

## 1.27 Second-Order Systems

Consider a second-order system with peak time  $t_p = 0.5$  sec. and 5% settling time  $t_s = 1.5$  sec. Determine the poles of this system.

**Solution:**

The poles of this system are given by  $s = -\sigma \pm j\omega_d$  where  $\sigma = \zeta\omega_n$  and  $\omega_d = \omega_n\sqrt{1-\zeta^2}$ .

Given information  $t_p = 0.5$  sec and 5% settling time  $t_s = 1.5$  sec leads to  $\omega_d = 2\pi$  and  $\sigma = 2$ . Therefore the poles of the system are  $s = -2 \pm 2\pi$

**GPT Answer :**

The calculations yield the damping ratio  $\zeta = 0.391$  and the natural frequency  $\omega_n = 6.826$  rad/sec. Using these values, the poles of the system are located at  $-2.667 + 6.283i$  and  $-2.667 - 6.283i$ .

Therefore, the poles of this second-order system are complex conjugates, indicating an under-damped system.

**Confidence Level: 95%**

**Calculation error**

**Claude Answer**

To determine the poles of the second-order system, we need to calculate the natural frequency ( $\omega_n$ ) and the damping ratio ( $\zeta$ ) using the given peak time ( $t_p$ ) and settling time ( $t_s$ ).

Given: - Peak time,  $t_p = 0.5$  sec

- Settling time (5%),  $t_s = 1.5$  sec

Step 1: Calculate the damping ratio ( $\zeta$ ) using the settling time. For a second-order system, the settling time is related to the damping ratio by the following equation:  $t_s \approx \frac{3}{\zeta\omega_n}$

Step 2: Calculate the natural frequency ( $\omega_n$ ) using the peak time. The peak time is related to the damping ratio and natural frequency by the following equation:  $t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

Step 3: Solve for  $\zeta$  and  $\omega_n$ . From the peak time equation:  $0.5 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$   $\omega_n = \frac{\pi}{0.5\sqrt{1-\zeta^2}}$

Substituting this into the settling time equation:  $1.5 \approx \frac{3}{\zeta \frac{\pi}{0.5\sqrt{1-\zeta^2}}}$   $1.5 \approx \frac{3 \cdot 0.5\sqrt{1-\zeta^2}}{\zeta\pi}$   $1.5\zeta\pi \approx$

$$1.5\sqrt{1-\zeta^2} \zeta^2\pi^2 \approx 1 - \zeta^2 \quad \zeta^2(\pi^2 + 1) \approx 1 \quad \zeta \approx \sqrt{\frac{1}{\pi^2 + 1}} \approx 0.3015$$

Now, calculate  $\omega_n$ :  $\omega_n = \frac{\pi}{0.5\sqrt{1-0.3015^2}} \approx 6.6579$  rad/s

Step 4: Calculate the poles using  $\zeta$  and  $\omega_n$ . The poles of a second-order system are given by:  $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

Substituting the values:  $s_{1,2} = -0.3015 \cdot 6.6579 \pm j \cdot 6.6579\sqrt{1-0.3015^2}$   $s_{1,2} = -2.0074 \pm j \cdot 6.2078$

Therefore, the poles of the given second-order system are  $s_1 = -2.0074 + j6.2078$  and  $s_2 = -2.0074 - j6.2078$ .

**Confidence level: 95%**

**Correct**