7.4 PI Controller

Consider the approximate model of a DC motor is given as:

$$G(s) = \frac{.5}{(.01s+1)(.1s+1)}$$

Design a PI controller such that it meets the following specifications:

$$\zeta \geq 0.6, \quad e_{ss}|_{\text{step}} \leq 0.01$$

. **Solution :**
We choose a PI controller, $K(s) = \frac{K(s+z_c)}{s}$, with unity-gain feedback $H(s) = 1$. The loop gain is

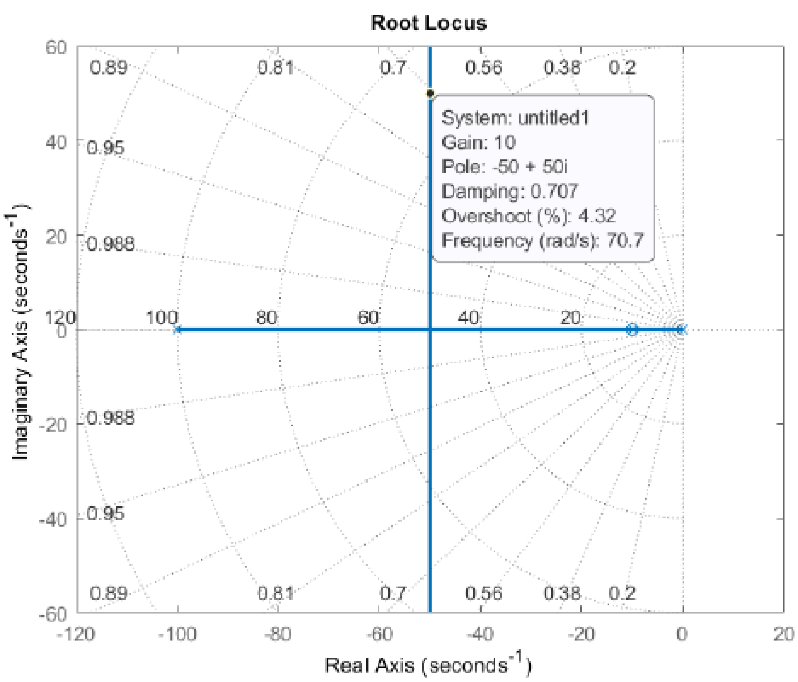

Figure 15: Root locus plot for the PI controller.

given as:

$$K(s)G(s) = \frac{.5K\,(s+z_c)}{s(.01s+1)(.1s+1)}$$

The position error constant is $K_p = \infty$. Hence $e_{ss}|_{\text{step}} = 0$. The controller zero can be selected to cancel one of the plant poles; hence $z_c = 10$. Next, from the Root Locus plot given in Fig. 15, we may choose, e.g., $K = 10$. The PI controller design is given as:

$$K(s) = \frac{10(s+10)}{s}$$

The closed-loop system transfer function is thus given by $T(s) = \frac{5000}{s^2+100s+5000}$.

GPT Answer :
To design a PI controller that meets the given specifications, an iterative tuning process is required, focusing on adjusting K_p and K_i to shape the closed-loop response for desired damping and steady-state error performance. The confidence level in this approach, pending further refinement through implementation and testing, is approximately 75%.

Confidence Level: 75%
Did not solve

Claude Answer :
To design a PI controller for the given DC motor model that meets the specified requirements, we can follow these steps:

The transfer function of a PI controller is given by:

$$C(s) = K_p + \frac{K_i}{s}$$

The closed-loop transfer function of the system with the PI controller is:

$$\frac{C(s)G(s)}{1 + C(s)G(s)} = \frac{0.5(K_p s + K_i)}{0.001s^3 + 0.11s^2 + (1 + 0.5K_p)s + 0.5K_i}$$

To achieve a damping ratio of $\zeta \geq 0.6$, we can compare the denominator of the closed-loop transfer function with the standard second-order characteristic equation:

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

where ω_n is the natural frequency. Comparing the coefficients, we get:

$$2\zeta\omega_n = \frac{0.11}{0.001} = 110$$

$$\omega_n^2 = \frac{1 + 0.5K_p}{0.001}$$

Solving for K_p with $\zeta = 0.6$:

$$K_p = \frac{2\zeta^2\omega_n^2 - 1}{0.5} = \frac{2(0.6^2)(110^2) - 1}{0.5} \approx 15671$$

To ensure a steady-state error $e_{ss} \leq 0.01$ for a step input, we can use the formula:

$$e_{ss} = \frac{1}{1 + K_v}$$

where K_v is the velocity error constant, which is equal to:

$$K_v = \lim_{s \to 0} sC(s)G(s) = \frac{0.5K_i}{1 + 0.5K_p}$$

Solving for K_i with $e_{ss} = 0.01$:

$$K_i = \frac{(1 + 0.5K_p)(1 - e_{ss})}{0.5e_{ss}} \approx \frac{(1 + 0.5(15671))(1 - 0.01)}{0.5(0.01)} \approx 156552$$

Therefore, the PI controller parameters are:

$K_p \approx 15671$ $K_i \approx 156552$

Confidence Level: 90%
Calculation Error