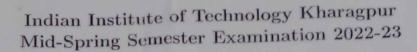
Duration: 2 Hrs,

Subject Name: RTSM

TOTAL MARKS: 30





Session FN/AN, Date of Examination:

Subject. No. MA60056 / MA60280

Department: Mathematics

Specific Chart, graph paper log book etc. required.... NO

No. of Registered Students: 62 (PGDBA)+64 (B.Tech)

INSTRUCTIONS: Answer all the questions. Answer all parts of a question in consecutive places. Numerical answers must be in decimal. Answer only within the error range ∓ 0.01 will get the credit

Numeric values might be of use: $\Phi(1.64) = 0.90$; $\Phi(1.96) = 0.95$, $\Phi(0.25) = 0.5987063$. $P(t_{18} < 2.1) = 0.975$, $P(t_{9} \le 1.833) = 0.95$, $P(t_{8} \le 2.306) = 0.975$, $P(t_{6} \le 2.447) = 0.975$

1. Let
$$S_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1^2 + x_2^2 + x_3^2 \leq 9\}$$
 and a subspace of \mathbb{R}^3 as $S_2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid 2x_1 + 5x_2 + 9x_3 = 0, 2x_1 + 4x_2 + 6x_3 = 0\}$. Find the area of $S_1 \cap S_2^{\perp}$. [4]

2. Let
$$(X, Y)$$
 follow a bivariate normal distribution with $(\mu_x = 2, \mu_y = 3, \sigma_x^2 = 4, \sigma_y^2 = 9, \rho = 1/3)$. Find $P(|3X - 2Y| \le \sqrt{3})$.

3. Let $y_i \sim N(\beta_0 + \beta_1 x_i, \sigma^2)$ for all i = 1, 2, ... 10 independently. Observed values of $\hat{\beta}_0 = 1.2$, $MSError = 3.6, \bar{x} = 2.3, S_{xx} = 5.7.$ Compute the observed absolute value of the t-statistic for $H_0: \beta_0 = 1.8 \text{ vs } H_1: \beta_0 \neq 1.8.$

Let for a simple linear regression model
$$MSError = 0.35$$
, $n = 10$, $S_{xx} = 5.7$, $\bar{x} = 3.5$. Find the length of the 95% prediction interval of y for $x = 3.3$

For the model
$$\mathbf{Y} = \mathbf{X}\beta + \epsilon$$
, where $\epsilon \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_n), \mathbf{Y} \in \mathbb{R}^n$, $\beta \in \mathbb{R}^{(k+1)}$ if $R^2 = 0.82$, $k = 6$, $n = 25$ find the value of F-statistic for the ANOVA of regression model. [4]

§. Prove or disprove: $(Y_1 - 2Y_2 + 3Y_3 - 1.5Y_4 - 0.5Y_6)$ is a liner zero function under multiple linear regression model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\epsilon \sim N(0, \sigma^2 \mathbf{I}_6), \mathbf{Y} \in \mathbb{R}^6, \beta \in \mathbb{R}^4$ observed \mathbf{X} matrix is

$$\mathbf{X} = \begin{bmatrix} 1 & 3.0 & -1.5 & -0.4 \\ 1 & 1.5 & 1.0 & 2.0 \\ 1 & 0.5 & 2.0 & 0.8 \\ 1 & 1.0 & 1.0 & -2.0 \\ 1 & 7.0 & 4.0 & 0.9 \\ 1 & 0.0 & 2.0 & 7.0 \end{bmatrix}$$

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For the model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$, where $\boldsymbol{\epsilon} \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_n)$, $\mathbf{Y} \in \mathbb{R}^n$, $\boldsymbol{\beta} \in \mathbb{R}^{(k+1)}$ test at 5% level for the hypothesis $H_0: \beta_1 - 2\beta_2 = 2.2$ against $H_1: \beta_1 - 2\beta_2 \neq 2.2$. It is given that n = 25, k = 6, estimated values of β_1 and β_2 are 3.73 and 0.75 respectively. Denoting $C = (\mathbf{X}^T \mathbf{X})^{-1}$ it is obtained from data that $C_{00} = 0.0839$, $C_{11} = 0.25$, $C_{22} = 0.64$, $C_{02} = 0.12$, $C_{12} = 0.025$ and $\hat{\sigma} = 0.125$. Find the observed value of t-statistic .

8. Consider the simple linear regression model E(y|x) = a + bx. Here x variable stands for the length of a pendulum in \log_{10} scale and y variable stands for the measured time period of it in the \log_{10} scale too. Under the i.i.d. normality assumption for random errors predict value of the time period (y_0) for length $x_0 = 1.06$ with justification.

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	11	0.818	0.845	0.899	0.865	0.890	0.946	0.950	0.935