

5.24 Controller Design

Consider a feedback control system under unity feedback setting with a controller $K(s) = K$ and the system $G(s)$ given as follows:

$$G(s) = \frac{1}{s^2(s+1)}$$

Suppose we would like to place a closed loop pole at $\bar{s} = -1 + j$. What is the phase of $K(\bar{s})$?

Solution:

The controller needs to contribute π rad since $\angle G(\bar{s}) + \angle K(\bar{s}) \equiv \pi$ which leads to:

$$\angle K(\bar{s}) = \pi - \left(0 - 2 \left(\frac{3\pi}{4} \right) - \frac{\pi}{2} \right) = \pi + 2\pi = \pi$$

GPT Answer :

To find the phase of $K(\bar{s})$ for the given feedback control system under unity feedback with a controller $K(s) = K$ and a system $G(s)$ as described, we first recognize that the overall transfer function $T(s)$ of the system in a unity feedback configuration is given by:

$$T(s) = \frac{KG(s)}{1 + KG(s)}$$

Given:

$$G(s) = \frac{1}{s^2(s+1)}$$

We want to place a closed loop pole at $\bar{s} = -1 + j$, so we need to analyze the phase of the system at this point to ensure the angle criterion of the Nyquist Stability Criterion is satisfied. The phase angle of $K(\bar{s})$ is simply the phase of K since K is a constant gain (i.e., its phase does not depend on s and is always zero degrees or zero radians). However, the relevant phase for determining the stability or the pole placement comes from $G(\bar{s})$.

To find the phase of $G(\bar{s})$ at $\bar{s} = -1 + j$, we substitute \bar{s} into $G(s)$:

$$G(\bar{s}) = \frac{1}{(-1+j)^2(-1+j+1)}$$

Let's compute this:

$$\begin{aligned} G(\bar{s}) &= \frac{1}{(-1+j)^2(j)} \\ G(\bar{s}) &= \frac{1}{(-1+2j-1)(j)} \\ G(\bar{s}) &= \frac{1}{(2j)(j)} \\ G(\bar{s}) &= \frac{1}{-2} \end{aligned}$$

This means that $G(\bar{s})$ at $\bar{s} = -1 + j$ has a phase of 180 degrees (or π radians) since it's a real negative number.

The phase of $K(\bar{s})$ is effectively the phase of K , which, being a constant gain, does not contribute to the phase shift introduced by the system itself. Hence, the phase shift that matters for the feedback loop comes from $G(\bar{s})$, which is 180 degrees or π radians.

Confidence Level: 90%

Correct

Claude Answer :

The phase of $K(\bar{s})$ is 0° .

To place a closed loop pole at $\bar{s} = -1 + j$, the angle condition from the root locus method states that:

$$\begin{aligned} \angle K(s)G(s) \Big|_{s=\bar{s}} &= (2k+1)180^\circ, \quad k = 0, \pm 1, \pm 2, \dots \\ \angle K(\bar{s}) + \angle G(\bar{s}) &= (2k+1)180^\circ \end{aligned}$$

Since $K(s) = K$ is a constant gain, its phase is always 0° , i.e., $\angle K(\bar{s}) = 0^\circ$. Therefore, to satisfy the angle condition, we must have:

$$\angle G(\bar{s}) = (2k+1)180^\circ, \quad k = 0, \pm 1, \pm 2, \dots$$

Confidence level: 100/100

Calculation Error