

2.3 Designing Minimal Complexity Stabilizing Controller

Consider a continuous time system with a transfer function $G_o(s)$ defined as

$$G_o(s) = \frac{2}{(s-1)(s+2)} = \frac{2}{s^2 + s - 2}. \quad (82)$$

Your task is to design a controller of minimal complexity that stabilizes the plant, ensures zero steady state error for step disturbances, and achieves a closed-loop response where natural modes decay at a rate no slower than e^{-3t} .

Solution :

We begin by recalling that we can choose an arbitrary set of closed loop natural frequencies if the closed loop characteristic polynomial, $A_{cl}(s)$, has degree at least equal to $2n - 1$ (the degree of $A_o(s)$, the plant nominal model denominator). However, since we want to force zero steady state error at d.c., an additional degree-of-freedom is needed. We are aiming for a minimum complexity controller; hence, we choose the degree of $A_{cl}(s)$ equal to 4.

In addition, the roots of $A_{cl}(s)$ should be to the left of $s = -3$ to ensure that the response time specification is met. Say we choose

$$A_{cl}(s) = (s^2 + 6s + 9)(s + 4)(s + 5) = s^4 + 15s^3 + 83s^2 + 201s + 180 \quad (83)$$

We can now solve the Diophantine equation

$$(s-1)(s+2) \underbrace{s(s+l_0)}_{L(s)} + 2 \underbrace{(p_2s^2 + p_1s + p_0)}_{P(s)} = s^4 + 15s^3 + 83s^2 + 201s + 180 \quad (84)$$

where $A_o(s)$ is the plant nominal model denominator, $B_o(s)$ is the plant numerator, $L(s)$ represents the controller's pole polynomial, and $P(s)$ represents the controller's zero polynomial. We finally obtain

$$P(s) = 35.5s^2 + 114.5s + 90 \quad (85)$$

$$L(s) = s^2 + 14s \quad (86)$$

$$C(s) = \frac{35.5s^2 + 114.5s + 90}{s^2 + 14s} \quad (87)$$

GPT Answer :

To design a controller that stabilizes the plant, ensures zero steady-state error for step disturbances, and achieves a closed-loop response where natural modes decay at a rate no slower than e^{-3t} , we can use a Proportional-Integral (PI) controller. The PI controller adds a zero to the open-loop transfer function and increases the system type, helping in eliminating the steady-state error for step inputs.

The general form of a PI controller is:

$$G_c(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s},$$

where K_p and K_i are the proportional and integral gains, respectively.

To ensure the natural modes of the closed-loop system decay at a rate no slower than e^{-3t} , the real parts of the closed-loop poles must be less than or equal to -3 . This requirement leads to a desired characteristic equation of the closed-loop system that can be represented as $(s + 3)^n$, with n being the order of the system plus the order of the controller.

Confidence Score: 90%

Incorrect Knowledge

Claude Answer :

To design a controller meeting the given requirements:

1. For stability and decay rate of at least e^{-3t} , all closed-loop poles must have real parts less than or equal to -3 .
2. For zero steady-state error to step disturbances, the controller must include an integrator (a pole at $s = 0$).
3. The simplest controller satisfying these criteria is a PI controller of the form:

$$G_c(s) = K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s} \quad (88)$$

Closed-loop transfer function:

$$\frac{Y(s)}{R(s)} = \frac{2(K_p s + K_i)}{s^3 + (2K_p + 1)s^2 + (2K_i - 2)s + 2K_i} \quad (89)$$

4. Choose K_p and K_i such that the closed-loop poles have real parts ≤ -3 . One possible choice: $K_p = 5$, $K_i = 11$. This yields closed-loop poles at $\{-3, -3, -5\}$.

Therefore, a PI controller with $K_p = 5$ and $K_i = 11$ is a minimal complexity controller that meets all stated requirements.

Confidence level: 95/100

Incorrect Knowledge