

1.16 Characterization of Second-Order Systems

Determine

- (a) the undamped natural frequency ω_n
- (b) the damping ratio ζ
- (c) the constant time $\tau = \frac{1}{\zeta\omega_n}$
- (d) the damped natural frequency ω_d
- (e) characteristic equation for the second-order system given by

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 9y = 9u \quad (54)$$

Solution :

- (a) $\omega_n^2 = 9$ or $\omega_n = 3$ rad/sec
- (b) $2\zeta\omega_n = 5$ or $\zeta = \frac{5}{2\omega_n} = \frac{5}{6}$
- (c) $\tau = \frac{1}{\zeta\omega_n} = \frac{2}{5}$ sec
- (d) $\omega_d = \omega_n\sqrt{1 - \zeta^2} = 1.66$ rad/sec
- (e) $s^2 + 5s + 9 = 0$

GPT Answer:

The results are:

- (a) The undamped natural frequency $\omega_n = 3$ rad/s
- (b) The damping ratio $\zeta = \frac{5}{6}$
- (c) The constant time $\tau = \frac{2}{5}$ sec.
- (d) The damped natural frequency $\omega_d = \frac{\sqrt{11}}{2}$ rad/s
- (e) The characteristic equation is $\lambda^2 + 5\lambda + 9 = 0$

Confidence level: 95%

Correct

Claude Answer:

1. The given second-order system is:

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 9y = 9u \quad (55)$$

- (a) The undamped natural frequency is given by $\omega_n = \sqrt{\frac{k}{m}}$, where k is the spring constant and m is the mass. From the equation, we have $k = 9$ and $m = 1$. Therefore, $\omega_n = \sqrt{\frac{9}{1}} = 3$ rad/s.
- (b) The damping ratio is given by $\zeta = \frac{c}{2\sqrt{mk}}$, where c is the damping coefficient. From the equation, we have $c = 5$. Therefore, $\zeta = \frac{5}{2\sqrt{1 \cdot 9}} = \frac{5}{6} \approx 0.833$.
- (c) The time constant is $\tau = \frac{1}{\zeta\omega_n} = \frac{1}{0.833 \cdot 3} = 0.4$ s.
- (d) The damped natural frequency is $\omega_d = \omega_n\sqrt{1 - \zeta^2}$. Substituting the values, we get $\omega_d = 3\sqrt{1 - 0.833^2} \approx 1.658$ rad/s.
- (e) The characteristic equation is obtained by setting the right-hand side of the differential equation to zero and replacing $\frac{d}{dt}$ with the Laplace variable s . Thus, the characteristic equation is:

$$s^2 + 5s + 9 = 0 \quad (56)$$

Confidence level: 95/100

Correct