

2.12 Routh-Hurwitz Stability

Use Hurwitz stability and show that for what range of values of K is the system with the following characteristic equation stable?

$$s^2 + Ks + 2K - 1 = 0 \quad (118)$$

Solution :

The Hurwitz determinants for this system are

$$\Delta_2 = \begin{vmatrix} K & 0 \\ 1 & 2K - 1 \end{vmatrix} = 2K^2 - K = K(2K - 1) \quad \Delta_1 = K \quad (119)$$

In order for these determinants to be positive, it is necessary that $K > 0$ and $2K - 1 > 0$. Thus the system is stable if $K > \frac{1}{2}$.

GPT Answer:

To determine the range of values of K for which the system is stable using the Hurwitz stability criterion, we examine the characteristic equation of the system, which is given by:

$$s^2 + Ks + 2K - 1 = 0 \quad (120)$$

For a second-order system, the Hurwitz stability criterion requires that all the coefficients of the characteristic equation are positive, and the determinant of the Hurwitz matrix (constructed from the coefficients) is also positive. The coefficients here are: 1 (coefficient of s^2), K (coefficient of s), and $2K - 1$ (constant term).

1. All coefficients positive: - The coefficient of s^2 is 1, which is always positive. - The coefficient of s is K , which means $K > 0$. - The constant term is $2K - 1$, which means $2K - 1 > 0 \Rightarrow K > \frac{1}{2}$.
2. The determinant of the Hurwitz matrix for a second-order system is simply the constant term, since the matrix is 1×1 . Therefore, we need $2K - 1 > 0$, as already established.

Combining these conditions, we find that the system is stable for:

$$K > \frac{1}{2} \quad (121)$$

Thus, the system is stable for K values greater than 0.5.

Confidence Level: 95%

Correct

Claude Answer:

To determine the range of values of K for which the given system is stable using the Hurwitz stability criterion, we need to construct the Hurwitz matrix and ensure that all its principal minors are positive.

Given: The characteristic equation of the system is $s^2 + Ks + 2K - 1 = 0$.

Step 1: Identify the coefficients of the characteristic equation. Let the characteristic equation be in the form: $s^2 + a_1s + a_0 = 0$. Comparing with the given equation, we have: $a_1 = K$ $a_0 = 2K - 1$

Step 2: Construct the Hurwitz matrix. For a second-order system, the Hurwitz matrix is:

$$H = \begin{bmatrix} a_1 & 0 \\ a_0 & a_1 \end{bmatrix}$$

Substituting the values:

$$H = \begin{bmatrix} K & 0 \\ 2K - 1 & K \end{bmatrix}$$

Step 3: Check the conditions for stability. For the system to be stable, all the principal minors of the Hurwitz matrix must be positive.

First principal minor (M1): $a_1 > 0$ $K > 0$

Second principal minor (M2): $a_1 \cdot a_0 > 0$ $K(2K - 1) > 0$ $2K^2 - K > 0$ $K(2K - 1) > 0$ $K > 0$ and $2K - 1 > 0$ $K > 0$ and $K > \frac{1}{2}$

Step 4: Combine the conditions. From M1 and M2, we have: $K > \frac{1}{2}$

Therefore, the system with the given characteristic equation is stable for $K > \frac{1}{2}$.

Confidence Level: 95%

Correct