PMAT 21562

#Year 02 #Semester 01 #problem sheet 6

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Q1 (a) $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n} + 3\sqrt{n}}$; take $a_n = \frac{1}{2(n)^{1/2} + (n)^{1/3}}$

Since largest n dominant term in derouinator is $2(n)^{1/2}$, for $n >> 1$ we expect a_n to behave like $\frac{1}{2(n)^{1/2}}$, so we let $b_n = \frac{1}{\sqrt{n}}$

Since b_n is p series with $P = \frac{1}{2} < 1$ $\therefore$ b_n diverges.

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{(n)^{1/2}}{2(n)^{1/2} + (n)^{1/3}} = \lim_{n\to\infty} \frac{1}{2 + 1/(n)^{1/6}} = \frac{1}{2} > 0$

By part 1 of the limit comparision rest,

Since $\sum b_n$ diverge also a_n diverge / $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n} + 3\sqrt{n}}$ diverge //

(b) $\sum_{n=1}^{\infty} \frac{3}{n + \sqrt{n}}$; take $a_n = \frac{3}{n + \sqrt{n}}$

Since largest n dominant term in detominator is n, for $n >> 1$ we expect a_n to behave like $3/n$, so we let $b_n = 1/n$

Since b_n is p $P = 1 \leq 1$ $\therefore$ b_n diverges

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{n + \sqrt{n}} = \lim_{n\to\infty} \frac{1}{1 + 1/\sqrt{n}} = 1 > 0$

By part 1 of the limit comparison test,

Since b_n diverges also $\sum a_n$ diverges / $\sum_{n=1}^{\infty} 3/(n + \sqrt{n})$ diverges //

(c) $\sum_{n=1}^{\infty} \frac{2n}{3n-1}$; $a_n = 2n/(3n-1)$

for $n >> 1$ we expect a_n to behave like $\frac{2n}{3n} = 2/3 = b_n$

$\sum_{n=1}^{\infty} b_n = 2/3 \sum_{n=1}^{\infty} 1 = \infty$ $\therefore$ b_n diverges

$\frac{2n}{3n-1} > 2/3$ ($\therefore$ since detominator of b_n is larger)

Thus given series $\sum_{n=1}^{\infty} 2n/(3n-1)$ is divergent by comparison test second test

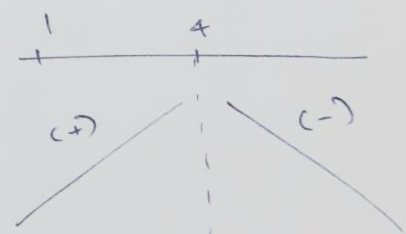
(d) $\sum_{n=1}^{\infty} \frac{1}{(\ln n)^2}$; $a_n = 1/(\ln n)^2$

take

$f(\ ) = \ln(n) - \sqrt{n}$

$f'(n) = \frac{1}{n} - \frac{1}{2} \times \frac{1}{\sqrt{n}} = \frac{2-\sqrt{n}}{2n} = 0$

$n = \pm 4$ since $n \geq 1$ $n = 4$



$\therefore$ when $n \geq 4$ $\ln(n) - \sqrt{n} < 0$

$\ln(n) < \sqrt{n}$

$(\ln(n))^2 < n$

$n = 4$, $(\ln(n) - \sqrt{n}) = -0.6$

since convergence of a series is not affected by a finite number of tens

$\therefore \frac{1}{(\ln(n))^2} > \frac{1}{n}$ $n \geq 4$

$\sum_{n=1}^{\infty} 1/n$ is divergent, series since $P = 1 \leq 1$

(e) $\sum_{n=1}^{\infty} \frac{(\ln n)^3}{n^3}$; $a_n = (\ln n)^3 / n^3$

$\frac{\ln n}{n} < \frac{\sqrt{n}}{n} \quad \forall n \geq$, since $\ln n > 0 \ \forall n > 1$

$\frac{(\ln n)^3}{n^3} < \left(\frac{\sqrt{n}}{n}\right)^3 \quad \forall n \geq 4$

Convergence of a series is not affected by a finite number of terms $\therefore$ let $b_n = \frac{1}{n^3}$

for $n \geq 4$ $\quad a_n < b_n$

$b_n = \frac{1}{n^{3/2}}$ is p series with $P = 3/2 > 1$, $\sum b_n$ is

Converging

Since $a_n < b_n$ by comparison test, $\sum a_n$ is also Convergents //

(f) $\sum_{n=1}^{\infty} \frac{1}{(1+\ln n)^2}$; $a_n = \frac{1}{(1+\ln n)^2}$

for $n \gg 1$ we expect a_n to behave like $1/(\ln n)^2 = b$

$\frac{1}{(1+\ln n)^2} < \frac{1}{\sum(\ln n)^2}$

we know that $\frac{1}{(\ln n)^2}$ is divergent by question Part (d)

$n \to \infty \quad \ln(n) \to \infty$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{\ln(n)\to\infty} \frac{(\ln(n))^2}{(1+\ln(n))^2} = \lim_{\ln(n)\to\infty} \frac{1}{\left(\frac{1}{\ln(n)}+1\right)^2} = 1 > 0$

By limit comparison test Part I both series diverges

$\sum a_n$ diverges //

(g) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$; $a_n = \sqrt{n}/(n^2+1)$

for $n \gg 1$ we expect a_n to behave like $b_n = \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}}$

Σb_n is a p-series where $P = 3/2 > 1$ This b_n converges

$\frac{\sqrt{n}}{n^2+1} < \frac{\sqrt{n}}{n^2} \ \forall n \equiv a_n < b_n \ \forall n$

$\therefore$ By comparison test part 1 Σa_n is also convergent series //

(h) $\sum_{n=1}^{\infty} \frac{1-n}{n2^n}$; $a_n = \frac{n-1}{n2^n}$ ($\because$ since we need series with (+) version)

for $n \gg 1$ we expect a_n to behave like $b_n = \frac{1}{2^n} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-1}$

b_n is a geometric series with $|r| = \frac{1}{2} < 1$

Thus Σb_n converges

$a_n = \frac{n-1}{n2^n} < \frac{n}{n2^n} = b_n \equiv a_n < b_n$

By comparison test part 1 Σa_n converges

$\sum_{n=1}^{\infty} \frac{1-n}{n} = (-1) \sum_{n=1}^{\infty} \frac{n-1}{n2^n}$

also $\sum_{n=1}^{\infty} \frac{1-n}{n2^n}$ converges //

(i) $\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n}$; $a_n = \frac{n+2^n}{n^2 2^n}$

$\frac{n+2^n}{2n^2 2^n} < \frac{2^{n+1}}{n^2 2^n} = b_n$

$b_n = \frac{2^{n+1}}{n^2 2^n} = \frac{2}{n^2}$ this is a p series with $P = 2 > 1$

since that Σb_n converges

by comparison test part one ~~seri~~ since b_n converge a_n also converges //

(j) $\sum_{n=1}^{\infty} \frac{10n+1}{n(n+1)(n+2)}$; $a_n = \frac{10n+1}{n(n+1)(n+2)}$

for $n \gg 1$ we expect a_n to behave like $\cancel{b_n = \frac{10n}{n^3}}$

$b_n = \frac{10n}{n^3} = 10/n^2$

$\sum b_n$ is p series with $p = 2 > 1$ Thus $\sum b_n$ Converges

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{(10n+1)n^2}{n(n+1)(n+2)\times 10} = \lim_{n\to\infty} \frac{(10+1/n)}{1(1+1/n)(1+2/n)\times 10} = \frac{10}{10} = 1 > 0$$

By part I of limit comparison test since $\sum b_n$ Converges $\sum a_n$

(02) (a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^4+1}}$; $a_n = \frac{1}{\sqrt{n^4+1}}$

for $n \gg 1$ we expect a_n behave like

$b_n = \frac{1}{\sqrt{n^4}} = 1/n^2$

$\sum b_n$ is p series with $p = 2 > 1$ since that, it converges.

$$\frac{1}{\sqrt{n^4+1}} < \frac{1}{n^2} \quad \forall n \geq 1$$

By comparison test, given series converges

$$S_{10} = \sum_{n=1}^{10} \frac{1}{\sqrt{n^4+1}} = 1.92486$$

$$R_{10} \leq T_{10} = \sum_{n=11}^{\infty} (1/n^2)$$

$$\sum_{n=0}^{\infty} (1/n^2) = \sum_{n=0}^{10} (1/n^2) + \sum_{n=11}^{\infty} (1/n^2)$$

$$\sum_{n=11}^{\infty} (1/n^2) = 1.6479 - 1.5498 = 0.0951$$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^4+1}} \simeq 1.2486$$

(d) $\sum_{n=1}^{\infty} n/(n+1)3^n$; $a_n = n/(n+1)3^n$

for $n >> 1$ we expect a_n to behave like $b_n = \frac{1}{3^n} = \underbrace{\frac{1}{3}}_{a} \times \underbrace{\left(\frac{1}{3}\right)}_{r}^{n-1}$

$$\frac{n}{(n+1)3^n} < \frac{1}{3^n}$$

b_n is geomatric series with $|r| = \frac{1}{3} < 1$ $\sum b_n$ converges
by comparison test I $\sum a_n$ converges

$S_{10} = 0.2836$

$$R_{10} \leq T_{10} = \sum_{n=11}^{\infty} 1/3^n = \sum_{n=1}^{\infty} 1/3^n - \sum_{n=1}^{10} 1/3^n$$

$$= \frac{1/3}{1-1/3} - 0.5 = 0.5 - 0.5 = 0$$

$\sum_{n=1}^{\infty} \frac{n}{(n+1)3^n} = 0.2836$ without an error //