

1.14 Linear Independence

Show that the function $1, t, t^2$ are linearly independent.

Solution :

The Wronskian of these three functions is

$$\begin{vmatrix} 1 & t & t^2 \\ 0 & 1 & 2t \\ 0 & 0 & 2 \end{vmatrix} \quad (48)$$

Since the Wronskian is nonzero, the functions are linearly independent.

GPT Answer:

To show that the functions $1, t, t^2$ are linearly independent, we must demonstrate that the only solution to the equation

$$c_1 \cdot 1 + c_2 \cdot t + c_3 \cdot t^2 = 0$$

for all t , is $c_1 = c_2 = c_3 = 0$, where c_1, c_2, c_3 are constants.

This equation represents a polynomial equation of degree 2. For it to be identically zero (i.e., equal to zero for all values of t), all the coefficients of the polynomial must be zero. This means $c_1 = 0$ (the coefficient of t^0), $c_2 = 0$ (the coefficient of t^1), and $c_3 = 0$ (the coefficient of t^2).

Since the only solution to the equation is $c_1 = c_2 = c_3 = 0$, the functions $1, t, t^2$ are linearly independent.

Confidence Level: 100 %

Correct

GPT Answer:

To show that the functions $1, t$, and t^2 are linearly independent, we need to prove that the only solution to the equation $c_1 \cdot 1 + c_2 \cdot t + c_3 \cdot t^2 = 0$ for all t is when $c_1 = c_2 = c_3 = 0$.

Suppose $c_1 \cdot 1 + c_2 \cdot t + c_3 \cdot t^2 = 0$ for all t . Then, we can choose three distinct values of t to form a system of linear equations:

1. $t = 0$: $c_1 \cdot 1 + c_2 \cdot 0 + c_3 \cdot 0^2 = 0 \implies c_1 = 0$
2. $t = 1$: $c_1 \cdot 1 + c_2 \cdot 1 + c_3 \cdot 1^2 = 0 \implies c_2 + c_3 = 0$
3. $t = -1$: $c_1 \cdot 1 + c_2 \cdot (-1) + c_3 \cdot (-1)^2 = 0 \implies -c_2 + c_3 = 0$

From equations 2 and 3, we can deduce that $c_2 = c_3 = 0$. Therefore, the only solution to the equation $c_1 \cdot 1 + c_2 \cdot t + c_3 \cdot t^2 = 0$ for all t is when $c_1 = c_2 = c_3 = 0$, proving that the functions $1, t$, and t^2 are linearly independent. Confidence level: 100

Correct