

## 2 Stability

### 2.1 Stability Analysis

In a feedback control loop, the open-loop transfer function  $G(s)$  and the controller  $C(s)$  are given by

$$G(s) = \frac{s-2}{(s-1)(s+4)}, \quad C(s) = K \frac{s+1}{s} \quad (66)$$

Determine  $K \in \mathbb{R}$ , if exists, such that the control loop is stable.

**Solution :**

The closed-loop characteristic polynomial is given by the numerator of  $1 + G(s)C(s)$ , i.e.,

$$p(s) = s^3 + (3+K)s^2 - (K+4)s - 2K \quad (67)$$

On applying the Routh criterion, we observe that  $p(s)$  has all its roots in the open Left Half Plane (LHP) if and only if the following three conditions are simultaneously satisfied:

$$3 + K > 0 \quad (68)$$

$$K^2 + 5K + 12 < 0 \quad (69)$$

$$-2K > 0 \quad (70)$$

The first and third conditions require that  $-3 < K < 0$ . On the other hand, the second condition can be re-written as  $(K + 2.5)^2 + 5.75 < 0$ . This form allows us to see that the condition cannot be met by any real value of  $K$ . In summary, there is no real value for  $K$  which stabilizes the closed loop.

**GPT Answer :**

The characteristic equation, after simplification, is given by

$$\frac{K(s-2)(s+1) + s(s-1)(s+4)}{s(s-1)(s+4)}$$

To determine stability, we look for values of  $K$  that ensure the roots of the numerator have negative real parts. The detailed stability analysis typically involves the Routh-Hurwitz criterion or specific sign analysis of the coefficients, which is not directly conducted here. Therefore, the exact value of  $K$  that ensures stability cannot be specified without further analysis.

**Confidence level: 50%**

**Calculation Complexity**

**Claude Answer :**

I apologize for the confusion in my previous response. Let me revise the solution and impose condition 2 more carefully.

Step 1 and Step 2 remain the same as in the previous solution.

Step 3: Apply the Routh-Hurwitz stability criterion. The characteristic equation is:  $s^3 + (3 + K)s^2 + (K - 4)s - 2K = 0$  The Routh-Hurwitz table is:

$s^3$	1	$K - 4$
$s^2$	$3 + K$	$-2K$
$s^1$	$\frac{(3+K)(K-4)+2K}{3+K}$	0
$s^0$	$-2K$	0

For the system to be stable, all elements in the first column of the Routh-Hurwitz table must be positive. This leads to the following conditions:

1.  $3 + K > 0$ , which implies  $K > -3$
2.  $\frac{(3+K)(K-4)+2K}{3+K} > 0$ , which simplifies to  $\frac{K^2-K-6}{3+K} > 0$
3.  $-2K > 0$ , which implies  $K < 0$

Let's focus on condition 2:  $\frac{K^2-K-6}{3+K} > 0$

For this fraction to be positive, the numerator and denominator must have the same sign (either both positive or both negative).

Case 1: If  $3 + K > 0$ , then  $K > -3$ , and we need  $K^2 - K - 6 > 0$ . Factoring the quadratic inequality:  $(K + 2)(K - 3) > 0$  This inequality is satisfied when  $K < -2$  or  $K > 3$ . However,  $K < -2$  contradicts the condition  $K > -3$ , so we only have  $K > 3$ .

Case 2: If  $3 + K < 0$ , then  $K < -3$ , and we need  $K^2 - K - 6 < 0$ . Factoring the quadratic inequality:  $(K + 2)(K - 3) < 0$  This inequality is satisfied when  $-2 < K < 3$ . However,  $K < -3$  contradicts this condition, so this case is not possible.

Combining the conditions from Case 1 with condition 3 ( $K < 0$ ), we find that there is no value of  $K$  that satisfies all the conditions for stability.

Therefore, there is no real value of  $K$  that makes the control loop stable.

**Confidence level: 95%**

**Correct**