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Faculty of Science
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Theoretical Atomic Physics and Synthetic Quantum Systems

Collective decay in a dissipative Rydberg-gas

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1 Rydberg atoms

- highly excited atoms with high quantum number n
- energy spectrum similar to hydrogen atom
- long lifetime and large radial extension
- Van der Waals (VdW)-interaction between atoms with distance R scales with n^{11}/R^6 [1, 2]

→ the excited state of an atom gets shifted upwards by excited neighbours

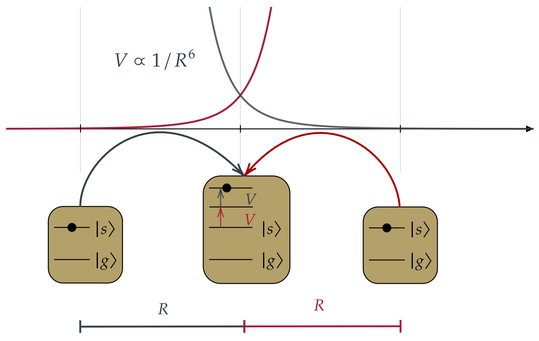


Figure 1: **An atom in the Rydberg potential of its neighbours.** The excited state gets shifted upwards by the potential of the neighbours.

2 The system

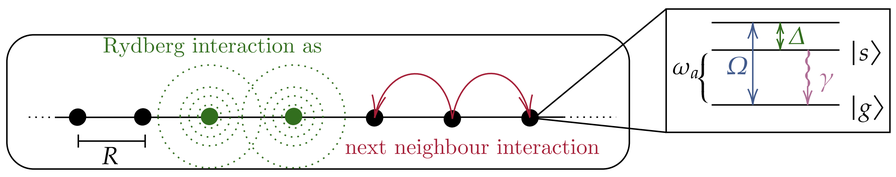


Figure 2: **Schematic description of the considered model of a strongly interacting Rydberg gas with nearest neighbour interaction.**
Lattice distance R, energy spacing ω_a, Rabi frequency Ω, detuning Δ, single atom decay rate γ

- 1D chain of strongly interacting Rydberg atoms (modelled as 2-level systems $|g\rangle, |s\rangle$)
- driven by a laser with Rabi frequency Ω
- assume cyclic boundary conditions
- lattice spacing R much larger than transition wavelength → neglecting collective decay processes [3]
- VdW potential treated as nearest-neighbour interaction [4, 5]

3 Local approach: Master equation

1. derive the master equation without Rydberg interaction and laser, see (eq. 2, black only)
2. add interaction afterwards [3] and obtain (eq. 1)

→ *jump operators* $L_k = \sigma_k$ act on single atom only

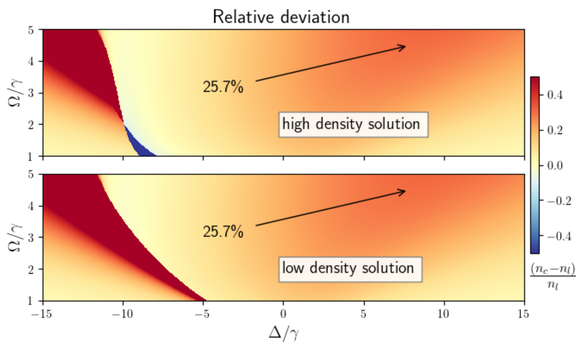
→ Is this really justified in a Rydberg gas with strong interactions?

$$\dot{\rho} = -\mathrm{i}[H,\rho] + \gamma \sum_k \left(\underbrace{\sigma_k}_{\uparrow} \rho \sigma_k^\dagger - \frac{1}{2}\left\{\sigma_k^\dagger \sigma_k, \rho\right\}\right) \qquad (1)$$
$$L_k$$

4 Consequence of strong interactions

- Consider strong interactions between nearest neighbours ⇒ 3 distinguishable configurations
- define an orthonormal set of projectors P_k^ξ for each neighbour configuration ξ of an atom k

In principle, the configurations can be distinguished by their different frequencies of emission!



5 Collective approach: Master equation

Derive the master equation in presence of nearest neighbour interactions [6].
We proceed analogously to [7–9] and obtain (eq. 4).

$$H = \overbrace{\underbrace{\omega_a \sum_k n_k}_{\text{bare atom energy}} + \underbrace{V \sum_k n_k n_{k+1}}_{\text{Rydberg interaction}} + \underbrace{\sum_q \omega_q a_q^\dagger a_q}_{\text{bath energy}}}^{H_0} + \underbrace{\sum_{q,k} g_{qk}(a_q \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_k} + a_q^\dagger \mathrm{e}^{-\mathrm{i}\mathbf{q}\cdot\mathbf{r}_k})\sigma_k^x}_{\text{system-bath interaction}} \qquad (2)$$

1. transform into interaction picture via unitary $U = \exp(\mathrm{i}H_0 t)$
2. apply Born-Markov approximation
3. apply rotating-wave approximation
4. assume large Rydberg interaction $V/\gamma \gg 1$
5. define the configuration dependent decay rate Γ^ξ with dipole matrix element $\mathbf{d_{gs}} = \langle g|e\mathbf{r}|s\rangle$

$$\Gamma^\xi = \frac{|\mathbf{d_{gs}}|^2}{3\pi\hbar\epsilon_0 c^3}(\omega_a + \xi V)^3 \qquad (3)$$

Collective master equation

$$\dot{\rho}_{\mathrm{int}} = \sum_{\xi=\{0,1,2\}} \Gamma^\xi \sum_k \underbrace{\sigma_k P_k^\xi}_{L_k^\xi} \rho \, P_k^\xi \sigma_k^\dagger + \left\{\sigma_k P_k^\xi \sigma_k^\dagger, \rho\right\} \qquad (4)$$

→ jump operators change to collective jump operators $L_k^\xi = \sigma_k P_k^\xi$ ⇒ dissipation is configuration dependent

6 Driven dissipative Rydberg gas within collective approach

- drive system with a laser with Rabi frequency Ω and detuning Δ
- study the excitation density $\langle n \rangle$ in the steady stationary state
- apply mean-field approximation $\langle n\sigma_x \rangle = \langle n \rangle \langle \sigma_x \rangle$ and homogenize atoms $\langle n_k \rangle = \langle n_{k'} \rangle$ [10]

→ for the collective approach we obtain an additional summand $4\langle n \rangle$ (in red) in the system of equations (eq. 5), which comes from the entry of the neighbouring atoms

→ depending on chosen parameters Ω and Δ either 1 or 3 solutions result

→ either 1 or 2 are stable → study only those

System of equations

$$\begin{aligned}
\langle \dot{n} \rangle &= \Omega \langle \sigma_y \rangle - \gamma \langle n \rangle \\
\langle \dot{\sigma}_x \rangle &= -\Delta \langle \sigma_y \rangle - \frac{\gamma}{2}(4\langle n \rangle + 1)\langle \sigma_x \rangle - 2V \langle n \rangle \langle \sigma_y \rangle \qquad (5) \\
\langle \dot{\sigma}_y \rangle &= -\Delta \langle \sigma_x \rangle - \frac{\gamma}{2}(4\langle n \rangle + 1)\langle \sigma_y \rangle + 2V \langle n \rangle \langle \sigma_x \rangle \\
&\quad - 2\Omega(2\langle n \rangle - 1)
\end{aligned}$$

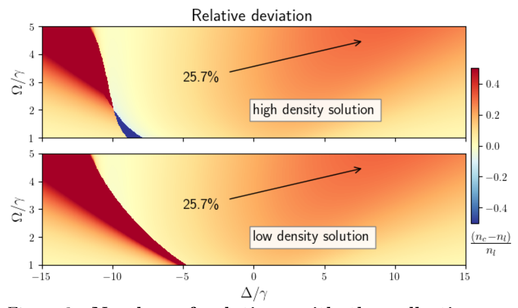


Figure 3: **Number of solutions with the collective approach** (c)**, the local approach** (l)**.** We set $V = 10\gamma$. We observe a shift in phase boundaries and critical points.

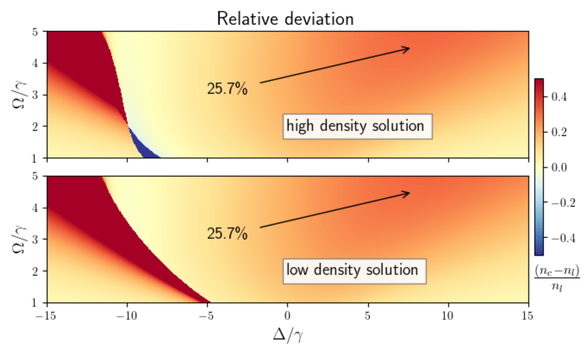


Figure 4: **Excitation density phase diagrams for collective and local approach.** $V = 10\gamma$. The low density solution corresponds to the smallest of the 3 solutions, the high density to the largest. We observe a shift in the phase boundaries for both solutions.

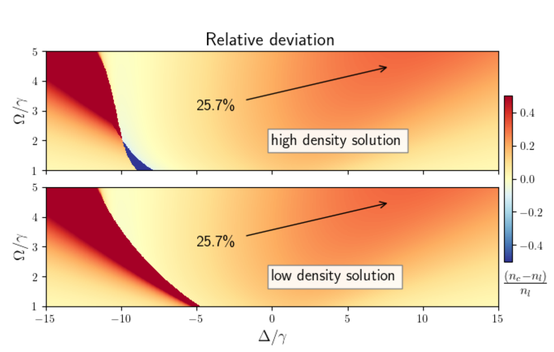


Figure 5: **Relative deviation of the phase diagrams** with collective vs. the local approach. We set $V = 10\gamma$. The low density solution corresponds to the smallest of the 3 solutions, the high density to the largest. In the area of the transitions from (fig. 3) we find a high relative deviation here.

Relative deviation for finite number of particles

- for particle numbers $N < 6$, we use the method of exact diagonalization [9]. For $N \geq 6$, we use the „continuous-time Monte Carlo“ algorithm from [11].
- simulations are done with Python framework QuTiP [12, 13].

→ in (fig. 6) similar parameter regimes for differences compared to the mean-field calculations can be identified

→ clear differences for $\Delta/\gamma < -5$: Excitation density is higher for the collective approach

- periodic effects in Monte Carlo calculations are most likely due to residual oscillations of the equilibrium state after finite simulation time

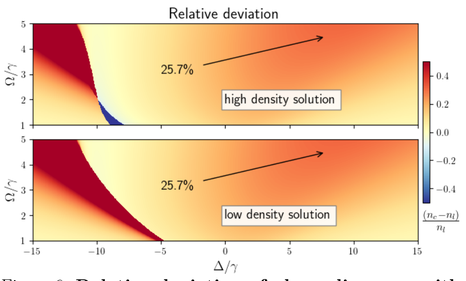


Figure 6: **Relative deviation of phase diagrams with finite number of particles** N at $V = 10\gamma$. Normalized by the number of particles.

7 Conclusion

→ The dissipation of a strongly interacting Rydberg gas changes due to its interaction.

→ Frequency and rate of spontaneous decay of an atom depends on configuration of neighbouring atoms.

→ Shift in phase boundaries and critical points occurs.

→ Excitation density $\langle n \rangle$ in mean-field approximation shows a significant deviation compared to the local approach for certain parameter choice.

→ With finite particle number, we observe a relative deviation up to 120 %, which grows with system size.

References

[1] N. Šibalić and C. S. Adams, *Rydberg physics*, 2399-2891 (IOP Publishing, 2018).
[2] A. Jahangiri, J. P. Shaffer, L. F. Gonçalves, and L. G. Marcassa, “Anisotropic interactions in electric field polarized ultracold rydberg gases”, Journal of Physics B: Atomic, Molecular and Optical Physics **53**, 014001 (2019).
[3] J. A. Needham, “Subradiance protected excitation transport”, PhD thesis (University of Nottingham, Dec. 2020).
[4] B. Olmos, R. González-Férez, and I. Lesanovsky, “Fermionic collective excitations in a lattice gas of rydberg atoms”, Phys. Rev. Lett. **103**, 185302 (2009).
[5] C. Ates, B. Olmos, J. P. Garrahan, and I. Lesanovsky, “Dynamical phases and intermittency of the dissipative quantum ising model”, Physical Review A **85**, 043620 (2012).
[6] J. O. González, L. A. Correa, G. Nocerino, J. P. Palao, D. Alonso, and G. Adesso, “Testing the validity of the ‘local’ and ‘global’ GKLS master equations on an exactly solvable model”, Open Systems & Information Dynamics **24**, 1740010 (2017).
[7] H.-P. Breuer and F. Petruccione, *The theory of open quantum systems* (Oxford University Press, Feb. 28, 2007).
[8] D. A. Lidar, *Lecture notes on the theory of open quantum systems*, Center for Quantum Information Science and Technology, University of Southern California, Feb. 21, 2020.
[9] G. Schaller, *Lecture notes: non-equilibrium master equations*, Technische Universität Berlin, Apr. 29, 2014.
[10] H. Weimer, R. Löw, T. Pfau, and H. P. Büchler, “Quantum critical behavior in strongly interacting rydberg gases”, Phys. Rev. Lett. **101**, 250601 (2008).
[11] M. B. Plenio and P. L. Knight, “The quantum-jump approach to dissipative dynamics in quantum optics”, Reviews of Modern Physics **70**, 101–144 (1998).
[12] J. R. Johansson, P. D. Nation, and F. Nori, “QuTiP: an open-source python framework for the dynamics of open quantum systems”, Computer Physics Communications **183**, 1760–1772 (2012).
[13] J. R. Johansson, P. D. Nation, and F. Nori, “QuTiP 2: a Python framework for the dynamics of open quantum systems”, Computer Physics Communications **184**, 1234–1240 (2013).

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