

12.3 Proportional Controller

We plan to design a controller $K(s) = K_p$ for a system with transfer function $G(s)$ given by:

$$G(s) = \frac{12}{3s + 5}$$

The controller and the system are in standard unity feedback setting. Let $e = r - y$ where r and y representing the reference and the output signals respectively. Let n represent the output noise i.e. if $n \neq 0$ then $e = r - (y + n)$. The specification is to design a controller so that the closed loop is stable and has gain less than 0.04 from input n to output y for frequencies above 200 rad/sec. What is the cross-over frequency.

Solution :

Firstly, we can show that the requirement of gain less than 0.04 from input n to output y for frequencies greater than 200 rad/sec translates to $|L(j\omega)| = |K(j\omega)G(j\omega)| \leq 0.04$ for $\omega \geq 200 \frac{\text{rad}}{\text{sec}}$. Thus $K_p \leq \frac{0.04}{|G(j200)|} \approx 2$ will ensure this requirement. Additionally, we can verify the closed loop pole of the system with $\frac{-5}{12} < K_p$ is in the LHP so the closed loop system is stable. Thus choosing $K_p = 2$ meet the both requirements.

The cross-over frequency is approximately 7.82 rad/sec.

GPT Answer :

Given the system transfer function:

$$G(s) = \frac{12}{3s + 5}$$

And the proportional controller:

$$K(s) = K_p$$

In a standard unity feedback system, the open-loop transfer function $L(s)$ is given by:

$$L(s) = K(s)G(s) = K_p \frac{12}{3s + 5}$$

The closed-loop transfer function $T(s)$ from r to y , ignoring n , is:

$$T(s) = \frac{L(s)}{1 + L(s)} = \frac{K_p \cdot 12}{3s + 5 + K_p \cdot 12}$$

To ensure stability, the denominator of $T(s)$ must have all its roots in the left half of the complex plane. Considering the gain from n to y , which is given by:

$$T_n(s) = \frac{G(s)}{1 + K(s)G(s)} = \frac{12}{3s + 5 + K_p \cdot 12}$$

The condition for the gain from n to y being less than 0.04 for frequencies above 200 rad/sec translates to ensuring that the magnitude of $T_n(j\omega)$ is less than 0.04 for $\omega > 200$.

Confidence Level: 75%

Calculation Complexity

Claude Answer:

To ensure closed-loop stability and a gain less than 0.04 from input n to output y for frequencies above 200 rad/sec, we can choose a proportional controller $K(s) = K_p$ such that:

$$|G(j\omega)K_p| < 0.04 \quad \text{for } \omega > 200 \text{ rad/sec}$$

Substituting $s = j\omega$ and $\omega = 200$ rad/sec in the given transfer function:

$$|G(j200)K_p| = \left| \frac{12K_p}{3(j200) + 5} \right| < 0.04$$

Solving for K_p :

$$K_p < 0.04 \cdot \left| \frac{3(j200) + 5}{12} \right| \approx 0.133$$

Choosing $K_p = 0.1$ satisfies the above condition.

The loop transfer function is $L(s) = G(s)K_p$. The loop crossover frequency ω_c is the frequency at which $|L(j\omega_c)| = 1$:

$$|L(j\omega_c)| = \left| \frac{12K_p}{3(j\omega_c) + 5} \right| = 1$$

Substituting $K_p = 0.1$ and solving for ω_c :

$$\omega_c \approx 24.5 \text{ rad/sec}$$

Therefore, with a proportional controller $K(s) = 0.1$, the closed-loop system is stable, has a gain less than 0.04 from input n to output y for frequencies above 200 rad/sec, and has a loop crossover frequency of approximately 24.5 rad/sec.

Confidence Level: 90%

Calculation Error