

1.23 Final Value Theorem

Consider the following transfer functions

$$G_1(s) = \frac{2}{s^2 - 2s + 4} \quad G_2(s) = \frac{2s - 3}{s^2 + 4s + 1}$$

Use the final value theorem to compute the DC gain for $G_1(s)$ and $G_2(s)$. In each case, explain whether the final value theorem gives the right answer and why.

Solution :

DC gain for $G_1(s) = G_1(s)|_{s=0} = \frac{1}{2}$

DC gain for $G_2(s) = G_2(s)|_{s=0} = -3$

The final value theorem computes the correct DC gain for $G_2(s)$ but not for $G_1(s)$ because the poles of $G_1(s)$ are not in LHP, hence Final Value Theorem does not apply.

GPT Answer :

To compute the DC gain of the given transfer functions $G_1(s)$ and $G_2(s)$ using the Final Value Theorem (FVT), we recall that the DC gain can be found by evaluating the limit of $sG(s)$ as s approaches zero. This is equivalent to evaluating the transfer function at DC (0 Hz), i.e., $G(0)$.

The Final Value Theorem states that for a function $f(t)$ with Laplace Transform $F(s)$, the final value of $f(t)$ as t approaches infinity is given by:

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

provided that all poles of $sF(s)$ are in the left half of the complex plane (i.e., the system is stable).

For $G_1(s)$:

Substituting $s = 0$ in $G_1(s)$:

$$G_1(0) = \frac{2}{0^2 - 2 \cdot 0 + 4} = \frac{2}{4} = 0.5$$

For $G_2(s)$:

Substituting $s = 0$ in $G_2(s)$:

$$G_2(0) = \frac{2 \cdot 0 - 3}{0^2 + 4 \cdot 0 + 1} = \frac{-3}{1} = -3$$

Analysis and Applicability of FVT:

- **For $G_1(s)$:** The FVT is applicable because the poles of $G_1(s)$, which are the roots of the denominator $s^2 - 2s + 4$, are both in the left half of the complex plane, indicating

Correct

Claude Answer :

The final value theorem states that for a stable system with transfer function $G(s)$ and input $U(s)$, the steady-state output value is given by:

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sG(s)U(s)$$

For a step input, $U(s) = \frac{1}{s}$, so the DC gain is:

$$\lim_{s \rightarrow 0} G(s)$$

For $G_1(s) = \frac{2}{s^2 - 2s + 4}$:

$$\lim_{s \rightarrow 0} G_1(s) = \frac{2}{4} = 0.5$$

However, $G_1(s)$ has poles at $s = 1 \pm j\sqrt{3}$, which lie in the right half-plane. This makes the system unstable, invalidating the use of the final value theorem. The result cannot be trusted.

For $G_2(s) = \frac{2s-3}{s^2+4s+1}$:

$$\lim_{s \rightarrow 0} G_2(s) = \frac{-3}{1} = -3$$

The poles of $G_2(s)$ are at $s = -2 \pm j$, which lie in the left half-plane. This system is stable, so the final value theorem applies, and the DC gain is indeed -3 .

Confidence Level: 95%

Correct