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Indian Institute of Technology Kharagpur
Mid-Spring Semester Examination 2022-23

Date of Examination: Session FN/AN,
Subject. No. **MA60056 / MA60280**
Department: Mathematics
Specific Chart, graph paper log book etc. required.... NO.
No. of Registered Students: **62 (PGDBA)+64 (B.Tech)**

Duration:2 Hrs,
Subject Name: **RTSM**
TOTAL MARKS: 30

INSTRUCTIONS: Answer all the questions. Answer all parts of a question in consecutive places. Numerical answers must be in decimal. Answer only within the error range ∓ 0.01 will get the credit.

Numeric values might be of use: $\Phi(1.64) = 0.90$; $\Phi(1.96) = 0.95$, $\Phi(0.25) = 0.5987063$. $P(t_{18} < 2.1) = 0.975$, $P(t_9 \leq 1.833) = 0.95$, $P(t_8 \leq 2.306) = 0.975$, $P(t_6 \leq 2.447) = 0.975$

1. Let $S_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1^2 + x_2^2 + x_3^2 \leq 9\}$ and a subspace of $\mathbb{R}^3$ as $S_2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid 2x_1 + 5x_2 + 9x_3 = 0, 2x_1 + 4x_2 + 6x_3 = 0\}$. Find the area of $S_1 \cap S_2^{\perp}$. [4]

2. Let (X, Y) follow a bivariate normal distribution with $(\mu_x = 2, \mu_y = 3, \sigma_x^2 = 4, \sigma_y^2 = 9, \rho = 1/3)$. Find $P(|3X - 2Y| \leq \sqrt{3})$. [4]

3. Let $y_i \sim N(\beta_0 + \beta_1 x_i, \sigma^2)$ for all $i = 1, 2, \ldots 10$ independently. Observed values of $\hat{\beta}_0 = 1.2$, $MSError = 3.6$, $\bar{x} = 2.3$, $S_{xx} = 5.7$. Compute the observed absolute value of the t-statistic for $H_0 : \beta_0 = 1.8$ vs $H_1 : \beta_0 \neq 1.8$. [4]

4. Let for a simple linear regression model $MSError = 0.35$, $n = 10$, $S_{xx} = 5.7$, $\bar{x} = 3.5$. Find the length of the 95% prediction interval of y for $x = 3.3$ [4]

5. For the model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\epsilon \sim N(0, \sigma^2 \mathbf{I}_n)$, $\mathbf{Y} \in \mathbb{R}^n$, $\beta \in \mathbb{R}^{(k+1)}$ if $R^2 = 0.82$, $k = 6$, $n = 25$ find the value of F-statistic for the ANOVA of regression model. [4]

6. Prove or disprove : $(Y_1 - 2Y_2 + 3Y_3 - 1.5Y_4 - 0.5Y_6)$ is a liner zero function under multiple linear regression model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\epsilon \sim N(0, \sigma^2 \mathbf{I}_6)$, $\mathbf{Y} \in \mathbb{R}^6$, $\beta \in \mathbb{R}^4$ observed $\mathbf{X}$ matrix is

$$\mathbf{X} = \begin{bmatrix} 1 & 3.0 & -1.5 & -0.4 \\ 1 & 1.5 & 1.0 & 2.0 \\ 1 & 0.5 & 2.0 & 0.8 \\ 1 & 1.0 & 1.0 & -2.0 \\ 1 & 7.0 & 4.0 & 0.9 \\ 1 & 0.0 & 2.0 & 7.0 \end{bmatrix}$$

[4]

P.T.O.

7. For the model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\epsilon \sim N(\mathbf{0}, \sigma^2\mathbf{I}_n)$, $\mathbf{Y} \in \mathbb{R}^n$, $\beta \in \mathbb{R}^{(k+1)}$ test at 5% level for the hypothesis $H_0 : \beta_1 - 2\beta_2 = 2.2$ against $H_1 : \beta_1 - 2\beta_2 \neq 2.2$. It is given that $n = 25$, $k = 6$, estimated values of β_1 and β_2 are 3.73 and 0.75 respectively. Denoting $C = (\mathbf{X}^T\mathbf{X})^{-1}$ it is obtained from data that $C_{00} = 0.0839, C_{11} = 0.25$, $C_{22} = 0.64$, $C_{02} = 0.12$, $C_{12} = 0.025$ and $\hat{\sigma} = 0.125$.. Find the observed value of t-statistic : [4]

8. Consider the simple linear regression model $E(y|x) = a + bx$. Here x variable stands for the length of a pendulum in $\log_{10}$ scale and y variable stands for the measured time period of it in the $\log_{10}$ scale too. Under the i.i.d. normality assumption for random errors predict value of the time period (y_0) for length $x_0 = 1.06$ with justification. [2]

x	1.04	1.08	1.02	1.10	1.07	1.05	1.03	1.09
y	0.818	0.845	0.899	0.865	0.890	0.946	0.938	0.935