

2.15 Assessing Stability (Discrete-time system)

What is the transfer function of a discrete system with a gain factor of 2, zeros at 0.2 and -0.5, and poles at 0.5, 0.6, and -0.4? Is it stable?

Solution :

The transfer function is

$$P(z) = \frac{2(z - 0.2)(z + 0.5)}{(z - 0.5)(z - 0.6)(z + 0.4)}$$

Since all the system poles are inside the unit circle, the system is stable.

GPT Answer:

The transfer function $H(z)$ of a discrete system can be represented as a ratio of its zeros and poles along with a gain factor. Given the gain factor $K = 2$, zeros at $z_1 = 0.2$ and $z_2 = -0.5$, and poles at $p_1 = 0.5$, $p_2 = 0.6$, and $p_3 = -0.4$, the transfer function is:

$$H(z) = K \cdot \frac{(z - z_1)(z - z_2)}{(z - p_1)(z - p_2)(z - p_3)}$$

Substituting the given values:

$$H(z) = 2 \cdot \frac{(z - 0.2)(z + 0.5)}{(z - 0.5)(z - 0.6)(z + 0.4)}$$

Stability of a discrete system is determined by the location of its poles. A discrete system is stable if all its poles are inside the unit circle in the z-plane, that is, if the magnitude of each pole is less than 1.

Given poles at 0.5, 0.6, and -0.4, all of them are within the unit circle ($|p_1| = 0.5 < 1$, $|p_2| = 0.6 < 1$, $|p_3| = 0.4 < 1$). Therefore, the system is stable.

Confidence Level: 95%

Correct

Claude Answer:

To determine the transfer function of the given discrete system and assess its stability, let's follow these steps:

Given information:

- Gain factor: 2
- Zeros: 0.2 and -0.5

- Poles: 0.5, 0.6, and -0.4

Step 1: Write the general form of the transfer function.

$$H(z) = K \times \frac{(z - z_1)(z - z_2)}{(z - p_1)(z - p_2)(z - p_3)}$$

where:

- K is the gain factor,
- z_1 and z_2 are the zeros,
- p_1, p_2 , and p_3 are the poles.

Step 2: Substitute the given values into the transfer function.

$$H(z) = 2 \times \frac{(z - 0.2)(z + 0.5)}{(z - 0.5)(z - 0.6)(z + 0.4)}$$

Step 3: Expand the numerator and denominator.

$$H(z) = 2 \times \frac{z^2 + 0.3z - 0.1}{z^3 - 0.7z^2 - 0.06z + 0.12}$$

Therefore, the transfer function of the discrete system is:

$$H(z) = 2 \times \frac{z^2 + 0.3z - 0.1}{z^3 - 0.7z^2 - 0.06z + 0.12}$$

Step 4: Assess the stability of the system. For a discrete system to be stable, all its poles must lie within the unit circle in the z-plane. In other words, the absolute value of each pole must be less than 1.

$$|p_1| = |0.5| = 0.5 < 1$$

$$|p_2| = |0.6| = 0.6 < 1$$

$$|p_3| = |-0.4| = 0.4 < 1$$

Since all the poles have absolute values less than 1, the system is stable.

Confidence Level: 95%

Correct