

12.4 Design a Stable Controller

Given a system's transfer function $G(s) = \frac{12}{3s+5}$, use loop-shaping procedure to design a controller $K(s)$ such that the closed loop system is stable and it satisfies:

- (a) has a loop crossover frequency near 20 rad/sec.
- (b) gain less than 0.01 from reference r to error e for frequencies below 0.1 rad/sec.
- (c) gain less than 0.04 from input n to output y for frequencies above 200 rad/sec.

Verify that the closed-loop is stable with your control design.

Solution :

We can use a proportional gain to set the loop cross-over frequency (requirement (a)). Then use an integral boost and roll-off to satisfy requirements (b) and (c) respectively. This may require some iteration to get values that meet all design specifications.

First we choose the proportional gain so that the loop gain has crossover $|L(j\omega_c)| = |K(j\omega_c)G(j\omega_c)| = 1$ at $\omega_c = 20$ rad/sec. This can be done with the gain $K_1 = \frac{1}{|G(j\omega_c)|} \approx 5.02$. This will give the first loop-shape $L_1(s) = G(s)K_1$. The first loop shape has DC gain $L_1(s) = K_1G(s)$. Thus we need to increase the gain by a factor of $\frac{100}{12} = 8.4$ to ensure that we satisfy requirement (b). We'll use an integral boost $K_2(s) = \frac{s+\bar{\omega}_2}{s}$ where $\bar{\omega}_2$ is the frequency below which the gain starts to increase. With some iteration the choice $\bar{\omega}_2 = 5$ rad/sec seem to easily exceed the low frequency requirement. Notice that $\bar{\omega}_2 < \omega_c$. For stability and robustness reasons we need the slope of $|L|$ to be shallow near ω_c . Thus we need to choose our low-frequency boost to start sufficiently below ω_c that it doesn't have a significant impact on the slope at ω_c . After this stage our controller is $K_1K_2(s)$ and our loop-shape is $L_2(s) = K_1K_2(s)G(s)$. Finally, we need to modify the loop shape to satisfy the design requirement iv. We can choose a roll-off to decrease the high frequency gain $K_3(s) = \frac{\bar{\omega}_3}{s+\bar{\omega}_3}$. Again, we want to choose $\bar{\omega}_3 > \omega_c$ so that the roll-off has negligible effect on the slope of $|L|$ near cross-over. However, we also want to choose $\bar{\omega}_3$ small enough that we decrease the high frequency gain enough to satisfy $|L(j200)| \leq 0.0385$. After some trial and error $\bar{\omega}_3 = 80$ rad/sec ensures that the requirements are satisfied. Our final controller is $K(s) = K_1K_2(s)K_3(s)$ and our final loop-shape is $L_2(s) = K(s)G(s)$. The Bode plot and step responses are shown below. The step response has a small steady state error and good noise rejection. The response also has a small overshoot.

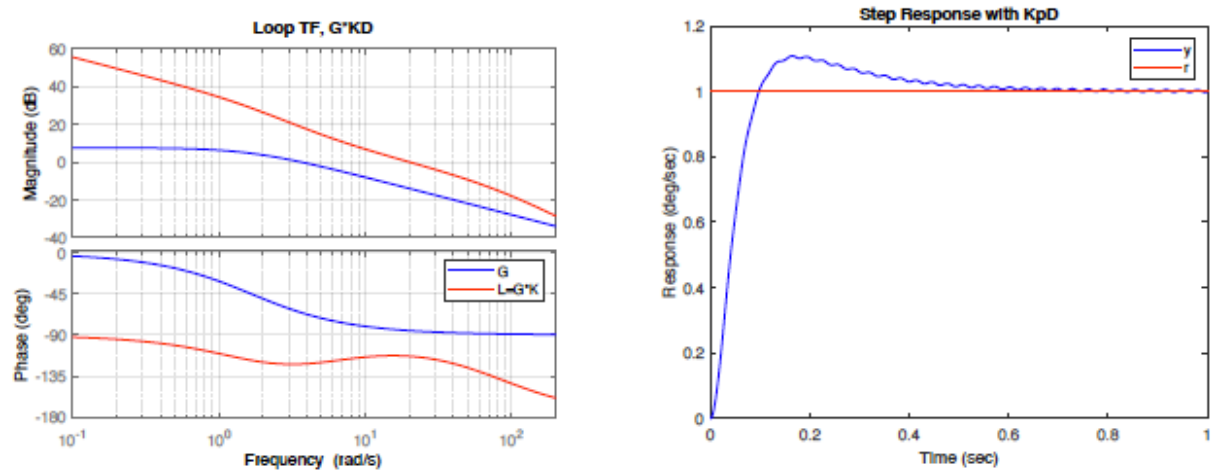


Figure 26: Bode Plot and step response for closed loop system.

GPT Answer:

To address this control system design challenge using the loop-shaping procedure, we'll construct a controller $K(s)$ that meets the specified requirements with respect to the given plant transfer function $G(s) = \frac{12}{3s+5}$. The design goals are focused on achieving a specific loop crossover frequency, and controlling the gain from reference to error and input to output across different frequency ranges.

Design Approach

- Loop Crossover Frequency near 20 rad/sec:** The loop crossover frequency ω_c is the frequency at which the magnitude of the open-loop transfer function $L(s) = K(s)G(s)$ equals 1 (or 0 dB). To achieve a crossover frequency near 20 rad/sec, we adjust $K(s)$ so that the magnitude of $L(j\omega)$ reaches 0 dB at $\omega = 20 \text{ rad/sec}$.
- Gain less than 0.01 from reference r to error e for frequencies below 0.1 rad/sec:** The transfer function from reference to error is given by $T_{re}(s) = \frac{e(s)}{r(s)} = \frac{1}{1+K(s)G(s)}$. To satisfy this condition, we ensure that $|T_{re}(j\omega)| < 0.01$ for $\omega < 0.1 \text{ rad/sec}$.
- Gain less than 0.04 from input n to output y for frequencies above 200 rad/sec:** The transfer function from input n to output y is given by $T_{ny}(s) = \frac{Y(s)}{N(s)} = \frac{K(s)G(s)}{1+K(s)G(s)}$. To satisfy this condition, we ensure that $|T_{ny}(j\omega)| < 0.04$ for $\omega > 200 \text{ rad/sec}$.

Controller Design

Given the plant $G(s) = \frac{12}{3s+5}$, a simple proportional controller may not suffice to meet all the design requirements due to the complexity of the desired frequency response characteristics. Instead, we consider a more complex controller form, such as $K(s) = k \cdot \frac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)}$, where k , z_1 , z_2 , p_1 , and p_2 are design parameters to be determined.

- To achieve the loop crossover frequency near 20 rad/sec, we ensure that the open-loop transfer function $L(s)$ has a phase margin greater than 0° at $\omega = 20 \text{ rad/sec}$ and that its magnitude is near 0 dB. - To satisfy the low-frequency gain requirement from r to e , we may need to introduce a high-pass filter characteristic in $K(s)$ to attenuate low-frequency signals. - To meet the high-frequency gain requirement from n to y , we ensure $K(s)G(s)$ rolls off sufficiently fast beyond 200 rad/sec, possibly by ensuring $K(s)$ includes enough high-frequency attenuation.

Stability Verification

Once $K(s)$ is designed, we verify the closed-loop stability by checking the poles of the closed-loop transfer functions $T_{re}(s)$ and $T_{ny}(s)$. The closed-loop system is stable if all poles have negative real parts.

Response Confidence Level: 75%

Did not solve

Claude Answer:

Given:

$G(s) = 12 / (3s + 5)$ Step 1: Shape the open-loop transfer function to achieve a crossover frequency near 20 rad/sec. To achieve this, we can use a simple gain controller $K(s) = K$. The open-loop transfer function becomes: $L(s) = G(s)K(s) = 12K / (3s + 5)$

To find the gain K that results in a crossover frequency near 20 rad/sec, we set $|L(j\omega)| = 1$ at $\omega = 20$: $|L(j20)| = |12K / (3j20 + 5)| = 1$ $K = |(3j20 + 5) / 12| \approx 5.14$

Step 2: Ensure the gain from reference r to error e is less than 0.01 for frequencies below 0.1 rad/sec. The transfer function from reference r to error e is given by: $E(s) / R(s) = 1 / (1 + L(s))$

At low frequencies ($\omega \ll 1$), we can approximate $L(s)$ as: $L(s) \approx 12K / 5 = 12.34$ (using $K = 5.14$ from Step 1)

Therefore, $|E(s) / R(s)| \approx 1 / (1 + 12.34) \approx 0.075$ at low frequencies, which is greater than the required 0.01. To reduce this gain, we can introduce an integrator in the controller $K(s)$: $K(s) = 5.14 (1 + 1/(0.01s))$

Step 3: Ensure the gain from input n to output y is less than 0.04 for frequencies above 200 rad/sec. The transfer function from input n to output y is given by: $Y(s) / N(s) = L(s) / (1 + L(s))$

At high frequencies ($\omega \gg 1$), we can approximate $L(s)$ as: $L(s) \approx 12K / 3s = 20.56 / s$ (using $K = 5.14$ from Step 1)

Therefore, $|Y(s) / N(s)| \approx 20.56 / (s + 20.56)$ at high frequencies. To reduce this gain below 0.04 at frequencies above 200 rad/sec, we can introduce a pole in the controller $K(s)$ at a frequency higher than 200 rad/sec. Let's choose a pole at 500 rad/sec: $K(s) = 5.14 (1 + 1/(0.01s)) / (1 + s/500)$

Final controller design: $K(s) = 5.14 (1 + 1/(0.01s)) / (1 + s/500)$

Stability verification: To check the closed-loop stability, we can use the Nyquist stability criterion. The open-loop transfer function with the designed controller is: $L(s) = G(s)K(s) = (12 \times 5.14 (1 + 1/(0.01s))) / ((3s + 5)(1 + s/500))$

The Nyquist plot of $L(s)$ should not encircle the -1 point for the closed-loop system to be stable. You can plot the Nyquist diagram using MATLAB or any other suitable software to verify the stability of the designed system.

Confidence Level: 90%

Correct