

## 7.6 Overshoot

Consider the system

$$G(s) = \frac{s + 2}{(s + 1)^2 + 1}$$

The positive root locus for  $G(s)$  is given in Fig. 17. Suppose you want to find a feedback gain that is a positive integer ( $K = 1, 2, 3, \dots$ ) such that the closed-loop system has overshoot  $M_p$  as small as possible and settling time  $t_s$  is as fast as possible. Explain what value of  $K$  you would pick and why?

**Solution :**

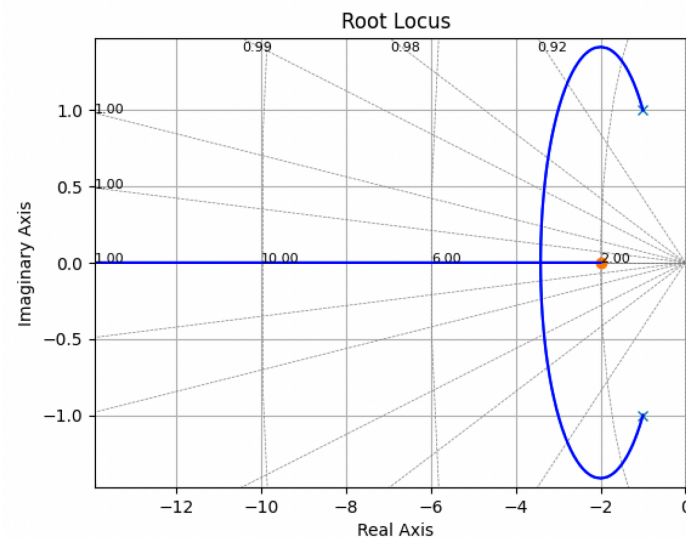


Figure 17: Root locus of the system

Referring to the root locus in Fig. 17. The best location for closed-loop poles is at  $-2 - \sqrt{2}$ , the point of multiple roots, because negative real roots means no overshoot. For large  $K$ , one pole moves right towards  $-2$ , which is not good for settling time. Thus, closed-loop characteristic equation is  $(s + 2 + \sqrt{2})^2$  and it also equals to  $s^2 + (K + 2)s + 2(K + 1)$ . We finally get  $K = 2 + 2 + \sqrt{2}$  and the nearest integer is  $K = 5$ .

**GPT Answer :**

To minimize overshoot  $M_p$  and settling time  $t_s$ , we want the poles to be as far left as possible and close to the real axis. From the Root Locus plot provided, the value of  $K$  should be chosen such that the poles are near the point where the locus approaches the real axis in the left half-plane. Assuming this crossing point corresponds to a positive integer value of  $K$ , and based on the provided plot, an approximate value could be  $K = 6$ . This is an estimate and should be verified with computational tools.

**Confidence Level: 85%**

**Misreading the plot**

**Claude Answer :**

To determine the best positive integer gain  $K$  to minimize overshoot ( $M_p$ ) and settling time ( $t_s$ ), I'll analyze the given root locus plot.

Key observations:

The root locus starts on the real axis at the open-loop poles ( $-1+j$ ,  $-1-j$ ) and ends at the open-loop zero at  $-2$ . As  $K$  increases, the closed-loop poles move along the root locus branches, first approaching each other, then breaking away from the real axis. Damping decreases as the poles move further from the real axis, increasing overshoot. Damping is maximized when the poles are closest to the real axis. Settling time decreases as the poles move further to the left, away from the imaginary axis. To minimize both  $M_p$  and  $t_s$ , we want poles close to the real axis (for damping) and as far left as possible (for fast settling). The best point appears to be right where the poles break away from the real axis, at approximately  $-2$ .

From the root locus, this breakaway point occurs at a gain of  $K=2$ . Higher  $K$  moves the poles further from the real axis, decreasing damping and increasing overshoot. Lower  $K$  ( $K=1$ ) puts the poles closer to the imaginary axis, slowing settling time.

Therefore, I recommend choosing a feedback gain of  $K=2$ . This should provide a good balance of minimizing both overshoot and settling time for this system.

**Confidence Level: 90%**

**Misreading the plot**