

1.25 Time-Domain Specs

Consider the transfer function

$$H(s) = \frac{16}{s^2 + 4s + 16}$$

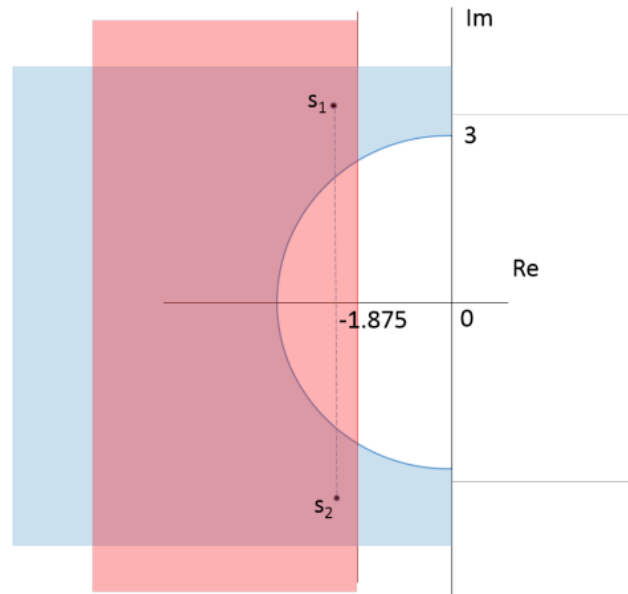
Suppose that you are given the following time-domain specs: $t_r \leq 0.6$, $t_s \leq 1.6$ (defined by the time system's response reach and stay within 5% of steady state). Does the given system satisfy these specs? Recall that $t_r = \frac{1.8}{\omega_n}$ and $t_s = \frac{3}{\zeta\omega_n}$.

Solution:

Recall that $t_r = \frac{1.8}{\omega_n}$ and $t_s = \frac{3}{\sigma}$. Therefore by the given specs $t_r \leq 0.6$ and $t_s \leq 1.6$ we can conclude that:

$$\omega_n \geq 3 \text{ and } \sigma \geq 1.875$$

The admissible pole locations are shown in the figure below, where the blue shaded region is due to rise time constraint and red shaded region is due to the settling time constraint.



The poles of $H(s)$ are the roots of $s^2 + 4s + 16$, which can be computed to be $s_{1,2} = -2 \pm 2\sqrt{3}$. They are plotted in the complex plane and found to be in both of shaded regions. Hence the given system satisfy these specs.

GPT Answer :

Given the transfer function:

$$H(s) = \frac{16}{s^2 + 4s + 16}$$

We identify $\omega_n = 4$ rad/s and $\zeta = 0.5$ from the standard form comparison. Substituting these values into the given time-domain specifications:

$$t_r = \frac{1.8}{\omega_n} = \frac{1.8}{4} = 0.45 \text{ seconds}$$

$$t_s = \frac{3}{\zeta\omega_n} = \frac{3}{0.5 \times 4} = 1.5 \text{ seconds}$$

Given the specifications $t_r \leq 0.6$ and $t_s \leq 1.6$, the system satisfies these since $t_r = 0.45 \leq 0.6$ and $t_s = 1.5 \leq 1.6$. **Confidence Level: 95%**

Correct

Claude Answer :

Given the transfer function:

$$H(s) = \frac{16}{s^2 + 4s + 16}$$

We can determine the natural frequency ω_n and damping ratio ζ as follows:

$$s^2 + 4s + 16 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

Comparing coefficients, we get:

$$2\zeta\omega_n = 4 \quad \text{and} \quad \omega_n^2 = 16$$

Solving for ω_n and ζ :

$$\omega_n = 4 \quad \text{and} \quad \zeta = 0.5$$

Using the given formulas for rise time and settling time:

$$t_r = \frac{1.8}{\omega_n} = \frac{1.8}{4} = 0.45$$

$$t_s = \frac{3}{\zeta\omega_n} = \frac{3}{0.5 \cdot 4} = 1.5$$

Comparing with the given specs:

$$t_r = 0.45 \leq 0.6 \quad (\text{satisfied})$$

$$t_s = 1.5 \leq 1.6 \quad (\text{satisfied})$$

Therefore, the given system satisfies both the rise time and settling time specifications.

Confidence level: 95%

Correct