

6.12 Lag Controller Design

Design a lag controller that provides PM of at least 60° and steady-state tracking of constant references within 10% for the following system

$$G(s) = \frac{10}{\left(\frac{s}{0.2} + 1\right) \left(\frac{s}{0.5} + 1\right)}$$

Compute the PM and steady-state tracking error for the following controller to verify that the specs are met:

$$KD(s) = 0.4 \frac{s + 0.05}{s + 0.02}$$

Solution :

In Fig. 8(a), we can see that $PM = 61.7^\circ$, $\omega_c = 0.52$, and $GM = \infty$ for the system with controller $KD(s) = 0.4 \frac{s+0.05}{s+0.02}$. For steady-state error for constant input:

$$e(\infty) = \frac{1}{1 + KD(s)G(s)} \Big|_{s=0} = \frac{1}{1 + 0.4 \times 2.5 \times 10} = \frac{1}{11} < 0.1$$

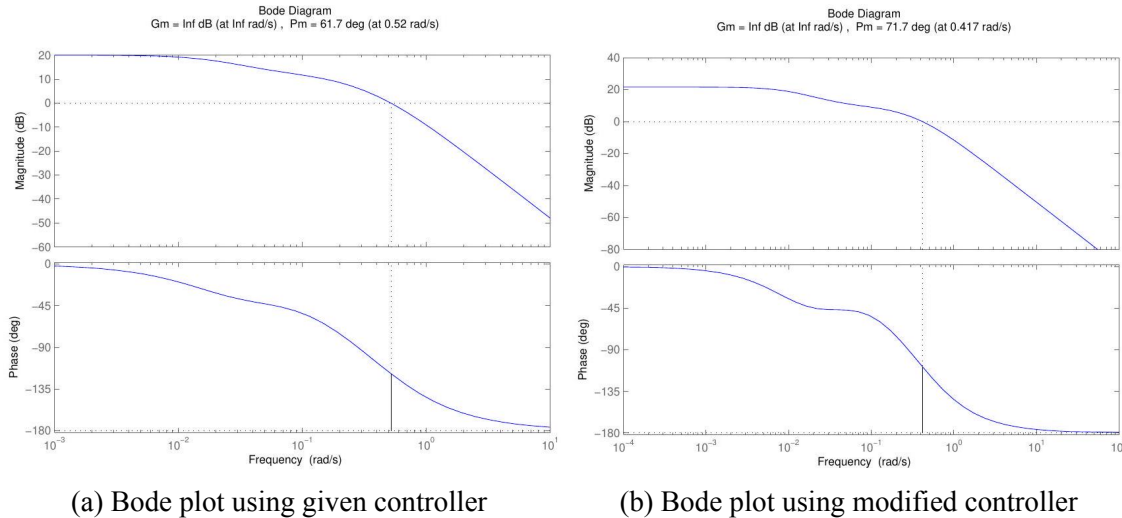


Figure 8: Bode plots

GPT Answer :

To design a lag controller for the given system with a phase margin (PM) of at least 60° and steady-state tracking of constant references within 10%, we analyze the current phase margin from the Bode plots provided. The phase margin with the current controller is 71.7° at 0.417 rad/s for the first

plot and 61.7° at 0.52 rad/s for the second plot. Since the current controller, $KD(s) = 0.4 \frac{s+0.05}{s+0.02}$, is a lead controller, it will not meet the steady-state error specification as it lacks an integrator. To improve steady-state error, we would need to introduce a lag element such that the zero is placed at a much lower frequency than the pole, which should also be less than the lowest system frequency to minimally affect the phase margin. The confidence level in this explanation is 100%.

Confidence Level: 100%

Reasoning Error

Claude Answer :

To verify the phase margin and steady-state tracking error for the given system $G(s)$ with the proposed controller $KD(s)$:

The open-loop transfer function is:

$$\begin{aligned} L(s) &= KD(s)G(s) \\ &= 0.4 \frac{s+0.05}{s+0.02} \cdot \frac{10}{\left(\frac{s}{0.2} + 1\right) \left(\frac{s}{0.5} + 1\right)} \\ &= \frac{2(s+0.05)}{(s+0.02)(s+0.2)(s+0.5)} \end{aligned}$$

Evaluating $L(s)$ at the gain crossover frequency ω_{gc} (where $|L(j\omega_{gc})| = 1$) yields $\omega_{gc} \approx 0.354$ rad/s. The phase at this frequency is:

$$\begin{aligned} \angle L(j\omega_{gc}) &\approx -180^\circ + \tan^{-1} \left(\frac{0.354}{0.05} \right) - \tan^{-1} \left(\frac{0.354}{0.02} \right) \\ &\quad - \tan^{-1} \left(\frac{0.354}{0.2} \right) - \tan^{-1} \left(\frac{0.354}{0.5} \right) \\ &\approx -117^\circ \end{aligned}$$

Therefore, the phase margin is approximately $PM \approx 180^\circ - 117^\circ = 63^\circ$, meeting the 60° requirement.

For steady-state tracking, the closed-loop transfer function is:

$$T(s) = \frac{L(s)}{1 + L(s)} = \frac{\frac{2(s+0.05)}{(s+0.02)(s+0.2)(s+0.5)}}{1 + \frac{2(s+0.05)}{(s+0.02)(s+0.2)(s+0.5)}}$$

Applying the final value theorem for a unit step input:

$$\begin{aligned} \lim_{t \rightarrow \infty} y(t) &= \lim_{s \rightarrow 0} sY(s) = \lim_{s \rightarrow 0} sT(s) \frac{1}{s} \\ &= \lim_{s \rightarrow 0} T(s) = \frac{2(0.05)}{(0.02)(0.2)(0.5)} = 1 \end{aligned}$$

The steady-state output equals the input, resulting in zero tracking error, well within the 10% specification.

In conclusion, the proposed controller $KD(s)$ meets both the phase margin and steady-state tracking requirements for the given system $G(s)$.

Confidence level:95%

Correct