

## 1.21 Dominant Pole Approximation

For each system:

- Construct a first-order or second-order approximation from the dominant pole
- Do you expect the dominant pole approximation to be accurate?

$$(a) \quad G_A(s) = \frac{5000}{(s+2)(s+20)(s^2+20s+500)}$$

$$(b) \quad G_B(s) = \frac{24}{(s+1)(s+2)^2(s+3)}$$

$$(c) \quad G_C(s) = \frac{15}{(s+1)^2(s+10)}$$

**Solution :**

(a)

- Poles are  $s_1 = -2$ ,  $s_2 = -20$ ,  $s_{3,4} = -10 \pm 20j$ . The dominant pole is  $s_1$ .

**DC Gain:**  $G(0) = \frac{5000}{2 \times 20 \times 500} = \frac{1}{4}$

**First-order approximation:**  $\hat{G}(s) = \frac{0.5}{s+2}$

- Accurate, since  $s_1 = -2$  is much slower than  $s_2$  and  $s_{3,4}$

(b)

- Poles are  $s_1 = -1$ ,  $s_{2,3} = -2$ ,  $s_4 = -3$ . The dominant pole is  $s_1$ .

**DC Gain:**  $G(0) = \frac{24}{1 \times 2^2 \times 3} = 2$

**First-order approximation:**  $\hat{G}(s) = \frac{2}{s+1}$

- Not that accurate, since all poles are of similar time scale.

(c)

- Poles are  $s_{1,2} = -1$ ,  $s_3 = -10$ . The dominant poles are  $s_{1,2}$ .

**DC Gain:**  $G(0) = \frac{15}{1^2 \times 10} = 1.5$

**Second-order approximation:**  $\hat{G}(s) = \frac{1.5}{(s+1)^2}$

- Accurate, since  $s_{1,2}$  are much slower than  $s_3$

**GPT Answer :**

For each system:

- (a) For  $G_A(s) = \frac{5000}{(s+2)(s+20)(s^2+20s+500)}$ , the dominant pole approximation is  $G_{A,approx}(s) = \frac{5000}{s+2}$ . The approximation should be relatively accurate.

- (b) For  $G_B(s) = \frac{24}{(s+1)(s+2)^2(s+3)}$ , the dominant pole approximation is  $G_{B,approx}(s) = \frac{24}{(s+2)^2}$ . The approximation is expected to be decent.
- (c) For  $G_C(s) = \frac{15}{(s+1)^2(s+10)}$ , the dominant pole approximation is  $G_{C,approx}(s) = \frac{15}{(s+1)^2}$ . This approximation is likely to be quite accurate.

**Confidence Level: Approximately 85%**

**Calculation Error for part (a) and (c). Reasoning Error for part (b)**

**Claude Answer::**

(a)  $G_A(s) = \frac{5000}{(s+2)(s+20)(s^2+20s+500)}$

First-order approximation:  $G_{A,approx}(s) \approx \frac{5000}{2(s+2)} = \frac{2500}{s+2}$

Accuracy expectation: The dominant pole approximation may not be very accurate, as the poles at  $s = -2$  and  $s = -10 \pm 10i$  are relatively close to each other. The second-order term ( $s^2 + 20s + 500$ ) will have a significant impact on the system response.

Confidence: 60/100

(b)  $G_B(s) = \frac{2}{(s+1)(0.5s+1)^2(s/3+1)}$

First-order approximation:  $G_{B,approx}(s) \approx \frac{2}{0.5^2(s+1)} = \frac{8}{s+1}$

Accuracy expectation: The dominant pole approximation should be reasonably accurate, as the pole at  $s = -1$  is closer to the origin than the other poles. However, the repeated pole at  $s = -2$  may have some impact on the system response.

Confidence: 75/100

(c)  $G_C(s) = \frac{15}{(s+1)^2(s+10)}$

First-order approximation:  $G_{C,approx}(s) \approx \frac{15}{10(s+1)^2} = \frac{1.5}{(s+1)^2}$

Accuracy expectation: The dominant pole approximation is likely to be very accurate, as the double pole at  $s = -1$  is much closer to the origin than the pole at  $s = -10$ . The first-order approximation should suffice in this case.

**Confidence Level: 90/100**

**Calculation Error for part (a) and (b)**