

### 3.21 Step Response

Consider the transfer function:

$$G(s) = \frac{1}{s^2 + s + 1}$$

Which of the following is the corresponding step response? Explain your choice and why you rejected the other two possibilities.

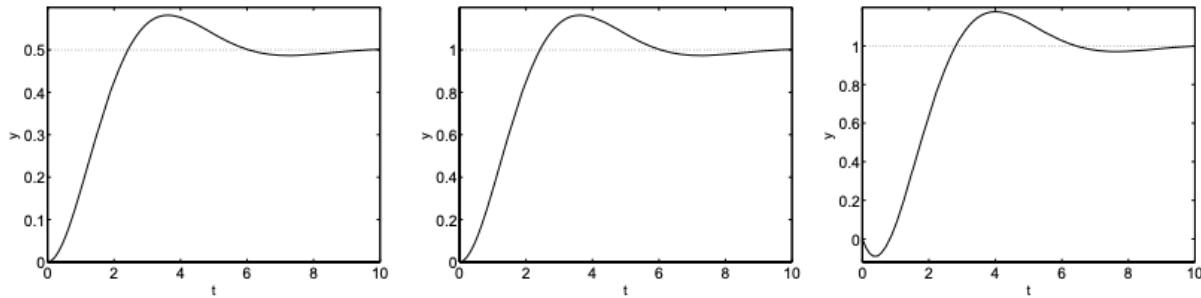


Figure 6: Sample step response plots

#### Solution:

The DC gain of  $H(s)$  equals 1, so the left plot is immediately rejected because it corresponds to DC gain of 0.5. The right plot shows the correct DC gain, but it has an initial dip which indicates the presence of a RHP zero, while  $H(s)$  has no zeros. The middle plot is consistent with a stable second-order transfer function with DC gain of 1 and with no zeros, so it is the correct one.

#### GPT Answer :

Given the transfer function  $G(s) = \frac{1}{s^2 + s + 1}$ , we identify the system as a second-order, underdamped system due to the quadratic polynomial in the denominator with no real roots. Such a system is expected to have an oscillatory step response with no steady-state error, and the oscillations should decay over time. Considering the provided step response plots:

- The first plot does not show any oscillations and does not settle to the final value of 1, which is inconsistent with an underdamped second-order system.
- The second plot, similarly, shows no oscillations and settles to a final value less than 1, which again does not match the expected behavior of the given system.
- The third plot exhibits an oscillatory response with a decay in amplitude, eventually settling to the final value of 1, which is consistent with the expected behavior of the given underdamped system.

Based on these observations, the third plot is the correct step response of the system represented by the given transfer function.

**Confidence Level: 95%**

**Reasoning error**

**Claude Answer :**

To determine which plot corresponds to the step response of the given transfer function  $G(s) = \frac{1}{s^2+s+1}$ , we need to analyze the characteristics of the transfer function and match them with the behavior shown in the plots.

**Given:**

- Transfer function:  $G(s) = \frac{1}{s^2+s+1}$

**Step 1: Analyze the transfer function**

- The denominator is a second-order polynomial, indicating a second-order system.
- The denominator can be factored as  $(s + 0.5 + j0.866)(s + 0.5 - j0.866)$ , which has complex conjugate poles.
- Complex conjugate poles in the left half-plane suggest an underdamped system with oscillatory behavior.

**Step 2: Examine the plots**

- **Plot 1:** Shows a smooth, overdamped response without oscillations. This is unlikely to be the correct plot for the given transfer function.
- **Plot 2:** Exhibits an underdamped response with oscillations that gradually decay towards a steady-state value. This behavior is consistent with the presence of complex conjugate poles in the transfer function.
- **Plot 3:** Displays an underdamped response with oscillations, but the oscillations do not seem to decay over time. This is not consistent with the expected behavior of the given transfer function.

Therefore, **Plot 2** is most likely the step response corresponding to the given transfer function  $G(s) = \frac{1}{s^2+s+1}$ . The presence of complex conjugate poles in the transfer function suggests an underdamped system with oscillatory behavior, which is clearly visible in Plot 2. The oscillations in Plot 2 gradually decay towards a steady-state value, which is consistent with the poles being in the left half-plane.

Plot 1 is rejected because it does not show any oscillations, indicating an overdamped system, which does not match the given transfer function. Plot 3 is rejected because, although it shows oscillations, they do not decay over time, which is inconsistent with the expected behavior of the given transfer function.

**Confidence Level: 95%**

**Correct**