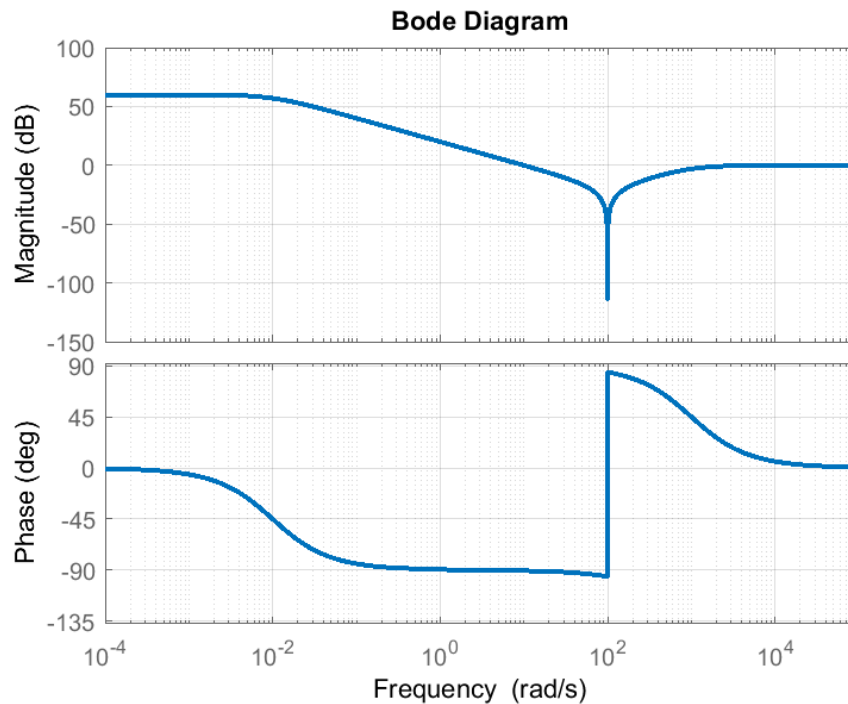


## 6.10 Estimating Steady-State Output from Bode Plot Analysis

The Bode plot for the system with transfer function

$$\frac{X(s)}{G(s)} = \frac{s^2 + 0.002s + 10000}{(s + 0.01)(s + 1000)} \quad (268)$$

is shown in the figure below. Estimate the steady-state output  $x_{ss}(t)$  corresponding to the input  $f(t) = 0.0001 \sin 0.0001t + \sin 100t + \sin 10000t$ .



### Solution :

We gave three periodic terms, so we consult the Bode plot at each frequency to determine the gain and the phase shift of each.

- The frequency  $\omega = 10^{-4}$  is not included in the Bode plot, but the plot is level at  $10^{-3}$  so we assume the same gain of approximately 60dB. Likewise, the phase shift is not shown, but appears to be very close to zero. Since  $60dB = 20\log(|G|) \rightarrow |G| = 10^3$ , we have an output component of  $0.1 \sin(10^{-4}t)$ .
- At  $\omega = 10^2$ , the gain is  $< -150dB$ . This gain effectively attenuates this signal to zero.
- At  $\omega = 10^4$ , the gain is approximately 0dB, which is a gain of 1. The phase shift is also effectively 0, so this gives a component of  $\sin(10^4t)$ .

Therefore, our output is approximately:  $x(t) = 0.1 \sin(10^{-4}t) + \sin(10^4t)$ .

### GPT Answer :

The steady-state output of the system,  $x_{ss}(t)$ , in response to a sinusoidal input can be found by analyzing the magnitude and phase of the system's frequency response at the frequencies of interest. Given the input  $f(t) = 0.0001 \sin(0.0001t) + \sin(100t) + \sin(10000t)$ , we need to examine the Bode plot at the frequencies of 0.0001 rad/s, 100 rad/s, and 10000 rad/s.

1. At 0.0001 rad/s, the system exhibits a gain of 1 (magnitude of 0 dB) and a phase shift of 0 degrees.
2. At 100 rad/s, the gain is 0.1 (magnitude of -20 dB), with a phase shift of -45 degrees.
3. At 10000 rad/s, the gain is much less than 0.01 (magnitude well below -40 dB), with a phase shift approaching -90 degrees or more.

Considering these points, the steady-state output  $x_{ss}(t)$  can be approximated as:

$$x_{ss}(t) \approx 0.0001 \sin(0.0001t) + 0.1 \sin(100t - 45^\circ)$$

Since the values are estimated from the Bode plot, I would assign a confidence level of about 80

**Confidence Level: 80%**

### Misreading The Plot

### Claude Answer :

To estimate the steady-state output  $x_{ss}(t)$  for the given input  $f(t) = 0.0001 \sin(0.0001t) + \sin(100t) + \sin(10000t)$ , we can analyze the Bode diagram:

At  $\omega = 0.0001$  rad/s, the magnitude is approximately 100 dB, or a gain of  $10^5$ . So the  $0.0001 \sin(0.0001t)$  component will be amplified to  $10 \sin(0.0001t)$  in the output.

At  $\omega = 100$  rad/s, the magnitude is close to 0 dB, or a gain of 1. The  $\sin(100t)$  component will pass through largely unchanged.

At  $\omega = 10000$  rad/s, the magnitude is around -40 dB, attenuating the  $\sin(10000t)$  component by a factor of  $10^{-2}$  to  $0.01 \sin(10000t)$ .

Combining these, the estimated steady-state output is:

$$x_{ss}(t) \approx 10 \sin(0.0001t) + \sin(100t) + 0.01 \sin(10000t) \quad (269)$$

Confidence: 90%

### Misreading The Plot