

1.26 Time-Domain Specs

Consider the transfer function

$$H(s) = \frac{16}{s^2 + 4s + 16}$$

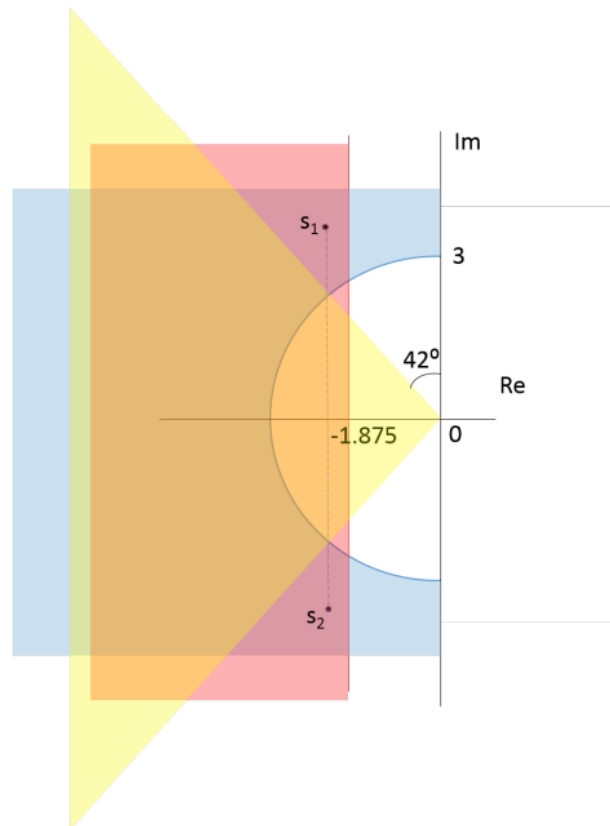
Suppose that you are given the following time-domain specs: $t_r \leq 0.6$, $t_s \leq 1.6$ (defined by the time system's response reach and stay within 5% of steady state). Suppose additionally, we have overshoot: $M_p \leq \frac{1}{e^2}$. Does the given system satisfy these specs?

Solution:

Recall that $t_r = \frac{1.8}{\omega_n}$, $t_s = \frac{3}{\sigma}$ and $M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = e^{-\pi \arctan \theta}$. Therefore by the given specs $t_r \leq 0.6$, $t_s \leq 1.6$ and $M_p \leq \frac{1}{e^2}$, we can conclude that:

$$\omega_n \geq 3, \text{ and } \sigma \geq 1.875, \text{ and } \theta \geq 42^\circ$$

The admissible region due to overshoot constraint is plotted as the yellow shaded area in the figure given below. Since the poles of the system are not in the yellow region, the given system does not satisfy the specs.



GPT Answer:

Based on the calculations:

- The rise time, $t_r \approx 0.605$ s, which is slightly above the specified maximum of 0.6 s.
- The settling time, $t_s \approx 2.0$ s, which exceeds the specified maximum of 1.6 s.
- The maximum overshoot, $M_p \approx 0.163$, which is greater than the specified maximum of $\frac{1}{e^2} \approx 0.135$.

Therefore, the given system does not satisfy the specified time-domain specs for rise time, settling time, and maximum overshoot.

Confidence Level: 90%

Correct

Claude Answer:

To determine if the given system satisfies the time-domain specifications, we need to calculate the rise time (t_r), settling time (t_s), and overshoot (M_p) from the transfer function.

Given: $H(s) = \frac{16}{s^2 + 4s + 16}$

Step 1: Determine the natural frequency (ω_n) and damping ratio (ζ). $s^2 + 4s + 16 = s^2 + 2\zeta\omega_n s + \omega_n^2$
 $\omega_n = 4$ and $\zeta = 0.5$

Step 2: Calculate the rise time (t_r). $t_r \approx \frac{1.8}{\omega_n} = \frac{1.8}{4} = 0.45$

Step 3: Calculate the settling time (t_s). $t_s \approx \frac{4}{\zeta\omega_n} = \frac{4}{0.5 \times 4} = 2$

Step 4: Calculate the overshoot (M_p). $M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = e^{-\frac{0.5\pi}{\sqrt{1-0.5^2}}} \approx 0.163$

Now, let's compare the calculated values with the given specifications:

1. Rise time: $t_r = 0.45 \leq 0.6$ (satisfies the spec) 2. Settling time: $t_s = 2 > 1.6$ (does not satisfy the spec) 3. Overshoot: $M_p = 0.163 > \frac{1}{e^2} \approx 0.135$ (does not satisfy the spec)

Therefore, the given system does not satisfy all the time-domain specifications. It meets the rise time requirement but fails to meet the settling time and overshoot requirements.

Confidence level: 95%

Correct