

10.6 Linearization

Consider the following nonlinear system:

$$\dot{x}(t) = f(x, u) = -4x(t) - 2u(t) + u(t)^3.$$

- (a) Find an equilibrium (\bar{x}, \bar{u}) with $\bar{u} = 2$.
- (b) Linearize the dynamics around (\bar{x}, \bar{u}) . Find the value of a_0 and b_0 of the linearized model in the form:

$$\begin{aligned} \dot{\delta}_x(t) + a_0 \delta_x(t) &= b_0 \delta_u(t), \\ \text{where } \delta_x(t) &:= x(t) - \bar{x} \text{ and } \delta_u(t) := u(t) - \bar{u}. \end{aligned}$$

Solution :

- (a) $\bar{x} = 1$.

Steps: Solve $-4\bar{x} - 2\bar{u} + \bar{u}^3 = 0$ with $\bar{u} = 2$ gives $\bar{x} = 1$.

- (b) $(a_0, b_0) = (4, 10)$.

Steps: $\delta_x = x - \bar{x} = x - 1$, $\delta_u = u - \bar{u} = u - 2$. Then we have

$$\dot{\delta}_x(t) \approx f(\bar{x}, \bar{u}) + \frac{\partial f}{\partial x}(\bar{x}, \bar{u})\delta_x + \frac{\partial f}{\partial u}(\bar{x}, \bar{u})\delta_u = -4\delta_x + 10\delta_u.$$

GPT Answer :

Given the nonlinear system:

$$\dot{x}(t) = f(x, u) = -4x(t) - 2u(t) + u(t)^3.$$

- (a) The equilibrium (\bar{x}, \bar{u}) with $\bar{u} = 2$ is found by solving $0 = -4\bar{x} - 4 + 8$, which gives $\bar{x} = 1$. Therefore, $(\bar{x}, \bar{u}) = (1, 2)$.
- (b) Linearizing the dynamics around (\bar{x}, \bar{u}) gives:

$$\begin{aligned} \frac{\partial f}{\partial x} &= -4, \quad \frac{\partial f}{\partial u} = -2 + 3u^2 \Big|_{u=2} = 10. \\ \therefore \dot{\delta}_x(t) + (-4)\delta_x(t) &= 10\delta_u(t), \end{aligned}$$

with $a_0 = -4$ and $b_0 = 10$.

Confidence Level: 95%

Correct

Claude Answer:

Let's solve this problem step by step.

(a) To find an equilibrium (\bar{x}, \bar{u}) with $\bar{u} = 2$, we set the derivative $\dot{x}(t) = 0$ and solve for \bar{x} :

$$0 = -4\bar{x} - 2\bar{u} + \bar{u}^3$$

Substituting $\bar{u} = 2$:

$$0 = -4\bar{x} - 2(2) + (2)^3 \quad 0 = -4\bar{x} - 4 + 8 \quad 4\bar{x} = 4 \quad \bar{x} = 1$$

Therefore, the equilibrium point is $(\bar{x}, \bar{u}) = (1, 2)$.

(b) To linearize the dynamics around (\bar{x}, \bar{u}) , we calculate the Jacobian matrix:

$$J(x, u) = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial u} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = -4$$

$$\frac{\partial f}{\partial u} = -2 + 3u^2$$

Evaluating the Jacobian at the equilibrium point $(\bar{x}, \bar{u}) = (1, 2)$:

$$J(1, 2) = \begin{bmatrix} -4 & -2 + 3(2)^2 \end{bmatrix} = \begin{bmatrix} -4 & 10 \end{bmatrix}$$

The linearized model is:

$$\dot{\delta}_x(t) = -4\delta_x(t) + 10\delta_u(t)$$

Comparing this with the given form:

$$\dot{\delta}_x(t) + a_0\delta_x(t) = b_0\delta_u(t)$$

We find that $a_0 = 4$ and $b_0 = 10$.

Confidence level: 95%

Correct