

## 1.20 Characterization of Second-Order Systems

For each of the system below, determine the followings:

- What is the natural frequency and damping ratio?
- Is the system under, over, or critically damped?
- For the unit step input, find the final value, settling time (5%) and overshoot (if underdamped)

(a)  $G_A(s) = \frac{20}{s^2 + 2s + 10}$

(b)  $G_B(s) = \frac{20}{s^2 + 11s + 10}$

**Solution :**

(a)

- $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \rightarrow \omega_n = \sqrt{10}/\text{rad/sec}, \quad \zeta = \frac{2}{2\omega_n} = \frac{1}{\sqrt{10}} \approx 0.316$
- $\zeta < 1 \rightarrow \text{underdamped}$
- $y_f = \frac{20 \cdot 1}{10} = 2 \quad t_s = \frac{3}{\zeta\omega_n} = 3 \quad Mp = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.35$

(b)

- $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \rightarrow \omega_n = \sqrt{10}/\text{rad/sec}, \quad \zeta = \frac{11}{2\omega_n} = \frac{1}{\sqrt{10}} \approx 1.74$
- $\zeta > 1 \rightarrow \text{overdamped}$
- $y_f = \frac{20 \cdot 1}{10} = 2 \quad \tau_1 = 1, \tau_2 = 10 \quad t_s = 3 * \min\{\tau_1, \tau_2\} = 3$

**GPT Answer :**

For each system:

(a)  $G_A(s) = \frac{20}{s^2 + 2s + 10}$

- Natural frequency  $\omega_n = \sqrt{10}$  rad/s
- Damping ratio  $\zeta = \frac{1}{\sqrt{10}}$
- System is underdamped.
- For a unit step input, final value = 2, settling time ( $T_s$ ) is approximately  $\frac{4.6}{\zeta\omega_n}$ , and percent overshoot can be calculated using  $PO = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$  times 100

(b)  $G_B(s) = \frac{20}{s^2 + 11s + 10}$

- Natural frequency  $\omega_n = \sqrt{10}$  rad/s
- Damping ratio  $\zeta = \frac{11}{2\sqrt{10}}$
- System is overdamped.
- For a unit step input, final value = 2. Settling time ( $T_s$ ) is generally larger than for underdamped systems, and there is no overshoot.

**Confidence level: 95%**

**Correct**

**Claude Answer :**

(a) For  $G_A(s) = \frac{20}{s^2 + 2s + 10}$ :

- Natural frequency:  $\omega_n = \sqrt{10} \approx 3.162$  rad/s  
Damping ratio:  $\zeta = \frac{2}{2\sqrt{10}} \approx 0.316$
- Underdamped system ( $0 < \zeta < 1$ )
- For a unit step input:
  - Final value:  $\lim_{s \rightarrow 0} sG_A(s) \frac{1}{s} = 2$
  - Settling time (5%):  $T_s \approx \frac{3}{\zeta\omega_n} \approx 3$  s
  - Overshoot:  $M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \approx 28.1\%$

(b) For  $G_B(s) = \frac{20}{s^2 + 11s + 10}$ :

- Natural frequency:  $\omega_n = \sqrt{10} \approx 3.162$  rad/s  
Damping ratio:  $\zeta = \frac{11}{2\sqrt{10}} \approx 1.737$
- Overdamped system ( $\zeta > 1$ )
- For a unit step input:
  - Final value:  $\lim_{s \rightarrow 0} sG_B(s) \frac{1}{s} = 2$
  - Settling time (5%):  $T_s \approx \frac{3}{(\zeta - \sqrt{\zeta^2 - 1})\omega_n} \approx 0.88$  s
  - No overshoot (overdamped)

Confidence level: 95/100

**Correct**