NATIONAL INSTITUTE OF TECHNOLOGY PATNA

Department of Mathematics

Mid-Semester Examination – March 2024

Subject: Linear Programming Problems & Game Theory **Code: MA47103**

Time: 2 hours **Maximum Marks:** $6 \times 5 = 30$

Answer any all questions

1. (a) Let T be a convex set in R^{n+1} and for every $x \in R^n$, let f be defined by

$$f(x) = \inf\{\alpha : \alpha \in R, (x, \alpha) \in T\}.$$

Show that f is convex on R^n.

(b) Verify that $f(x_1, x_2) = 3x_1^2 - 2x_1x_2 + 2x_2^2$ is a convex function on R^2. Let $g_1(x_1) = x_1 - ax_1^3$ and $g_2(x_2) = x_2 - bx_2^3$, where $a > 0$ and $b > 0$. Show that the composite function $f(g_1(x_1), g_2(x_2))$ is not convex.

2. Prove that if a standard linear program with the constraints $Ax = b$ and $x \geq 0$, where A is an $m \times n$ matrix of rank m, has a feasible solution, then it also has a basic solution.

3. (a) Reduce the linear program:

$$\text{Maximize } v = -3x_1 + 5x_2 - 2x_3 + 10x_4$$

subject to

$$\begin{aligned} -5x_1 + 10x_2 - 3x_3 + 6x_4 &\leq 5 \\ -2x_1 + 5x_2 - x_3 + 7x_4 &\leq 8 \\ x_1 \leq 0,\ x_2 \geq 0,\ x_3 &\leq 0,\ x_4 \geq 0 \end{aligned}$$

to the standard form. Find all basic feasible solutions to the problem and identify the optimal solutions.

(b) Use the simplex method to solve the problem

$$\text{Maximize } z = 5x_1 + 2x_2$$

subject to

$$\begin{aligned} x_1 + 4x_2 &\geq 4 \\ 5x_1 + 2x_2 &\geq 10 \\ x_1 \geq 0,\ x_2 &\geq 0 \end{aligned}$$

4. (a) Consider the linear program:

$$\text{Minimize } z = 2x_1 + x_2 + 3x_3 + x_4$$

subject to

$$\begin{aligned} 2x_1 + x_2 - 4x_3 + 2x_4 &= 3/2 \\ -x_1 + x_2 + 2x_3 - 3x_4 &= 1/4 \\ x_i \geq 0 \ \ \forall i & \end{aligned}$$

Prove that an optimal basic feasible solution to the program exists.

NATIONAL INSTITUTE OF TECHNOLOGY PATNA

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

MID-SEMESTER EXAMINATION - MARCH, 2024

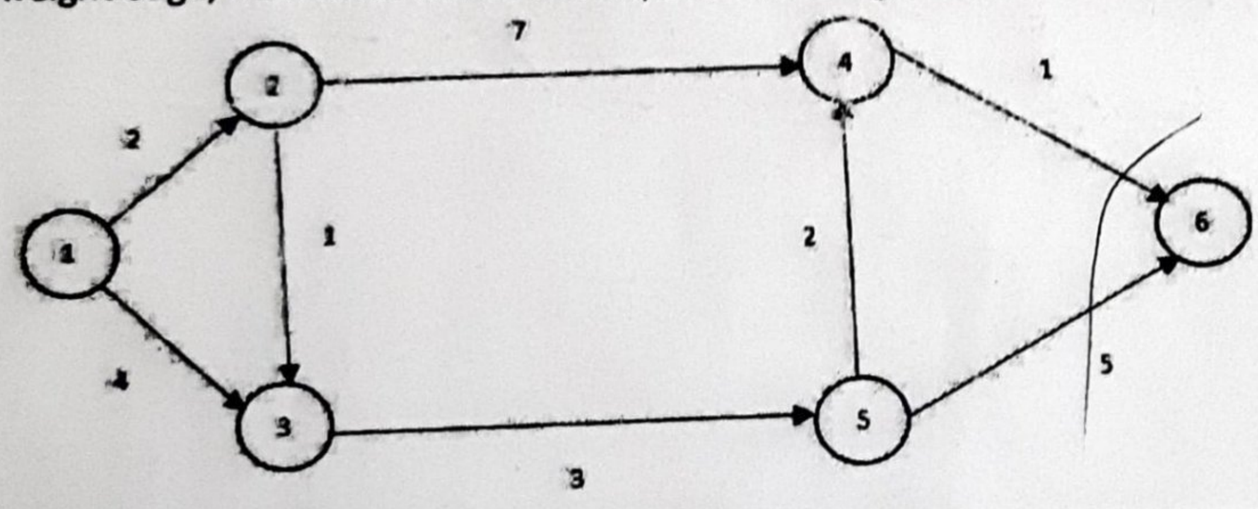
B. Tech (Computer Science & Engineering) IVth Semester **(SECTION – A, B, and C)**

Course: Design and Analysis of Algorithms Max. Marks:30 Max. Time: 2 Hrs.

Course Code: CS44115 (for B.Tech-CSE and DD-CSE-CS & DS) / CS47101 (for DD-MA-MCT)

Instruction:

1. Answer all questions.
2. The Marks, CO (Course Outcome) and BL (Bloom's Level) related to questions are mentioned on the right-hand side margin.

Q. No	Question	Marks	CO	BL
1	a. Solve the following recurrence relations using Master's Theorm. $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{log^2 n}$ $T(n) = 2T\left(\frac{n}{2}\right) + n^2$	3M	CO1	Understand Apply
	b. Analyze the the worst-case, average-case, and best-case time complexity of binary search algorithm.	3M	CO1	Analyze
2	Use the Dijkstra's algorithm to find the shortest path from starting vertex 1 to all other vertices in a given graph where each edge 'e' has a length $l_e \geq 0$, indicating the time (or distance, or cost) it takes to traverse 'e'. Show all the steps of finding the shortest path and also analyse the worst case time complexity of Dijkstra's algorithm. If there is the negative weight edge, then what will be the impact on the algorithm? 	6M	CO1 & CO4	Evaluate Analyze
3	We are given '*n*' objects and a knapsack. Object '*i*' has a weight 'w_i' and the knapsack has a capacity '*m*'. If a fraction of x_i, $0 \leq x_i \leq 1$, of object '*i*' is placed into the knapsack, then a profit of $p_i x_i$ is earned. The objective is to obtain a filling of the knapsack that maximizes the total profit earned. Formally, maximise $\sum_{i=1}^{n} p_i x_i$ subject to $\sum_{i=1}^{n} w_i x_i \leq m$ and $0 \leq x_i < 1, \quad 1 \leq i \leq n$. Here, the profit and weight are positive numbers. With above description, consider the following instance of knapsack problem. Show that what are the feasible solutions available? Which solution gives the optimal value of profit? ***n=3, m=20, (p1, p2, p3) = (25, 24, 15) , and (w1, w2, w3) = (18, 15, 10)*** .	6M	CO4 & CO5	Apply Create
4	Derive a solution for matrix chain multiplication problem using dynamic programming with necessary equation and computational complexity analysis. Determine the minimum number of multiplications using dynamic programming of a matrix-chain product whose sequence of dimensions is <5, 10, 3, 12, 5, 50, 6>.	6M	CO3	Understand Evaluate
5	a. A file contains the following characters with the frequencies as shown. Construct the Huffman Tree and write the Huffman Code for each character.	3M	CO4	Remember Apply

a	b	c	d	e	f
6	2	3	3	4	9

b. Find Minimum and Maximum elements of an array X[0 : 9] = (45, 83, 75, 17, 43, 37, 80, 53, 61, 22) using divide and conquer strategy. Show all the steps clearly.	3M	CO2 & CO5	Evaluate

National Institute of Technology Patna

Computer Science and Engineering Mid semester Exaamination March 2024

Course CS: 4564 Artificial Intelligence 4th Semester M. Sc. Mathematics and Computing

Time 2 hrs. **Marks: 30**

Answer All questions

Qs. No.	Question	Marks	CO	BO
1.	b) Describe three dimensional architecture of AI . c) Describe the BFS and DFS heuristic search methods.	5 5	CO1	Comprehensive
2.	a) Describe the rule based model and write in predicate form the following prepositions • Ram reads book and Shyam plays football. • Ali women love their children. • Some students like games and all students like reading. b) Explain Analogical and case base reasoning and its processes.	5 5	CO1 CO1	Comprehensive Comprehensive
3.	Derive an expression for delta rule. There are three inputs X1=.1, X2=.2 and X3=.3 and two output desired Y1=1.0 and Y2= 0.01. Taking initial weights of one calculate the change of weight up to two iteration for Y1.	10	CO2	Analytical

National Institute of Technology Patna

Department of Mathematics

Mid Semester Examination :March 2024

MA47101 : Numerical Methods for Engineers

Branch: B. Tech + M. Tech (Dual Degree) M&C Technology

Maximum Marks: 30

Time: 02.00 hours

Semester: 4th

Answer Any Six Questions

1. (a) Obtain the number of significant digits in x_A when compared to x for $x_A = 233.1223$ and $x = 233.12543$

   (b) Find the condition number at a point $x = c$ for the function $f(x) = 12.12 * x^2 + 1$

2. Let $x < 0 < y$ be such that the approximate numbers x_A and y_A has seven and nine significant digits with x and y respectively. Show that $z_A := x_A - y_A$ has at least six significant digits when compared to $z := x - y$.

3. Draw the graph of a function that satisfies the hypothesis of bisection method on the interval $[0, 1]$ such that the errors e_1, e_2, e_3 satisfy $e_1 > e_2$, and $e_2 < e_3$. Give formula for one such function.

4. Let bisection method be used to solve the nonlinear equation (x is in radians) $x \sin x - 1 = 0$ starting with the initial interval $[0, 2]$. In order to approximate a solution of the non- linear equation with an absolute error less than or equal to 10^{-3}, what is the minimum number of iterations required? Also find the corresponding approximate solution.

5. Use Cholesky factorization, calculate the decomposition $A = LL^T$ for $A = \begin{bmatrix} 2.25 & -3.0 & 4.5 \\ -3.0 & 5.0 & -10.0 \\ 4.5 & -10.0 & 34 \end{bmatrix}$.

   Also fine x such that $Ax = b$ where $b = [1, 1, 1]^t$

6. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$. Find the norm of A_∞, A_2 and A_1. Also verify that $\|Ax\| \leq \|A\|\|x\|$ for the any of the two norm defined above (The vector norm is understood in the sense that the matrix norm is subordinate to it.).

7. Consider the following system of linear equations,

$$0.8647x_1 + 0.5766x_2 = 0.2885$$
$$0.4322x_1 + 0.2882x_2 = 0.1442.$$

   (a) Find the exact solution of the linear system.

   (b) Using 4-digit chopping, do one iteration of residual corrector method to obtain $x(1)$. Use the initial guess to be $(0, 0)^T$. Compute the residual vector for the solution obtained.

   (c) Solve using Gauss-Jacobi method and tabulate the results upto first 5 iteration. Use the initial guess to be $(0.0)^T$

--- All the Best ---

MID SEMESTER EXAMINATION March'2024
DEPARTMENT OF MATHEMATICS
Subject: Graph Theory and Applications (MA47102)

Course Mathematics & Computing
Semester: 4th sem.

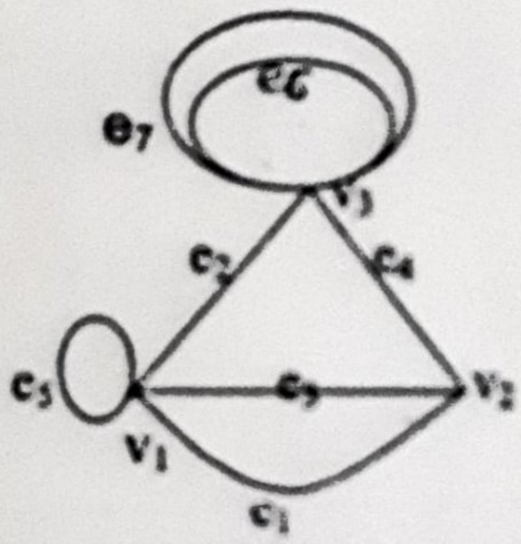
Time:2hrs
F.M.:30

Answer any Four questions

1(a) Two graphs G_1 and G_2 are isomorphic if and if only if their complement $\overline{G}_1$ and $\overline{G}_2$ are isomorphic.

(b) If G and H are isomorphic graphs, then the degree s of the vertices of G are the same as the degrees of the vertices of H.

2(a) Write down the adjacency and incidence matrix of the graph using the ordering.



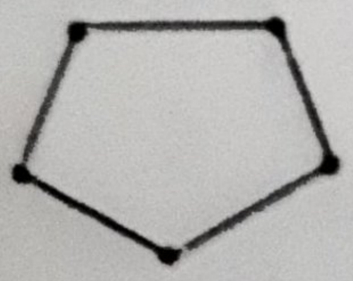
(b) Define adjacency and incidence matrices. Draw the graph of the adjacency matrix:

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

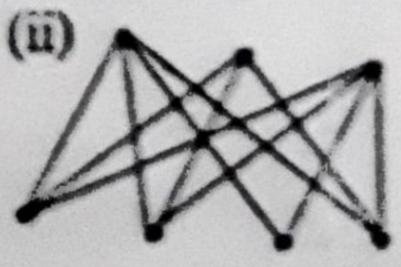
3(a) Define Regular graph. Construct a 5-regular graph on 10 vertices and 8 vertices

(b) Find Complement of the following graph:

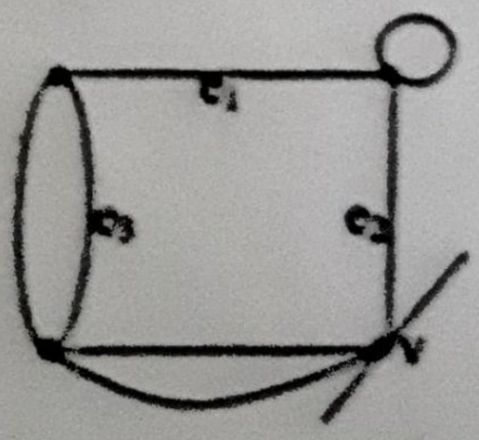
(i)



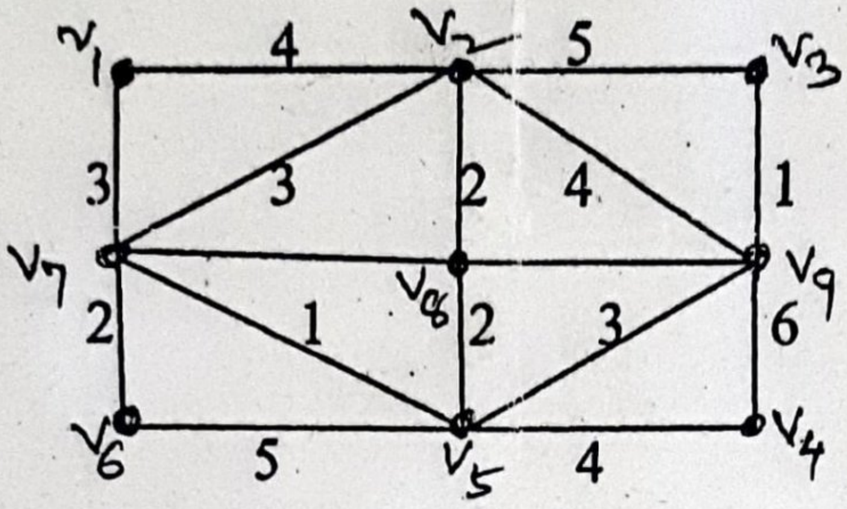
(ii)



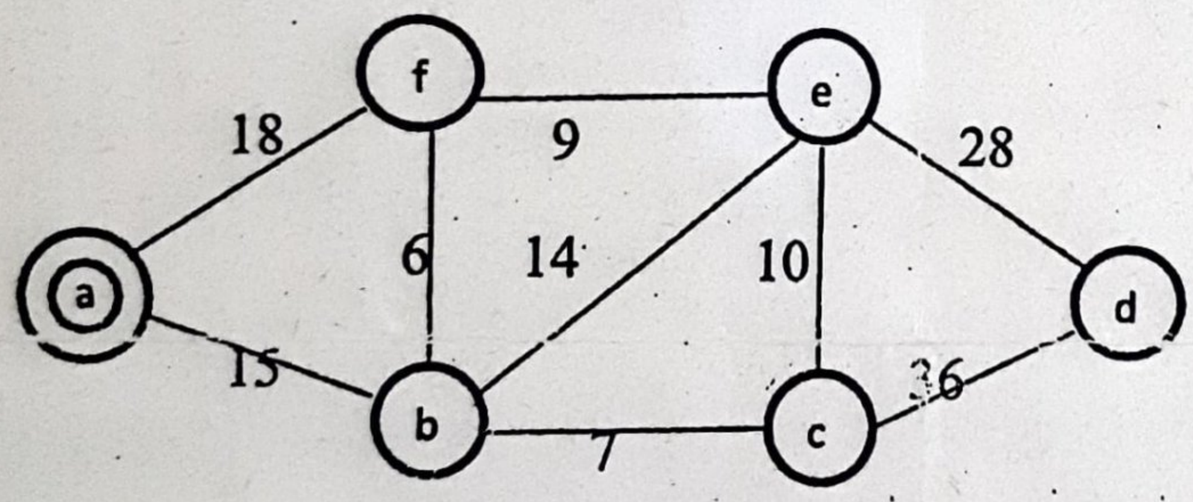
(c) Draw edge deleted graph and vertex deleted graph of the following graph:



4 Write Kruskal's algorithm.
Apply Kruskal's Algorithm to determine a minimal spanning tree of the following weighted graph G with nine vertices.



5 Write Djkastra's algorithm and apply the same to find source shortest paths problem for the following graph taking vertex 'a' as source in following figure:



6(a) Prove that a graph G is disconnected iff its vertex set V can be partitioned into two nonempty disjoint subsets V_1 and V_2 such that there exists no edges in G whose one end vertex is in subset V_1 and the other in subset V_2.

(b) Prove that If a graph (connected/disconnected) has exactly two vertices of odd degrees, there must be a path joining these two vertices.

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