LAB 4

ECE356

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Contents

1 Preparation

The plant has no poles at zero, so we need to have a pole at zero for the controller.

Write the transfer function from $R(s)$ to $E(s)$, assuming $D(s) = 0$. And the transfer function from $D(s)$ to $E(s)$, assuming $R(s) = 0$.

For the first case:

$$E(s) = R(s) - C(s)G(s)E(s)$$

$$E(s) = \frac{R(s)}{1 + C(s)G(s)}$$

For the second case:

$$E(s) = 0 - \left(C(s)E(s) + D(s)\right)G(s)$$

$$E(s) = -\frac{D(s)G(s)}{1 + C(s)G(s)}$$

Subbing in $G(s) = \frac{a}{s+b}$ and $C(s) = K(1 + \frac{1}{T_I s})$:

$$E(s) = \frac{R(s)}{1 + \frac{a}{s+b} \cdot K(1 + \frac{1}{T_I s})}$$

$$E(s) = \frac{R(s)(s+b)(T_I s)}{(s+b)(T_I s) + aK(T_I s + 1)}$$

$$E(s) = \frac{-D(s)\frac{a}{s+b}}{1 + \frac{a}{s+b} \cdot K(1 + \frac{1}{T_I s})}$$

$$E(s) = \frac{-D(s)a(T_I s)}{(s+b)(T_I s) + aK(T_I s + 1)}$$

Setting $R(s) = \frac{\overline{v}}{s}$ and $D(s) = \frac{\overline{d}}{s}$, we get:

$$E(s) = \frac{\overline{v}(s+b)T_I}{(s+b)(T_I s) + aK(T_I s + 1)}$$

$$E(s) = \frac{-\overline{d}aT_I}{(s+b)(T_I s) + aK(T_I s + 1)}$$

If the limit of $\lim_{t\to\infty} e(t)$ exists, then we can use the final value theorem:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot \frac{\overline{v}(s+b)T_I}{(s+b)(T_I s) + aK(T_I s + 1)} = 0$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot \frac{-\overline{d}aT_I}{(s+b)(T_I s) + aK(T_I s + 1)} = 0$$

The controller asymptotically tracks the reference even if the disturbance is present, as error converges to zero.