

### 3.16 Zeros or Poles

Let  $y(t)$  be the response of a stable, LTI system  $G(s)$  due to a sinusoidal input  $u(t) = \sin(2t)$ . If  $y(t)$  converges back to zero in steady-state, then what can we say about the zeros or poles of  $G(s)$ ?

**Solution :**

$$y(t) = |G(2j)|\sin(2t + \angle G(2j)) = 0 \rightarrow |G(2j)| = 0$$

Which we can conclude that  $G(s)$  has a zero at  $2j$ .

**GPT Answer:**

Given that  $y(t)$  converges back to zero in steady-state in response to a sinusoidal input  $u(t) = \sin(2t)$  for a stable, Linear Time-Invariant (LTI) system  $G(s)$ , we can infer the following about the system's zeros or poles:

For a system to be stable and LTI, the poles of  $G(s)$  must be in the left half of the complex plane. The input  $u(t) = \sin(2t)$  has a Laplace transform of  $\frac{2}{s^2+4}$ , indicating a frequency component at  $s = \pm j2$ .

If  $y(t)$  converges to zero in steady-state, it implies that the system does not have a resonance at the frequency of the input signal (2 rad/s). This could mean two things regarding the system's transfer function  $G(s)$ :

1. **\*\*Poles:\*\*** The system does not have a pole at  $s = \pm j2$ , since having a pole there would imply an infinite or sustained response at the input frequency, not convergence to zero.
2. **\*\*Zeros:\*\*** The system could have a zero at  $s = \pm j2$ . A zero at the input signal's frequency would attenuate the response at this frequency, leading to a convergence to zero in the steady-state output.

Hence, we can say that  $G(s)$  does not have a pole at  $s = \pm j2$ , and it might have a zero at  $s = \pm j2$  to ensure that the steady-state response to the sinusoidal input converges back to zero.

**Confidence level: 85%**

**Reasoning Error:** Since GPT says that the system "might" have a zero at  $s = \pm 2j$ , whereas the system **\*must\*** have a zero there given the info in the problem statement.

**Claude Answer:**

To answer this question, let's think about the properties of the system and the input signal:

1. The system  $G(s)$  is stable and LTI (Linear Time-Invariant).
2. The input signal is a sinusoid:  $u(t) = \sin(2t)$ .
3. In steady-state, the output  $y(t)$  converges back to zero.

Now, let's analyze:

1. For a stable system, all poles of  $G(s)$  must lie in the left half of the complex plane (i.e., they must have negative real parts).
2. The input signal  $u(t) = \sin(2t)$  has a frequency of  $\omega = 2$  rad/s. In the Laplace domain, this sinusoid corresponds to  $\frac{2}{s^2+4}$  (ignoring the phase shift).
3. If the output  $y(t)$  converges to zero in steady-state, it means that the system  $G(s)$  has zero gain at the input frequency  $\omega = 2$  rad/s. This can only happen if  $G(s)$  has a zero at  $s = \pm j2$  (i.e., on the imaginary axis at  $\pm j2$ ).

Therefore, we can conclude that the system  $G(s)$  must have at least one zero at  $s = \pm j2$  to ensure that the output converges to zero in steady-state for the given sinusoidal input.

Note that we cannot say anything specific about the poles of  $G(s)$  other than they must lie in the left half-plane for stability. The location of the poles does not directly influence the steady-state response to a sinusoidal input, as long as the system is stable.

**Confidence level: 95%**

**Correct**