5.9 PI Controller

Consider the following plant:

$$2\dot{y}(t) + 6y(t) = 8u(t) \tag{215}$$

with a PI controller in the following form:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau \tag{216}$$

where $e(t) = r(t) - y(t)$

(a) What is the ODE model for the closed-loop from $r(t)$ to $y(t)$?

(b) Choose (K_p, K_i) so that the closed-loop :

- Stable
- Settling time $t_s = \frac{3}{\zeta\omega_n}$ is 3
- damping ratio $\zeta = 0.5$

(c) How will rising time, overshoot and settling time change if K_i is increased further?

Solution :

(a) $2\ddot{y}(t) + (6 + 8K_p)\dot{y}(t) + 8K_i y(t) = 8K_p\dot{r}(t) + 8K_i r(t)$

(b) $\zeta\omega_n = 1 \quad \text{and} \quad \zeta = 0.5 \rightarrow \omega_n = 2$
Thus,

$$K_p = -\frac{1}{4} \quad K_i = 1 \tag{217}$$

(c) $2\zeta\omega_n$ remains unchanged and ω_n increases. Increasing K_i makes the following change:

- rising time $\uparrow$
- overshoot $\downarrow$
- settling time, does not change

GPT Answer :

Part (a): ODE Model for the Closed-Loop System

Given:

$$2\dot{y}(t) + 6y(t) = 8u(t) \tag{218}$$

and the PI controller:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau \tag{219}$$

where $e(t) = r(t) - y(t)$, substituting $u(t)$ and $e(t)$ into the plant equation yields:

$$2\dot{y}(t) + (6 + 8K_p)y(t) = 8K_p r(t) + 8K_i \int_0^t (r(\tau) - y(\tau)) d\tau \tag{220}$$

Part (b): Choosing (K_p, K_i)

For stability, settling time of 3, and $\zeta = 0.5$, we look for values of K_p and K_i that achieve the desired system characteristics without explicit calculation here.

Part (c): Effect of Increasing K_i

Increasing K_i may decrease rising time initially, increase overshoot, and initially decrease then potentially increase settling time due to instability.

Confidence Level: 85%

Reasoning Error

Claude Answer :

(a) To find the ODE model for the closed-loop from r(t) to y(t), let's first find the closed-loop transfer function.

The plant transfer function is:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{8}{2s+6} = \frac{4}{s+3}$$

The PI controller transfer function is:

$$C(s) = K_p + \frac{K_i}{s}$$

The closed-loop transfer function is:

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)} = \frac{4K_p s + 4K_i}{2s^2 + (6 + 4K_p)s + 4K_i}$$

The closed-loop ODE can be obtained by taking the inverse Laplace transform of the transfer function:

$$2\ddot{y}(t) + (6 + 4K_p)\dot{y}(t) + 4K_i y(t) = 4K_p r(t) + 4K_i \int_0^t r(\tau) d\tau$$

(b) To choose (K_p, K_i) based on the given specifications, we can use the standard second-order system parameters:

$$\omega_n = \frac{3}{t_s \zeta} = \frac{3}{3 \cdot 0.5} = 2$$

$$2\zeta\omega_n = 6 + 4K_p \implies K_p = 0.5$$

$$\omega_n^2 = \frac{4K_i}{2} \implies K_i = 2$$

Therefore, the controller gains are $K_p = 0.5$ and $K_i = 2$.

(c) If K_i is increased further:

Rise time will decrease because the system will respond faster to the error. Overshoot will increase because the system will become more oscillatory due to the increased integral action. Settling time will increase because the system will take longer to settle down due to the increased oscillations. In summary, we derived the closed-loop ODE model, chose the appropriate controller gains to meet the given specifications, and analyzed the effect of increasing the integral gain on the system response.

Reasoning Error and Calculation Error