

5.22 Proportional Controller

Consider the system

$$G(s) = \frac{s+1}{s^2+2s+3}$$

is in standard feedback configuration using a controller K . Is it possible to make the settling time t_s arbitrarily small by choosing the gain K large enough for the closed-loop system?

Solution :

It is worth noting that the closed loop poles for the given system are:

$$s = \frac{-(K+2) \pm \sqrt{K^2-8}}{2}$$

As $K \rightarrow \infty$, both of the poles for the closed loop equation are real with one of them going to $-\infty$ and at the same time the other root goes to -1. Thus, the rate limiting step is going to be -1. This limits our ability to make t_s arbitrarily small. Therefore, it's not possible to make t_s arbitrarily small by choosing large enough gain K .

GPT Answer :

To answer whether it's possible to make the settling time t_s arbitrarily small by increasing the controller gain K for the given system $G(s) = \frac{s+1}{s^2+2s+3}$ in a standard feedback configuration, we analyze the closed-loop transfer function and its relation to the settling time.

The closed-loop transfer function $T(s)$ is given by $T(s) = \frac{KG(s)}{1+KG(s)}$, where $G(s) = \frac{s+1}{s^2+2s+3}$. This simplifies to $T(s) = \frac{K(s+1)}{s^2+(2+K)s+(3+K)}$.

The settling time t_s is related to the system's poles, which are the roots of the characteristic equation $s^2 + (2+K)s + (3+K) = 0$. While increasing K can influence the poles and potentially reduce t_s , there are practical and theoretical limits due to stability and performance considerations.

Confidence Level: 85%

Did not solve

Claude Answer :

1) As we saw before, the poles of the closed-loop system are the roots of the characteristic equation:

$$s^2 + (K+2)s + (K+3) = 0$$

2) Using the quadratic formula, we can find these poles:

$$s = \frac{-(K+2) \pm \sqrt{(K+2)^2 - 4(K+3)}}{2}$$

3) Now, let's consider what happens as K becomes very large. In this case, the term under the square root simplifies to:

$$(K + 2)^2 - 4(K + 3) \approx K^2 - 4K = K(K - 4)$$

4) Therefore, for large K , the poles are approximately:

$$s \approx \frac{-K \pm \sqrt{K(K - 4)}}{2}$$

5) This can be further simplified to:

$$s \approx \frac{-K \pm K\sqrt{1 - \frac{4}{K}}}{2}$$

6) As K tends to infinity, the term $\frac{4}{K}$ tends to zero, so the poles approach:

$$s \approx \frac{-K \pm K}{2} = \begin{cases} -K & \text{or} \\ 0 \end{cases}$$

7) This means that as K becomes very large, one pole moves far into the left half of the s -plane, while the other pole approaches the origin.

8) The pole near the origin is the dominant pole, as it will have the largest influence on the system's response. The settling time is inversely proportional to the distance of the dominant pole from the origin.

9) As K increases, the dominant pole gets closer and closer to the origin, but it never actually reaches the origin (unless K is literally infinite, which is not practically possible).

10) This means that while increasing K can reduce the settling time, it can never make the settling time exactly zero. There will always be some finite settling time, no matter how large K is made.

In summary, it is impossible to make the settling time arbitrarily small by increasing K , because one of the poles will always remain close to the origin, preventing the settling time from becoming zero. The physical limitations of the system will also limit how large K can practically be made.

Confidence Level: 95%

Correct