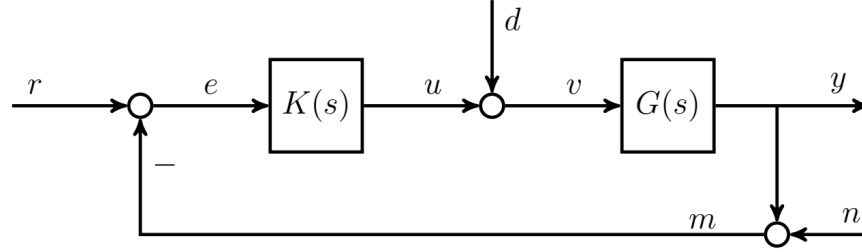


4.2 Feedback System Design for Error Specifications

Consider the general feedback system in the below figure.



- The closed-loop should have zero steady-state error for step reference commands. Translate this into a requirement on the sensitivity $S(s)$. What does this imply about the $L(s) = G(s)K(s)$?
- The closed-loop should have an error ≤ 0.1 for reference commands $r(t) = 2 \sin(\omega t)$ with $\omega \leq 10$ rad/sec. Translate this into a requirement on the sensitivity $S(s)$.
- The closed-loop should have an error ≤ 0.05 for noise $n(t) = 10 \sin(\omega t)$ with $\omega \geq 1000$ rad/sec. Translate this into a requirement on the complementary sensitivity $T(s)$.

Solution :

- Since $S(s) = \frac{1}{1+L(s)}$ and we desire that $S(0) = 0$, then we must have $L(0) = +\infty$. $L(s)$ must have a pole at $s = 0$.
- $e(t) \rightarrow 2|S(j\omega)| \sin(\omega t + \angle S(j\omega)) \leq 0.1$ implies that $|S(j\omega)| \leq 0.05$ for $\omega \leq 10$.
- $y(t) \rightarrow -10|T(j\omega)| \sin(\omega t + \angle T(j\omega)) \leq 0.05$ implies that $|T(j\omega)| \leq 0.05/10 = 0.005$ for $\omega \geq 1000$.

GPT Answer :

The closed-loop system requirements can be translated into requirements on the sensitivity $S(s)$ and complementary sensitivity $T(s)$ as follows:

- For zero steady-state error for step reference commands, the sensitivity function $S(s)$ at the limit as $s \rightarrow 0$ should satisfy $\lim_{s \rightarrow 0} sS(s) = 0$. This implies that the open-loop transfer function $L(s) = G(s)K(s)$ should have at least one integrator, i.e., a pole at the origin.
- For an error ≤ 0.1 for reference commands $r(t) = 2 \sin(\omega t)$ with $\omega \leq 10$ rad/sec, the sensitivity function $S(s)$ should be such that $|S(j\omega)| \leq 0.05$ for $\omega \leq 10$ rad/sec.

- (c) For an error ≤ 0.05 for noise $n(t) = 10 \sin(\omega t)$ with $\omega \geq 1000$ rad/sec, the complementary sensitivity function $T(s)$ should be such that $|T(j\omega)| \leq 0.005$ for $\omega \geq 1000$ rad/sec.

Confidence :Level: 90%

Correct

Claude Answer :

The image shows a general feedback system block diagram. The input signal is r , the error signal is e , the control signal is u , the output signal is y , the disturbance is d , and the noise is n . The controller transfer function is $K(s)$ and the plant transfer function is $G(s)$.

To address the questions:

- (a) For zero steady-state error to step reference commands, the sensitivity $S(s)$ should go to 0 as s goes to 0, i.e., $S(0) = 0$. This implies that the loop gain $L(s) = G(s)K(s)$ should have a pole at $s = 0$, meaning $L(s)$ should include an integrator term.
- (b) For an error ≤ 0.1 for sinusoidal references with frequencies $\omega \leq 10$ rad/sec, we need $|S(j\omega)| \leq 0.1/2 = 0.05$ for $\omega \leq 10$. In other words, the sensitivity magnitude should be less than -26 dB ($20 \log_{10}(0.05)$) at frequencies up to 10 rad/sec.
- (c) For an error ≤ 0.05 due to sinusoidal noise with frequencies $\omega \geq 1000$ rad/sec, we need $|T(j\omega)| \leq 0.05/10 = 0.005$ for $\omega \geq 1000$. This means the complementary sensitivity magnitude should be less than -46 dB ($20 \log_{10}(0.005)$) at frequencies of 1000 rad/sec and higher.

Confidence :Level: 95%

Correct