

9.4 Gain/Phase Margins

Calculate the gain and phase margin for $GH = \frac{432}{s(s^2+13s+115)}$.

Solution :

Given the open-loop transfer function $GH(s) = \frac{432}{s(s^2+13s+115)}$, we can write $GH(j\omega)$ in polar form

- The magnitude $|GH(j\omega)| = \frac{432}{\sqrt{(115-\omega^2)^2 + (-13\omega)^2}}$
- The phase $\angle GH(j\omega) = -\tan^{-1}\left(\frac{115-\omega^2}{-13\omega}\right)$

Then $-\tan^{-1}\left(\frac{115-\omega_\pi^2}{-13\omega_\pi}\right) = -\pi$, $\omega_\pi = \sqrt{115}$. Hence, gain margin is

$$\frac{1}{|GH(j\omega_\pi)|} = \frac{13 \times 115}{432} = 3.46$$

Also,

$$|GH(j\omega_1)| = \frac{432}{\sqrt{(115-\omega_1^2)^2 + (-13\omega_1)^2}} = 1$$

happens only at $\omega_1 = 3.86$, therefore

$$\phi_{PM} = 180^\circ - \tan^{-1}\left(\frac{115-\omega_1^2}{-13\omega_1}\right) = 64^\circ$$

GPT Answer:

The calculations yield the following results:

- The gain crossover frequency (ω_{gc}) is approximately 3.86 rad/s.
- The phase margin (PM) is approximately 63.39 degrees.
- The phase crossover frequency (ω_{pc}) is approximately 10.72 rad/s.
- The gain margin (GM) is approximately 10.78 dB.

Therefore, the gain margin is 10.78 dB, and the phase margin is 63.39 degrees.

Confidence level: 95%

Correct

Clause Answer

Step 1: Rewrite GH in the standard second-order form (this step is correct): $GH = \frac{432}{s(s^2+13s+115)} = \frac{432}{s(s^2+2\zeta\omega_n s + \omega_n^2)}$

where $\omega_n = \sqrt{115} = 10.724$ and $\zeta = \frac{13}{2\sqrt{115}} = 0.606$.

Step 2: Calculate the phase crossover frequency (ω_{pc}) by setting the phase to -180° : $\angle GH(j\omega) = -180^\circ = -\pi \text{ rad}$

$$\angle GH(j\omega) = -90^\circ - \tan^{-1} \left(\frac{2\zeta\omega_{pc}}{\omega_n} \right) - \tan^{-1} \left(\frac{\omega_{pc}}{\omega_n} \right) = -\pi$$

Solving this equation numerically yields $\omega_{pc} = 11.398 \text{ rad/s}$.

Step 3: Calculate the gain margin: $GM = \frac{1}{|GH(j\omega_{pc})|} |GH(j\omega_{pc})| = \frac{432}{\omega_{pc} \sqrt{(\omega_{pc}^2 - \omega_n^2)^2 + (2\zeta\omega_n\omega_{pc})^2}} = 0.195$

Therefore, $GM = \frac{1}{0.195} = 5.128 = 14.19 \text{ dB}$.

Step 4: Calculate the gain crossover frequency (ω_{gc}) by setting the magnitude to 1: $|GH(j\omega_{gc})| = 1$

$$\frac{432}{\omega_{gc} \sqrt{(\omega_{gc}^2 - \omega_n^2)^2 + (2\zeta\omega_n\omega_{gc})^2}} = 1$$

Solving this equation numerically yields $\omega_{gc} = 6.611 \text{ rad/s}$ (this step is correct).

Step 5: Calculate the phase margin: $PM = 180^\circ + \angle GH(j\omega_{gc})$ $\angle GH(j\omega_{gc}) = -90^\circ - \tan^{-1} \left(\frac{2\zeta\omega_{gc}}{\omega_n} \right) - \tan^{-1} \left(\frac{\omega_{gc}}{\omega_n} \right) = -142.8^\circ$

Therefore, $PM = 180^\circ - 142.8^\circ = 37.2^\circ$.

In conclusion, for the given transfer function $GH = \frac{432}{s(s^2 + 13s + 115)}$, the gain margin is 14.19 dB and the phase margin is 37.2° . I apologize for the errors in my previous response.

Confidence level: 85%

Calculation Error