1. ATQ, final we want a qubit which gives Head (0), 60% of times & tails (1) 40% of times when measured.

Ans. or $p(0) = 0.6$ $\quad p(1) = 0.4$

Let a final qubit be $|q\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$

Here probability of q to measure out to be 0 will be $\rightarrow \dfrac{a^2}{a^2+b^2}$ (Born Rule)

$$\therefore \quad 0.6 = \frac{a^2}{a^2+b^2} \quad \Rightarrow \quad 0.6a^2 + 0.6b^2 = a^2$$

$$\text{or} \quad 0.4a^2 = 0.6b^2$$

$$\text{or} \quad a^2 = \frac{3}{2}b^2$$

$$\text{or} \quad a = \sqrt{\frac{3}{2}}\, b$$

$$\therefore \text{Matrix} \rightarrow |q\rangle = \begin{bmatrix} \sqrt{\frac{3}{2}} \\ 1 \end{bmatrix} b \quad \text{or} \quad |q\rangle = \sqrt{\tfrac{3}{2}}\, b|0\rangle + b|1\rangle$$

Also using $RY(\theta)$, gate, we can transform qubit from state $|0\rangle$ to:

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle$$

Comparing with $|q\rangle = \sqrt{\frac{3}{2}}\, b\,|0\rangle + b\,|1\rangle$

$$\Rightarrow \quad \cos\frac{\theta}{2} = \sqrt{\frac{3}{2}}\, b, \quad \sin\frac{\theta}{2} = b$$

$$\Rightarrow \quad \tan\frac{\theta}{2} = \frac{\sin\theta/2}{\cos\theta/2} = \sqrt{\frac{2}{3}}$$

$$\text{or} \quad \frac{\theta}{2} = \tan^{-1}\sqrt{\frac{2}{3}} = 0.6847$$

$$\text{or} \quad \underline{\theta = 1.3694}$$

$\therefore$ We apply $RY(1.3694)$ gate to $|0\rangle$ qubit.

2. Initially $|q\rangle$ is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

first we apply X gate : $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\therefore \; |qX\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ or $|1\rangle$

Now applying H gate :

$$|q|H\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ or } \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

or $|-\rangle$

(Superposition state)