

2.17 Stability and Time Constant

For each of the systems below:

- What are the poles? Is the system stable?
- What is the time constant associated with each pole? Which time constant is the slowest? (Provide a time constant regardless of stability.)

(a) $G(s) = \frac{s-2}{s+7}$

(b) $G(s) = \frac{s+2}{s-7}$

(c) $G(s) = \frac{-9}{s^2+2s-8}$

(d) $G(s) = \frac{5}{(s^2+4s+13)(s-5)}$

Solution :

(a) **Poles:** $s = -7$ system is stable

Time constant: $\tau = \frac{1}{|Re(s)|} = \frac{1}{7}$ sec

(b) **Poles:** $s = 7$ system is unstable

Time constant: $\tau = \frac{1}{|Re(s)|} = \frac{1}{7}$ sec

(c) **Poles:** $s_1 = -4, s_2 = 2$ system is unstable

Time constant: $\tau_1 = \frac{1}{|Re(s)|} = \frac{1}{4}$ sec, $\tau_2 = \frac{1}{|Re(s)|} = \frac{1}{2}$ sec, τ_2 is slowest time constant

(d) **Poles:** $s_{1,2} = -2 \pm 3j, s_3 = +5$ system is unstable

Time constant: $\tau_{1,2} = \frac{1}{|Re(s)|} = \frac{1}{2}$ sec, $\tau_3 = \frac{1}{|Re(s)|} = \frac{1}{5}$ sec, $\tau_{1,2}$ is slowest time constant

GPT Answer :

Here's the analysis for each system:

(a) $G(s) = \frac{s-2}{s+7}$

- Poles: $[-7]$
- Stability: Yes
- Time Constants: $[\frac{1}{7}]$
- Slowest Time Constant: $\frac{1}{7}$

(b) $G(s) = \frac{s+2}{s-7}$

- Poles: $[7]$
- Stability: No
- Time Constants: $[\frac{1}{7}]$
- Slowest Time Constant: $\frac{1}{7}$

(c) $G(s) = \frac{-9}{s^2+2s-8}$

- Poles: $[-4, 2]$
- Stability: No
- Time Constants: $[\frac{1}{4}, \frac{1}{2}]$
- Slowest Time Constant: $\frac{1}{2}$

(d) $G(s) = \frac{5}{(s^2+4s+13)(s-5)}$

- Poles: $[5, -2 - 3i, -2 + 3i]$
- Stability: No
- Time Constants: $[\frac{1}{5}, \frac{\sqrt{13}}{13}, \frac{\sqrt{13}}{13}]$
- Slowest Time Constant: $\frac{\sqrt{13}}{13}$

This analysis provides the poles, stability status, time constants associated with each pole, and the slowest time constant for each given system. Systems (a) and (b) have real poles and straightforward time constants. Systems (c) and (d) involve more complex poles or quadratic factors, but we have still identified the real part of these poles to determine stability and associated time constants. Note that system (a) is the only stable system, while all others are unstable due to the presence of poles with positive real parts or, in the case of system (d), a pole in the right half of the s-plane.

Confidence Level: 95%

Reasoning Error: In part (d), while the poles are correctly calculated, the error lies in using the poles' magnitude rather than their real parts to compute the time constant.

Claude Answer :

Here is the analysis for each of the given systems:

(a) $G(s) = \frac{s-2}{s+7}$ - The pole is at $s = -7$. Since the pole is in the left half-plane (LHP), the system is stable. - The time constant associated with the pole is $\tau = \frac{1}{7} \approx 0.143$ seconds.

(b) $G(s) = \frac{s+2}{s-7}$ - The pole is at $s = 7$. Since the pole is in the right half-plane (RHP), the system is unstable. - The time constant associated with the pole is $\tau = \frac{1}{7} \approx 0.143$ seconds.

(c) $G(s) = \frac{-9}{s^2+2s-8}$ - The poles are found by solving the characteristic equation: $s^2 + 2s - 8 = 0$
 $s = \frac{-2 \pm \sqrt{4+32}}{2} = \frac{-2 \pm 6}{2} = -4, 2$ - The system has one pole in the LHP at $s = -4$ and one pole in the RHP at $s = 2$. Since there is a pole in the RHP, the system is unstable. - The time constants associated with the poles are: For $s = -4$: $\tau_1 = \frac{1}{4} = 0.25$ seconds For $s = 2$: $\tau_2 = \frac{1}{2} = 0.5$ seconds - The slowest time constant is $\tau_2 = 0.5$ seconds.

(d) $G(s) = \frac{5}{(s^2+4s+13)(s-5)}$ - The poles are found by solving the characteristic equations: $s^2 + 4s + 13 = 0$ and $s - 5 = 0$
 $s = \frac{-4 \pm \sqrt{16-52}}{2} = \frac{-4 \pm \sqrt{-36}}{2} = -2 \pm j\sqrt{9} = -2 \pm j3$ $s = 5$ - The system has a pair of complex conjugate poles in the LHP at $s = -2 \pm j3$ and a pole in the RHP at $s = 5$. Since there is a pole in the RHP, the system is unstable. - The time constants associated with the poles are: For $s = -2 \pm j3$: $\tau_1 = \frac{1}{2} = 0.5$ seconds For $s = 5$: $\tau_2 = \frac{1}{5} = 0.2$ seconds - The slowest time constant is $\tau_1 = 0.5$ seconds.

Confidence Level: 95%

Correct