# 2.11 Routh-Hurwitz Stability

Construct a Routh table and determine the number of roots with positive real parts for the equation

$$2s^3 + 4s^2 + 4s + 12 = 0 ag{117}$$

### **Solution:**

The Routh table for this equation is given below. Here the  $s^2$  row was divided by 4 before the  $s^1$  row was computed. The  $s^1$  row was then divided by 2 before the  $s^0$  was computed.

Since there are two changes of sign in the first column of the Routh table, the equation above has two roots with positive real parts.

### **GPT Answer:**

To answer your question, let's first construct the Routh table for the given equation:

$$2s^3 + 4s^2 + 4s + 12 = 0$$

The polynomial coefficients are arranged in descending powers of s, thus we have:

- $a_3 = 2$
- $a_2 = 4$
- $a_1 = 4$
- $a_0 = 12$

The Routh table is constructed as follows:

$$\begin{array}{c|cccc}
s^3 & 2 & 4 \\
s^2 & 4 & 12 \\
s^1 & \frac{(4 \times 4 - 2 \times 12)}{4} = 0
\end{array}$$

Since the computation for the first element of the  $s^1$  row results in 0, we have a special case. To continue, we need to use an  $\varepsilon$  approach or consider the auxiliary equation. However, the primary goal here is to determine the number of roots with positive real parts, which can be inferred directly from the Routh table without needing to resolve the  $s^1$  row ambiguity.

From the Routh table, the change in sign in the first column indicates the number of roots with positive real parts. In this case, assuming the  $s^1$  row could potentially lead to a root with positive

real part due to the special handling required for 0, but since the signs before and after are consistent (positive), we observe no changes in sign.

Thus, the equation

$$2s^3 + 4s^2 + 4s + 12 = 0$$

has **0 roots** with positive real parts.

**Confidence level: 95%** 

### **Calculation Error**

#### **Claude Answer**

To determine the number of roots with positive real parts, we will use the Routh-Hurwitz stability criterion. First, let's construct the Routh table for the given equation:

$$2s^3 + 4s^2 + 4s + 12 = 0$$

$$\begin{array}{c|ccccc} s^3 & 2 & 4 \\ \hline s^2 & 4 & 12 \\ s^1 & \frac{4\cdot 4 - 2\cdot 12}{4} = -2 & 12 \\ s^0 & \frac{-2\cdot 12 - 4\cdot 12}{-2} = 12 & - \end{array}$$

Now, let's analyze the Routh table:

- 1. The first column of the Routh table contains the coefficients of the even powers of s (including  $s^0$ ).
- 2. The number of sign changes in the first column indicates the number of roots with positive real parts.

In this case, the first column of the Routh table is [2, 4, -2, 12]. There are two sign changes (from 4 to -2 and from -2 to 12).

Therefore, the equation  $2s^3 + 4s^2 + 4s + 12 = 0$  has two roots with positive real parts.

**Confidence level: 95%** 

## Correct