Investigating the Relationship Between Horizontal Tube Length and Flow Rate

Research Question: How does the length of a horizontal tube affect the flow rate of water when connected to the bottom a cylindrical water tank with water level higher than the tube?

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Research Question
How does the length of a horizontal tube affect the flow rate of water when connected to the bottom a cylindrical water tank with water level higher than the tube?

Introduction

In physics, fluid dynamics can be a complex area of study with many applications in many different fields such as engineering.

In one's daily life, there are many different methods to transport fluids such as forcing fluids through a tube with a pressure difference. Some common examples are garden hoses, fuel pumps and straws, which all seem to be able to transport fluid through a long distance with high efficiency.

One commonly used way to calculate fluid flow rate from a constant pressure difference is the use of Bernoulli's equation, which produces accurate results in many scenarios. However, in some scenarios, the flow rate calculated from Bernoulli's equation could be different from the actual flow rate. The reason behind this is that Bernoulli's equation assumes the fluid to be frictionless, and therefore the resistance from the tube on the fluid is ignored (Moebs et al., 2016). Generally, the resistance would not be significant, since commonly used tubes have relatively large radius and short tube length. However, in thin and long tubes, the resistance can be significant and must be taken into consideration while predicting fluid flow.

Poiseuille's law is an equation that allows the calculation of fluid flow that considers the resistance between the tube and the fluid. This resistance is affected by the radius of the tube, the viscosity of the fluid, and the length of the tube. Since this law only applies to laminar flow, where layers of fluid flow without mixing, Reynolds' number, an indicator of laminar flow must be small. Larger values of Reynolds' number are indicative of turbulent flow (Moebs et al., 2016). For this essay, a maximum Reynolds' number of 2300 will be used for laminar flow (Moxey et al., 2010).

If the tube is short, the resistance from the tube becomes insignificant and the flow rate predicted by Poiseuille's law becomes unreasonably high, and thus Bernoulli's equation will be used to calculate the maximum flow rate.

Theoretical Equations

To determine the flow rate of laminar flows which considers the resistance caused by the tube in which the fluid flows through, Poiseuille's law will be used. Thus, the flow rate Q is given by (Moebs et al., 2016):

$$Q = \frac{(p_2 - p_1)\pi r^4}{8\eta l}$$

Where $p_2 - p_1 \equiv$ pressure difference between the two ends of the tube, $r \equiv$ tube radius, $\eta \equiv$ dynamic viscosity of the fluid, $l \equiv$ tube length.

From the equation above, the flow rate increases as the length of tube decreases. However, the flow rate approaches infinity as the length of the tube approaches zero, which is nonphysical. Therefore, there must be a maximum flow rate where there is no resistance caused by the tube. This is calculated using Bernoulli's equation, which assumes the fluid is incompressible and frictionless. This law states (Moebs et al., 2016):

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Where $p \equiv$ pressure, $\rho \equiv$ fluid density, $v \equiv$ fluid velocity, $g \equiv$ acceleration due to gravity, $h \equiv$ height of fluid.

In addition, since Poiseuille's law assumes laminar flow, Reynolds' number can be calculated to determine if the flow is laminar. This occurs when Reynolds' number is below about 2300 (Moxey et al., 2010). The equation for fluid flows in a tube is given by (Moebs et al., 2016):

$$N_R = \frac{2\rho v r}{\eta}$$

Where $N_R \equiv$ Reynolds' number, $\rho \equiv$ fluid density, $v \equiv$ fluid velocity, $r \equiv$ tube radius, $\eta \equiv$ dynamic viscosity of fluid.

Experimental Design

This investigation aims to measure the flow rate of water out of a water tank, which is done by connecting a long, thin cylindrical tube horizontally to the side of a wide cylindrical water tank near the bottom. The tank and the tube are then filled with water up to an initial height above the connection point of the tube, which provides a pressure difference between the two ends of the tube and allow the flow of water. The end of the tube that is not connected to the water tank is placed at the opening of a graduated cylinder. This setup is shown in the diagram below.

Figure 1: The experimental setup used in data collection, with a large water tank on the right and wooden blocks to ensure horizontal flow of water

The independent variable of this investigation is the length of the tube, which is changed by removing the extra length of the tube from the end that is not connected to the tank after all the trials for one particular length has been completed. A ruler placed parallel to the tube before the tube is connected to the tank is used to mark where the tube should be cut.

The dependent variable of this experiment is the flow rate of water through the tube, which is measured by recording the time taken for the volume of the water in the graduated cylinder to reach a set volume. The time measurement is accomplished by using a stopwatch with 0.01s uncertainty. The volume is measured using a graduated cylinder with an uncertainty of $\pm\ 0.5\text{mL}$. A general fixing of the flow rate can be observed for longer tube lengths.

Controlling the Variables

The purpose of the experiment was to determine the relationship between the independent variable and the dependent variable, meaning that any additional factor that can affect the experiment must be measured and controlled.

The first variable to be controlled would be the radius of the cylindrical tube. From Poiseuille's law, the flow rate of the fluid has a direct correlation with the radius of the tube raised to the fourth power, meaning that even a slight change in this radius can affect the flow rate when there is resistance caused by the tube (Moebs et al., 2016). In addition, the radius of the tube could also affect the flow rate from Bernoulli's equation. Since the equation determines the velocity of the fluid, the flow rate can be determined by multiplying the fluid velocity with the cross-section area, which has a direct correlation with the square of the radius of the tube. The tube used in this experiment has a radius of 1.19 ± 0.01mm (converted from an inner diameter of 3/32 inch as stated by the manufacturer and an uncertainty of $0.01\mathrm{mm}$ is assumed), which is a relatively small radius compared to tubes used for household purposes. The reason to use such a small radius is to make the effect of flow resistance easier to detect, since a smaller radius would result in more flow resistance, and a small radius makes the Reynolds' number smaller, meaning that the flow can be laminar even with higher velocity. To control the tube radius, the same tube is used throughout the experiment, and the only the length of the tube is altered as the independent variable.

The second variable to control is the initial height of the water. This variable should be controlled because the initial height of the water above the connection between the tube and the tank provides the pressure difference between the two ends of the tube. If this is not controlled, the pressure difference between trials would differ and would ultimately affect the flow rate. To control the initial height, a ruler is used to measure the water level before each trial, with the placement of one end of the ruler so that it corresponds to the centre of the tube.

The radius of the cylindrical tank was also controlled because as water flows from the tank, the height of the water level can decrease. Therefore, it is necessary to use the same radius so that the decrease of the water level is the same between trials. If different radii were used between trials, the difference of water level can lead to a change in flow rate. This variable was controlled by using the same cylindrical tank with radius 13.25 ± 0.05cm. Note that tanks with larger radii are not sensitive to changes in water level when water is siphoned off.

The volume to measure the time taken for the water flow to fill up is another variable to be controlled. The reason to control this variable is similar to the reason to control the radius of the tank, so that the effect of the change in water level due to water flowing out of the tank remains constant. The volume selected for this experiment was 25.0mL due to limitations of the volume measuring device. A smaller volume would result in less change in the water level, which would mean a more accurate result compared to the theory, however, the smallest marking on the graduated cylinder is at 10.0mL, and therefore, the volume selected is selected to be larger than 10.0mL and set to be 25.0mL.

The other variables needed to be controlled are the properties of the fluid used in this experiment, specifically the density and the dynamic viscosity of the fluid. Under the environment in which this experiment is conducted, the temperature is estimated to be 20℃.

At this temperature, the dynamic viscosity of liquid water is 1.002mPa·s, and the density of liquid water is $998.2\,\frac{\mathrm{kg}}{\mathrm{m}^3}$ (Moebs et al., 2016). This is controlled by using the same source of water at the same temperature for all the trials.

Definitions of Symbols

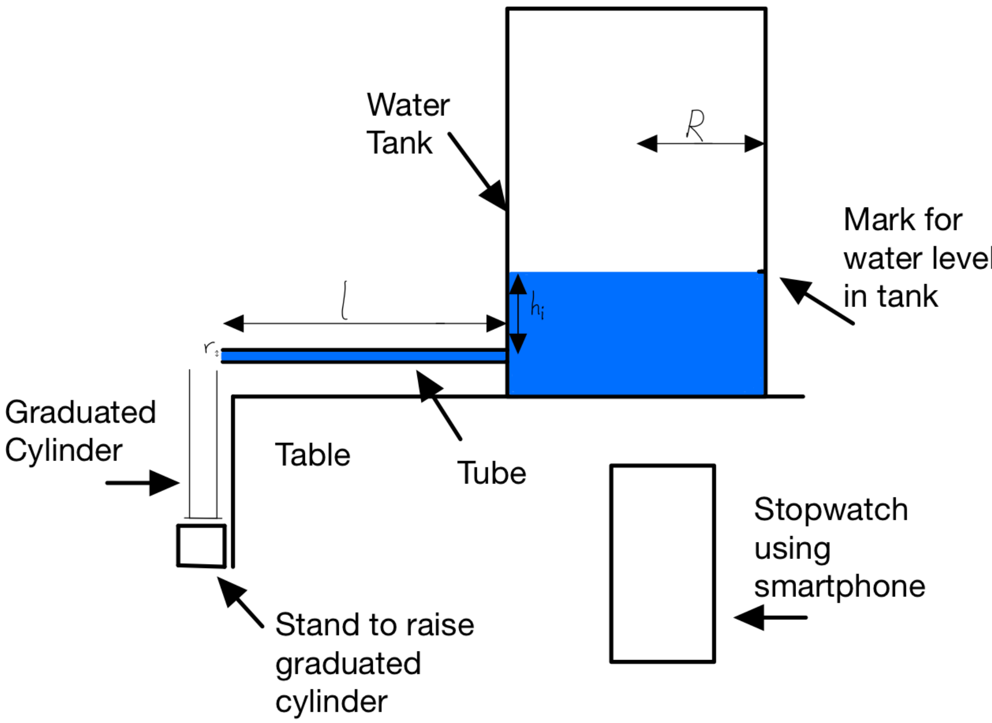


Figure 2: Drawn diagram of the experimental setup with labels of variables and devices used

r ≡ radius of the tube

l ≡ length of the tube

h_i ≡ initial height of water in the tank above the connection to the tube

R ≡ radius of the water tank

η ≡ dynamic viscosity of water

ρ ≡ density of water

Additional Calculations and Predictions

For this experiment design, the flow rate of water is calculated by Poiseuille's law, while a maximum flow rate is calculated using Bernoulli's equation which assumes no resistance on the fluid. In addition, to ensure that the water flow is laminar flow, which is required to apply Poiseuille's law, the Reynolds' number required must correspond to that of laminar flow.

To begin, the maximum flow rate for water to maintain in laminar flow is set to a maximum Reynolds' number of 2300 (Moebs et al., 2016; Moxey et al., 2010).

$$N_R = \frac{2\rho v r}{\eta}$$

Substituting the values in this experiment: $\rho = 998.2\,\frac{\text{kg}}{\text{m}^3}$, $\eta = 0.001002$Pa·s, $r =$ 0.00119m, and $N_R = 2300$ gives:

$$v = \frac{N_R \eta}{2\rho v r}$$

$$= 0.970\,\frac{\text{m}}{\text{s}}$$

Thus, maximum fluid velocity must be less than or equal to $0.970\,\frac{\text{m}}{\text{s}}$ to maintain laminar flow within the tube.

To calculate the maximum fluid velocity, Bernoulli's equation is used (Moebs et al., 2016):

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Where $p_1 = p_2$, $v_1 = 0$, and $h_2 = 0$. Cancelling out the variables gives:

$$gh_1 = \frac{1}{2}v_2^2 \text{ and thus } h_1 = \frac{1}{2}\frac{v_2^2}{g}$$

Substituting the maximum velocity to maintain laminar flow $0.970\,\frac{\text{m}}{\text{s}}$ and $g = 9.81\,\frac{\text{m}}{\text{s}^2}$ gives:

$$h_1 = 0.0480 \text{ m}$$

From the calculation, the initial height of water in the tank above the connection to the tube should be lower than 0.0480m to maintain laminar flow in the tube. Therefore, for this experiment, an initial height of $h_i = 0.0450$m was selected.

To calculate Q_{max}, the maximum flow rate when using initial height of 0.0450m, the fluid velocity is multiplied by the cross-sectional area of the tube, $A = \pi r^2$, thus:

$$\begin{aligned} Q_{max} &= v\pi r^2 \\ &= \sqrt{2gh_i}\pi r^2 \\ &= \left(\sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.0450\text{ m})}\right)\pi(0.00119\text{ m})^2 \\ &= 4.18\,\frac{\text{mL}}{\text{s}} \end{aligned}$$

Using Poiseuille's law, the flow rate with fluid resistance can be calculated by substituting a pressure difference of $p_2 - p_1 = \rho g h_i$, $\rho = 998.2\,\frac{\text{kg}}{\text{m}^3}$, $g = 9.81\,\frac{\text{m}}{\text{s}^2}$, $h_i = 0.0450 \pm 0.0005$m, $r = 0.00119 \pm 0.00001$m, and $\eta = 0.001002$Pa·s (Moebs et al., 2016):

$$\begin{aligned} Q &= \frac{\rho g h_i \pi r^4}{8\eta l} \\ &= \frac{\left(998.2\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.0450\text{ m} \pm 1.11\%)\pi(0.00119\text{ m} \pm 0.840\%)^4}{8(0.001002\text{ Pa·s})l} \\ &= \left(0.346\,\frac{\text{mL·m}}{\text{s}} \pm 4.47\%\right) \times \frac{1}{l} \\ &= 0.35 \pm 0.02\,\frac{\text{mL·m}}{\text{s}} \times \frac{1}{l} \end{aligned}$$

To calculate the tube length where the flow rate is maximum, we set:

$$Q = Q_{max}$$

$$\frac{\rho g h_i \pi r^4}{8\eta l} = \sqrt{2gh_i}\pi r^2$$

$$\Rightarrow l = \frac{\rho r^2}{8\eta}\sqrt{\frac{gh_i}{2}}$$

Substituting values into the equation gives:

$$l = 0.0828 \text{ m}$$

This shows that in theory, when the tube is shorter than 0.0828m, the flow rate calculated by Poiseuille's law would be greater than the flow rate calculated using Bernoulli's equation, where no resistance is considered. Therefore, the flow rate should follow Poiseuille's law for tube lengths longer than 0.0828m and approach the maximum flow rate of $4.18\,\frac{\text{mL}}{\text{s}}$ when the tube is shorter than that.

From Poiseuille's equation, the relationship between the tube length and the flow rate is shown as:

$$Q \propto \frac{1}{l}$$

Therefore, according to the theoretical calculations, the flow rate will have a positive, linear correlation with the inverse of the tube length, and if plotted against each other, the gradient would be $0.35 \pm 0.02\,\frac{\text{mL}\cdot\text{m}}{\text{s}}$.

To consider the change in pressure difference caused by water flowing away from the tank, another calculation is done using Poiseuille's law, substituting Q with $\frac{dV}{dt}$ and h with $h_i - \frac{V}{\pi R^2}$, where $R =$ radius of tank:

$$\frac{dV}{dt} = \frac{\rho g(h_i - \frac{V}{\pi R^2})\pi r^4}{8\eta l}$$

Substituting $V = 0$ at $t = 0$, this equation becomes:

$$V = \pi R^2 h_i \left(1 - e^{-\frac{\rho g r^4 t}{8\eta l R^2}}\right)$$

The negative exponent of $\frac{\rho g r^4 t}{8\eta l R^2}$ is calculated using the same values used before and $R = 0.1325 \pm 0.0005$m. Values of l and t are not used as they would change between manipulations of the independent variable:

$$\frac{\rho g r^4 t}{8\eta l R^2} = \frac{\left(998.2\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.00119 \pm 0.00001\ \text{m})^4}{8(0.001002\ \text{Pa·s})(0.1325 \pm 0.0005\ \text{m})^2} \times \frac{t}{l}$$

$$= \frac{\left(998.2\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.00119\ \text{m} \pm 0.840\%)^4}{8(0.001002\ \text{Pa·s})(0.1325\ \text{m} \pm 0.377\%)^2} \times \frac{t}{l}$$

$$= \left(1.40 \times 10^{-4}\ \frac{\text{m}}{\text{s}} \pm 4.11\%\right) \times \frac{t}{l}$$

$$= \left((1.40 \pm 0.06) \times 10^{-4}\ \frac{\text{m}}{\text{s}}\right) \times \frac{t}{l}$$

Since the order of magnitude of one component of the exponent is 10^{-4} and time would be short while the length would not be shorter than 0.0828m for laminar flow, the argument of the exponential is significantly smaller than 1, and the full equation can be approximated using $1 - e^{-x} \approx x$ for $x \ll 1$:

$$V = \pi R^2 h_i \left(\frac{\rho g r^4 t}{8\eta l R^2}\right)$$

$$\Rightarrow \frac{V}{t} = \frac{\rho g h_i \pi r^4}{8\eta l}$$

This approximated equation is the same as Poiseuille's law, showing that for this experiment, the effect on water pressure due to water flowing away from the tank is insignificant. Therefore, the initial formulation of Poiseuille's law should hold true.

Thesis

The flow rate of water will be inversely proportional to the length of the horizontal tube for long tubes. When the tube is short, the flow rate deviates from the inverse proportional relationship and approaches a maximum flow rate.

Experimental Procedure

1. The equipment and devices were set up according to Figure 2 with an initial tube length of 0.5000 ± 0.0005m.
2. The tube was marked from 1.00 ± 0.05cm to 50.00 ± 0.05cm with 1.00cm increments using a marker.
3. Water was filled into the tank until the water level was 4.50 ± 0.05cm above the connection between the tube and the tank. During this process, the end of the tube not connected to the tank was covered once water filled the tube.
4. The opening of the tube was placed at the opening of the graduated cylinder.
5. The covering on the tube was released, and when the water flow first reached the bottom of the graduated cylinder, the stopwatch with 0.01s uncertainty was started.
6. The stopwatch was stopped when the water level in the graduated cylinder reached 25.0 ± 0.5mL, observed from a horizontal perspective. The values of the tube length, trial number, and the number displayed on the stopwatch were recorded.
7. The water in the graduated cylinder was poured back into the tank.

8. Steps 3 to 7 was repeated four more times.

9. The tube was cut to the marked length 1.00cm shorter than the current length.

10. Steps 3 to 9 was repeated until the tube could not be further cut.

Data Collection

Table 1: Raw data for the time taken for 25.0 ± 0.5 mL of water to flow into the graduated cylinder recorded using stopwatch with 0.01 s uncertainty, with tube lengths with 1.00 cm increments from 1.00 ± 0.05 cm to 50.00 ± 0.05 cm.

Tube length, l / ± 0.05 cm	Time taken for 25.0 ± 0.5 mL of water to flow into the graduated cylinder, t / ± 0.01 s				
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
1.00	9.17	9.20	9.06	9.06	9.20
2.00	9.69	9.35	9.39	9.22	9.39
3.00	9.29	9.50	9.50	9.36	9.46
4.00	10.12	9.91	10.30	10.18	10.13
5.00	10.50	10.40	10.77	10.52	11.03
6.00	11.35	11.46	10.58	11.52	11.49
7.00	11.63	11.95	12.00	11.85	11.35
8.00	12.88	12.19	12.50	12.81	12.43
9.00	13.46	13.67	13.80	13.51	13.78
10.00	14.20	13.62	13.56	13.40	14.08
11.00	14.03	13.95	13.92	14.13	13.92
12.00	14.66	14.76	14.39	14.29	15.05
13.00	15.00	15.12	14.55	14.67	14.75
14.00	15.10	14.90	15.39	15.25	15.23
15.00	15.53	15.23	15.44	15.23	15.70
16.00	15.85	16.43	15.82	15.85	16.39
17.00	16.65	16.72	16.73	16.63	16.96
18.00	17.89	17.60	17.35	17.23	17.25
19.00	17.59	17.53	17.73	17.63	17.32
20.00	17.96	17.66	17.85	18.53	18.16
21.00	18.69	18.66	18.97	18.52	19.32
22.00	19.08	19.55	19.65	19.80	19.71
23.00	20.30	20.07	20.40	20.20	20.73
24.00	21.86	21.82	21.67	21.02	20.76
25.00	21.72	20.85	20.89	21.86	22.26
26.00	22.02	21.55	21.43	22.89	22.35
27.00	23.53	25.15	23.65	23.02	23.10
28.00	29.02	24.06	24.28	24.11	24.84

Table 1 (continued).

29.00	25.53	24.07	25.23	25.22	24.86
30.00	25.17	26.55	25.06	25.73	25.59
31.00	25.79	26.87	25.40	27.44	26.16
32.00	28.02	26.73	28.00	27.48	26.45
33.00	27.48	27.00	27.28	26.43	28.69
34.00	29.57	28.41	27.78	26.78	28.21
35.00	28.95	29.49	29.18	29.50	28.46
36.00	29.93	29.53	28.72	29.37	29.58
37.00	29.94	28.95	30.58	30.23	30.38
38.00	31.82	30.90	31.87	30.81	31.35
39.00	34.43	32.95	31.56	33.66	32.44
40.00	35.61	34.98	34.42	34.69	35.36
41.00	35.78	33.86	37.86	34.22	35.23
42.00	36.17	35.63	35.43	36.35	36.76
43.00	38.53	38.65	38.82	38.21	38.44
44.00	37.43	38.89	39.83	40.06	38.84
45.00	42.67	41.48	41.36	40.49	40.37
46.00	45.36	45.51	45.68	46.49	44.71
47.00	45.85	44.86	45.90	46.01	46.48
48.00	48.38	49.68	46.35	46.50	48.19
49.00	49.77	49.72	49.71	50.17	50.60
50.00	47.01	47.62	50.15	49.62	50.03

Qualitative observations:

Bubbles were noticed in the tube for trials with tube length between 46.00cm and 50.00cm. The bubbles were removed by clearing all water from the tube and re-filling it with water slowly. The water level in the graduated cylinder did not increase steadily if water drops were formed instead of water flowing down the side of graduated cylinder.

Data Processing

We show sample calculations for $l = 50.00 \pm 0.05$cm:

The average time recorded of the five trials, t_{ave} is:

$$t_{ave} = \frac{t_1 + t_2 + t_3 + t_4 + t_5}{5}$$

$$= \frac{(47.01\text{ s}) + (47.62\text{ s}) + (50.15\text{ s}) + (49.62\text{ s}) + (50.03\text{ s})}{5}$$

$$= 48.89\text{ s}$$

The uncertainty of the average time recorded of the five trials, Δt:

$$\Delta t = \pm \frac{t_{max} - t_{min}}{2}$$

$$= \pm \frac{(50.15\text{ s}) - (47.01\text{ s})}{2}$$

$$= \pm 2\text{ s}$$

Therefore, the value for t_{ave} for the manipulation with tube length $l = 50.00 \pm 0.05$cm is 49 ± 2 s.

The average flow rate of water from the five trials, Q_{ave}, using the volume of water flow $V = 25.0 \pm 0.5$mL:

$$Q_{ave,best} = \frac{V}{t_{ave}}$$

$$= \frac{25.0\text{ mL}}{49\text{ s}}$$

$$= 0.51 \frac{\text{mL}}{\text{s}}$$

$$\Delta Q_{ave} = \frac{\Delta V}{t_{ave}} + \frac{V \Delta t_{ave}}{(t_{ave})^2}$$

$$= \frac{(0.5\text{ mL})}{(49\text{ s})} + \frac{(25.0\text{ mL})(2\text{ s})}{(49\text{ s})^2}$$

$$= 0.03\ \frac{\text{mL}}{\text{s}}$$

$$Q_{ave} = 0.51 \pm 0.03\ \frac{\text{mL}}{\text{s}}$$

To plot flow rate against the inverse of tube length, $\frac{1}{l}$ is calculated and will be plotted on the horizontal axis:

$$\left(\frac{1}{l}\right)_{best} = \frac{1}{0.5000\text{ m}}$$

$$= 2.000\text{ m}^{-1}$$

$$\Delta\left(\frac{1}{l}\right) = \frac{\Delta l}{l^2}$$

$$= \frac{(0.0005\text{ m})}{(0.5000\text{ m})^2}$$

$$= 0.002\text{ m}$$

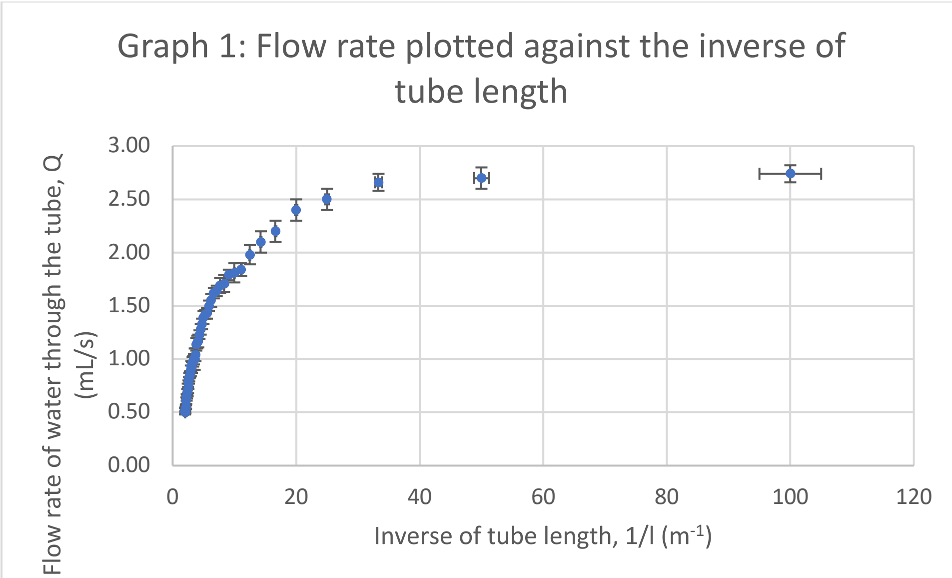
$$\frac{1}{l} = 2.000 \pm 0.002\text{ m}^{-1}$$

Table 2: Data table for the processed data of tube length and the flow rate of water.

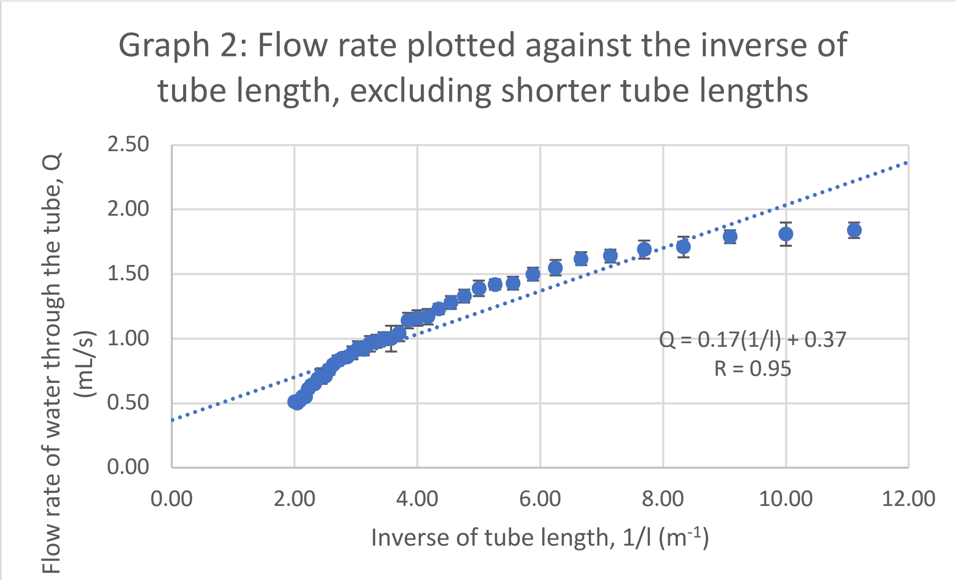
Inverse of tube length, $\frac{1}{l}$ / m^{-1}	Flow rate of water through the tube, Q / $\frac{\text{mL}}{\text{s}}$
$(1.00 \pm 0.05) \times 10^2$	2.74 ± 0.08
$(5.0 \pm 0.1) \times 10$	2.7 ± 0.1
33.3 ± 0.6	2.66 ± 0.08
25.0 ± 0.3	2.5 ± 0.1
20.0 ± 0.2	2.4 ± 0.1
16.7 ± 0.1	2.2 ± 0.1
14.3 ± 0.1	2.1 ± 0.1
12.50 ± 0.08	1.98 ± 0.09
11.11 ± 0.06	1.84 ± 0.06

Table 2 (continued).

10.00 ± 0.05	1.81 ± 0.09
9.09 ± 0.04	1.79 ± 0.05
8.33 ± 0.03	1.71 ± 0.08
7.69 ± 0.03	1.69 ± 0.07
7.14 ± 0.03	1.64 ± 0.05
6.67 ± 0.02	1.62 ± 0.05
6.25 ± 0.02	1.55 ± 0.06
5.88 ± 0.02	1.50 ± 0.05
5.56 ± 0.02	1.43 ± 0.05
5.26 ± 0.01	1.42 ± 0.04
5.00 ± 0.01	1.39 ± 0.06
4.76 ± 0.01	1.33 ± 0.05
4.55 ± 0.01	1.28 ± 0.05
4.348 ± 0.009	1.23 ± 0.04
4.167 ± 0.009	1.17 ± 0.06
4.000 ± 0.008	1.16 ± 0.06
3.846 ± 0.007	1.14 ± 0.06
3.704 ± 0.007	1.04 ± 0.06
3.571 ± 0.006	1.0 ± 0.1
3.448 ± 0.006	1.00 ± 0.05
3.333 ± 0.006	0.98 ± 0.05
3.226 ± 0.005	0.96 ± 0.06
3.125 ± 0.005	0.92 ± 0.05
3.030 ± 0.005	0.93 ± 0.05
2.941 ± 0.004	0.89 ± 0.05
2.857 ± 0.004	0.86 ± 0.03
2.778 ± 0.004	0.85 ± 0.03
2.703 ± 0.004	0.83 ± 0.04
2.632 ± 0.003	0.80 ± 0.03
2.564 ± 0.003	0.76 ± 0.04
2.500 ± 0.003	0.71 ± 0.03
2.439 ± 0.003	0.71 ± 0.06
2.381 ± 0.003	0.69 ± 0.03
2.326 ± 0.003	0.65 ± 0.02
2.273 ± 0.003	0.64 ± 0.03
2.222 ± 0.002	0.61 ± 0.03
2.174 ± 0.002	0.55 ± 0.02
2.128 ± 0.002	0.55 ± 0.02
2.083 ± 0.002	0.52 ± 0.03
2.041 ± 0.002	0.50 ± 0.01
2.000 ± 0.002	0.51 ± 0.03



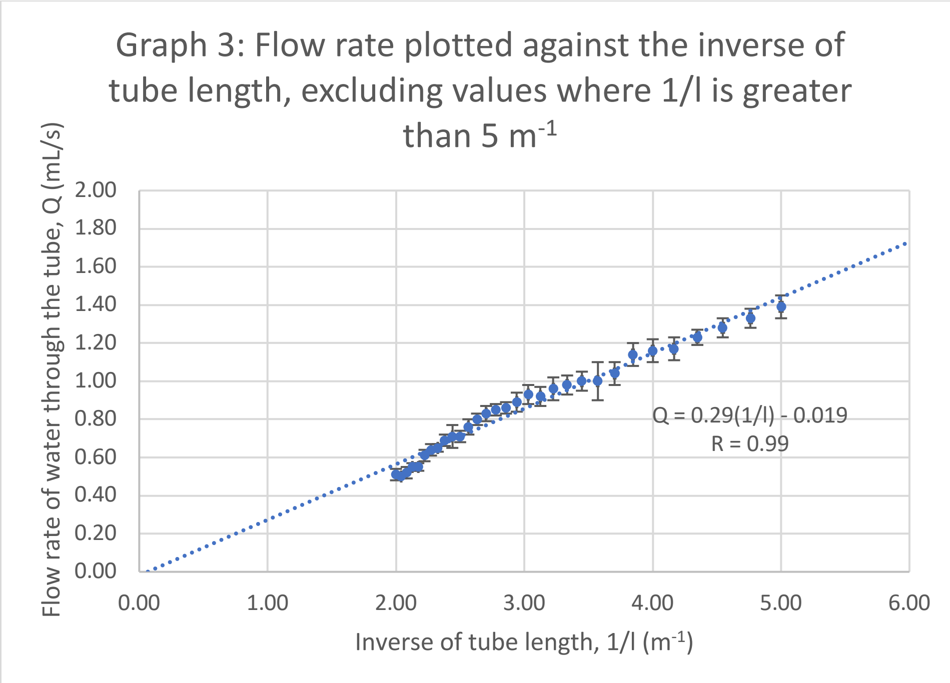
After plotting the flow rate of water through the tube against the inverse of the tube, there appears to be a positive linear correlation where the inverse of tube length is small, and as the inverse of tube length increases, the change of flow rate eventually slows down and results in a curve. The reason for change in the change of flow rate was explained by the fact that when the tube length decreases, Poiseuille's law produces an unreasonable flow rate, and therefore there will be a maximum flow rate, meaning that flow rate is no longer governed by Poiseuille's law when the tube is too short. The data points for tube length shorter than the tube length for Poiseuille's law and Bernoulli's equation to produce the same flow rate, 0.0828m, was removed.



With points removed, the relationship now appears more linear with the line of best fit of:

$$Q = \left(0.17\frac{\text{mL m}}{\text{s}}\right)\left(\frac{1}{l}\right) + 0.37\frac{\text{mL}}{\text{s}}$$

However, despite the high r coefficient of 0.95, a non-linear trend can still be seen where the rate of change of flow rate slows down. This shows that the transition between the flow following Poiseuille's law and following Bernoulli's equation is not instantaneous, and there is a transition phase. Therefore, to further eliminate the effect of the slowing down in the change of flow rate as the flow approaches maximum flow rate for shorter tube lengths, only data points with $\frac{1}{l}$ lower than 5m^{-1} are used. The reason for this chosen value is to remove data from short points that are affected by the transition state, while not removing too much data points so that the more than half the original data points remains:



Observing this graph, a strong linear trend can be seen between flow rate and inverse of tube length, with a r-coefficient of 0.99 and linear line of best fit of:

$$Q = \left(0.29\,\frac{\text{mL m}}{\text{s}}\right)\left(\frac{1}{l}\right) - 0.019\,\frac{\text{mL}}{\text{s}}$$

Calculation for the uncertainty of gradient:

$$\Delta m = \ |m| \times \left(\frac{2\Delta(Q_{ave})}{|Q_{max} - Q_{min}|} + \frac{2\Delta(\frac{1}{l_{ave}})}{\left|\frac{1}{l_{max}} - \frac{1}{l_{min}}\right|}\right)$$

$$= \left(0.29\,\frac{\text{mL m}}{\text{s}}\right) \times \left(\frac{2\left(0.04\,\frac{\text{mL}}{\text{s}}\right)}{\left(1.39\,\frac{\text{mL}}{\text{s}}\right) - \left(0.50\,\frac{\text{mL}}{\text{s}}\right)} + \frac{2(0.005\ \text{m}^{-1})}{(5.00\ \text{m}^{-1}) - (2.000\ \text{m}^{-1})}\right)$$

$$= 0.03\,\frac{\text{mL m}}{\text{s}}$$

Where m represents the gradient and Δm represents the absolute uncertainty, the final value for the gradient, including uncertainty, would be $0.29 \pm 0.03\,\frac{\text{mL m}}{\text{s}}$.

Comparing this value with theoretical gradient of $0.35 \pm 0.02\,\frac{\text{mL·m}}{\text{s}}$, the percentage error of this gradient would be:

$$\%Error = \left|\frac{0.35\,\frac{\text{mL·m}}{\text{s}} - 0.29\,\frac{\text{mL m}}{\text{s}}}{0.35\,\frac{\text{mL·m}}{\text{s}}}\right| \times 100\%$$

$$= 20\%$$

A percentage error of 20% appears to be a relatively big error, however, since fluid dynamics can be unpredictable, another percentage error using the closest values between both theoretical gradient and experimental gradient, within their uncertainties, is calculated:

$$\%Error_{closest} = \left|\frac{0.33\,\frac{\text{mL·m}}{\text{s}} - 0.32\,\frac{\text{mL m}}{\text{s}}}{0.33\,\frac{\text{mL·m}}{\text{s}}}\right| \times 100\%$$

$$= 3\%$$

Conclusion

Using the positive linear correlation between flow rate and inverse of tube length shown in Graph 3, it can be concluded that flow rate of water is inversely proportional to length of the horizontal tube when connected to the bottom of a cylindrical water tank to create a pressure difference between the two ends of the tube. The line of best fit from Graph 3 is a good fit and strengthens the theory as it passes through the error bars of most of data points shown, has a high r-coefficient of 0.99, and no apparent non-linear trend is seen.

Graph 1 shows that as the inverse of tube length increases (tube length decreases), the relationship between flow rate of water and inverse of tube length deviates from linear, where the increase in flow rate slows down and appears to approach a maximum value. This shows that the flow rate does approach a maximum value as described earlier. However, Graph 2 shows that the transition between water flow completely following Poiseuille's law and water flowing at maximum flow rate does not only happen when the flow rates calculated using Poiseuille's law and Bernoulli's equation equal each other, but it would start affecting the water flow rate before that (before flow rate equals the maximum flow rate and before tube length becomes shorter than where the flow rate from Poiseuille's law equals the maximum flow rate). Nevertheless, the effect of flow rate approaching the maximum value become insignificant when more data from short tubes are removed, as shown in Graph 3.

The slope of the best-fit line drawn in Graph 3 is $0.29 \pm 0.03 \frac{\text{mL m}}{\text{s}}$, which has a percentage error of 20% when compared to the theoretical value of $0.35 \pm 0.02 \frac{\text{mL}\cdot\text{m}}{\text{s}}$. However, since fluid dynamics involves a lot of uncertainties, the maximum value of the experimental gradient and the minimum value of theoretical gradient is compared and has a 3% error. This low percentage error strengthens the theory that predicted this close value to the experimental gradient. The Y-intercept of Graph 3 is negative, which does not follow the direct correlation as in theory, showing that there are certain systematic errors that shifted the values for the flow rates downwards.

Another potential error can be seen in the maximum flow rate obtained from the experiment. According to Bernoulli's equation, the maximum flow rate should be $4.18\ \frac{\text{mL}}{\text{s}}$, while the maximum flow rate from the experiment is only $2.74 \pm 0.08\ \frac{\text{mL}}{\text{s}}$. This shows that there may be some other factors affecting the flow of water, which will be explained in the evaluation.

Application

The results of this investigation could be applied in the field of long-range fluid transportation as well as in the medical field. The relationship between tube length and fluid flow rate indicates that long-range fluid transportation becomes inefficient because the flow rate slows down for longer tubes. This suggests that shorter tube length can provide faster fluid transportation. The transition between the flow following Poiseuille's law and the flow approaching the maximum flow rate shows that although shorter tube lengths would allow faster fluid flow rate, there is a length where shortening the tube would not increase flow rate significantly. In the medical field, many injections happen by using thin tubes to transport fluids. When using thin tubes, the effects of resistance from the tube on the fluid are more significant and would have to be considered if an accurate rate of injection is required for successful medical treatment. Longer tubes can result in a slower rate of injection and shorter tube can result in faster rate of injection up to a threshold value.

An extension that can be taken for this investigation would be to investigate both the length and the radius of tubes and how they affect fluid flow rate, allowing a more useful correlation to be found since in many actual applications of fluid transportation, both radius and the length can be changed. Also, it could be useful to systematically study the transition state between fluid flow following Poiseuille's law and Bernoulli's equation to allow more accurate predictions of fluid flow.

Evaluation

From the experiment, several errors can be found, which could have potentially affected the results of this investigation.

During the experiment, the imperfections of the horizontal tube could have caused changes to the experimental flow rate. First, many visible bubbles formed for trials with tube length between 46.00cm and 50.00cm, which can be seen in the data collected, where the flow rates from these data points are significantly less than data collected for tube length of 45.00cm and shorter. This indicates that the flow rates can be heavily affected by imperfections in the tube such as bubbles formation. It would be reasonable to assume that even after the bubbles are removed for tube length shorter than 45.00cm, there may be other factors in the tube, such as smaller bubbles that cannot be seen without careful inspection and bending in the flexible tube. The result of this error would be a decrease in flow rate for all data points, which could potentially explain the negative y-intercept in Graph 3. To solve this, clear and solid plastic or glass tubes could be used. This would reduce bending happening in the tube, as well as allowing easier observations in the tube for anything blocking fluid flow, such as bubbles.

Another problem in this experiment is the maximum expected flow rate of water. Using Bernoulli's equation, the predicted flow rate is almost two times the experimental flow rate collected using tube with a length of 1.00cm. This shows that some opposing force is acting on the water and slowing the flow rate. Since Bernoulli's equation assumes the fluid to be frictionless, one possible explanation would be the very real effect of resistance inside the tube. It is possible that the surface tension of water could also lead to forces resisting water flow from the tank. To solve this problem, liquids with minimal surface tension and viscosity can be used so that less force would be applied to the flowing fluid.

The collection of data for this experiment can have high uncertainties. The cause of this uncertainty would be that when measuring the time taken for 25.0mL of water to flow out of the tube, sometimes the flow is not a continuous stream, and individual water drops can cause the water level to oscillate quickly, making an accurate measurement difficult. Also, for trials with faster flow rates, the reaction time for the experimenter to stop the stopwatch could be significant and affect the uncertainty of the data. Therefore, to reduce these uncertainties, a thinner graduated cylinder with higher precision could be used while a high-speed camera could be placed level to the 25.0mL label on the graduated cylinder to determine the precise time when water level reaches 25.0mL. In addition, thinner graduated cylinder could allow the water to flow down along the side of the graduated cylinder and not form individual water droplets, thus reducing the uncertainties caused by oscillating water level.

Word Count: 3981

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