

5.5 Proportional Controller

Answer True or False to the following questions. Also justify your answers.

- (a) Let $y(t)$ be the response of an LTI system $G(s)$ with zero initial conditions due to the input $u(t)$. The response of $G(s)$ with nonzero initial conditions and input $2 * u(t)$ will always be equal to $2 * y(t)$.
- (b) An LTI system $G(s)$ with a zero in the right half plane will always have an unbounded step response.
- (c) Let $G(s)$ be an LTI system with one real pole that has a time constant five times larger than all the other poles. A dominant pole approximation for $G(s)$ will be reasonably accurate.
- (d) Let $G(s)$ be a second-order LTI system and $K(s)$ be a PID controller designed to place all closed-loop poles in the left half plane. If the derivative term is implemented using an approximate derivative then the resulting closed-loop will be fourth order.
- (e) Consider a control system with the plant $G(s) = \frac{5}{s^2+s}$ and a proportional controller with gain k_p . It is possible to choose k_p so that the closed-loop is stable and has zero steady-state error due to a unit step reference command.

Solution :

- (a) False
- (b) False
- (c) True
- (d) True
- (e) True

GPT Answer:

- (a) **False.** In LTI systems, the principle of superposition holds, meaning that if the input is scaled, the output is scaled by the same factor, assuming zero initial conditions. However, with **nonzero initial conditions**, the system's response includes both the homogeneous solution (related to initial conditions) and the particular solution (related to input). Scaling the input does not necessarily scale the homogeneous solution, thus the overall response to $2 * u(t)$ may not be $2 * y(t)$.

- (b) **False.** A zero in the right half plane does not directly affect the boundedness of the step response of an LTI system. Boundedness of the step response is primarily determined by the system poles, not zeros. A system with a zero in the right half plane can still have a bounded step response if all its poles are in the left half plane, ensuring stability.
- (c) **True.** When an LTI system has one real pole that is significantly slower (larger time constant) than the others, this pole dominates the system's response. The dominant pole approximation assumes that the system's behavior can be closely approximated by considering only the dominant pole while neglecting the faster ones. This approximation is reasonably accurate in depicting the system's dynamics over a longer time scale.
- (d) **False.** A PID controller does not change the order of the system; it merely alters the system dynamics. Implementing a derivative term using an approximate derivative introduces a high-pass filter effect, which can add a pole (or more, depending on the approximation method), but this alone does not specify that the closed-loop system will be fourth order. The order of the closed-loop system is determined by the sum of the orders of the plant and the controller. If $G(s)$ is second-order and $K(s)$ is a PID controller (typically not increasing the system order), the closed-loop order depends on how the derivative is approximated and could potentially increase but not necessarily to fourth order.
- (e) **True.** For the given plant $G(s) = \frac{5}{s^2+s}$, introducing a proportional controller with gain k_p modifies the closed-loop transfer function. The stability depends on the location of poles of the closed-loop system, which can be influenced by k_p . A proportional controller cannot alter the type of the system, and since the given system is Type 1 (single integrator in the denominator), it inherently has zero steady-state error to a step input. Therefore, it is possible to choose k_p to ensure stability without introducing steady-state error to a unit step reference.

Confidence Level: 90%

Incorrect Knowledge

Claude Answer:

- (a) False. The response of an LTI system to an input depends linearly on the input, but not on the initial conditions. With nonzero initial conditions and input $2u(t)$, the response will be $2y(t)$ plus an additional term due to the initial conditions. Confidence: 95/100
- (b) False. A system with a right half plane zero may have a bounded step response if the system also has poles that pull the response back down. The zero causes an initial inverse response, but does not necessarily lead to unbounded behavior. Confidence: 90/100
- (c) True. If one pole has a time constant much larger than the others, that "dominant pole" will largely determine the system's response after the faster poles have decayed. A first-order approximation using just the dominant pole can capture the key dynamics.

Confidence: 85/100

- (d) False. An ideal PID controller has a pure derivative term, making it improper. The approximate derivative adds a pole, so the closed-loop system would be third order (second order plant plus first order controller). Confidence: 80/100
- (e) False. This plant has two poles, one at $s = 0$ and one at $s = -1$. Proportional control adds a zero but no poles, so it cannot stabilize the plant or eliminate steady-state error. A PI or PID controller would be needed.

Confidence score: 95/100

Incorrect Knowledge