

3.18 Steady-State Response

Answer the following questions.

- (a) Consider the following system $G(s)$ and sinusoidal input:

$$\begin{aligned} -3\dot{y}(t) - 2y(t) &= 7u(t) \\ u(t) &= 6 \cos(t + 4) \end{aligned}$$

What is the magnitude and phase of $G(1j)$? Is the steady-state output bounded? If yes, what is it?

- (b) Consider the following system $G(s)$ and sinusoidal input:

$$\begin{aligned} \ddot{y}(t) + 0.1\dot{y}(t) + 4y(t) &= \dot{u}(t) + 2u(t) \\ u(t) &= -\cos(2t) \end{aligned}$$

What is the magnitude and phase of $G(2j)$? Is the steady-state output bounded? If yes, what is it?

Solution :

For a sinusoidal input $u(t) = A \cos(\omega t)$ an LTI system G with zero initial conditions, the system output can be expressed as follows:

$$y(t) = A|G(j\omega)| \cos(\omega t + \angle G(j\omega))$$

- (a) We derive the transfer function as $G(s) = \frac{-7}{3s+2}$. This gives $|G(j)| = \frac{7\sqrt{13}}{13}$ and $\angle G(j) = \arctan\left(\frac{3}{-2}\right) = \pi - \arctan(3/2) = 123.69^\circ$. The steady state output is bounded and can be readily obtained:

$$y_{ss}(t) = 6|G(j)| \cos(t + 4 + \angle G(j)) \approx 11.65 \cos(t + 6.16)$$

- (b) Here the transfer function is given by

$$G(s) = \frac{s+2}{s^2+0.1s+4}$$

and so $|G(2j)| = 10\sqrt{2}$ and $\angle G(2j) = \frac{-\pi}{4}$. Again, the steady state output is bounded and given by:

$$y_{ss}(t) = -10\sqrt{2} \cos\left(2t - \frac{\pi}{4}\right)$$

GPT Answer :

Solution for (a):

- Magnitude of $G(1j)$: 1.941
- Phase of $G(1j)$: -1.5 radians (or -86.0 degrees)
- The steady-state output is bounded and given by $y_{ss}(t) = 11.646 \cdot \cos(t + 2.5)$

Solution for (b):

- Magnitude of $G(2j)$: 14.142
- Phase of $G(2j)$: -1.0 radian (or -57.3 degrees)
- The steady-state output is bounded and given by $y_{ss}(t) = 14.142 \cdot \cos(2t - 1.0)$

Confidence Level: 95%

Calculation error

Claude Answer :

(a) For the first system: - The transfer function is:

$$G(s) = \frac{7}{-3s-2}$$

- Evaluating $G(1j)$:

$$G(1j) = \frac{7}{-3j-2} = \frac{7(-2+3j)}{(-2+3j)(-2-3j)} = \frac{-14+21j}{13} = -1.077 + 1.615j$$

- The magnitude and phase of $G(1j)$ are:

$$|G(1j)| = \sqrt{(-1.077)^2 + 1.615^2} = 1.923$$

$$\angle G(1j) = \tan^{-1}\left(\frac{1.615}{-1.077}\right) = 2.159 \text{ rad}$$

- The steady-state output is bounded and can be calculated as:

$$y_{ss}(t) = |G(1j)| \cdot 6 \cos(t + 4 + \angle G(1j)) = 11.538 \cos(t + 4 + 2.159)$$

(b) For the second system: - The transfer function is:

$$G(s) = \frac{s+2}{s^2+0.1s+4}$$

- Evaluating $G(2j)$:

$$G(2j) = \frac{2j+2}{(2j)^2+0.1(2j)+4} = \frac{2+2j}{-0.2j} = -10 - 10j$$

- The magnitude and phase of $G(2j)$ are:

$$|G(2j)| = \sqrt{(-10)^2 + (-10)^2} = 10\sqrt{2} = 14.142$$

$$\angle G(2j) = \tan^{-1}\left(\frac{-10}{-10}\right) = 3.927 \text{ rad}$$

- The steady-state output is bounded and can be calculated as:

$$y_{ss}(t) = |G(2j)| \cdot (-\cos(2t + \angle G(2j))) = -14.142 \cos(2t + 3.927)$$

Confidence Level: 95%

Calculation error