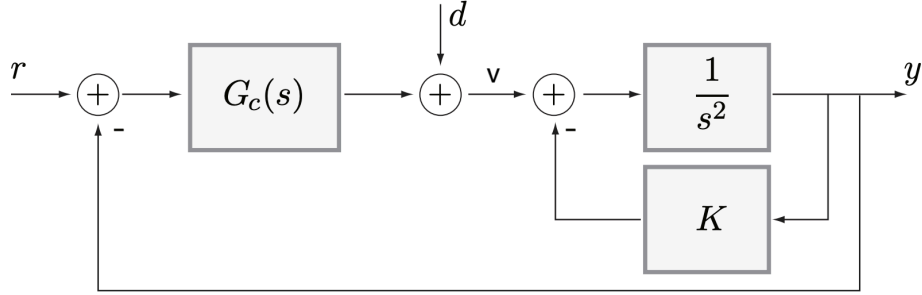


4.4 Tracking a Ramp Reference

The plant transfer function $G_p(s) = \frac{1}{s^2}$ shown below describes the angle of a pendulum without damping.



- What conditions must G_c satisfy to ensure that the system can track a ramp reference input with finite steady-state error?
- Find a compensator that satisfies the conditions of (a). What class of disturbances $d(t)$ will the system reject perfectly?

Solution :

- The plant transfer function G_p is given by:

$$G_p(s) = \frac{Y}{V} = \frac{1}{s^2 + K}$$

Let $G = G_c(s)G_p(s)$ then with $e = r - y$, we have $\frac{E}{R} = \frac{1}{1+G}$.

Since $r(t) = t \implies R(s) = \frac{1}{s^2}$, the steady state error is given by:

$$e_\infty = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} \frac{1}{s(1 + G(s))}$$

Therefore, $G_c(s)$ must have an integrator controller and we can choose a PID controller.

- We can try a PID controller with $G_c(s) = K_p + \frac{K_i}{s} + K_d s$. The closed-loop transfer function $\frac{Y}{d}$ is given by:

$$\frac{Y}{d} = \frac{G_p}{1 + G} = \frac{s}{s^3 + K_d s^2 + (K + K_p)s + K_i}$$

In order to place all the poles at -1, we set $K_i = 1$, $K_p = 3 - K$ and $K_d = 3$. This PID compensator will perfectly reject DC disturbances.

GPT Answer :

(a) For a system to track a ramp reference input (which is a type of first-order input as its Laplace transform is $\frac{1}{s^2}$) with finite steady-state error, the system's type number must be at least 2. This is achieved by ensuring that the open-loop transfer function has at least two poles at the origin, i.e., the system must have at least two integrators. The plant $G_p(s) = \frac{1}{s^2}$ already includes two integrators. Hence, the controller $G_c(s)$ does not need to add more integrators to achieve a system type of 2. However, the controller must not introduce any zeros that would cancel the poles at the origin since this would reduce the system type.

(b) A simple compensator that would satisfy the conditions mentioned in part (a) is a proportional controller $G_c(s) = K_p$ where $K_p > 0$. This compensator would not alter the number of poles at the origin and hence would not affect the system's ability to track a ramp input with finite steady-state error.

The class of disturbances that the system will reject perfectly are constant disturbances (step-type), as the system with a proportional controller still maintains a type 1 system for disturbance rejection. This is due to the fact that the open-loop transfer function with respect to the disturbance $d(t)$ will have one integrator due to the plant, which is sufficient to reject step disturbances in steady state.

Confidence Level: 85%

Misreading the plot (in this case the block diagram)

Claude Answer :**(a) Conditions for $G_c(s)$ to track a ramp input with finite steady-state error:**

For the system to track a ramp input ($r(t) = t \cdot u(t)$, where $u(t)$ is the unit step function) with finite steady-state error, the compensator $G_c(s)$ must contain at least one integrator (a pole at $s = 0$). This ensures that the open-loop transfer function has a pole at the origin, which is necessary for a finite steady-state error when tracking a ramp input.

(b) Compensator design and disturbance rejection:

A simple compensator that satisfies the condition in (a) is a proportional-integral (PI) controller:

$$G_c(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s}$$

where K_p and K_i are the proportional and integral gains, respectively.

With this compensator, the open-loop transfer function becomes:

$$G_{OL}(s) = G_c(s) \cdot G_p(s) = \frac{K_p s + K_i}{s} \cdot \frac{1}{s^2} = \frac{K_p s + K_i}{s^3}$$

The closed-loop transfer function from the disturbance $d(t)$ to the output $y(t)$ is:

$$\frac{Y(s)}{D(s)} = \frac{1}{1 + G_{OL}(s)} = \frac{s^3}{s^3 + K_p s + K_i}$$

The system will perfectly reject step disturbances ($d(t) = u(t)$) because the numerator of the disturbance transfer function has a higher degree than the disturbance input.

Confidence level: 90%

Misreading the plot (Do not detect K in the loop of the system) and also do not detect that using PI controller can not stabilize the system (Reasoning Error)