The effect of different initial heights of a falling mass hung by a string on the angular velocity of a rotating wheel.

Aim

The aim of this experiment was to investigate the effect of different initial heights above the ground of a mass hung (30.00 cm, 60.00 cm, 90.00 cm, 120.00 cm, 150.00 cm, 180.00 cm) tied to a string, that was wrapped around the outside of a bicycle wheel without the tire, on the angular velocity ($\frac{\text{rad}}{\text{s}}$) of the wheel when the wheel was rotating after the mass hung was released and landed on the ground, by measuring the period of one revolution (s). The mass of the falling mass (300. g), the mass of the wheel (1140. g), the inner radius of the wheel (29.11 cm), the outer radius of the wheel (30.87 cm), and the radius of where the string was wrapped around the wheel (30.87 cm) were controlled.

Introduction

Wheels appear very frequently throughout one's daily life, and there are many applications of wheels. One of such is the pulley, where strings are wrapped around a wheel to change the direction of a force. Wells can be a good example of this application, where a wheel (the pulley) rotates with strings attached to it, in order to lift a bucket (object hung).

When the wheel is released with the object hung above the ground, the wheel will experience angular acceleration, which would make the wheel rotate as the object hung falls to the ground. The higher the initial height of the object hung, the faster the wheel will be rotating after the object lands. Therefore, this experiment will be manipulating the initial height of an object hung on a wheel with a string to explore how this would affect the angular velocity of the wheel once the object landed on the ground. Through this experiment, it could be possible to help model some real-life scenarios involving a mass tied to a pulley.

In this investigation, the equipment was set up by removing the tire from the bicycle wheel and wrapping a string to the outer side of the rim. The other end of the string was tied to a mass hung (a filled plastic jar with a mass of 300. g). This could be demonstrated by the following image:

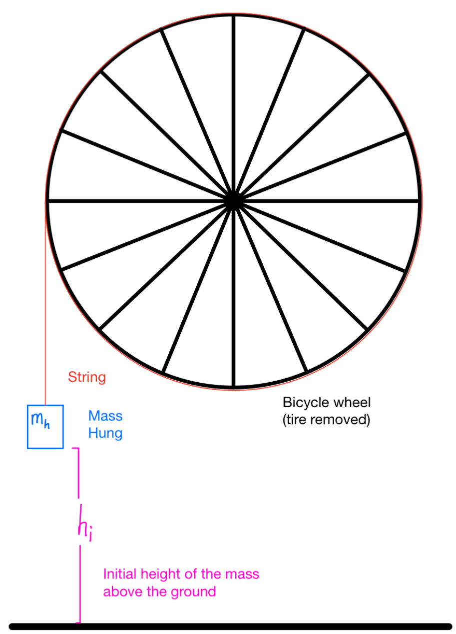


Figure 1: Diagram of the setup of the investigation: a mass hung tied to a string that was wrapped around a bicycle wheel

In this experiment, the mass hung was released from different heights, which means that the mass hung had different initial gravitational potential energy. When the mass was released, the falling of this mass released gravitational potential energy and it was transformed into kinetic energy of the mass hung and the angular kinetic energy of the wheel by exerting tension force on the string connecting the mass hung and the wheel. Once the mass hung landed on the ground, all the gravitational potential energy of the mass hung from its initial height was converted. The mass would no longer be exerting tension force on the string, and the wheel would continue rotating with an approximately constant angular velocity until the friction exerted on the wheel from the axle eventually stopped the rotation.

To calculate the final angular velocity of the wheel at the moment before the mass hung landed on the ground, the conservation of energy was used:

$$E_{total,initial} = E_{total,final}$$
$$E_{g,i} + E_{k,i} + E_{R,i} + W_{ext} = E_{g,f} + E_{k,f} + E_{R,f}$$

Where:
$E_{total,initial}$ = the total energy before the mass was released
$E_{total,final}$ = the total energy at the exact moment before the falling mass landed on the ground
$E_{g,i}$ = the initial gravitational potential energy of the mass hung
$E_{k,i}$ = the initial kinetic energy of the mass hung
$E_{R,i}$ = the initial rotational kinetic energy of the wheel
W_{ext} = the external work done on the system
$E_{g,f}$ = the final gravitational potential energy of the mass hung
$E_{k,f}$ = the final kinetic energy of the mass hung
$E_{R,f}$ = the final rotational kinetic energy of the wheel

Before the mass was released, there was no kinetic energy in the mass hung and no rotational kinetic energy since none of the objects in the system was moving. The major source of external work done on the system, the friction between the wheel and the axle, was considered insignificant due to the purpose of a bicycle wheel, which was to be able to rotate with low frictional force.

At the moment before the mass hung landed on the ground, there would be no more gravitational potential energy from the mass hung because at this moment, the final height of the mass hung would be 0 m above the ground.

Therefore, the energies in this experiment would be:

$$E_{g,i} = E_{k,f} + E_{R,f}$$

Substituting the equation for the initial gravitational potential energy of the mass hung ($E_{g,i} = m_h g h_i$), the final kinetic energy of the mass hung ($E_{k,f} = \frac{1}{2} m_h v_f^2$), and the final rotational kinetic energy of the wheel ($E_{R,f} = \frac{1}{2} I_w \omega_f^2$) into the equation (Moebs et al., 2016):

$$m_h g h_i = \frac{1}{2} m_h v_f^2 + \frac{1}{2} I_w \omega_f^2$$

Where:
m_h = mass of the mass hung
g = acceleration due to gravity
h_i = initial height above the ground of the mass hung
v_f = final velocity of the mass hung
I_w = rotational inertia of the wheel
ω_f = final angular velocity of the wheel

Since in this experiment, the string was wrapped around the wheel, the velocity of the mass hung can be related to the angular velocity of the wheel using the following formula:

$$v = \omega r$$

Where:
v = velocity of the mass hung
ω = angular velocity of the wheel
r = radius of where the string was wrapped around the wheel

Substituting the equation for the velocity of the mass hung into the equation of $m_h g h_i = \frac{1}{2} m_h v_f^2 + \frac{1}{2} I_w \omega_f^2$:

$$m_h g h_i = \frac{1}{2} m_h \left(\omega_f r\right)^2 + \frac{1}{2} I_w \omega_f^2$$

Through some manipulations of the equation, the final angular velocity can be expressed as:

$$\frac{2m_h g}{m_h r^2 + I_w} \times h_i = \omega_f^2$$

From the equation, the relationship between the final angular velocity and the initial height above the ground of the mass hung, while all other variables were held constant, can be shown as:

$$\omega_f^2 \propto h_i$$

Therefore, it was expected that if ω_f^2 was plotted against h_i, there would be a linear graph passing through the origin.

Since $\omega_f^2 \propto h_i$:

$$\omega_f \propto \sqrt{h_i}$$

Therefore, it was expected that if ω_f was plotted against h_i, there would be a square root graph passing through the origin.

For the experiment, the gradient of the line for ω_f^2 versus h_i would be:

$$Gradient = \frac{2m_h g}{m_h r^2 + I_w}$$

The equation used to calculate the rotational inertia of an object was as following (Moebs et al., 2016):

$$I = \sum_i m_i r_i^2$$

Where:
m_i = a point mass that makes up the object
r_i = the distance from the axis of rotation to the point mass m_i

For this experiment, to find the rotational inertia of the wheel, I_w, the wheel was assumed to be a hollow cylinder rotating about its central axis, and the equation for it was (Jones, 2019):

$$I_w = \frac{1}{2} m_w (r_1^2 + r_2^2)$$

Where:
m_w = the mass of the wheel
r_1 = the inner radius of the wheel (from the centre to the inner side of the rim)
r_2 = the outer radius of the wheel (from the centre to the outer side of the rim)

Substituting the equation for rotational inertia into the equation of the gradient:

$$Gradient = \frac{2m_h g}{m_h r^2 + \frac{1}{2} m_w (r_1^2 + r_2^2)}$$

Using the controlled variables of this experiment and constants, $m_h = 0.300$ kg, $g = 9.81 \frac{\mathrm{m}}{\mathrm{s}^2}$, $r = 0.3087$ m, $m_w = 1.140$ kg, $r_1 = 0.2911$ m, and $r_2 = 0.3087$ m were substituted:

$$Gradient = \frac{2(0.300\ \mathrm{kg})(9.81 \frac{\mathrm{m}}{\mathrm{s}^2})}{(0.300\ \mathrm{kg})(0.3087\ \mathrm{m})^2 + \frac{1}{2}(1.140\ \mathrm{kg})[(0.2911\ \mathrm{m})^2 + (0.3087\ \mathrm{m})^2]}$$
$$= 44.9\ \mathrm{m}^{-1}\mathrm{s}^{-2}$$

Therefore, the gradient of the graph for ω_f^2 versus h_i was expected to be $44.9\ \mathrm{m}^{-1}\mathrm{s}^{-2}$.

To measure the angular velocity of the wheel, the period of one revolution was recorded. The reason to not measure longer periods was that even though the friction was considered insignificant, over time it was still possible for it to decrease the angular velocity significantly, and that by measuring only one period, the effect of friction could be minimized. However, using only one period can cause a lot of uncertainty due to the short time interval and the reaction time of the person conducting the experiment. Therefore, this experiment was recorded using a high-speed camera and was analyzed by video analysis in the Logger Pro software to eliminate the potential errors that could be caused by human reaction time, and by using video analysis, higher precision of time measurement could be achieved.

To calculate the angular velocity from the period, the following equation was used:

$$\omega = \frac{2\pi}{T}$$

Where:
ω = angular velocity
T = period of one revolution

<u>Variables</u>

Independent Variable

The initial height of the mass hung above the ground (30.00 cm, 60.00 cm, 90.00 cm, 120.00 cm, 150.00 cm, 180.00 cm).

Dependent Variable

The angular velocity of the bicycle wheel after the mass hung landed on the ground ($\frac{\text{rad}}{\text{s}}$), as measured by the period of one revolution of the rotation of the wheel after the mass hung landed on the ground.

Controlled Variables

- The mass of the mass hung (300. g)
- The inner radius of the wheel (29.11 cm)
- The outer radius of the wheel (30.87 cm)
- The mass of the wheel without the tire (1140. g)
- The radius of where the string was wrapped around the wheel (30.87 cm)

<u>Materials</u>

Bicycle, 1 (the wheel had an inner radius of 29.11 cm, an outer radius of 30.87 cm, and a mass of 1140. g)
Table, 1 (height of approximately 1.6 m)
Plastic jar, 1
Rice (approximately 300. g)
Hook, 1
String, 1 (approximately 5 m)
Tape
Whiteboard
Whiteboard marker
Right-angle ruler, 1
Tape measure, 1 ($\pm$ 0.05 cm)
Electronic balance, 1 ($\pm$ 1 g)
Camera, 1 (240 frames per second)
Tripod, 1
Logger Pro software

Diagram

Figure 2: Setup of the materials before the hung plastic jar was adjusted to the correct height, and the brake was held.

Method

1. The hook was attached to the centre of the lid of the plastic jar.
2. The plastic jar was filled with rice until the total mass measured by the electronic balance ($\pm$ 1 g) was 300. g.
3. The rubber tire of the bicycle wheel was removed, the remaining parts of the wheel were weighted, the inner radius and the outer radius of the wheel (which was the radius of where the string was wrapped around the wheel) were also measured (mass = 1140. g, inner radius = 29.11 cm, outer radius = radius of where the string was wrapped = 30.87 cm).
4. The wheel, after being attached to the bicycle, was positioned on the top of a high table (approximately 1.6 m) with the bicycle inverted, part of the wheel was reaching over the edge of the table perpendicular to the floor, and nothing other than the axle was touching the wheel.
5. The string was tied to the wheel through the hole that was originally for the valve.
6. The other end of the string was tied to the plastic jar (300. g).
7. The string was wrapped around the outer edge of the wheel (radius of 30.87 cm).
8. The tape measure ($\pm$ 0.05 cm) was positioned vertically on the wall next to the plastic jar that was hung (300. g) with the 0 cm mark on the ground and was taped onto the wall.
9. The high-speed camera (240 fps) was positioned on the tripod at a location that can both record the wheel and the ground directly below the hung plastic jar (300. g).
10. The initial height of the trial and the number of the trial were labelled on the whiteboard, and the whiteboard was then positioned next to the bicycle where it could be recorded by the camera.
11. The wheel was rotated until the lowest point of the hung plastic jar (300. g) was 30.00 cm above the ground, measured by placing the right-angle ruler next to the tape measure ($\pm$ 0.05 cm) and was then held by the brake of the bicycle.
12. The recording of the high-speed camera (240 fps) was started.
13. The wheel was released.
14. The recording was stopped after the wheel rotated for more than one revolution after the hung plastic jar (300. g) landed on the ground.
15. Steps 10 to 14 were repeated 4 more times. The mass of the wheel, the radius of the wheel, the radius of where the string was wrapped around the wheel, and the mass of the hung plastic jar were not changed.
16. Steps 10 to 15 were repeated 5 more times, each time the initial height above the ground of the hung plastic jar (300. g) was increased by 30.00 cm, measured by the tape measure ($\pm$ 0.05 cm).
17. All the recorded videos were analyzed using the software Logger Pro, by going through the individual frames of every video to record the time in the video of the frames of when the hung

plastic jar (300. g) landed on the ground and the time when the wheel completed one revolution after the jar had landed.

Data Collection

Table 1: Data table for the recorded time in the recorded videos of when the mass hung (the hung plastic jar) landed on the ground and for when the bicycle wheel completed one revolution after that, as measured by video analysis of Logger Pro using the recorded videos, and the initial height of the lowest point of the mass hung above the ground before it was released, as measured by the tape measure ($\pm$ 0.05 cm). The total mass of the mass hung was controlled at 300. g, the inner radius of the wheel was controlled at 29.11 cm, the outer radius of the wheel was controlled at 30.87 cm, the mass of the wheel without the tire was controlled at 1140. g, and the radius of where the string was wrapped around the wheel was controlled at 30.87 cm.

The initial height above the ground of the mass hung, h_i / cm $\pm$ 0.05	The time in the video of when the mass hung landed on the ground in the slow-motion video recording, t_i / s $\pm$ 0.004					The time in the video of when the wheel completed one revolution after the mass hung landed on the ground in the slow-motion video recording, t_f / s $\pm$ 0.004				
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
30.00	1.951	1.234	2.605	1.843	0.554	3.711	2.956	4.394	3.686	2.480
60.00	9.372	1.792	6.169	5.557	4.202	10.545	2.956	7.336	6.736	5.398
90.00	2.701	3.902	4.019	3.156	1.363	3.643	4.832	4.949	4.085	2.302
120.00	2.151	2.331	2.451	3.090	3.568	2.956	3.136	3.256	3.891	4.364
150.00	2.668	7.948	5.407	1.606	8.907	3.381	8.665	6.128	2.324	9.619
180.00	4.081	3.031	2.106	3.943	2.648	4.736	3.694	2.769	4.598	3.315

The uncertainty of the times in the video was ± 0.004 s because the videos were recorded using a 240 frames per second camera, and by analyzing each video frame by frame, the uncertainty of all time taken would be the smallest increment of time, which would be 1/240 of a second, which would be 0.004 s when rounded to 1 significant digit.

Data Processing

Table 2: Data table for the period of one revolution of the wheel after the mass hung landed on the ground and the initial height above the ground of the mass hung:

The initial height above the ground of the mass hung, h_i / m $\pm$ 0.0005	The period of one revolution of the wheel after the mass hung landed on the ground, T / s $\pm$ 0.008				
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
0.3000	1.760	1.722	1.789	1.843	1.926
0.6000	1.173	1.164	1.167	1.179	1.196
0.9000	0.942	0.930	0.930	0.929	0.939
1.2000	0.805	0.805	0.805	0.801	0.796
1.5000	0.713	0.717	0.721	0.718	0.712
1.8000	0.655	0.663	0.663	0.655	0.667

Sample calculation of T for trial 1 of $h_i = 0.3000 \pm 0.0005$ m:

$$\begin{aligned} T &= t_f - t_i \\ &= (3.711 \pm 0.004 \text{ s}) - (1.951 \pm 0.004 \text{ s}) \\ &= 1.760 \pm 0.008 \text{ s} \end{aligned}$$

Table 3: Data table for the average period of one revolution of the wheel after the mass hung landed on the ground and the initial height above the ground of the mass hung:

The initial height above the ground of the mass hung, h_i / m $\pm$ 0.0005	The average period of one revolution of the wheel after the mass hung landed on the ground, T_{ave} / s
0.3000	1.8 ± 0.1
0.6000	1.18 ± 0.01
0.9000	0.934 ± 0.008
1.2000	0.802 ± 0.008
1.5000	0.716 ± 0.008
1.8000	0.661 ± 0.008

Sample calculation of T_{ave} for $h_i = 0.3000 \pm 0.0005$ m:

$$T_{ave} = \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5}$$

$$= \frac{(1.760 \pm 0.008\text{ s}) + (1.722 \pm 0.008\text{ s}) + (1.789 \pm 0.008\text{ s}) + (1.843 \pm 0.008\text{ s}) + (1.926 \pm 0.008\text{ s})}{5}$$

$$= 1.808 \pm 0.008\text{ s}$$

$$Random\ Error = \frac{(1.926\text{ s}) - (1.722\text{ s})}{2}$$

$$= \pm 0.1\text{ s}$$

Between the random error and the measurement error, the higher one was used for each manipulation.

Table 4: Data table for the angular velocity of the wheel after the mass hung landed on the ground and the initial height above the ground of the mass hung:

The initial height above the ground of the mass hung, h_i / m $\pm$ 0.0005	The angular velocity of the wheel after the mass hung landed on the ground, ω / $\frac{rad}{s}$
0.3000	3.5 ± 0.2
0.6000	5.32 ± 0.05
0.9000	6.73 ± 0.06
1.2000	7.83 ± 0.08
1.5000	8.8 ± 0.1
1.8000	9.5 ± 0.1

Sample calculation of ω for $h_i = 0.3000 \pm 0.0005$ m:

$$\omega = \frac{2\pi}{T}$$

$$= \frac{2\pi\text{ rad}}{1.8 \pm 0.1\text{ s}}$$

$$= \frac{2\pi\text{ rad}}{1.8\text{ s} \pm 5.6\%}$$

$$= 3.5\frac{\text{rad}}{\text{s}} \pm 5.6\%$$

$$= 3.5 \pm 0.2\ \frac{\text{rad}}{\text{s}}$$

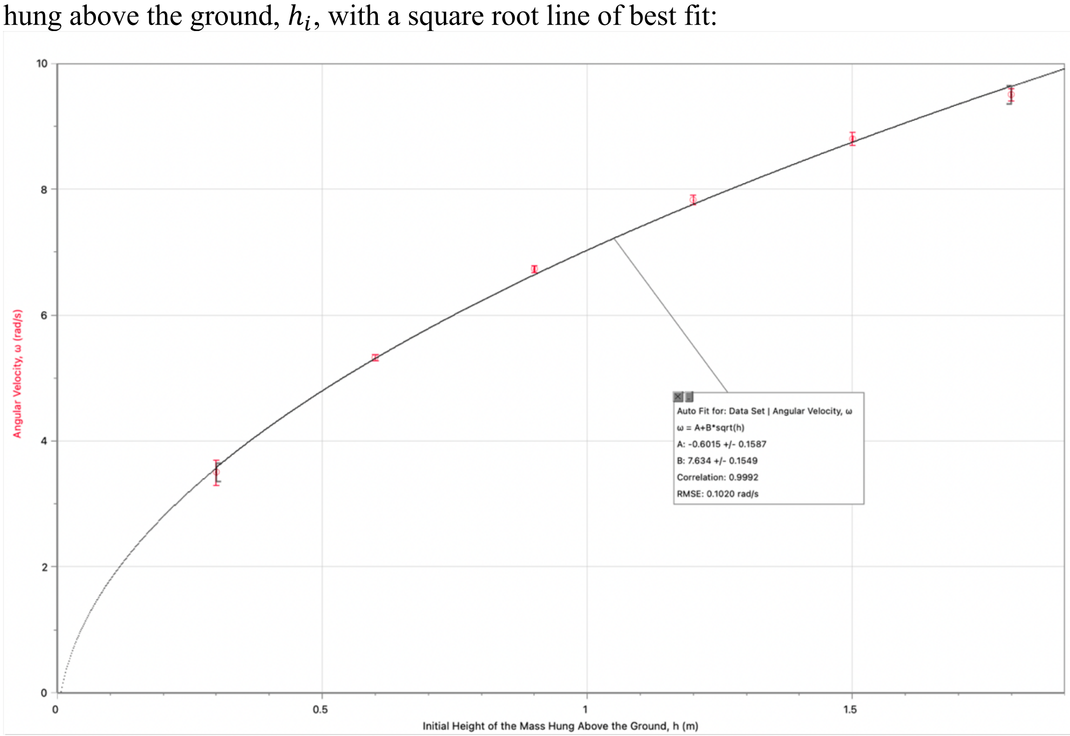
Table 5: Data table for the square of the angular velocity of the wheel after the mass hung landed on the ground and the initial height above the ground of the mass hung:

The initial height above the ground of the mass hung, h_i / m $\pm$ 0.0005	The square of the angular velocity of the wheel after the mass hung landed on the ground, ω^2 / $\frac{rad^2}{s^2}$
0.3000	12 ± 1
0.6000	28.3 ± 0.5
0.9000	45.3 ± 0.8
1.2000	61 ± 1
1.5000	77 ± 2
1.8000	90 ± 2

Sample calculation of ω^2 for $h_i = 0.3000 \pm 0.0005$ m:

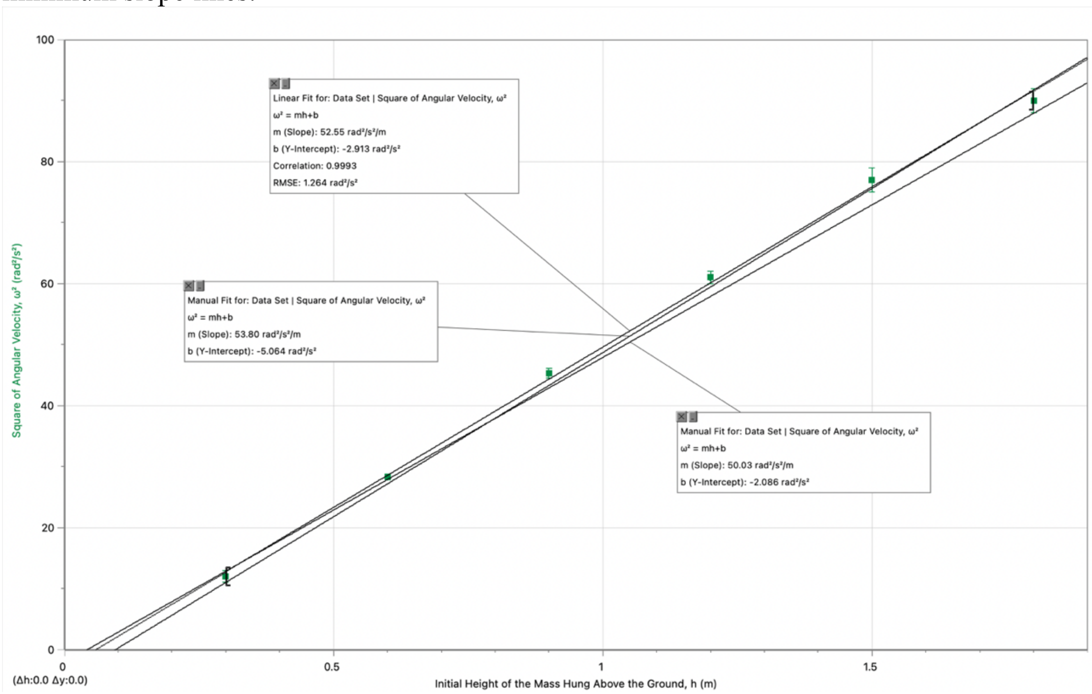
$$\omega^2 = \left(3.5 \pm 0.2\ \frac{\text{rad}}{\text{s}}\right)^2$$
$$= \left(3.5\ \frac{\text{rad}}{\text{s}} \pm 5.7\%\right)^2$$
$$= 12\ \frac{rad^2}{s^2} \pm 11.4\%$$
$$= 12 \pm 1 \frac{rad^2}{s^2}$$

Graph 1: The relationship between the angular velocity of the wheel, ω, and the initial height of the mass hung above the ground, h_i, with a square root line of best fit:



From this graph, it can be seen that a square root line of best fit can fit the graph of the angular velocity plotted against the initial height, with its y-intercept very close to the origin, showing a square root relationship between ω and h_i.

Graph 2: The relationship between the square of the angular velocity of the wheel, ω^2, and the initial height of the mass hung above the ground, h_i, with a linear line of best fit, along with maximum and minimum slope lines:



From the linear line of best fit for the square of the angular velocity plotted against the initial height with its y-intercept close to the origin, a direct proportionality can be seen between ω^2 and h_i. In this line of best fit, the gradient was $52.55\,\frac{\mathrm{rad}^2}{\mathrm{s}^2\mathrm{m}}$. Using the maximum and minimum slope lines, the uncertainty of the gradient was calculated:

$$\begin{aligned} Uncertainty_{Gradient} &= \frac{Gradient_{max} - Gradient_{min}}{2} \\ &= \frac{53.80\,\frac{\mathrm{rad}^2}{\mathrm{s}^2\mathrm{m}} - 50.03\,\frac{\mathrm{rad}^2}{\mathrm{s}^2\mathrm{m}}}{2} \\ &= \pm\, 2\,\frac{\mathrm{rad}^2}{\mathrm{s}^2\mathrm{m}} \end{aligned}$$

Therefore, the gradient of the line for ω^2 versus h_i was:

$$Gradient = 53 \pm 2\,\frac{\mathrm{rad}^2}{\mathrm{s}^2\mathrm{m}}$$

Using the same graph, the y-intercept of this graph and its uncertainty was found:

$$\begin{aligned} Uncertainty_{Y\text{-}Intercept} &= \frac{Y\text{-}Intercept_{max} - Y\text{-}Intercept_{min}}{2} \\ &= \frac{\left(-2.086\,\frac{\mathrm{rad}^2}{\mathrm{s}^2}\right) - \left(-5.064\,\frac{\mathrm{rad}^2}{\mathrm{s}^2}\right)}{2} \\ &= \pm\, 1\,\frac{\mathrm{rad}^2}{\mathrm{s}^2} \end{aligned}$$

Therefore, using the y-intercept of the best fit line:

$$Y\text{-}Intercept = -3 \pm 1\,\frac{\mathrm{rad}^2}{\mathrm{s}^2}$$

Using the gradient calculated from the equation $\omega_f^2 = \frac{2m_h g h_i}{m_h r^2 + I_w}$, the percentage error of the gradient from the data collected was found:

$$Percentage\ Error = \left|\frac{Theoretical\ Gradient - Measured\ Gradient}{Theoretical\ Graident}\right| \times 100\%$$

$$= \left|\frac{44.9\ \mathrm{m^{-1}s^{-2}} - 53\ \frac{\mathrm{rad^2}}{\mathrm{s^2m}}}{44.9\ \mathrm{m^{-1}s^{-2}}}\right| \times 100\%$$

$$= 18\%$$

Conclusion

From the processed data, it can be concluded that the relationship between the square of the final angular velocity and the initial height of the mass hung was directly proportional, and there was a square root relationship between the final angular velocity and the initial height. This was seen from the linear line of best fit between ω^2 and h_i, $\omega^2 = \left(53\frac{\mathrm{rad^2}}{\mathrm{s^2m}}\right)h_i - 3\frac{\mathrm{rad^2}}{\mathrm{s^2}}$, and the square root line of best fit between ω and h_i. The graph drawn were good fits for the data collected because both the linear line of best fit and the square root line of best fit passed through the error bars of most of the data, and the data points which the line did not pass through the error bars were relatively close to the line.

In the linear line of best fit for ω^2 versus h_i, the gradient was $53 \pm 2\frac{\mathrm{rad^2}}{\mathrm{s^2m}}$. This showed that for every metre increased in the initial height of the mass hung above the ground, the square of the final angular velocity of the wheel would be increased by $53 \pm 2\frac{\mathrm{rad^2}}{\mathrm{s^2}}$. However, the percentage error of 18% when compared to the theoretical value of the gradient suggested that there were certain factors significantly affecting the angular velocity of the trials and caused the gradient to be higher. This will be discussed in the evaluation section.

The y-intercept of $-3 \pm 1\frac{\mathrm{rad^2}}{\mathrm{s^2}}$ from the line of best fit for ω^2 versus h_i suggested that there was some systematic error that shifted all the data downwards since the expected y-intercept for this experiment was $0\ \frac{\mathrm{rad^2}}{\mathrm{s^2}}$ for the direct proportionality between ω^2 and h_i. The positive x-intercept showed that the angular velocity would be 0 even if the initial height above the ground of the mass hung was not 0 m. The cause of this error will be discussed in the evaluation section.

Evaluation and Improvement:

One systematic error in this experiment is the friction that was exerted on the wheel from its axle. The effect of this was shown in the data collected in two ways. The first was, as mentioned in the conclusion section, the negative y-intercept in the graph for ω^2 versus h_i, which showed that all the data collected were slightly shifted downwards. This was not possible in an ideal experiment as the square of angular velocity cannot be a negative value. Also, this produced a positive x-intercept in the graph, which would mean that the angular velocity would become zero even when the initial height of the mass hung was above the ground, while the wheel should have been able to rotate due to the presence of even a relatively small value of change in gravitational potential energy of the mass hung when it was falling. In addition, the negative y-intercept in the graph for ω^2 versus h_i could be related to the graph for ω versus h_i, in which a negative y-intercept would imply that the wheel would be rotating in the opposite direction with an initial height of 0 m. The second effect of this systematic error was the percentage error of 18% in the gradient of the line of best fit in the graph for ω^2 versus h_i. Due to the manipulations with lower initial heights having longer periods for one rotation, while the manipulations with higher initial heights having shorter periods, the square of angular velocities of the data points with lower initial heights would have been decreased more than the data points with higher initial heights. In the graph for ω^2 versus h_i, since the left side of the graph (data points with lower initial heights) was shifted downwards while the right side (data points with higher initial heights) was not shifted as much relative to the data points on the left, the gradient would be a larger value than expected. To reduce the errors caused by this, some lubricant could be applied to the axle to decrease the friction, or by using a different wheel with better bearing to also reduce the friction. Therefore, the angular velocities of the trials would not be affected as much.

Another systematic error of this experiment would be the measurements of the bicycle wheel. In the equation used to calculate the rotational inertia of the wheel for the calculation of the expected gradient of this experiment, the bicycle wheel was assumed to be a perfect hollow cylinder, and the equation $I_w = \frac{1}{2} m_w (r_1^2 + r_2^2)$ was used (Jones, 2019). However, the bicycle wheel was not a perfect hollow cylinder, as some parts of the mass of this wheel were in the axle and the spoke. Using the equation $I = \sum_i m_i r_i^2$ (Moebs et al., 2016), it could be seen that for the masses located near the centre of the wheel which has a smaller radius to the centre of rotation, the rotational inertia from these masses would have been smaller. In this experiment, the mass of the wheel was measured as a whole and not by individual parts of the wheel, meaning that the actual total rotational inertia for the wheel would have been a smaller value than the one that was calculated since the mass. Since the rotational inertia of the wheel used in this experiment was smaller than the theoretical value, the gradient of the data collected in this experiment was higher because the rotational inertia was in the denominator of the equation $Gradient = \frac{2m_h g}{m_h r^2 + I_w}$, and potentially caused the 18% percentage error of this gradient. To reduce this error, a different wheel with the shape of a thin, solid cylinder with uniform density could be used to reduce the complexity of the shape of the rotating object, so that the rotational inertia of the rotating object could be calculated accurately, and the gradient of the data collected would be closer to the theoretical value.

One random error in this experiment would be how the data was collected. Using video analysis for videos with high frames per second, the uncertainties from the measurement devices were insignificant, but the method used to measure one period could have some random errors. When determining the period of one revolution of the wheel, a point on the wheel in the recorded video displayed by the Logger Pro software was marked, and the time in the video of when the point was marked was recorded. Then, when the same point on the wheel returned to the point marked after one revolution, the second time was recorded. The human conducting the experiment might have uncertainties to determine whether the same point of the wheel returned to the marked spot in the video after one revolution. For example, the human might record the time one or two frames before or after the point on the wheel returned to the same marked spot, which would make the period shorter or longer. This error could be seen in the data points for $h_i = 0.9000 \pm 0.0005$ m and $h_i = 1.2000 \pm 0.0005$ m in the graph of ω^2 versus h_i, which the line of best fit did not pass through their error bars, this was potentially due to the measured period for these points being slightly shorter than the actual period because of the reasons mentioned above, which would lead to a higher angular velocity, and cause the points to be above the line of best fit. However, this was not a significant error because the data points were very close to the lines. To reduce this error, the camera could be placed at a location closer to the wheel, even if only a portion of the wheel would be present in the video (as long as there are marks on the wheel to determine if the wheel completed one revolution), so that when marking the point on the wheel during video analysis, it would be easier for the person since, with a closer camera, every point on the wheel would be larger in the recorded video, and it would be less likely for the person to make a mistake determining if one revolution was completed.

References

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