

7 Root-Locus Design

7.1 Root Locus Sketches of Transfer Functions

Sketch the root loci for the following $L(s)$ by hand. Recall that the root locus plots how the solutions of $1 + KL(s) = 0$ vary as K goes from 0 to $+\infty$.

$$(a) \quad L(s) = \frac{1}{s^2 + 2s + 20} \quad (b) \quad L(s) = \frac{s - 3}{s^2 + 2s + 20}$$

$$(c) \quad L(s) = \frac{(s + 1)(s + 2)}{s(s^2 + 4)(s^2 + 5)} \quad (d) \quad L(s) = \frac{s + 3}{s^5 + 1}$$

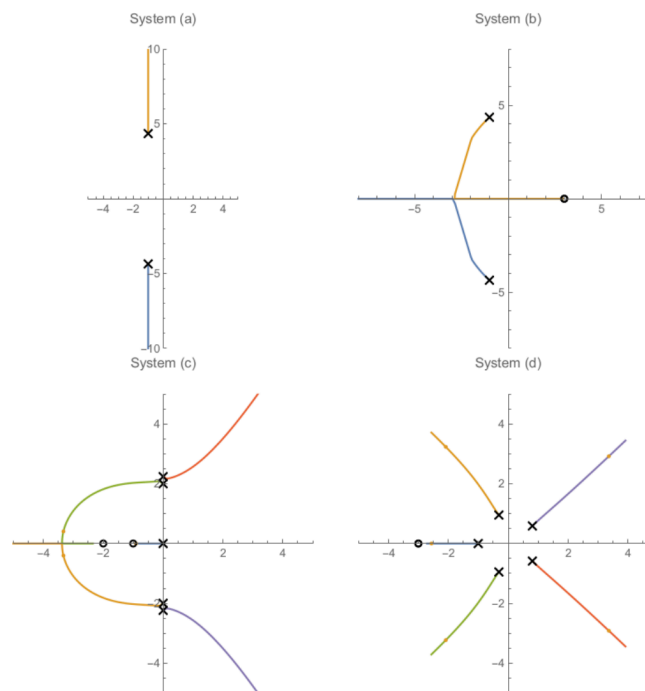


Figure 1: The root-locus plots for the given systems

Figure 12: Root locus for systems in Problem 3

Solution :

The application of Rules for the given systems results in the following table:

The required plots are shown in Fig. 12.

GPT Answer :

Sketching the root loci for each given $L(s)$ involves analyzing the pole-zero configuration and applying the basic rules of root locus.

- (a) $L(s) = \frac{1}{s^2 + 2s + 20}$ has no zeros and poles at $s = -1 \pm j\sqrt{3}$. The root locus starts at the poles and heads to infinity.

Rule / System System	System (a)	System (b)	System (c)	System (d)
No. of branches	2	2	5	5
Branch starts	$-1 \pm \sqrt{19}j$	$-1 \pm \sqrt{19}j$	$0, \pm 2j, \pm \sqrt{5}j$	$-1, \pm \sqrt{2}/2, \pm \sqrt{2}j/2$
Branch ends	∞	$3, \pm \infty$	$-1, -2, \infty$	$-3, \pm \infty$
Real RL	None	$(\infty, 3)$	$(-\infty, -2) \cup (-1, 0)$	$(-3, -1)$
Exit angles	$90^\circ, 270^\circ$	180°	$\frac{2\pi k}{3} \quad k = 1, 2, 3$	$\frac{\pi k}{4} \quad k = 1, 3, 5, 7$
$j\omega$ - crossing	None	$\omega = 0$	None	None

Table 1: Root Locus rules for systems in Problem 7.1

- (b) $L(s) = \frac{s-3}{s^2+2s+20}$ introduces a zero at $s = 3$. The root locus starts at the poles and ends at the zero.
- (c) $L(s) = \frac{(s+1)(s+2)}{s(s^2+4)(s^2+5)}$ has zeros at $s = -1, -2$ and poles at $s = 0, \pm j2, \pm j\sqrt{5}$. The locus starts at the poles and ends at the zeros and infinity.
- (d) $L(s) = \frac{s+3}{s^5+1}$ introduces poles at the roots of $s^5 + 1 = 0$ and a zero at $s = -3$. The locus starts at these poles and one branch ends at the zero.

GPT-4 is unable to draw the root loci.

Confidence Level: 90%

Did not solve

Claude Answer :

I can provide you with step-by-step instructions on how to draw the root loci for each given transfer function.

For (a) $L(s) = \frac{1}{s^2+2s+20}$:

Find the poles: $s^2 + 2s + 20 = 0 \rightarrow s = -1 \pm j\sqrt{19}$. Place 'x' marks at these pole locations on the complex plane. As there are no zeros, the root loci will start from the poles and end at infinity. The asymptotes will have angles of $\pm 90^\circ$ since there are two poles and no zeros. The root loci will be symmetric about the real axis due to complex conjugate poles. The breakaway point is at $s = -1$ (centroid of the poles).

For (b) $L(s) = \frac{s-3}{s^2+2s+20}$:

Find the poles: $s^2 + 2s + 20 = 0 \rightarrow s = -1 \pm j\sqrt{19}$. Place 'x' marks at these pole locations on the complex plane. Find the zero: $s - 3 = 0 \rightarrow s = 3$. Place an 'o' mark at this zero location on the real axis. The root loci will start from the poles and end at the zero and infinity. The asymptote will have an angle of 0° since there are two poles and one zero. The root loci will be symmetric about the real axis due to complex conjugate poles. The breakaway point is at $s = -1$ (centroid of the poles).

For (c) $L(s) = \frac{(s+1)(s+2)}{s(s^2+4)(s^2+5)}$:

Find the poles: $s = 0, s = \pm j2, s = \pm j\sqrt{5}$. Place 'x' marks at these pole locations on the complex plane. Find the zeros: $s = -1, s = -2$. Place 'o' marks at these zero locations on the real axis. The root loci will start from the poles and end at the zeros. There are no asymptotes as the number of poles equals the number of zeros. The root loci will be symmetric about the real axis due to complex conjugate poles and zeros. The breakaway points can be found using the breakaway point formula.

For (d) $L(s) = \frac{s+3}{s^5+1}$:

Find the poles: $s^5 + 1 = 0 \rightarrow s = e^{j(2k+1)\pi/5}, k = 0, 1, 2, 3, 4$. Place 'x' marks at these pole locations on the complex plane. Find the zero: $s + 3 = 0 \rightarrow s = -3$. Place an 'o' mark at this zero location on the real axis. The root loci will start from the poles and end at the zero and infinity. There will be 4 asymptotes with angles $\pm 36^\circ, \pm 108^\circ$ since there are 5 poles and 1 zero. The root loci will be symmetric about the real axis due to complex conjugate poles. The breakaway points can be found using the breakaway point formula.

Confidence Level: 80%

Did not solve