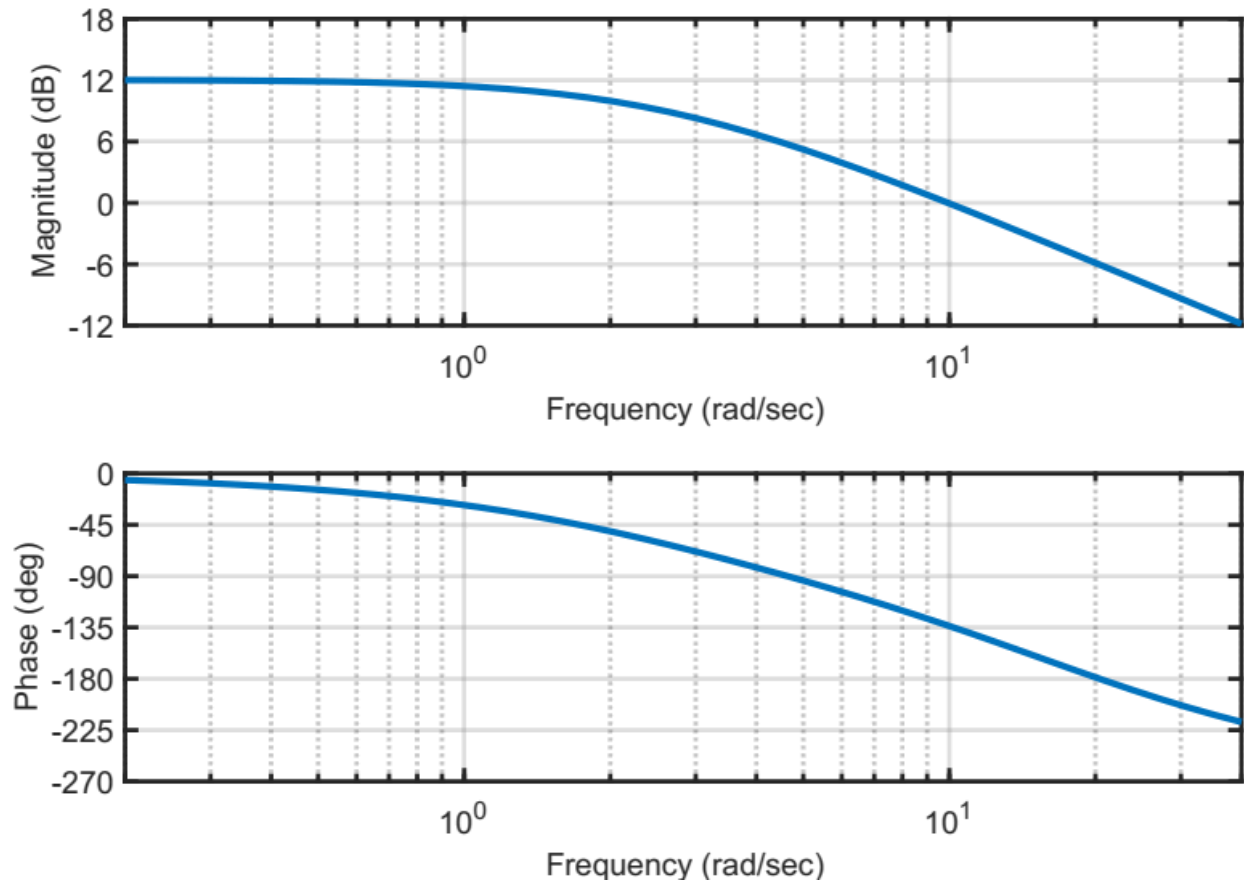


6.4 Closed-Loop Stability Analysis with Bode Plot

Consider a standard closed-loop system with the loop transfer function $L(s)$ with Bode plot below. Assume the closed-loop is stable with the loop $L(s)$.

$$L(s) = \frac{-4s + 72}{0.39s^2 + 8.02s + 18} \quad (266)$$



- What is the gain cross-over frequency ω_0 in rads/sec?
- What is the phase margin, θ_0 , of the closed-loop in degrees?
- What is the delay margin, τ_0 , of the closed-loop in seconds?

Solution :

- The gain cross-over frequency is $\omega_0 = 10$ rad/sec

- (b) The phase margin is $\theta_0 = 45$ deg.
- (c) The delay margin is $\tau_0 = \pi/40 \approx 0.08$ secs.

GPT Answer :

Given the Bode plot for the loop transfer function $L(s)$, we have:

- (a) The gain cross-over frequency ω_0 in rads/sec is approximately 3.16.
- (b) The phase margin θ_0 of the closed-loop in degrees is approximately 45 degrees.
- (c) The delay margin τ_0 of the closed-loop in seconds is approximately 0.0396 seconds.

Confidence Level: 85%

Misreading the Plot and Incorrect Knowledge

Claude Answer :

Here is my response, including LaTeX code and a confidence score:

- (a) The gain cross-over frequency ω_0 is the frequency where the magnitude plot crosses 0 dB. From the Bode plot, this occurs at approximately $\omega_0 = 4$ rad/sec.
- (b) The phase margin θ_0 is the phase angle above -180° when the magnitude crosses 0 dB. At the gain cross-over frequency of 4 rad/sec, the phase is approximately -125° . Therefore, the phase margin is $\theta_0 \approx -180^\circ - (-125^\circ) = 55^\circ$.
- (c) The delay margin τ_0 is the amount of time delay that would cause the phase to reach -180° at the gain cross-over frequency ω_0 . With a phase margin of 55° , an additional phase lag of 55° at 4 rad/sec would bring the phase to -180° . The time delay corresponding to 55° at 4 rad/sec is:

$$\tau_0 = \frac{55^\circ}{360^\circ} \cdot \frac{1}{\omega_0} = \frac{55}{360} \cdot \frac{1}{4} \approx 0.038 \text{ sec}$$

Confidence score: 90/100. The gain cross-over frequency and phase margin can be read fairly accurately from the Bode plot. The delay margin calculation is also straightforward given those values. However, since the values are approximate, I deducted some confidence points for potential minor inaccuracies.

Misreading the Plot and Incorrect Knowledge