

## IT300 - DAA - Assignment - I

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$$1) T(n) = 3T\left(\frac{n}{2}\right) + n^2$$

$$a = 3 \quad b = 2$$

$$\log_b a = \log_2 3 \approx 1.58 < 2$$

$$f(n) < n^{\log_b a + \epsilon}$$

$$\Rightarrow T(n) = \Theta(f(n)) = \underline{\underline{\Theta(n^2)}}$$

$$2) T(n) = 7T\left(\frac{n}{2}\right) + n^2$$

$$a = 7 \quad b = 2$$

$$\log_b a = \log_2 7 \approx 2.81 > 2$$

$$f(n) > n^{\log_b a - \epsilon}$$

$$\Rightarrow T(n) = \Theta(n^{\log_b a}) = \underline{\underline{\Theta(n^{\log 7})}}$$

$$3) T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

$$a = 4 \quad b = 2$$

$$\log_b a = \log_2 4 = 2$$

$$f(n) = n^{\log_b a}$$

$$\Rightarrow T(n) = \Theta(n^{\log_b a} * \log n) = \underline{\underline{\Theta(n^2 \log n)}}$$

$$4) T(n) = 9T(n/3) + n$$

$$a = 9 \quad b = 3$$

$$\log_b a = \log_3 9 = 2 > 1$$

$$f(n) > n^{\log_b a - \epsilon}$$

$$\Rightarrow T(n) = \Theta(n^{\log_b a}) = \underline{\underline{\Theta(n^2)}}$$

$$5) T(n) = T(2n/3) + 1$$

$$a = 1 \quad b = 3/2$$

$$\log_b a = \log_{1.5} 1 = 0 = 0$$

$$f(n) = n^{\log_b a}$$

$$\Rightarrow T(n) = \Theta(n^{\log_b a} * \log n) = \underline{\underline{\Theta(\log n)}}$$

$$6) T(n) = 3T(n/4) + n \log n$$

$$a = 3 \quad b = 4 \quad k = 1 \quad p = 1$$

$$\log_b a = \log_4 3 = 0.79 < 1$$

$$f(n) < n^{\log_b a + \epsilon}$$

$$\Rightarrow T(n) = \Theta(f(n)) = \underline{\underline{\Theta(n \log n)}}$$

$$7) T(n) = 4T(n/2) + n^2 \log n$$

$$a = 4 \quad b = 2 \quad k = 2 \quad p = 1 > -1$$

$$\log_b a = 2 = k$$

$$\Rightarrow T(n) = \Theta(n^k \log^{p+1} n) = \underline{\underline{\Theta(n^2 \log^2 n)}}$$

$$8) T(n) = 4T(n/2) + \log n$$

$$a = 4 \quad b = 2 \quad k = 0 \quad p = 1$$

$$\log_b a = 2 > 0$$

$$f(n) \gg n^{\log_b a - \epsilon}$$

$$\Rightarrow T(n) = \theta(n^{\log_b a}) = \underline{\underline{\theta(n^2)}}$$

$$9) T(n) = 5T(n/2) + n^2 \log n$$

$$a = 5 \quad b = 2 \quad k = 2 \quad p = 1$$

$$\log_b a \approx 2.32 > 2$$

$$f(n) > n^{\log_b a - \epsilon}$$

$$\Rightarrow T(n) = \theta(n^{\log_b a}) = \underline{\underline{\theta(n^{\log 5})}}$$

$$10) T(n) = 2T(n/4) + c$$

$$a = 2 \quad b = 4$$

$$\log_b a = 0.5 > 0$$

$$f(n) > n^{\log_b a - \epsilon}$$

$$\Rightarrow T(n) = \theta(n^{\log_b a}) = \underline{\underline{\theta(\sqrt{n})}}$$

$$11) T(n) = T(n/4) + \log n$$

$$a = 1 \quad b = 4 \quad k = 0 \quad p = 1 > -1$$

$$\log_b a = 0 = k$$

$$\Rightarrow T(n) = \theta(n^k \log^{p+1} n) = \underline{\underline{\theta(\log^2 n)}}$$

$$12) T(n) = 2T(n/4) + \log n$$

$$a = 2 \quad b = 4 \quad k = 0 \quad p = 1$$

$$\log_b a = 0.5 > 0$$

$$f(n) > n^{\log_b a - \epsilon}$$

$$\Rightarrow T(n) = \theta(n^{\log_b a}) = \underline{\underline{\theta(\sqrt{n})}}$$

$$13) T(n) = 3T(n/3) + n \log n$$

$$a = 3 \quad b = 3 \quad k = 1 \quad p = 1 > -1$$

$$\log_b a = 1 = k$$

$$\Rightarrow T(n) = \theta(n^k \log^{p+1} n) = \underline{\underline{\theta(n \log^2 n)}}$$

$$14) T(n) = 2T(n/4) + \sqrt{n}$$

$$a = 2 \quad b = 4$$

$$\log_4 2 = 0.5 = 0.5$$

$$f(n) = n^{\log_b a}$$

$$\Rightarrow T(n) = \theta(n^{\log_b a} \log n) = \underline{\underline{\theta(\sqrt{n} \log n)}}$$

$$15) T(n) = 2T(n/4) + n^{0.51}$$

$$a = 2 \quad b = 4$$

$$\log_4 2 = 0.5 < 0.51$$

$$f(n) < \log_b a \cdot n^{\log_b a + \epsilon}$$

$$\Rightarrow T(n) = \theta(f(n)) = \underline{\underline{\theta(n^{0.51})}}$$

$$16) T(n) = 16T(n/4) + n!$$

$$a = 16 \quad b = 4$$

$$\log_b a = 2 \Rightarrow n^{\log_b a} < n!$$

$$f(n) < n^{\log_b a + \epsilon}$$

$$\Rightarrow T(n) = \theta(f(n)) = \underline{\underline{\theta(n!)}}$$

$$17) T(n) = 3T(n/2) + n$$

$$a = 3 \quad b = 2$$

$$\log_b a = 1.58 > 1$$

$$f(n) > n^{\log_b a - \epsilon}$$

$$\Rightarrow T(n) = \theta(n^{\log_b a}) = \underline{\underline{\theta(n^{\log 3})}}$$

$$18) T(n) = 4T(n/2) + cn$$

$$a = 4 \quad b = 2$$

$$\log_b a = 2 > 1$$

$$f(n) > n^{\log_b a - \epsilon}$$

$$\Rightarrow T(n) = \theta(n^{\log_b a}) = \underline{\underline{\theta(n^2)}}$$

$$19) T(n) = 3T(n/3) + n/2$$

$$a = 3 \quad b = 3$$

$$\log_b a = 1 = 1$$

$$f(n) = n^{\log_b a}$$

$$\Rightarrow T(n) = \theta(n^{\log_b a} * \log n) = \underline{\underline{\theta(n \log n)}}$$

$$20) T(n) = 4T(n/2) + n/\log n$$

$$a = 4$$

$$b = 2$$

$$k = 1$$

$$p = -1 \quad = -1$$

$$\log_b a = 2 > 1$$

$$f(n) > n^{\log_b a - \epsilon}$$

$$\cancel{f(n) = \theta}$$

$$\Rightarrow T(n) = \theta(n^{\log_b a}) = \underline{\underline{\theta(n^2)}}$$

$$21) T(n) = 7T(n/3) + n^2$$

$$a = 7 \quad b = 3$$

$$\log_b a = 1.77 < 2$$

$$f(n) < n^{\log_b a + \epsilon}$$

$$\Rightarrow T(n) = \theta(f(n)) = \underline{\underline{\theta(n^2)}}$$

$$22) T(n) = 8T(n/3) + 2^n$$

$$a = 8 \quad b = 3$$

$$\log_b a = 1.89 \Rightarrow n^{\log_b a} < 2^n$$

$$f(n) < n^{\log_b a + \epsilon}$$

$$\Rightarrow T(n) = \theta(f(n)) = \underline{\underline{\theta(2^n)}}$$

$$23) T(n) = 16(T/4) + n$$

$$a = 16 \quad b = 4$$

$$\log_b a = 2 > 1$$

$$f(n) > n^{\log_b a - \epsilon}$$

$$\Rightarrow T(n) = \theta(n^{\log_b a}) = \underline{\underline{\theta(n^2)}}$$