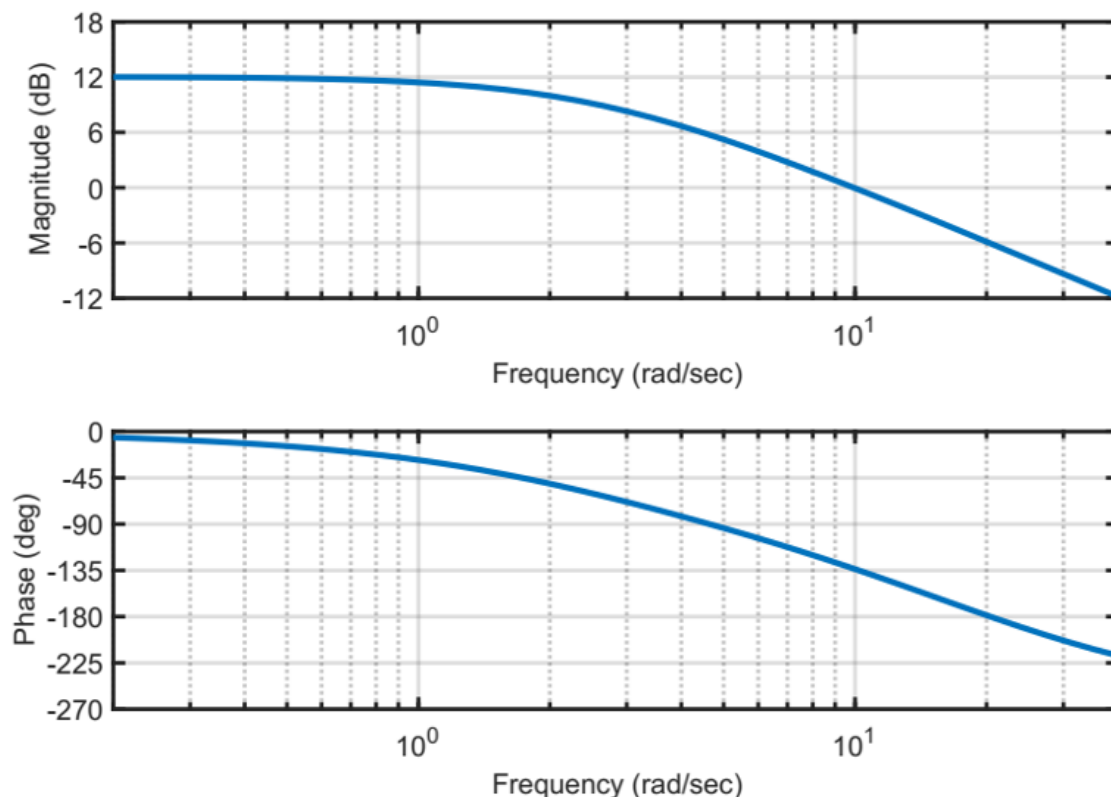


## 6 Bode Analysis

### 6.1 Stability Analysis of a Closed-Loop System Using Bode Plot

Consider a standard closed-loop system with the loop transfer function  $L(s)$  with the Bode plot below. Assume the closed-loop is stable with the loop  $L(s)$ :

$$L(s) = \frac{-4s + 72}{0.39s^2 + 8.02s + 18}$$



- What is the phase crossover frequency,  $\omega_0$ ?
- What is the gain margin,  $g_0$ , of the closed-loop?
- Is the closed-loop stable if the open-loop transfer function is  $1.5L(s)$ ?

**Solution :**

- $\omega_0 = 20 \text{ rad/sec}$ .

- (b)  $g_0 = 2$ .
- (c) Yes. Because  $1.5 < g_0 = 2$ .

**GPT Answer :**

Based on the Bode plot provided:

- (a) The phase crossover frequency,  $\omega_0$ , appears to be slightly less than  $10^1$  rad/sec, as it is the frequency at which the phase angle crosses  $-180^\circ$ .
- (b) The gain margin,  $g_0$ , seems to be slightly above 6 dB, indicating a positive gain margin at the phase crossover frequency.
- (c) If the open-loop transfer function is scaled to  $1.5L(s)$ , we increase the magnitude by  $20 \log(1.5)$  dB, approximately 3.52 dB. Even with this increase, the magnitude at the phase crossover frequency would still be below 0 dB, suggesting that the system should remain stable.

**Confidence Level: 85%**

**Misreading the plot**

**Claude Answer :**

- (a) The phase crossover frequency  $\omega_0$  is the frequency where the phase plot crosses  $-180^\circ$ . From the Bode plot, this occurs at approximately  $\omega_0 = 3$  rad/sec.

Confidence: 95/100

- (b) The gain margin  $g_m$  is the negative of the magnitude in dB at the phase crossover frequency. From the magnitude plot at  $\omega_0 = 3$ , the magnitude is approximately 8 dB. Therefore, the gain margin is  $g_m = 8$  dB.

Confidence: 90/100

- (c) If the open-loop transfer function is multiplied by 1.5, this increases the magnitude plot by  $20 \log_{10}(1.5) \approx 3.5$  dB. With an original gain margin of 8 dB, the new gain margin would be approximately 4.5 dB, which is still positive. Therefore, the closed-loop system remains stable with  $1.5L(s)$ .

Confidence: 85/100

Overall, I have high confidence in parts (a) and (b) as the phase crossover frequency and gain margin can be read fairly accurately from the Bode plot. I have slightly lower confidence in part (c) since the stability assessment relies on the accuracy of the 8 dB gain margin estimate and the mental calculation of the updated margin.

## **Misreading the plot**