

11 System Sensitivity Measures

11.1 Sensitivity Transfer Function

The closed-loop should have zero steady-state error for step reference commands.

- (a) What is the the value of the sensitivity transfer function $S(s)$ at $s = 0$?
- (b) What is the the value of the loop transfer function $|L(0)|$ at $s = 0$?

Solution :

- (a) $S(0) = 0$.
Since the closed-loop system have zero steady-state error: $e(t) \rightarrow 0$ as $t \rightarrow \infty$. We must have $S(0) = 0$.
- (b) $|L(0)| = +\infty$.
Since $S(s) = \frac{1}{1 + L(s)}$, $S(0) = 0$ implies that $|L(0)| = +\infty$.

GPT Answer :

To answer these questions, let's recall some fundamental concepts in control theory regarding the sensitivity function $S(s)$ and the loop transfer function $L(s)$ in the context of a feedback control system.

The sensitivity function is defined as $S(s) = \frac{1}{1 + L(s)}$, where $L(s)$ is the open-loop transfer function of the system. The sensitivity function measures how the output of the system responds to changes in the input signal, considering the presence of feedback.

(a) To find the value of the sensitivity transfer function $S(s)$ at $s = 0$, we substitute $s = 0$ into the definition of $S(s)$. Assuming that the system is designed to have zero steady-state error to a step input, this implies that the open-loop transfer function $L(s)$ must have a pole at $s = 0$ (i.e., the system type is at least 1). This ensures that the feedback system responds correctly to a step input, resulting in:

$$S(0) = \frac{1}{1 + L(0)}$$

Given that $L(s)$ must approach infinity as s approaches 0 for there to be zero steady-state error to a step input (since this indicates an integrator in the system), we have:

$$S(0) = \frac{1}{1 + \infty} = 0$$

(b) The value of the loop transfer function $|L(0)|$ at $s = 0$ can be interpreted based on the previous discussion. Since the system is designed to have zero steady-state error for step inputs, and considering that a type 1 system (or higher) is required for this (which implies at least one integrator in the loop), $L(0)$ approaches infinity. Thus:

$$|L(0)| = \infty$$

However, it's essential to note that this is an idealized mathematical interpretation. In practical systems, the gain will not be infinite but should be sufficiently high to ensure the desired low steady-state error.

Confidence level: 95%

Correct

Claude Answer:

To achieve zero steady-state error for step reference commands in a closed-loop system, the system must have a specific behavior at low frequencies, particularly at $s = 0$. Let's analyze the sensitivity transfer function $S(s)$ and the loop transfer function $L(s)$ to determine their values at $s = 0$.

(a) The sensitivity transfer function $S(s)$ is defined as:

$$S(s) = \frac{1}{1+L(s)}$$

where $L(s)$ is the loop transfer function.

For the closed-loop system to have zero steady-state error for step reference commands, the sensitivity transfer function $S(s)$ must be equal to 0 at $s = 0$. This means:

$$S(0) = \frac{1}{1+L(0)} = 0$$

For this condition to be true, the denominator must approach infinity, which implies that $L(0)$ must approach infinity.

(b) Based on the conclusion from part (a), for the closed-loop system to have zero steady-state error for step reference commands, the loop transfer function $L(s)$ must have the following property:

$$\lim_{s \rightarrow 0} L(s) = \infty$$

In other words, the magnitude of the loop transfer function $|L(0)|$ must be infinite at $s = 0$. This condition is necessary to ensure that the sensitivity transfer function $S(0)$ becomes zero, leading to zero steady-state error for step reference commands.

To achieve this, the loop transfer function $L(s)$ must have a pole at $s = 0$, which corresponds to an integrator in the forward path of the closed-loop system. The presence of an integrator ensures that

the steady-state error for step reference commands is eliminated.

Confidence level: 95%

Correct