

7.7 Stability Analysis

Consider a feedback control system under unity feedback setting with a controller $K(s) = K$ and the system $G(s)$ given as follows:

$$G(s) = \frac{s(s+10)(s+12)}{(s-1)(s+11)(s+13)(s+14)}$$

Is there a non-negative value of K for which the closed loop system is stable? Explain why or why not?

Solution:

No, there is no non-negative value of K for which the closed loop system is stable. Check the root locus given below. The root locus branch between the RHP pole and the imaginary axis zero keeps the system unstable.

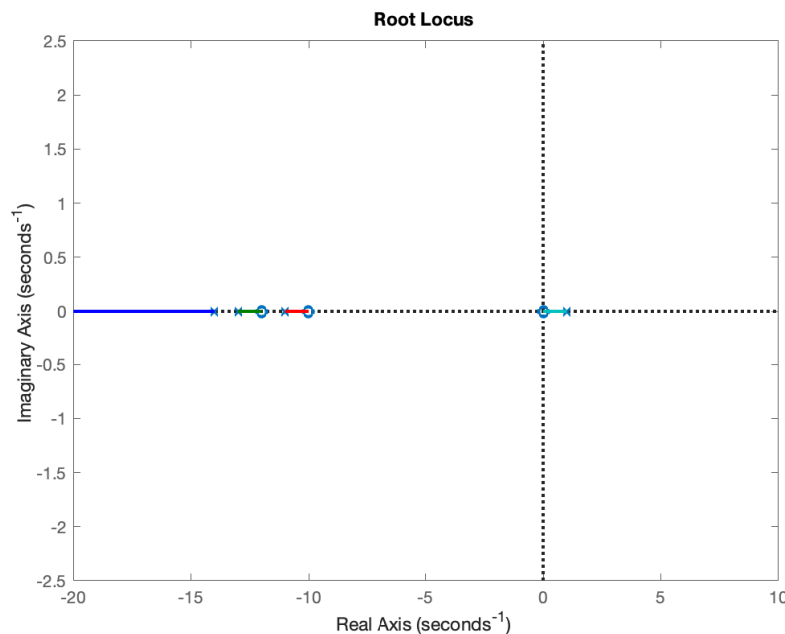


Figure 18: Root locus of $G(s)$

GPT Answer:

To analyze the stability of the closed-loop system under unity negative feedback with a proportional controller $K(s) = K$, we use the characteristic equation of the closed-loop transfer function. The closed-loop transfer function is given by:

$$T(s) = \frac{KG(s)}{1 + KG(s)}$$

Substituting $G(s)$ into this equation gives:

$$T(s) = \frac{K \frac{s(s+10)(s+12)}{(s-1)(s+11)(s+13)(s+14)}}{1 + K \frac{s(s+10)(s+12)}{(s-1)(s+11)(s+13)(s+14)}}$$

The characteristic equation is obtained by setting the denominator equal to zero:

$$1 + K \frac{s(s+10)(s+12)}{(s-1)(s+11)(s+13)(s+14)} = 0$$

Simplifying, we find the characteristic equation is:

$$(s-1)(s+11)(s+13)(s+14) + Ks(s+10)(s+12) = 0$$

To ensure the closed-loop system is stable, all roots of the characteristic equation must have negative real parts. The presence of a pole at $s = 1$ in the open-loop system indicates inherent instability, and no non-negative value of K can achieve stability in the closed-loop system.

Confidence Level: 95%

Correct

Claude Answer :

Let's approach this problem step by step:

1) For a closed loop system to be stable under unity feedback, all the poles of the closed loop transfer function must lie in the left half of the s-plane.

2) The closed loop transfer function is given by:

$$\frac{Y(s)}{R(s)} = \frac{G(s)K(s)}{1+G(s)K(s)}$$

3) Substituting the given $G(s)$ and $K(s) = K$, we get:

$$\frac{Y(s)}{R(s)} = \frac{Ks(s+10)(s+12)}{(s-1)(s+11)(s+13)(s+14)+Ks(s+10)(s+12)}$$

4) The characteristic equation of the system is the denominator of the closed loop transfer function equated to zero:

$$(s-1)(s+11)(s+13)(s+14) + Ks(s+10)(s+12) = 0$$

5) For stability, the roots of the characteristic equation (i.e., the poles of the closed loop system) must all have negative real parts.

6) In this case, we see that the characteristic equation has a term $(s-1)$, which means that one of

the poles is at $s = 1$ regardless of the value of K . This pole is in the right half of the s-plane.

7) Since there is a pole in the right half plane for any value of K , there is no non-negative value of K for which the closed loop system is stable.

Therefore, there is no non-negative value of K that can make this closed loop system stable. This is because the open loop system $G(s)$ itself is unstable due to the pole at $s = 1$, and no amount of proportional control can stabilize an inherently unstable system.

Confidence Level: 95%

Correct