

## 1.2 Initial Value Theorem

Consider a system with the transfer function given by

$$H(s) = \frac{2(s+1)}{s^2 + 4s + 9} \quad (6)$$

and denote by  $g(t)$  the unit step response with zero initial conditions. Compute the initial slope of  $g(t)$ , i.e., the slope at  $t = 0+$ .

### Solution :

We first note that the derivative of  $g(t)$  is the response to the derivative of the unit step, i.e., it is equal to the response to a Dirac delta. Hence, we have

$$\mathcal{L}\left\{\frac{dg(t)}{dt}\right\} = \mathcal{L}\{h(t)\} = H(s) \quad (7)$$

Therefore, the answer to the question translates into computing  $h(0+)$ . This can be computed via the Initial Value Theorem. This yields

$$\dot{g}(0+) = h(0+) = \lim_{s \rightarrow \infty} sH(s) = \lim_{s \rightarrow \infty} \frac{2s(s+1)}{s^2 + 4s + 9} = 2 \quad (8)$$

### GPT Answer :

Given the transfer function:

$$H(s) = \frac{2(s+1)}{s^2 + 4s + 9} \quad (9)$$

The initial slope of the unit step response  $g(t)$  at  $t = 0+$  is computed using the Final Value Theorem applied to the derivative of the Laplace-transformed response, leading to:

$$\lim_{s \rightarrow 0} s \frac{d}{ds} \left[ \frac{2(s+1)}{s^2 + 4s + 9} \right] = 0 \quad (10)$$

**Confidence Level: 95%**

### Calculation Error

### Claude Answer:

Then after the prompt "Your answer does not seem correct. Fix it," Claude-3 responds as: To find the initial slope of the unit step response  $g(t)$  at  $t = 0+$ , we can use the following property of the Laplace transform:

$$\lim_{t \rightarrow 0+} \frac{dg(t)}{dt} = \lim_{s \rightarrow \infty} sG(s) - g(0+) \quad (11)$$

where  $G(s)$  is the Laplace transform of  $g(t)$ , and  $g(0+)$  is the initial value of the response. Given the transfer function:

$$H(s) = \frac{2(s+1)}{s^2 + 4s + 9} \quad (12)$$

The Laplace transform of the unit step response is:

$$G(s) = \frac{1}{s}H(s) = \frac{2(s+1)}{s(s^2 + 4s + 9)} \quad (13)$$

Now, we can calculate the initial slope:

$$\begin{aligned} \lim_{t \rightarrow 0+} \frac{dg(t)}{dt} &= \lim_{s \rightarrow \infty} sG(s) - g(0+) \\ &= \lim_{s \rightarrow \infty} s \cdot \frac{2(s+1)}{s(s^2 + 4s + 9)} - \lim_{s \rightarrow \infty} \frac{2(s+1)}{s^2 + 4s + 9} \\ &= \lim_{s \rightarrow \infty} \frac{2(s+1)}{s^2 + 4s + 9} - 0 \\ &= 2 \end{aligned}$$

Therefore, the initial slope of the unit step response  $g(t)$  at  $t = 0+$  is 2.

Confidence score: 98/100

**Correct**