

2.7 Routh-Hurwitz Stability

A system has the second-order characteristic equation

$$s^2 + as + b = 0, \quad (112)$$

where a and b are constant parameters. Determine the necessary and sufficient conditions for the system to be stable. Is it possible to determine the stability of a second-order system just by inspecting the coefficients of the characteristic equation?

Solution :

The characteristic equation is $s^2 + as + b = 0$, so, the Routh array is

$$\begin{array}{l} s^2 : 1 \quad b \\ s^1 : a \quad 0 \\ s^0 : b \end{array}$$

The system is stable if and only if $a > 0$ and $b > 0$. For a second-order system, a necessary and sufficient condition for stability is that all the coefficients have the same sign.

GPT Answer :

To determine the necessary and sufficient conditions for stability in a second-order system described by the characteristic equation

$$s^2 + as + b = 0, \quad (113)$$

where a and b are constant parameters, the conditions are: 1. $a > 0$, 2. $b > 0$.

These conditions ensure the system's stability by confirming positive damping and a real, positive natural frequency. It is possible to determine the stability of a second-order system by inspecting these coefficients, following the Routh-Hurwitz criterion.

Confidence Level: 95%

Correct

Claude Answer :

To determine the stability of a second-order system using its characteristic equation, we can use the Routh-Hurwitz stability criterion. For a second-order system, the Routh-Hurwitz criterion states that the system is stable if and only if all the coefficients of the characteristic equation are positive.

Given the characteristic equation:

$$s^2 + as + b = 0$$

where a and b are constant parameters, the necessary and sufficient conditions for the system to be stable are:

1. $a > 0$
2. $b > 0$

Proof: The roots of the characteristic equation are given by:

$$s = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

For the system to be stable, the real parts of both roots must be negative. This is only possible when:

1. $a > 0$: This ensures that the real part of the roots is always negative, regardless of the value of the discriminant ($a^2 - 4b$).
2. $b > 0$: This ensures that the roots are either real and negative or complex with negative real parts. If $b < 0$, the roots would be real with opposite signs, making the system unstable.

Therefore, by inspecting the coefficients of the characteristic equation, we can determine the stability of a second-order system. If both a and b are positive, the system is stable. If either a or b is negative, the system is unstable.

It's important to note that this method only applies to second-order systems. For higher-order systems, the Routh-Hurwitz stability criterion can be used, but the conditions become more complex and require the construction of the Routh-Hurwitz table.

Confidence Level: 95%

Correct