

1.17 Transfer Function

What is the transfer function of a system whose input and output are related by the following differential equation?

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = u + \frac{du}{dt} \quad (57)$$

Solution :

Taking the Laplace transform of this equation, ignoring terms due to initial conditions, we obtain

$$s^2Y(s) + 3sY(s) + 2Y(s) = U(s) + sU(s) \quad (58)$$

This equation can be written as

$$Y(s) = \left[\frac{s+1}{s^2+3s+2} U(s) \right] \quad (59)$$

The transfer function of this system is therefore given by

$$P(s) = \frac{s+1}{s^2+3s+2} \quad (60)$$

GPT Answer:

The transfer function of the system, given the corrected differential equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = u + \frac{du}{dt}$$

is found by taking the Laplace transform and assuming initial conditions are zero, resulting in:

$$H(s) = \frac{s+1}{s^2+3s+2}$$

Confidence level: 95%

Correct

Claude Answer:

The transfer function of the given system is:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{s+2}{s^2+3s+2}$$

To derive this, we take the Laplace transform of the differential equation, assuming zero initial conditions:

$$\begin{aligned} s^2Y(s) + 3sY(s) + 2Y(s) &= U(s) + sU(s) \\ Y(s)(s^2 + 3s + 2) &= U(s)(s + 1) \\ \frac{Y(s)}{U(s)} &= \frac{s + 1}{s^2 + 3s + 2} \end{aligned}$$

Confidence level: 95/100

Correct