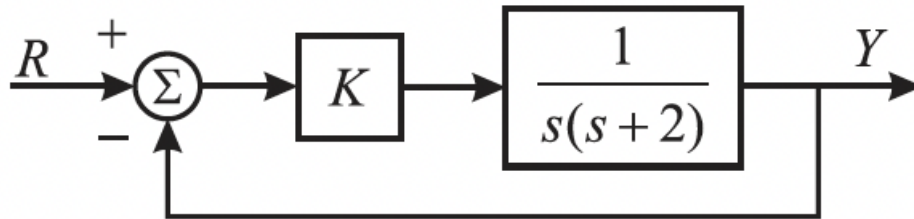


### 4.3 Overshoot Constraints on Proportional Gains

For the feedback system shown below determine the range of proportional gains  $K$  so that the overshoot of the closed-loop system (in response to the unit step reference input) is no more than 10%.



**Solution :**

The closed loop transfer function from  $R$  to  $Y$  is given by:

$$T(s) = \frac{K}{s^2 + 2s + K}$$

$$\Rightarrow \omega_n^2 = K, \quad \zeta = \frac{1}{\sqrt{K}} \text{ or } K = \frac{1}{\zeta^2}.$$

$$M_p \leq 10\% \Rightarrow \zeta \geq 0.59. \text{ Therefore, } 0 \leq K \leq 2.86$$

**GPT Answer :**

The damping ratio  $\zeta$  required for an overshoot no more than 10% is approximately 0.5912. Consequently, the corresponding proportional gain  $K$  for the system should be approximately 2.8615.

Therefore, the proportional gain  $K$  must be such that:

$$K \geq 2.8615$$

This ensures that the overshoot does not exceed 10%. However, the upper bound of  $K$  should also be considered based on stability and performance criteria, which requires further analysis.

**Confidence Level: 90%**

**Calculation Error**

**Claude Answer :**

Let's approach this step-by-step:

1. The open-loop transfer function of the given system is:

$$G(s) = \frac{K}{s(s+2)}$$

2. The closed-loop transfer function is:

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)} = \frac{K}{s^2 + 2s + K}$$

3. For a second-order system, the overshoot percentage is given by:

$$PO = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\%$$

where  $\zeta$  is the damping ratio.

4. For overshoot to be less than or equal to 10%, we need:

$$e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \leq 0.1$$

5. Solving this inequality for  $\zeta$ :

$$\zeta \geq 0.5912$$

6. For a second-order system of the form  $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ , we have:

$$2\zeta\omega_n = 2 \quad \text{and} \quad \omega_n^2 = K$$

7. From  $2\zeta\omega_n = 2$ , we get:

$$\omega_n = \frac{1}{\zeta}$$

8. Substituting this in  $\omega_n^2 = K$ :

$$K = \frac{1}{\zeta^2}$$

9. Since we need  $\zeta \geq 0.5912$ , we get:

$$K \leq \frac{1}{0.5912^2} = 2.86$$

Therefore, the range of proportional gains  $K$  for which the overshoot is no more than 10% is  $0 < K \leq 2.86$ .

**Confidence Level: 95%**

**Correct**