

9.9 Comparing the Contributions of Poles and Zeros in Magnitude and Phase Plots

Consider the transfer function

$$G(s) = \frac{1}{(s-1)(s+1)}$$

The transfer function $G(j\omega)$ in this problem has an unstable real pole of the form $(j\omega\tau - 1)^{-1}$ and a stable real pole of the form $(j\omega\tau + 1)^{-1}$. Compare the role of both poles and explain their contribution both to the magnitude and to the phase plot.

Solution :

$(j\omega\tau - 1)^{-1}$ and $(j\omega\tau + 1)^{-1}$ have the same magnitude $\frac{1}{\sqrt{(\omega\tau)^2 + 1}}$.

Before the break point $\omega = \frac{1}{\tau}$, $(j\omega\tau + 1)^{-1} \approx 1$. After the break point, $(j\omega\tau + 1)^{-1} \approx (j\omega\tau)^{-1}$. Therefore, its phase changes from 0° to -90° .

The phase of unstable real pole is trickier. Before the break point $\omega = \frac{1}{\tau}$, $(j\omega\tau - 1)^{-1} \approx -1$. After the break point, $(j\omega\tau - 1)^{-1} \approx (j\omega\tau)^{-1}$. Therefore, its phase changes from -180° to -90° .

GPT Answer :

To provide a comprehensive explanation of the roles of both poles in the given transfer function $G(s) = \frac{1}{(s-1)(s+1)}$ and explain their contribution to the magnitude and phase plot, we begin by recognizing that the given transfer function represents a system with a pole at $s = 1$ (unstable real pole) and a pole at $s = -1$ (stable real pole). When evaluating the transfer function on the imaginary axis, $s = j\omega$, where ω is the frequency, the transfer function becomes $G(j\omega) = \frac{1}{(j\omega-1)(j\omega+1)}$.

The poles contribute to the transfer function as follows:

- **Magnitude Plot Contribution**:** - The pole at $s = 1$ (or $(j\omega - 1)^{-1}$ when substituted) contributes to the magnitude of the transfer function by decreasing the magnitude as the frequency ω increases from 0 to 1, and increasing the magnitude as ω further increases beyond 1. This is because the distance in the complex plane from $j\omega$ to the pole at $s = 1$ decreases initially and then increases. - The pole at $s = -1$ (or $(j\omega + 1)^{-1}$ when substituted) behaves similarly but in the opposite direction. The magnitude decreases as the frequency moves away from this pole.
- **Phase Plot Contribution**:** - The pole at $s = 1$ contributes a phase shift that starts at 0 degrees for low frequencies and approaches -90 degrees as the frequency increases. This is because the vector from the pole to the point on the imaginary axis ($j\omega$) rotates clockwise as the frequency increases. - The pole at $s = -1$ similarly contributes a phase shift from 0 degrees and moves towards -90 degrees as the frequency increases, due to the counterclockwise rotation of the vector from the pole to $j\omega$.

The combined effect of these poles on the magnitude plot is to create a system response that decreases in magnitude around the frequencies near the poles and then flattens out or increases slightly as we move away from the poles. For the phase plot, the combined effect tends towards a phase shift of -180 degrees at high frequencies, combining the contributions from both poles.

It is critical to note that the pole at $s = 1$ makes the system inherently unstable, as any input can lead to an unbounded output, whereas the pole at $s = -1$ does not contribute to system instability.

Confidence Level: 90%

Reasoning Error

Claude Answer :

To analyze the role of the poles in the given transfer function and their contribution to the magnitude and phase plots, let's first consider the general form of the transfer function:

$$G(s) = \frac{1}{(s - 1)(s + 1)}$$

Unstable real pole: $(s - 1)^{-1}$ Stable real pole: $(s + 1)^{-1}$

Magnitude plot contribution:

The unstable real pole $(s - 1)^{-1}$ contributes to a resonance peak in the magnitude plot. As the frequency ω approaches the pole location $\omega = 1$, the magnitude of the transfer function increases significantly, leading to a peak in the magnitude plot. The stable real pole $(s + 1)^{-1}$ contributes to a low-pass filtering effect in the magnitude plot. As the frequency ω increases, the magnitude of the transfer function decreases, resulting in a roll-off in the magnitude plot at higher frequencies. Phase plot contribution:

The unstable real pole $(s - 1)^{-1}$ contributes a phase shift of -90° (or $-\pi/2$ radians) as the frequency ω approaches the pole location $\omega = 1$. This phase shift occurs rapidly around the pole frequency. The stable real pole $(s + 1)^{-1}$ also contributes a phase shift of -90° (or $-\pi/2$ radians) as the frequency ω increases. However, this phase shift occurs more gradually compared to the unstable pole. The overall phase shift of the transfer function is the sum of the phase shifts contributed by both poles. In this case, the total phase shift at high frequencies will approach -180° (or $-\pi$ radians) due to the combined effect of the two poles.

In summary, the unstable real pole contributes to a resonance peak in the magnitude plot and a rapid phase shift of -90° around its pole frequency. The stable real pole contributes to a low-pass filtering effect in the magnitude plot and a gradual phase shift of -90° at higher frequencies. The combination of these poles shapes the overall frequency response of the transfer function.

Confidence Level: 90%

Reasoning Error