

3.19 Unbounded Response

Consider the transfer function

$$G(s) = \frac{1}{s^2 + \omega_n^2} \quad \omega_n > 0$$

and the input $u(t) = \sin(\omega t)$, $\omega > 0$. What value of ω in $u(t)$ leads to an unbounded response and why it does so?

Solution :

$$G(s) = \frac{1}{s^2 + \omega_n^2} \implies G(j\omega) = \frac{1}{(j\omega)^2 + \omega_n^2} = \frac{1}{\omega_n^2 - \omega^2}$$

$$\implies |G(j\omega)| = \left| \frac{1}{\omega_n^2 - \omega^2} \right| \rightarrow \infty \text{ when } \omega \rightarrow \omega_n.$$

GPT Answer :

To determine the value of ω that leads to an unbounded response, we look at the characteristics of the transfer function and the input. An unbounded response, or resonance, occurs when the input frequency matches the system's natural frequency, causing the system to oscillate at an increasing amplitude. The natural frequency of the system is given by ω_n . For the input $u(t) = \sin(\omega t)$, the frequency of the input signal is ω . When this frequency matches the system's natural frequency ω_n , the denominator of the transfer function $s^2 + \omega_n^2$ becomes zero at $s = j\omega_n$, leading to resonance. Thus, the value of ω that leads to an unbounded response is $\omega = \omega_n$. This causes the denominator of the transfer function to approach zero, and the system's response to the input signal becomes unbounded, indicating a resonance condition.

Confidence Level: 95%

Correct

Claude Answer :

To determine the value of ω in the input $u(t) = \sin(\omega t)$ that leads to an unbounded response, we need to analyze the system's behavior in the frequency domain.

Given: - Transfer function: $G(s) = \frac{1}{s^2 + \omega_n^2}$, where $\omega_n > 0$ - Input: $u(t) = \sin(\omega t)$, where $\omega > 0$

Step 1: Analyze the transfer function in the frequency domain. The transfer function has poles at $s = \pm j\omega_n$. These poles lie on the imaginary axis, which means the system is marginally stable.

Step 2: Consider the input frequency ω . When the input frequency ω is equal to the natural frequency ω_n of the system, the system experiences resonance. At resonance, the system's response is amplified, and the amplitude of the output becomes unbounded.

Therefore, when $\omega = \omega_n$, the input frequency matches the system's natural frequency, leading to an unbounded response.

Explanation: The unbounded response occurs because the input frequency excites the system at its natural frequency, causing resonance. At resonance, the system continuously absorbs energy from the input, and the output amplitude grows without bound. In reality, physical systems have damping or nonlinearities that limit the amplitude, but in this ideal case, the response becomes unbounded.

In summary, when the input frequency ω is equal to the system's natural frequency ω_n , the response becomes unbounded due to resonance.

Confidence Level: 95%

Correct