

3.5 Step Response

What is the unit step response of a continuous system whose transfer function has a zero at -1 , a pole at -2 , and a gain factor of 2?

Solution :

The Laplace transform of the output is given by $Y(s) = P(s)U(s)$. Here

$$P(s) = \frac{2(s+1)}{s+2} \quad U(s) = \frac{1}{s} \quad Y(s) = \frac{2(s+1)}{s(s+2)} = \frac{1}{s} + \frac{1}{s+2} \quad (146)$$

Evaluating the inverse transform of the partial fraction expansion of $Y(s)$ gives $y(t) = 1 + e^{-2t}$

GPT Answer:

Given: $H(s) = 2 \frac{s+1}{s+2}$

Unit step response, $y(t)$, is found by taking the inverse Laplace transform of $Y(s) = 2 \frac{s+1}{s(s+2)}$

After correct partial fraction decomposition and inverse Laplace transform:

$$y(t) = 1 + e^{-2t}, \quad t \geq 0$$

Confidence Level: 95%

Correct

Claude Answer:

To find the unit step response of the given system, we need to perform the following steps:

1. Write the transfer function of the system.
2. Apply the final value theorem to find the steady-state value of the response.
3. Find the inverse Laplace transform of the output function to obtain the time-domain response.

Given:

- Zero at $s = -1$
- Pole at $s = -2$
- Gain factor of 2

Step 1: Write the transfer function of the system.

$$G(s) = 2 \cdot \frac{s+1}{s+2}$$

Step 2: Apply the final value theorem to find the steady-state value of the response.

$$y_{ss} = \lim_{s \rightarrow 0} sY(s) = \lim_{s \rightarrow 0} s \cdot G(s) \cdot \frac{1}{s} = \lim_{s \rightarrow 0} 2 \cdot \frac{s+1}{s+2} = 2 \cdot \frac{1}{2} = 1$$

Step 3: Find the inverse Laplace transform of the output function to obtain the time-domain response.

$$Y(s) = G(s) \cdot \frac{1}{s} = 2 \cdot \frac{s+1}{s(s+2)}$$

Using partial fraction decomposition,

$$Y(s) = \frac{A}{s} + \frac{B}{s+2} = \frac{A(s+2) + Bs}{s(s+2)}$$

Equating the numerators and solving for A and B ,

$$A(s+2) + Bs = 2(s+1) \Rightarrow A = 1, \quad B = 1$$

Therefore,

$$Y(s) = \frac{1}{s} + \frac{1}{s+2}$$

Taking the inverse Laplace transform,

$$y(t) = 1 + e^{-2t} \quad \text{for } t \geq 0$$

The unit step response of the given continuous system is:

$$y(t) = \begin{cases} 1 + e^{-2t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Confidence Level: 95%

Correct