

5.8 Proportional Controller

Consider the following plant:

$$2\dot{y}(t) + 3y(t) = -4u(t) + d(t) \quad (207)$$

- What is the model from inputs (r, d) to output y if we use an proportional controller $u(t) = K_p(r(t) - y(t))$?
- Select K_p so that the steady-state error $\bar{e} = \bar{r} - \bar{y}$ is less than 0.1 when $r(t) = \bar{r} = 2$ and $d(t) = \bar{d} = 1$.
- What is the time constant of the closed-loop?

Solution :

$$(a) \quad 2\dot{y}(t) + (3 - 4K_p)y(t) = -4K_p r(t) + d(t)$$

- The closed-loop is stable if and only if $K_p < \frac{3}{4}$. Also, in steady-state we have:

$$\bar{e} = \bar{r} - \bar{y} = \left(1 + \frac{4K_p}{3 - 4K_p}\right) \bar{r} - \frac{1}{3 - 4K_p} \bar{d} \quad (208)$$

$$= \left(1 + \frac{4K_p}{3 - 4K_p}\right) 2 - \frac{1}{3 - 4K_p} 1 \quad (209)$$

$$= 2 + \frac{8K_p - 1}{3 - 4K_p} < 0.1 \quad (210)$$

Thus,

$$K_p < -11.75 \quad (211)$$

$$(c) \quad \tau = \frac{2}{3 - 4K_p}$$

GPT Answer:

Part (a) - Model: $2\dot{y}(t) + (3 + 4K_p)y(t) = 4K_p r(t) + d(t)$

Part (b) - K_p for $\bar{e} < 0.1$: $K_p = 11.75$

Part (c) - Time constant: Conceptual, based on system coefficients.

Confidence Level: 95% for (a) and (b), 90% for (c).

Calculation Error

Claude Answer:

- The closed-loop transfer function from (r, d) to y is:

$$\frac{Y(s)}{R(s)} = \frac{-4K_p}{2s + 3 + 4K_p}, \quad \frac{Y(s)}{D(s)} = \frac{1}{2s + 3 + 4K_p} \quad (212)$$

(b) For a steady-state error less than 0.1 with $\bar{r} = 2$ and $\bar{d} = 1$:

$$\bar{e} = \bar{r} - \bar{y} = \bar{r} - \left(\frac{-4K_p}{3 + 4K_p} \bar{r} + \frac{1}{3 + 4K_p} \bar{d} \right) < 0.1 \quad (213)$$

Solving for K_p gives $K_p > 6.75$. Choosing $K_p = 7$ satisfies the requirement.

(c) The time constant of the closed-loop system with $K_p = 7$ is:

$$\tau = \frac{2}{3 + 4K_p} = \frac{2}{31} \approx 0.065 \text{ seconds} \quad (214)$$

Confidence Score: 95/100

Calculation Error