

5.7 Proportional Controller

Consider the following plant:

$$\ddot{y}(t) + 2\dot{y}(t) + 10y(t) = 20u(t) + 10d(t) \quad (192)$$

- (a) What is the model from inputs (r, d) to output y if we use an open-loop controller $u(t) = K_{ol}r(t)$?
- (b) Can the gain K_{ol} be selected so that the control system is overdamped from r to y ?
- (c) Select K_{ol} so that $y(t) \rightarrow \bar{r}$ when $r(t) = \bar{r}$ and $d(t) = 0$
- (d) What is the impact of the disturbance $d(t)$ when $r(t) = 2$ and $d(t) = 1$

Solution :

- (a) $\ddot{y}(t) + 2\dot{y}(t) + 10y(t) = 20K_{ol}r(t) + 10d(t)$
- (b) No, it is underdamped for any choice of K_{ol} .
- (c) Set $\ddot{y} = \dot{y} = 0 \rightarrow 10\bar{y} = 20K_{ol}\bar{r}$, thus $K_{ol} = \frac{1}{2}$
- (d) It causes error in steady state solution.

GPT Answer:

To address this question, let's first analyze the given differential equation and then proceed to answer each part systematically.

Given:

$$\ddot{y}(t) + 2\dot{y}(t) + 10y(t) = 20u(t) + 10d(t) \quad (193)$$

- (a) With an open-loop controller $u(t) = K_{ol}r(t)$, the equation becomes:

$$\ddot{y}(t) + 2\dot{y}(t) + 10y(t) = 20K_{ol}r(t) + 10d(t) \quad (194)$$

The model from inputs (r, d) to output y is defined by this equation with $u(t) = K_{ol}r(t)$.

- (b) The system's nature (overdamped, critically damped, underdamped) is determined by the roots of the characteristic equation, which, in this case, are not affected by the gain K_{ol} since it does not appear in the homogeneous part of the differential equation. The characteristic equation derived from the homogeneous part is:

$$s^2 + 2s + 10 = 0 \quad (195)$$

The roots of this equation, given by the quadratic formula, are complex because the discriminant $2^2 - 4 \cdot 1 \cdot 10 < 0$, indicating the system is underdamped. Thus, K_{ol} cannot be selected to make the system overdamped.

(c) To ensure $y(t) \rightarrow \bar{r}$ as $r(t) = \bar{r}$ and $d(t) = 0$, the steady-state gain from r to y needs to be unity. The transfer function from r to y (ignoring $d(t)$) can be represented as:

$$\frac{Y(s)}{R(s)} = \frac{20K_{ol}}{s^2 + 2s + 10} \quad (196)$$

For a unity steady-state gain, we require the DC gain ($s=0$) to be 1:

$$\frac{20K_{ol}}{10} = 1 \Rightarrow K_{ol} = \frac{1}{2} \quad (197)$$

(d) When $r(t) = 2$ and $d(t) = 1$, substituting $K_{ol} = \frac{1}{2}$ and these values into the differential equation, we get:

$$\ddot{y}(t) + 2\dot{y}(t) + 10y(t) = 20\left(\frac{1}{2} \cdot 2\right) + 10 \cdot 1 = 20 \quad (198)$$

This shows that the disturbance $d(t) = 1$ affects the system by adding to the input, potentially affecting the output depending on the system's response characteristics.

Confidence level: 95%

Correct

Claude Answer:

Let's approach this problem step by step:

(a) To find the model from inputs (r, d) to output y with the open-loop controller $u(t) = K_{ol}r(t)$, substitute $u(t)$ into the plant equation:

$$\ddot{y}(t) + 2\dot{y}(t) + 10y(t) = 20K_{ol}r(t) + 10d(t) \quad (199)$$

Taking the Laplace transform (assuming zero initial conditions):

$$(s^2 + 2s + 10)Y(s) = 20K_{ol}R(s) + 10D(s) \quad (200)$$

The transfer function from $R(s)$ to $Y(s)$ is:

$$G_{yr}(s) = \frac{20K_{ol}}{s^2 + 2s + 10} \quad (201)$$

And the transfer function from $D(s)$ to $Y(s)$ is:

$$G_{yd}(s) = \frac{10}{s^2 + 2s + 10} \quad (202)$$

(b) The system is overdamped if the poles of the closed-loop system are real and distinct. The poles are determined by the characteristic equation:

$$s^2 + 2s + 10 = 0 \quad (203)$$

This equation has a discriminant of $2^2 - 4 \cdot 10 = -36 < 0$, which means the poles are complex conjugates. Therefore, the system cannot be made overdamped by selecting K_{ol} .

(c) For $y(t) \rightarrow \bar{r}$ when $r(t) = \bar{r}$ and $d(t) = 0$, we need:

$$\lim_{s \rightarrow 0} G_{yr}(s) = \lim_{s \rightarrow 0} \frac{20K_{ol}}{s^2 + 2s + 10} = 1 \quad (204)$$

Solving for K_{ol} :

$$K_{ol} = \frac{1}{2} \quad (205)$$

(d) When $r(t) = 2$, $d(t) = 1$, and $K_{ol} = \frac{1}{2}$, the steady-state output is:

$$y_{ss} = \lim_{s \rightarrow 0} (G_{yr}(s)R(s) + G_{yd}(s)D(s)) = \lim_{s \rightarrow 0} \left(\frac{20 \cdot \frac{1}{2} \cdot 2}{s^2 + 2s + 10} + \frac{10 \cdot 1}{s^2 + 2s + 10} \right) = 2 + 1 = 3 \quad (206)$$

So, the disturbance $d(t)$ causes a steady-state error of 1 in the output.

Confidence level: 95%

Correct