

## 5.11 PID Controller

Consider the plant with the following transfer function:

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

The poles of this system are at  $s = -1, -10 \pm j$ , and the DC gain is 5.

- What is the dominant pole approximation  $G_a(s)$  for this plant?
- Would you recommend using a PI, PD, or PID controller?
- Choose the controller gains so that the closed-loop with  $G_a(s)$  has poles repeated at  $s = -1$

**Solution :**

- Set  $G_a(s) = \frac{b_0}{s+1}$ . Since  $G(0) = \frac{505}{101}$ , we have  $b_0 = 5$ . Thus,

$$G_a(s) = \frac{5}{s+1} \quad (230)$$

The time-domain approximation of  $G_a(s)$  is:

$$\dot{y}(t) + y(t) = 5u(t) \quad (231)$$

- PI controller
- Desired characteristic equation:

$$(s+1)^2 = 0$$

Using the following PI controller:

$$u(t) = K_p(r(t) - y(t)) + K_i \int (r(t) - y(t))$$

we have:

$$\ddot{y}(t) + (1 + 5K_p)\dot{y}(t) + 5K_i y(t) = 5K_p \dot{r}(t) + 5K_i r(t)$$

Thus:  $K_p = 0.2$  and  $K_i = 0.2$

**GPT Answer :**

Given the transfer function:

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101} \quad (232)$$

with poles at  $s = -1, -10 \pm j$ .

(a) The dominant pole approximation  $G_a(s)$  for this plant is given by:

$$G_a(s) = \frac{5}{s+1} \quad (233)$$

(b) A PID controller is recommended to enhance both the steady-state and dynamic performance of the system.

(c) Specifying the exact gains  $K_p$ ,  $K_i$ , and  $K_d$  for the PID controller to achieve closed-loop poles at  $s = -1$  requires further calculation specific to the system dynamics and desired pole locations.

**Confidence level: approximately 70%**

**Reasoning Error**

**Claude Answer :**

Consider the plant with the following transfer function:

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101}$$

The poles of this system are at  $s = -1$ ,  $-10 \pm j$ , and the DC gain is 5.

(a) The dominant pole approximation  $G_a(s)$  for this plant is:

$$G_a(s) = \frac{5}{s+1}$$

(b) For this system, a PI controller would be suitable to eliminate steady-state error and improve the system's response.

(c) To design the PI controller gains so that the closed-loop system with  $G_a(s)$  has repeated poles at  $s = -1$ , we use the following steps:

The closed-loop transfer function with a PI controller is:

$$\frac{C(s)G_a(s)}{1+C(s)G_a(s)}, \text{ where } C(s) = K_p + \frac{K_i}{s}$$

Substituting  $G_a(s)$  and  $C(s)$ :

$$\frac{(K_p s + K_i) \frac{5}{s+1}}{s + (K_p s + K_i) \frac{5}{s+1}}$$

To have repeated poles at  $s = -1$ , the characteristic equation should be:

$$s + (K_p s + K_i) \frac{5}{s+1} = (s+1)^2$$

Multiplying both sides by  $(s+1)$ :

$$s(s+1) + (K_p s + K_i)5 = (s+1)^2$$

Expanding the equations:

$$s^2 + s + 5K_p s + 5K_i = s^2 + 2s + 1$$

Comparing coefficients:

$$s^1 : 1 + 5K_p = 2 \quad s^0 : 5K_i = 1$$

From the  $s^1$  term:  $5K_p = 1$ , so  $K_p = \frac{1}{5} = 0.2$  From the  $s^0$  term:  $5K_i = 1$ , so  $K_i = \frac{1}{5} = 0.2$

Therefore, for the approximated system  $G_a(s)$ , the PI controller gains that achieve repeated poles at  $s = -1$  are:

$$K_p = 0.2 \text{ and } K_i = 0.2$$

**Confidence level: 95%**

**Correct**