Space and Congruence Compression of Proofs

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European Master in Computational Logic

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Space and Congruence Compression of **Proofs**

Resolution

Resolution Rule

$$\frac{C \lor x \quad D \lor \neg x}{C \lor D}$$

Resolution

Resolution Rule

$$\begin{tikzcd}
C, x & & D, \neg x \\
& C, D \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

Example

$$(x_1 \lor x_2 \lor x_3) \land (x_1 \lor \neg x_2) \land (x_1 \lor \neg x_3) \land (\neg x_1)$$

Example

$$(x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2) \wedge (x_1 \vee \neg x_3) \wedge (\neg x_1)$$

$\neg x_1$

Example

$$(x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2) \wedge (x_1 \vee \neg x_3) \wedge (\neg x_1)$$

$$\begin{tikzcd}
x_1, x_2, x_3 & & \neg x_1 \\
& x_2, x_3 \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

Example

$$(x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2) \wedge (x_1 \vee \neg x_3) \wedge (\neg x_1)$$

$$\begin{tikzcd}
x_1, x_2, x_3 & & \neg x_1 & & x_1, \neg x_2 \\
& x_2, x_3 \arrow[ul] \arrow[ur] & & \neg x_2 \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

Example

$$(x_1 \lor x_2 \lor x_3) \land (x_1 \lor \neg x_2) \land (x_1 \lor \neg x_3) \land (\neg x_1)$$

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x_1, x_2, x_3 & & \neg x_1 & & x_1, \neg x_2 & & x_1, \neg x_3 \\
& x_2, x_3 \arrow[ul] \arrow[ur] & & \neg x_2 \arrow[ul] \arrow[ur] & & \neg x_3 \arrow[ulll] \arrow[ur] &
\end{tikzcd}$$

Example

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$$\begin{tikzcd}
x_1, x_2, x_3 & & \neg x_1 & & x_1, \neg x_2 & & x_1, \neg x_3 \\
& x_2, x_3 \arrow[ul] \arrow[ur] & & \neg x_2 \arrow[ul] \arrow[ur] & & \neg x_3 \arrow[ulll] \arrow[ur] & \\
& & x_3 \arrow[ul] \arrow[ur] & & & &
\end{tikzcd}$$

Example

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x_1, x_2, x_3 & & \neg x_1 & & x_1, \neg x_2 & & x_1, \neg x_3 \\
& x_2, x_3 \arrow[ul] \arrow[ur] & & \neg x_2 \arrow[ul] \arrow[ur] & & \neg x_3 \arrow[ulll] \arrow[ur] & \\
& & x_3 \arrow[ul] \arrow[ur] & & & & \\
& & & \bot \arrow[ul] \arrow[uurr] & & &
\end{tikzcd}$$

Example

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x_1, x_2, x_3 & & \neg x_1 & & x_1, \neg x_2 & & x_1, \neg x_3 \\
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& & x_3 \arrow[ul] \arrow[ur] & & & & \\
& & & \bot \arrow[ul] \arrow[uurr] & & &
\end{tikzcd}$$

Length: 9
Best Space: 4

Example

$$(x_1 \lor x_2 \lor x_3) \land (x_1 \lor \neg x_2) \land (x_1 \lor \neg x_3) \land (\neg x_1)$$

$$\begin{tikzcd}
x_1, x_2, x_3 & & x_1, \neg x_2 & & \\
& x_1, x_3 \arrow[ul] \arrow[ur] & & x_1, \neg x_3 & \\
& & x_1 \arrow[ul] \arrow[ur] & & \neg x_1 \\
& & & \perp \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

Length: 7
Best Space: 3

Space and Congruence **Compression** of Proofs

Proof Compression

- Smaller unsat cores, interpolants
- Easier proof processing
- Smaller proofs libraries
- Easier trusted interaction of deductive systems
- Proof generalization

Synthesizing Multiple Boolean Functions using Interpolation on a Single Proof

- Georg Hofferek, et al, 2013, TU Graz

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Method

1. Formulate problem in SMT theory of uninterpreted functions
2. Obtain proof of unsatisfiability from SMT solver
3. Transform proof (local first, colorable)
4. Extract a single *n*-interpolant from the proof
5. Extract multipple interpolants the single interpolant

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- Worst case exponential runtime in proof size

Proof Compression using Skeptik

- Input proof: 1,870,407 nodes
- Output proof: 868,760 nodes (53,6% compression)

Space and **Congruence** Compression of Proofs

Example

Knowledge

1. $f(a) = a$
2. $a = b$
3. $b = f(b)$
4. $f(a) \neq f(b)$

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Proof

Equality is transitive, therefore from $f(a) = a$, $a = b$ and $b = f(b)$ follows $f(a) = f(b)$, which contradicts $f(a) \neq f(b)$

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A different Proof

$f(.)$ is a function, therefore from $a = b$ follows $f(a) = f(b)$, which contradicts $f(a) \neq f(b)$

A proof

$$\begin{tikzcd}
f(a) \neq a, a \neq b, b \neq f(b), f(a) = f(b) & & f(a) = a & & \\
& a \neq b, b \neq f(b), f(a) = f(b) \arrow[ul] \arrow[ur] & & a = b & \\
& & b \neq f(b), f(a) = f(b) \arrow[ul] \arrow[ur] & & b = f(b) \\
& f(a) \neq f(b) & & f(a) = f(b) \arrow[ul] \arrow[ur] & \\
& & \perp \arrow[ul] \arrow[ur] & &
\end{tikzcd}$$

A different proof

$$\begin{tikzcd}
 & a \neq b, f(a) = f(b) & & a = b \\
f(a) \neq f(b) & & f(a) = f(b) \arrow[ul] \arrow[ur] & \\
 & \perp \arrow[ul] \arrow[ur] & &
\end{tikzcd}$$

Definitions

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Ground Terms

- Constants $a, b, c, \ldots$
- Compound Terms $f(t_1, \ldots, t_n)$

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Congruence Relation R

- Reflexive: $\forall t (t, t) \in R$
- Symmetric: $(s, t) \in R \Rightarrow (t, s) \in R$
- Transitive: $(t_1, t_2) \in R \ldots (t_{m-1}, t_m) \in R \Rightarrow (t_1, t_m) \in R$
- Compatible: $\forall i (t_i, s_i) \in R \Rightarrow (f(t_1, \ldots, t_n), f(s_1, \ldots, s_n)) \in R$

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Congruence Closure E^* of set of equations E

- Smallest Congruence Relation containing E
- Computable in $O(n \log(n))$
- E is explanation for $(s, t) \in E^*$

Example revisited

Knowledge

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Example revisited

Knowledge

1. $f(a) = a$
2. $a = b$
3. $b = f(b)$
4. $f(a) \neq f(b)$

Explanation for $f(a) = f(b)$

$\{\ f(a) = a,\ a = b\ ,\ b = f(b)\ \}$

Example revisited

Knowledge

1. $f(a) = a$
2. $a = b$
3. $b = f(b)$
4. $f(a) \neq f(b)$

Explanation for $f(a) = f(b)$

$\{ \qquad a = b \, , b = f(b) \, \}$

Example revisited

Knowledge

1. $f(a) = a$
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3. $b = f(b)$
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Explanation for $f(a) = f(b)$

$\{ \quad a = b \quad \}$

Example revisited

Knowledge

1. $f(a) = a$
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4. $f(a) \neq f(b)$

Explanation for $f(a) = f(b)$

$\{ \qquad a = b \qquad \}$

Short explanation $\rightsquigarrow$ short (sub)proof

Short Explanation Decision Problem

Given a set of input equations E, a target equation $s = t$ and $k \in \mathbb{N}$, does there exist an explanation $E' \subseteq E$ of $s = t$ with $|E'| \leq k$?

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NP-complete

NP-completeness proof sketch

From a propositional logic formula Φ obtain ...

- a set of equations E_Φ
- a target equation $s_\Phi = t_\Phi$
- $k_\Phi \in \mathbb{N}$

such that ...

Φ is satisfiable if and only if there is an explanation $E' \subseteq E_\Phi$ of $s_\Phi = t_\Phi$ with $|E'| \leq k_\Phi$

NP-completeness proof sketch example

Formula

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2)$$

Translation to equations

NP-completeness proof sketch example

Formula

$$(x_1 \lor x_2 \lor \neg x_3) \land (\neg x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2)$$

Translation to equations

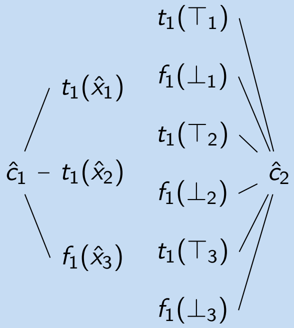
$$\begin{tikzcd}
 & t_1(\hat{x}_1) \\
\hat{c}_1 \arrow[ur, dash] \arrow[r, dash] \arrow[dr, dash] & t_1(\hat{x}_2) \\
 & f_1(\hat{x}_3)
\end{tikzcd}$$

NP-completeness proof sketch example

Formula

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2)$$

Translation to equations



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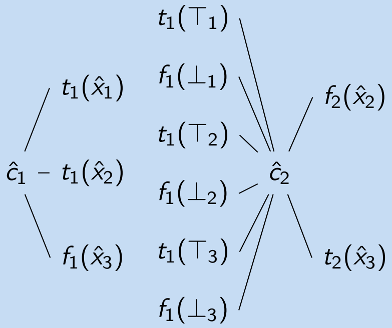

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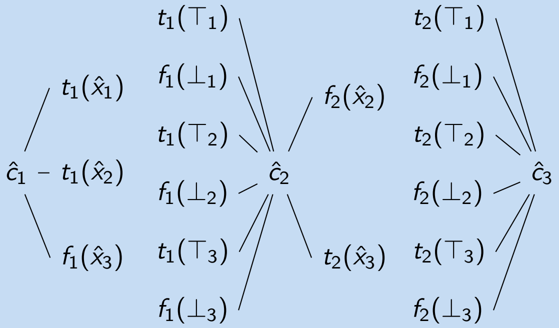

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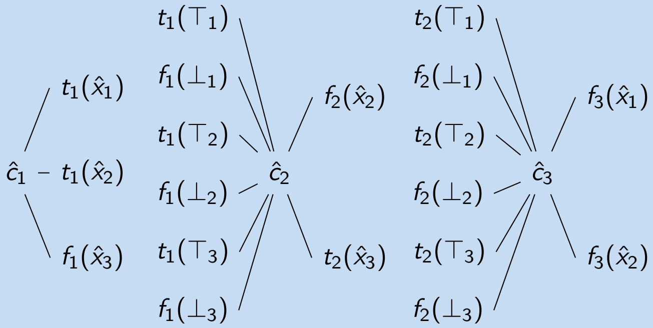
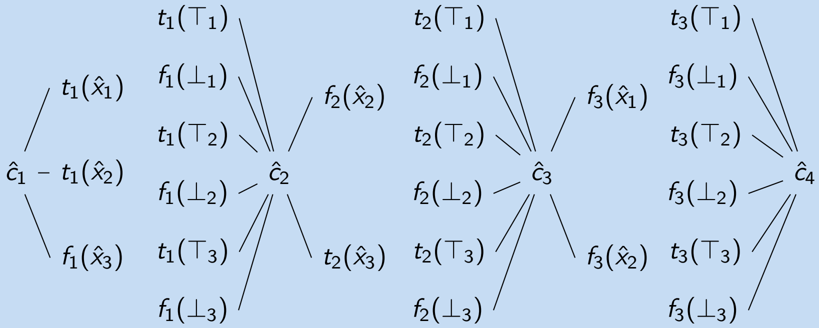


NP-completeness proof sketch example

Formula

$$(x_1 \lor x_2 \lor \neg x_3) \land (\neg x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2)$$

Translation to equations



NP-completeness proof sketch example

Formula

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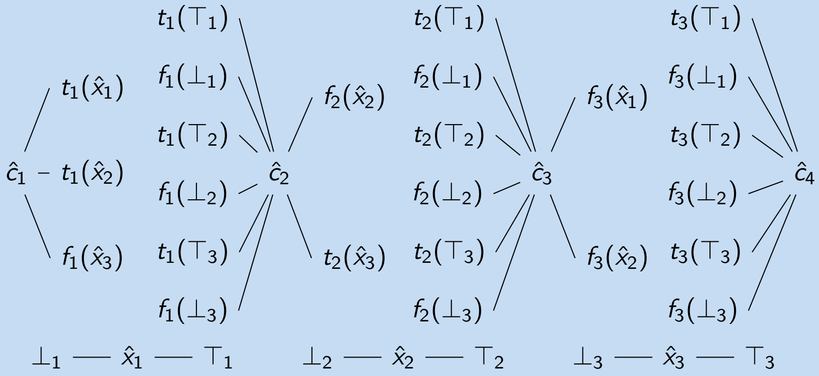
Translation to equations



NP-completeness proof sketch example

Formula

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2)$$

Small subset corresponding to satisfying assignment

$\hat{c}_1 - t_1(\hat{x}_1) \quad t_1(\top_1) - \hat{c}_2 - f_2(\hat{x}_2) \quad f_2(\bot_2) - \hat{c}_3 - f_3(\hat{x}_2) \quad f_3(\bot_2) - \hat{c}_4$

$\hat{x}_1$ —— $\top_1$ $\qquad$ $\bot_2$ —— $\hat{x}_2$ $\qquad$ $\hat{x}_3$ ——— $\top_3$

Space and Congruence Compression of Proofs

Is it possible to remove parts of the proof from memory during proof processing?

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Which parts?

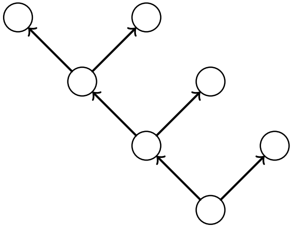
Space Measure Example

$$\begin{tikzcd}
x_1, x_2, \neg x_3 & & x_1, x_2 & & \\
& x_1, \neg x_3 \arrow[ul] \arrow[ur] & & x_1, x_3 & \\
& & x_1 \arrow[ul] \arrow[ur] & & \neg x_1 \\
& & & \bot \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

Space Measure Example

Maximum number of nodes in memory: 0

○	Not in memory
●	In memory

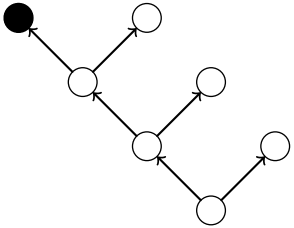


Space Measure Example

Maximum number of nodes in memory: 1

○ Not in memory

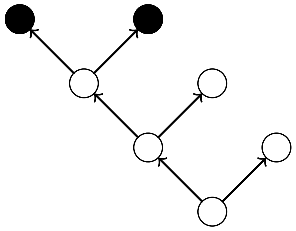
● In memory



Space Measure Example

Maximum number of nodes in memory: 2

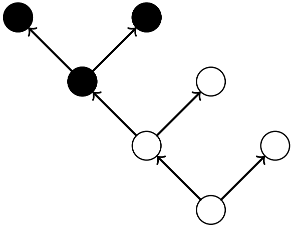
○	Not in memory
●	In memory



Space Measure Example

Maximum number of nodes in memory: 3

○	Not in memory
●	In memory



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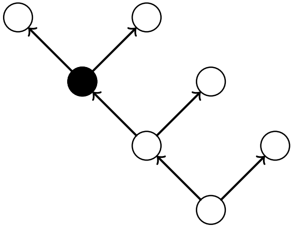
●	In memory



Space Measure Example

Maximum number of nodes in memory: 3

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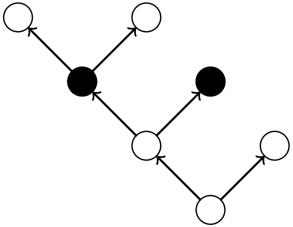
● In memory



Space Measure Example

Maximum number of nodes in memory: 3

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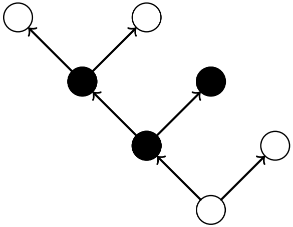
●	In memory



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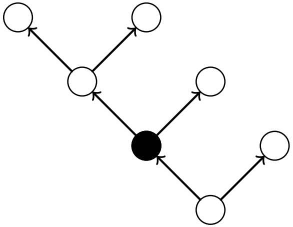

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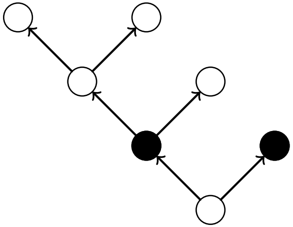

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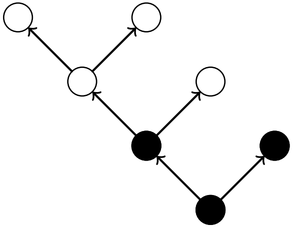

Space Measure Example

Maximum number of nodes in memory: 0

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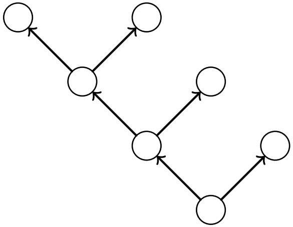

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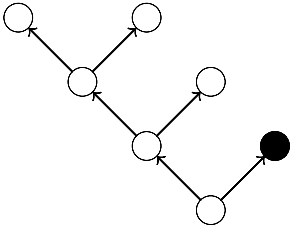

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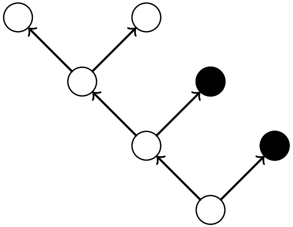

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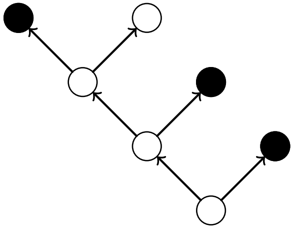

Space Measure Example

Maximum number of nodes in memory: 4

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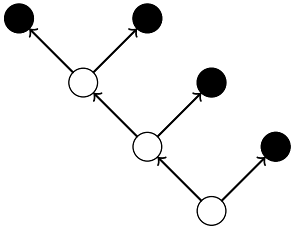

Space Measure Example

Maximum number of nodes in memory: 5

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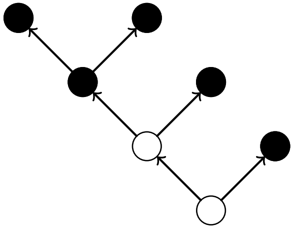

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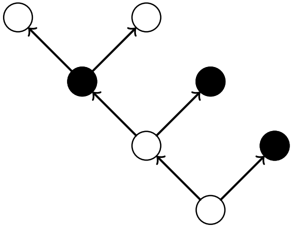

Space Measure Example

Maximum number of nodes in memory: 5

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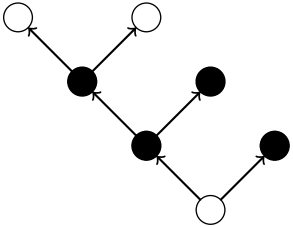

Space Measure Example

Maximum number of nodes in memory: 5

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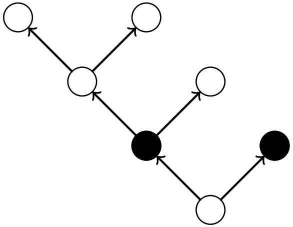

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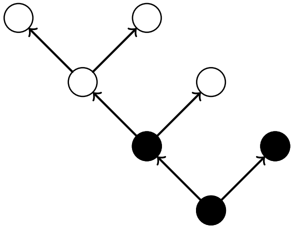
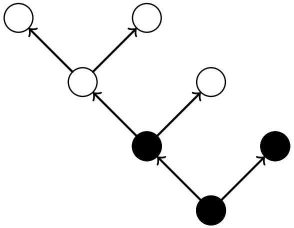


Space Measure Example

Maximum number of nodes in memory: 5

○ Not in memory

● In memory



Good traversal orders are essential!

Space Measure

Space measure of a proof and a traversal order

Maximal amount of nodes that have to be kept in memory at once while processing the proof following the traversal order

Construct Traversal Orders

Construct Optimal Order

- NP-complete
- Optimal strategy in some pebbling game

Construct Good Order

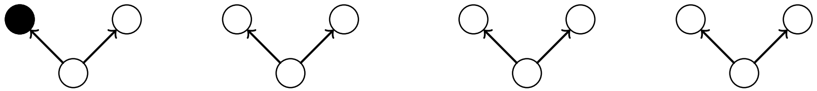
- Top-Down
- Bottom-Up
- Heuristic choices

Construct Order Top-Down

Maximum number of nodes in memory: 0

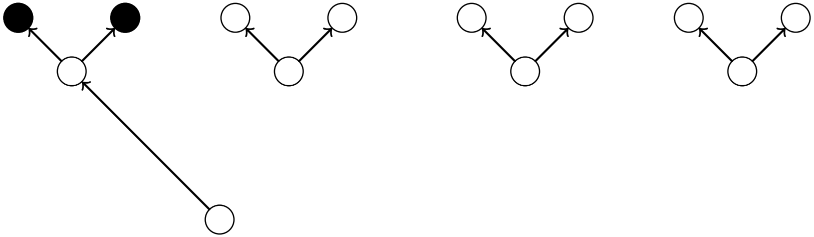
Construct Order Top-Down

Maximum number of nodes in memory: 1



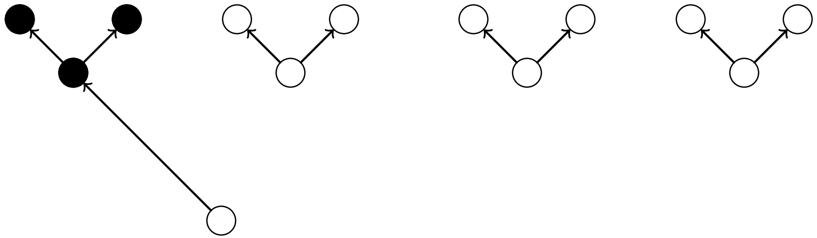
Construct Order Top-Down

Maximum number of nodes in memory: 2



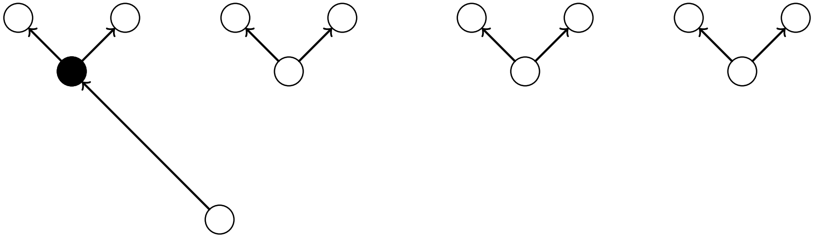
Construct Order Top-Down

Maximum number of nodes in memory: 3



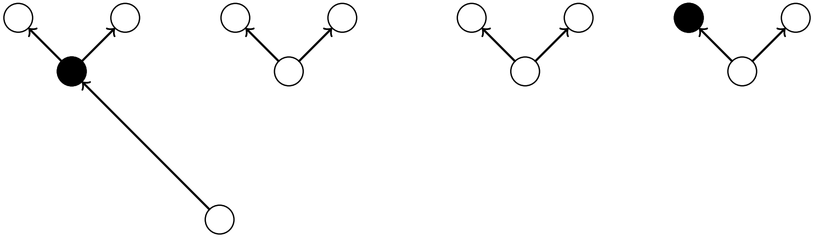
Construct Order Top-Down

Maximum number of nodes in memory: 3



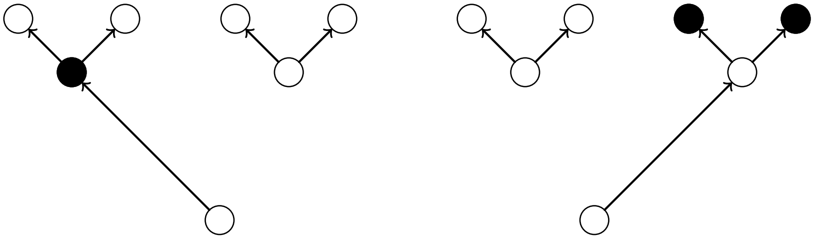
Construct Order Top-Down

Maximum number of nodes in memory: 3



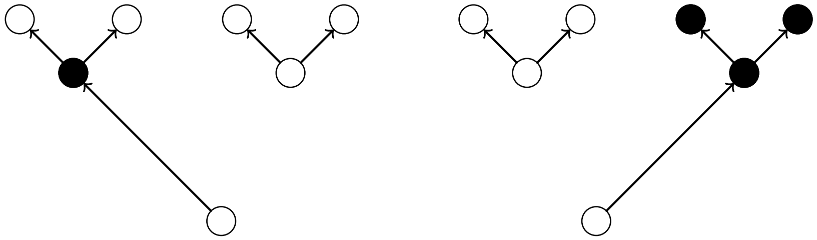
Construct Order Top-Down

Maximum number of nodes in memory: 3



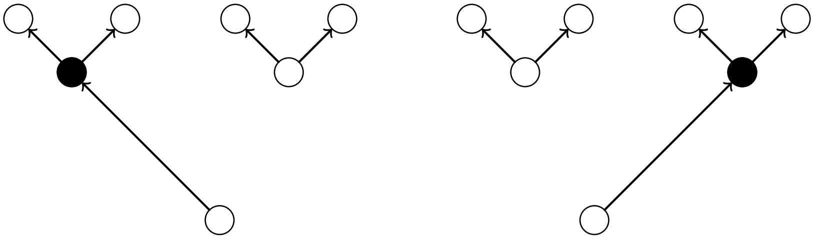
Construct Order Top-Down

Maximum number of nodes in memory: 4



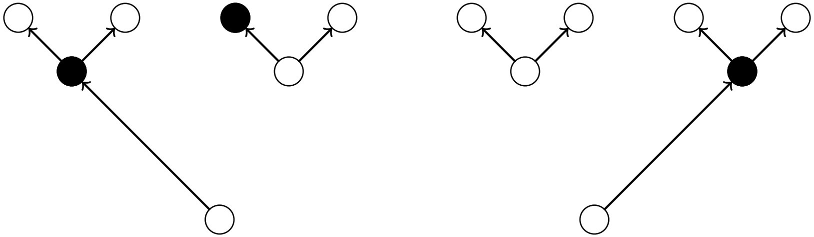
Construct Order Top-Down

Maximum number of nodes in memory: 4



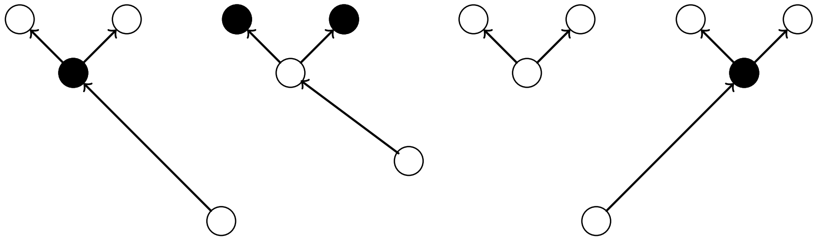
Construct Order Top-Down

Maximum number of nodes in memory: 4



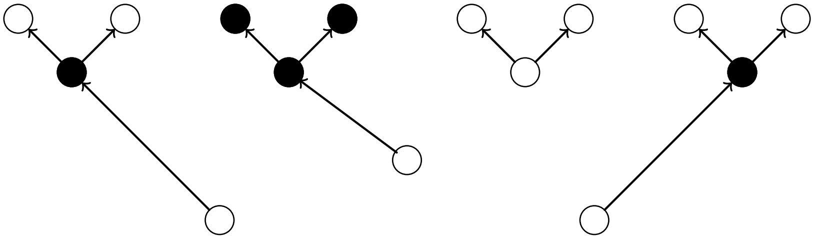
Construct Order Top-Down

Maximum number of nodes in memory: 4



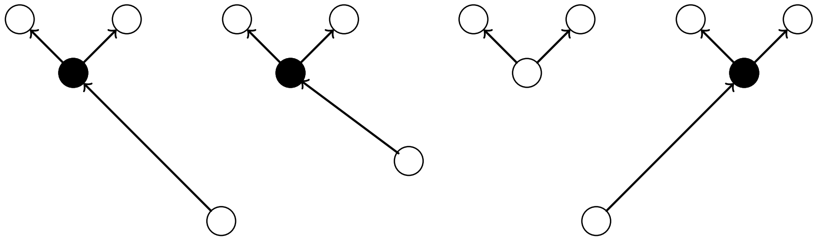
Construct Order Top-Down

Maximum number of nodes in memory: 5



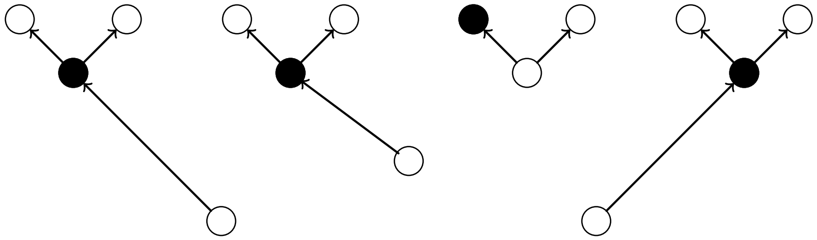
Construct Order Top-Down

Maximum number of nodes in memory: 5



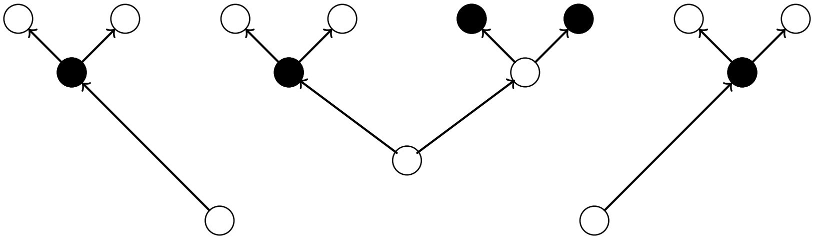
Construct Order Top-Down

Maximum number of nodes in memory: 5



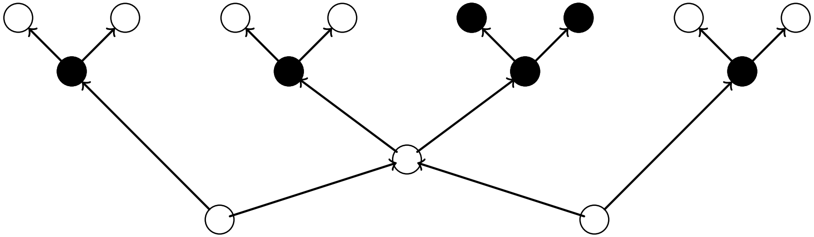
Construct Order Top-Down

Maximum number of nodes in memory: 5



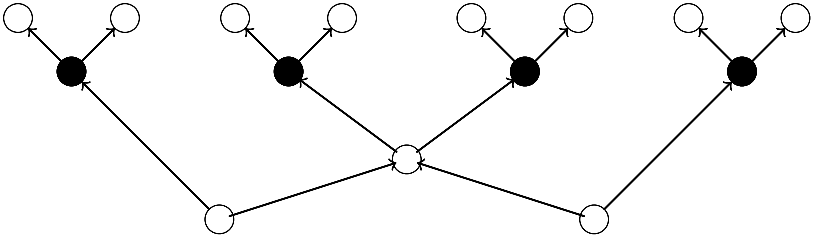
Construct Order Top-Down

Maximum number of nodes in memory: 6



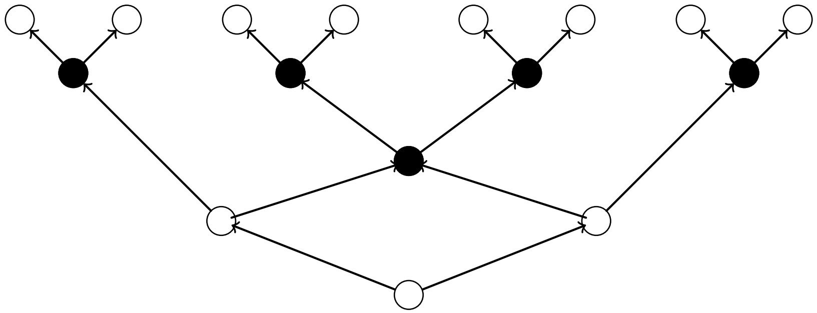
Construct Order Top-Down

Maximum number of nodes in memory: 6



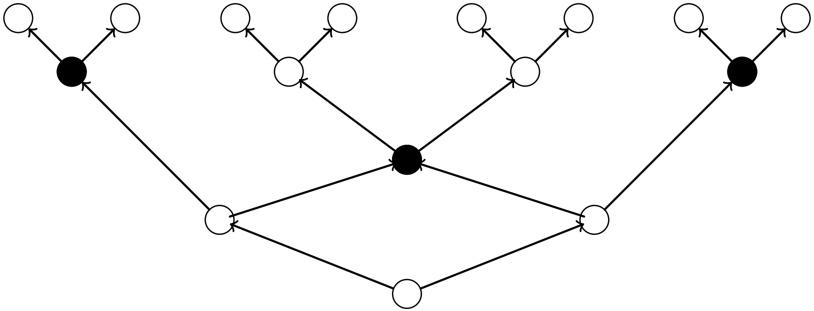
Construct Order Top-Down

Maximum number of nodes in memory: 6



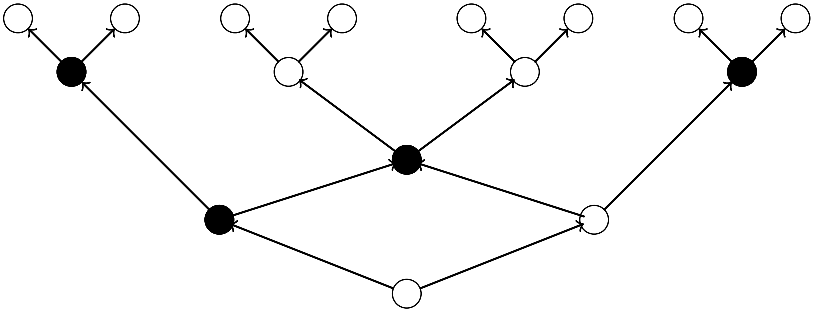
Construct Order Top-Down

Maximum number of nodes in memory: 6



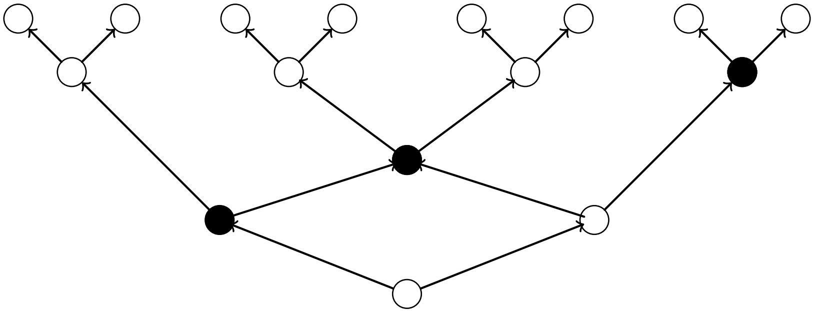
Construct Order Top-Down

Maximum number of nodes in memory: 6



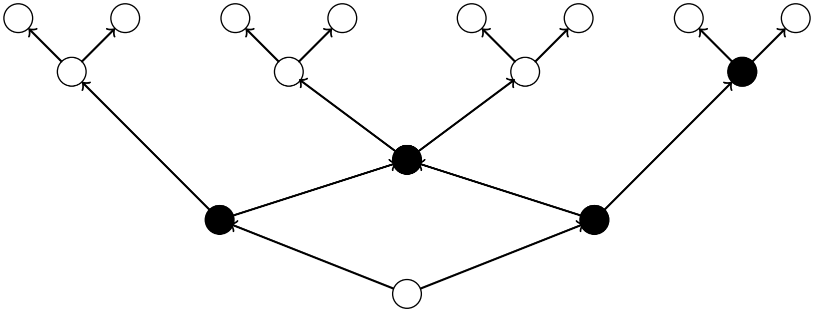
Construct Order Top-Down

Maximum number of nodes in memory: 6



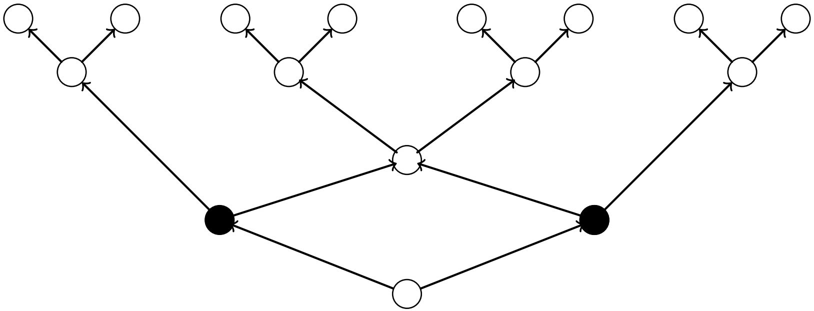
Construct Order Top-Down

Maximum number of nodes in memory: 6



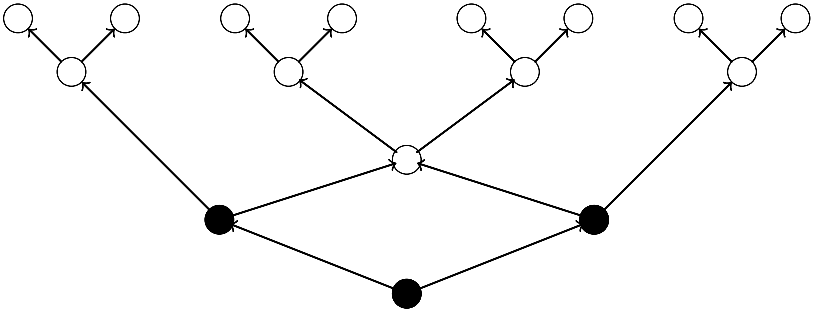
Construct Order Top-Down

Maximum number of nodes in memory: 6



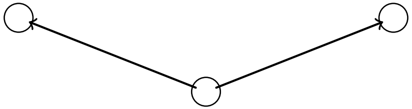
Construct Order Top-Down

Maximum number of nodes in memory: 6



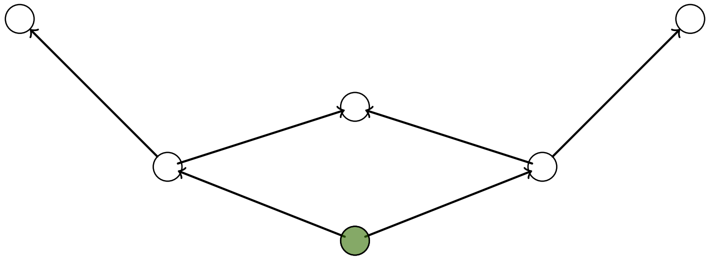
Construct Order Bottom-Up

Maximum number of nodes in memory: 0



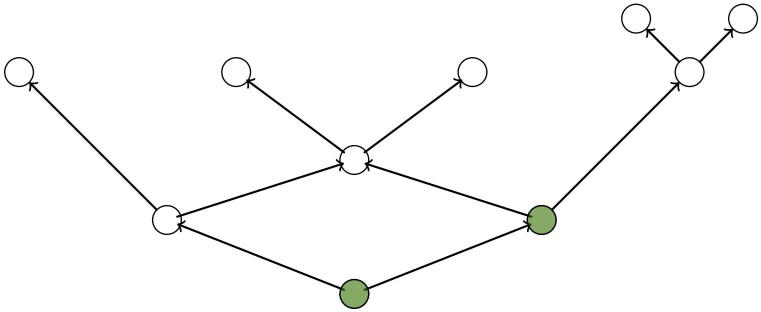
Construct Order Bottom-Up

Maximum number of nodes in memory: 0



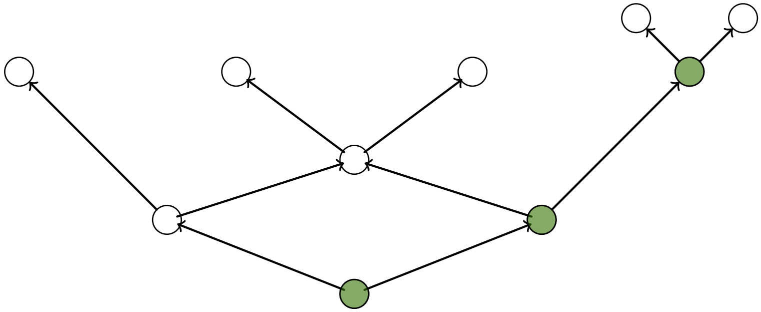
Construct Order Bottom-Up

Maximum number of nodes in memory: 0



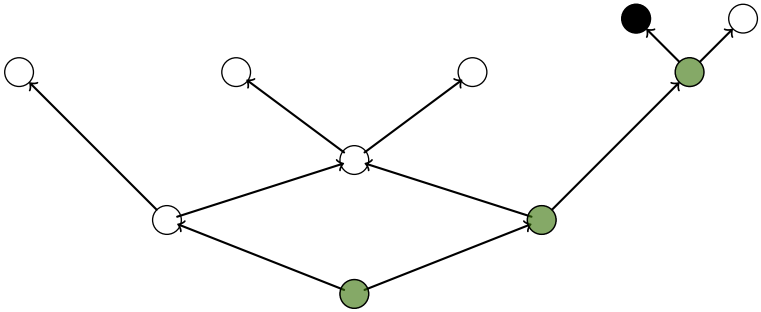
Construct Order Bottom-Up

Maximum number of nodes in memory: 0



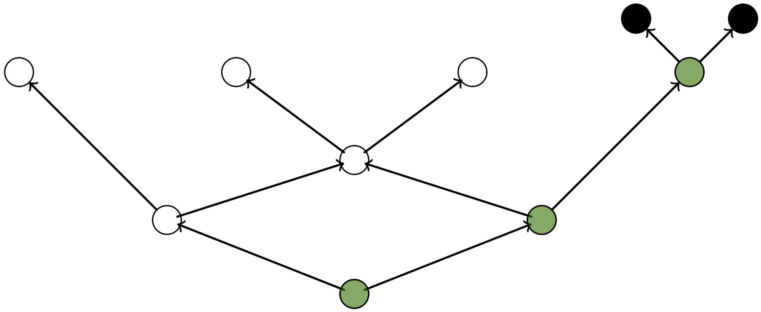
Construct Order Bottom-Up

Maximum number of nodes in memory: 1



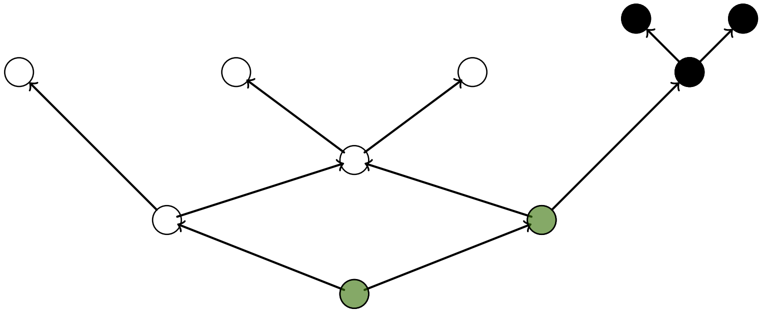
Construct Order Bottom-Up

Maximum number of nodes in memory: 2



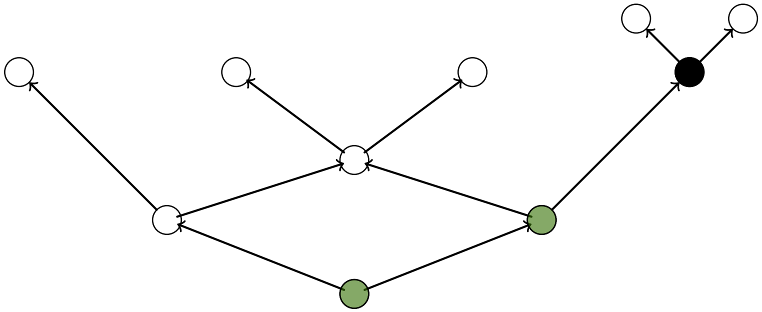
Construct Order Bottom-Up

Maximum number of nodes in memory: 3



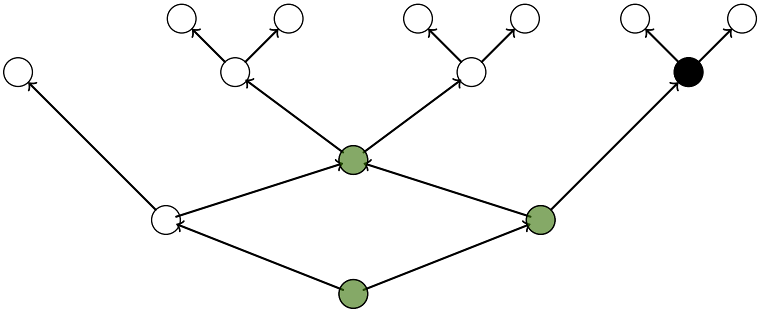
Construct Order Bottom-Up

Maximum number of nodes in memory: 3



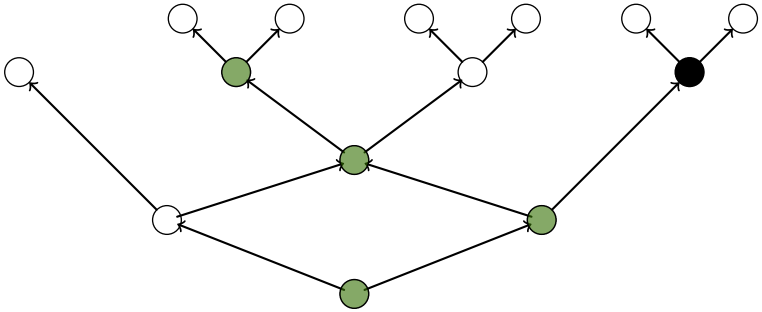
Construct Order Bottom-Up

Maximum number of nodes in memory: 3



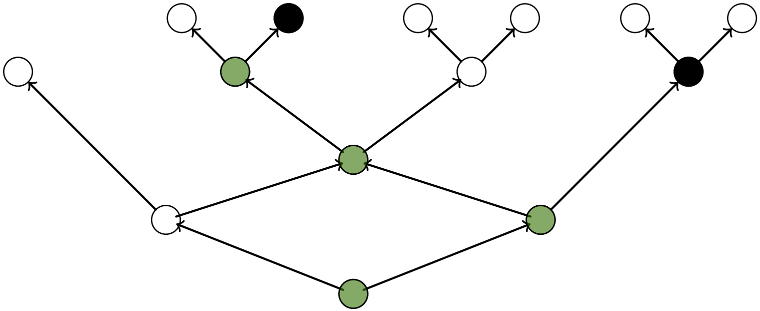
Construct Order Bottom-Up

Maximum number of nodes in memory: 3



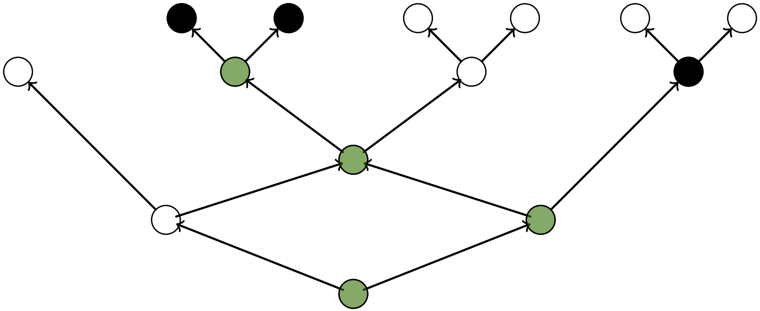
Construct Order Bottom-Up

Maximum number of nodes in memory: 3



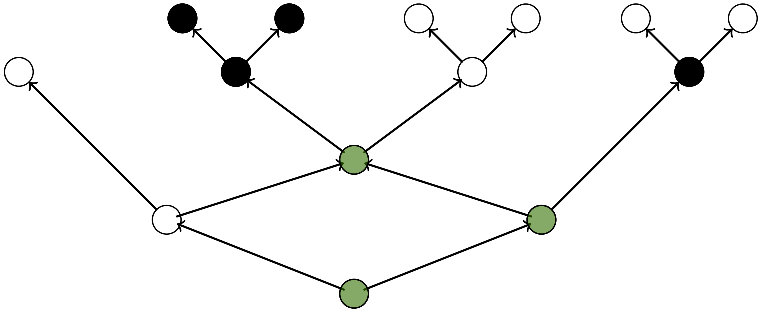
Construct Order Bottom-Up

Maximum number of nodes in memory: 3



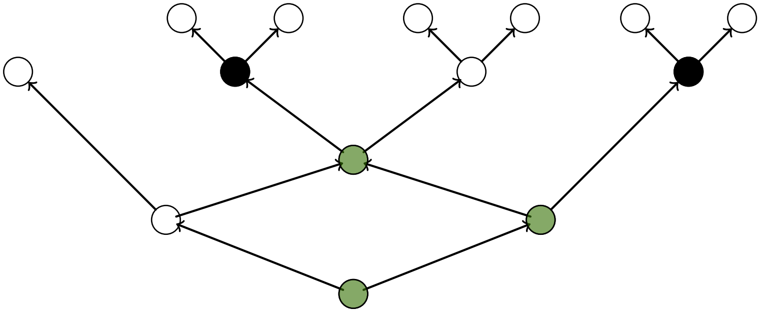
Construct Order Bottom-Up

Maximum number of nodes in memory: 4



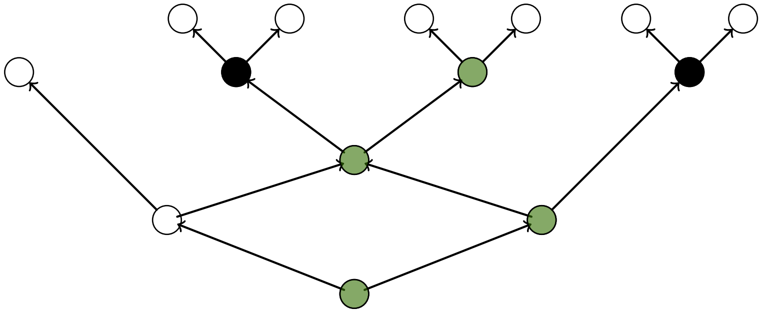
Construct Order Bottom-Up

Maximum number of nodes in memory: 4



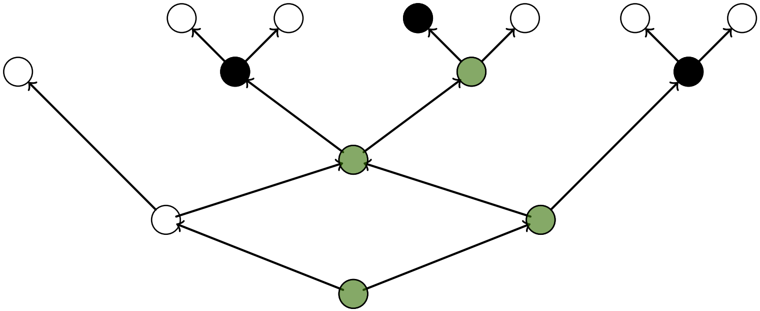
Construct Order Bottom-Up

Maximum number of nodes in memory: 4



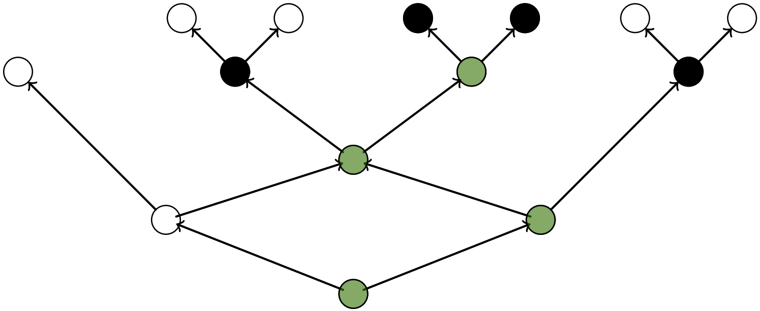
Construct Order Bottom-Up

Maximum number of nodes in memory: 4



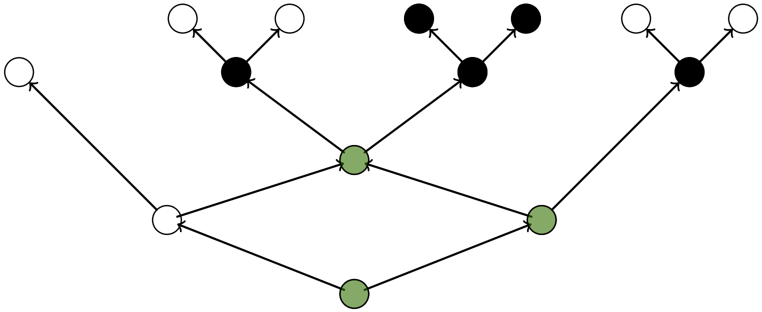
Construct Order Bottom-Up

Maximum number of nodes in memory: 4



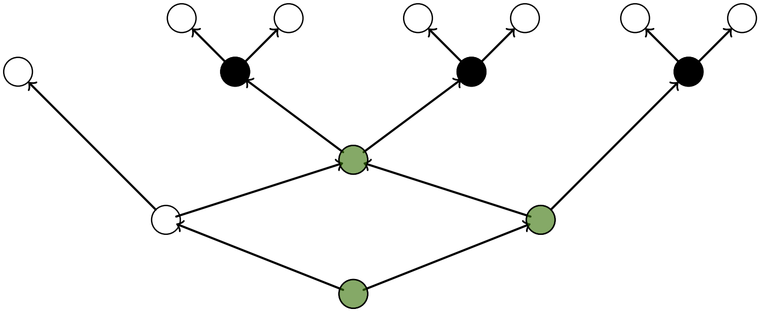
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



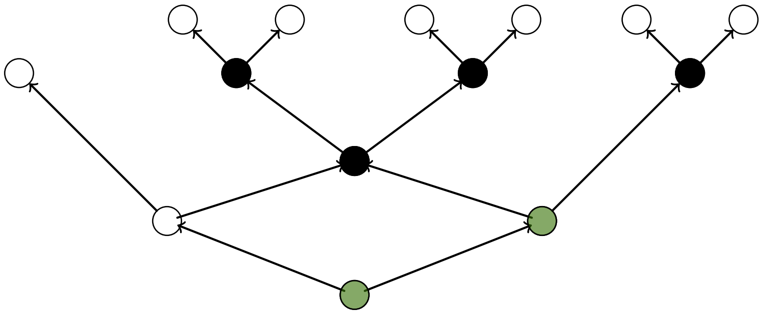
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



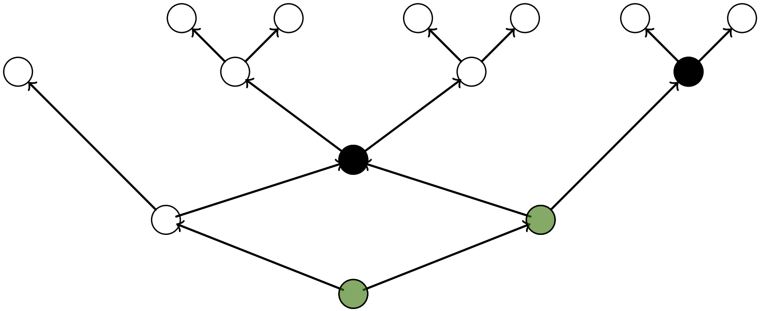
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



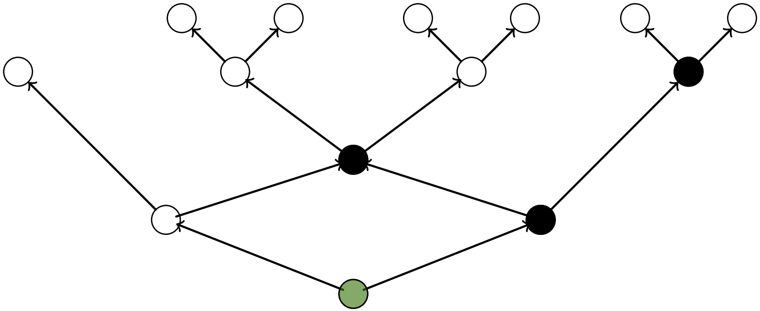
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



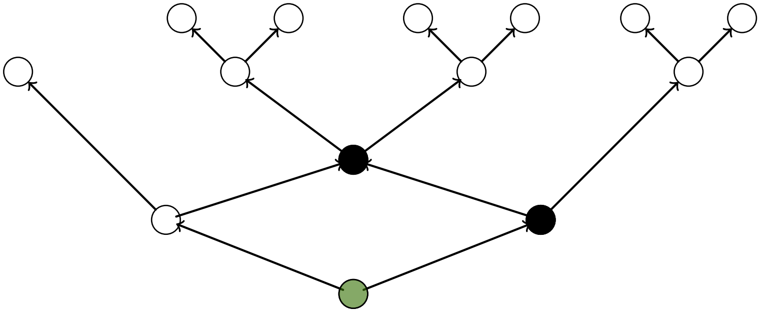
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



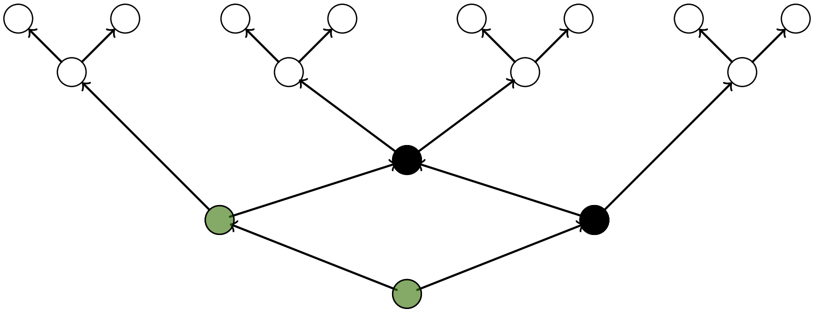
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



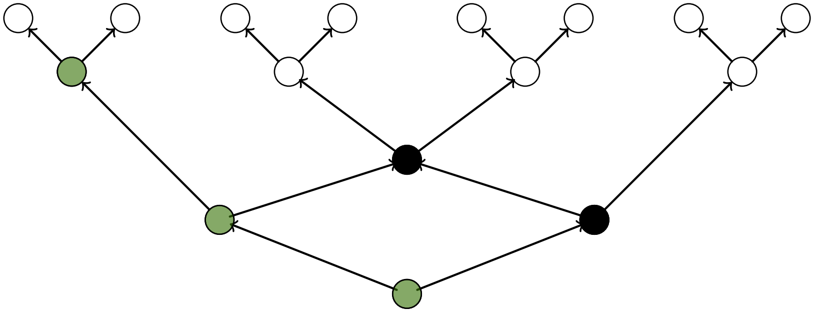
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



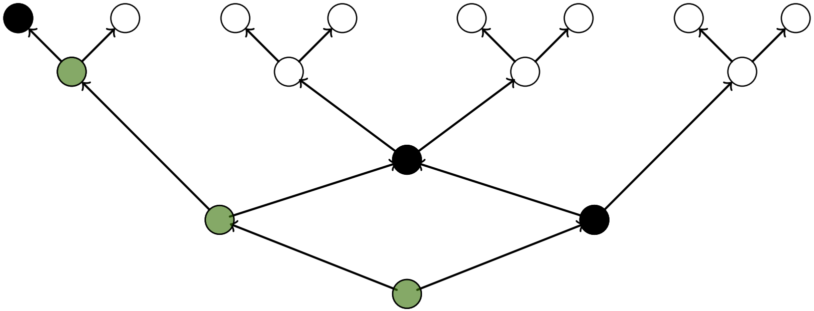
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



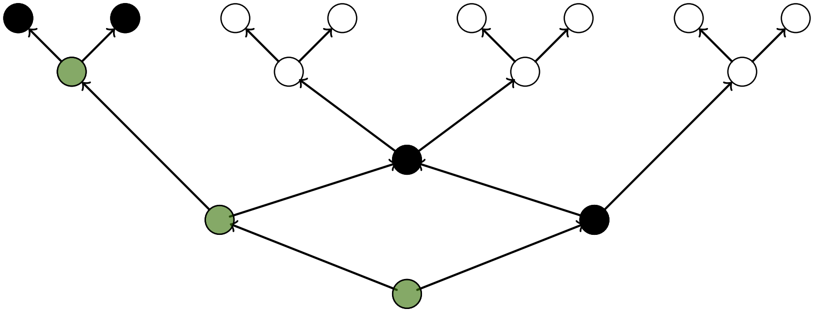
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



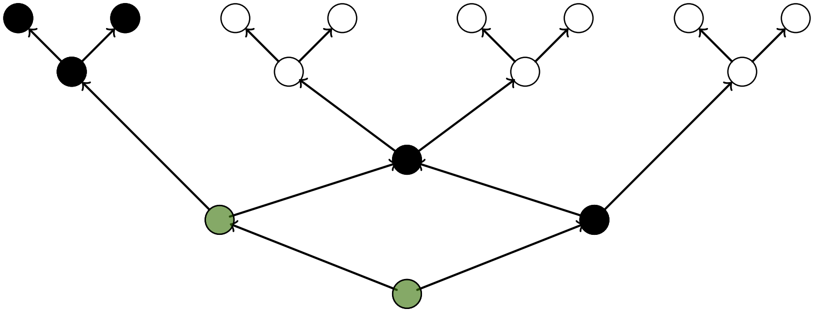
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



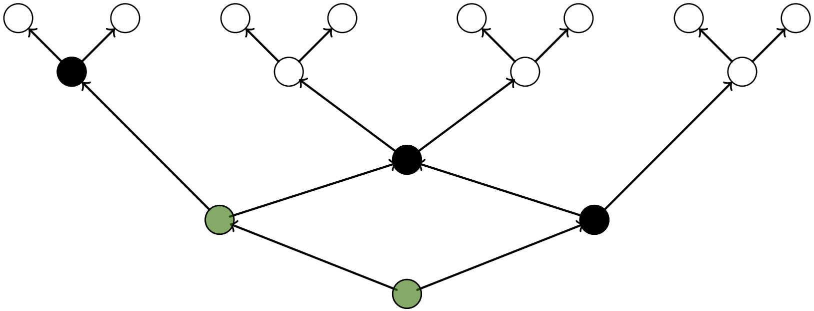
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



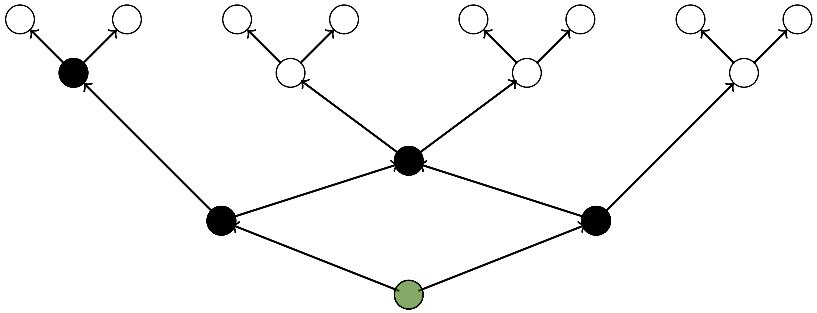
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



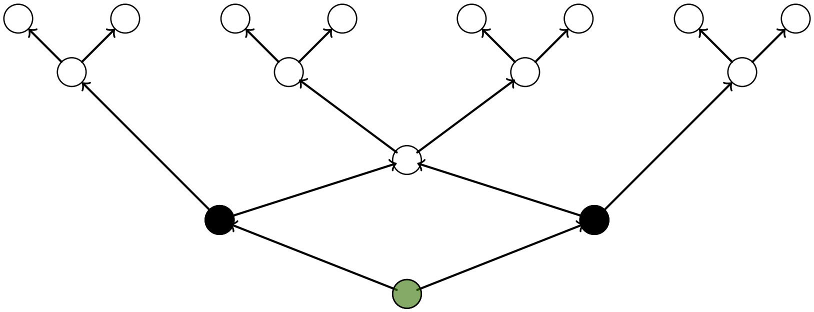
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



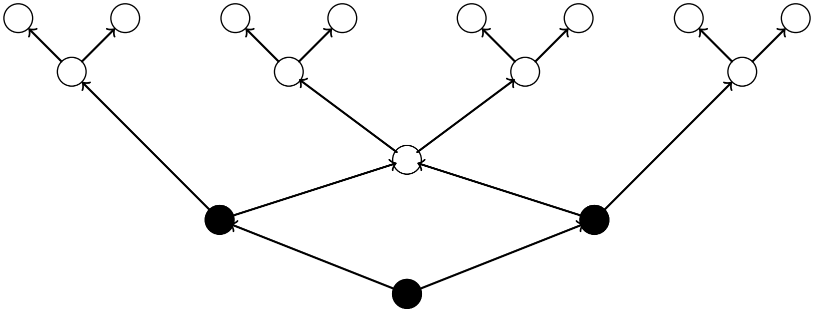
Construct Order Bottom-Up

Maximum number of nodes in memory: 5



Construct Order Bottom-Up

Maximum number of nodes in memory: 5



Experiments, Unsung Heroes & Conclusion

Experimental Results

Congruence Compression

- 2% average effective compression in proof length
- 28% compression in explanation length

Space Compression

- Bottom-Up outperforms Top-Down
- Average space measure is 44.1 times smaller than proof length

Unsung Heroes

- Explanation producing congruence closure algorithm
    - Using immutable data structures
    - Modified version of Dijkstra's shortest path algorithm
- Proof producing algorithm
- Resolution calculus extended with equality
- SAT translation of optimal traversal order
- Correct- & soundness proofs
- Implementation of all presented methods

Conclusion

- Proofs can be compressed in length and space
- Finding the shortest explanation is NP-complete
- Proof production is tricky
- Construct traversal orders Bottom-Up

Thank you for your attention!