Exam

February 5, 2024

Paper document allowed
duration: 1h30

1 Basic computation

For the given function: $f(x) = (x_1 - 3)^2 + 4(x_1^2 - x_2)^2$

1. Calculate $\nabla f(x, y)$
2. Calculate $\nabla^2 f(x, y)$

2 Newton's method

Let us consider the following function: $f(x) = (x_1 - 3)^2 + 4(x_1^2 - x_2)^2$ with $x_0 = [-2, 3]^T$.

1. Calculate the first three iterations of the Newton's method.

3 Quadratic programming

For the given function: $f(x) = 4x_1^2 + 6x_2^2 - 5x_1x_2 - 2x_1 + x_2$

1. Show that $f(x_1, x_2) = \frac{1}{2}x^T Ax - b^T x$
2. Calculate A and b
3. Detail the algorithm to solve this problem
4. Calculate the theoretical solution.

4 Optimization problem with constraints

Solve the following problem:

$$\underset{x}{\text{minimize}}\ x_1^2 - x_2$$

Subject to:

$$\begin{aligned} x_1 + x_2 &= 3 \\ x_1^2 - 4 &\geq 0 \\ x_1^2 + x_2^2 &\leq 29 \end{aligned}$$

5 Lesson questions

1. Explain the differences between Gradient descent method with optimal step size and Newton's method.
2. What happens when using Newton's method and the Hessian matrix is not invertible. Which alternative to this method should be used to avoid this problem? Explain this method in a few lines.

philippe.fraisse@umontpellier.fr

6 Appendix

6.1 Reminder of matrix and vector derivation

Let x be a vector of dimension $(n \times n)$, A a matrix of dimension $(n \times n)$ and b a coefficient vector of dimension $(n \times 1n)$:

$$\frac{\partial x^T A x}{\partial x} = (A + A^T)x$$

$$\frac{\partial x^T b}{\partial x} = \frac{\partial b^T x}{\partial x} = b$$

6.2 Gradient and Hessian Matrix

Calculation of the gradient of $f(x)$:

$$\vec{\nabla} f(x) = \begin{pmatrix} \frac{\partial f(x)}{\partial x_1} \\ \\ \frac{\partial f(x)}{\partial x_2} \end{pmatrix}^T$$

Calculation of the hessian matrix of $f(x)$:

$$H(f) = \begin{pmatrix} \frac{\partial^2 f(x)}{\partial x_1 \partial x_1} & \frac{\partial^2 f(x)}{\partial x_1 \partial x_2} \\ \\ \frac{\partial^2 f(x)}{\partial x_2 \partial x_1} & \frac{\partial^2 f(x)}{\partial x_2 \partial x_2} \end{pmatrix}$$

with $x = (x_1\ x_2)^T$.

Master EEA Semester 1 - Exam - February 2022

HAE916E - C++ LANGUAGE

Duration : 1h.00 - All printed documents authorized.
All numerical devices (Computer, Tablet, Phone, etc.) ***Prohibited.***
The style of the code will be part of the evaluation.

- Exercise : Functions -

I.1 - Basic Function

We give the following C++ Code :

```
#include <iostream>
#include <vector>
using namespace std;

// Base class
class Function {
public:
    virtual double evaluate(double x) const;
    virtual void display() const ;
};

double Function::evaluate(double x) const {}
void Function::display() const {}
```

This class is designed to be inherited the following way :

- `evaluate` method should be used to calculate the value of the function for the input x
- `display` method may display the function

1 - Write an inheriter class of Function that may represent a Gaussian function like :

$$f(x) = A \exp\left(-\frac{(x-C)^2}{B^2} + D\right)$$

At this level, we only want the class itself. You should add the necessary data members.

2 - Write a useful constructor and destructor

3 - Write the evaluate member function

4 - Write the display member function. The function may be displayed with the value of parameters, and not symbols for the parameters.

5 - Make a basic `main` for the function

$$g(x) = 2 \exp\left(-\frac{(x-3)^2}{1.5^2} + 7\right)$$

and make it display the result for g(8.5)

I.2 - Polynomials

We want to make a new inheriter for `Function` that may be a `Polynomial`, i.e. something like :

$$h(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0$$

The polynomial will be modelized as a `vector<term>`, where `term` is a class that you will have to design to contain both the coefficient and the exponent of x for a single term.

1 - Create the class `term`

2 - Create the inheritor class called `Polynomial`

3 - Create a member function that adds a new term to the polynom, by taking the coefficient and the exponent of x for the term to add. We will not try to maintain a given order regarding the exponents. However, if the exponent already exists, a new term should not be created : only the coefficient must be changed. In all other cases, you may add a new term to the polynom.

4 - Write both evaluate and display member Functions

5 - Give a basic `main` for the polynom :

$$x^2 + 3x - 5$$

And make the program display the result for $x = -1$.

Master2 Robotics 2023-2024

Exam of Applied Robotics (1st session)

Documents not authorized
Duration: 2h

1- In Micro-robotics, why are articulated structures replaced by deformable structures? Give a reasoned answer.

2- Magnetic and electrostatic forces are two candidates for actuation in the microworld. In the hypothesis of extreme miniaturization, which forces seem to you the most adapted for a good efficiency?

3- Explain the impact of the scale effect (size reduction) on microworld physics. Compare the physical interactions between the microworld and the macroworld.

4- What is actuation redundancy? What are the advantages of a redundant actuation system?

5- How many solutions does the inverse kinematics of a cable-driven parallel robot possess?

6- What is the main specificity of actuation with cables (compared to actuation with rigid links)?

7- Give three advantages of cable-driven parallel robots compared to a rigid-link parallel robot.

8- Explain the difference between virtual reality and mixed reality?

9- Explain the principle of the localization algorithm used by the Vive Lighthouse?

10- In your opinion, what technologies unlocked the development of VR headsets?

11- What is the main effect of time delay to bilateral teleoperation system? What are the main methods to handle the time delay problem?

12- What is the difference between unilateral and bilateral teleoperation systems? List out the main components of a typical bilateral teleoperation system.

13- Draw a general 4-channel bilateral teleoperation architecture

14- What are the two main control objectives to achieve for bilateral teleoperation? Can these two control objectives be guaranteed at the same time? Provide some analysis tools/methods to evaluate each objective respectively.

15- What are the main challenges of minimally invasive surgery from an engineer's point of view? What is the contribution of robotics for this type of surgery?

16- If you have to develop a robotic assistant that can automatically position, orient and insert a needle based on images acquired by MRI, how many degrees of freedom does the robot need to perform this task? knowing that needle rotation is not necessary. Justify your answer.

17- Define the principle of tele-operation. What is its interest in microsurgery (name at least 2)?

18- Enumerate three different CAD operations for the design of a part, explaining their principles.

19- Explain how the concepts of PDEs and Algebraic Equations are related in the context of Finite Element Methods.

20- Enumerate two typical problems when using 3D printing and some respective solutions.