

2.6 Routh-Hurwitz Stability

A system has a characteristic equation given by

$$s^3 + Ks^2 + (K + 1)s + 6 = 0. \quad (109)$$

Determine the range of K for a stable system. You can use Routh-Hurwitz Stability Criteria.

Solution :

The Routh array is structured as follows:

$$\begin{array}{l} s^3 : 1 \quad K + 1 \\ s^2 : K \quad 6 \\ s^1 : b \quad 0 \\ s^0 : 6 \end{array}$$

where $b = \frac{K(K+1)-6}{K}$ for the s^1 row. For stability, we require $K > 0$ and $b > 0$. Therefore, using the condition that $b > 0$, we obtain

$$K^2 + K - 6 > 0, \quad (110)$$

and solving for K yields $K > 2$ and $K < -3$. We select $K > 2$, since we also have the condition that $K > 0$.

GPT Answer :

To determine the range of K for which the system described by the characteristic equation

$$s^3 + Ks^2 + (K + 1)s + 6 = 0 \quad (111)$$

is stable, we use the Routh-Hurwitz stability criterion. According to this criterion, for the system to be stable, all the elements of the first column of the Routh array must be positive.

The Routh array for this system is constructed as follows:

$$\begin{array}{l} s^3 \quad 1 \quad K + 1 \\ s^2 \quad K \quad 6 \\ s^1 \quad E \\ s^0 \quad 6 \end{array}$$

where $E = \frac{-6+K^2+K}{K}$. For stability, $K > 0$ and $E > 0$, leading to $K > 2$.

Confidence Level: 95%

Correct

Claude Answer :

To determine the range of K for a stable system using the Routh-Hurwitz Stability Criteria, we need to construct the Routh array and ensure that all elements in the first column have the same sign. Let's solve this step by step.

Given the characteristic equation:

$$s^3 + Ks^2 + (K + 1)s + 6 = 0$$

Step 1: Arrange the coefficients of the characteristic equation in descending order of s :

$$\begin{aligned} s^3 &: 1 \\ s^2 &: K \\ s^1 &: K + 1 \\ s^0 &: 6 \end{aligned}$$

Step 2: Construct the Routh array:

$$\begin{array}{c|cc} s^3 & 1 & K + 1 \\ s^2 & K & 6 \\ s^1 & b_1 & 0 \\ s^0 & 6 & 0 \end{array}$$

Where $b_1 = \frac{K(K+1)-6}{K} = K + 1 - \frac{6}{K}$

Step 3: For stability, all elements in the first column must have the same sign. Therefore:

$$K > 0 \quad (\text{from the } s^2 \text{ row})$$

$$K + 1 - \frac{6}{K} > 0$$

Solving the second inequality:

$$K^2 + K - 6 > 0$$

$$(K + 3)(K - 2) > 0$$

This inequality is satisfied when $K < -3$ or $K > 2$.

Step 4: Combining the conditions from Step 3 and considering that $K > 0$, we get:

$$K > 2$$

Therefore, the range of K for a stable system is $K > 2$.

Confidence Level: 95%

Correct