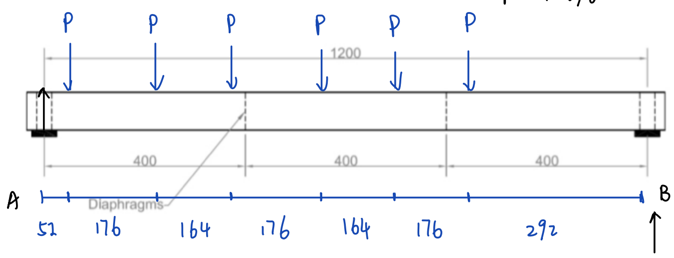
Hand Calculations for Design 0

Train @ 0 mm

$P = 400/6$



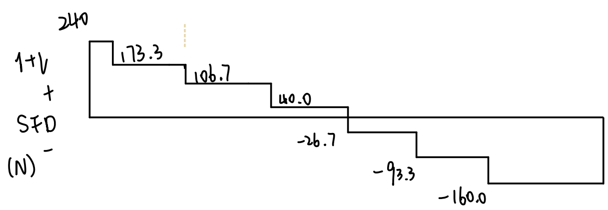
$\Sigma M_A = 0$

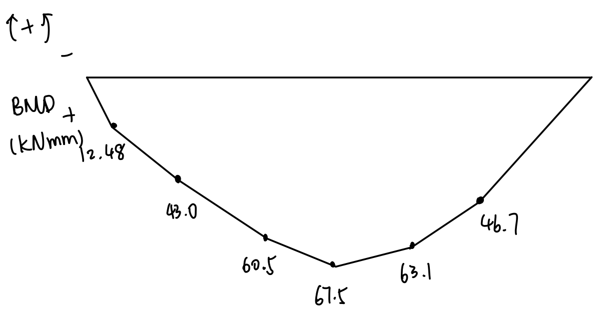
$52P + (52+176)P + (52+176+164)P +$
$(52+176+164+176)P +$
$(52+176+164+176+164)P +$
$(52+176+164+176+164+176)P - B_y(1200)$
$= 0$

$B_y = 160\,N$

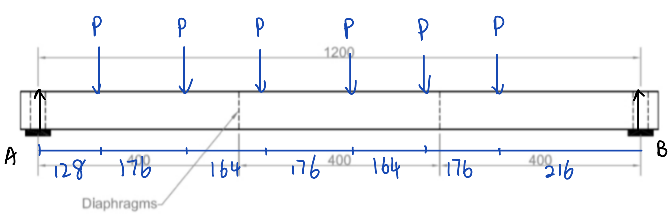
$\Sigma F_y = 0$

$A_y = 400 - 160 = 240\,N$





Train @ 76mm



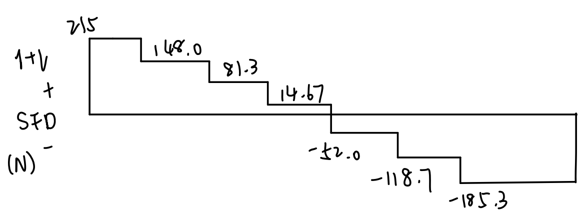
$\Sigma M_A = 0$

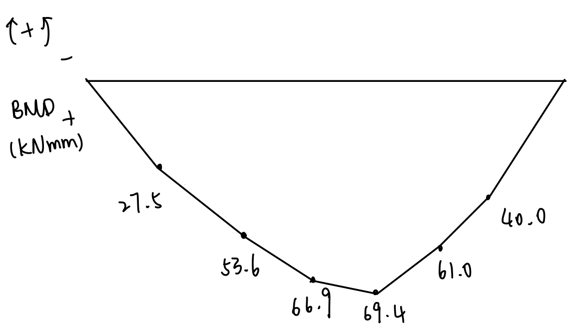
$128P + (128+176)P + (128+176+164)P +$
$(128+176+164+176)P +$
$(128+176+164+176+164)P +$
$(128+176+164+176+164+176)P - B_y(1200)$
$= 0$

$B_y = 185.3\,N$

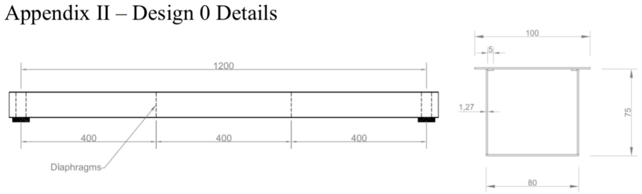
$\Sigma F_y = 0$

$A_y = 400 - 185.3 = 215\,N$

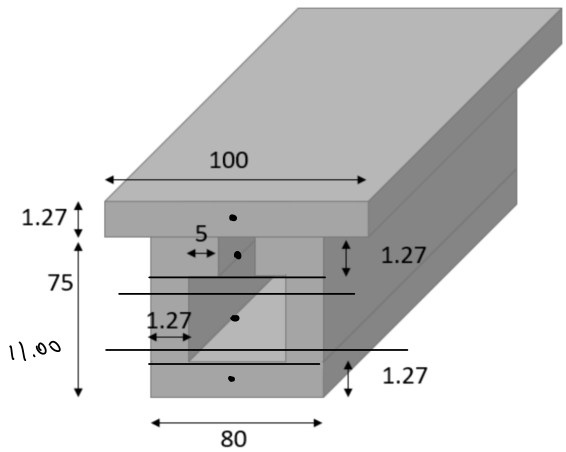




Appendix II – Design 0 Details



Elevation view and cross-section view of Design 0 (same cross-section view along the entire span)



Centroidal Axis

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{(100mm)(1.27mm)(75mm + \frac{1.27mm}{2}) + 2(5mm)(1.27mm)(75mm - \frac{1.27mm}{2}) + 2(1.27mm)(75mm-1.27mm)(\frac{75mm-1.27mm}{2} + 1.27mm) + (80mm)(1.27mm)(\frac{1.27mm}{2})}{(100mm)(1.27mm) + 2(5mm)(1.27mm) + (80mm)(1.27mm)}$$

$$= 41.43109\ mm \simeq 41.4\ mm$$

Cross-sectional Area

$$A = (100mm)(1.27mm) + 2(5mm)(1.27mm) + 2(1.27mm)(75mm - 1.27mm) + (80mm)(1.27mm)$$

$$= 428.5742 mm^2 \simeq 429 mm^2$$

Second Moment of Inertia

$$I = \Sigma I_0 + A_i d_i^2$$

$$= \frac{(100mm)(1.27mm)^3}{12} + (100mm)(1.27mm)(75mm + \frac{1.27mm}{2} - 41.4mm)^2$$

$$+ 2\left(\frac{(5mm)(1.27mm)^3}{12} + (5mm)(1.27mm)(75mm - \frac{1.27mm}{2} - 41.4mm)^2\right)$$

$$+ 2\left(\frac{(1.27mm)(75mm-1.27mm)^3}{12} + (1.27mm)(75mm-1.27mm)(\frac{75mm-1.27mm}{2} + 1.27mm - 41.4mm)^2\right)$$

$$+ \frac{(80mm)(1.27mm)^3}{12} + (80mm)(1.27mm)(\frac{1.27mm}{2} - 41.4mm)^2$$

$$= 418352.20899 mm^4 \simeq 418 \times 10^3 mm^4$$

Centroid Q

$$Q = 2(41.4mm - 1.27mm)(1.27mm)(\frac{41.4mm - 1.27mm}{2}) + (80mm)(1.27mm)(41.4mm - \frac{1.27mm}{2})$$

$$= 6193.2833 mm^3 = 6.19 \times 10^3 mm^3$$

Glue Q

$$Q = (1.27mm)(100mm)(75mm + \frac{1.27mm}{2} - 41.4mm) = 4343.8960 mm^3 = 4.34 \times 10^3 mm^3$$

$A = 428.5742\ mm^2$

$I = 418352.20899\ mm^4$

$Q_{cent} = 6193.2833\ mm^3$

$Q_{glue} = 4343.8960\ mm^3$

$SF_{max} = 240\ N$

$BM = 69445.333\ Nmm$

$E = 4000 MPa$

$\bar{y} = 41.43109$

Calculations used unrounded values

$\sigma_{tension} = 30MPa$ $\tau_{cent} = 4MPa$

$\sigma_{comp} = 6MPa$ $\tau_{glue} = 2MPa$

1) Flexural Stress Failure (tension)

$\sigma_b = \frac{My}{I} = \frac{(69445.333)(41.4)}{I} = 6.87MPa$ (tension) $FOS_{tension} = \frac{\sigma_{tension}}{\sigma_b} = \frac{30}{6.87} = 4.36$

2) Flexural Stress Failure (compression)

$\sigma_t = \frac{My}{I} = \frac{(69445.333)(75+1.27-41.4)}{I} = 5.79MPa$ (compression) $FOS_{compression} = \frac{\sigma_{compression}}{\sigma_t} = \frac{6}{5.79} = 1.037$

3) Shear failure (centroid):

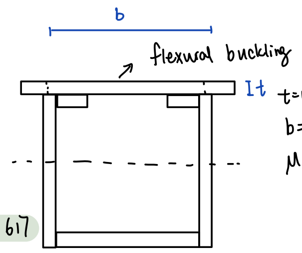
$\tau_{cent} = \frac{VQ_{cent}}{Ib}$ $b = 1.27 \times 2$ $\tau_{cent} = 1.399$ $FOS_{cent} = \frac{\tau_{board}}{\tau_{cent}} = \frac{4}{1.399} = 2.86$

4) Shear failure (glue):

$\tau_{glue} = \frac{VQ_{glue}}{Ib}$ $b = 1.27 \times 2 + 10$ $\tau_{glue} = 0.1987$ $FOS_{glue} = \frac{\tau_{glue}}{\tau_{glue}} = \frac{2}{0.1989} = 10.06$

5)

① Top flange middle buckling



$t = 1.27$

$b = 80 - 1.27$

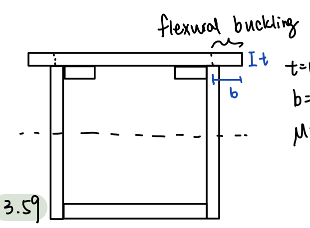
$\mu = 0.2$

$\sigma_{buck,1} = \frac{k\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$

$\sigma_{buck,1} = 3.57\ MPa$

$FOS_{buck,1} = \frac{\sigma_{buck,1}}{\sigma_t} = \frac{3.57}{5.79} = 0.617$

② Top flange edge buckling



$t = 1.27$

$b = 10 + \frac{1.27}{2} = 10.635$

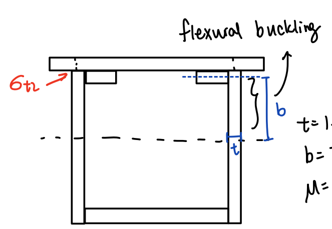
$\mu = 0.2$

$\sigma_{buck,2} = \frac{0.425\ \pi^2 E}{12(1-\mu^2)}\left(\frac{1.27}{10.635}\right)^2$

$= 20.8MPa$

$FOS_{buck,2} = \frac{\sigma_{buck,2}}{\sigma_t} = \frac{20.8}{5.79} = 3.59$

③ Web buckling



$t = 1.27$

$b = 75 - 41.43109 - \frac{1.27}{2} = 32.9$

$\mu = 0.2$

$\sigma_{buck,3} = \frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{1.27}{32.9339}\right)^2$

$= 30.6MPa$

Compression stress at $y = (75 - 41.4) = 33.6mm$

$\sigma_{t2} = \frac{My}{I} = \frac{(69445.333)(75-41.4)}{I} = 5.58\ MPa$

$FOS_{buck,3} = \frac{\sigma_{buck,3}}{\sigma_{t2}} = \frac{30.6}{5.58} = 5.48$

6)

Shear buckling

$$\tau = \frac{k\pi^2 E}{12(1-\mu^2)}\left[\left(\frac{t}{b}\right)^2 + \left(\frac{t}{a}\right)^2\right] = 5.27\ \text{MPa}$$

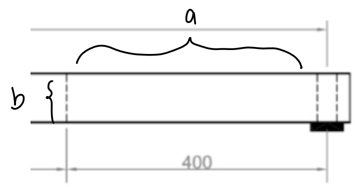
$k=5$ $a=385$ $b=75-1.27$

$$\text{FOS shear} = \frac{\tau_{buck}}{\tau_{cent}} = \frac{5.27}{1.399} = 3.77$$

min FOS is buckling at the middle of the top flange.

$0.617 \times 400\,N = 247N$

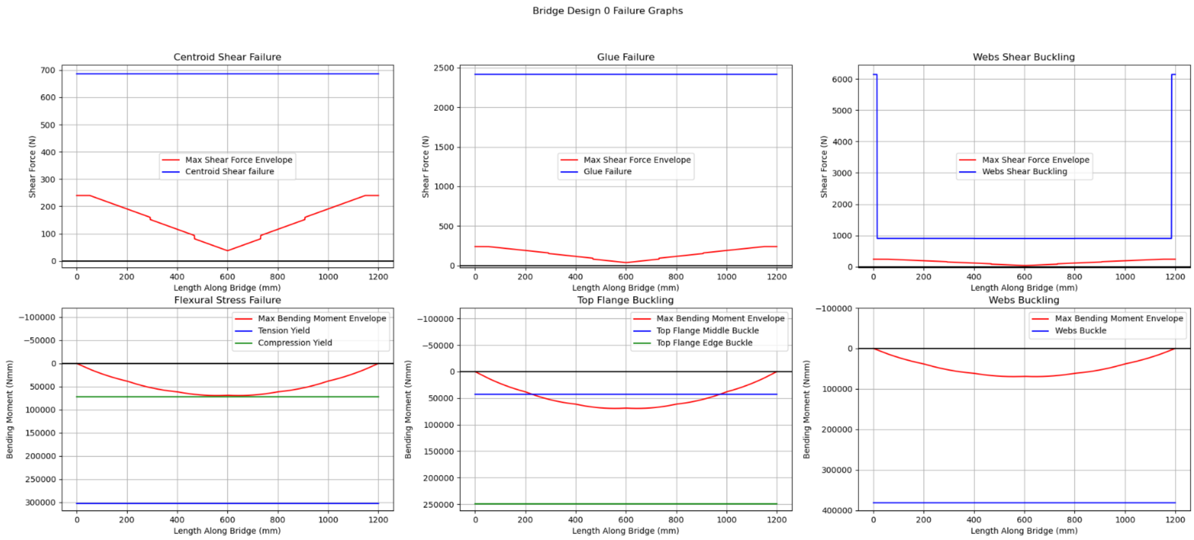
[Design 0 can take a 247N train]



Note: At the edge, diaphragms are 30mm apart, the shear buckling stress will be significantly higher due to small "a" value. The remaining part, considering the 30mm at the edge, will have an "a" value of 385mm.

Computer Output For Design 0

Computer Output:



- Cross Sectional Area = 428.57419999999996 mm^2
- Second Moment of Area = 418352.20899942354 mm^4
- Q at centroid = 6193.283330576374 mm^3
- Q at glue = 4343.896017305755 mm^3
- Centroid Axis y = 41.43109435192319 mm
- Max compressive Flexural Stress At Top = 5.783163955284812 MPa
- Max Tensile Flexural Stress At Bottom = 6.877449421183901 MPa
- Max Shear Stress At Centroid = 1.3988025236519268 MPa
- Shear Stress At Glue = 0.19872433823234437 MPa
- Top Flange Middle Buckling Stress = 3.5669267268124725 MPa
- Top Flange Edge Buckling Stress = 20.769624320288845 MPa
- Webs Buckling Stress = 30.57591595827078 MPa
- Max Compressive Flexural Stress on Webs = 5.572347395849709 MPa
- Shear Buckling Stress on Webs Near Edge = 5.270341432147787 MPa

- Board shear FOS = 2.8595887785196377
- Glue failure FOS = 10.064192528152448
- Shear buckling FOS = 3.7677523045842327
- Tension FOS 4.362082243395943
- Compression FOS 1.0374943623234194
- Top flange middle buckling FOS 0.616777728314778
- Top flange edge buckling FOS 3.5913946899791767
- Webs buckling FOS 5.48708000169619

The maximum train load this bridge can hold is 246.7110913259 N

Computer Output (Rounded):

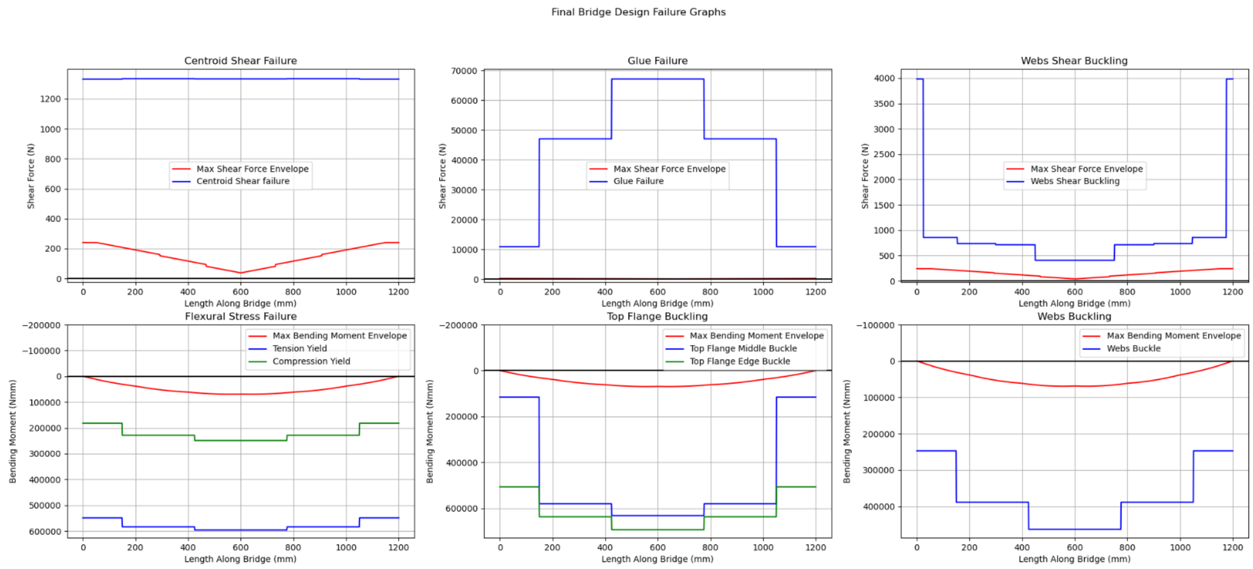
- Cross Sectional Area = 429 mm^2
- Second Moment of Area = 418000 mm^4
- Q at centroid = 6190 mm^3
- Q at glue = 4340 mm^3
- Centroid Axis y = 41.4 mm
- Max compressive Flexural Stress At Top = 5.78 MPa
- Max Tensile Flexural Stress At Bottom = 6.88 MPa
- Max Shear Stress At Centroid = 1.399 MPa
- Shear Stress At Glue = 0.1987 MPa
- Top Flange Middle Buckling Stress = 3.57 MPa
- Top Flange Edge Buckling Stress = 20.8 MPa
- Webs Buckling Stress = 30.6 MPa
- Max Compressive Flexural Stress on Webs = 5.57 MPa
- Shear Buckling Stress on Webs Near Edge = 5.27 MPa
- Board shear FOS = 2.86
- Glue failure FOS = 10.06

- Shear buckling FOS = 3.77
- Tension FOS 4.36
- Compression FOS 1.037
- Top flange middle buckling FOS 0.617
- Top flange edge buckling FOS 3.59
- Webs buckling FOS 5.49

The maximum train load this bridge can hold is 247 N

Calculations for Final Design

Computer Output:



board shear FOS: 5.547303849577916 at x=0
glue failure FOS: 45.29755951143525 at x=0
shear buckling FOS: 3.5638078015539616 at x=25
tension FOS: 8.581376898757934 at x=556
compression FOS: 3.5863854774831045 at x=556
top flange middle buckle FOS: 3.7259908002777107 at x=1051
top flange edge buckle FOS: 9.974103289382535 at x=556
webs buckle FOS: 6.104646363211872 at x=424

- Minimum FOS is 3.5638078015539616 From Shear Buckling at x=25
- Max Train Load this bridge can support is: 1425.5231206215847 N

Rounded Values:

board shear FOS: 5.55 at x=0
glue failure FOS: 45.3 at x=0
shear buckling FOS: 3.56 at x=25
tension FOS: 8.58 at x=556
compression FOS: 3.59 at x=556
top flange middle buckle FOS: 3.73 at x=1051
top flange edge buckle FOS: 9.97 at x=556
webs buckle FOS: 6.10 at x=424

- Minimum FOS is 3.56 From Shear Buckling at x=25
- Max Train Load this bridge can support is: 1426 N