

12 Loop-Shaping

12.1 Loop Gain

We plan to design a controller $K(s)$ for a system with transfer function given by $G(s)$. The controller and the system are in standard unity feedback setting. Let $e = r - y$ where r and y representing the reference and the output signals respectively. Let n represent the output noise i.e. if $n \neq 0$ then $e = r - (y + n)$. The specifications are to design a controller so that the closed loop is stable and has:

- (a) gain less than 0.01 from reference r to error e for frequencies below 0.1 rad/sec.
- (b) gain less than 0.04 from input n to output y for frequencies above 200 rad/sec.

Translate the above specifications into requirements on the loop gain $|L(j\omega)|$ where loop gain is given by $L(s) = K(s)G(s)$.

Solution :

The transfer function from r to e is $S(s) = \frac{1}{1+L(s)}$. The steady error due to a unit step reference input $r(t) = \bar{r}$ is given by $e = S(0)\bar{r}$. Thus specification (a) is equivalent to $|S(j\omega)| \leq 0.01$ for $\omega \leq 0.1 \frac{\text{rad}}{\text{sec}}$. This is approximately equivalent to $|L(j\omega)| \geq 99$ for $\omega \leq 0.1 \frac{\text{rad}}{\text{sec}}$.

The transfer function from n to y is $-T(s) = \frac{-L(s)}{1+L(s)}$. Specification (b) corresponds to $|T(j\omega)| \leq 0.04$ for $\omega \geq 200 \frac{\text{rad}}{\text{sec}}$ and this is approximately equivalent to $|L(j\omega)| \leq 0.04$ for $\omega \geq 200 \frac{\text{rad}}{\text{sec}}$.

GPT Answer :

To translate the given specifications into requirements on the loop gain $|L(j\omega)|$, where $L(s) = K(s)G(s)$, we must consider the relationship between the open-loop gain and the closed-loop behavior in a standard unity feedback system. The closed-loop transfer function from reference r to output y is given by:

$$T(s) = \frac{K(s)G(s)}{1 + K(s)G(s)} = \frac{L(s)}{1 + L(s)}$$

And the transfer function from the noise n to the output y in the presence of feedback is:

$$H(s) = \frac{1}{1 + K(s)G(s)} = \frac{1}{1 + L(s)}$$

(a) For the gain from reference r to error e to be less than 0.01 for frequencies below 0.1 rad/sec, considering that $e = r - y$, the transfer function for e can be derived as:

$$E(s) = \frac{1}{1 + L(s)}$$

The specification means that $|E(j\omega)| < 0.01$ for $\omega < 0.1$ rad/sec, which implies $|1 + L(j\omega)| > 100$ for $\omega < 0.1$ rad/sec. This is because $|E(j\omega)| = \frac{1}{|1+L(j\omega)|}$.

(b) For the gain from input n to output y to be less than 0.04 for frequencies above 200 rad/sec, we already have the transfer function $H(s)$ from n to y . This means that $|H(j\omega)| < 0.04$ for $\omega > 200$ rad/sec, which implies $|1 + L(j\omega)| > 25$ for $\omega > 200$ rad/sec.

Combining both conditions, we get the requirements on the loop gain $|L(j\omega)|$ as follows: - For $\omega < 0.1$ rad/sec, $|1 + L(j\omega)| > 100$, implying a high loop gain to ensure low error due to reference signal changes. - For $\omega > 200$ rad/sec, $|1 + L(j\omega)| > 25$, implying a sufficiently high loop gain to attenuate the effect of noise n on the output y .

Confidence Level: 90%

Calculation Error

Claude Answer:

To translate the given specifications into requirements on the loop gain $|L(j\omega)|$, we need to consider the relationship between the closed-loop transfer functions and the loop gain. Let's analyze each specification separately.

(a) Gain less than 0.01 from reference r to error e for frequencies below 0.1 rad/sec:

The closed-loop transfer function from reference r to error e is given by:

$$\frac{E(s)}{R(s)} = \frac{1}{1+L(s)}$$

To achieve a gain less than 0.01 for frequencies below 0.1 rad/sec, we require:

$$\left| \frac{E(j\omega)}{R(j\omega)} \right| = \left| \frac{1}{1+L(j\omega)} \right| < 0.01 \text{ for } \omega < 0.1 \text{ rad/sec}$$

Rearranging the inequality, we get:

$$|1 + L(j\omega)| > 100 \text{ for } \omega < 0.1 \text{ rad/sec}$$

Since $|1 + L(j\omega)| \approx |L(j\omega)|$ for large $|L(j\omega)|$, the requirement can be approximated as:

$$|L(j\omega)| > 100 \text{ for } \omega < 0.1 \text{ rad/sec}$$

(b) Gain less than 0.04 from input n to output y for frequencies above 200 rad/sec:

The closed-loop transfer function from input n to output y is given by:

$$\frac{Y(s)}{N(s)} = \frac{L(s)}{1+L(s)}$$

To achieve a gain less than 0.04 for frequencies above 200 rad/sec, we require:

$$\left| \frac{Y(j\omega)}{N(j\omega)} \right| = \left| \frac{L(j\omega)}{1+L(j\omega)} \right| < 0.04 \text{ for } \omega > 200 \text{ rad/sec}$$

Rearranging the inequality, we get:

$$|L(j\omega)| < \frac{0.04}{1-0.04} \approx 0.0417 \text{ for } \omega > 200 \text{ rad/sec}$$

Therefore, the requirements on the loop gain $|L(j\omega)|$ can be summarized as:

$$- |L(j\omega)| > 100 \text{ for } \omega < 0.1 \text{ rad/sec} - |L(j\omega)| < 0.0417 \text{ for } \omega > 200 \text{ rad/sec}$$

These requirements on the loop gain will help in designing a controller $K(s)$ that meets the given specifications for the closed-loop system.

Confidence level: 95%

Correct