

## 1.9 Convolution Integral

Let  $f_1(t) = 3e^{-3t}u_s(t)$  and  $f_2(t) = te^{-2t}u_s(t)$ . Compute the following convolution:

$$g(t) = f_1(t) * f_2(t).$$

**Solution :**

Calculating the Laplace transform of  $f_1(t)$  and  $f_2(t)$ , we have:

$$F_1(s) = \frac{3}{s+3}, \quad F_2(s) = \frac{1}{(s+2)^2} \quad (41)$$

It is known that  $G(s) = F_1(s) \cdot F_2(s)$ . Using partial fraction, we have

$$G(s) = \frac{3}{(s+3)(s+2)^2} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{s+3} \quad (42)$$

Solving for  $A, B$  and  $C$ , we have

$$A = -3, \quad B = 3, \quad C = 3 \quad (43)$$

we have:

$$g(t) = \mathcal{L}^{-1}(G(s)) = 3(e^{-3t} + te^{-2t} - e^{-2t}) \quad (44)$$

**GPT Answer :**

The correct convolution  $g(t) = f_1(t) * f_2(t)$ , after rectifying previous errors, is given by:

$$g(t) = 3((t-1)e^t + 1)e^{-3t}$$

This result accurately represents the convolution of the two functions, taking into account the effect of the unit step function and properly integrating the product of  $f_1(t)$  and  $f_2(t)$  over the interval from 0 to  $t$ .

**Confidence Level: 100%**

**Correct**

**Claude Answer :**

I apologize for my mistake. Let me redo the calculation.

To compute the convolution  $g(t) = f_1(t) * f_2(t)$ , we use the convolution integral:

$$g(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau) d\tau$$

Substituting the given functions:

$$\begin{aligned} g(t) &= \int_{-\infty}^{\infty} 3e^{-3\tau} u_s(\tau) \cdot (t - \tau) e^{-2(t-\tau)} u_s(t - \tau) d\tau \\ &= 3e^{-2t} \int_0^t (t - \tau) e^{-\tau} d\tau \end{aligned}$$

Evaluating the integral using integration by parts:

Let  $u = t - \tau$  and  $dv = e^{-\tau} d\tau$ , then  $du = -d\tau$  and  $v = -e^{-\tau}$ .

$$\begin{aligned} \int_0^t (t - \tau) e^{-\tau} d\tau &= -(t - \tau) e^{-\tau} \Big|_0^t + \int_0^t e^{-\tau} d\tau \\ &= -te^{-t} + t - (-e^{-\tau}) \Big|_0^t \\ &= -te^{-t} + t - (-e^{-t} + 1) \\ &= t - te^{-t} + e^{-t} - 1 \end{aligned}$$

Substituting back into the original equation:

$$\begin{aligned} g(t) &= 3e^{-2t} (t - te^{-t} + e^{-t} - 1) \\ &= 3te^{-2t} - 3te^{-3t} + 3e^{-3t} - 3e^{-2t} \end{aligned}$$

Therefore, the convolution is:

$$g(t) = 3te^{-2t} - 3te^{-3t} + 3e^{-3t} - 3e^{-2t}, \quad t \geq 0$$

**Confidence Level: 98/100**

**Calculation error**