ECON2500 Spring 2025
Paper Presentation

Arellano, Cristina, Mateos-Planas, Xavier and Ríos-Rull, José-Víctor, (2023), Partial Default, Journal of Political Economy, 131, issue 6, p. 1385 - 1439.

Adrien Foutelet

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Introduction

Motivation

Financial Times, March 7 2025. "What a Mar-a-Lago accord could look like":

> *[...] On September 22 1985 [in Plaza Hotel in New York], the US government persuaded Britain, Japan, Germany and France to jointly devalue the dollar, to boost America's industrial competitiveness.*
> *[...] There has been speculation about a new Plaza Accord — dubbed the "Mar-a-Lago Accord" — to depreciate the US dollar.*
> *[...] consider a must-read essay from Stephen Miran, Trump's pick for chair of the Council of Economic Advisers.*
> *[...] while Miran's essay warns that tariffs might initially strengthen the dollar, he thinks Washington can offset this. [...] one idea floating around is that other nations will be "encouraged" to swap holdings of dollars, short-term Treasuries or even gold for long-term or perpetual dollar bonds suitable for repurchase deals at the Federal Reserve.*

Formally, this is default: a sovereign does not honor its contractual payment schedule.

Motivation

- Standard theory: Eaton and Gersovitz (1981).

  A sovereign defaults completely and then faces complete exclusion from borrowing.

  No further default or accumulation of defaulted debt.

  Problem: most of the time, default is partial and is followed by ad-hoc negociations with the lenders to pay later, such that defaulted debt accumulates.

Research question

- Novel paradigm.

  Partial default is an alternative and rational way to intertemporally transfer resources. It raises current resources (benefit) and increases future liabilities (cost).

  Difference with standard borrowing:
  - Flexibility: it does not require the acsquiescence of the lender (flexibility),
  - Expensiveness: it impacts future resources through the interest rate.

Research question:
For a small open economy facing a portfolio choice, when is partial default in long-term bonds rational alongside borrowing? What are the effcts of partial default on the economy?

Accounting framework

Definitions

Let:
a_t the total amount owed by the sovereign to the lender,
d_t the flexible partial default policy of the sovereign,
q_t the perpetuity bond price,
b_t the perpetuity bond value,
δ the rate of decay of the coupon payment,
κ the defaulted debt accumulation and restructuration parameter,
R the risk-free gross interest rate.

We define:

$(1 - d_t)a_t$ the debt service,

$d_t a_t$ the defaulted coupon,

$\{q_t; b_t; \delta\}$ the borrowing contract (the sovereign receives $q_t b_t$ at t and pays back $\delta^{j-1} b_t$ at $t + j$),

Definitions

We define:

$\kappa d_t a_t$ the present value of futur obligations resulting from the defaulted coupon.

By annuitization, we get:

$(R-\delta)\kappa d_t a_t$ the face value of the long-term perpetuity contract resulting from the defaulted coupon.

This takes us to the law of motion of debt due accumulation:

$$a_t = \underbrace{\delta a_t}_{\text{legacy debt}} + \underbrace{(R-\delta)\kappa d_t a_t}_{\text{defaulted coupon}} + \underbrace{b_t}_{\text{new borrowing}}$$

Accounting variables

We define:

${\tilde{a}_t}^{t+j}$ the contractual payment due at $t+j$.

We get:

$\sum_{j=1}^{+\infty} \frac{{\tilde{a}_t}^{t+j}}{R^j}$ the debt level at t,

$\frac{1}{\sum_{j=1}^{+\infty} \frac{{\tilde{a}_t}^{t+j}}{R^j}} \sum_{j=1}^{+\infty} j \frac{{\tilde{a}_t}^{t+j}}{R^j}$ the duration of the debt at t.

We define:

s_t the sovereign's spread ($R+s_t$ the yield to maturity).

We get:

$\sum_{j=1}^{+\infty} \frac{{\tilde{a}_t}^{t+j}}{(R+s_t)^j}$ the market value of debt.

Accounting variables

The perpetuity bond price must be the price of both the borrowing contract and the default coupon (same risk of future default).

Therefore the market value of the long term perpetuity contract must be equal to $q_t a_{t+1}$.

Therefore:

$$s_t = \frac{1}{q_t} + \delta - R$$

Accounting variables

For a default episode of length $N+1$, we have:

$\sum_{j=0}^{N} \frac{d_{t+j} a_{t+j}}{R^j}$ the defaulted debt at t,

$n_{t+j+1} = \begin{cases} (R-\delta)\kappa d_{t+j} + \delta n_{t+j} & \text{if j} \in [\![0; N]\!] \\ \delta^{j-N-1} & else \end{cases}$ the obligations to pay the defaulted debt back,

$\sum_{j=0}^{N} \frac{(1-d_{t+j})n_{t+j}}{R^j} + n_{t+N+1} \sum_{j=0}^{+\infty} \frac{\delta^j}{R^{N+j+1}}$ the restructured debt at t,

$1 - \frac{\sum_{j=0}^{N} \frac{(1-d_{t+j})n_{t+j}}{R^j} + n_{t+N+1} \sum_{j=0}^{+\infty} \frac{\delta^j}{R^{N+j+1}}}{\sum_{j=0}^{N} \frac{d_{t+j} a_{t+j}}{R^j}}$ the haircut at t.

Data and Empirics

Data

World Bank:

- World Development Indicators,
- International Debt Statistics,
- Debt Reporting System,
- Global Financial Database.

37 emerging countries from 1970 to 2019, annual.

$\longrightarrow$ Panel data of debt service, defaulted coupons, debt due, partial defaut, spreads, linearily detrended log-GDP.

Empirics

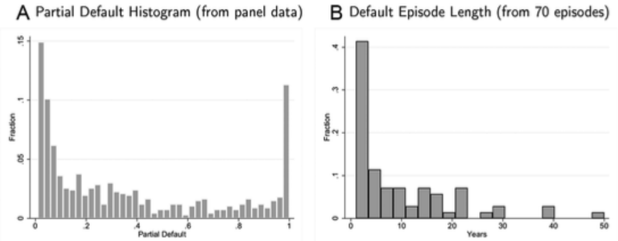

Figure: Partial default and default episode length.

Empirics

A. Partial Default			
Frequency			36
Mean | partial default > 0			38
Standard deviation | partial default > 0			22
B. Default Episodes			
Episode length (years)			9
Fraction of short episodes (≤2 years)			36
Haircut (%)			36
Maturity extension (years)			6
C. Default Episodes Dynamics			
	Partial Default	Debt	Output
Before episode	0	32	0
Beginning of episode	22	34	−2
Middle of episode	33	40	−5
After episode	0	33	−3

Figure: Partial default and default episodes in percentages.

Empirics

	No Default	Partial default > 0		
		Small (0–25%)	Medium (25%–75%)	Large (25%–75%)
Partial default	0	3	28	91
Spreads	4	6	8	18
Debt to output	24	33	43	60
Output	1	−1	−2	−4

Figure: Comovements.

- Partial default is common,
- Higher partial default $\leftrightarrow$ Higher spreads, higher debts, more depressed output,
- Hump-shape of the debt-to-output ratio, no reduction in debt.

Model

Setting

Consider a small open economy with:

- A stream of stochastic endowments,
- A sovereign who:
  - Borrows in long-term bonds,
  - Chooses to partially default on the debt due.

The sovereign's problem

Let:

z_t a stochastic endowment that follows a Markov process (transition probability: $\pi(z_{t+1}, z_t)$),

$E\left\{\sum_{t=0}^{+\infty} \beta^t u(c_t)\right\}$ the objective function of the sovereign,

$c_t = y_t - a_t(1-d_t) + q(a_{t+1}, d_t, z_t)b_t$ the budget of the sovereign,

$y_{t+1} = z_{t+1}\Psi(d_t, z_{t+1}) \leq z_{t+1}$ the "production function" of income, with Ψ decreasing and concave in d_t and $\Psi(0, z_{t+1} = 1)$ (default carries a resource cost).

Recall the accounting law of motion:

$$a_t = \delta a_t + (R-\delta)\kappa d_t a_t + b_t$$

Each period, the sovereign chooses $\{d_t; b_t\}$ facing $\{q_t; z_t\}$.

The lenders' model

- Purchase sovereign bonds by issuing securities at the risk-free world gross interest rate R.
- The value of 1 unit of existing debt today is equal to the discounted stream of payments, which is:
  - The amount paid today (partially defaulted or not),
  - The expected discounted continuation value,
- Free entry in the market drives profit to 0 and specifies $q(a_{t+1}, d_t, z_t)$,
- Partial default:
  - Reduces the value held by lenders today,
  - Increases the value held by lenders tomorrow,
  - Affects the value lent by lenders tomorrow through its effect on output.

Equilibrium in recursive terms

The recursive Markov equilibrium consists in:

- The borrower's decision rules for:
  - Borrowing $b(a, y, z)$,
  - Partial default $d(a, y, z)$,
  - Consumption $c(a, y, z)$,

  Which induce debt due $a'(a, y, z)$,
- The value of existing debt $H(a, y, z)$,
- The bond price function $q(a', d, z)$,

Such that:

- Taking $q(a', d, z)$ as given, the borrower's decision rule holds,
- Taking $b(a, y, z)$, $d(a, y, z)$, $c(a, y, z)$ as given, the lenders' decision rule holds,
- $q(a', d, z)$ yields zero profit.

Teachings from the optimality conditions for the sovereign

Debt and borrowing are capped diffently.

FOC with respect to b:

- 1 extra unit of b increases consumption by q but reduces the price of debt (raising the raising the debt due) by $q_{a'}$,
- This marginal gain from borrowing equals the marginal cost, which is the discounted expected value of increasing the debt burden.

Resources raised with borrowing are shaped by a laffer. curve

FOC with respect to d:

- 1 extra unit of d increases consumption by a but reduces the price of debt (due to increasing future debt obligations and decreasing future resources) by $(q_{a'}(R-\delta)\kappa a + q_d)b$.

Resources raised with borrowing are not shaped by a laffer curve but have upper bound a.

Teachings from the optimality conditions for the sovereign

The combination of both yields, for interior solutions (partial default), the portfolio condition: $R^b = R^d = \frac{u_c}{\beta E[u_c']}$ (the expected return of marginal utility of consumption)

For corner solutions (no partial default): $R^b < R^d$

In case of partial default, the next period sees $R^d < R^b$ and, by virtue of the portfolio condition, the sovereign is incentivized to exit default

Borrowing is an attractive way to smooth consumption when:

- The price of new debt is high,
- The price of new debt decreases little marginally.

Partial default is an attractive way to smooth consumption when:

- The price of new debt is low,
- The price of new debt decreases a lot marginally.

Teachings from the zero profit condition and the decision rule

This specifies:

- $q_{a'}$:
  - Negative effect from the higher likelyhood of a loss from not paying,
  - The diluted continuation effect (continuation amount diluted because of the future borrowing and defaults).
- $q_{d'}$
  - Negative effect from the output cost of default,
  - The diluted continuation effect (continuation amount diluted because of the future borrowing and defaults caused by the output decrease).

Quantitative results

Model calibration

Set:

$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$,

$log(z)$ follows an 11-state approximation of an autoregressive process,

$\Psi(z', d) = (1 - \phi_0 d^\gamma)(1 - \hat{\phi}_1(z' - z*))$ with $\phi_0 > 0$, $\gamma > 1$, $\hat{\phi}_1 = \phi_1$ if $d > 0$.

Other parameters estimated by minimum distance (sum of the proportional square residuals of 11 moments), averaged across the 37 countries.

Model simulated for 750,000 periods with the first 10% discarded.

Simulation

	Data	Model
A. Target Moments		
Partial default (%):		
Frequency	36	37
Mean | partial default > 0	38	39
Standard deviation | partial default > 0	22	19
Debt to output (%):		
Mean	32	32
Standard deviation	18	25
Debt service to output (%):		
Mean	3.6	3.5
Standard deviation	2.1	2.2
Debt due to output mean	4.9	5
Spread standard deviation	4.1	3.7
Output:		
Persistence	.89	.88
Standard deviation (%)	10	12
B. Other Moments in Panel		
Defaulted coupons to output (%):		
Mean | partial default > 0	5.2	4.0
Standard deviation | partial default > 0	6.4	3.6
Spreads:		
Mean	5.3	1.6
Correlation with output	−17	−38
Correlation with debt	24	56
Consumption standard deviation (relative to output)	1.0	.91

Decision rules to enter partial default

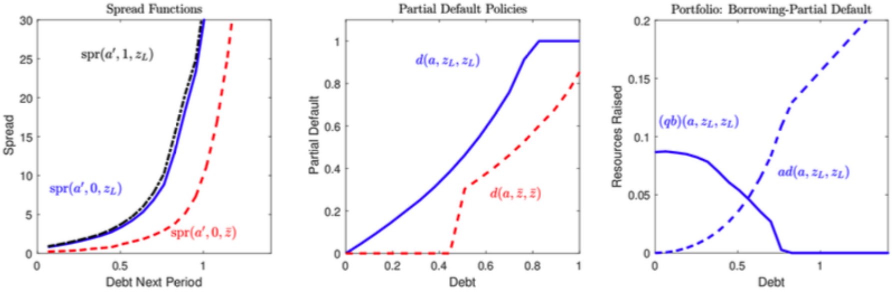

Figure: Spreands, partial default, and portfolio.

Bins for quantitative implications

	Partial Default Bins			
	No Default	Small	Medium	Large
	A. Data			
Partial default	0	3	28	91
Debt to output	24	33	43	60
Spreads	4	6	8	18
Output	1	−1	−2	−4
	B. Model			
Partial default	0	20	35	66
Debt to output	18	33	55	82
Spreads	1	1	2	8
Output	6	−11	−10	−18

Figure: Partial default, spreads, debt, and output.

Partial default episodes: length, haircuts, muaturity extensions

	Data	Model
A. Properties of Episodes		
Mean episode length (years)	9	8
Percentage of short episodes (≤2)	36	42
Coefficient of variation for episode length	1.1	1.5
Haircut (%)	36	37
Maturity extension	6	7
Correlation (length, partial default)	26	75
B. Default Episode Dynamics		
Partial default:		
Before	0	0
Beginning	22	21
Middle	33	28
End	0	0
Output:		
Before	0	0
Beginning	−2	−7
Middle	−5	−9
End	−3	3
Debt:		
Before	32	32
Beginning	34	35
Middle	40	44
End	33	42

Partial default episodes: dynamics

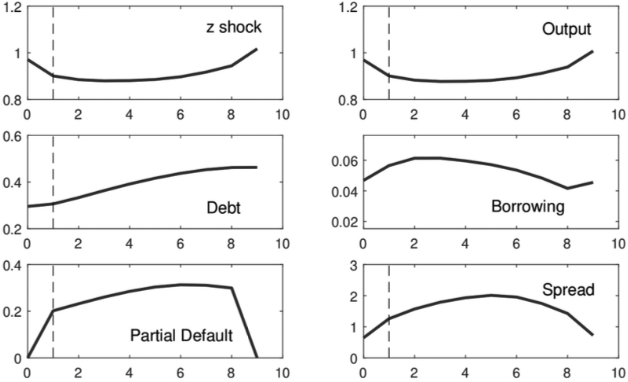

Figure: Default episode properties.

Conclusion