

3.11 Time Response of ODEs

For the given system $G(s) = \frac{5}{s^2 - 2s + 5}$:

- What are the poles, zeros, and DC gain?
- What is the general form of the free response?
- What is the general form of the forced response?

If any poles are complex then express the free/forced response in its “real” form.

Solution :

Poles: $s_{1,2} = 1 \pm 2j$

Zeros: No zeros

DC Gain: $G(0) = \frac{5}{5}$

Free response: $y(t) = C_1 e^t \cos(2t) + C_2 e^t \sin(2t)$

Forced response: $y_p(t)$ depends on $u(t)$

GPT Answer:

he given system is $G(s) = \frac{5}{s^2 - 2s + 5}$.

- The zeros of the system are none, since the numerator is a constant 5.
- The poles are found by solving $s^2 - 2s + 5 = 0$, resulting in $s = 1 \pm 2j$.
- The DC gain of the system is 1, calculated by evaluating $G(0) = 1$.

The general form of the free response, given the complex poles $1 \pm 2j$, is:

$$y(t) = e^t (A \cos(2t) + B \sin(2t))$$

The general form of the forced response depends on the input to the system but will be influenced by the system's characteristics.

Confidence Level: 90%

Correct

Claude Answer:

Let's solve this problem step by step:

1) Poles, zeros, and DC gain:

The poles are the roots of the characteristic equation (the denominator of the transfer function):

$$s^2 - 2s + 5 = 0$$

Using the quadratic formula, we get:

$$s = \frac{2 \pm \sqrt{4-20}}{2} = 1 \pm 2i$$

So the poles are $1 + 2i$ and $1 - 2i$.

There are no zeros as the numerator is a constant.

The DC gain is the value of the transfer function when $s = 0$:

$$G(0) = \frac{5}{0^2 - 2(0) + 5} = 1$$

2) General form of the free response:

Since the poles are complex, the general form of the free response is:

$$y(t) = e^{\sigma t}(A \cos(\omega t) + B \sin(\omega t))$$

where $\sigma + i\omega$ is the pole in the upper half-plane. In this case, $\sigma = 1$ and $\omega = 2$.

So the free response is:

$$y(t) = e^t(A \cos(2t) + B \sin(2t))$$

3) General form of the forced response:

The general form of the forced response depends on the input. Let's assume the input is a step function, i.e., $u(t)$. The Laplace transform of a step function is $\frac{1}{s}$.

The forced response is:

$$Y(s) = G(s)U(s) = \frac{5}{s^2 - 2s + 5} \cdot \frac{1}{s}$$

Using partial fraction expansion:

$$Y(s) = \frac{1}{s} - \frac{s-2}{s^2 - 2s + 5}$$

Taking the inverse Laplace transform:

$$y(t) = 1 - e^t(\cos(2t) - \sin(2t))$$

This is the general form of the forced response to a step input.

Therefore, for the given system, the poles are $1 + 2i$ and $1 - 2i$, there are no zeros, and the DC gain is 1. The free response is $y(t) = e^t(A \cos(2t) + B \sin(2t))$, and the forced response to a step input is $y(t) = 1 - e^t(\cos(2t) - \sin(2t))$.

Confidence Level: 95%

Correct