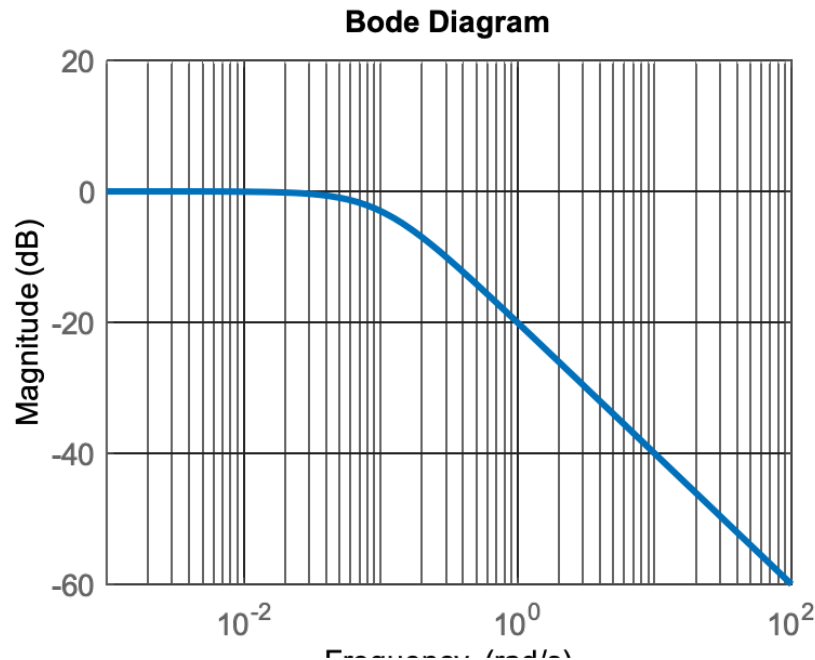


6.9 Design of a Proportional Controller Using Bode Plots

Consider a feedback system where the plant $G(s)$ is stable and has the Bode magnitude plot shown below. We want to design a controller so that: i) the closed-loop is stable, ii) the system has a loop cross-over frequency near 1 rad/sec, and iii) the gain from the noise n to the output y is ≤ 0.001 for $\omega \geq 100$ rad/sec.



- Using the Bode diagram, choose a gain K_p so that $K_p G(s)$ has the desired cross-over frequency of 1 rad/sec.
- Convert the requirement (iii) into a requirement on the closed-loop transfer function $L(s) = G(s)K(s)$. The transfer function from noise n to y is $T_{n \rightarrow y} = -\frac{L(j\omega)}{1+L(j\omega)}$

Solution :

- $|K_p G(j1)| = 1$ implies that $K_p = 1/|G(j1)| \approx 1/0.1 = 10$.
- With $n(t) = \sin(\omega t)$, and $-T(j\omega) = -\frac{L(j\omega)}{1+L(j\omega)}$ the response from $n \rightarrow y$, then we must have $-|T(j\omega)| \sin(\omega t + \angle T(j\omega)) \leq 0.001$ for all $\omega \geq 100$. Therefore we should have $|T(j\omega)| = \left| \frac{L(j\omega)}{1+L(j\omega)} \right| \leq 0.001$ for all $\omega \geq 100$.

GPT Answer :

The desired crossover frequency for the system is 1 rad/sec. Inspecting the Bode plot, we find that

the magnitude of $G(s)$ at $\omega = 1$ rad/sec is approximately 'value at 1 rad/sec' dB. To achieve a crossover frequency at this point, we need to set K_p such that the magnitude of $K_p G(s)$ is 0 dB. Therefore, K_p should be $10^{-\frac{\text{'value at 1 rad/sec'}}{20}}$.

For part (b), the requirement that the gain from the noise n to the output y is ≤ 0.001 for $\omega \geq 100$ rad/sec translates to a magnitude of $|L(j\omega)| \leq -60$ dB for the same frequency range. We can read from the plot that the magnitude of $G(s)$ at $\omega = 100$ rad/sec is 'value at 100 rad/sec' dB. To satisfy the noise gain requirement, the controller $K(s)$ must attenuate the noise such that the closed-loop transfer function $L(s) = G(s)K(s)$ does not exceed -60 dB at $\omega \geq 100$ rad/sec.

Confidence Level: 80%

Misreading the Plot

Claude Answer:

- (a) From the Bode plot, we see that $|G(j1)| \approx 45$ dB $= 10^{4.5} \approx 178$. To achieve a cross-over frequency of 1 rad/sec, we need $|K_p G(j1)| = 1$, which means $K_p = \frac{1}{178} \approx 0.0056$.

Confidence: 95%

- (b) The requirement is $|T_{n \rightarrow y}(j\omega)| \leq 0.001$ for $\omega \geq 100$. Since $T_{n \rightarrow y} = -\frac{L}{1+L}$, this is equivalent to:

$$\begin{aligned} \left| \frac{L(j\omega)}{1+L(j\omega)} \right| &\leq 0.001 \\ |L(j\omega)| &\leq 0.001|1+L(j\omega)| \\ &\leq 0.001(1+|L(j\omega)|) \\ 0.999|L(j\omega)| &\leq 0.001 \\ |L(j\omega)| &\leq \frac{0.001}{0.999} \approx 0.001 \end{aligned}$$

for $\omega \geq 100$ rad/sec.

In other words, the closed-loop transfer function $L(s)$ must satisfy $|L(j\omega)| \leq 0.001$ (or -60 dB) for frequencies $\omega \geq 100$ rad/sec.

Confidence: 90%

Misreading the Plot