

3.12 Sketching Time Response

(a) Roughly sketch the response for the following:

$$\dot{y}(t) + 2y(t) = 4u(t) \quad y(0) = 0, \quad u(t) = 3 \text{ for all } t \geq 0 \quad (147)$$

(b) Roughly sketch the response for the following:

$$\dot{y}(t) - 3y(t) = 2u(t) \quad y(0) = 0, \quad u(t) = 1 \text{ for all } t \geq 0 \quad (148)$$

Solution :

(a)

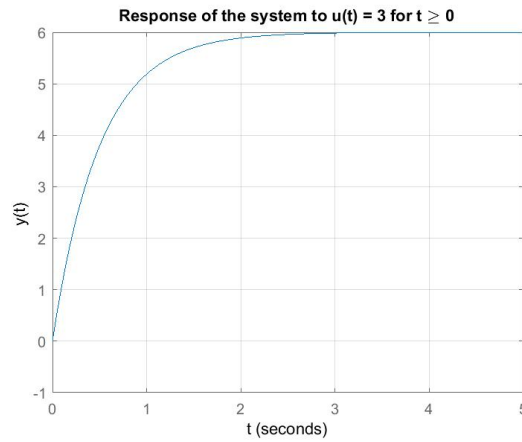


Figure 2: The system response over time.

(b)

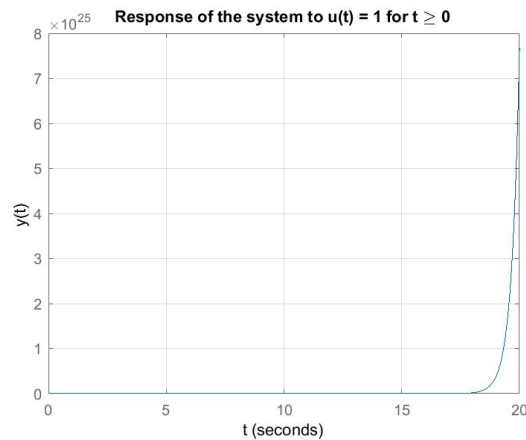


Figure 3: The system response over time.

GPT Answer :

The responses for the given differential equations are as follows:

Part (a): The solution, after applying the initial condition $y(0) = 0$, is $y(t) = -6e^{-2t} + 6$. This leads to a response that starts at 0 and asymptotically approaches 6 as $t \rightarrow \infty$.

Part (b): The solution, after applying the initial condition $y(0) = 0$, is $y(t) = \frac{2}{3}e^{3t} - \frac{2}{3}$. The response starts at 0 and diverges, significantly influenced by the exponential growth, yet it is asymptotically tethered towards the value $-\frac{2}{3}$ due to the particular solution.

Confidence Level: 90%

Correct**Claude Answer :**

To find the response (solution) to these first-order linear differential equations, we can use the Laplace transform method.

$$(a) \dot{y}(t) + 2y(t) = 4u(t) \quad y(0) = 0, \quad u(t) = 3 \text{ for all } t \geq 0$$

Taking the Laplace transform of both sides, we get: $sY(s) - y(0) + 2Y(s) = 4 \cdot \frac{3}{s}$

Substituting the initial condition $y(0) = 0$ and simplifying: $(s + 2)Y(s) = \frac{12}{s}$

$$\text{Solving for } Y(s): Y(s) = \frac{12}{s(s+2)}$$

Using partial fraction decomposition and taking the inverse Laplace transform, we obtain the solution: $y(t) = 6(1 - e^{-2t})$ for $t \geq 0$

$$(b) \dot{y}(t) - 3y(t) = 2u(t) \quad y(0) = 0, \quad u(t) = 1 \text{ for all } t \geq 0$$

Taking the Laplace transform of both sides, we get: $sY(s) - y(0) - 3Y(s) = 2 \cdot \frac{1}{s}$

Substituting the initial condition $y(0) = 0$ and simplifying: $(s - 3)Y(s) = \frac{2}{s}$

$$\text{Solving for } Y(s): Y(s) = \frac{2}{s(s-3)}$$

Using partial fraction decomposition and taking the inverse Laplace transform, we obtain the solution: $y(t) = \frac{2}{3}(e^{3t} - 1)$ for $t \geq 0$

Therefore, the responses (solutions) to the given differential equations are:

$$(a) y(t) = 6(1 - e^{-2t}) \quad \text{for } t \geq 0 \quad (b) y(t) = \frac{2}{3}(e^{3t} - 1) \quad \text{for } t \geq 0$$

Confidence Level: 95%

Correct