

## 5.4 Pole Placement

Consider a nominal model given by:

$$G_o(s) = \frac{3s + 1}{(s + 2)(s - 3)} \quad (179)$$

The goal is to design a control law that tracks a constant reference and cancels the pole at  $s = -2$  in  $G_o(s)$ . Design a suitable controller. If pole placement is required for the characteristic equation, start with a pole at  $s = -2$  and for any additional poles required, increase their magnitude sequentially (e.g., for three required poles, assign them at  $-2$ ,  $-3$ , and  $-4$ ; and so forth). Provide the transfer function of the designed controller in the format: " $C(s) = \frac{\text{Numerator}}{\text{Denominator}}$ ", where Numerator and Denominator are fully expanded polynomials.

### Solution :

The minimum degree of  $A_{cl}(s)$  is four since we need to force integration in the controller, then

$$C(s) = \frac{p_2 s^2 + p_1 s + p_0}{s(s + \lambda_1)} \quad (180)$$

Since  $(s + 2)$  is cancelled if and only if  $(s + 2)$  is a factor of  $A_{cl}(s)$ , we choose

$$A_{cl}(s) = (s + 2)(s + 3)(s + 4)(s + 5) \quad (181)$$

The pole assignment equation then becomes

$$A_o(s)L(s) + B_o(s)P(s) = (s + 2)(s + 3)(s + 4)(s + 5) \quad (182)$$

$$(s + 2)(s - 3)(s + \lambda_1)s + (3s + 1)(p_2 s^2 + p_1 s + p_0) = (s + 2)(s + 3)(s + 4)(s + 5) \quad (183)$$

In this polynomial identity we note that  $(s + 2)$  has to be a factor of  $P(s)$  (a plant pole can only be cancelled by a controller zero). Thus we define

$$(s + 2)(\tilde{p}_1 s + \tilde{p}_0) = p_2 s^2 + p_1 s + p_0 \quad (184)$$

The pole assignment equation then simplifies as follows:

$$(s - 3)(s + \lambda_1)s + (3s + 1)(\tilde{p}_1 s + \tilde{p}_0) = (s + 3)(s + 4)(s + 5) \quad (185)$$

$$s^3 + (3\tilde{p}_1 + \lambda_1 - 3)s^2 + (\tilde{p}_1 + 3\tilde{p}_0 - 3\lambda_1)s + \tilde{p}_0 = s^3 + 12s^2 + 47s + 60 \quad (186)$$

This leads to equations

$$3\tilde{p}_1 + \lambda_1 - 3 = 12 \quad (187)$$

$$\tilde{p}_1 + 3\tilde{p}_0 - 3\lambda_1 = 47 \quad (188)$$

$$\tilde{p}_0 = 60 \quad (189)$$

The solution is  $\tilde{p}_0 = 60$ ,  $\tilde{p}_1 = -\frac{44}{5}$  and  $\lambda_1 = \frac{207}{5}$ . Finally the controller is

$$C(s) = \frac{(s + 2)(-44s + 300)}{s(5s + 207)} \quad (190)$$

**GPT Answer :**

To design a controller  $C(s)$  for the nominal model  $G_o(s) = \frac{3s+1}{(s+2)(s-3)}$ , with the objectives of tracking a constant reference and canceling the pole at  $s = -2$ , a basic approach involves incorporating a zero in the controller at  $s = -2$  to cancel the pole in  $G_o(s)$  and ensuring a pole at the origin for PI action. A suggested controller form, without specific tuning, is:

$$C(s) = \frac{K(s+2)}{s(s+3)}$$

where  $K$  is a gain that can be adjusted to meet additional design specifications.

**Confidence Level: 75%**

**Reasoning Error**

**Claude Answer :**

Consider a controller with a zero and two poles:

$$C(s) = \frac{K(s+2)(s+a)}{s(s+b)}$$

The closed-loop transfer function is:

$$\frac{C(s)G_o(s)}{1 + C(s)G_o(s)} = \frac{\frac{K(s+2)(s+a)}{s(s+b)} \cdot \frac{3s+1}{(s+2)(s-3)}}{1 + \frac{K(s+2)(s+a)}{s(s+b)} \cdot \frac{3s+1}{(s+2)(s-3)}}$$

The closed-loop characteristic equation becomes:

$$s(s+b)(s-3) + K(s+2)(s+a)(3s+1) = 0$$

$$s^3 + (b+a-3)s^2 + (ab-3b+6K+2Ka)s + (2Kab-6K) = 0$$

Comparing with the desired characteristic equation  $(s+2)(s+3)(s+4) = s^3 + 9s^2 + 26s + 24$ :

$$b+a-3=9 \implies a=12-b$$

$$ab-3b+6K+2Ka=26 \implies 12b-b^2-3b+6K+2K(12-b)=26$$

$$2Kab-6K=24$$

Choose  $b=3$  to simplify the equations. Then,  $a=9$  and:

$$6K+2K(12-3)=26 \implies K=1$$

Substituting the values of  $K$ ,  $a$ , and  $b$ , the controller transfer function is:

$$C(s) = \frac{(s+2)(s+9)}{s(s+3)} = \frac{s^2 + 11s + 18}{s^2 + 3s}$$

(191)

**Confidence score: 95/100**

**Calculation Error**