

1.11 Time-Invariance vs. Time-Variance in ODEs

Classify the following linear differential equations according to whether they are time-variable or time-invariant. Indicate any time-variable terms.

- (a) $\frac{d^2y}{dt^2} + 2y = 0$
- (b) $\frac{d}{dt}(t^2y) = 0$
- (c) $\left(\frac{1}{t+1}\right) \frac{d^2y}{dt^2} + \left(\frac{1}{t+1}\right) y = 0$
- (d) $\frac{d^2y}{dt^2} + (\cos t)y = 0$

You may assume $t \geq 0$ in these differential equations.

Solution :

- (a) Time-invariant.
- (b) $\frac{d}{dt}(t^2y) = 2ty + t^2 \left(\frac{dy}{dt}\right) = 0$. Dividing through by t , $t \left(\frac{dy}{dt} + 2y\right) = 0$ which is time-variable.
The time-variable term is $t \left(\frac{dy}{dt}\right)$
- (c) Multiplying through by $t + 1$, we obtain $\frac{d^2y}{dt^2} + y = 0$ which is time-invariant.
- (d) Time-variable. The time-variable term is $(\cos t)y$

GPT Answer:

Classify the following linear differential equations according to whether they are time-variable or time-invariant. Indicate any time-variable terms.

- (a) $\frac{d^2y}{dt^2} + 2y = 0$
This equation is **time-invariant** because the coefficients of $\frac{d^2y}{dt^2}$ and y are constants.
- (b) $\frac{d}{dt}(t^2y) = 0$
This equation is **time-variable**, with time-variable terms being t^2 and $2t$.
- (c) $\left(\frac{1}{t+1}\right) \frac{d^2y}{dt^2} + \left(\frac{1}{t+1}\right) y = 0$
This equation is **time-variable**, with the time-variable term being $\frac{1}{t+1}$.
- (d) $\frac{d^2y}{dt^2} + (\cos t)y = 0$
This equation is **time-variable**, with the time-variable term being $\cos t$.

Confidence Level: 95%

Reasoning Error

Claude Answer:

- (a) Time-invariant. There are no time-variable terms in the equation $\frac{d^2y}{dt^2} + 2y = 0$.
- (b) Time-variable. The term t^2 multiplying y inside the derivative is time-variable in the equation $\frac{d}{dt}(t^2y) = 0$.
- (c) Time-invariant. Although the term $\frac{1}{t+1}$ is time-variable, it is multiplying both $\frac{d^2y}{dt^2}$ and y equally, so it can be factored out and canceled, leaving the equation in a time-invariant form: $\frac{d^2y}{dt^2} + y = 0$.
- (d) Time-variable. The term $\cos t$ multiplying y is time-variable in the equation $\frac{d^2y}{dt^2} + (\cos t)y = 0$.

Confidence level: 100/100

Correct