

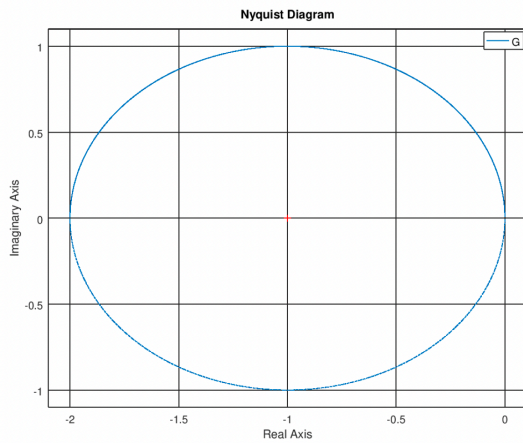
8.5 Closed-Loop Poles of the Feedback

Consider the system $G(s)$ and the controller $K(s)$ are in standard unity feedback setting. Define $L(s) := G(s)K(s)$. Use the Nyquist plot for the following loop transfer functions and apply the Nyquist stability theorem to predict the number of closed-loop poles of the feedback system in the right-half plane.

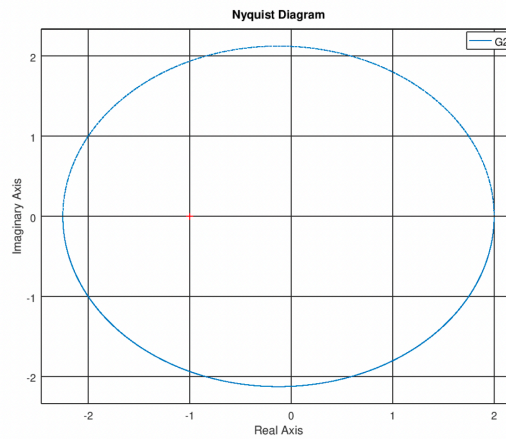
$$(a) \quad L(s) = \frac{-8}{s+4}$$

$$(b) \quad L(s) = \frac{2s+9}{s-4}$$

Solution :



(a) Nyquist plot of $L(s)$ in (a)



(b) Nyquist plot of $L(s)$ in (b)

Figure 23: Nyquist plot for the systems

The Nyquist plot for the two $L(s)$ is given in Fig. 23. The Nyquist stability theorem says that if the number of RHP poles of the closed loop transfer function $\frac{L(s)}{1+L(s)}$ is Z , the number of RHP poles of the open loop transfer function $L(s)$ is P , and N is the number of clockwise rotations of the Nyquist plot around -1 , then $Z = N + P$. Therefore,

$$(a) \quad N = 1; P = 0 \text{ so } Z = 1.$$

$$(b) \quad N = -1; P = 1 \text{ so } Z = 0.$$

GPT Answer :

To address your question, we analyze each part (a) and (b) separately.

Part (a): $L(s) = \frac{-8}{s+4}$

1. *Nyquist Plot for $L(s)$:* A first-order system with a pole at -4 and a negative gain. The plot starts from -8 on the real axis and approaches the origin, not enclosing $-1 + 0j$.

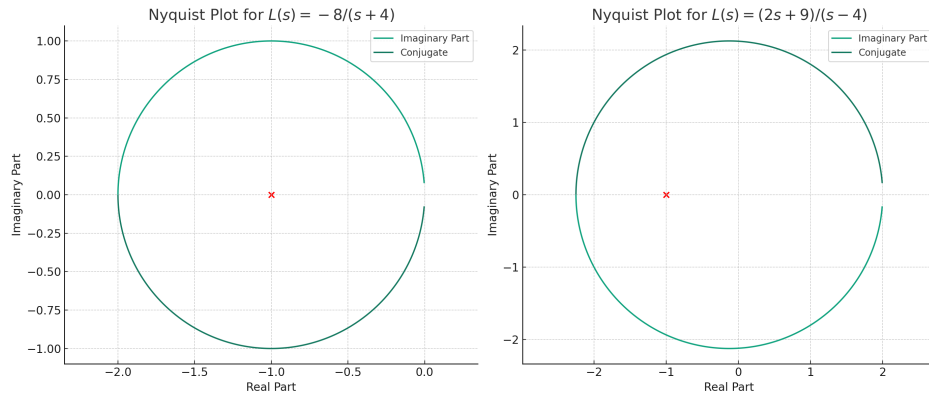


Figure 24: Nyquist Plots from GPT 4

2. *Nyquist Stability Criterion*: No encirclements of $-1 + 0j$ and no poles on the imaginary axis, thus $N = 0$, indicating stability.

Part (b): $L(s) = \frac{2s+9}{s-4}$

1. *Nyquist Plot for $L(s)$* : Has a zero and a RHP pole, with a plot starting at 180° phase and circling large, possibly around $-1 + 0j$.

2. *Nyquist Stability Criterion*: With one RHP pole ($P = 1$), the system shows potential instability, suggesting at least one RHP closed-loop pole.

Confidence Level: 85%

Reasoning error

Claude Answer :

The Nyquist stability theorem says that if the number of RHP poles of the closed loop transfer function $\frac{L(s)}{1+L(s)}$ is Z , the number of RHP poles of the open loop transfer function $L(s)$ is P , and N is the number of clockwise rotations of the Nyquist plot around -1 , then $Z = N + P$. Therefore,

- (a) $N = 1$; $P = 0$ so $Z = 1$. This means the Nyquist plot for $L(s) = \frac{-8}{s+4}$ makes one clockwise encirclement of the -1 point. There are no open-loop poles in the right-half plane. Applying the Nyquist stability criterion, the closed-loop system will have one pole in the right-half plane, making it unstable.
- (b) $N = -1$; $P = 1$ so $Z = 0$. This means the Nyquist plot for $L(s) = \frac{2s+9}{s-4}$ makes one counter-clockwise encirclement of the -1 point. There is one open-loop pole in the right-half plane at $s = 4$. Applying the Nyquist stability criterion, the number of poles in the right-half plane for the closed-loop system is zero, indicating a stable system.

Confidence Level: 95%

Correct