

5.16 PI Controller

Consider a car whose longitudinal motion is modeled by the following ODE:

$$2085\dot{v}(t) + 23.2v(t) = 40u(t) + 108.4 - F_{grav}(t)$$

The input is the throttle u and the output is the velocity v . The gravitational force F_{grav} is a disturbance. Let $e(t) = v_{des} - v(t)$ denote the tracking error between the desired velocity $v_{des} = 29$ m/s and actual velocity $v(t)$. Consider a PI controller of the following form:

$$u(t) = \bar{u} + K_p e(t) + K_i \int_0^t e(\tau) d\tau$$

where $\bar{u} = 14.11$ is the open-loop input to maintain v_{des} when on at road $\theta = 0^\circ$. Choose the PI gains so that the cruise control system is stable and rejects disturbances due to changing road slopes within ≈ 10 sec. The closed-loop should also be over or critically damped as oscillations are uncomfortable for the driver.

Solution :

This particular problem is of interest as it is one of the first design problems encountered by undergraduate control engineer. Moreover, it requires reasoning about the impact of the control gains on the closed-loop poles and the connection to transient response properties. There is more than one design that satisfies the given objectives. One approach is to note that the closed-loop characteristic equation is:

$$2085s^2 + (23.2 + 40K_p)s + 40K_i = 0$$

We can place the closed-loop poles in the LHP to have any desired damping ratio ζ and natural frequency ω_n by choosing the controller gains to satisfy $\omega_n^2 = \frac{40K_i}{2085}$ and $2\zeta\omega_n = \frac{23.2+40K_p}{2085}$. Select $\zeta = 1$ to obtain two critically damped closed-loop poles placed at $s = -\omega_n$. The time constant for these poles is $\tau = \frac{1}{\omega_n}$ sec and, for critically damped poles, the 5% settling time is approximately $4.75\tau = \frac{4.75}{\omega_n}$. Thus we select $\omega_n = \frac{4.75}{10} = 0.475$ to obtain a settling time near 10sec. Finally, use $\zeta = 1$ and $\omega_n = 0.475$ to solve for the PI gains using the relations above. This yields $K_p = 48.94$ and $K_i = 11.76$. The corresponding transfer function from F_{grav} to v is

$$T_{F_{grav} \rightarrow v}(s) = \frac{-s}{2085s^2 + 1981s + 470.4}$$

This has critically damped poles at $s = -0.475$ as expected. A step change increase in gravitational force (due to a step increase in the road slope) will cause the velocity to initially drop from the desired value. However, the PI controller rejects the disturbance in ≈ 10 sec with the velocity converging to the desired value with a nice overdamped response.

GPT Answer :

The given system's differential equation is:

$$2085\dot{v}(t) + 23.2v(t) = 40u(t) + 108.4 - F_{grav}(t)$$

with the desired velocity $v_{des} = 29$ m/s.

Steps for PI Controller Design include Linearization, Transfer Function derivation, and PI Controller Design to ensure Stability, Disturbance Rejection, and Damping criteria are met.

Given the objectives and constraints, the exact values of K_p and K_i are dependent on detailed analysis. The confidence level in providing specific gain values without further computation is low, but the approach for control design is accurate.

Confidence Level: 40%

Did not solve

Claude Answer:

To design a PI controller for the given car longitudinal motion model, we need to choose the gains K_p and K_i such that the closed-loop system is stable, rejects disturbances within approximately 10 seconds, and has an overshoot less than 1% in the step response.

First, let's write the closed-loop transfer function from the desired velocity v_{des} to the actual velocity $v(t)$:

$$\frac{V(s)}{V_{des}(s)} = \frac{40K_p s + 40K_i}{2085s^2 + (23.2 + 40K_p)s + 40K_i}$$

For a stable system with an overshoot less than 1%, the damping ratio should be $\zeta \geq 0.826$. We can choose the poles to be $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$, with $\zeta = 0.9$ and $\omega_n = 0.4$ to achieve a settling time of approximately 10 seconds (settling time $\approx \frac{4}{\zeta\omega_n}$).

Comparing the denominator of the closed-loop transfer function with the standard second-order polynomial $s^2 + 2\zeta\omega_n s + \omega_n^2$, we get:

$$\begin{aligned} 2\zeta\omega_n &= \frac{23.2 + 40K_p}{2085} \\ \omega_n^2 &= \frac{40K_i}{2085} \end{aligned}$$

Solving for K_p and K_i with the chosen poles:

$$\begin{aligned} K_p &= \frac{2085 \cdot 2\zeta\omega_n - 23.2}{40} \approx 71.6 \\ K_i &= \frac{2085\omega_n^2}{40} \approx 33.4 \end{aligned}$$

Therefore, setting $K_p \approx 71.6$ and $K_i \approx 33.4$ should result in a stable closed-loop system that rejects disturbances within approximately 10 seconds and has an overshoot less than 1% in the step response.

Confidence Level: 90%

Correct