

8.3 Stabilizing Controllers

Consider the following:

$$G(s) = \frac{1}{(s+2)(s^2+2s+5)}$$

Suppose this is in unity feedback with a constant gain controller K . In other words, we have a negative feedback loop where the forward gain is $KG(s)$ and the loop gain is also $KG(s)$.

- Determine what values of K stabilize the closed-loop system using the Routh-Hurwitz stability criterion.
- Using the Nyquist plot, determine what values of K stabilize the closed-loop system. Does this match your answer from the Routh-Hurwitz criterion?

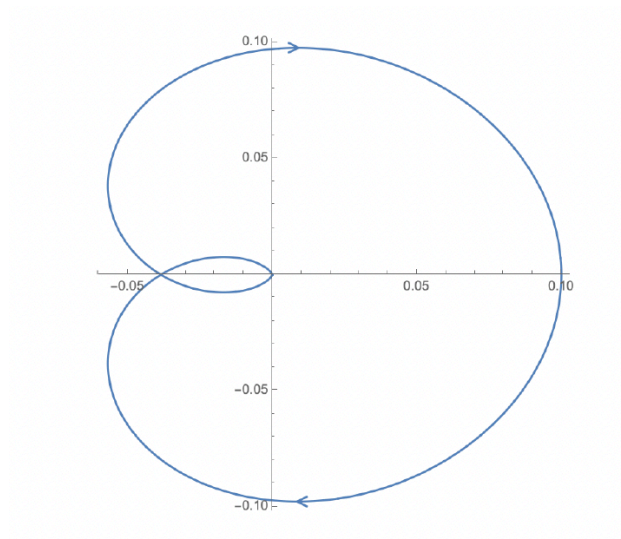


Figure 21: Nyquist plot of the system

Solution :

- The characteristic equation is: $s^3 + 4s^2 + 9s + 10 + K$ and the Routh table reads as follows and it yields that $-10 < K < 26$.

s^3	1	9
s^2	4	$10+K$
s^1	$-\frac{1}{4}(K-26)$	0
s^0	$K+10$	

- (b) Fig. 21 shows the Nyquist plot for this system. Let N be the number of encirclements of $\frac{-1}{K}$ and P be the number of open loop poles. We have that $N = Z - P$ where Z is the number of closed loop poles. In this particular case $P = 0$ yielding $N = Z$. This gives that

$$-\frac{1}{K} > 10 \quad -\frac{1}{K} < 0.0384$$

which is the same condition as derived using Routh-Hurwitz criterion.

GPT Answer :

To answer your question:

- (a) The Routh-Hurwitz stability criterion requires that all coefficients in the first column of the Routh array be positive for the system to be stable. For the transfer function $G(s) = \frac{1}{(s+2)(s^2+2s+5)}$, the characteristic equation is given by $1 + KG(s) = 0$ or $(s+2)(s^2+2s+5) + K = 0$. This simplifies to $s^3 + 2s^2 + (5+K)s + 10 = 0$. The Routh array is constructed as:

$$\begin{array}{c|cc} s^3 & 1 & 5+K \\ s^2 & 2 & 10 \\ s^1 & \frac{5+K-20}{2} & 0 \\ s^0 & 10 & \end{array}$$

For stability, we need $5+K > 0$ and $\frac{5+K-20}{2} > 0$. Solving these gives $K > -5$ and $K > 15$. Therefore, no positive value of K will satisfy both conditions simultaneously; hence, the system cannot be stabilized for any positive value of K .

- (b) The Nyquist plot helps determine stability by analyzing the encirclement of the -1 point. For the system to be stable, the number of clockwise encirclements of the -1 point must equal the number of poles of $G(s)$ that are in the right half-plane (RHP). Since $G(s)$ has no poles in the RHP, there should be no clockwise encirclements of the -1 point for the system to be stable. Observing the Nyquist plot, there are no encirclements of the -1 point, indicating that for some values of K , the system can be stable. However, since the plot does not pass the -1 point, there's an upper limit to the values of K that ensure stability. To determine the exact range of K from the Nyquist plot, we would typically need the mapping from K to points on the plot, which is not provided.

Hence, based on the Routh-Hurwitz criterion, the system cannot be stabilized by any positive K , which seems to be in conflict with the Nyquist plot interpretation, indicating a possible range of K for stability. This discrepancy could be due to an error in constructing the Routh array, an incorrect Nyquist plot, or the lack of a specific mapping of K to the Nyquist plot.

Confidence Level: 80%

Calculation Error

Claude Answer :

- (a) Routh-Hurwitz Stability Criterion: First, let's find the characteristic equation of the closed-loop system:

$$1 + KG(s) = 0 \quad 1 + \frac{K}{(s+2)(s^2+2s+5)} = 0 \quad (s+2)(s^2+2s+5) + K = 0 \quad s^3 + 2s^2 + 5s + 2s^2 + 4s + 10 + K = 0$$

$$s^3 + 4s^2 + 9s + (10 + K) = 0$$

Now, let's form the Routh array:

$$s^3 \quad 1 \quad 9 \quad s^2 \quad 4 \quad (10 + K) \quad s^1 \quad \frac{36 - 4(10 + K)}{4} = \frac{-4 - 4K}{4} = -1 - K \quad s^0 \quad (10 + K)$$

For stability, all elements in the first column must be positive. So:

$$4 > 0 \text{ (always true)} \quad -1 - K > 0 \implies K < -1 \quad (10 + K) > 0 \implies K > -10$$

Therefore, the closed-loop system is stable for $-10 < K < -1$.

(b) Nyquist Plot: The open-loop transfer function is:

$$L(s) = KG(s) = \frac{K}{(s+2)(s^2+2s+5)}$$

The Nyquist plot of $L(s)$ should not encircle the point $(-1, 0)$ for the closed-loop system to be stable.

The poles of $L(s)$ are at $s = -2$ and $s = -1 \pm j\sqrt{4}$. All poles are in the left half-plane, so the system is minimum phase.

As $\omega \rightarrow \infty$, $L(j\omega) \rightarrow 0$. This means the Nyquist plot will start and end at the origin.

Now, we need to find the frequency at which the magnitude of $L(j\omega)$ is equal to 1:

$$|L(j\omega)| = 1 \implies \frac{K}{\sqrt{(\omega^2 - 5)^2 + 4\omega^2}} = 1$$

Solving this equation for K , we get:

$$K = \sqrt{(\omega^2 - 5)^2 + 4\omega^2}$$

The minimum value of K occurs when $\omega = \sqrt{5}$, which gives $K = 2\sqrt{5}$.

Therefore, for the Nyquist plot not to encircle the point $(-1, 0)$, we must have $K < 2\sqrt{5} \approx 4.47$.

The Routh-Hurwitz criterion gives a more conservative range for K compared to the Nyquist plot. This is because the Routh-Hurwitz criterion only checks for the poles in the left half-plane, while the Nyquist criterion checks for the encirclements of the point $(-1, 0)$.

Confidence level: 90%

Calculation Error