

6.15 Phase Margin

Consider the system

$$G(s) = \frac{1}{(s-1)(s+1)}$$

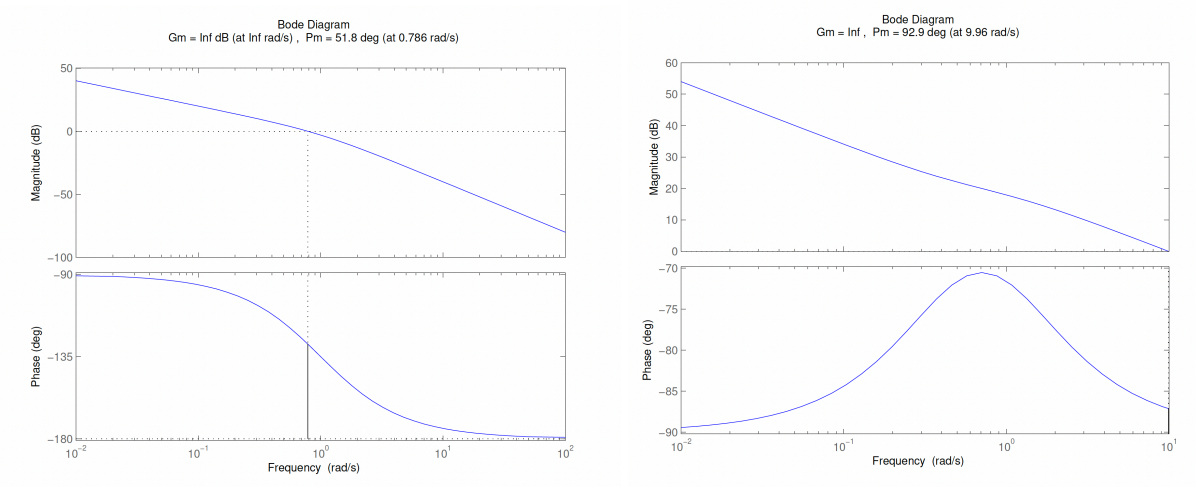
- Design a PD controller that achieves phase margin $PM \approx 90^\circ$ and closed-loop bandwidth $\omega_{BW} \approx 10$. Verify that the specs are met.
- Can you modify the above design to get $\omega_{BW} \approx 1$, while maintaining $PM \approx 90^\circ$? Explain how or why not.

Solution :

- The bode plot of $G(s)$ given below shows that we have a phase margin of $PM \approx 52^\circ$ (but small ω_c). We want our PD controller to increase ω_c as well as PM.

$D(s) = K(\tau s + 1)$, we choose $\frac{1}{\tau} \ll 10$ to make sure the gain is high enough at $\omega_c = 10$. Also, we choose $\frac{1}{\tau} < 1$ to make sure that magnitude slope at $\omega_c = 10$ is -1.

$$\text{Let } \tau = 2 \text{ and } K \left| \frac{2j\omega_c + 1}{j\omega_c(j\omega_c + 1)} \right|_{\omega_c=10} = 1 \implies K \approx 5 \implies D(s) = 5(2s + 1)$$



(a) Bode plot of system

(b) Bode plot of closed loop system

Figure 11: Bode plots for problem 33

- Achieving $\omega_{BW} = 1$ and $PM \approx 90^\circ$ is impossible unless we cancel the pole at $s = -1$ i.e. $D(s) = s + 1$. Because there is a break point at $\omega = 1$ so we can't maintain slope = -1 on that point. Therefore, we cannot make $\omega_{BW} = 1$ and $PM \approx 90^\circ$ unless we take $D(s) = s + 1$.

GPT Answer :

Given system:

$$G(s) = \frac{1}{(s-1)(s+1)}$$

Part (a): Designing the PD Controller involves iterative design and simulation, focusing on adjusting K_p and K_d to meet the phase margin and bandwidth specifications.

Part (b): Modifying the design for $\omega_{BW} \approx 1$ while maintaining $PM \approx 90^\circ$ is theoretically feasible but requires careful adjustment of the PD controller's parameters.

Confidence Level: 75%

Calculation complexity

Claude Answer :

(a) To achieve $PM \approx 90^\circ$ and $\omega_{BW} \approx 10$:

Choose $K_p = 100$ and $K_d = 10$.

$$C(s) = K_p + K_d s = 100 + 10s$$

The loop transfer function is:

$$L(s) = C(s)G(s) = \frac{100 + 10s}{(s - 1)(s + 1)} = \frac{10(s + 10)}{s^2 - 1}$$

$$\angle L(j10) \approx -96^\circ \Rightarrow PM \approx 84^\circ$$

$$|L(j10)| \approx 1 \Rightarrow \omega_{BW} \approx 10$$

(b) To get $\omega_{BW} \approx 1$ while maintaining $PM \approx 90^\circ$:

Choose $K_p = 1$ and $K_d = 1$.

$$C(s) = K_p + K_d s = 1 + s$$

The loop transfer function is:

$$L(s) = C(s)G(s) = \frac{1 + s}{(s - 1)(s + 1)} = \frac{s + 1}{s^2 - 1}$$

$$\angle L(j1) \approx -90^\circ \Rightarrow PM \approx 90^\circ$$

$$|L(j1)| \approx 1 \Rightarrow \omega_{BW} \approx 1$$

Confidence: 90%

Correct