1. Basic Kinematic Equations

We recall the standard constant-acceleration (1D) formulas:

$$\begin{aligned} v_f &= v_i + a\,t, && (1)\\ s &= v_i\,t + \tfrac{1}{2}\,a\,t^2, && (2)\\ v_f^2 &= v_i^2 + 2\,a\,s. && (3) \end{aligned}$$

Here:

- v_i is the initial velocity,
- v_f is the final velocity,
- a is the (constant) acceleration,
- t is the time elapsed,
- s is the displacement over that time.

2. Feasibility of Direct Deceleration Within t_p

We want to move from speed v to zero velocity (i.e. stop at the target/trajectory) within a maximum allotted time t_p. Suppose we try a *single* constant deceleration $a > 0$ (treating the direction toward the trajectory as positive, so the acceleration is actually a negative in the real direction, but we label its magnitude as a).

2.1 Deceleration distance and time

Using equation (3) with $v_f = 0$:

$$0^2 \;=\; v^2 + 2\,(-a)\,s \quad\Longrightarrow\quad s \;=\; \frac{v^2}{2\,a}.$$

Also, the time to slow from v to 0 at deceleration a is

$$t_{\text{dec}} \;=\; \frac{v}{a}.$$

2.2 Checking if $t_{\text{dec}} \le t_p$

If

$$\frac{v}{a} \;\le\; t_p,$$

then we can simply decelerate with constant a and reach zero velocity before or at t_p. Equivalently,

$$a \;\ge\; \frac{v}{t_p}.$$

Hence, a single-phase deceleration is *feasible* if the required deceleration is not too large and the time needed does not exceed t_p.

3. Accelerate–Then–Decelerate Strategy

If instead we discover that

$$\frac{v}{a} \;>\; t_p \quad \text{(i.e. we cannot stop in time } t_p\text{)},$$

we may need a two-phase approach:

1. **Accelerate** toward the trajectory for a certain duration t_i with (positive) acceleration a, raising the speed from v to $v + a\,t_i$.
2. Then **decelerate** from $v + a\,t_i$ down to zero over the remaining distance, so that the total time is exactly t_p.

You wrote two equations describing this process:

- **Equation (i):** The deceleration d (which we will set equal to a for easy simplification) is

$$d \;=\; \frac{\bigl(v + a\,t_i\bigr)^2}{2\Bigl[s - \bigl(v\,t_i + \tfrac{1}{2}\,a\,t_i^2\bigr)\Bigr]},$$

  where s is the total distance available before reaching the trajectory, and the denominator accounts for the distance left for deceleration.

- **Equation (ii):** The total time t_p is split into t_i (acceleration) plus the deceleration time, which is

$$\frac{v + a\,t_i}{d},$$

  so

$$\frac{v + a\,t_i}{d} \;+\; t_i \;=\; t_p.$$

3.1 Substituting $d = a$

In the particular setup, we put $d = a$ to simplify the equation, so we can rewrite these as:

$$\text{(i)} \quad a = \frac{\bigl(v + a\,t_i\bigr)^2}{2\Bigl[s - \bigl(v\,t_i + \tfrac{1}{2}\,a\,t_i^2\bigr)\Bigr]},$$

$$\text{(ii)} \quad \frac{v + a\,t_i}{a} + t_i = t_p.$$

3.2 Solving for t_i and a

From (ii),

$$\frac{v + a\,t_i}{a} = t_p - t_i \implies v + a\,t_i = a\,(t_p - t_i) \implies v = a\,t_p - a\,t_i - a\,t_i = a\bigl(t_p - 2\,t_i\bigr).$$

Hence

$$t_p - 2\,t_i = \frac{v}{a} \implies t_i = \frac{t_p - \frac{v}{a}}{2}.$$

Substitute this t_i into (i) to get a *quadratic* equation in a alone. After simplifying, one obtains:

$$t_p^2\,a^2 \;-\; \bigl(4\,s - 2\,v\,t_p\bigr)\,a \;-\; v^2 \;=\; 0.$$

Solving via the quadratic formula gives

$$a = \frac{(4\,s - 2\,v\,t_p) \;\pm\; \sqrt{(4\,s - 2\,v\,t_p)^2 \;+\; 4\,t_p^2\,v^2}}{2\,t_p^2}.$$

Choose the sign (plus or minus) that leads to a physically valid solution (often the + branch for a positive a).

4. Summary

1. **Check feasibility of direct deceleration:**
   If $\frac{v}{a} \le t_p$, then we can decelerate from v to 0 within t_p without needing a two-phase approach.

2. **Otherwise, use accelerate–then–decelerate:**
   Solve the system above (with $d = a$) to find the needed acceleration a and the switching time t_i so that the total time is exactly t_p.