

2.10 Routh-Hurwitz Stability

The characteristic equation of a given system is

$$s^4 + 6s^3 + 11s^2 + 6s + K = 0 \quad (115)$$

What restrictions must be placed upon the parameter K in order to ensure that the system is stable?

Solution :

The Routh table for this system is

$$\begin{array}{c|ccc} s^4 & 1 & 11 & K \\ s^3 & 6 & 6 & 0 \\ s^2 & 10 & K & 0 \\ s^1 & \frac{60-6K}{10} & 0 & \\ s^0 & K & & \end{array}$$

For the system to be stable. $60 - 6K > 0$. or $K < 10$, and $K > 0$. Thus $0 < K < 10$.

GPT Answer:

For a system to be stable, all roots of its characteristic equation must have negative real parts. This requirement ensures that any response of the system to an initial condition or an input will eventually decay to zero as time approaches infinity. Given the characteristic equation of the system

$$s^4 + 6s^3 + 11s^2 + 6s + K = 0, \quad (116)$$

we apply the Routh-Hurwitz criterion, which provides a systematic method to determine the stability of a system without explicitly calculating the roots of its characteristic equation.

The Routh-Hurwitz criterion involves constructing the Routh array and ensuring that there are no sign changes in the first column of the array. The array is constructed from the coefficients of the characteristic equation.

Given the equation coefficients, the Routh array for this system would look something like this:

$$\begin{array}{ccc} s^4 & 1 & 11 \\ s^3 & 6 & 6 \\ s^2 & \frac{66-6K}{6} & 0 \\ s^1 & 6 - \frac{6K}{66-6K} & \\ s^0 & K & \end{array}$$

For the system to be stable, all elements in the first column must be positive, which means:

1. $\frac{66-6K}{6} > 0$ simplifying to $11 - K > 0$, hence $K < 11$. 2. The calculation for s^1 must also be positive. However, since the direct computation involves K in both the numerator and the

denominator, the critical insight is ensuring that the denominator itself is positive, which is covered by the first condition, and thus, K must be positive for the s^0 term to be positive. Therefore, $K > 0$.

Combining these conditions, we conclude that for the system to be stable, $0 < K < 11$.

Confidence Level: 95%

Calculation Error

Claude Answer:

To determine the stability of the system, we need to apply the Routh-Hurwitz stability criterion. The system is stable if all the roots of the characteristic equation have negative real parts.

Given the characteristic equation:

$$s^4 + 6s^3 + 11s^2 + 6s + K = 0$$

Step 1: Write the coefficients of the characteristic equation in descending order of powers of s .

$$a_0 = K, a_1 = 6, a_2 = 11, a_3 = 6, a_4 = 1$$

Step 2: Construct the Routh-Hurwitz array.

s^4	1	11	K
s^3	6	6	0
s^2	b_1	b_2	0
s^1	c_1	0	0
s^0	d_1	0	0

where:

$$b_1 = \frac{6 \times 11 - 1 \times 6}{6} = 10,$$

$$b_2 = \frac{6K - 1 \times 0}{6} = K,$$

$$c_1 = \frac{10 \times 6 - 6 \times K}{10} = 6 - \frac{3K}{5},$$

$$d_1 = K$$

Step 3: For the system to be stable, all the elements in the first column of the Routh-Hurwitz array must be positive. This leads to the following conditions:

1. $a_1 > 0$: This condition is already satisfied as $a_1 = 6 > 0$.
2. $b_1 > 0$: This condition is also satisfied as $b_1 = 10 > 0$.
3. $c_1 > 0$: $6 - \frac{3K}{5} > 0$, which implies $K < 10$.

4. $d_1 > 0$: $K > 0$.

Therefore, for the system to be stable, the parameter K must satisfy the condition:

$$0 < K < 10$$

Confidence Level: 95%

Correct