Estimating a simpel model by simulated minimum distance (SMD)

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1 Estimating a simpel model by simulated minimum distance (SMD)

Inspiration: This notebook is based on SMD.ipynb notebook from last lecture which is again based on a Bradley Setzler's blog post.

1.1 Getting started

- Download:
  - Open source version
  - JuliaPro from Julia Computing (bundled with IDE and notebook support)
- Documentation (language and about 1900 packages)
- Community:
  - Discourse
  - Slack

For introductory material on Julia for Economists, see https://lectures.quantecon.org/jl/

Some initial setup

```julia
using Random, Statistics, PyPlot, Optim
include("model.jl"); # Use trailing semi-colon to suppress output
```

1.2 The economic model

Let c_i denote consumption and $0 \leqslant l_i \leqslant 1$ denote leisure. Consider an agent who wishes to maximize Cobb-Douglas utility over consumption and leisure subject to a budget constraint. That is

$$\max_{c_i,l_i} c_i^{\gamma} l_i^{1-\gamma} \text{s.t.} c_i \leqslant (1-\tau)w_i(1-l_i) + \varepsilon_i$$

where $0 \leqslant \gamma \leqslant 1$ is the relative preference for consumption, w_i is wage earned from supplying labor, τ is the tax rate and ε_i is a non-labor income shock.

This model has a closes form solution given by:

$$c^{\star}(w_i, \varepsilon_i; \gamma) = \gamma(1-\tau)w_i + \gamma\varepsilon_i l^{\star}(w_i, \varepsilon_i; \gamma) = (1-\gamma) + \frac{(1-\gamma)\varepsilon_i}{(1-\tau)w_i}$$

1.3 The empirical goal

We will impose a set of simplifying assumptions:

- The unobserved income shock is iid with known distribution, $\varepsilon_i \sim N(0, \sigma^2)$.
- The individual wages, w_i, are observed along with consumption and labor choices for $n = 10,000$ individuals.

The goal is to estimate the relative preference for consumption and leisure, γ, and the tax rate, τ in this model. The set of parameters to be estimated thus is $\theta = (\gamma, \tau, \sigma)$.

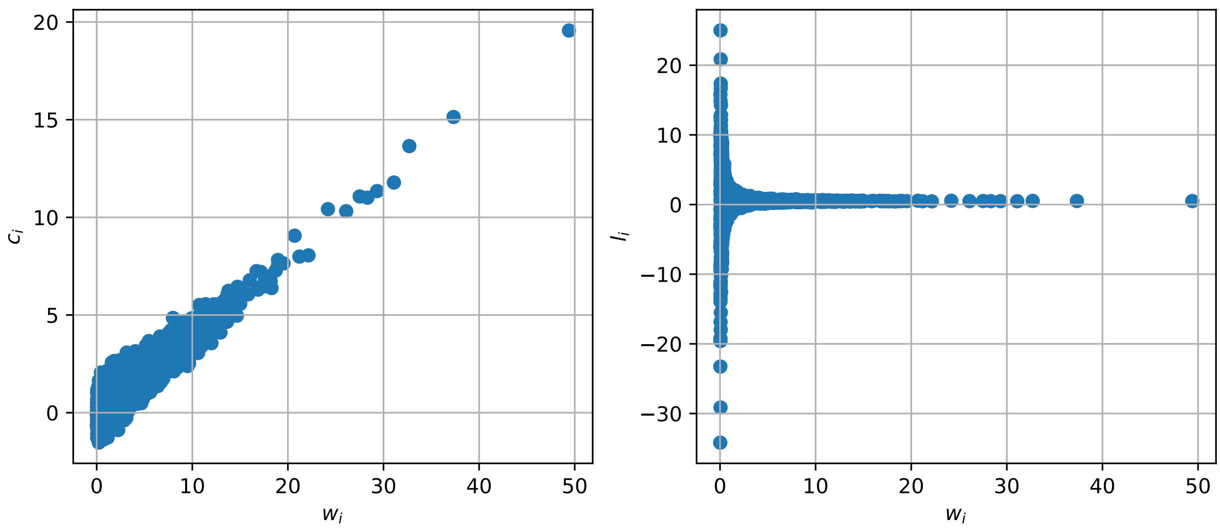
To this end, we assume that we have a dataset consisting of $\{w_i, c_i, l_i\}_{i=1}^{n}$. To simulate such a dataset we run the following code.

```
# a.  true parameters in the data generating process
### In Julia, it is often convenient to use NamedTuples for storing parameters
θ = (γ=0.5, τ=0.2, σ=1.0)

# b.  simulate observed dataset
### For reproducibility, it is a good idea, to explicitly pass a RNG (here
    MersenneTwister)
### object to the random number functions
rng = MersenneTwister(123)
n = 10_000

## stochastic variables
### distribution of log-wages are (arbitrarily) chosen to be standard normal
w = exp.(randn(rng, n))
e = θ.σ*randn(rng, n)

# optimal choices
### We use the dot-syntax `foo.`` to make the function *broadcast* over all elements of
    the
### arrays and we use `Ref` to signal that an argument should be treated as a scalar in
    the
### broadcast operation.
con = Model.c_star.(w, e, Ref(θ))
lab = Model.l_star.(w, e, Ref(θ));
```

1.4 Visualizing the data

We could imagine plotting the scatter of consumption and labor supply against wages.

```
### There are several plotting packages in Julia but for here we use the PyPlot.jl
    package
### which wraps Python's matplotlib.  Notice, that it is possible to reuse much of the
### Python syntax dispite this being evaluated as normal Julia code.
fig = plt.figure(figsize=(10,4))
ax = fig.add_subplot(1,2,1)
ax.scatter(w, con)
ax.grid(true)
ax.set_xlabel(raw"$w_i$")
ax.set_ylabel(raw"$c_i$")

ax = fig.add_subplot(1,2,2)
ax.scatter(w, lab)
ax.grid(true)
ax.set_xlabel(raw"$w_i$")
ax.set_ylabel(raw"$l_i$")
fig
```



We note that there is a clear relationship between consumption and wages. We could thus imagine estimating the two parameters in θ using the correlation between the consumption and wages, the average labor supply and the variance of consumption.

1.5 Identification

From the solution we note that the correlation between wages and consumption depends on γ and τ, the mean of the labor supply should be $1-\gamma$ and that the variance of consumption should be proportional to the variance of $varepsilon$, σ^2. These moments, in turn, should be able to identify the parameters in θ.

We can denote the moments in the data as $\Lambda^{\text{data}} = (\text{corr}(w_i, c_i), \text{mean}(l_i), \text{var}(c_i))$ and calculate them as:

```
moments_fun = (w, con, lab) -> (cor(w, con), mean(lab), var(con))
mom_data = moments_fun(w, con, lab)
mom_data
```

```
(0.8673269168418695, 0.4810790538523146, 1.0050937857814541)
```

1.6 An SMD Estimator

We can then estimate θ by minimizing the squared distance between the empirical moments in the data, Λ^{data}, and the same moments calculated from $S = 100$ simulated agents for each of the n values of w_i from the model for each guess of θ, $\Lambda^{\text{sim}}(\theta; w)$.

Specifically, we simulate draws of $\{\varepsilon_i^{(s)}\}_{i=1,s=1}^{n,S}$,calculate the synthetic dataset for a value of θ, $\{w_i, s_i^{(s)}, l_i^{(s)}\}_{i=1,s=1}^{n,S}$, and stack these $n \times S$ observations to calculate the moments above to get $\Lambda^{\text{sim}}(\theta; w)$.

Our **SMD estimator** is

$$\hat{\theta} = \arg\min_{\theta} Q(\theta; w)$$

where the objective function is $Q(\theta, w) = ||\Lambda^{\text{data}} - \Lambda^{\text{sim}}(\theta, w)||^2$

The objective function could be coded as done in the `obj_fun`. In the `model.jl` file.

1.7 Objective function

We specify which parameters to be estimated and construct initial starting values as

```
θ_0 = (γ=0.4, τ=0.15, σ=0.9)   # initial guesses
```

```
(γ = 0.4, τ = 0.15, σ = 0.9)
```

And we can evaluate the objective function as

```
obj_at_θ_0 = Model.obj_fun(θ_0, w, mom_data, moments_fun)
```

```
0.12185671913345228
```

2 Call optimizer

We then call an unconstained optimizer using numerical gradients with option to print iterations

```julia
### The optimization function passes a `Vector` to the objective function so first we
    have
### to construct a `NamedTuple` from the `Vector`
obj_fun = vθ -> Model.obj_fun((γ=vθ[1], τ=vθ[2], σ=vθ[3]), w, mom_data, moments_fun)

results = optimize(obj_fun, [θ_0...], BFGS()) # [θ_0...]  converts the NamedTuple to a
    Vector
```

```
Results of Optimization Algorithm
 * Algorithm: BFGS
 * Starting Point: [0.4,0.15,0.9]
 * Minimizer: [0.520726900724164,0.23427069484126523, ...]
 * Minimum: 9.384266e-20
 * Iterations: 9
 * Convergence: true
   * |x - x'| ≤ 0.0e+00: false
     |x - x'| = 3.52e-07
   * |f(x) - f(x')| ≤ 0.0e+00 |f(x)|: false
     |f(x) - f(x')| = 1.76e+05 |f(x)|
   * |g(x)| ≤ 1.0e-08: true
     |g(x)| = 3.69e-10
   * Stopped by an increasing objective: false
   * Reached Maximum Number of Iterations: false
 * Objective Calls: 24
 * Gradient Calls: 24
```

```julia
println("True value:", θ_0)
```

```
True value:(γ = 0.4, τ = 0.15, σ = 0.9)
```

```julia
println("Estimated value:", NamedTuple{keys(θ_0)}(round.(results.minimizer, digits=2)))
```

```
Estimated value:(γ = 0.52, τ = 0.23, σ = 0.96)
```