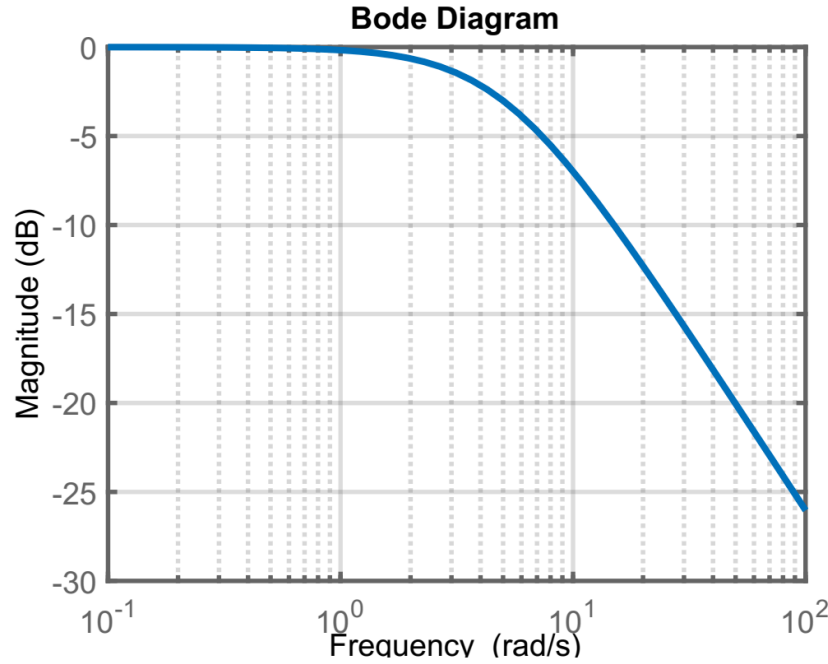


## 6.8 Design of a Proportional Controller Using Bode Plots

Consider a feedback system where the plant  $G(s)$  is stable and has the Bode magnitude plot shown below. We want to design a controller so that: i) the closed-loop is stable, ii) the system has a loop cross-over frequency near 50 rad/sec, and iii) the closed-loop can track  $r(t) = \sin(0.1t)$  with less than 1% error.



- Using the Bode diagram, choose a gain  $K_p$  so that  $K_p G(s)$  has the desired cross-over frequency of 50 rad/sec.
- Convert the requirement (iii) into a requirement on the closed-loop transfer function  $L(s) = G(s)K(s)$ .

**Solution :**

- $|K_p G(j50)| = 1$  implies that  $K_p = 1/|G(j50)| \approx 1/0.1 = 10$
- With  $r(t) = \sin(0.1t)$ , and  $S(j\omega) = \frac{1}{1+L(j\omega)}$  the response from  $r \rightarrow e$ , then we must have  $|S(j\omega)| \sin(0.1t + \angle S(j\omega)) \leq 0.01$ . Therefore  $|S(j\omega)| = \left| \frac{1}{1+L(j\omega)} \right| \leq 0.01$  and so  $100 \leq |1 + L(j\omega)| \approx |L(j\omega)|$ .

**GPT Answer :**

To design a controller that satisfies the given requirements based on the provided Bode plot, we can proceed as follows:

(a) To achieve the desired crossover frequency of 50 rad/sec, we need to adjust the proportional gain  $K_p$  such that the magnitude of the open-loop transfer function  $|K_p G(s)|$  is 0 dB at 50 rad/sec. From the Bode plot, we can observe that the magnitude at 50 rad/sec is below 0 dB. To find the exact value, we would ideally read the magnitude directly from the plot at 50 rad/sec and calculate  $K_p$  by setting  $20 \log(K_p) = -\text{Magnitude at 50 rad/sec}$ . However, since I cannot directly measure values from the plot, I can only suggest that this method be used to determine  $K_p$ .

(b) To ensure that the closed-loop can track  $r(t) = \sin(0.1t)$  with less than 1% steady-state error, we need to consider the frequency of the input signal, which is 0.1 rad/sec. For a sinusoidal input, the steady-state error can be related to the magnitude of the frequency response of the open-loop transfer function at the input frequency. The requirement can be translated to ensuring that the magnitude of  $|L(j0.1)|$  is large enough to guarantee less than 1% steady-state error. This typically involves ensuring a sufficiently high magnitude (in dB) at the input frequency of 0.1 rad/sec in the Bode plot.

**Confidence Level: 80%**

### Misreading the Plot

#### Claude Answer:

(a) To achieve a desired crossover frequency of 50 rad/s, we need to choose a gain  $K_p$  such that  $|K_p G(j50)| = 1$  (0 dB). From the Bode plot,  $|G(j50)| \approx 0.14 = -17$  dB. Therefore, we need  $K_p$  to provide 17 dB of gain to bring the magnitude up to 0 dB at 50 rad/s.  $17 \text{ dB} = 20 \log_{10}(K_p)$   
 $K_p = 10^{17/20} \approx 7.08$

(b) To track  $r(t) = \sin(0.1t)$  with less than 1% error, we need  $\left| \frac{Y(j0.1)}{R(j0.1)} - 1 \right| < 0.01$  where  $Y(s)/R(s)$  is the closed-loop transfer function. For unity feedback,  $Y(s)/R(s) = L(s)/(1+L(s))$ . Substituting:  $\left| \frac{L(j0.1)}{1+L(j0.1)} - 1 \right| < 0.01$  This can be approximated as:  $|L(j0.1)| > 100$  or equivalently  $|L(j0.1)| > 40$  dB

Therefore, the requirement on the loop transfer function  $L(s)$  is  $|L(j0.1)| > 40$  dB.

Confidence level: 90

### Misreading the Plot