

1.19 State-Space Representation of ODEs

Show the equation $\frac{d^2x}{dt^2} = f(x, \frac{dx}{dt})$ can be equivalently described by a pair of first-order differential equations.

Solution :

We define a set of new variables: $x_1 = x$ and $x_2 = \frac{dx_1}{dt} = \frac{dx}{dt}$.

$$\frac{d^2x}{dt^2} = \frac{d^2x_1}{dt^2} = \frac{dx_2}{dt} = f(x, \frac{dx}{dt}) = f(x_1, \frac{dx_1}{dt}) = f(x_1, x_2) \quad (62)$$

The two desired equations are therefore:

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= f(x_1, x_2) \end{aligned}$$

GPT Answer:

To show that the second-order differential equation $\frac{d^2x}{dt^2} = f(x, \frac{dx}{dt})$ can be equivalently described by a pair of first-order differential equations, we introduce a new variable $v = \frac{dx}{dt}$. This gives us our first first-order differential equation:

$$\frac{dx}{dt} = v$$

Then, we can express the original second-order differential equation in terms of v and x , which gives us our second first-order differential equation:

$$\frac{dv}{dt} = f(x, v)$$

Therefore, the pair of first-order differential equations that equivalently describe the original second-order differential equation is:

$$\frac{dx}{dt} = v, \quad \frac{dv}{dt} = f(x, v)$$

Confidence level: 100%

Correct

Claude Answer:

To show that the second-order differential equation $\frac{d^2x}{dt^2} = f(x, \frac{dx}{dt})$ can be equivalently described by a pair of first-order differential equations, let's introduce a new variable, $y = \frac{dx}{dt}$. Now we have:

$$\begin{aligned}\frac{dx}{dt} &= y \\ \frac{dy}{dt} &= \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d^2x}{dt^2} = f(x, y)\end{aligned}$$

Therefore, the original second-order differential equation can be written as a system of two first-order differential equations:

$$\begin{aligned}\frac{dx}{dt} &= y \\ \frac{dy}{dt} &= f(x, y)\end{aligned}$$

Confidence level: 100/100

Correct