

5.2 Pole Assignment

A plant with nominal model

$$G_o(s) = \frac{1}{(s+1)^2} \quad (162)$$

is in a feedback loop under control with a PI controller having transfer function $C(s) = \frac{K_p s + K_i}{s}$. Determine whether this controller configuration can be used to achieve full pole placement.

Solution :

The plant is of second-order, and the controller introduces an additional third pole. However, the controller only have two gains, corresponding to two numerator coefficients, which generally precludes it from being used to place all three poles effectively.

GPT Answer :

To determine if the PI controller can be used to achieve full pole placement for the given plant with nominal model $G_o(s) = \frac{1}{(s+1)^2}$, we must analyze the closed-loop transfer function and the nature of the closed-loop poles.

First, let's establish the closed-loop transfer function using the given plant and controller. The PI controller has the transfer function $C(s) = \frac{K_p s + K_i}{s}$. The open-loop transfer function $L(s)$ is the product of the controller and the plant:

$$L(s) = C(s)G_o(s) = \left(\frac{K_p s + K_i}{s} \right) \left(\frac{1}{(s+1)^2} \right) = \frac{K_p s + K_i}{s(s+1)^2}$$

The closed-loop transfer function $T(s)$ using the standard feedback formula $\frac{L(s)}{1+L(s)}$ is:

$$T(s) = \frac{\frac{K_p s + K_i}{s(s+1)^2}}{1 + \frac{K_p s + K_i}{s(s+1)^2}} = \frac{K_p s + K_i}{s(s+1)^2 + K_p s + K_i}$$

Simplifying the denominator, we get:

$$T(s) = \frac{K_p s + K_i}{s^3 + 2s^2 + (1 + K_p)s + K_i}$$

The closed-loop characteristic equation, which determines the stability and dynamics of the system, is then given by:

$$s^3 + 2s^2 + (1 + K_p)s + K_i = 0$$

To see if full pole placement is possible, we need to determine if we can arbitrarily assign the poles of the closed-loop system by selecting appropriate values for K_p and K_i . The characteristic equation is a cubic polynomial, and theoretically, we can choose K_p and K_i to place its roots (poles) anywhere in the s-plane, assuming no constraints such as non-negativity of coefficients or other practical limits.

****Analysis:**** 1. The coefficients of s^2 , s , and the constant term are directly influenced by K_p and K_i . We can select K_p and K_i to modify these coefficients, thus affecting the location of the poles. 2. There are no terms missing in the polynomial, and no constraints on the signs or values of K_p and K_i are specified, so we can manipulate them freely. 3. Through an appropriate selection of K_p and K_i , any set of desired pole locations (provided they satisfy the necessary conditions for a real, implementable system, such as real coefficients) can be targeted.

****Conclusion:**** Yes, using the PI controller configuration $C(s) = \frac{K_p s + K_i}{s}$ for the given plant $G_o(s) = \frac{1}{(s+1)^2}$, full pole placement is theoretically achievable. You can manipulate the values of K_p and K_i to place the poles of the closed-loop system anywhere desired in the s-plane, as long as the resulting values lead to a physically realizable and stable system.

Confidence Level: 85%

Reasoning Error

Claude Answer :

To determine if the given PI controller configuration can be used to achieve full pole placement, we need to analyze the closed-loop transfer function of the system and check if we can arbitrarily place the poles by adjusting the controller gains K_p and K_i .

Given: - Plant transfer function: $G_o(s) = \frac{1}{(s+1)^2}$ - PI controller transfer function: $C(s) = \frac{K_p s + K_i}{s}$

Step 1: Find the closed-loop transfer function.

The closed-loop transfer function is given by:

$$\frac{Y(s)}{R(s)} = \frac{C(s)G_o(s)}{1+C(s)G_o(s)}$$

Substituting the given transfer functions:

$$\frac{Y(s)}{R(s)} = \frac{\frac{K_p s + K_i}{s} \cdot \frac{1}{(s+1)^2}}{1 + \frac{K_p s + K_i}{s} \cdot \frac{1}{(s+1)^2}}$$

Simplifying:

$$\frac{Y(s)}{R(s)} = \frac{K_p s + K_i}{s(s+1)^2 + K_p s + K_i}$$

Step 2: Analyze the characteristic equation.

The characteristic equation is the denominator of the closed-loop transfer function:

$$s(s+1)^2 + K_p s + K_i = 0$$

Expanding:

$$s^3 + 2s^2 + s + K_p s + K_i = 0$$

For full pole placement, the system must have three poles that can be arbitrarily placed by adjusting K_p and K_i . However, the characteristic equation is a third-order polynomial, and we only have two parameters (K_p and K_i) to adjust. This means that we cannot achieve full pole placement with the given PI controller configuration.

Conclusion: The given PI controller configuration cannot be used to achieve full pole placement for the plant with transfer function $G_o(s) = \frac{1}{(s+1)^2}$. The system has three poles, but the PI controller only provides two adjustable parameters, which is insufficient for arbitrarily placing all three poles.

Confidence Level: 95%

Correct