

7.2 Closed-Loop Stability

Suppose the closed loop transfer function is given by:

$$\frac{KL(s)}{1 + KL(s)}$$

where K is some constant control gain.

- If $L(s)$ has 3 LHP poles and 1 LHP zero, is the closed-loop system stable for very large values of $K > 0$?
- If $L(s)$ has 2 LHP poles, 1 RHP poles, and 3 LHP zeros, is the closed-loop system stable for very large values of $K > 0$?
- If $L(s)$ has 5 LHP poles, 4 LHP zeros, and 1 RHP zeroes, is the closed-loop system stable for very large values of $K > 0$?

Give detailed justification for each answer.

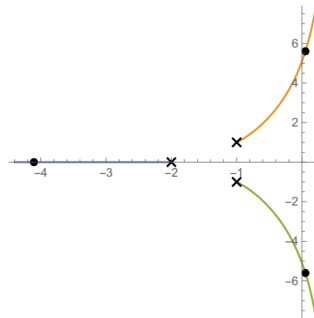


Figure 13: All poles and zeros in LHP and still high gain can make a system unstable.

Solution :

- Although, all poles and zeros are in the LHP, it does not necessary mean that all values of gain, K , will keep the system stable. Suppose that the transfer function is strictly proper, then the asymptotes should be taken into consideration. If there are $j\omega$ -crossings, then some asymptotes may escape into RHP with high gain, causing the system to be unstable (see Fig. 13). For example, consider a system with a zero at $s = -5$ and poles at $s = -2, s = -1 \pm j$. Although, it has 3 LHP poles and 1 LHP zero, the root locus plot of this system shows that there are $j\omega$ -crossings and the system becomes unstable with high gain.
- Root locus branches start at the poles and end at the zeros. If the number of poles is greater than the number of zeros, then some branch(es) will go to infinity. In this case, there is one

unstable pole (RHP pole). Since there are equal numbers of poles and zeros, all branches will end at the zeros and because all the zeros are in the LHP, a large enough K can stabilize the closed-loop system.

- (c) Similar to the previous problem, all branches will start at the poles and end at the zeros. This case also has equal number of both. However, one of the zeros is in the RHP. Therefore, a large value of K can cause the closed-loop system to be unstable.

GPT Answer :

To answer these questions, we can apply the Nyquist stability criterion, which is a graphical technique used in control systems to determine the stability of a closed-loop system. It involves plotting the Nyquist plot of the open-loop transfer function $L(s)$ and then using this plot to determine the number of times the plot encircles the point $-1 + j0$ in the complex plane. For a system to be stable, the number of clockwise encirclements of the point $-1 + j0$ by the Nyquist plot must equal the number of poles of $L(s)$ in the right half of the complex plane (RHP).

Let's analyze each scenario:

- (a) If $L(s)$ has 3 Left Half Plane (LHP) poles and 1 LHP zero, then the open-loop system is stable, as all poles of $L(s)$ are in the LHP. For large values of $K > 0$, the closed-loop system tends to have a pole-zero cancellation effect for the LHP zero and one of the LHP poles, leaving the closed-loop system with poles in the LHP, which implies stability. Thus, the closed-loop system is stable for very large values of $K > 0$.
- (b) If $L(s)$ has 2 LHP poles, 1 RHP pole, and 3 LHP zeros, the presence of a RHP pole in $L(s)$ indicates that the open-loop system is unstable. As K increases, the closed-loop system's stability is determined by the encirclements of the point $-1 + j0$. In this case, the RHP pole contributes to an encirclement that cannot be canceled by zeros in the LHP. Therefore, the closed-loop system remains unstable for very large values of $K > 0$.
- (c) If $L(s)$ has 5 LHP poles, 4 LHP zeros, and 1 RHP zero, the open-loop system is stable, as all poles are in the LHP. However, the presence of a RHP zero complicates the stability of the closed-loop system for large K . In the high-gain limit ($K \rightarrow \infty$), the effect of the RHP zero becomes prominent, potentially affecting stability. However, since zeros do not directly determine the stability but rather the encirclements of the $-1 + j0$ point, the crucial factor is the number of RHP poles, which remains zero. Therefore, the closed-loop system should remain stable for very large values of $K > 0$, assuming the RHP zero does not introduce a destabilizing effect through excessive phase lag.

Given these analyses, here is the concise response with confidence levels:

- (a) The closed-loop system is stable for very large values of $K > 0$.
- (b) The closed-loop system is unstable for very large values of $K > 0$.

- (c) The closed-loop system is likely to be stable for very large values of $K > 0$, though the RHP zero may affect response characteristics.

Confidence Level: 90%

Reasoning Error

Claude Answer :

I apologize for my mistake. Let me review the problems and provide the correct answers.

(a) $L(s)$ has 3 LHP poles and 1 LHP zero. For large K , the Nyquist plot will start in quadrant 4, cross into quadrant 3, and end in quadrant 2 due to the surplus of LHP poles over LHP zeros ($3 - 1 = 2$). This results in an encirclement of $-1+j0$, making $N = -1$. However, there are no RHP poles ($P = 0$), so the closed-loop system is unstable for large $K > 0$.

(b) $L(s)$ has 2 LHP poles, 1 RHP pole, and 3 LHP zeros. There is 1 RHP pole ($P = 1$). For stability, $N = P - Z = 1 - 3 = -2$. The Nyquist plot needs to make 2 clockwise encirclements of $-1+j0$. For large K , the surplus of LHP zeros over LHP poles ($3 - 2 = 1$) will cause the plot to make 2 clockwise encirclements. Therefore, the closed-loop system is stable for large $K > 0$.

(c) $L(s)$ has 5 LHP poles, 4 LHP zeros, and 1 RHP zero. There are no RHP poles ($P = 0$). For stability, $N = -Z = -1$. The Nyquist plot needs to make 1 clockwise encirclement of $-1+j0$. For large K , the extra LHP pole over LHP zeros will cause the plot to start in quadrant 4, cross into quadrant 3, and end in quadrant 2 due to the RHP zero. However, this results in no encirclements ($N = 0$), so the closed-loop system is unstable for large $K > 0$.

Confidence Level: 95%

Correct