Comparing Behaviour of Pendulum With Damped Harmonic Oscillator

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1 Introduction

This report aims to measure a pendulum and compare the behaviour to the model of a damped harmonic oscillator [1]. First using pendulum with length 0.447 ± 0.005 m, relationship of period versus various starting amplitude measured by 240 FPS video camera was found to generally follow a quadratic model. Then by measuring the same pendulum's oscillations using 30 FPS video camera and *Tracker* [2] starting at approximately $\pm 20°$ where the effect of angle on period is found to be negligible, the change of amplitude over time follows that of the model which predicted an exponential decay. Using the same amplitude versus time data, quality factor at that length was found to be 272 ± 4 using method of curve-fitting, and 270 ± 2 using method of counting, where both method produced results agreeing to each other. Later by varying length of pendulum between 5.1 cm to 50.2 cm, period was found to be depending on length in a square root relationship by measurements using 30 FPS video camera, with the exponent of power law model found to be 0.495 ± 0.009, supported by log-log linear plot with coefficient of 0.50 ± 0.01. Lastly, the quality factor was found to have a general trend of initially increasing with length for shorter pendulum, then decreases with increasing length after $L > 0.348 \pm 0.005$ m.

2 Theory

This section first discusses the theoretical relationship between release amplitude and period for a pendulum with length 0.447 ± 0.005 m. Then, the quality factor determination methods are discussed with consideration of amplitude's effects on period. Followed by theoretical predictions of the effects of pendulum length on the period and quality factor.

2.1 Period and Amplitude

Assuming $\sin\theta \simeq \theta$ at small angles, an approximation for a pendulum's period is:

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{1}$$

Where T is the period (s), L is the pendulum length (m), and g is the acceleration due to gravity ($\mathrm{m \cdot s^{-2}}$).

Since this equation is based on the approximation of $\sin\theta \simeq \theta$, period is underestimated at high angles. Therefore, the actual period will be high than the predicted constant period (for the same length) at higher angles. This relationship should be symmetrical for both positive and negative angles, thus period should not depend on the sign of release amplitude.

To model the period dependency on release amplitude, a power-series can be used. In this experiment, an approximation is made using a quadratic function to analyse the data:

$$T = A\theta_0^2 + B\theta_0 + C \tag{2}$$

Where θ_0 is the initial amplitude (rad), while A ($\mathrm{s \cdot rad^{-2}}$), B ($\mathrm{s \cdot rad}$), and C (s), are the coefficients of the quadratic function approximation.

A should be a non-zero number since it indicates the symmetrical effect of release angle on the period, and the theory supports that the release angle has symmetrical effects on period.

B should be zero, since a non-zero B indicates asymmetry in the pendulum. It can be claimed "experimentally zero" if the value is less than its uncertainty.

Since $T = C$ at zero-angle, C approximates small-amplitude period. Following Equation 1, C for this pendulum using $L = 0.447 \pm 0.005$ m and $g = 9.81\ \mathrm{m \cdot s^{-2}}$ is expected to be:

$$\begin{aligned} C \approx T &= \frac{2\pi}{\sqrt{g}}\sqrt{L} \\ &= 1.34 \pm 0.02 \text{ s} \end{aligned}$$

2.2 Quality Factor

Oscillating pendulums experience resistance and the amplitude is thus damped.

For pendulums, the general model for angle-time relationship, assuming resistance linearly proportional to velocity, is [1]:

$$\theta(t) = \theta_0 e^{-\frac{t}{2\tau}} \cos(\omega t + \phi) \tag{3}$$

Where $\theta(t)$ is the angle (rad) at time t (s), τ is the decay time (s), ω is the angular frequency ($\frac{\text{rad}}{\text{s}}$), and ϕ is the phase shift (rad).

Removing the cosine term gives the damped amplitude in the form of an exponential function:

$$A_d(t) = \theta_0 e^{-\frac{t}{2\tau}} \tag{4}$$

Where $A_d(t)$ is the damped amplitude (rad).

The quality factor Q is a parameter characterizing the rate of energy loss in an oscillator:

$$Q = \pi \frac{2\tau}{T} \tag{5}$$

From this, Q can be calculated by finding 2τ from the parameters of oscillation data's best fit curve to Equation 4.

By substituting Q into Equation 4 with number of oscillation, N, replacing time with $t = NT$:

$$\frac{A_d\left(\frac{Q}{2}\right)}{\theta_0} = e^{-\frac{\pi}{2}} \approx 0.2079 \tag{6}$$

Counting N required for the amplitude to decrease to 0.2079 of the initial amplitude gives $\frac{Q}{2}$.

Note: This derivation assumes $N = \frac{t}{T}$ which requires the period to be constant, or that changes to period is insignificant. Angle range for constant period is determined by the amplitude-period dependency experiment.

2.3 Period and Length

Using Equation 1, the relationship between pendulum length and period can be shown as:

$$T = \frac{2\pi}{\sqrt{g}} \sqrt{L}$$

Using $g = 9.81\ \text{m} \cdot \text{s}^{-2}$:

$$T = 2.01\ \frac{\text{s}}{\text{m}^{\frac{1}{2}}} \sqrt{L} \tag{7}$$

To verify this relationship, period can be plotted against length in a power law function:

$$T = kL^n \tag{8}$$

For this equation to match Equation 7, K should be 2.01 $\frac{\text{s}}{\text{m}^{\frac{1}{2}}}$ and n should be 0.5.

Taking the natural log of both sides of Equation 7 should give following linear equation:

$$\ln T = \ln k + n \ln L \tag{9}$$

2.4 Quality Factor and Length

Two values determining Q from Equation 5 are **not** obviously constant: the period T, and the decay time τ, which is the time it takes for the amplitude to decrease by a factor of $\frac{1}{e}$.

It is expected that τ remains constant throughout the experiment since it can be calculated using [1]:

$$\tau = \frac{m}{b} \tag{10}$$

Equation 10 depends only on two factors: a damping constant b and the mass of pendulum m. Both are expected to remain constant regardless of the pendulum length (Through experimentation it was found that τ is not constant. This will be discussed in Section 4.4).

Since τ is expected to remain constant, Q is expected to have an inverse relationship with T, which itself is related to length shown through Equation 7.

Therefore, Q should depend on pendulum length and is expected to show a relationship of:

$$Q \propto \frac{1}{\sqrt{L}} \tag{11}$$

3 Methods and Procedures

3.1 Pendulum Design

The initial pendulum tied a string to a hollow wheel and fixed the other end of string to a solid board. Release amplitude was measured by protractor in front of pendulum that is not touching the string, with protractor's base aligned to centre of rotation.

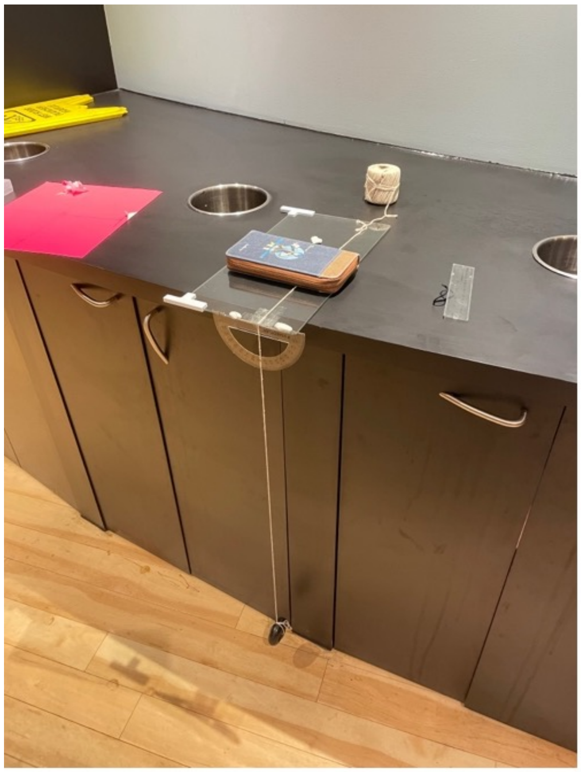

Figure 1: Initial pendulum design. Which was a rubber wheel tied to a string taped to a solid board. The string had to be held with a heavy object to prevent slipping.

This design had oscillations out-of-plane for angle measurement. Therefore, new wooden pendulum was designed to be suspended from a top board at two points so that out-of-plane movements is reduced/prevented. To hold the board in place, four legs were designed to extend out of the board at 30° angle to withstand force the pendulum exerts to top board.

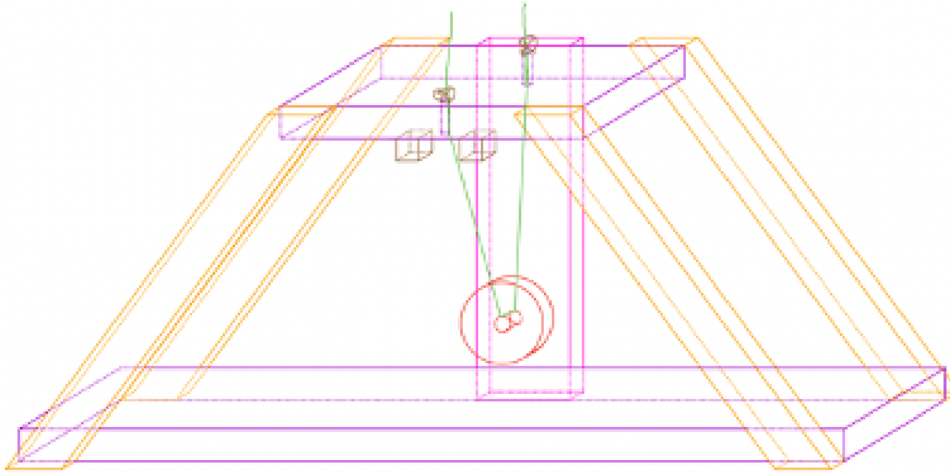

Figure 2: Initial digital design of the new pendulum. The support board (coloured in pink) was removed from the final design. The purple bottom board was replaced with smaller boards only connecting adjacent legs on the short sides. The way the boards connects were also changed, which is shown in later figures.

The material was later changed from wood to foam-boards and honeycomb cardboards out of consideration for cost and weight.

The pendulum was constructed:

Top board (foam-board): 6in×13in with rubber tubes fit through two holes at centre of the board 1in away from longer side to hold strings in place and prevent damages to board. Rubber tubes also reduces friction from strings contacting with other materials.

Support legs (honeycomb cardboard): Four 3.5in×24in boards with 30° cut from opposite corners. One side of legs connects to top board, and adjacent legs on short side were connected using a smaller foam-board at bottom to improve structural stability. Connections used glue gun.

Top board is enhanced using a 2in×13in foam-board glued sideways to prevent top board bending.

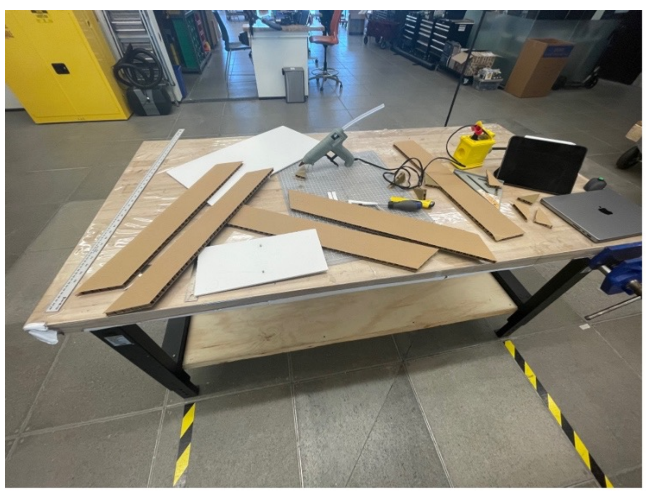

Figure 3: Components of the pendulum prior to combining them together.

Rubber wheel was suspended using one thin string as shown in Figure 4. A protractor was stuck to side of frame and aligned with the pendulum's centre-of-rotation with "90° mark" aligning with string when pendulum was at rest. Thin wire was chosen to allow more accurate angle measurements using protractor. The radius of rotation of the pendulum between the centre of rotation and the centre of mass of the rubber wheel was measured to be 0.447 ± 0.005 m.

Figure 4: The final pendulum of a rubber wheel held by a thin string that passes through it, and suspended by a foam-board held up by angled honeycomb cardboard supports.

Using digital level, top board was adjusted to have 0° incline to prevent asymmetry in oscillation.

Figure 5: The pendulum's level being tested using the built-in level in a smart-phone.

3.2 Period and Amplitude

A 240 frames-per-second (FPS) slow-motion camera was aligned to "90° mark" of protractor and the two strings of the pendulum at rest such that the pendulum oscillates in-plane with the camera. This ensures accurate angle measurement and eliminates effects of differences in camera's perspective.

Aligning eyesight with the two strings and angle markings on the protractor, the amplitude of pendulum release was determined for each trial. The range of release amplitude spans from -90° to +90° in 5° increments, excluding 0°. For each amplitude, five trials were done.

240 FPS recordings was imported to *Tracker* [2] and the software was used to find the difference in frame numbers for one oscillation of the pendulum after its string first reaches angle 0°. This was due to high uncertainty determining the frame number for when pendulum's motion begun from pendulum's low initial velocity. When pendulum passes through angle 0°, it is more certain. It was assumed that friction when pendulum moves from initial angle to 0° is insignificant and does not affect period measurement.

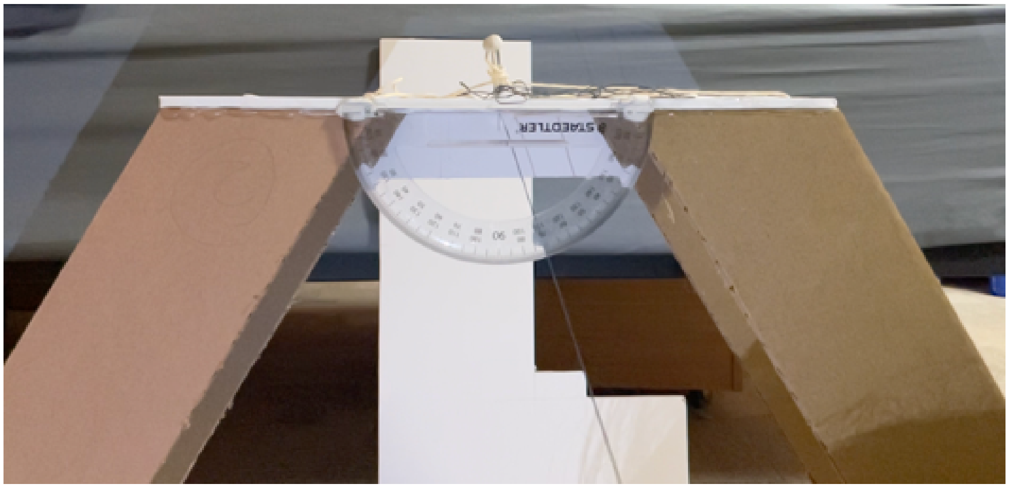

Figure 6: Screenshot of one of the slow-motion videos used for measuring period of one oscillation.

3.3 Quality Factor

A 30 FPS camera was aligned in-plane to the centre of pendulum's oscillation. This ensures pendulum's angle shown in video recording is consistent to actual pendulum's angle. A lower FPS was used because time uncertainty for one specific data becomes less significant as larger collection of data can reduce random errors in values generated from best fit curve. For Q factor obtained from counting, time uncertainty was insignificant compared to period.

The pendulum was released at approximately 28° and allowed to oscillate. The recorded "start time" was when the pendulum's amplitude first decreased below 20° because at angle below 20°, period's dependency on amplitude becomes insignificant compared to the measured time uncertainty, fulfilling the constant period assumption to compute Q factor by counting oscillations. The amplitude was measured through plotting pendulum position in the recording using *Tracker* [2] for each maximum angle reached to get a high precision of angle measurement.

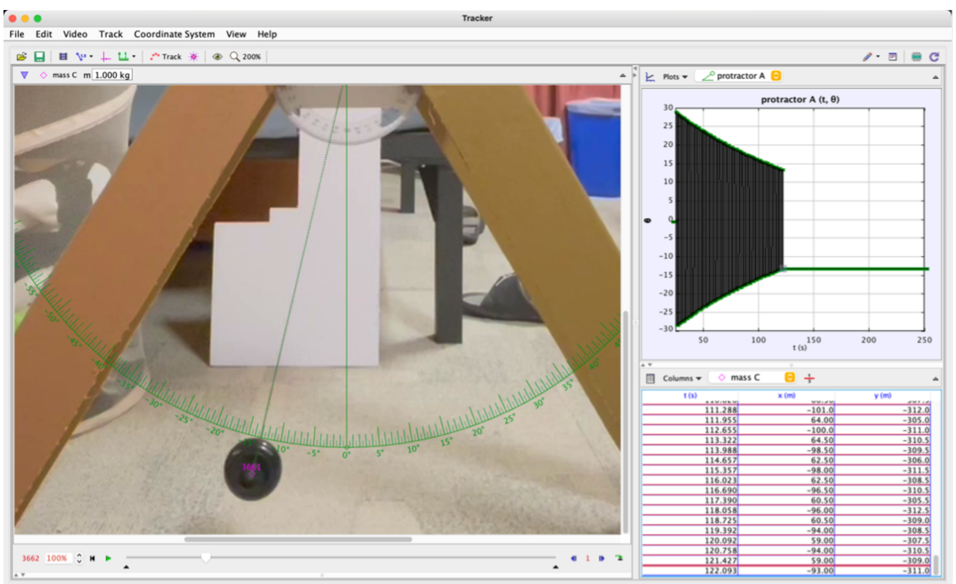

Figure 7: Screenshot of *Tracker* [2] while taking measurements for the amplitude of oscillation.

Prior to analysis, all angle measurements were converted to radians.

3.4 Period and Length

For this portion, the pendulum has been slightly modified to allow the ease of adjusting the pendulum length. The method of connecting the pendulum string to the base of rotation was switched from tying a knot to using small clamps so that there is no need to re-tie the knot for every length, and the length adjustment can be more precise.

Figure 8: The base of the pendulum using clamps to hold the strings in place, allowing easy and more precise length adjustments

The range of length spans from 0.051 m (5.1 cm) to 0.502 m (50.2 cm) in increments of approximately 5 cm. After each length adjustment, the pendulum's length was re-measured from the base of rotation to the centre of mass of the pendulum.

The period for each length was measured similarly to Section 3.2. The difference is that for this section, only one measurement of period was done for each of the angles ranging from -20° to +20° in 5° increments, since periods at these angles have been determined to show little to no dependency to the release amplitude. Furthermore, 30 FPS camera was used for this section's determination of period.

3.5 Quality Factor and Length

At each length, Q was measured using the method of counting oscillations since the two methods of determining Q agree with each other, and that counting oscillations for a large set of data can be completed in shorter time.

Measurements are done similarly to section 3.3, using *Tracker* [2] to find angle of pendulum to determine the how many oscillations it takes for pendulum to oscillate $\frac{Q}{2}$ times.

4 Results and Analysis

4.1 Period and Amplitude

Data for period and release amplitude was plotted in the following graph, with quadratic curve of best generated by python script:

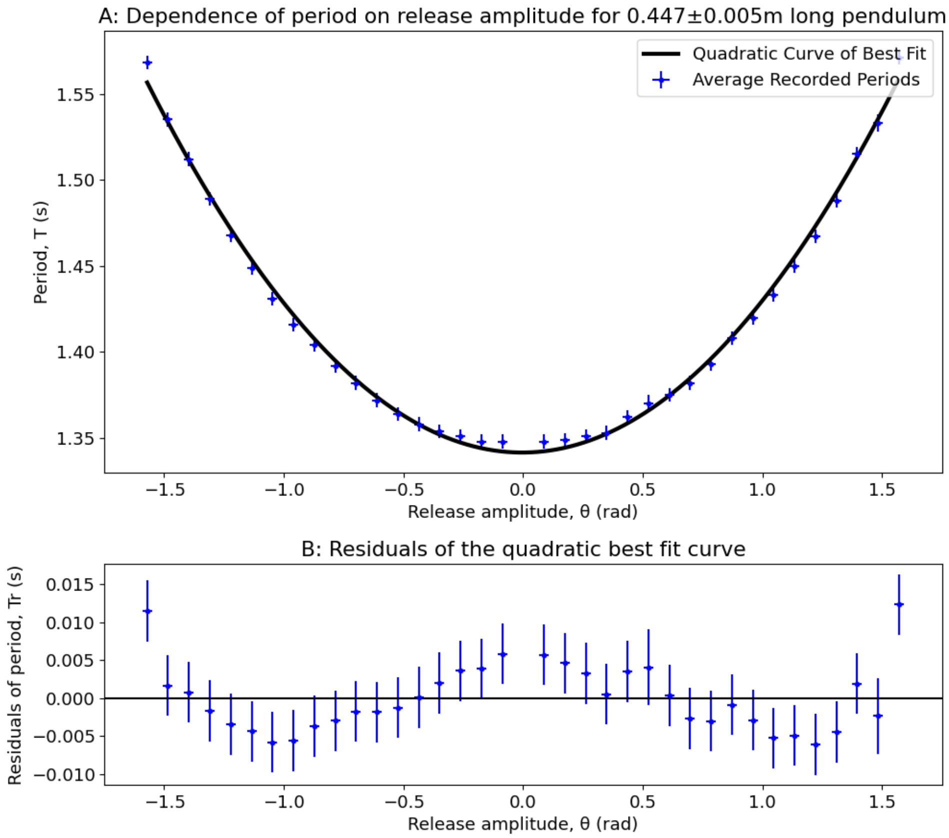

Figure 9: Plot of the periods of one oscillation (averaged from five separate measures of periods) of pendulum released at different amplitudes. A quadratic best fit curve was used (A), and the residual can be seen at the bottom (B).

The measurement uncertainties associated with the data are:

- Amplitude: ± 0.009 rad - Thickness of string compared to minimum increment of analogue protractor makes angle measurement only certain to $\pm 0.5°$. Converting to radians gives: $\Delta\theta_0 = 0.5° \times \frac{\pi}{180°} = 0.009$ rad
- Period: ± 0.004 s and ± 0.005 s - 240 FPS video used to measure time has uncertainty of one frame, which is ± 0.004 s. Additionally, the standard deviation of the five trials used to find average period was calculated. The largest of two uncertainties are used.

Other sources of uncertainties includes:

- The act of releasing pendulum was done using hand, which may potentially affect the period of pendulum if the pendulum was not released in the exact same ways. This should not be a significant error since the period measurements are all very close to the average, and most of the standard deviations of the periods was smaller than the camera's own uncertainty.
- The camera angle might not be exactly aligned with the 90° mark on the protractor, so the period recorded may be the time interval of when the pendulum passes through a different point other than the bottom. This should not be significant since even if camera is not recording the absolute bottom of the protractor, it should still capture the period of one oscillation because the pendulum's oscillation should not be affected by external factors such as frictions in this short period of time.

Overall, the largest uncertainty would be the amplitude measurements, since other errors are insignificant compared to it, and inaccurate amplitude measurements can cause a shift in all data points to left or right of the graph.

Using Equation 2, quadratic best fit curve follows: $T = A\theta_0^2 + B\theta_0 + C$. The generated parameters are:

$$A = 0.088 \pm 0.001 \text{ s} \cdot \text{rad}^{-2} \tag{12}$$

$$B = 0.0007 \pm 0.0008 \text{ s} \cdot \text{rad} \tag{13}$$

$$C = 1.342 \pm 0.001 \text{ s} \tag{14}$$

B is experimentally zero because it is less than its uncertainty, indicating that the pendulum has no asymmetry in dependence of period on release amplitude.

A is a small non-zero value, showing symmetrical dependency between period and amplitude, and that period increases with release amplitudes.

C, the small angle period, fitted from data, is 1.342 ± 0.001 s, is consistent with theoretical prediction for low angles periods since C lies within the uncertainty of theoretical $T = 1.34 \pm 0.02$ s.

This quadratic curve is a good fit to the data, since it matches the symmetrical dependency of period on release amplitude. Also, the calculated R^2 value is as high as 0.9652, further supporting that this is a good fit.

To determine quality factor, period needs to be independent of amplitude since both damped harmonic oscillation equation and equation for Q assumes constant period.

For amplitude between -1.57 rad and +1.57 rad , periods span from 1.348 ± 0.002 s to 1.571 ± 0.004 s . This range was too big to have period assumed constant, thus a smaller angle range must be used for constant period. A determined angle range was between -0.349 rad and +0.349 rad because:

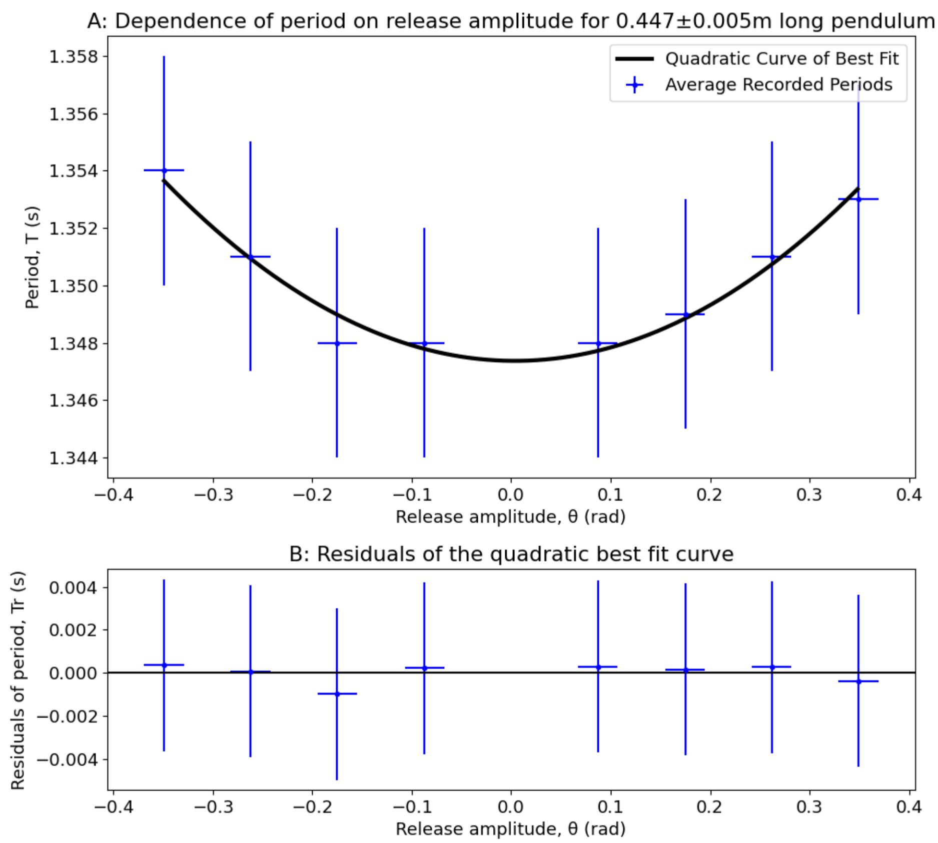

Figure 10: Plot of the periods of one oscillation (averaged from five separate measures of periods) of the pendulum released at different amplitudes between -0.349 and +0.349. A quadratic best fit curve was used (A), and the residual can be seen at the bottom (B).

Using best fit curve for data in this angle range, the best fit equation for this curve is:

$$T = \left(0.050 \pm 0.004 \text{ s} \cdot \text{rad}^{-2}\right) \theta_0^2 + \left(-0.0003 \pm 0.0008 \text{ s} \cdot \text{rad}\right) \theta_0 + \left(1.3474 \pm 0.0003 \text{ s}\right) \tag{15}$$

Between -0.349 rad and +0.349 rad ($\pm 20°$), difference between maximum and minimum period calculated by this equation is 0.0067 s. Since this difference is less than time measurement uncertainty for method used in determining quality factor (± 0.03 s), the differences in period can be claimed as insignificant.

4.2 Quality Factor

Data for amplitude against time after initial release was plotted in the following graph, with exponential curve of best fit, and a linear best fit line for comparison generated by python script:

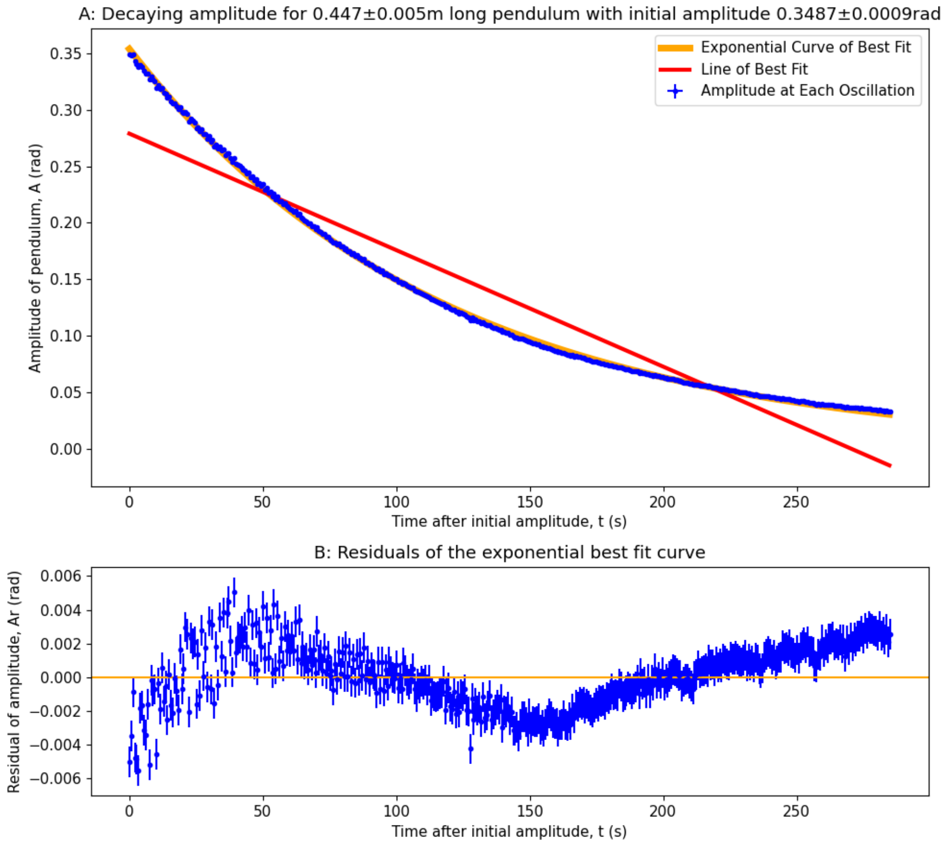

Figure 11: Plot of the amplitude of every oscillation of the pendulum and the time after the initial amplitude was reached (B). An exponential best fit curve was used, and the residual can be seen at the bottom (B). A linear best fit line was plotted for comparison of models.

The uncertainties associated with the data are:

- Amplitude: ± 0.0009 rad - *Tracker* [2] records angle of any chosen point with a high precision. However, the limitation of angle determination lies on the person marking the position of pendulum. The maximum and minimum measured angle for a given point in time was determined based on the thickness of the pendulum string, which is $0.05°$. Converting to radians gives: $\Delta\theta_0 = 0.05° \times \frac{\pi}{180°} = 0.0009$ rad
- Time: ± 0.03 s - 30 FPS video used to measure time has uncertainty of one frame, thus the uncertainty is the time of one frame.

Other sources of uncertainties includes:

- Similar to the previous section, when releasing pendulum by hand, there may be small vibrations and oscillations in and out of plane of intended oscillation, affecting the angle measurements. This should not be significant since the pendulum was allowed to oscillate for multiple oscillations prior to recording angle data. The two-string set up of the pendulum should have minimized the effect of initial disruptions.

Overall, the largest uncertainty would only be the angle measurement, especially when the amplitude decays to a small value, since then the relative uncertainty of the angle becomes increasingly significant.

Using Equation 4 for exponential fit:

$$A_d(t) = (0.3537 \pm 0.0003 \text{ rad})\, e^{-\frac{t}{116.2 \pm 0.1 \text{ s}}} \tag{16}$$

Generated θ_0 is 0.3537 ± 0.0003 rad, the generated initial amplitude based on this model and the data, was slightly higher than the actual initial amplitude 0.3487 ± 0.0009 rad. The percentage error is:

$$\frac{(0.3537 \pm 0.0003 \text{ rad}) - (0.3487 \pm 0.0009 \text{ rad})}{(0.3487 \pm 0.0009 \text{ rad})} \times 100\% = 1.4 \pm 0.8\% \tag{17}$$

The percentage error of 1.4% indicates a difference between reality and the collected data that is not very significant.

Exponential curve appears to be a "good fit" since the general trend of data follows the curve, supported by a high R^2 value of 0.9996, showing that the model fits the data.

It can also be concluded from the graph that amplitude certainly does not decay following a linear model, since linear best fit deviates from the data significantly.

The generated $2\tau = 116.2 \pm 0.1$ s, and theoretical period of oscillation at low angles, $T = 1.34 \pm 0.02$ s, were used to compute Q using Equation 5:

$$Q = 272 \pm 4 \tag{18}$$

Counting the number of oscillations to $A_d\left(\frac{Q}{2}\right) = (0.3487 \pm 0.0009 \text{ rad})e^{-\frac{\pi}{2}} = 0.0725 \pm 0.0002$ rad following Equation 6:

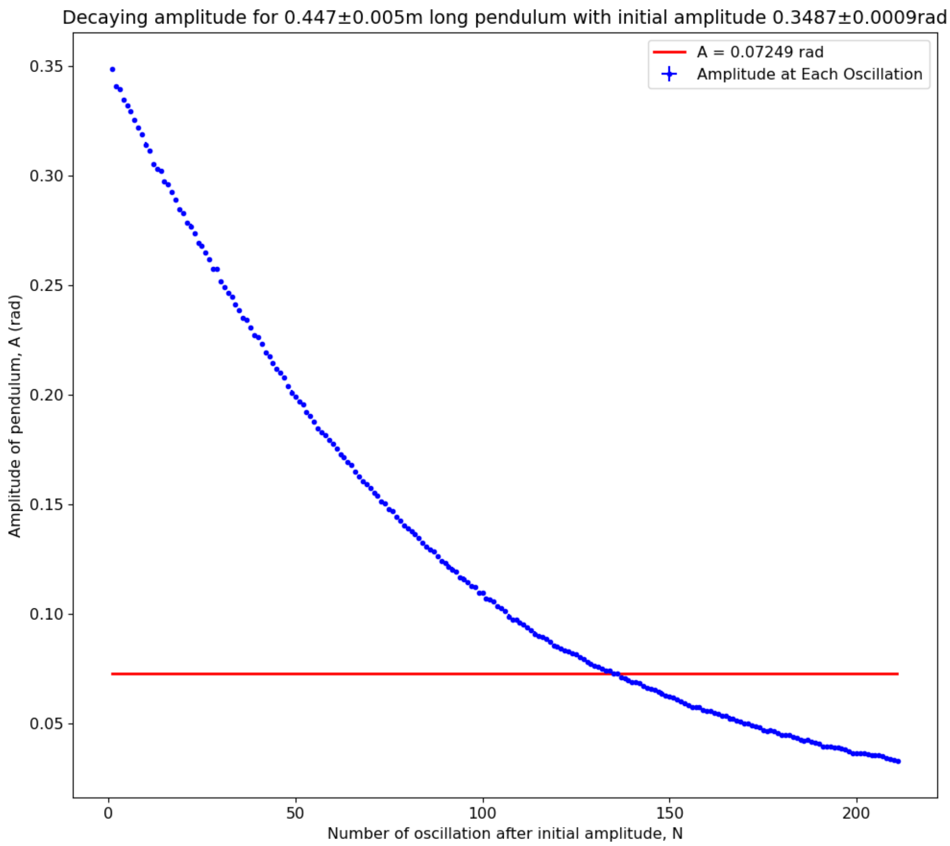

Figure 12: Plot of the amplitude of every oscillation of the pendulum and the number of oscillations after the initial amplitude was reached. A horizontal line was drawn to indicate the oscillation number $= \frac{Q}{2}$

From this, it was found that oscillation number 134 and oscillation number 135 have amplitudes 0.0727 ± 0.0009 rad and 0.0725 ± 0.0009 rad, which overlaps with 0.0725 ± 0.0002 rad and any other oscillations have amplitude that does not fall within this range. This shows that for number of oscillations to be equal to $\frac{Q}{2}$, $\frac{Q}{2}$ falls within 134 and 135 with uncertainty of 1 (since $\frac{Q}{2}$ is a counted value).

Therefore:

$$\frac{Q}{2} = 135 \pm 1 \tag{19}$$

$$Q = 270 \pm 2 \tag{20}$$

The largest uncertainty associated with the counting method is the uncertainty of angles, which causes overlaps of measured angles and the theoretical expected angle. This causes uncertainty with the counted oscillation number and carries towards calculation of Q.

Comparing Q obtained with different methods:

$$Q_{calculated} = 272 \pm 4 \tag{21}$$

$$Q_{counted} = 270 \pm 2 \tag{22}$$

The results from both methods agree with each other, since their uncertainties overlaps with each other. For future experiments, the counting method was used since it both produces less uncertainty and requires less process to collect data.

4.3 Period and Length

Data for period against pendulum length was plotted in the following graph, with power law curve of best fit following Equation 8:

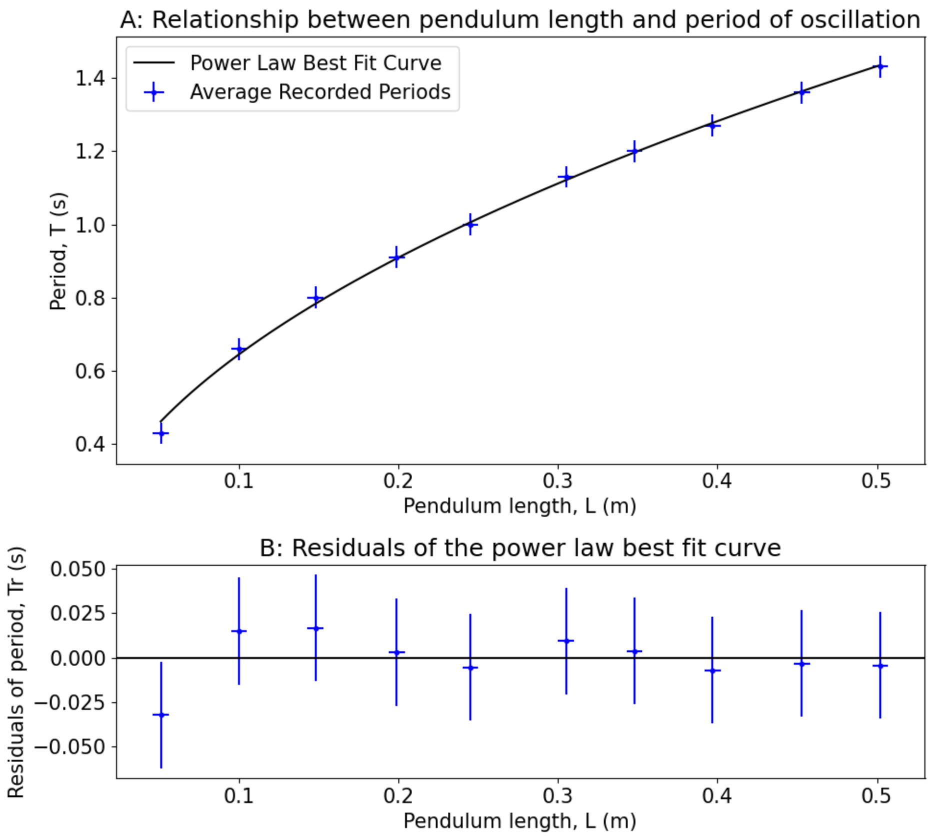

Figure 13: Plot of the period of oscillation of the pendulum (averaged from eight different angles) with different pendulum length. A power law best fit curve was used (A) and the residual can be seen at the bottom (B).

The measurement uncertainties associated with the data are:

- Period: ± 0.3 s - 30 FPS video used to measure time. Duration of one frame is ± 0.3 s. Additionally, the standard deviation of the eight periods measurement was calculated. Since no standard deviation is larger than 0.3 s, this uncertainty was used for all data points.
- Length: ± 0.005 m - The rubber wheel used as the oscillating mass has a hole with diameter approximately 1 cm at the centre, causing uncertainty of length of ± 0.5 cm which was then converted to m.

Other sources of uncertainty are generally the same as Section 4.1, which includes hand-shaking when releasing pendulum and camera angle error.

Overall, for this section, largest source of uncertainty would mainly be the length measurements. Period measurements can bring uncertainty, but averaging multiple (8) measurements should have reduced errors of measurement.

Following Equation 8, the generated best fit curve is:

$$T = \left(2.02 \pm 0.02 \ \frac{\mathrm{s}}{\mathrm{m}^{0.495}}\right) L^{0.495 \pm 0.009} \tag{23}$$

K is $2.02 \pm 0.02\ \frac{\text{s}}{\text{m}^{0.495}}$, which includes the theoretical value $2.01\ \frac{\text{s}}{\text{m}^{\frac{1}{2}}}$ within its uncertainty range. n is 0.495 ± 0.009, which includes the theoretical value $\frac{1}{2}$ within its uncertainty range. This shows that the collected data agrees with the theoretical prediction of period's dependence on pendulum length.

The best fit curve matches the data collected as it passes through the uncertainty bar of almost all data points (even for the data point it does not pass through the uncertainty bar, it was still reasonably close). Furthermore, R^2 value of this curve fit is 0.9981, further showing that it is a good fit.

Taking the natural log of both the recorded period and length gives:

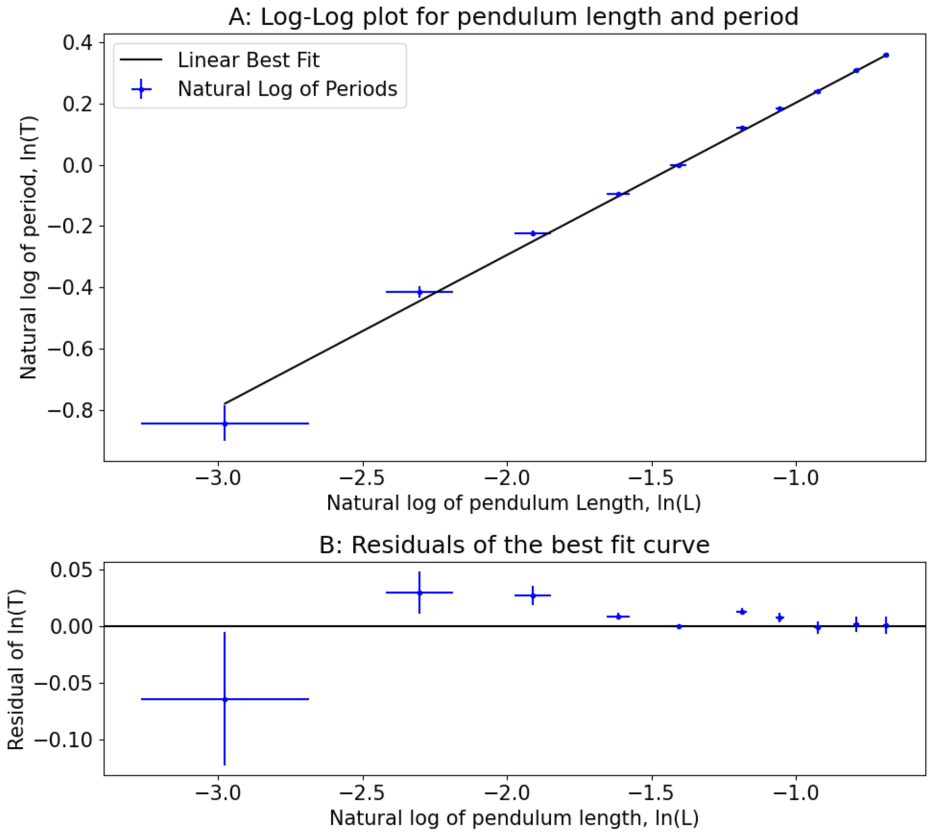

Figure 14: Plot of natural log of period of oscillation of the pendulum in seconds (averaged from eight different angles) against natural log of pendulum length in metres. A linear best fit was used (A) and the residual can be seen at the bottom (B).

Following Equation 9, the computer generated parameters of the linear fit is:

$$\ln k = 0.70 \pm 0.02 \tag{24}$$

$$n = 0.50 \pm 0.01 \tag{25}$$

These two values further shows that the theory is consistent with data collected, since the value n is exactly $\frac{1}{2}$ as predicted (plus uncertainty) and $\ln k$ matches natural log of expected $k = 2.01\ \frac{\text{s}}{\text{m}^{\frac{1}{2}}}$ since $\ln 2.01 = 0.698$ falls within the uncertainty of the generated $\ln k$.

The line of best fit matches the data since it can be seen in Figure 14-A, that the best fit line passes through the uncertainty bars of all data. Also, the R^2 value of this best fit line is 0.9952, showing that it is a good fit.

Overall, from both the power law best fit curve and the linear fit of the log-log graph, it can be seen that the theory is consistent with the collected data.

4.4 Quality Factor and Length

Collected quality factors are plotted against pendulum length in the following graph, fitted to a general sinusoidal best fit curve:

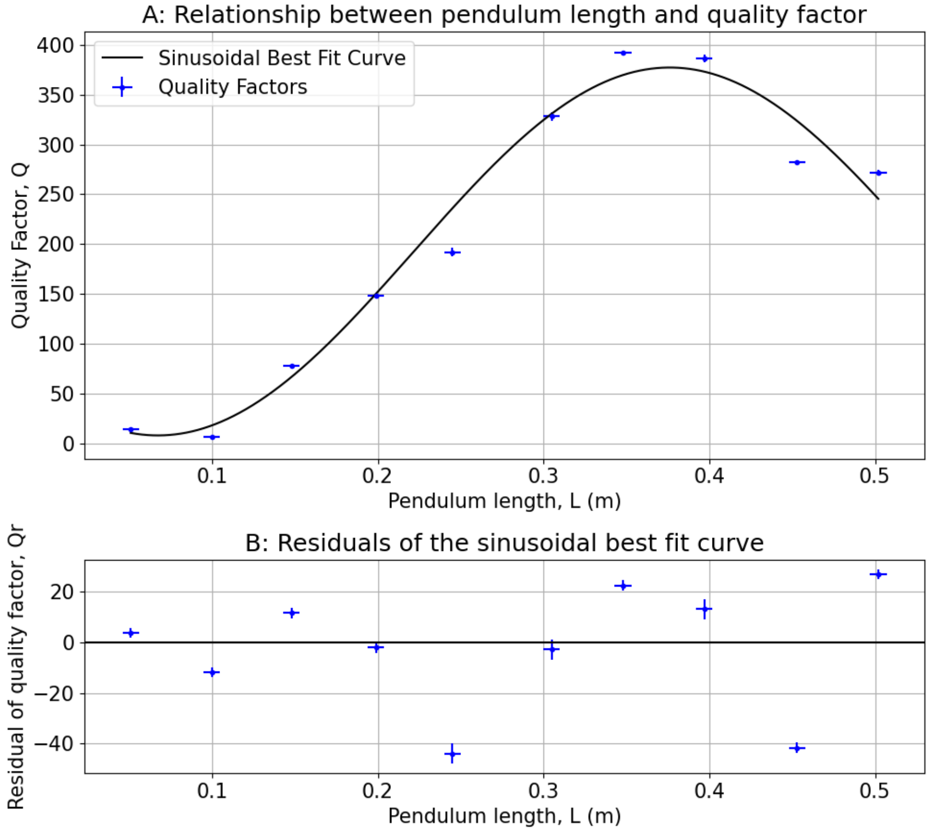

Figure 15: Plot of quality factors against pendulum length. A sinusoidal best fit curve was used (A) and the residual can be seen at the bottom (B).

The measurement uncertainties associated with the data are:

- Quality Factor: ± 2 to ± 4 - quality factors was determined using the "counting" method used in Section 4.2, and the uncertainty was determined the same way.
- Length: ± 0.005 m - same as Section 4.3.

Other sources of uncertainty are the same as Section 4.2, since the same method was used.

Overall, the largest uncertainty is the length measurements, since the relative uncertainties for Q are mostly smaller than the relative uncertainty in L. Also, relative uncertainty of pendulum length being more significant can directly be observed from Figure 15-A.

The parameters of the best-fit sinusoidal curve, following a general formula of $A\sin(\omega x + \phi) + D$ are:

$$A = -190 \pm 10 \tag{26}$$

$$\omega = 10.4 \pm 0.6\ \frac{\text{rad}}{\text{s}} \tag{27}$$

$$\phi = 0.8 \pm 0.2\ \text{rad} \tag{28}$$

$$D = 200 \pm 10 \tag{29}$$

A sinusoidal best fit curve was used due to the observed general relationship between Q and L, which the Q first increases with L, then decreases when L keeps increasing. Different best-fit curves were compared, and the sinusoidal best fit curve produces the largest R^2 value of 0.9673. Additionally, this curve is a good fit not only because of the high R^2 value, but also because it matches the trend of data within the range of collected data and is close to most data points.

The existence of a trend shows that Q has a clear dependence on pendulum length since the differences in Q at different length is significantly larger than their uncertainties. However, this trend does not agree with the prediction shown in Equation 11, indicating potential flaws in the theory used.

One possible explanation would be that the prediction using Equation 5 assumed that τ is constant, but it is not constant.

τ is a measurement of the effect of resistance force on pendulums' oscillations, and in a real pendulum, there should be two main sources of resistance: the air drag, and the friction at the base of the pendulum.

When pendulum is long, it oscillates at low frequency (due to longer period) while travelling through air over longer distance at a higher average speed. This means higher resistance due to drag because of its high speed travelling through more air, and lower friction at the base because the string slides against the base less frequently.

When pendulum is short, it oscillates at a high frequency (due to shorter period) while travelling through air over shorter distance at a lower average speed. This means lower resistance due to drag because of its low speed travelling through less air, and higher friction at the base because the string slides against the base more frequently.

An additional factor affecting the friction at the base is the "two-strings" set-up of the pendulum used in this experiment. The two-string set-up initially aimed to reduce in-out plane oscillation, but it could have potentially increased the friction at lower pendulum length:

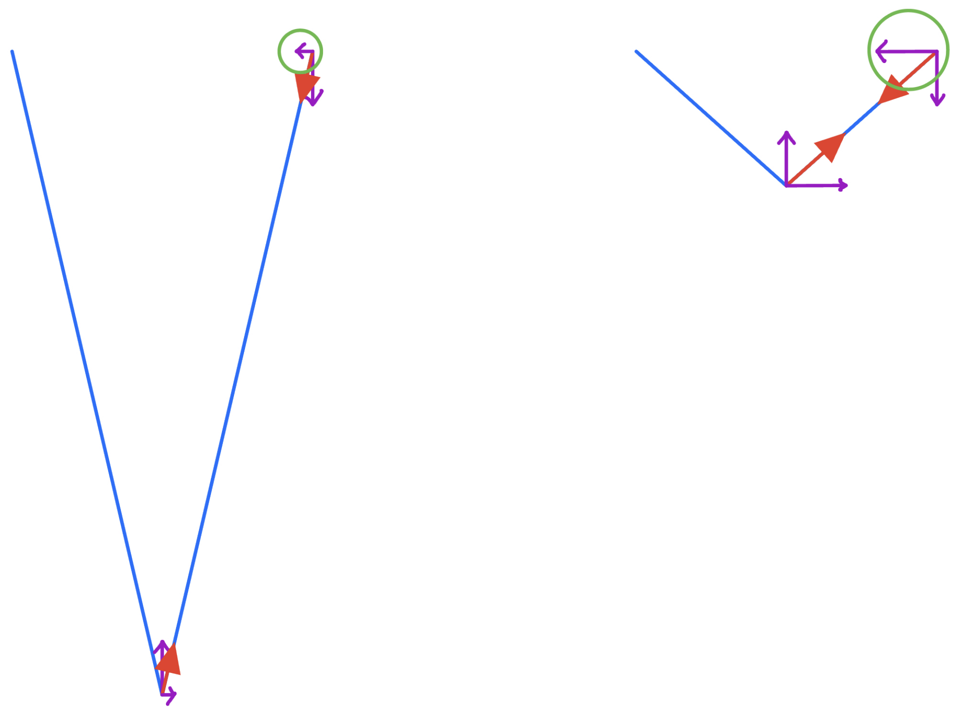

Figure 16: Diagram showing the strings of this pendulum at different lengths, with longer pendulum on the left and shorter pendulum on the right. Forces are labelled for the string on the right side, with x-y components coloured in purple and overall force coloured in red.

- The pendulum in Figure 16 oscillates in and out of page with two strings to prevent unwanted oscillation in the "wrong" direction (in this diagram, the "unwanted" oscillation direction is left and right)
- Pendulum on the left has longer length. As seen from Figure 16, it forms a larger angle with the base of oscillation, which provides supports to the string to hold them in place. The purple force inside the green circle is the horizontal component of the string's tension, which is supported by the base through normal force.
- Pendulum on the right has shorter length, which forms a smaller angle with the base of oscillation. As can be seen from Figure 16, the horizontal force is larger and thus more normal force is provided by the base to support the pendulum.
- Since right pendulum's base provides more normal force to string than the left pendulum, the pendulum with shorter length experiences more friction because friction scales with normal force.

Overall, since friction with base is significant at lower length, the resistance increases as length decreases. The result is that Q decreases as length decreases. Since air drag in significant at higher length, resistance increases with length, thus decreasing Q with increasing length. This explanation matches the relationship observed in Figure 15, where Q first increases, then decreases for $L > 0.348 \pm 0.005$ m.

5 Conclusion

In conclusion, this experiment explored behaviour of pendulum through first measuring period's dependency on release amplitude as well as angle range where effect of angle on period is negligible using 240 FPS camera and *Tracker* [2], then measuring the behaviour of pendulum and quality factor, period's dependency on length, and quality factor's dependency on length using 30 FPS camera and *Tracker* [2].

Section 4.1 shows that period depends on release amplitude symmetrically, and that period is approximate constant for release amplitude between ± 0.349 rad ($\pm 20°$).
Largest uncertainty for this section is the amplitude (angle) measurements.

Section 4.2 shows that pendulum's amplitude decreases following exponential decay, and that both methods of determining Q produces results that match each other since their uncertainties overlaps with each other. It was determined that the method of counting oscillation should be used for future Q determination.
Largest uncertainty for this section is the angle measurement for both methods of determining Q.

Section 4.3 shows a square-root relationship between T and L, as demonstrated by both a power law graph and a log-log graph which generated parameters with that of a square root relationship within the uncertainty range.
Largest uncertainty for this section is the length measurements.

Section 4.4 shows that Q has dependency on L, and that Q increases with L with shorter lengths, but decreases with L when length is longer than 0.348 ± 0.005 m.
Largest uncertainty for this section is the length measurements.

In future experiments, to reduce the uncertainty, the following can be done:

- To make more accurate measurements of angle for amplitude versus period data, the angle should be measured digitally after pendulum is released instead of manually measuring angles using a protractor.
- To reduce angle uncertainty in digital angle measurements, a thinner string can be used so the position marked in *Tracker* [2] can be more precise and thus give precise angle measurements.
- To reduce uncertainty in length measurements, the centre of mass of the swinging object can be first marked, and measure length from that point. This way the length measurement will not be limited by the size of the hole in the centre of the rubber wheel.

References

[1] J Poonyawatpornkul and P Wattanakasiwich 2013 High-speed video analysis of damped harmonic motion Phys. Educ. 48 782. doi:10.1088/0031-9120/48/6/782

[2] Douglas Brown, Wolfgang Christian and Robert M Hanson. Tracker. https://physlets.org/tracker/