

### 3.10 Time Response of ODEs

For the given system  $G(s) = \frac{4s-6}{s^2+2s-3}$ :

- What are the poles, zeros, and DC gain?
- What is the general form of the free response?
- What is the general form of the forced response?

If any poles are complex then express the free/forced response in its “real” form.

**Solution :**

**Poles:**  $s_1 = -3, s_2 = 1$

**Zeros:**  $s = \frac{6}{4}$

**DC Gain:**  $G(0) = \frac{-6}{-3}$

**Free response:**  $y(t) = C_1 e^{-3t} + C_2 e^t$

**Forced response:**  $y_p(t)$  depends on  $u(t)$

**GPT Answer:**

Given the system  $G(s) = \frac{4s-6}{s^2+2s-3}$ :

- The zeros are at  $s = \frac{3}{2}$ .
- The poles are at  $s = -3$  and  $s = 1$ .
- The DC gain is 2.

The general form of the free response involves exponential terms due to the real poles, represented as a combination of  $e^{-3t}$  and  $e^t$ . The general form of the forced response depends on the input, shaped by the system’s impulse response.

**Confidence Level: 95%**

**Correct**

**Claude Answer:**

To analyze the given system  $G(s) = \frac{4s-6}{s^2+2s-3}$ , let’s find the poles, zeros, DC gain, and the general forms of the free and forced responses.

**1. Poles, zeros, and DC gain:**

**Poles:** To find the poles, set the denominator equal to zero and solve for  $s$ .

$$s^2 + 2s - 3 = 0 \implies (s + 3)(s - 1) = 0$$

The poles are at  $s = -3$  and  $s = 1$ .

**Zeros:** To find the zeros, set the numerator equal to zero and solve for  $s$ .

$$4s - 6 = 0 \implies s = \frac{6}{4} = 1.5$$

The zero is at  $s = 1.5$ .

**DC gain:** To find the DC gain, evaluate  $G(s)$  at  $s = 0$ .

$$G(0) = \frac{4(0) - 6}{0^2 + 2(0) - 3} = \frac{-6}{-3} = 2$$

The DC gain is 2.

**2. General form of the free response:**

The free response is given by:

$$y_f(t) = c_1 e^{-3t} + c_2 e^t$$

where  $c_1$  and  $c_2$  are constants determined by the initial conditions.

**3. General form of the forced response:**

The forced response depends on the input function  $u(t)$ . For a general input  $U(s)$ , the forced response in the  $s$ -domain is:

$$Y_F(s) = G(s)U(s) = \frac{4s - 6}{s^2 + 2s - 3}U(s)$$

To find the forced response in the time domain, you would need to perform an inverse Laplace transform on  $Y_F(s)$ . The specific form of the forced response will depend on the input function  $u(t)$ .

Since there are no complex poles in this system, the free and forced responses are already in their "real" forms.

**Confidence Level: 95%**

**Correct**