

5.13 PID Controller

Consider the plant with the following transfer function:

$$G(s) = \frac{20}{s^2 - 6s + 10} \quad (245)$$

- (a) What is the closed-loop ODE from reference r to output y if we use the following PID controller?

$$u(t) = K_p e(t) + K_i \int e(t) + K_d \dot{e}(t)$$

where $e(t) = r(t) - y(t)$

- (b) Choose the controller gains so that the closed-loop has poles repeated at $s = -3$. Hint: $(s + 3)^3 = s^3 + 9s^2 + 27s + 27$
- (c) What is the impact of the implementing the derivative term $K_d \dot{e}(t)$ as versus the rate feedback from $-K_d \dot{y}(t)$?

Solution :

- (a) $G(s)$ in the time-domain can be written as

$$\ddot{y}(t) - 6\dot{y}(t) + 10y(t) = 20u(t)$$

using the PID controller $u(t)$, we have:

$$\ddot{y}(t) + (20K_d - 6)\dot{y}(t) + (10 + 20K_p)y(t) + 20K_i \int y(t) = 20K_d \ddot{r}(t) + 20K_p \dot{r}(t) + 20K_i r(t)$$

- (b) we have:

$$K_p = \frac{17}{20}, K_i = \frac{27}{20}, K_d = \frac{15}{20}$$

- (c) $K_d \dot{e}(t)$ causes an extra term in the numerator of the transfer function (extra zero). If $\dot{r}(t)$ changes fast, this causes overshoot/oscillatory.

GPT Answer :

Given the plant transfer function: $G(s) = \frac{20}{s^2 - 6s + 10}$, and the PID controller $u(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{d}{dt} e(t)$, where $e(t) = r(t) - y(t)$, the calculated PID gains for poles at $s = -3$ are $K_p = \frac{17}{20}$, $K_i = \frac{27}{20}$, and $K_d = \frac{3}{4}$. The implementation of the derivative term as $K_d \dot{e}(t)$ versus $K_d \dot{y}(t)$ impacts noise sensitivity, with $K_d \dot{y}(t)$ potentially offering better noise management.

Confidence Level: 95%

Reasoning Error

Claude Answer :

Let's tackle this problem step by step.

(a) To find the closed-loop ODE from reference r to output y , we first need to derive the closed-loop transfer function.

The PID controller in the Laplace domain is: $U(s) = K_p E(s) + \frac{K_i}{s} E(s) + K_d s E(s)$

where $E(s) = R(s) - Y(s)$

The closed-loop transfer function is: $\frac{Y(s)}{R(s)} = \frac{G(s)C(s)}{1+G(s)C(s)}$

where $C(s) = K_p + \frac{K_i}{s} + K_d s$

Substituting the plant transfer function and the PID controller: $\frac{Y(s)}{R(s)} = \frac{\frac{20}{s^2-6s+10}(K_p + \frac{K_i}{s} + K_d s)}{1 + \frac{20}{s^2-6s+10}(K_p + \frac{K_i}{s} + K_d s)}$

The closed-loop ODE can be obtained by taking the inverse Laplace transform of the closed-loop transfer function. However, the resulting ODE will depend on the values of the PID gains, which we will determine in part (b).

(b) To have the closed-loop poles repeated at $s = -3$, the closed-loop characteristic equation should be: $(s + 3)^3 = s^3 + 9s^2 + 27s + 27$

The closed-loop characteristic equation is: $1 + G(s)C(s) = 0$

Substituting the plant transfer function and the PID controller: $1 + \frac{20}{s^2-6s+10}(K_p + \frac{K_i}{s} + K_d s) = 0$

Multiplying both sides by $(s^2 - 6s + 10)$: $s^2 - 6s + 10 + 20(K_p + \frac{K_i}{s} + K_d s) = 0$

Simplifying: $s^3 + (20K_d - 6)s^2 + (20K_p + 10)s + 20K_i = 0$

Comparing coefficients with the desired characteristic equation: $20K_d - 6 = 9 \Rightarrow K_d = \frac{15}{20} = 0.75$
 $20K_p + 10 = 27 \Rightarrow K_p = \frac{17}{20} = 0.85$ $20K_i = 27 \Rightarrow K_i = \frac{27}{20} = 1.35$

(c) The impact of implementing the derivative term as $K_d \dot{e}(t)$ versus $-K_d \dot{y}(t)$ is as follows:

- $K_d \dot{e}(t)$ is the "ideal" derivative term, which directly acts on the rate of change of the error. This implementation requires the reference signal to be differentiable and known in advance. - $-K_d \dot{y}(t)$ is a more practical implementation, as it only requires the measurement of the output $y(t)$. This is known as "derivative on measurement" or "rate feedback." However, it can amplify high-frequency noise in the output signal.

In summary, while both implementations can provide similar performance in ideal conditions, the choice between them depends on the availability and quality of the reference and output signals, as well as the noise characteristics of the system.

Confidence Level: 95%

Reasoning Error