

9.7 Gain/Phase Margin

Suppose we have the open-loop transfer function $G(s) = \frac{K}{s(s+20)(s+85)}$, and we put it through unity feedback, i.e. the closed loop transfer function is $\frac{G(s)}{1+G(s)}$.

- Set the gain K so that the magnitude is 1 (0 dB) at $\omega = 1$. What K achieves this?
- We wish to achieve 15% overshoot in the transient response for a step input. What phase margin is required to achieve this?
- What frequency on the Bode phase diagram yields this phase margin?
- Find the adjusted gain necessary to produce the required phase margin.

Solution :

- For 0 db at $\omega = 1$, we have that,

$$|G(j1)| = \left| \frac{K}{j(j+20)(j+85)} \right| = 1$$

$$\text{so, } K = |j(j+20)(j+85)| = \sqrt{2897626} = 1702.24$$

- For 15% overshoot, we have that,

$$\zeta = \frac{-\ln(0.15)}{\sqrt{\pi^2 + \ln^2(0.15)}} = 0.5169$$

using this damping ratio, we can obtain the desired phase margin

$$\Phi_M = \tan^{-1} \left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}} \right) = 53.17^\circ$$

- The phase which yields this phase margin is $\phi = -180 + \Phi_M = -127$. The Bode plot shows the phase is -127 when $\omega = 11.2 \text{ rad/s}$.
- We achieve the desired phase margin by shifting the magnitude at $\omega_{PM} = 11.2$ to 0 dB. This can be done by changing the gain:

$$\begin{aligned} 0 \text{ dB} &= 20 \log |K^* G(j\omega_{PM})| \\ 1 &= |K^* G(j\omega_{PM})| \end{aligned}$$

Therefore $K^* = 1/|G(j\omega_{PM})| = 12.93$ and the adjusted gain is $K^* = \sqrt{2897626} \times 12.93 \approx 22010$.

GPT Answer :

Given the open-loop transfer function $G(s) = \frac{K}{s(s+20)(s+85)}$, with unity feedback, the task was to adjust the system's gain K and analyze its behavior for certain specifications. The process involved the following steps:

- To set the gain K so that the magnitude is 1 (0 dB) at $\omega = 1$, the calculation yielded $K \approx 1702.24$.
- For a desired transient response with 15% overshoot, the phase margin required was determined to be approximately 53.17 degrees.
- The frequency on the Bode phase diagram that yields this phase margin was found to be around 10.72 rad/s, by analyzing the phase curve on the generated Bode plot.
- Adjusting the gain K to produce the required phase margin involves ensuring that the system's gain and phase characteristics at the identified frequency align with the desired performance criteria. Initially, K was set based on the magnitude condition, with further tuning potentially required based on detailed Bode plot analysis.

The analysis incorporated both analytical solutions and practical Bode plot interpretation to address the specified requirements. The approach was grounded in control system theory, applying established principles to derive and verify the system's performance characteristics.

Confidence Level: 90%

Reasoning Error**Claude Answer :**

Let's solve this problem step by step:

- To find the gain K that makes the magnitude 1 (0 dB) at $\omega = 1$:

Substitute $s = j\omega$ and $\omega = 1$ into $G(s)$: $G(j1) = \frac{K}{j1(j1+20)(j1+85)}$ Find the magnitude of $G(j1)$ and set it equal to 1: $|G(j1)| = \frac{K}{\sqrt{1^2}\sqrt{1^2+20^2}\sqrt{1^2+85^2}} = 1$ Solve for K : $K = \sqrt{1^2}\sqrt{1^2+20^2}\sqrt{1^2+85^2} = 1717.2$ (b) To achieve 15

- To find the frequency that yields the required phase margin:

The phase of $G(s)$ is: $\angle G(s) = -90^\circ - \tan^{-1}\left(\frac{\omega}{20}\right) - \tan^{-1}\left(\frac{\omega}{85}\right)$ Set the phase equal to the required phase margin (PM) and solve for ω : $-90^\circ - \tan^{-1}\left(\frac{\omega}{20}\right) - \tan^{-1}\left(\frac{\omega}{85}\right) = -180^\circ + PM - \tan^{-1}\left(\frac{\omega}{20}\right) - \tan^{-1}\left(\frac{\omega}{85}\right) = -90^\circ + PM$ Using $PM = 50^\circ$ and solving numerically, we get $\omega \approx 7.78$ rad/s. (d) To find the adjusted gain necessary to produce the required phase margin:

Substitute $s = j\omega$ and the frequency found in (c) into $G(s)$: $G(j7.78) = \frac{K}{j7.78(j7.78+20)(j7.78+85)}$ Find the magnitude of $G(j7.78)$ and set it equal to 1: $|G(j7.78)| = \frac{K}{\sqrt{7.78^2}\sqrt{7.78^2+20^2}\sqrt{7.78^2+85^2}} = 1$

Solve for the adjusted gain K: $K = \sqrt{7.78^2} \sqrt{7.78^2 + 20^2} \sqrt{7.78^2 + 85^2} = 6674.9$ Therefore, the adjusted gain necessary to produce the required phase margin of 50° is approximately 6674.9.

Confidence Level: 95%

Incorrect Knowledge