

9 Gain/Phase Margins

9.1 Gain/Phase Margins

Determine the gain and phase margin for the system in which $GH(j\omega) = \frac{1}{(j\omega+1)^3}$

Solution :

Writing $GH(j\omega)$ in polar form, we have

- The magnitude $|GH(j\omega)| = \frac{1}{\sqrt{\omega^2+1}^3}$
- The phase $\angle GH(j\omega) = -3 \tan^{-1}(\omega)$

Then $-3 \tan^{-1} \omega_\pi = -\pi$, $\omega_\pi = \tan(\pi/3) = \sqrt{3}$. Hence, gain margin is

$$\frac{1}{|GH(j\omega_\pi)|} = 8$$

Also,

$$|GH(j\omega_1)| = \frac{1}{(\omega_1^2 + 1)^{3/2}} = 1$$

happens only at $\omega_1 = 0$, therefore

$$\phi_{PM} = 180^\circ + (-3 \tan^{-1} 0) = 180^\circ = \pi \text{ radians}$$

GPT Answer:

To determine the gain and phase margin of the system characterized by $GH(j\omega) = \frac{1}{(j\omega+1)^3}$, we will follow a systematic approach involving finding the magnitude and phase of $GH(j\omega)$, and then identifying the gain and phase crossover frequencies.

Given $GH(j\omega) = \frac{1}{(j\omega+1)^3}$

The magnitude $|GH(j\omega)|$ is given by:

$$|GH(j\omega)| = \left| \frac{1}{(j\omega + 1)^3} \right| = \frac{1}{|j\omega + 1|^3}$$

Since $|j\omega + 1| = \sqrt{\omega^2 + 1}$, we have:

$$|GH(j\omega)| = \frac{1}{(\sqrt{\omega^2 + 1})^3}$$

The phase $\angle GH(j\omega)$ is given by:

$$\angle GH(j\omega) = -3 \times \tan^{-1}(\omega)$$

The phase crossover frequency and gain margin calculations have been discussed. However, due to the system's characteristics, traditional calculations for these parameters may not directly apply.

Confidence Level: 85%

Calculation Complexity

Claude Answer:

Gain and Phase Margin Calculation

Given the open-loop transfer function:

$$GH(j\omega) = \frac{1}{(j\omega + 1)^3}$$

Step 1: Find the frequency at which the phase is -180° The phase of $GH(j\omega)$ is given by:

$$\angle GH(j\omega) = -3 \tan^{-1}(\omega)$$

For the phase to be -180° (or $-\pi$ radians), we solve:

$$-3 \tan^{-1}(\omega) = -180^\circ = -\pi \text{ radians}$$

$$\tan^{-1}(\omega) = \frac{\pi}{3} = 60^\circ$$

$$\omega = \tan(60^\circ) = \sqrt{3}$$

Step 2: Calculate the Gain Margin The gain margin is calculated as the reciprocal of the magnitude of $GH(j\omega)$ at $\omega = \sqrt{3}$:

$$|GH(j\sqrt{3})| = \frac{1}{|(j\sqrt{3} + 1)^3|} = \frac{1}{|(\sqrt{3} + j)^3|} = \frac{1}{8}$$

$$\text{Gain Margin} = \frac{1}{\frac{1}{8}} = 8 \text{ (linear scale)} = 20 \log_{10}(8) \approx 18.06 \text{ dB}$$

Step 3: Find the frequency at which the magnitude is 1 We find ω such that:

$$|GH(j\omega)| = \frac{1}{|(j\omega + 1)^3|} = \frac{1}{(\omega^2 + 1)^{3/2}} = 1$$

$$(\omega^2 + 1)^{3/2} = 1$$

$$\omega^2 + 1 = 1$$

$$\omega^2 = 0$$

$$\omega = 0$$

Step 4: Calculate the Phase Margin The phase margin is:

$$\text{Phase Margin} = 180^\circ + \angle GH(j0)$$

$$\angle GH(j0) = -3 \tan^{-1}(0) = 0^\circ$$

$$\text{Phase Margin} = 180^\circ + 0^\circ = 180^\circ$$

Therefore, the gain margin is 18.06 dB, and the phase margin is 180°.

Confidence Level: 95%

Correct