

### 5.23 Dynamic Output Feedback Controller

Consider the following system:

$$\dot{x} = \begin{pmatrix} 0 & -1 & 2/3 \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} u \quad y = x_2$$

Design a dynamic output feedback controller such that the poles of the closed-loop system are at  $-10, -10 \pm 5j$ .

**Solution:**

We can obtain a dynamic output feedback controller by combining state feedback controller with a full state observer design. The state-space model for the resulting controller will be:

$$\begin{aligned} \dot{\hat{x}} &= (A - LC - BK)\hat{x} + Ly \\ u &= -K\hat{x} \end{aligned} \quad (265)$$

where in order to place the poles of closed-loop system at  $-10, -10 \pm 5j$  we need to use  $K = (325 \quad -1251 \quad 735)$  and  $L = (26769 \quad 59 \quad 28032)^T$  to place the observer poles at  $-20, -20 \pm 2j$ .

**GPT Answer :**

Upon recalculating and combining all results for both the feedback gain  $K$  and the observer gain  $L$ , we confirm the following:

- The feedback gain  $K$  to place the closed-loop system poles at  $-10, -10 + 5j$ , and  $-10 - 5j$  is given by:

$$K = (325 \quad -1251 \quad 735.333) .$$

- The observer gain  $L$ , designed to place the observer's poles at  $-20, -25$ , and  $-30$ , is determined as:

$$L = \begin{pmatrix} 48851 \\ 74 \\ 50775 \end{pmatrix} .$$

Combining these results ensures that the control system is robust, with the desired pole placements for both the system response and the observer dynamics, leading to a confidence level in these results of **100%**.

**Correct**

**Claude Answer :**

The MATLAB commands and the resulting calculations for determining the state feedback gain matrix  $K$  and the observer gain matrix  $L$  are presented below:

Using the `place` function in MATLAB to place the poles of the state feedback system:

```
>> A = [0 -1 2/3; -1 -2 1; 0 -3 1];  
>> B = [1; 2; 3];  
>> C = [0 1 0];  
>> desired_poles = [-10, -10+5j, -10-5j];  
>> K = place(A, B, desired_poles)
```

The computed state feedback gain matrix  $K$  is:

$$K = \begin{bmatrix} 325 & -1251 & 735.3333 \end{bmatrix}$$

Using the acker function in MATLAB to place the poles of the observer:

```
>> observer_poles = desired_poles * 5;  
>> L = acker(A', C', observer_poles)'
```

The computed observer gain matrix  $L$  is:

$$L = \begin{bmatrix} 485301.0000 \\ 149.0000 \\ 493575.0000 \end{bmatrix}$$

**Confidence Level: 95%**

**Correct**