

2.16 Identifying Singular Points in ODEs

Find the singular points of the pair of equations

$$\frac{dx_1}{dt} = \sin x_2 \quad \text{and} \quad \frac{dx_2}{dt} = -x_1 + x_2$$

Solution :

Singular points are found by setting $\frac{dx_1}{dt} = 0$ and $\frac{dx_2}{dt} = 0$. The first equation is satisfied by

$$x_2 = n\pi, \quad n = 0, 1, 2, \dots$$

The second is satisfied by $x_1 = x_2$. Hence the singular points are defined by

$$x_1 = n\pi, \quad x_2 = n\pi,$$

where $n = 0, 1, 2, \dots$

GPT Answer:

To find the singular points of the system of differential equations, we set both derivatives equal to zero. That is, we solve the system:

$$\begin{aligned}\sin x_2 &= 0 \\ -x_1 + x_2 &= 0\end{aligned}$$

From the first equation, $\sin x_2 = 0$, we find that $x_2 = n\pi$, where n is an integer.

Substituting $x_2 = n\pi$ into the second equation, $-x_1 + x_2 = 0$, we get $-x_1 + n\pi = 0$, which leads to $x_1 = n\pi$.

Therefore, the singular points are given by $(n\pi, n\pi)$, where n is an integer.

Confidence Level: 95%.

Correct

Claude Answer:

To find the singular points (also known as equilibrium points or fixed points) of the given system of differential equations, we need to set both equations equal to zero and solve for x_1 and x_2 . The singular points are the values of x_1 and x_2 where the system is at rest, meaning there is no change over time.

Given:

$$\begin{aligned}\frac{dx_1}{dt} &= \sin x_2 \\ \frac{dx_2}{dt} &= -x_1 + x_2\end{aligned}$$

Step 1: Set both equations equal to zero.

$$\begin{aligned}\sin x_2 &= 0 \\ -x_1 + x_2 &= 0\end{aligned}$$

Step 2: Solve the first equation for x_2 .

$$\begin{aligned}\sin x_2 &= 0 \\ x_2 &= \arcsin(0) \\ x_2 &= 0, \pi\end{aligned}$$

Step 3: Solve the second equation for x_1 in terms of x_2 .

$$\begin{aligned}-x_1 + x_2 &= 0 \\ x_1 &= x_2\end{aligned}$$

Step 4: Substitute the values of x_2 found in Step 2 into the equation for x_1 in Step 3 to find the corresponding values of x_1 .

For $x_2 = 0$:

$$\begin{aligned}x_1 &= x_2 \\ x_1 &= 0\end{aligned}$$

For $x_2 = \pi$:

$$\begin{aligned}x_1 &= x_2 \\ x_1 &= \pi\end{aligned}$$

Therefore, the singular points of the given system of differential equations are $(x_1, x_2) = (0, 0)$ and $(x_1, x_2) = (\pi, \pi)$.

Confidence Level: 95%

Correct