

1.5 Ordinary Differential Equation Representation of Transfer Functions

Consider two systems $G_1(s) = \frac{-3}{s+11}$ and $G_2(s) = \frac{2}{s+9}$.

- (a) What is the ODE that represents the serial interconnection $H(s) = G_2(s)G_1(s)$?
- (b) What is the ODE that represents the parallel interconnection $H(s) = G_1(s) + G_2(s)$?

Solution :

- (a) Multiplication of the transfer functions yields:

$$H(s) = \frac{2}{s+9} \times \left(\frac{-3}{s+11} \right) = \frac{-6}{s^2 + 20s + 99} \quad (16)$$

The corresponding ODE from input u to output y is:

$$\ddot{y}(t) + 20\dot{y}(t) + 99y(t) = -6u(t) \quad (17)$$

- (b) Addition of the transfer functions yields:

$$H(s) = \frac{-3}{s+11} + \frac{2}{s+9} = \frac{-3(s+9) + 2(s+11)}{(s+11)(s+9)} = \frac{-s-5}{s^2 + 20s + 99} \quad (18)$$

The corresponding ODE from input u to output y is:

$$\ddot{y}(t) + 20\dot{y}(t) + 99y(t) = -\dot{u}(t) - 5u(t) \quad (19)$$

GPT Answer :

Part (a): Serial Interconnection

Given $G_1(s) = \frac{-3}{s+11}$ and $G_2(s) = \frac{2}{s+9}$, the overall transfer function for the serial interconnection is:

$$H(s) = \frac{-6}{(s+9)(s+11)}$$

The corresponding ODE is:

$$(D^2 + 20D + 99)y(t) = -6x(t)$$

Part (b): Parallel Interconnection

The overall transfer function for the parallel interconnection is:

$$H(s) = \frac{-s - 5}{(s + 9)(s + 11)}$$

The corresponding ODE is:

$$(D^2 + 20D + 99)y(t) = (-D - 5)x(t)$$

Confidence Level: 90%

Correct

Claude Answer :

(a) The ODE representing the serial interconnection $H(s) = G_2(s)G_1(s) = \frac{2}{s+9} \cdot \frac{-3}{s+11} = \frac{-6}{s^2+20s+99}$ is:

$$\ddot{y}(t) + 20\dot{y}(t) + 99y(t) = -6x(t)$$

(b) The ODE representing the parallel interconnection $H(s) = G_1(s) + G_2(s) = \frac{-3}{s+11} + \frac{2}{s+9} = \frac{-3s-27+2s+22}{(s+11)(s+9)} = \frac{-s-5}{s^2+20s+99}$ is:

$$\ddot{y}(t) + 20\dot{y}(t) + 99y(t) = -\dot{x}(t) - 5x(t)$$

Confidence Level: 95%

Correct