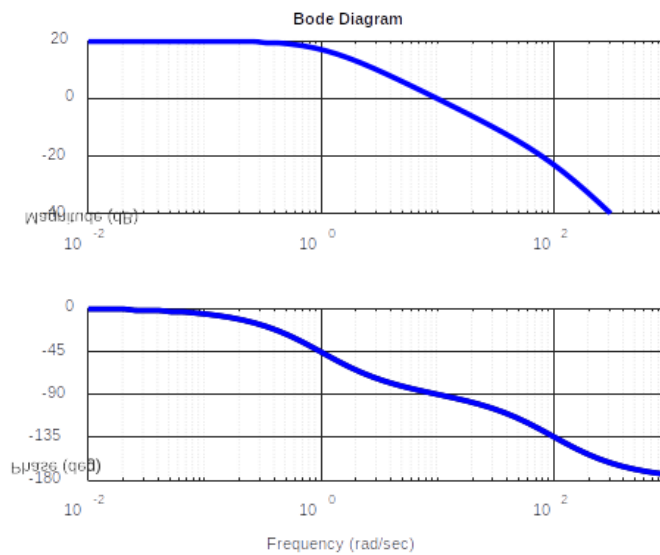


6.3 Bode Plot Analysis and Steady-State Response of a Linear System

A linear system $G(s)$ with input u and output y has the bode plot shown below:

- What is $|G(10j)|$ in dB and actual units?
- What is $\angle G(10j)$ in degs and radians?
- What is the output response $y(t)$ in steady-state for the input $u(t) = 2 \cos(10t)$?
- What is the steady-state value of $y(t)$ if the input is a unit step $u(t) = 1$ for all $t \geq 0$?



Solution :

- $|G(j10)| = 0dB = 1$
- $\angle G(j10) = -90deg = -\frac{\pi}{2}rad$
- $y(t) = 2|G(j10)| \cos(10t + \angle G(j10)) = 2 \cos(10t - \frac{\pi}{2})$
- $u(t) = \cos(\omega t)$ with $\omega = 0$

$$|G(j0)| = 20dB = 10 \quad \text{and} \quad \angle G(j0) = 0deg = 0rad$$

Thus, we have:

$$y(t) = |G(j0)| \cos(\omega t + \angle G(j0)) = 10$$

GPT Answer :

Based on the Bode plot provided:

- (a) $|G(10j)|$ in dB is approximately -6 dB. In actual units, $|G(10j)| = 10^{\frac{-6}{20}} \approx 0.5012$.
- (b) $\angle G(10j)$ is approximately -45 degrees, or in radians, $\frac{-45 \times \pi}{180} \approx -0.7854$ radians.
- (c) The output response $y(t)$ in steady-state for the input $u(t) = 2 \cos(10t)$ is $y(t) = 0.5012 \cdot 2 \cos(10t - 0.7854)$.
- (d) The steady-state value of $y(t)$ for a unit step input cannot be determined precisely without additional information from the Bode plot at $\omega = 0$.

Confidence Level: 80%

Misreading the Plot

Claude Answer :

- (a) From the magnitude plot, $|G(10j)| \approx -45$ dB. Converting to actual units: $|G(10j)| = 10^{-45/20} \approx 0.00562$
- (b) From the phase plot, $\angle G(10j) \approx -90^\circ$. Converting to radians: $\angle G(10j) \approx -\frac{\pi}{2}$ rad
- (c) For an input $u(t) = 2 \cos(10t)$, the steady-state output will be: $y(t) = 2|G(10j)| \cos(10t + \angle G(10j)) = 2(0.00562) \cos(10t - \frac{\pi}{2}) = 0.01124 \sin(10t)$
- (d) The steady-state value for a step input is given by the Final Value Theorem: $\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) = \lim_{s \rightarrow 0} sG(s) \frac{1}{s} = \lim_{s \rightarrow 0} G(s)$ From the magnitude plot, as $\omega \rightarrow 0$, $|G(j\omega)| \rightarrow 0$ dB = 1. Therefore, the steady-state value of $y(t)$ for a unit step input is 1.

Confidence level: 95%

Misreading the Plot