

1.18 Time-Delayed ODEs: Transfer Function Approach

A particular system containing a time delay has the differential equation $\frac{dy(t)}{dt} + y(t) = u(t - T)$. Find the transfer function of this system.

Solution :

The Laplace transform of the differential equation ignoring terms due to initial conditions, is $sY(s) + Y(s) = e^{-sT}U(s)$. The system transfer function would be

$$P(s) = \frac{Y(s)}{U(s)} = \frac{e^{-sT}}{s + 1} \quad (61)$$

GPT Answer:

To find the transfer function of the system described by the differential equation $\frac{dy(t)}{dt} + y(t) = u(t - T)$, where T is the time delay, we first take the Laplace transform of both sides of the equation. Remember, the Laplace transform of $y(t)$ is $Y(s)$ and for $u(t - T)$ with a time delay T , it is $U(s)e^{-Ts}$.

The Laplace transform of the left side, using the linearity property, gives:

$$\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} + \mathcal{L}\{y(t)\} = sY(s) - y(0) + Y(s)$$

Given that initial conditions are typically zero for finding transfer functions ($y(0) = 0$), the equation simplifies to:

$$sY(s) + Y(s) = U(s)e^{-Ts}$$

Combining like terms yields:

$$(s + 1)Y(s) = U(s)e^{-Ts}$$

The transfer function $H(s)$ is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input, under zero initial conditions. Thus, we have:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{e^{-Ts}}{s + 1}$$

Therefore, the transfer function of the system is $\frac{e^{-Ts}}{s+1}$.

Confidence level: 95%

Correct

Claude Answer:

To find the transfer function, we take the Laplace transform of the differential equation:

$$\mathcal{L}\left\{\frac{dy(t)}{dt} + y(t)\right\} = \mathcal{L}\{u(t - T)\}$$

$$sY(s) - y(0) + Y(s) = U(s)e^{-sT}$$

$$(s + 1)Y(s) - y(0) = U(s)e^{-sT}$$

Assuming zero initial conditions, $y(0) = 0$:

$$(s + 1)Y(s) = U(s)e^{-sT}$$

$$\frac{Y(s)}{U(s)} = \frac{e^{-sT}}{s + 1}$$

$$\boxed{G(s) = \frac{e^{-sT}}{s + 1}}$$

Confidence level: 95/100

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Correct