

Sudden stops, asset prices: the role of financial market participation

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 - Empirically, I show that lower financial market participation is associated with a higher drop in asset prices.
 - I enrich a model of sudden stops with limited financial market participation.

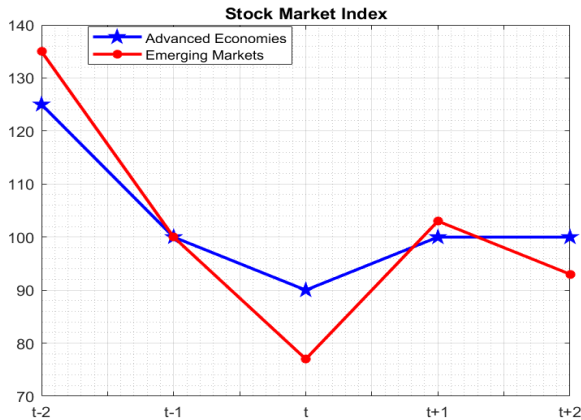
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 - Empirically, I show that lower financial market participation is associated with a higher drop in asset prices.
 - I enrich a model of sudden stops with limited financial market participation.
- **Findings**
 - Limited financial market participation amplifies the drop in asset prices by **20%**.

Data

- **Data on sudden stops are from Korinek and Mendoza (2014)**
 - Time span: 1980-2012;
 - Countries span: 59 countries (35 Emerging Markets and 23 Advanced Economies) have experienced at least one sudden stops;
 - Sudden stop: quarterly capital flows exceeding two standard deviations from its mean.
- **IMF data on Financial Development index**
 - Time span: 1980-2017;
 - Financial Market Access index: total number of issuers of debt (domestic and external, non financial and financial corporations);
 - Covers more than 170 countries in the world, including OECD countries, Latin American countries and African countries.

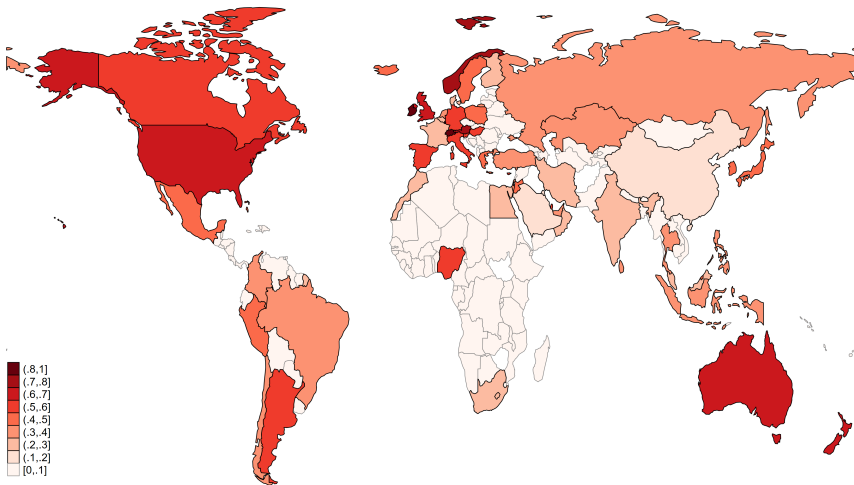
Fact 1: Drop in asset prices is higher in Emerging Markets than in Advanced Economies.



Source: Korinek and Mendoza (2014)

Fact 2: Financial market participation is lower in Emerging Markets than in Advanced Economies.

Median of the Financial Market Access Index from 1980-2017



Source: Author's calculation using the financial development index from IMF

Fact 3: Lower financial market participation is associated with a higher drop in asset prices.

Asset price growth	(1) Aggregate	(2) Advanced Economies	(3) Emerging Market
Sudden stops (SS)	-0.438***	-0.334***	-0.649***
Financial market participation (FMA)	-0.110	0.0104	-0.269
FMAxSS	0.416**	0.295*	0.985**
Observations	631	366	265
R-squared	0.352	0.533	0.421
Number of countries	29	15	14

*** p<0.01, ** p<0.05, * p<0.1

Regression is done with country and year fixed effect. SS is a dummy variable which takes 1 if a country is in a sudden stop for a given year. Data on sudden stops are from Korinek and Mendoza (2014). **FMA is the Financial Market Access index from IMF.** FMA.SS is a cross product of FMA and SS. We control for capital flow. The data covers 1980-2012. We drop the sudden stops events with an increase in asset prices.

Summary of the facts

- **Fact 1:** Drop in asset prices is higher in Emerging Markets than in Advanced Economies.
- **Fact 2 :** Financial market participation is lower in Emerging Markets than in Advanced Economies.
- **Fact 3:** Lower financial market participation is associated with a higher drop in asset prices.

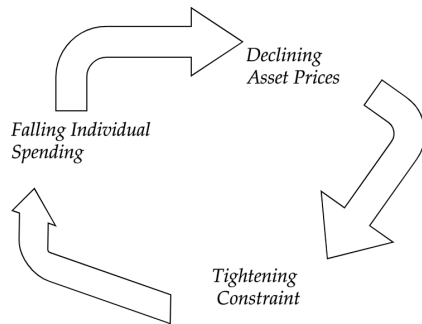
Sudden stops

- **High leverage**

Framework : DSGE + Collateral constraint

Sudden stops

- High leverage

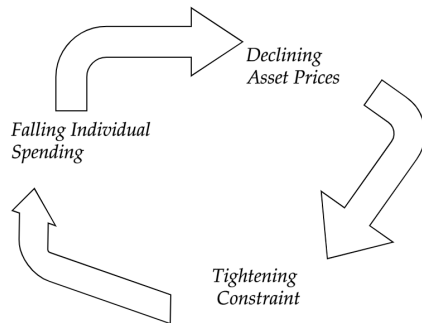


Framework : DSGE + Collateral constraint

Sudden stops

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⇒ ↓ asset prices and ↓ economic activities.



Framework : DSGE + Collateral constraint

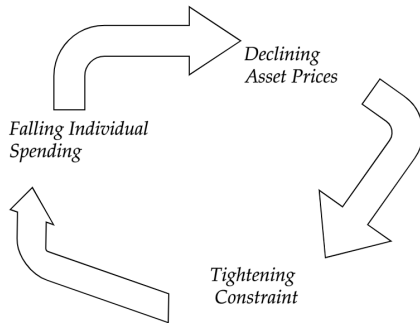
Sudden stops

- **High leverage**

⇒ ↓ asset prices and ↓ economic activities.

- **Limited financial market participation**

⇒ wealth concentration ⇒ ↑ burden on asset holder ⇒ **high ↓ asset prices.**



Framework : DSGE + Collateral constraint + heterogeneous agents.

Sudden stops

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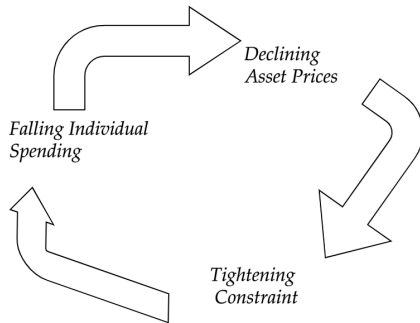
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- **Limited financial market participation**

⇒ wealth concentration ⇒ ↑ burden on asset holder ⇒ **high ↓ asset prices.**

- **Consequences for capital control**

⇒ **more capital control.**



Framework : DSGE + Collateral constraint + heterogeneous agents.

Literature review

- **Collateral constraint:** Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1998).
- **Sudden stops and aggregate effect:** Calvo (1998), V.V.Chari et al. (2005), Mendoza (2002, 2006, 2010).
- **Macroprudential policy:** Bianchi and Mendoza (2018), Bengui and Bianchi (2019), Arce et al. (2019)

Two agents framework

- **Non Hand to Mouth consumer**

- Infinitely lived of measure $1 - \theta \in (0, 1)$;
- Work, consume, and save into two assets: stock k_t and foreign bond b_t ;
- Household face a collateral borrowing limit;
- Receives dividends from firm's profit.

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 - Infinitely lived of measure $\theta \in (0, 1)$;
 - Work and consume all their labor income.
- **Firms**
 - Perfect competition on the good market and labor market;
 - Firms finance production via equity or debt;
 - Debt is subject to collateral constraint;
 - Use imported input v_t to produce.

Firm-Non Hand to Mouth Problem

$$\max_{C_{1t}, b_{t+1}, k_{t+1}, v_t, L_{1t}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U(C_{1t} - G(L_{1t}))$$
$$C_{1t} + \frac{b_{t+1}}{R_t} + q_t k_{t+1} = F(k_t, L_t, v_t) + b_t + q_t k_t - T_t$$

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$$\text{Euler 1 : } \underbrace{U'(t)}_{\text{marginal benefit of borr.}} = \underbrace{\beta R_t E_t U'(t+1)}_{\text{marginal cost of borr.}} + \underbrace{\mu_t}_{\text{shadow price of relaxing the constr.}}$$

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Hand to Mouth Problem

$$\begin{aligned} \max_{C_{2t}, L_{2t}} \quad & \mathbb{E} \sum_{t=0}^{\infty} \beta^t U(C_{2t} - G(L_{2t})) \\ \theta C_{1t} \quad &= \theta w_t L_{2t} + T_t \end{aligned}$$

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$$\text{Labor :} \quad \underbrace{G'(L_{2t})}_{\text{marginal disutility of labor}} = \underbrace{w_t}_{\text{real wage}}$$

Consumption inequality wedge

- Let $\frac{\lambda_t^R}{E_t[\beta\lambda_{t+1}^R]}$ be the marginal rate of substitution of consumption with $\theta = 0$.

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- $\lambda_t = a_t^\sigma \lambda_t^R$ where $a_t = \frac{(1-\theta)\omega + (\omega\theta - 1)\frac{c_{2t}}{c_{1t}}}{\omega - \frac{c_{2t}}{c_{1t}}}$.
- Define $\frac{\lambda_t^R}{E_t[\beta\lambda_{t+1}^R]} - \frac{\lambda_t}{E_t[\beta\lambda_{t+1}]}$ as **consumption inequality wedge**.

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- Define $\frac{\lambda_t^R}{E_t[\beta\lambda_{t+1}^R]} - \frac{\lambda_t}{E_t[\beta\lambda_{t+1}]}$ as **consumption inequality wedge**.
- The consumption inequality wedge is zero when $\theta = 0$ and/or $\frac{c_{2t}}{c_{1t}}$ constant over time.
- If $\theta > 0$ the consumption inequality wedge depends on the consumption inequality $\frac{c_{1t}}{c_{2t}}$.

Sudden stop crisis

- Equity premium

$$E_t \left[R_{t+1}^q - R_t \right] = \underbrace{\frac{(1 - \kappa) \mu_t^R}{E_t \left[\beta \lambda_{t+1}^R \right]}}_{\text{liquidity premium}} - (1 - \kappa) \underbrace{\left(\frac{\lambda_t^R}{E_t \left[\beta \lambda_{t+1}^R \right]} - \frac{\lambda_t}{E_t \left[\beta \lambda_{t+1} \right]} \right)}_{\text{inequality wedge}} - \underbrace{\frac{\text{Cov} \left(\lambda_{t+1}, R_{t+1}^q \right)}{E_t \left[\lambda_{t+1} \right]}}_{\text{risk premium}}$$

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- Higher equity premium leads to lower asset prices

Financial market participation and consumption inequality 1/2

Claim 1: If consumption inequality constant over time then the financial market participation does not matter for the sudden stops crises. **Proof**

- **Necessary condition:** decline in aggregate consumption is exactly ω times the decline in aggregate labor. **Detail**
- **Intuition:** risk sharing with HtM consumers via the labor market.

Financial market participation and consumption inequality 2/2

Claim 2: Let's suppose perfect foresight (no uncertainty) $\mathbb{E}_t[X_{t+1}] = X_{t+1}$.

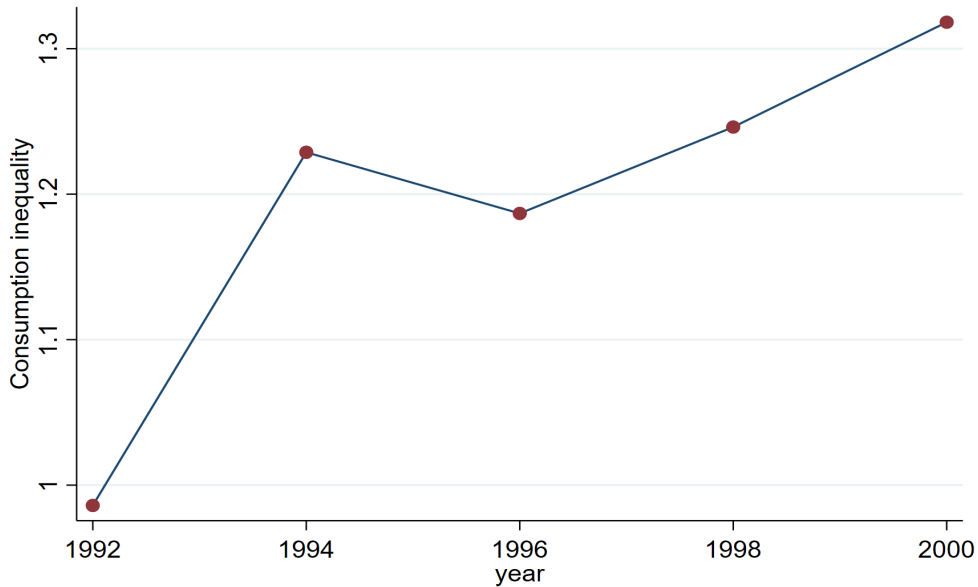
If at the time of financial shock (binding collateral constraint), consumption inequality is higher (lower), the economy will generate less (higher) amplification effect. **Proof**

- **Intuition:** Non-Hand to Mouth bears higher cost.

What about consumption inequality in the data: Mexico's case

- **National Household Consumption and Income (ENIGH) survey in Mexico**
 - Representative household survey (more than 10 000 household for each survey);
 - Every two years since 1992;
 - Information on consumption, income and wealth.
- **Measuring consumption inequality**
 - Determine HtM status: Htm consumers have zero net liquid wealth;
 - Consumption inequality: ratio of NHtM consumption to HtM consumption.

Empirical fact: Consumption inequality decreases



Quantitative results: Identification of sudden stops in the model

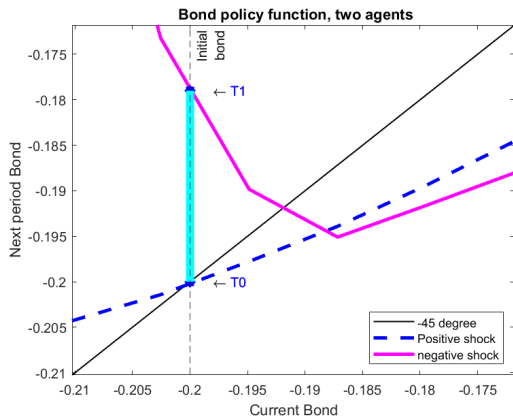
- Three aggregate shocks: real interest rate, imported input price, and TFP
- Define sudden stops as:
 - Collateral binds and,
 - the trade balance is two deviation (cyclical component) above its mean.
- Comparative analysis: introduce financial shock κ to κ_t

Calibration

Parameters set Independently	Value	Source/Target
Risk aversion	$\sigma = 2$	Standard value
Share of labor in gross output	$\alpha = 0.592$	Mexico GDP labor share 0.66
Share of input in gross output	$\eta = 0.10229$	Mexico data
Share of asset in output	$\gamma = 0.043$	steady state asset return
Frisch elasticity	$\omega = 1.846$	Mendoza (2010)
Working capital coefficient	$\phi = 0.13$	Working capital/ GDP ratio = 10%
Share of HtM	$\theta = 0.5$	Mexico data
Transfer	$T_t = 0.14$	Avr cons ineq of 1.25
Parameters set by Simulation	Value	Target
Discount factor	$\beta = 0.920$	Net foreign asset of 20%
Fraction of collateral value	$\kappa = 0.43$	Financial crisis of 4 %

Policy function

(a) Limited financial market participation



(b) Full financial market participation

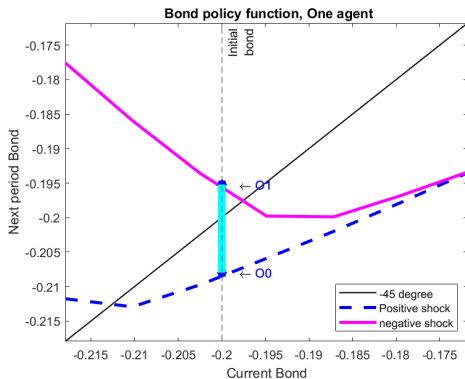
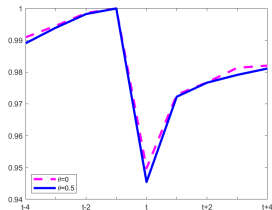


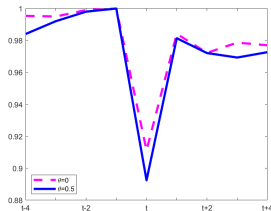
Figure: Policy function for private debt

Quantitative results: Sudden stop events

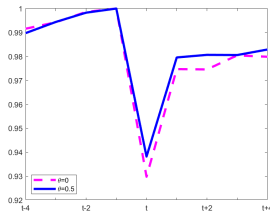
(a) Output



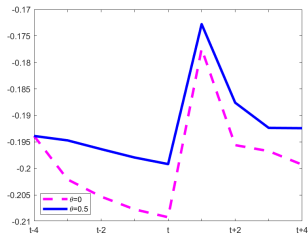
(b) Consumption of Asset holders



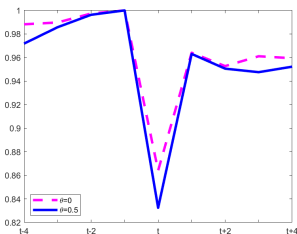
(c) Consumption of Hand to Mouth



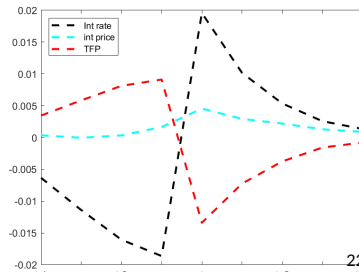
(d) Private Debt



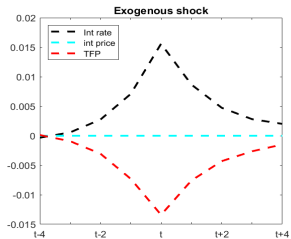
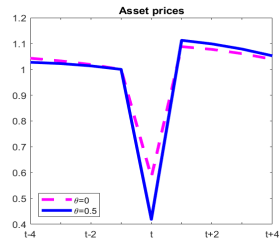
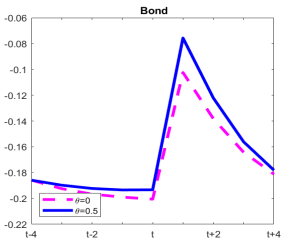
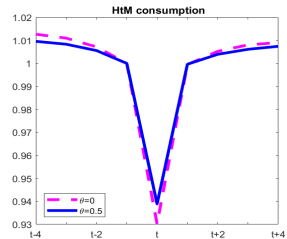
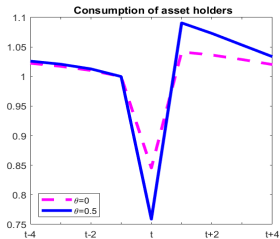
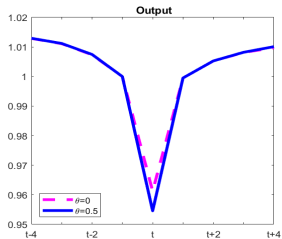
(e) Asset prices



(f) Exogenous shocks



Sudden stops with financial shock



Financial market participation affects the asset prices drop

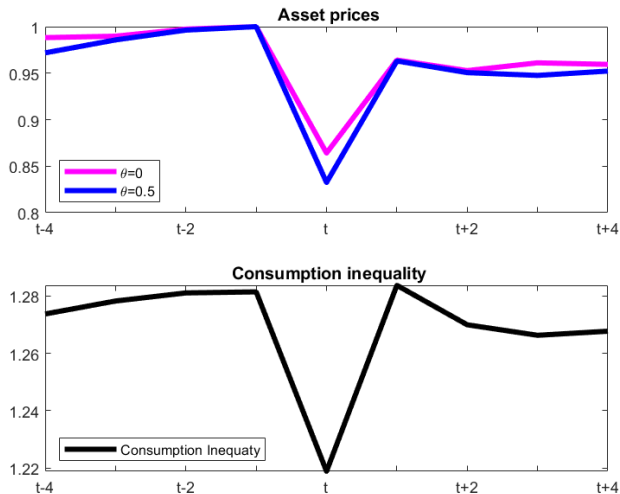


Figure: Asset prices and inequality

Business cycles

Table: Business cycle moments

	Standard deviation			Correlation with output		
	$\theta = 0$	$\theta = 0.5$	data	$\theta = 0$	$\theta = 0.5$	data
GDP	2.68	2.73	2.72	1.00	1.00	1.00
Consumption	3.55	3.51	3.39	0.94	0.95	0.89
Trade balance/gdp	1.33	1.26	2.1	-0.51	-0.49	-0.68
Asset prices	5.63	6.68	14.64	0.89	0.89	0.57
Interest rate	1.95	1.95	1.95	-0.64	-0.65	-0.59

Take away

- **Empirical results**
 - Limited financial market participation is associated with the decline in asset prices.
- **Analytical results**
 - If consumption inequality drops, there is a high amplification in the sudden stops.
- **Quantitative results**
 - Limited financial market participation explains 20% of the difference in the drop of asset prices between Emerging markets and Advanced Economies.
- **Next: Optimal Policy.**

Time-consistent Planner's Problem

- Maximize the weighted average of lifetime utility of hand to mouth consumers and non-hand to mouth consumers.
- Pecuniary externality: the choice of debt affect the price which determines the value of the collateral.

Solution to the planner's problem

- Use the planner's solution to decentralise the Competitive Equilibrium (CE) via a debt tax (the tax is high during good time and almost 0 during bad time).

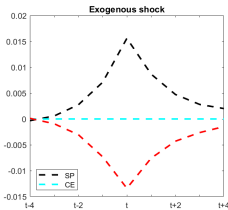
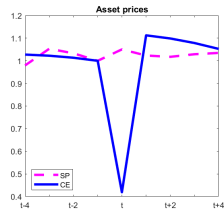
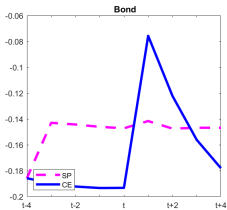
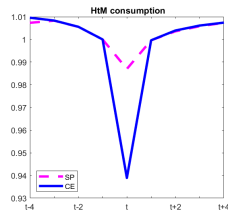
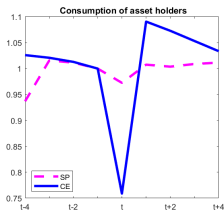
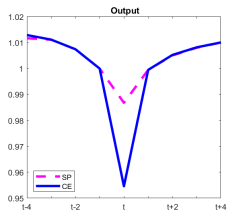
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- Average tax on bond is 1.2% with $\theta = 0$ and 4 % with $\theta = 0.5$.

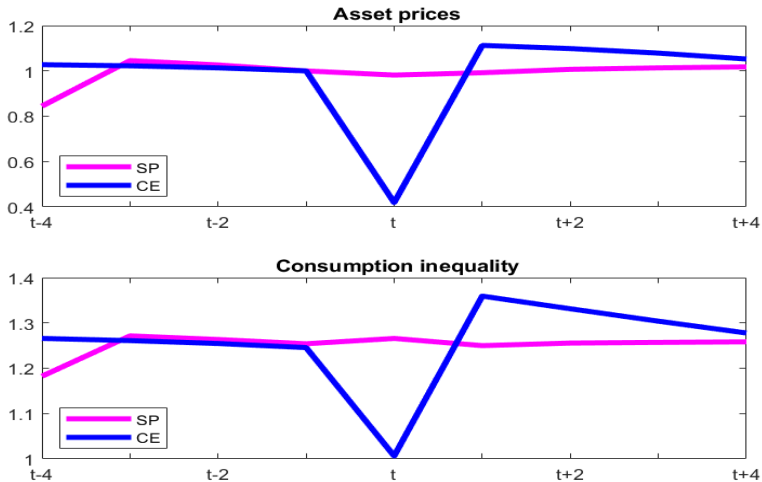
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- Average tax on bond is 1.2% with $\theta = 0$ and 4 % with $\theta = 0.5$.
- Dynamics of the financial crisis with optimal debt tax policy:
 - construct 9 years window for the financial crisis in the (CE);
 - identify the initial bond level 4 years prior to the financial crisis in the CE;
 - identify the exogenous aggregate shocks that hit the economy;
 - use the identified initial level of bond and the shocks and apply it to the planner's policy functions.

Social Planner vs Competitive Equilibrium



Capital control and consumption inequality



Conclusion

- **Empirical results**

- Limited financial market participation is associated with the decline in asset prices.

- **Analytical results**

- If consumption inequality drops, there is a high amplification in the sudden stops.

- **Quantitative results**

- Limited financial market participation explains 20% of the difference in the drop of asset prices between Emerging markets and Advanced Economies.

- **Optimal solution.** Debt tax can reduce the severity of the financial crises but may increase **consumption inequality at the short run** .

Time-consistent Planner's Problem

$$V(b, s) = \max_{c_1, c_2, b', L, v, q, \mu} \left\{ \theta u(c_2 - G(L) + (1 - \theta)u(c_1 - G(L))) + \beta \mathbb{E}_{s', s} V(b', s') \right\}$$

$$(1 - \theta) c_1 + \frac{b'}{R} = F(1, L, v) - p^v v - \theta w L - \phi r (w L + p^v v) + b - T.$$

$$\theta c_2 = \theta w L + T$$

$$-\phi R (w L + p^v v) + \frac{b'}{R} \geq -\kappa q$$

$$AF_v(1, L, v) = p^v + \phi \left(r + R \frac{\mu}{u'(c_1 - G(L))} \right) p^v$$

$$AF_l(1, L, v) = G'(L) + \phi \left(r + R \frac{\mu}{u'(c_1 - G(L))} \right) w$$

$$w = G'(L)$$

$$q u'(c_1 - G(L)) = \beta \mathbb{E} [(\mathbb{D}(b', s') + \mathbb{Q}(b', s')) u'(\mathbb{C}(b', s'))] + \kappa q \mu$$

How does a binding collateral constraint ($\mu_t > 0$) induce a sudden stops crisis 2/3

- External financing premium on working capital,

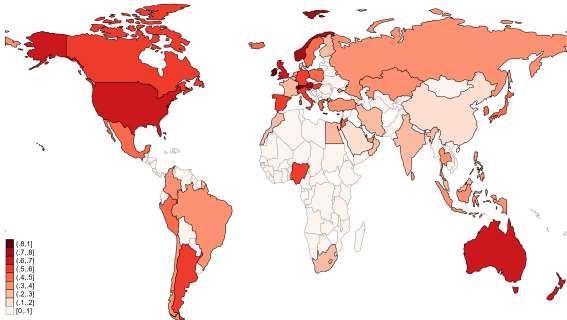
$$\underbrace{F_l(k_t, L_t, v_t)}_{\text{marginal benefit of labor}} = \underbrace{\left(1 + \phi r_t + \phi R_t \frac{\mu_t}{\lambda_t}\right)}_{\text{marginal cost of labor}} w_t \quad (1)$$

$$\underbrace{F_l(k_t, L_t, v_t)}_{\text{marginal benefit of imp input}} = \underbrace{\left(1 + \phi r_t + \phi R_t \frac{\mu_t}{\lambda_t}\right)}_{\text{marginal cost of imp input}} p_t^v \quad (2)$$

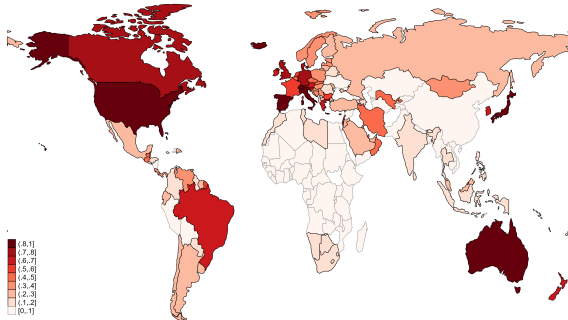
where $R_{t+1}^q \equiv \frac{d_t + q_{t+1}}{q_t}$, the equity return ; $\lambda_t^R = U'(C_t - G(L_t))$ and $\lambda_t = U'(C_{1t} - G(L_t))$

Fact 2: Financial market participation is lower in Emerging Markets than in Advanced Economies.

Median of the Financial Market Access Index from 1980-2017



Median of the Financial Institution Access Index from 1980-2017



Source: Author's calculation using the financial development index from IMF

Non Hand to Mouth Problem

$$\max_{C_{1t}, b_{1t+1}, s_{t+1}, N_{1t}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U(C_{1t} - G(N_{1t}))$$

$$C_{1t} + \frac{b_{1t+1}}{R_t} + q_t s_{t+1} = w_t N_{1t} + b_{1t} + (d_t + q_t) s_t$$

$$\frac{b_{1t+1}}{R_t} \geq -\kappa q_t s_{t+1}$$

$EE1 :$
 $\underbrace{U'(t)}_{\text{marginal benefit of borr.}} = \underbrace{\beta R_t E_t U'(t+1)}_{\text{marginal cost of borr.}} + \underbrace{\mu_t^{nh}}_{\text{shadow price of relaxing the constr.}}$

$EE2 :$
 $\underbrace{q_t U'(t)}_{\text{marginal cost of buy.}} = \underbrace{\beta E_t [(d_{t+1} + q_{t+1}) U'(t+1)]}_{\text{marginal benef. of buy.}} + \underbrace{k q_t \mu_t^{nh}}_{\text{gain of relax. the constr.}}$

$Lab :$
 $\underbrace{G'(N_{1t})}_{\text{marginal disutility of labor}} = \underbrace{w_t}_{\text{real wage}}$

Hand to Mouth Problem

$$\begin{aligned} \max_{C_{2t}, N_{2t}} \quad & \mathbb{E} \sum_{t=0}^{\infty} \beta^t U(C_{2t} - G(N_{2t})) \\ C_{1t} = \quad & w_t N_{2t} \end{aligned}$$

$$\text{Labor :} \quad \underbrace{G'(N_{2t})}_{\text{marginal disutility of labor}} = \underbrace{w_t}_{\text{real wage}}$$

Firm problem

$$\max_{d_t, k_{t+1}^f, b_{t+1}^f, v_t, L_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U'(C_{1t} - G(N_{1t})) d_t$$

$$d_t + \frac{b_{t+1}^f}{R_t} + i_t = F(k_t^f, L_t, v_t) - (1 + \theta r_t)(w_t L_t + p_t^v v_t) + b_t^f$$

$$i_t = k_{t+1}^f - k_t^f + \delta k_t^f + (k_{t+1}^f - k_t^f) \psi \left(\frac{k_{t+1}^f - k_t^f}{k_t^f} \right)$$

$$\frac{b_{t+1}^f}{R_t} - \theta R_t (w_t L_t + p_t^v v_t) \geq -\kappa^f q_t k_{t+1}^f$$

Opt condition

How does a binding collateral ($\mu_t > 0$) induce a sudden stops crises : debt deflation mechanism 3/3

- Debt deflation mechanism

$$q_t = \mathbb{E} \left[\sum_{i=0}^{\infty} \left(\prod_{j=0}^i \frac{1}{\mathbb{E} [R_{t+1+j}^q]} \right) d_{t+1+i} \right] \quad (3)$$

where $\tilde{R}_{t+1} = \frac{\beta \lambda_{t+1}}{\lambda_t - k \mu_t} d_t = F_k(k_t, L_t, v_t)$

- **Intuition:** Binding collateral constraint increases the expected equity return then decreases current asset price q_t .

Size of the inequality

- Ratio c_1 to c_2 is 2.3 implies for instance
- $c_1 = 23\,000$ \$ per year $c_2 = 10\,000$ \$ $\text{Ineq}_1 = 2.3$

Optimality conditions for firm

$$\begin{aligned} [b_{t+1}^f] &:: U'(t) = R_t E_t [U'(t+1)] + U'(t) \mu_t^f \\ [k_{t+1}^f] &:: U'(t) \frac{\partial i_t}{\partial k_{t+1}} = E_t \left[U'(t+1) \left\{ F_k(k_t^f, L_t, v_t) - \frac{\partial i_{t+1}}{\partial k_{t+1}} \right\} \right] \\ &+ k^f q_t U'(t) \mu_t^f \\ [L_t] &:: F_L(k_t^f, L_t, v_t) = (1 + \phi r_t + \phi R_t \mu_t^f) w_t \\ [v_t] &:: F_v(k_t^f, L_t, v_t) = (1 + \phi r_t + \phi R_t \mu_t^f) p_t^v \\ KT &:: \mu_t^f \left(\frac{b_{t+1}^f}{R_t} - \phi R_t (w_t L_t + p_t v_t) + k^f q_t k_{t+1}^f \right) \\ \mu_t^f &\geq 0 \end{aligned}$$

Proof of proposition 2

- **First step:** Show that if $x_t = \frac{c_{2t}}{c_{1t}}$ is constant then $\frac{\mu_t}{\lambda_t} = \frac{\mu_t^R}{\lambda_t^R}$ and ($\lambda_t = b\lambda_t^R$ and $\mu_t = b\mu_t^R$) where b is a constant
- **Second step:** Using above result, show that equations 1-6 depend only on λ_t^R and μ_t^R .

First step: We Know that $\lambda_t = a_t^\sigma \lambda_t^R$ where $a_t = \frac{(1-\lambda)\omega - (\omega\lambda - 1)\frac{c_{2t}}{c_{1t}}}{\omega - \frac{c_{2t}}{c_{1t}}}$.

If $x_t = \frac{c_{2t}}{c_{1t}}$ is constant then $\lambda_t = b\lambda_t^R$ for every t where b is a constant. Since the bond EE in the two agents model is $\lambda_t = \beta R_t E_t \lambda_{t+1} + \mu_t$ then $b\lambda_t^R = \beta b R_t E_t \lambda_{t+1}^R + \mu_t$. But in the representative agent case $\lambda_t^R = \beta R_t E_t \lambda_{t+1}^R + \mu_t^R$. Then $\frac{1}{b}\mu_t = \mu_t^R$. So $\mu_t = b\mu_t^R$. We can then conclude that $\frac{\mu_t}{\lambda_t} = \frac{\mu_t^R}{\lambda_t^R}$.

Second step: Using $\frac{\mu_t}{\lambda_t} = \frac{\mu_t^R}{\lambda_t^R}$, and ($\lambda_t = b\lambda_t^R$ and $\mu_t = b\mu_t^R$) it is straightforward to show that eq 1-6 depend only on λ_t^R and μ_t^R . Which correspond exactly to Mendoza (2010) economy. [back](#)

Proof of proposition 3

- **First part:** higher consumption inequality leads to lower amplification effect that is less distortion and small effect on the asset price q_t which limit the debt deflation mechanism
- **second part:** Lower consumption inequality leads to higher amplification effect that is more distortion and high effect on the asset price q_t which increase the debt deflation mechanism.

First part: Suppose the consumption inequality increases at the time of financial crises.

That is $x_t < x_{t+1}$ with $x_t = \frac{c_{2t}}{c_{1t}}$.

We know that $\lambda_t = b_t \lambda_t^R$ where $b_t = \left[\frac{(1-\lambda)\omega - (\omega\lambda-1)x_t}{\omega - x_t} \right]^\sigma$.

Note that b_t is an increasing function of x_t . *That is : if $x_t < x_{t+1}$ then $b_t < b_{t+1}$* back

Proof proposition 3

$$\begin{aligned}
 \lambda_t &= \beta R_t \lambda_{t+1} + \mu_t \\
 b_t \lambda_t^R &= \beta R_t b_{t+1} \lambda_{t+1}^R + \mu_t \\
 b_{t+1} \lambda_t^R &> \beta R_t b_{t+1} \lambda_{t+1}^R + \mu_t \\
 b_{t+1} \left(\lambda_t^R - \beta R_t b_{t+1} \lambda_{t+1}^R \right) &> \mu_t \\
 b_{t+1} \mu_t^R &> \mu_t \\
 \frac{\mu_t^R}{\lambda_{t+1}^R} &> \frac{\mu_t}{\lambda_{t+1}}
 \end{aligned}$$

So we have $\frac{\mu_t}{\lambda_{t+1}} < \frac{\mu_t^R}{\lambda_{t+1}^R}$

$$\begin{aligned}
 \lambda_t &= \beta R_t \lambda_{t+1} + \mu_t \\
 b_t \lambda_t^R &= \beta R_t b_{t+1} \lambda_{t+1}^R + \mu_t \\
 b_t \lambda_t^R &> \beta R_t b_t \lambda_{t+1}^R + \mu_t \\
 b_t \left(\lambda_t^R - \beta R_t b_t \lambda_{t+1}^R \right) &> \mu_t \\
 b_t \mu_t^R &> \mu_t \\
 \frac{\mu_t^R}{\lambda_t^R} &> \frac{\mu_t}{\lambda_t}
 \end{aligned}$$

So we have $\frac{\mu_t}{\lambda_{t+1}} < \frac{\mu_t^R}{\lambda_{t+1}^R}$ and $\frac{\mu_t}{\lambda_t} < \frac{\mu_t^R}{\lambda_t^R}$

Proof proposition 3

Inspecting the following equations we can conclude that because $\frac{\mu_t}{\lambda_{t+1}} < \frac{\mu_t^R}{\lambda_{t+1}^R}$ and $\frac{\mu_t}{\lambda_t} < \frac{\mu_t^R}{\lambda_t^R}$, we have lower amplification effect than the representative agent case. Note that when the collateral constraint is not bind ($\mu_t = 0$) and there is no amplification effect.

$$E_t \left[R_{t+1}^q - R_t \right] = \frac{(1-k)\mu_t}{E_t [\beta \lambda_{t+1}]} - \frac{\text{Cov}(\lambda_{t+1}, R_{t+1}^q)}{E_t [\lambda_{t+1}]}$$

$$F_l(k_t, L_t, v_t) = \left(1 + \phi r_t + \phi R_t \frac{\mu_t}{\lambda_t} \right) w_t$$

$$F_v(k_t, L_t, v_t) = \left(1 + \phi r_t + \phi R_t \frac{\mu_t}{\lambda_t} \right) p_t^v$$

$$q_t = \mathbb{E} \left[\sum_{i=0}^{\infty} \left(\prod_{j=0}^i \frac{1}{\mathbb{E} \left[R_{t+1+j}^q \right]} \right) d_{t+1+i} \right]$$

Second part: The Proof is similar to the first part.[back](#)

labor decline vs aggregate decline

Note that $C_{2t} = L_t^\omega$. Let $x_t = \frac{C_{2t}}{C_{1t}}$.

$$\begin{aligned} C_t &= \lambda C_{2t} + (1 - \lambda) C_{1t} \\ &= \lambda L_t^\omega + (1 - \lambda) \frac{1}{x_t} L_t^\omega \end{aligned}$$

$$C_t = \left[\lambda + (1 - \lambda) \frac{1}{x_t} \right] L_t^\omega$$

$$\log(C_t) = \log \left[\lambda + (1 - \lambda) \frac{1}{x_t} \right] + \omega \log(L_t)$$

if x_t constant then $\frac{\partial \log(C_t)}{\partial \log(L_t)} = \omega$ [back 1](#)

On average the decline in asset price index is higher for less developed market: FIA

equity price growth	(1) Aggregate	(2) Advanced Market	(3) Emerging Market
SS	-0.474*** (0.118)	-0.159 (0.195)	-0.581*** (0.128)
FIA	-0.200 (0.143)	-0.0453 (0.122)	-0.443* (0.208)
FIA.SS	0.467** (0.217)	-0.0511 (0.321)	1.325*** (0.422)
Observations	631	366	265
R-squared	0.355	0.529	0.426
Number of id	29	15	14
Country FE	YES	YES	YES
Year FE	YES	YES	YES

*** p<0.01, ** p<0.05, * p<0.1

SS is a dummy variable which takes 1 if a country is in a sudden stop for a given year. Data on sudden stops are from Korinek and Mendoza (2014). **FIA is the Financial Institution Access index from IMF**. FIA.SS is a cross product of FIA and SS. co We control for capital flow. The data covers 1980-2012. We drop the sudden stops events with an increase in equity price