

3.17 Steady State Response

Consider a first-order LTI system $G(s) = \frac{b_0}{s+a_0}$ where a_0 and b_0 are non-zero numbers. If the phase of $G(j\omega)$ at $\omega = 0$ is expressed in degrees, what values can it have?

Solution :

It can only be 0 degrees or 180 degrees.

GPT Answer:

To find the phase of the given first-order Linear Time-Invariant (LTI) system $G(s) = \frac{b_0}{s+a_0}$ at $\omega = 0$, we substitute s with $j\omega$ and evaluate at $\omega = 0$:

$$G(j\omega) = \frac{b_0}{j\omega + a_0}$$

At $\omega = 0$, this becomes:

$$G(0) = \frac{b_0}{a_0}$$

Since a_0 and b_0 are real and non-zero, the phase of $G(0)$ is dependent on the sign of $\frac{b_0}{a_0}$. The phase $\angle G(0)$ is:

- 0 degrees if $\frac{b_0}{a_0} > 0$ (i.e., b_0 and a_0 have the same sign) - 180 degrees or -180 degrees if $\frac{b_0}{a_0} < 0$ (i.e., b_0 and a_0 have opposite signs)

Thus, the phase of $G(j\omega)$ at $\omega = 0$ can be either 0 degrees or ± 180 degrees.

Confidence level: 95%

Correct

Claude Answer:

Let's approach this step-by-step.

1) First, let's understand what the phase of a complex number means. If a complex number is represented in polar form as $z = re^{j\theta}$, then θ is the phase of the complex number.

2) In our case, we want to find the phase of $G(j\omega)$ at $\omega = 0$. So, let's calculate $G(j\omega)$ first:

$$G(j\omega) = \frac{b_0}{j\omega + a_0}$$

3) Now, let's substitute $\omega = 0$:

$$G(j0) = \frac{b_0}{j0 + a_0} = \frac{b_0}{a_0}$$

4) $\frac{b_0}{a_0}$ is a real number because both a_0 and b_0 are non-zero real numbers. And the phase of a real number can only be either 0° or 180° .

5) If $\frac{b_0}{a_0} > 0$, then the phase is 0° . This happens when a_0 and b_0 have the same sign (both positive or both negative).

6) If $\frac{b_0}{a_0} < 0$, then the phase is 180° . This happens when a_0 and b_0 have opposite signs (one positive and one negative).

Therefore, the phase of $G(j\omega)$ at $\omega = 0$, when expressed in degrees, can only be either 0° or 180° , depending on whether a_0 and b_0 have the same or opposite signs.

Confidence level: 95%

Correct