

5.10 PD Controller

Consider the following plant:

$$\ddot{y}(t) - 2\dot{y}(t) + y(t) = u(t) \quad (221)$$

with a PD controller in the following form:

$$u(t) = K_p(r(t) - y(t)) - K_d\dot{y}(t) \quad (222)$$

- What is the ODE model for the closed-loop from $r(t)$ to $y(t)$?
- Choose (K_p, K_d) so that the closed-loop is:
 - stable
 - has $(\omega_n, \zeta) = (2\text{rad/sec}, 0.5)$
- What is the steady-state error if r is a unit step reference?
- Would you increase or decrease K_p to reduce the steady-state error?

Solution :

$$(a) \ddot{y}(t) + (K_d - 2)\dot{y}(t) + (K_p + 1)y(t) = K_p r(t)$$

$$(b) K_p = 3 \text{ and } K_d = 4$$

$$(c) 4\bar{y} = 3\bar{r} = 3 \rightarrow \bar{y} = \frac{3}{4}$$

Thus,

$$\bar{e} = \bar{r} - \bar{y} = \frac{1}{4} \quad (223)$$

$$(d) \bar{y} = \frac{K_p}{K_p + 1} \bar{r}. \text{ Thus, increasing } K_p \text{ reduce steady-state error.}$$

GPT Answer :

Given the plant equation:

$$\ddot{y}(t) - 2\dot{y}(t) + y(t) = u(t) \quad (224)$$

and the PD controller:

$$u(t) = K_p(r(t) - y(t)) - K_d\dot{y}(t) \quad (225)$$

- The ODE model for the closed-loop from $r(t)$ to $y(t)$ is:

$$\ddot{y}(t) + (K_d - 2)\dot{y}(t) + (1 - K_p)y(t) = K_p r(t)$$

(b) To achieve stability and specified $(\omega_n, \zeta) = (2 \text{ rad/sec}, 0.5)$, we choose:

$$K_d = 4, \quad K_p = -3$$

Note: This result indicates a calculation error, as K_p and K_d should be positive for the specified system characteristics.

(c) For a unit step reference, the steady-state error calculation approach indicates an inconsistency with the expected system behavior.

(d) Typically, increasing K_p reduces the steady-state error for a type 0 system.

Confidence Level: 60%

Calculation Error

Claude Answer :

(a) The closed-loop ODE from $r(t)$ to $y(t)$ is:

$$\ddot{y}(t) + (K_d - 2)\dot{y}(t) + (K_p + 1)y(t) = K_p r(t) \quad (226)$$

(b) For the closed-loop to be stable and have $(\omega_n, \zeta) = (2 \text{ rad/sec}, 0.5)$:

$$K_p + 1 = \omega_n^2 = 2^2 = 4 \Rightarrow K_p = 3 \quad (227)$$

$$K_d - 2 = 2\zeta\omega_n = 2 \times 0.5 \times 2 = 2 \Rightarrow K_d = 4 \quad (228)$$

(c) The steady-state error for a unit step reference is:

$$e_{ss} = \frac{1}{1 + K_p} = \frac{1}{1 + 3} = \frac{1}{4} = 0.25 \quad (229)$$

(d) To reduce the steady-state error, you should increase K_p .

Confidence level: 98/100

The revised answer corrects the calculation for K_d and maintains the same steady-state error calculation and recommendation for reducing the error.

The provided answer assumes a second-order system with the given natural frequency and damping ratio. The steady-state error calculation assumes a type-0 system (no integrators or free integrators in the forward path).

Correct