

1.28 Time-Domain Specs

Consider a second-order system with peak time $t_p = 0.5$ sec. and 5% settling time $t_s = 1.5$ sec. Does this system satisfy rising time $t_r \leq 0.18$ sec?

Solution:

Recall that $t_r = \frac{1.8}{\omega_n}$. Therefore this system must have $\omega_n \geq 10$ in order to satisfy the rise time requirement.

We know that $\omega_n^2 = \sigma^2 + \omega_d^2$. Given that $t_p = 0.5$ sec and 5% settling time $t_s = 1.5$ sec, we can compute the corresponding $\omega_d = 2\pi$ and $\sigma = 2$.

Therefore this system's $\omega_n^2 = 2^2 + (2\pi)^2 = 68$. Since $\omega_n^2 < 100$, therefore this system can't satisfy $t_r \leq 0.18$ sec.

GPT Answer :

To determine whether the system satisfies $t_r \leq 0.18$ sec, let's analyze the given parameters of the second-order system:

- Peak time (t_p) = 0.5 sec - 5

For a second-order system, the peak time (t_p) and settling time (t_s) can be related to the system's natural frequency (ω_n) and damping ratio (ζ) as follows:

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$t_s = \frac{4}{\zeta \omega_n}$$

From the given t_p and t_s , we can solve for ω_n and ζ .

However, the relationship between the rise time (t_r) and the system parameters for a second-order system underdamped ($0 < \zeta < 1$) condition is approximated by:

$$t_r \approx \frac{1.8}{\omega_n}$$

Given that we don't have explicit values for ω_n and ζ without solving the equations, we proceed with a general analysis to check if it's plausible to directly determine if $t_r \leq 0.18$ sec based on the given t_p and t_s without calculating ω_n and ζ .

Given that t_r depends on ω_n , and seeing as both t_p and t_s also depend on ω_n and ζ , there's a direct but complex relationship between t_r , t_p , and t_s .

However, without calculating ω_n and ζ explicitly, we can use the given t_p and t_s to infer that for typical second-order systems:

- t_p gives us a direct measure of how quickly the system reaches its first peak, and a lower t_p generally indicates a faster system which could suggest a smaller t_r . - t_s gives an indication of how quickly the system's output settles within a certain percentage of the final value. A shorter t_s indicates a quicker settling and potentially a quicker rise time as well.

Given these relationships, while we can infer that a system with a shorter t_p and t_s is generally "faster" in response, directly concluding that $t_r \leq 0.18$ sec based solely on the given $t_p = 0.5$ sec and $t_s = 1.5$ sec requires solving for ω_n and then evaluating the approximate relationship for t_r .

Therefore, without the explicit calculation showing t_r based on derived ω_n and ζ , we cannot conclusively say whether $t_r \leq 0.18$ sec just from t_p and t_s with high confidence. However, based on typical relationships between these time parameters in second-order systems, it might suggest a system that could potentially meet the $t_r \leq 0.18$ sec criterion if ω_n is sufficiently high.

Confidence Level: 50%

Reasoning error

Claude Answer :

To determine if the given second-order system satisfies the rising time requirement, we need to calculate the natural frequency (ω_n) and damping ratio (ζ) using the peak time (t_p) and settling time (t_s). Then, we can calculate the rising time (t_r) and compare it with the given requirement.

Given: - Peak time, $t_p = 0.5$ sec - Settling time (5- Required rising time, $t_r \leq 0.18$ sec

Step 1: Calculate the damping ratio (ζ) using the settling time formula. $t_s = \frac{3}{\zeta\omega_n} \quad 1.5 = \frac{3}{\zeta\omega_n}$
 $\zeta\omega_n = 2$

Step 2: Calculate the natural frequency (ω_n) using the peak time formula. $t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \quad 0.5 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$
 $\omega_n\sqrt{1-\zeta^2} = \frac{\pi}{0.5} = 2\pi$

Step 3: Substitute $\zeta\omega_n = 2$ into the equation for ω_n . $\omega_n = \frac{\pi}{0.5\sqrt{1-(\frac{2}{\omega_n})^2}}$

Solve this equation numerically to find $\omega_n \approx 7.2552$ rad/s.

Step 4: Calculate the damping ratio (ζ). $\zeta = \frac{2}{\omega_n} \approx \frac{2}{7.2552} \approx 0.2758$

Step 5: Calculate the rising time (t_r) using the approximate formula. $t_r \approx \frac{1.8}{\omega_n} = \frac{1.8}{7.2552} \approx 0.2482$ sec

Conclusion: The calculated rising time ($t_r \approx 0.2482$ sec) is greater than the required rising time ($t_r \leq 0.18$ sec). Therefore, this second-order system does not satisfy the given rising time requirement.

Confidence level: 95%

Correct