

# 1 Differential Equations, Laplace Transform and Preliminaries

## 1.1 Differential Equations

Determine the transfer function of a linear time invariant (LTI) system given the following information:

1. The system has relative degree 3.
2. It has 3 poles, of which 2 are at -2 and -4.
3. The impulse response resembles a step response for a stable linear system with a steady state value of 0.25.

### **Solution :**

Given a linear time-invariant (LTI) system with specific characteristics, we aim to determine its transfer function. The details and solution steps are as follows:

#### **System Characteristics**

- The system has a relative degree 3 with 3 poles, hence it has no finite zeros.
- It has 3 poles, hence it takes the form:

$$G(s) = \frac{K}{A(s)(s+2)(s+4)} \quad (1)$$

- Since the impulse response resembles a step response with a steady state value, we conclude the system must contain a pole at zero. Therefore, the transfer function is of the form:

$$G(s) = \frac{K}{s(s+2)(s+4)} \quad (2)$$

#### **Determination of $K$**

Using the final value theorem to determine  $K$ , we have:

$$\lim_{s \rightarrow 0} G(s) = \lim_{s \rightarrow 0} \frac{sK}{s(s+2)(s+4)} \quad (3)$$

$$= \frac{K}{8} \quad (4)$$

Given the steady state value of 0.25, i.e.,  $\lim_{s \rightarrow 0} sG(s) = 0.25$ , we find  $K = 2$ . Therefore, the transfer function of the system is:

$$G(s) = \frac{2}{s(s+2)(s+4)} \quad (5)$$

### GPT Answer :

To determine the transfer function  $H(s)$  of a Linear Time Invariant (LTI) system given the conditions, we need to consider each piece of information provided:

1. **Relative degree 3**: This implies that the degree of the denominator (the poles) of the transfer function is three more than the degree of the numerator (the zeros).
2. **3 poles, with 2 at -2 and -4**: If there are three poles and we know the location of two of them, we denote these poles as  $s = -2$  and  $s = -4$ . The third pole's location is not specified, so we'll denote it as  $s = p$ .
3. **Impulse response resembles a step response for a stable linear system with a steady state value of 0.25**: A system's steady state value in response to a step input (which is essentially the system's DC gain or the gain as  $s \rightarrow 0$ ) can be found by evaluating the transfer function at  $s = 0$ . Since the impulse response resembles the step response with a steady state value of 0.25, the transfer function's value at  $s = 0$  should be 0.25.

Given these conditions, the general form of the transfer function  $H(s)$  for an LTI system is:

$$H(s) = \frac{K}{(s+2)(s+4)(s+p)}$$

Where: -  $K$  is the gain of the system, which we need to find. -  $p$  is the location of the third pole, which we need to determine based on the system being stable (i.e., the real part of  $p$  must be negative).

Since the system is stable and has a steady state value of 0.25 for a step input, we can use the Final Value Theorem which states that:

$$\lim_{s \rightarrow 0} sH(s) = 0.25$$

Plugging the general form of  $H(s)$  into this theorem:

$$\lim_{s \rightarrow 0} s \frac{K}{(s+2)(s+4)(s+p)} = 0.25$$

Solving for  $K$  when  $s = 0$  gives:

$$K = 0.25(0 + 2)(0 + 4)(0 + p) = 2p$$

However, without the exact value of  $p$ , we cannot specify  $K$  exactly. We do know that  $p$  must be negative for stability, and without further information, we can't identify  $p$ 's exact value. Typically, the step of finding  $K$  would involve knowing  $p$  to maintain the system's stability and meet the given steady state value condition.

Given these constraints and without the specific value of  $p$ , we cannot provide a singular transfer function or the exact value of  $K$ . The information leads to a partially determined transfer function:

$$H(s) = \frac{K}{(s + 2)(s + 4)(s + p)}$$

with  $K$  related to  $p$  by  $K = 2p$ , and  $p < 0$  for system stability.

**Confidence Level: 75%**

The confidence level reflects the partial determination of the transfer function due to the unknown third pole and the assumption made about the system's stability to derive  $K$ .

**Calculation Error****Claude Answer :**

The transfer function of the given LTI system is:

$$H(s) = \frac{K}{(s+2)(s+4)(s+b)}$$

where  $b$  is a positive real number, and  $K$  is a constant gain term.

From the given information:

1. Relative degree 3 implies that the numerator is of degree 0 (a constant) and the denominator is of degree 3.
2. Two poles are at  $s = -2$  and  $s = -4$ , so the denominator contains terms  $(s + 2)$  and  $(s + 4)$ .
3. The steady-state value of the impulse response is 0.25, which means  $\lim_{s \rightarrow 0} sH(s) = 0.25$ . Solving this limit gives  $K/(2 \cdot 4 \cdot b) = 0.25$ , or  $K = 2b$ .

To determine the value of  $b$ , we need more information about the system, such as the location of the third pole.

**Confidence Level: 95/100****Calculation Error**