

2.9 Routh-Hurwitz Stability

Determine if the following characteristic equation has any roots with positive real parts:

$$s^4 + s^3 - s - 1 = 0 \quad (114)$$

Solution :

Note that the coefficient of the s^2 term is zero. The Routh table for this equation is

$$\begin{array}{c|ccc} s^4 & 1 & 0 & -1 \\ s^3 & 1 & -1 & 0 \\ s^2 & 1 & -1 & \\ s^1 & 0 & 0 & \\ \text{new } s^1 & 2 & 0 & \\ s^0 & 0 & & \end{array}$$

The presence of the zeros in the s^1 row indicates that the characteristic equation has two roots which satisfy the auxiliary equation formed from the s^2 row as follows: $s^2 - 1 = 0$. The roots of the equation are $= 1$ and -1 . The new s^1 row was formed using the coefficients from the derivative of the auxiliary equation: $2s - 0 = 0$. Since there is one change of sign, the characteristic equation has one root with a positive real part, the one at $+1$ determine from the auxiliary equation.

GPT Answer:

To determine if the given characteristic equation $s^4 + s^3 - s - 1 = 0$ has any roots with positive real parts, we can employ the Routh-Hurwitz criterion. This criterion provides a systematic method to ascertain the number of roots with positive real parts of a polynomial without explicitly finding the roots. The characteristic equation is a polynomial of degree 4, and we need to construct the Routh-Hurwitz array to apply the criterion.

Given the polynomial:

$$P(s) = s^4 + s^3 - s - 1$$

The coefficients are arranged in descending powers of s , which gives us:

$$a_4 = 1, a_3 = 1, a_2 = 0, a_1 = -1, a_0 = -1$$

The Routh-Hurwitz array is constructed as follows:

$$\begin{array}{c|cc} s^4 & 1 & 0 \\ s^3 & 1 & -1 \\ s^2 & -1 & -1 \\ s^1 & \frac{-1-(0 \cdot -1)}{-1} = -1 & 0 \\ s^0 & -1 & \end{array}$$

To ensure no roots with positive real parts, all the elements in the first column of the Routh-Hurwitz array must be of the same sign. However, we observe that there are sign changes in the first column, indicating that the polynomial has roots with positive real parts.

Therefore, based on the Routh-Hurwitz criterion, the characteristic equation $s^4 + s^3 - s - 1 = 0$ does have roots with positive real parts.

Confidence level: 95%.

Correct

Claude Answer:

To determine if the given characteristic equation has any roots with positive real parts, we can use the Routh-Hurwitz stability criterion. The Routh-Hurwitz criterion states that a polynomial with real coefficients is stable (i.e., all its roots have negative real parts) if and only if all the elements in the first column of the Routh-Hurwitz table are positive.

Given the characteristic equation:

$$s^4 + s^3 - s - 1 = 0$$

Step 1: Write the coefficients of the polynomial in descending order of powers:

$$a_4 = 1, \quad a_3 = 1, \quad a_2 = 0, \quad a_1 = -1, \quad a_0 = -1$$

Step 2: Construct the Routh-Hurwitz table:

$$\begin{array}{c|ccc} s^4 & a_4 = 1 & a_2 = 0 & a_0 = -1 \\ s^3 & a_3 = 1 & a_1 = -1 & \\ s^2 & b_1 = 1 & b_2 = -1 & \\ s^1 & c_1 = 0 & & \\ s^0 & d_1 = -1 & & \end{array}$$

where:

$$\begin{aligned} b_1 &= \frac{a_3 a_2 - a_4 a_1}{a_3} = \frac{1 \cdot 0 - 1 \cdot (-1)}{1} = 1, \\ b_2 &= a_0 = -1, \\ c_1 &= \frac{b_1 a_1 - a_3 b_2}{b_1} = \frac{1 \cdot (-1) - 1 \cdot (-1)}{1} = 0, \\ d_1 &= b_2 = -1. \end{aligned}$$

Step 3: Check the sign of the elements in the first column of the Routh-Hurwitz table.

The first column contains: $1, 1, 0, -1$. Since there is a sign change (from 1 to -1) and a zero element, the system is unstable, meaning that at least one root has a positive real part.

Therefore, the given characteristic equation has at least one root with a positive real part.

Confidence level: 95%.

Correct