/5, 2/5, 1/5, 1/5)

= 3/5

- This corresponds to rule 1
- Rule 1: Dirt is medium and Grease is medium has maximum strength (3/5)
- To find out the final defuzzified value, we now take average (mean) of $\mu_M(z)$.

$$\mu_M(z) = \frac{z - 10}{15}$$

$$\mu_M(z) = \frac{40 - z}{15}$$

$$\frac{3}{} = \frac{z - 10}{}$$

$$\frac{3}{} = \frac{40 - z}{}$$

SIS

Watch Time

$\therefore z = 19$

$\therefore z = 31$

$$\therefore Z = \frac{19 + 31}{2}$$

$$\boxed{Z = 25\,min}$$