

1.4 Superposition Principle

Consider a linear system with transfer function $G(s) = \frac{4}{s+2}$. Let u denote the input and y denote the output. The response of $G(s)$ with $u(t) = 2$ for $t \geq 0$, $u(t) = 0$ for $t < 0$, and zero initial conditions is $y(t) = 4(1 - e^{-2t})$.

- What is the response y_A from zero initial conditions if $u_A(t) = -1$ for $t \geq 0$? Provide your answer as an explicit function of $y(t)$.
- Analyze the response $y_B(t)$ from zero initial conditions for a piecewise input:

$$u_B(t) = \begin{cases} 2 & 0 \leq t < 10, \\ 4 & t \geq 10. \end{cases}$$

Express $y_B(t)$ as a mathematical expression in terms of $y(t)$, using the principle of superposition and time-invariance.

Solution :

- Note that $u_A = -\frac{1}{2}u$. By the principle of superposition (scaling), the response due to u_A is

$$y_A = -\frac{1}{2}y = -2(1 - e^{-2t}).$$

- Note that $u_B(t) = u(t) + u(t - 10)$ where $u(t - 10)$ is the original input shifted by 10 sec. By time-invariance, the response due to $u(t - 10)$ is given by $y(t - 10)$. By the principle of superposition (addition), the response due to u_B is given by $y_B(t) = y(t) + y(t - 10)$. This plot is shown below.

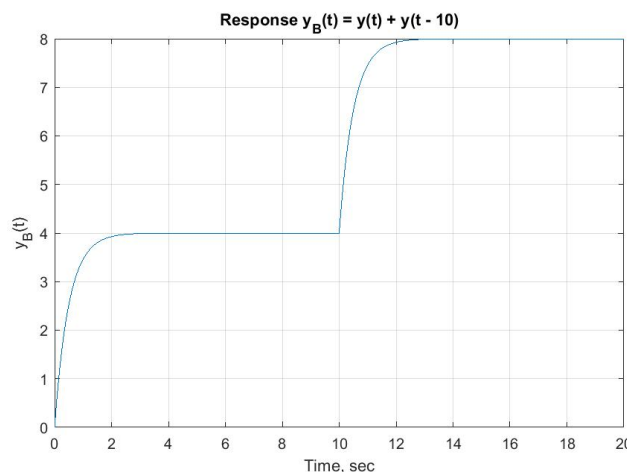


Figure 1: The system response $y_B(t)$ over time.

GPT Answer :

Given the linear time-invariant system $G(s) = \frac{4}{s+2}$ and its response to a unit step input of $u(t) = 2$, we have $y(t) = 4(1 - e^{-2t})$.

For part (a), the response y_A for an input $u_A(t) = -1$ can be derived using the property of linearity. The original response was for $u(t) = 2$, hence the response for $u_A(t) = -1$ is simply the original response scaled by $\frac{-1}{2}$, since $u_A(t) = -\frac{1}{2} \cdot 2$.

Therefore, the response $y_A(t)$ is given by:

$$y_A(t) = \frac{-1}{2} \cdot 4(1 - e^{-2t}) = -2(1 - e^{-2t})$$

For part (b), considering $u_B(t)$ which changes from 2 to 4 at $t = 10$, we can express $y_B(t)$ using the superposition principle and taking into account time-invariance.

- For $0 \leq t < 10$, the input is 2, and the response is $y(t) = 4(1 - e^{-2t})$. - For $t \geq 10$, the input increases by an additional 2, making it equivalent to applying a step of magnitude 2 at $t = 10$. This can be thought of as an additional input starting at $t = 10$, whose response can be shifted in time to start at 10. This yields an additional response of $4(1 - e^{-2(t-10)})$.

Combining both, for $t \geq 10$, the response is the sum of the ongoing response up to $t = 10$ plus the response due to the new input starting at $t = 10$. Therefore,

$$y_B(t) = \begin{cases} 4(1 - e^{-2t}) & 0 \leq t < 10, \\ 4(1 - e^{-2t}) + 4(1 - e^{-2(t-10)}) & t \geq 10. \end{cases}$$

Confidence level: 95%

Correct

Claude Answer :

- (a) Since the system is linear, we can use scaling to determine the response to $u_A(t) = -1$, $t \geq 0$. The response will be:

$$y_A(t) = -\frac{1}{2} \cdot y(t) = -2(1 - e^{-2t}), \quad t \geq 0$$

- (b) Using the principle of superposition and time-invariance, we can express $y_B(t)$ as:

$$y_B(t) = \begin{cases} y(t), & 0 \leq t < 10, \\ y(t) + y(t-10), & t \geq 10. \end{cases}$$

Substituting the expression for $y(t)$:

$$y_B(t) = \begin{cases} 4(1 - e^{-2t}), & 0 \leq t < 10, \\ 4(1 - e^{-2t}) + 4(1 - e^{-2(t-10)}), & t \geq 10. \end{cases}$$

Confidence Score: 95/100

Correct