

1.3 Undershoot

Consider a system with a transfer function given by

$$H(s) = \frac{2}{s+1} + \frac{\alpha}{s+2} \quad (14)$$

where α is a real number. Is there a range of real values for α such that the system's unit step response exhibits undershoot?

Solution :

We first note that if $\alpha > 0$, the unit step response is always positive, thus no undershoot occurs.

To have undershoot in the unit step response, it is sufficient that a linear system have a real zero in the open Right Half Plane (RHP) – a Non-Minimum Phase (NMP) zero. To investigate that possibility, we add the two fractions in $H(s)$ to yield:

$$H(s) = \frac{2s + 4 + \alpha s + \alpha}{(s+1)(s+2)} = (4 + \alpha) \frac{\frac{2+\alpha}{4+\alpha}s + 1}{(s+1)(s+2)} \quad (15)$$

Then, we have a NMP zero if and only if $-4 < \alpha < -2$.

GPT Answer :

To address whether the system's unit step response exhibits undershoot, we need to examine the system's response characteristics, particularly focusing on the behavior determined by its poles and zeroes and how these relate to the damping ratio and natural frequency of the system.

The given transfer function is:

$$H(s) = \frac{2}{s+1} + \frac{\alpha}{s+2}$$

This system is a sum of two first-order systems, each of which can be analyzed independently. The first term, $\frac{2}{s+1}$, represents a system with a pole at $s = -1$, and the second term, $\frac{\alpha}{s+2}$, represents a system with a pole at $s = -2$. Both poles are real and negative, indicating that each subsystem is stable.

For a system to exhibit undershoot in its step response, it typically needs to have a complex conjugate pair of poles with a non-zero imaginary part, which indicates oscillatory behavior. The damping ratio (ζ) of the system affects the degree of undershoot; specifically, for a second-order system or higher, undershoot occurs when $0 < \zeta < 1$.

Given that both poles of the system described by $H(s)$ are real and distinct, there are no complex conjugate pairs, and thus, no oscillatory behavior intrinsic to this system's natural response. Therefore, the system's unit step response will not exhibit undershoot for any real value of α , as undershoot is associated with systems that have at least one pair of complex conjugate poles.

Confidence Level: 95%

This conclusion is drawn based on the nature of the poles of the system and the fundamental principles of control theory regarding system response to input signals.

Incorrect Knowledge**Claude Answer :**

For undershoot to occur in the unit step response, the transfer function should have a zero in the right half-plane (RHP). In this case,

$$H(s) = \frac{2}{s+1} + \frac{\alpha}{s+2} = \frac{2s+4+\alpha s+\alpha}{(s+1)(s+2)}.$$

The zero is located at $s = -\frac{4+\alpha}{2+\alpha}$. For the zero to be in the RHP, $-\frac{4+\alpha}{2+\alpha} > 0$.

Solving the inequality yields:

$$4 + \alpha < 0 \text{ and } 2 + \alpha > 0, \text{ or } 4 + \alpha > 0 \text{ and } 2 + \alpha < 0.$$

The solution that satisfies both conditions is: $-4 < \alpha < -2$.

Therefore, the system will exhibit undershoot in its unit step response for $-4 < \alpha < -2$.

Correct