3.14 Steady State Response

For the following first-order systems with the sinusoidal input $u(t) = 5\sin(4t + 0.1)$

- What is the magnitude and phase of $G(j\omega)$?
- Is the steady-state response bounded? If yes, what is it?

(a) $-2\dot{y}(t) - y(t) = 3u(t)$

(a) $-2\dot{y}(t) + y(t) = 3u(t)$

Solution :

(a)

- $G(s) = \frac{3}{-2s-1} \rightarrow s_1 = \frac{-1}{2} \rightarrow$ system is stable
  $\omega = 4 \rightarrow |G(4j)| = 0.37 \quad \text{and} \quad \angle G(j\omega) = 1.7$
- Yes, $y(t) = 1.85\sin(4t + 1.8)$

(b)

- $G(s) = \frac{3}{-2s+1} \rightarrow s_1 = \frac{1}{2} \rightarrow$ system is unstable
  $\omega = 4 \rightarrow |G(4j)| = 0.37 \quad \text{and} \quad \angle G(j\omega) = 1.44$
- No, response is not bounded.

GPT Answer :
The calculated magnitude and phase for the given systems with input $u(t) = 5\sin(4t + 0.1)$ are:

For system (a):

$$|G(j4)| = \frac{3}{\sqrt{65}}, \quad \angle G(j4) = 1.45\,\text{radians}$$

For system (b):

$$|G(j4)| = \frac{3}{\sqrt{65}}, \quad \angle G(j4) = -1.45\,\text{radians}$$

These results confirm that the steady-state response is bounded for both systems.

Confidence Level: 95%

Calculation Error for part (a). Calculation Error and Reasoning Error for part (b).

Claude Answer :
Here's the complete solution for both parts, considering everything in radians and returning the answer in radians.

(a) $-2\dot{y}(t) - y(t) = 3u(t)$

1. Transfer function: $G(s) = \frac{Y(s)}{U(s)} = \frac{3}{-2s-1}$

2. Magnitude and phase of G(jω): $G(j\omega) = \frac{3}{-2j\omega-1}$

Magnitude: $|G(j\omega)| = \sqrt{\frac{3^2}{(-2\omega)^2+1^2}} = \frac{3}{\sqrt{4\omega^2+1}}$

Phase: $\angle G(j\omega) = -\tan^{-1}(2\omega) - \pi$

3. Steady-state response: $u(t) = 5\sin(4t + 0.1)$, so $\omega = 4$

Magnitude at $\omega = 4$: $|G(j4)| = \frac{3}{\sqrt{4(4)^2+1}} = \frac{3}{\sqrt{65}} \approx 0.372$

Phase at $\omega = 4$: $\angle G(j4) = -\tan^{-1}(2(4)) - \pi \approx -4.587$ rad

The steady-state response is bounded and given by: $y_{ss}(t) = 5|G(j4)|\sin(4t + 0.1 + \angle G(j4)) \approx 1.86\sin(4t - 4.487)$

(b) $-2\dot{y}(t) + y(t) = 3u(t)$

1. Transfer function: $G(s) = \frac{Y(s)}{U(s)} = \frac{3}{-2s+1}$

2. Magnitude and phase of G(jω): $G(j\omega) = \frac{3}{-2j\omega+1}$

Magnitude: $|G(j\omega)| = \sqrt{\frac{3^2}{(-2\omega)^2+1^2}} = \frac{3}{\sqrt{4\omega^2+1}}$

Phase: $\angle G(j\omega) = \tan^{-1}(2\omega)$

3. Steady-state response: $u(t) = 5\sin(4t + 0.1)$, so $\omega = 4$

Magnitude at $\omega = 4$: $|G(j4)| = \frac{3}{\sqrt{4(4)^2+1}} = \frac{3}{\sqrt{65}} \approx 0.372$

Phase at $\omega = 4$: $\angle G(j4) = \tan^{-1}(2(4)) \approx 1.445$ rad

The steady-state response is bounded and given by: $y_{ss}(t) = 5|G(j4)|\sin(4t + 0.1 + \angle G(j4)) \approx 1.86\sin(4t + 1.545)$

Both systems have bounded steady-state responses with the same frequency as the input signal. The magnitudes of the responses are the same, but the phase shifts differ due to the sign difference in the transfer functions.

Confidence Level: 95%
Correct