LAB 3

ECE356

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Assuming $\theta = 0$ and hence $D(s) = 0$, find final value of v if input is $v_m(t) = V_0 \cdot \mathbb{1}(t)$, and $v(0) = 0$.

$$V(s) = V_m(s) \cdot \frac{a}{s+b}$$

$$V(s) = V_0 \cdot \frac{a}{s \cdot (s+b)}$$

Using FVT:

$$\lim_{t\to\infty} v(t) = \lim_{s\to 0} s \cdot V(s) = \lim_{s\to 0} s \cdot V_0 \cdot \frac{a}{s \cdot (s+b)} = V_0 \cdot \frac{a}{b}$$

Subbing in $a = \frac{K_m}{M}$ and $b = \frac{M}{B}$:

$$\lim_{t\to\infty} v(t) = V_0 \cdot \frac{K_m}{M} \cdot \frac{M}{B} = V_0 \cdot \frac{K_m}{B}$$

$$\Delta v = 0.457873 - (-0.40167) = 0.859543$$

$$\Delta v = \frac{V_0 a}{b} = \frac{3a}{b} \implies a = \frac{\Delta v b}{3} = 0.286514b$$

For $b = 10$, $a = 2.86514$

In a feedback system, the output can be modelled as:

$$Y(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} \cdot R(s)$$

Where in a proportional controller, $C(s) = K$ and $G(s) = \frac{a}{s+b}$, with the reference as a step function $R(s) = \frac{\overline{R}}{s}$.

$$Y(s) = \frac{K \cdot \frac{a}{s+b}}{1 + K \cdot \frac{a}{s+b}} \cdot \frac{\overline{R}}{s} = \frac{K \cdot a}{s + b + K \cdot a} \cdot \frac{\overline{R}}{s}$$

This system is BIBO stable with only negative poles, so we can use the FVT to find the step response:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s \cdot Y(s) = \lim_{s\to 0} s \cdot \frac{K \cdot a}{s + b + K \cdot a} \cdot \frac{\overline{R}}{s} = \frac{K \cdot a}{b + K \cdot a} \cdot \overline{R}$$

This means that, as $K \to \infty$, the steady state output will approach $\overline{R}$. However, if K is not sufficiently large, the steady state output will be smaller than $\overline{R}$. This result matches what was observed, where increasing K brought the steady state output closer to the reference, but did not reach it.

For a PI controller, $C(s) = K\frac{T_I s+1}{T_I s}$.
Plugging into the system, we get:

$$Y(s) = \frac{K\frac{T_I s+1}{T_I s}\frac{a}{s+b}}{1+K\frac{T_I s+1}{T_I s}\frac{a}{s+b}} \cdot \frac{\overline{R}}{s} = \frac{K \cdot a \cdot (T_I s+1)}{T_I s(s+b)+K \cdot a(T_I s+1)} \cdot \frac{\overline{R}}{s}$$

Using the FVT, we get:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s \cdot Y(s) = \lim_{s\to 0} s \cdot \frac{K \cdot a \cdot (T_I s+1)}{T_I s(s+b)+K \cdot a(T_I s+1)} \cdot \frac{\overline{R}}{s} = \frac{K \cdot a}{K \cdot a} \cdot \overline{R} = \overline{R}$$