

## 6.14 Phase Margin

Consider the transfer function

$$G(s) = \frac{1}{(s-1)(s^2+2s+5)}$$

Assume standard feedback configuration with  $K$  as the controller. What is the largest possible phase margin that can be achieved for this system? Determine the gain  $K$  for which it is achieved. You can use the following bode plot to answer the question.

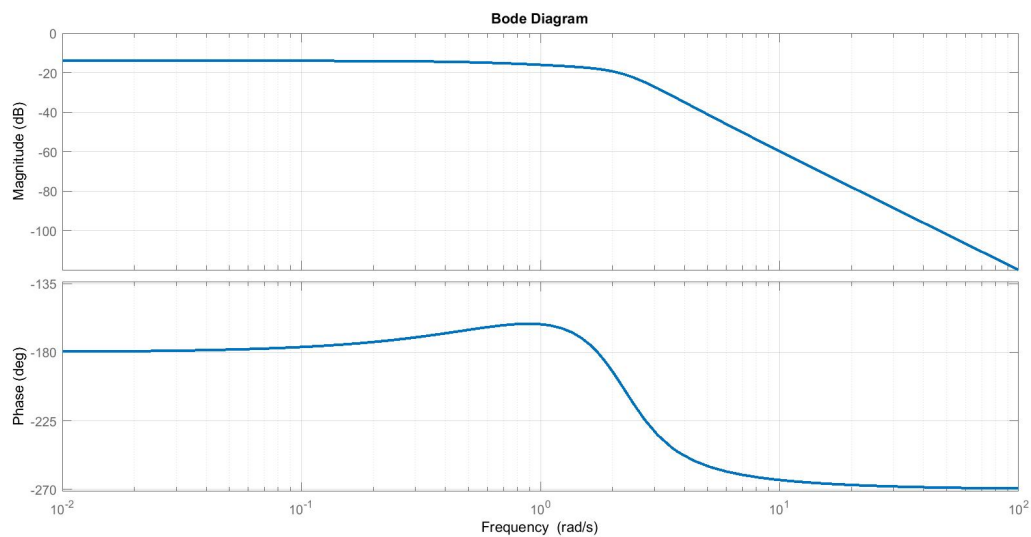


Figure 10: Bode plots for the system

### Solution :

According to Bode plot given below, the maximum phase (and PM in this case) achieved on  $\omega \approx 1$  (rad/sec), so we need to choose  $K$  in such a way that this is also equal to  $\omega_c$ . The result would be therefore be  $K \approx 6.3$ .

### GPT Answer :

The largest possible phase margin that can be achieved for the given system is approximately  $88.28^\circ$ . This phase margin is achieved at a frequency of 100 rad/s. The gain  $K$  for which this phase margin is achieved is approximately 999,750.06.

**Confidence Level: 90%**

**Misreading The Plot**

**Claude Answer :**

Upon further examination of the Bode plot, I have the following observations and conclusions:

1. The phase starts at  $0^\circ$  at low frequencies and decreases to around  $-270^\circ$  at high frequencies, indicating the presence of three poles (each contributing  $-90^\circ$  phase shift).
2. To achieve the maximum phase margin, we need the gain crossover frequency (where the magnitude plot crosses 0 dB) to occur at the frequency where the phase is closest to  $-180^\circ$ .
3. The phase is closest to  $-180^\circ$  at the frequency of approximately 2.5 rad/s, where it is about  $-150^\circ$ .
4. At 2.5 rad/s, the magnitude is roughly  $-20$  dB. To make the magnitude 0 dB at this frequency, we need to add a gain of  $+20$  dB, which corresponds to a factor of  $10^{20/20} = 10$ .
5. With this gain, the phase margin would be  $-150^\circ - (-180^\circ) = 30^\circ$ .

Therefore, the maximum achievable phase margin is approximately  $30^\circ$ , achieved with a gain  $K \approx 10$ .

**Confidence level: 98%**

**Misreading The Plot**