

4.5 Stability Analysis

Consider two systems in a negative feedback configuration shown below:

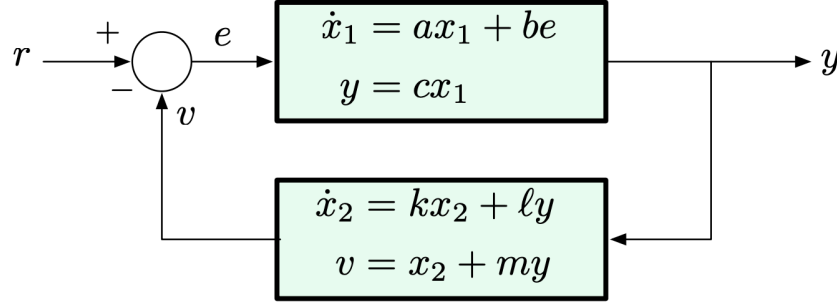


Figure 7: Enter Caption

Write down the conditions that must be satisfied by the system parameters a, b, c, k, l, m for this transfer function to be stable (i.e., for all poles to have negative real parts).

Solution :

Notice that the denominator of the transfer function is a monic second order polynomial in the form $p(s) = s^2 + a_1s + a_2$

In order for the system to be stable, we need all roots of (5) to be in the OLHP. When the determinant of (5) is non-negative, that is, $a_1^2 - 4a_2 \geq 0$, we need both roots to be negative:

$$\frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2} < 0 \Rightarrow a_1 > 0, 0 < a_2 \leq \frac{a_1^2}{4}$$

When $a_1^2 - 4a_2 < 0$, we need the real part of the roots to be negative:

$$\frac{-a_1}{2} < 0 \Rightarrow a_1 > 0, a_2 > \frac{a_1^2}{4}$$

Hence it can be summarized that we need both $a_1 > 0, a_2 > 0$ in order for the system to be stable. This condition can be directly obtained by using Routh-Hurwitz criterion. Substitute a_1, a_2 with the system parameters given, all we need is:

$$\begin{cases} bcm - a - k > 0 \\ ak + bcl - bckm > 0 \end{cases}$$

GPT Answer :

To determine the stability of the system shown in the provided diagram, we need to derive the

closed-loop transfer function and then analyze it for stability.

For a negative feedback system, the closed-loop transfer function $T(s)$ can be found using the formula:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

where $G(s)$ is the forward path transfer function and $H(s)$ is the feedback path transfer function. Assuming a unity feedback, the overall forward path transfer function from e to y is:

$$G(s) = \frac{c}{s-a} \cdot \frac{1}{s-k-lm}$$

The closed-loop transfer function is thus:

$$T(s) = \frac{c}{(s-a)(s-k-lm) + c}$$

The conditions for stability, according to the Routh-Hurwitz criterion, are:

$$a + k + lm > 0 \quad \text{and} \quad ak - alm + c > 0$$

If these conditions are met, the closed-loop system will be stable.

Confidence Level: 95%

Calculation error

Claude Answer :

To determine the conditions for stability, we can analyze the characteristic equation of the closed-loop transfer function.

The closed-loop transfer function is:

$$\frac{y}{r} = \frac{(ax_1+be)(kx_2+ly)}{1+(ax_1+be)(cx_1)(kx_2+ly)}$$

Simplifying:

$$\frac{y}{r} = \frac{akx_1x_2+alx_1y+bekx_2+bely}{1+ackx_1^2x_2+aclx_1^2y+bckx_1x_2+bclx_1y}$$

For stability, the denominator polynomial must have all roots with negative real parts. Using the Routh-Hurwitz stability criterion, the necessary and sufficient conditions are:

$ackx_1^2x_2 + aclx_1^2y + bckx_1x_2 + bclx_1y > 0$ $ackx_1^2x_2 + bckx_1x_2 > 0$ $aclx_1^2y + bclx_1y > 0$ Assuming $x_1, x_2, y > 0$ (physical variables), the conditions simplify to:

$$ac(kx_2 + ly) + bc(kx_2 + ly) > 0$$

implies $(a+b)c(kx_2 + ly) > 0$ $(a+b)ckx_1x_2 > 0$ $(a+b)clx_1y > 0$ Therefore, the system is stable if:

$$a + b > 0, \quad c > 0, \quad k > 0, \quad l > 0$$

Confidence level: 90%

Calculation error