

## 1.10 Classification of Differential Equations

Classify the following differential equations according to whether they are ordinary or partial. Indicate the dependent and independent variables.

(a)  $\frac{dx}{dt} + \frac{dy}{dt} + x + y = 0 \quad x = x(t) \quad y = y(t)$

(b)  $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + x + y = 0 \quad f = f(x, y)$

(c)  $\frac{d}{dt} \left[ \frac{\partial f}{\partial x} \right] = 0 \quad f = x^2 + \frac{dx}{dt}$

(d)  $\frac{df}{dx} = x \quad f = y^2(x) + \frac{dy}{dx}$

**Solution :**

(a) Ordinary; independent variable  $t$ ; dependent variables  $x$  and  $y$ .

(b) Partial; independent variables  $x$  and  $y$ ; dependent variable  $f$ .

(c) Since  $\frac{\partial f}{\partial x} = 2x$ , then  $\frac{d}{dt} \left[ \frac{\partial f}{\partial x} \right] = 2 \frac{dx}{dt} = 0$ , which is an ordinary differential equation independent variable  $t$ ; dependent variable  $x$

(d)  $\frac{df}{dx} = 2y \frac{dy}{dx} + \frac{d^2 y}{dx^2} = x$ , which is an ordinary differential equation; independent variable  $x$ ; dependent variable  $y$ .

**GPT Answer:**

Classify the following differential equations according to whether they are ordinary or partial. Indicate the dependent and independent variables.

(a) The differential equation  $\frac{dx}{dt} + \frac{dy}{dt} + x + y = 0 \quad x = x(t) \quad y = y(t)$  is an **ordinary differential equation (ODE)** with dependent variables  $x$  and  $y$ , both functions of the independent variable  $t$ .

(b) The equation  $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + x + y = 0 \quad f = f(x, y)$  is a **partial differential equation (PDE)** with the dependent variable  $f$ , a function of the independent variables  $x$  and  $y$ .

(c) The equation  $\frac{d}{dt} \left[ \frac{\partial f}{\partial x} \right] = 0 \quad f = x^2 + \frac{dx}{dt}$  leans towards an **ordinary differential equation (ODE)** with mixed characteristics, having  $f$  as the dependent variable, primarily a function of  $x$ , and  $t$  as the independent variable.

(d) The equation  $\frac{df}{dx} = x \quad f = y^2(x) + \frac{dy}{dx}$  is an **ordinary differential equation (ODE)** with  $f$  as the dependent variable, a function of the independent variable  $x$ , and indirectly  $y$ , as  $y$  is a function of  $x$ .

**Confidence Level: 95%.**

**Correct**

**Claude Answer:**

- (a) This is an ordinary differential equation (ODE) system. The dependent variables are  $x$  and  $y$ , and the independent variable is  $t$ .
- (b) This is a partial differential equation (PDE). The dependent variable is  $f$ , and the independent variables are  $x$  and  $y$ .
- (c) This is an ordinary differential equation (ODE). The dependent variable is  $f$ , and the independent variable is  $t$ . Note that although  $f$  depends on  $x$ ,  $x$  is not an independent variable in this context.
- (d) This is an ordinary differential equation (ODE). The dependent variable is  $f$ , and the independent variable is  $x$ . The function  $y$  is also dependent on  $x$ .

**Confidence Level: 95/100**

**Correct**