

3.4 Inputs for Unbounded Outputs in Systems

Determine a bounded input which will produce an unbounded output from an integrator.

Solution :

The input $u = 1$ will produce the output $y = t$, which is unbounded.

GPT Answer:

The task is to identify a bounded input which, when applied to an integrator, yields an unbounded output. A suitable example of such input is a constant positive value, denoted by $c > 0$. When this input is fed into an integrator, the resulting output is given by the equation $ct + C$, where C represents the initial condition of the integrator, and t symbolizes time. As t extends towards infinity, the output progressively becomes unbounded, clearly demonstrating that a bounded input can indeed lead to an unbounded output under these conditions.

Confidence Level: 95%

Correct

Claude Answer:

To determine a bounded input that will produce an unbounded output from an integrator, we need to consider the properties of the integration operation. The output of an integrator is the integral of its input signal over time. If the input signal has a non-zero average value, the output will grow unbounded over time.

One example of a bounded input that produces an unbounded output is a constant signal, such as a step function.

Consider a unit step function $u(t)$ defined as:

$$u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases}$$

The output of the integrator, $y(t)$, with the unit step function as its input, will be:

$$y(t) = \int u(t) dt$$

Solving the integral:

$$y(t) = \begin{cases} 0 & \text{for } t < 0 \\ t & \text{for } t \geq 0 \end{cases}$$

As t approaches infinity, the output $y(t)$ will grow unbounded, even though the input $u(t)$ is

bounded between 0 and 1.

Another example of a bounded input that produces an unbounded output is a sinusoidal function with a non-zero DC offset. The DC offset ensures that the average value of the input signal is non-zero, causing the output to grow unbounded over time.

In summary, any bounded input signal with a non-zero average value will produce an unbounded output when passed through an integrator. Examples include a constant signal (like a step function) or a sinusoidal signal with a non-zero DC offset.

Confidence Level: 95%

Correct