NATIONAL INSTITUTE OF TECHNOLOGY PATNA

Department of Mathematics

End-Semester Examination – May 2022

Subject: Linear Programming Problem & Game Theory **Code: MA6701**

Time: 3 hours **Maximum Marks:** $10 \times 6 = 60$

Answer any **all** questions

1. Prove that a set $S \subset R^n$ is convex if and only if every convex combination of any finite number of points of S is contained in S.

2. (a) If $f(x)$ and $g(x)$ are convex functions, show that $f(x)g(x)$ is not necessarily a convex function. Justify with an example.

   (b) If $f(x)$ is both convex and concave on R^n, show that $f(x)$ must be a linear function.

3. Use simplex method to solve:

$$\text{Maximize } z = x_2$$

subject to

$$\begin{aligned} x_1 + x_2 &\geq 2 \\ x_1 + x_2 &\leq 3 \\ x_1 - x_2 &\leq 2 \\ -x_1 + x_2 &\leq 2 \\ x_i \geq 0 \ \ &(i = 1, 2). \end{aligned}$$

Plot the feasible region in the x_1x_2-plane and show the steps graphically. On the graph so obtained, interpret the shift from one basic feasible solution to the next.

4. Solve the following program by the Charnes M-Technique:

$$\text{Maximize } z = 2x_3 + 2x_4 + 2x_5$$

subject to

$$\begin{aligned} x_1 + x_3 - x_4 + 2x_5 &= 2 \\ x_2 - x_3 + 2x_4 - x_5 &= 1 \\ x_i \geq 0 \ \ (i = 1, 2, 3, 4, 5)&. \end{aligned}$$

5. Solve the following LPP by the two-phase simplex method:

$$\text{Minimize } z = -x_1 - x_2$$

subject to

$$\begin{aligned} x_1 - x_2 - x_3 &= 1 \\ -x_1 + x_2 + 2x_3 - x_4 &= 1 \\ x_i \geq 0 \ \ (i = 1, 2, 3, 4)&. \end{aligned}$$

6. Use the duality theory to prove that the following linear programming problem is feasible but has no optimal solution.

$$\text{Minimize } z = 3x_1 - 5x_2 + x_3$$

subject to

$$x_1 - 2x_3 \geq 4$$
$$2x_1 - x_2 + x_3 \geq 2$$
$$x_i \geq 0 \quad (i = 1, 2, 3).$$

7. There are five pumps available for developing five wells. The efficiency of each pump in producing the maximum yield at each well is shown in the following table:

	Well				
Pump	(1)	(2)	(3)	(4)	(5)
1	45	40	65	30	55
2	50	30	25	60	30
3	25	20	15	20	40
4	35	25	30	25	20
5	80	60	60	70	50

In what way should the pumps be assigned so as to maximize the overall efficiency?

8. A petroleum company has three refineries A, B, and C which produce 150 units, 220 units and 130 units of petrol, respectively. The company owns four warehouses (1), (2), (3), and (4), each of which must receive 110 units, 120 unit, 150 unit, and 120 units of petrol, respectively. The transportation costs (per unit) are given in the following table:

	To warehouse			
From Refinery	(1)	(2)	(3)	(4)
A	65	45	35	75
B	60	55	20	80
C	60	50	30	85

(a) Find an initial basic feasible solution by the least cost rule.

(b) Find an optimal solution starting with the obtained initial basic feasible solution.

9. Prove that every symmetric game has the value $v = 0$, and each player has the same set of optimal strategies.

10. Solve the game by linear programming with the pay-off matrix

$$\begin{pmatrix} 2 & -3 & -1 & 1 \\ 0 & 2 & 1 & 2 \end{pmatrix}$$

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