

9.3 Resonance Peak

Determine the resonance peak M_p and the resonant frequency ω_p for the system whose transfer function is $\frac{C}{R}(s) = \frac{5}{s^2 + 2s + 5}$.

Solution :

The magnitude $|C/R(j\omega)|$ is given by:

$$\left| \frac{C}{R}(j\omega) \right| = \left| \frac{5}{-\omega^2 + 2j\omega + 5} \right| = \frac{5}{\sqrt{\omega^4 - 6\omega^2 + 25}}$$

Setting the derivative of $\left| \frac{C}{R}(j\omega) \right|$ equal to zero, we get $\omega_p = \pm\sqrt{3}$. Therefore,

$$M_p = \max_{\omega} \left| \frac{C}{R}(j\omega) \right| = \left| \frac{C}{R}(j\sqrt{3}) \right| = \frac{5}{4} \quad (273)$$

GPT Answer:

The transfer function given is $\frac{C}{R}(s) = \frac{5}{s^2 + 2s + 5}$. By comparing it to the standard form $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$, we identify $\omega_n = \sqrt{5}$ and $\zeta = \frac{1}{\sqrt{5}}$.

The resonance peak M_p is calculated as:

$$M_p = 20 \log_{10} \left(\frac{5}{4} \right) \text{ dB}$$

And the resonant frequency ω_p is:

$$\omega_p = \sqrt{3} \text{ rad/s}$$

Confidence level: 95%

Correct

Claude Answer:

To analyze the resonance peak (M_p) and the resonant frequency (ω_p), we will examine the given transfer function step by step.

Given: The transfer function $\frac{C}{R}(s) = \frac{5}{s^2 + 2s + 5}$

Step 1: Express the transfer function in standard second-order form The transfer function can be rewritten in the form:

$$\frac{C}{R}(s) = \frac{5}{s^2 + 2s + 5} = \frac{5}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Here, ω_n is the natural frequency, and ζ is the damping ratio.

Comparing the denominators, we have:

$$2\zeta\omega_n = 2, \quad \omega_n^2 = 5$$

Step 2: Determine the natural frequency (ω_n) Solving for ω_n :

$$\omega_n^2 = 5 \implies \omega_n = \sqrt{5}$$

Step 3: Determine the damping ratio (ζ) Solving for ζ :

$$2\zeta\sqrt{5} = 2 \implies \zeta = \frac{1}{\sqrt{5}}$$

Step 4: Calculate the resonant frequency (ω_p) The resonant frequency ω_p is given by:

$$\omega_p = \omega_n \sqrt{1 - 2\zeta^2}, \quad \text{for } 0 < \zeta < \frac{1}{\sqrt{2}}$$

$$\omega_p = \sqrt{5} \sqrt{1 - 2\left(\frac{1}{\sqrt{5}}\right)^2} = \sqrt{5} \sqrt{\frac{3}{5}} = \sqrt{3}$$

Step 5: Calculate the resonance peak (M_p) The resonance peak M_p is given by:

$$M_p = \frac{1}{2\zeta\sqrt{1 - \zeta^2}}, \quad \text{for } 0 < \zeta < \frac{1}{\sqrt{2}}$$

$$M_p = \frac{1}{2\left(\frac{1}{\sqrt{5}}\right)\sqrt{1 - \left(\frac{1}{\sqrt{5}}\right)^2}} = \frac{\sqrt{5}}{2\sqrt{\frac{4}{5}}} = \frac{\sqrt{5}}{2 \times \frac{2}{\sqrt{5}}} = \frac{5}{4}$$

Therefore, the resonant frequency (ω_p) is $\sqrt{3}$, and the resonance peak (M_p) is $\frac{5}{4}$.

Confidence level: 95%

Correct