5.12 PI Controller

Consider the plant with the following transfer function:

$$G(s) = \frac{505}{s^3 + 21s^2 + 121s + 101} \tag{234}$$

The poles of this system are at $s = -1, -10 \pm j$.

(a) What is the dominant pole approximation $G_a(s)$ for this plant?

(b) Choose the controller gains so that the closed-loop with $G_a(s)$ has poles repeated at $s = -2$

Solution :

(a) Set $G_a(s) = \frac{b_0}{s+1}$. Since $G(0) = \frac{505}{101}$, we have $b_0 = 5$. Thus,

$$G_a(s) = \frac{5}{s+1} \tag{235}$$

The time-domain approximation of $G_a(s)$ is:

$$\dot{y}(t) + y(t) = 5u(t) \tag{236}$$

(b) Desired characteristic equation:

$$(s+2)^2 = 0$$

Using the following PI controller:

$$u(t) = K_p(r(t) - y(t)) + K_i \int (r(t) - y(t))$$

we have:

$$\ddot{y}(t) + (1 + 5K_p)\dot{y}(t) + 5K_i y(t) = 5K_p \dot{r}(t) + 5K_i r(t)$$

Thus: $K_p = 0.6$ and $K_i = 0.8$

GPT Answer :

Part (a): Dominant Pole Approximation

The dominant pole approximation $G_a(s)$ of the given transfer function is:

$$G_a(s) = \frac{5}{s+1}$$

Part (b): Controller Design for Repeated Poles at $s = -2$

The controller gain K to make the closed-loop system have repeated poles at $s = -2$ is:

$$K = \frac{3}{5}$$

Reasoning Error

Claude Answer :

(a) Dominant Pole Approximation The dominant pole approximation $G_a(s)$ is:

$$G_a(s) = \frac{5}{s+1} \tag{237}$$

(b) Controller Design for Repeated Poles at $s = -2$ To achieve repeated poles at $s = -2$ with a proportional-derivative (PD) controller, we use:

$$C(s) = K_p + K_d s \tag{238}$$

The closed-loop transfer function with $G_a(s)$ and $C(s)$ is:

$$T(s) = \frac{C(s)G_a(s)}{1 + C(s)G_a(s)} = \frac{5(K_p + K_d s)}{s + 1 + 5(K_p + K_d s)} \tag{239}$$

The characteristic equation for having repeated poles at $s = -2$ is:

$$s + 1 + 5(K_p + K_d s) = (s+2)^2 = s^2 + 4s + 4 \tag{240}$$

Equating coefficients from both sides, we obtain:

$$5K_d = 1 \tag{241}$$
$$1 + 5K_p = 4 \tag{242}$$

Solving these equations:

$$K_d = \frac{1}{5} \tag{243}$$
$$K_p = \frac{3}{5} \tag{244}$$

Therefore, the controller gains should be chosen as $K_p = \frac{3}{5}$ and $K_d = \frac{1}{5}$ to achieve the desired closed-loop pole locations with the dominant pole approximation $G_a(s)$.

Confidence level: 95%

Calculation Error