

## 5 Control System Design

### 5.1 PI Control Design

Consider a plant with a nominal model given by

$$G(s) = \frac{1}{s+2} \quad (149)$$

Compute the parameters  $K_p$  and  $K_i$  of a PI controller so that the natural modes of the closed loop response decay as fast as  $e^{-5t}$ .

**Solution :**

A PI controller has a transfer function given by

$$C(s) = \frac{K_p s + K_i}{s}; \quad (150)$$

The closed-loop characteristic polynomial,  $A_{cl}(s)$ , is derived as

$$A_{cl}(s) = \text{numerator of } \{1 + G(s)C(s)\} = s^2 + (2 + K_p)s + K_i. \quad (151)$$

To achieve a closed-loop transient response that decays as fast as  $e^{-5t}$ , the controller must generate a pair of complex conjugate poles with real parts equal to  $-5$ . This requirement determines the values of  $K_p$  and  $K_i$ , with  $K_p$  needing to be 8 and  $K_i$  needing to be greater than or equal to 25. Therefore, the appropriate PI controller parameters are  $K_p = 8$  and  $K_i \geq 25$ .

**GPT Answer:**

To design a PI controller that ensures the natural modes of the closed loop response decay as fast as  $e^{-5t}$ , we start with the plant model:

$$G(s) = \frac{1}{s+2}$$

A PI (Proportional-Integral) controller can be represented as:

$$C(s) = K_p + \frac{K_i}{s}$$

where  $K_p$  is the proportional gain and  $K_i$  is the integral gain. The closed-loop transfer function  $T(s)$  when a PI controller is used, is given by:

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

Substituting  $C(s)$  and  $G(s)$  into the above equation, we get:

$$T(s) = \frac{\left(K_p + \frac{K_i}{s}\right) \left(\frac{1}{s+2}\right)}{1 + \left(K_p + \frac{K_i}{s}\right) \left(\frac{1}{s+2}\right)}$$

Simplifying, the characteristic equation of the closed-loop system is derived from the denominator of the closed-loop transfer function:

$$1 + \left(K_p + \frac{K_i}{s}\right) \left(\frac{1}{s+2}\right) = 0$$

Further simplification leads to:

$$s + 2 + K_p(s + 2) + \frac{K_i}{s} = 0$$

$$s^2 + (2 + K_p)s + K_i = 0$$

Given that we want the natural modes to decay as fast as  $e^{-5t}$ , the desired closed-loop poles are at  $s = -5$ . Thus, the characteristic equation of the desired closed-loop system is:

$$(s + 5)^2 = s^2 + 10s + 25 = 0$$

Equating the coefficients, we get:

$$2 + K_p = 10 \implies K_p = 8$$

$$K_i = 25$$

Therefore, the PI controller parameters required to achieve a decay rate of  $e^{-5t}$  in the closed-loop system are  $K_p = 8$  and  $K_i = 25$ .

**Confidence level: 95%**

**Correct**

**Claude Answer:**

To design a PI controller that achieves the desired closed-loop response, we need to determine the values of the proportional gain ( $K_p$ ) and the integral gain ( $K_i$ ). Let's follow these steps:

1. The transfer function of a PI controller is:

$$C(s) = K_p + \frac{K_i}{s} \quad (152)$$

2. The closed-loop transfer function with the plant and the PI controller is:

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)} \quad (153)$$

3. Substituting  $C(s)$  and  $G(s)$ :

$$\frac{Y(s)}{R(s)} = \frac{(K_p + \frac{K_i}{s}) \cdot \frac{1}{s+2}}{1 + (K_p + \frac{K_i}{s}) \cdot \frac{1}{s+2}} \quad (154)$$

4. Simplify the equation:

$$\frac{Y(s)}{R(s)} = \frac{K_p s + K_i}{s^2 + (2 + K_p)s + K_i} \quad (155)$$

5. The characteristic equation of the closed-loop system is:

$$s^2 + (2 + K_p)s + K_i = 0 \quad (156)$$

6. For the natural modes to decay as fast as  $e^{-5t}$ , the poles of the closed-loop system should be at  $s = -5$ . Therefore, the desired characteristic equation is:

$$(s + 5)^2 = s^2 + 10s + 25 = 0 \quad (157)$$

7. Comparing the coefficients of the desired characteristic equation with the actual characteristic equation:

$$2 + K_p = 10 \quad (158)$$

$$K_i = 25 \quad (159)$$

8. Solving for  $K_p$  and  $K_i$ :

$$K_p = 8 \quad (160)$$

$$K_i = 25 \quad (161)$$

Therefore, to achieve the desired closed-loop response where the natural modes decay as fast as  $e^{-5t}$ , the PI controller parameters should be:

- Proportional gain,  $K_p = 8$
- Integral gain,  $K_i = 25$

**Confidence level: 95%**

**Correct**