

9.2 Bandwidth

Determine the bandwidth for the system with transfer function $(C/R)(s) = 1/(s + 1)$.

Solution :

we have

$$\left| \frac{C}{R}(j\omega) \right| = \frac{1}{\sqrt{\omega^2 + 1}} \quad (272)$$

The bandwidth is the frequency at which this magnitude drops to $\frac{1}{\sqrt{2}}$ of its maximum value. Since $|(C/R)(j\omega)|$ is a strictly decreasing function of positive frequency, maximum happens at $\omega = 0$. Setting the magnitude equal to $\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{0+1}}$ and solving for ω yields:

$$\frac{1}{\sqrt{\omega^2 + 1}} = \frac{1}{\sqrt{2}}$$

Solving this equation:

$$\sqrt{\omega^2 + 1} = \sqrt{2}$$

$$\omega^2 + 1 = 2$$

$$\omega^2 = 1$$

$$\omega = 1$$

Therefore, the bandwidth of the system is $\omega = 1$ rad/s.

GPT Answer:

To determine the bandwidth of a system with the transfer function $(C/R)(s) = \frac{1}{s+1}$, we consider the bandwidth as the frequency at which the magnitude of the transfer function decreases to $\frac{1}{\sqrt{2}}$ of its maximum value. The maximum value of the magnitude for a first-order system like this is at DC ($s = 0$), where the magnitude is 1.

Given:

$$(C/R)(s) = \frac{1}{s+1}$$

To find the bandwidth, we substitute s with $j\omega$, where ω is the angular frequency in radians per second, and solve for the frequency ω when the magnitude of the transfer function equals $\frac{1}{\sqrt{2}}$.

$$|(C/R)(j\omega)| = \left| \frac{1}{j\omega + 1} \right| = \frac{1}{\sqrt{(\omega^2 + 1)}}$$

Set the magnitude equal to $\frac{1}{\sqrt{2}}$ to find the bandwidth:

$$\frac{1}{\sqrt{\omega^2 + 1}} = \frac{1}{\sqrt{2}}$$

Solving for ω^2 :

$$\omega^2 + 1 = 2$$

$$\omega^2 = 1$$

$$\omega = 1 \text{ rad/s}$$

Thus, the bandwidth of the system is 1 rad/s.

Confidence level: 95%

Correct

Claude Answer:

To determine the bandwidth of the given system, we need to find the frequency at which the magnitude of the transfer function drops to $\frac{1}{\sqrt{2}}$ (approximately 0.707) of its maximum value. Let's solve this problem step by step.

Given:

$$\left(\frac{C}{R}\right)(s) = \frac{1}{s + 1}$$

Step 1: Find the magnitude of the transfer function The magnitude of the transfer function at a complex frequency $j\omega$ is:

$$\left|\left(\frac{C}{R}\right)(j\omega)\right| = \left|\frac{1}{j\omega + 1}\right|$$

To simplify this, we rationalize the denominator:

$$\left|\left(\frac{C}{R}\right)(j\omega)\right| = \left|\frac{1}{j\omega + 1} \cdot \frac{-j\omega + 1}{-j\omega + 1}\right| = \left|\frac{-j\omega + 1}{\omega^2 + 1}\right|$$

The magnitude is then the square root of the sum of the squares of the real and imaginary parts:

$$\left|\left(\frac{C}{R}\right)(j\omega)\right| = \sqrt{\frac{1 + \omega^2}{\omega^2 + 1}} = \frac{1}{\sqrt{\omega^2 + 1}}$$

Step 2: Find the maximum value of the magnitude The maximum value occurs at $\omega = 0$:

$$\left| \left(\frac{C}{R} \right) (j0) \right| = \frac{1}{\sqrt{0^2 + 1}} = 1$$

Step 3: Set the magnitude equal to $\frac{1}{\sqrt{2}}$ times the maximum value and solve for ω

$$\frac{1}{\sqrt{\omega^2 + 1}} = \frac{1}{\sqrt{2}}$$

Cross-multiply and square both sides:

$$\omega^2 + 1 = 2$$

$$\omega^2 = 1$$

$$\omega = 1 \text{ rad/s}$$

Therefore, the bandwidth of the system is 1 rad/s.

Confidence level: 95%

Correct