

5.14 PI Controller

Consider the following plant and PI controller:

$$2\dot{y}(t) + 6y(t) = 8u(t) \quad u(t) = 2.5e(t) + 9 \int_0^t e(\tau) d\tau$$

- (a) What sampling time Δt would you recommend for a discrete-time implementation?
- (b) The value of $u(t)$ at $t = \Delta t$ is:

$$u(\Delta t) = 2.5e(\Delta t) + 9 \int_0^{\Delta t} e(\tau) d\tau$$

Approximate $u_1 := u(\Delta t)$ in terms $e_0 := e(0)$ and $e_1 := e(\Delta t)$.

Solution :

(a)

$$2\ddot{y}(t) + 26\dot{y}(t) + 72y(t) = 20\dot{r}(t) + 72r(t)$$

Thus,

$$T_{r \rightarrow y}(s) = \frac{20s + 72}{2s^2 + 26s + 72}$$

The poles of the transfer function are $s_1 = -4$ and $s_2 = -9$. The corresponding time constants are $\tau_1 = \frac{1}{4}$ and $\tau = \frac{1}{9}$. One good approximation for Δt would be

$$\Delta t = \frac{1/9}{10} \approx \frac{1}{100} = 10\text{msec}$$

- (b) Using this approximation $\int_0^{\Delta t} e(\tau) d\tau \approx \frac{1}{2}(e_0 + e_1)\Delta t$, we have:

$$u_1 \approx 2.5e_1 + 9\left(\frac{1}{2}(e_0 + e_1)\Delta t\right)$$

GPT Answer :

Part (a): Recommended Sampling Time (Δt)

Given the dynamics of the system represented by $2\dot{y}(t) + 6y(t) = 8u(t)$, the selection of Δt for a discrete-time implementation should consider the system's response characteristics and computational capabilities. A detailed analysis or simulation would be necessary for a precise recommendation.

Part (b): Approximation of $u_1 := u(\Delta t)$

For the approximation of u_1 in terms of e_0 and e_1 , the corrected expression is:

$$u_1 = 2.5e_1 + 4.5\Delta t(e_0 + e_1)$$

This approximation assumes linear behavior of $e(t)$ within the interval $[0, \Delta t]$.

Confidence Level: 75%

Calculation Complexity

Claude Answer :

- (a) For a discrete-time implementation, a good rule of thumb is to select a sampling time Δt that is 10 to 20 times faster than the dominant time constant of the closed-loop system. The closed-loop transfer function is:

$$Y(s) = \frac{20 + 72/s}{s^2 + 3s + 20 + 72/s}U(s)$$

The dominant time constant is approximately $\tau \approx 1/3 = 0.33$ seconds. Therefore, I would recommend a sampling time in the range of $\Delta t = 0.02$ to 0.03 seconds.

Confidence: 90%

- (b) Using trapezoidal integration to approximate the integral term:

$$\begin{aligned} u_1 &= 2.5e_1 + 9 \int_0^{\Delta t} e(\tau) d\tau \\ &\approx 2.5e_1 + 9 \left(\frac{e_0 + e_1}{2} \right) \Delta t \\ &= 2.5e_1 + 4.5e_0\Delta t + 4.5e_1\Delta t \\ &= (2.5 + 4.5\Delta t)e_1 + 4.5e_0\Delta t \end{aligned}$$

Confidence: 98%

Calculation Error