

5.15 Proportional Controller

Consider the transfer function

$$T(s) = \frac{b_0 K_p}{s^2 + a_1 s + (a_0 + b_0 K_p)} \quad (246)$$

where $b_0 > 0$ and the closed-loop is critically damped. What happens to the settling time if K_p is increased? K_p is decreased?

Solution :

We have $a_1 = 2\zeta\omega_n$ and $a_0 + b_0 K_p = \omega_n^2$.

- Increasing K_p does not change the real part and thus settling time t_s remain unchanged.
- Decreasing K_p brings one of the poles closer to the origin, thereby increasing the settling time t_s .

GPT Answer :

To determine the effect of increasing or decreasing K_p on the settling time, let's analyze the given transfer function in the context of a second-order system. The transfer function is:

$$T(s) = \frac{b_0 K_p}{s^2 + a_1 s + (a_0 + b_0 K_p)}$$

A second-order system is critically damped when its damping ratio, ζ , is equal to 1. The general form of a second-order system is:

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Comparing the given transfer function to the general form, we can relate:

$$2\zeta\omega_n = a_1, \quad \omega_n^2 = a_0 + b_0 K_p$$

For the system to be critically damped, $\zeta = 1$. Therefore, we focus on ω_n (the natural frequency of the system) to determine the effect on settling time.

Settling time (T_s) of a second-order system, especially when it is critically damped or overdamped, is inversely proportional to the natural frequency ω_n , usually approximated by:

$$T_s \approx \frac{4}{\zeta \omega_n}$$

Given that $\zeta = 1$ for critical damping, we see that as ω_n increases, T_s decreases. From the equation $\omega_n^2 = a_0 + b_0 K_p$, it is clear that increasing K_p increases ω_n^2 , and thus ω_n , which leads to a decrease in settling time. Conversely, decreasing K_p decreases ω_n , which increases the settling time.

Therefore:

- Increasing K_p decreases the settling time. - Decreasing K_p increases the settling time.

Confidence Level: 95%

Reasoning Error

Claude Answer :

System Analysis with Critically Damped Response

Given the transfer function:

$$T(s) = \frac{b_0 K_p}{s^2 + a_1 s + (a_0 + b_0 K_p)} \quad (247)$$

where $b_0 > 0$ and the system is critically damped.

Characterization of Critically Damped Systems For a critically damped system, the characteristic equation is:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \quad (248)$$

with $\zeta = 1$ for critical damping, where ω_n is the natural frequency.

Comparing with the denominator of the transfer function:

$$s^2 + a_1 s + (a_0 + b_0 K_p) = s^2 + 2\omega_n s + \omega_n^2 \quad (249)$$

This implies:

$$2\omega_n = a_1 \quad \text{and} \quad \omega_n^2 = a_0 + b_0 K_p \quad (250)$$

For the system to remain critically damped, the damping ratio ζ must be:

$$\zeta = \frac{a_1}{2\sqrt{a_0 + b_0 K_p}} = 1 \quad (251)$$

Settling Time for the Critically Damped System The approximate settling time t_s for the critically damped system is:

$$t_s \approx \frac{4}{a_1} \quad (252)$$

Effect of Changing K_p 1. **Increasing K_p :** - The settling time t_s theoretically remains constant as it depends only on a_1 . - However, increasing K_p raises $b_0 K_p$, potentially making the system underdamped.

2. **Decreasing K_p :** When K_p is decreased, the system becomes overdamped, and the settling time increases.

Confidence Level: 95%

Correct