

2.13 Routh-Hurwitz Stability

Determine the Hurwitz conditions for stability of the following general fourth-order characteristic equation, assuming a_4 is positive.

$$a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0 = 0 \quad (122)$$

Solution :

The Hurwitz determinants are:

$$\begin{aligned} \Delta_1 &= a_3, \\ \Delta_2 &= a_3a_2 - a_4a_1, \\ \Delta_3 &= \begin{vmatrix} a_3 & a_1 & 0 \\ a_4 & a_2 & a_0 \\ 0 & a_3 & a_1 \end{vmatrix}, \\ \Delta_4 &= \begin{vmatrix} a_3 & a_1 & 0 & 0 \\ a_4 & a_2 & a_0 & 0 \\ 0 & a_3 & a_1 & 0 \\ 0 & a_4 & a_2 & a_0 \end{vmatrix}. \end{aligned}$$

For the system to be stable:

$$\Delta_1 > 0, \quad \Delta_2 > 0, \quad \Delta_3 > 0, \quad \Delta_4 > 0$$

GPT Answer:

The Hurwitz criterion provides a necessary and sufficient condition for the stability of a linear time-invariant (LTI) system by examining its characteristic equation. For a fourth-order characteristic equation like the one given:

$$a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0 = 0$$

assuming $a_4 > 0$ for system stability, all the roots of the characteristic equation must have negative real parts. The Hurwitz conditions for this equation can be determined by constructing the Hurwitz matrix from the coefficients of the polynomial and then ensuring that all the leading principal minors of this matrix are positive.

The Hurwitz matrix for a fourth-order characteristic equation is:

$$\begin{pmatrix} a_3 & a_1 & 0 & 0 \\ a_4 & a_2 & a_0 & 0 \\ 0 & a_3 & a_1 & 0 \\ 0 & a_4 & a_2 & a_0 \end{pmatrix}$$

The Hurwitz conditions for stability are that all the coefficients a_i must be positive (which ensures that there are no sign changes in the sequence of coefficients, a necessary condition for stability) and that the determinants of the leading principal minors of the Hurwitz matrix (denoted as $\Delta_1, \Delta_2, \Delta_3, \Delta_4$) must all be positive. These minors are:

- $\Delta_1 = a_3$
- $\Delta_2 = a_3a_2 - a_4a_1$
- $\Delta_3 = (a_3a_2 - a_4a_1)a_1 - a_3^2a_0$
- $\Delta_4 = a_0\Delta_3$

Therefore, for the given characteristic equation to be stable, the Hurwitz conditions are:

1. $a_i > 0$ for all i (where $i = 0, 1, 2, 3, 4$)
2. $\Delta_1 > 0$
3. $\Delta_2 > 0$
4. $\Delta_3 > 0$
5. $\Delta_4 > 0$

These conditions ensure that all roots of the characteristic equation have negative real parts, indicating a stable system.

Confidence Level: 95%

Correct

Claude Answer:

To determine the Hurwitz conditions for stability of the given fourth-order characteristic equation, we need to construct the Hurwitz matrix and ensure that all its principal minors are positive. Assuming $a_4 > 0$, the Hurwitz matrix for a fourth-order system is:

$$H = \begin{bmatrix} a_3 & a_1 & 0 & 0 \\ a_4 & a_2 & a_0 & 0 \\ 0 & a_3 & a_1 & 0 \\ 0 & a_4 & a_2 & a_0 \end{bmatrix} \quad (123)$$

The principal minors of the Hurwitz matrix are:

$$\Delta_1 = a_3 \quad (124)$$

$$\Delta_2 = \begin{vmatrix} a_3 & a_1 \\ a_4 & a_2 \end{vmatrix} = a_3 a_2 - a_4 a_1 \quad (125)$$

$$\Delta_3 = \begin{vmatrix} a_3 & a_1 & 0 \\ a_4 & a_2 & a_0 \\ 0 & a_3 & a_1 \end{vmatrix} = a_3(a_2 a_1 - a_3 a_0) \quad (126)$$

$$\Delta_4 = \begin{vmatrix} a_3 & a_1 & 0 & 0 \\ a_4 & a_2 & a_0 & 0 \\ 0 & a_3 & a_1 & 0 \\ 0 & a_4 & a_2 & a_0 \end{vmatrix} = a_0(a_3 a_2 a_1 - a_4 a_1^2 - a_3^2 a_0) \quad (127)$$

For the system to be stable, all the principal minors must be positive. Therefore, the Hurwitz conditions for stability of the given fourth-order characteristic equation are:

1. $a_4 > 0$ (given)
2. $a_3 > 0$
3. $a_3 a_2 - a_4 a_1 > 0$
4. $a_3(a_2 a_1 - a_3 a_0) > 0$
5. $a_0(a_3 a_2 a_1 - a_4 a_1^2 - a_3^2 a_0) > 0$

If all these conditions are satisfied, the system represented by the given characteristic equation will be stable.

Confidence Level: 95%

Correct