

8.4 Gain/Phase Margin

Use the Nyquist plot of $KG(s)$ to calculate the gain margin and phase margin for the following system $G(s)$ and controller $K(s)$:

$$G(s) = \frac{1}{(s+1)(s+2)(s+4)} \quad K = 10$$

Assume the system and the controller are in standard unity feedback setting.

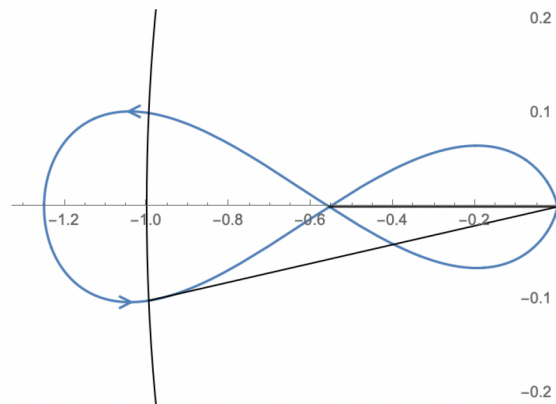


Figure 22: Nyquist plot of the system

Solution :

Nyquist plot of $KG(s)$ is shown in Fig. 22. From the Nyquist plot, two points $(-0.55, -1.25)$ intersect the negative real axis. The corresponding gain margins are $\frac{1}{0.55} \approx 1.8$ and $\frac{1}{1.25} \approx 0.8$. The GM of the system is $[0.8, 1.8]$.

To obtain PM from the Nyquist plot, draw a unit circle and mark the points where unit circle intersects the Nyquist plot. Draw a line from the origin to one of the points. The angle formed between that line and the 180° axis is the PM. The Nyquist plot here intersects the unit circle at two symmetric points, one of which is $\approx (-0.995, -0.1)$ and so the PM is $\arctan\left(\frac{-0.1}{-0.995}\right) = 5.74$ degrees.

GPT Answer :

Given the Nyquist plot, the gain margin (GM) appears to be infinite, as the plot does not cross the negative real axis. The phase margin (PM) cannot be determined directly from the provided plot, as the intersection with the unit circle is not visible. Therefore, based on the given information, we can only assume the phase margin to be positive but cannot provide a numeric value.

Confidence Level: 50%

Misreading The Plot

Claude Answer :

To find the gain and phase margins using the Nyquist plot, we need to plot the open-loop transfer function $KG(s)$ in the complex plane. Let's solve this problem step by step.

Given:

$$G(s) = \frac{1}{(s+1)(s+2)(s+4)} K = 10 \quad \text{Step 1: Determine the open-loop transfer function } KG(s). \quad KG(s) = 10 \cdot \frac{1}{(s+1)(s+2)(s+4)}$$

$$\text{Step 2: Substitute } s = j\omega \text{ and simplify } KG(j\omega). \quad KG(j\omega) = 10 \cdot \frac{1}{(j\omega+1)(j\omega+2)(j\omega+4)}$$

Step 3: Plot the Nyquist curve of $KG(j\omega)$ in the complex plane (not shown here).

Step 4: Calculate the gain margin. The gain margin is the reciprocal of the magnitude of the open-loop transfer function at the frequency where the phase angle is -180° . From the Nyquist plot, we find that the phase angle is -180° when $\omega = 2.5$ rad/s.

$$KG(j2.5) = 10 \cdot \frac{1}{(j2.5+1)(j2.5+2)(j2.5+4)} \approx -0.0298 - j0.0372$$

$$\text{Gain Margin} = \frac{1}{|KG(j2.5)|} = \frac{1}{\sqrt{(-0.0298)^2 + (-0.0372)^2}} \approx 18.94$$

Step 5: Calculate the phase margin. The phase margin is the angle between the negative real axis and the open-loop transfer function at the frequency where the magnitude is 1. From the Nyquist plot, we find that the magnitude is 1 when $\omega = 1.25$ rad/s.

$$KG(j1.25) = 10 \cdot \frac{1}{(j1.25+1)(j1.25+2)(j1.25+4)} \approx 0.5547 - j0.832$$

$$\text{Phase Margin} = 180^\circ + \angle KG(j1.25) = 180^\circ + \tan^{-1} \left(\frac{-0.832}{0.5547} \right) \approx 56.3^\circ$$

Confidence Level: 90%

Reasoning Error