

# 第 1 讲：非寿险精算简介

高光远

中国人民大学 统计学院

Non-life insurance in Continental Europe is also known as property and casualty insurance (P&C) in the US and Canada, and general insurance in the UK and Australia.

- The China Association of Actuaries (CAA).
- The Casualty Actuarial Society (CAS) is a North American based actuarial association, specialized in non-life insurance.
- The Society of Actuaries (SOA) is a North American based actuarial association, providing various tracks such as life and annuity, retirement, quantitative finance, general insurance, etc.



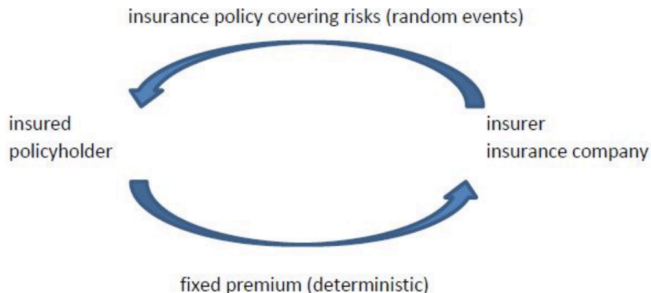
- The Institute and Faculty of Actuaries (IFoA) is the British actuarial association.
- The Institute of Actuaries of Australia (IAA).



- Insurance originates from a general demand of society who asks for protection against **unforeseeable events** which might cause **financial damage** to individuals and society.
- The general solution is to build a community to which everybody contributes a **fixed deterministic premium** and then the **unforeseeable financial damage** is financed by the means of this community.

# 保险公司 (insurer) 和被保险人 (insured)

This community is developed to **insurer**, and the contributors are called **insured** or **policyholders**.



# 保险合同 (insurance contracts (policies))

- Insurance contracts for these products always specify an insurance period (typically of one year).
- The insured events must occur within this insurance period, and cause financial damage.
- The financial damage is indemnified by insurer's payment.
- Such random payments are called insurance claims.

# 大数定理 (the law of large numbers)

- Typically, the insurance **premium** is paid at the **beginning** of the insurance period (upfront).
- To determine this insurance premium, the insurance company pools **similar risks** whose individual **insurance claims** can be described by a sequence  $Y_1, \dots, Y_n$  of **random variables**.
- These insurance claims  $Y_i$  are **random** at the beginning of the insurance period, and therefore need to be described with **probability theory**.



# 大数定理 (the law of large numbers)

- Assume  $Y_1, \dots, Y_n$  are **uncorrelated** and **identically** distributed random variables with **finite mean**  $\mu = \mathbb{E}[Y_i]$ .
- **The weak law of large numbers (LLN)** says that for all  $\varepsilon > 0$

$$\lim_{n \rightarrow \infty} \mathbb{P} \left[ \left| \frac{1}{n} \sum_{i=1}^n Y_i - \mu \right| \geq \varepsilon \right] = 0 \quad (1)$$

- This means that the **average** claim amount  $1/n \sum_{i=1}^n Y_i$  becomes more “**predictable**” with increasing **portfolio size**  $n$ .
- Therefore, we can calculate the insurance premium quite **accurately** for **large** portfolio sizes  $n$ .



The weak law of large numbers is considered to be a [theoretical cornerstone of insurance](#). It goes back to the Swiss mathematician [Jakob Bernoulli](#) (1655-1705) of the famous Bernoulli family.

# 中心极限定理 (central limit theorem)

- For **independent** and **identically** distributed random variables  $Y_1, Y_2, \dots$  with **finite variances**  $\sigma^2$ , the weak law of large numbers can further be refined by **the central limit theorem (CLT)** which provides the **asymptotic limit distribution**.
- The CLT states that we have the following **convergence in distribution**:

$$\frac{\sum_{i=1}^n Y_i - n\mu}{\sqrt{n}\sigma} \rightarrow \mathcal{N}(0, 1) \quad \text{as } n \rightarrow \infty \quad (2)$$

# 中心极限定理 (central limit theorem)

- We obtain a standard Gaussian distribution (normal distribution).
- The denominator increases of order  $\sqrt{n}$ , a slower rate than  $n$ .
- This implies that the confidence bounds of average claims amount get narrower with the larger portfolio size.
- These are the basics why insurance works.



The **Gaussian distribution** (normal distribution) is named after the German mathematician **Carl Friedrich Gauss** (1777-1855). He was one of the greatest mathematicians and has contributed to many different fields in mathematics and physics.

# 非寿险的种类

- **财产保险 (property insurance)**是以财产及其相关利益为**保险标的**, 当保险事故发生导致被保险财产遭受损失时, 由**保险人**以**金钱或实物**对被保险人进行补偿的一种保险. 如**车损险 (body damage)**, **盗抢险 (theft and robbery)**.
- **责任保险 (liability insurance)**以被保险人依法应负的民事损害赔偿**责任**或经过特别约定的**合同责任**为**保险标的**. 其**保险责任**包括两类: 被保险人应负的经济赔偿责任; 因**责任判定**赔偿纠纷引起的应由被保险人支付的费用. 如**交强险 (compulsory third party, CTP)**, **第三者责任险 (third party liability)**.
- **短期健康和意外伤害保险 (short-term health insurance, casualty insurance)**.

注: **红色**一般表示最重要的概念, **蓝色**次之.

# 非寿险和寿险的比较

Table 1: 非寿险 (non-life, general, property&casualty insurance) 与寿险 (life insurance) 的比较

	寿险	非寿险
退保转保	业务稳定	业务极不稳定
合同期限	长期合同	短期合同
赔付金额	赔付金额具有可预期性	赔付金额有很强的随机性
概率依据	生命表	历史赔付

# 非寿险精算中的主要数学和统计知识

- ① 集体风险模型 (collective risk modelling)
- ② 个体风险模型 (individual risk modelling)
- ③ 破产理论 (ruin theory)
- ④ 费率厘定 (pricing)
- ⑤ 贝叶斯模型和信度理论 (Bayesian models and credibility theory)
- ⑥ 准备金评估模型 (claims reserving)
- ⑦ 机器学习 (machine learning). 参考资料 [3].

其中 4-7 点为本课程主要介绍的内容.

注: **加粗**一般用来强调或者对比.



- 以下按照**优先级**列出. [1],[2] 为必读, 其它为选读.
- 选读中, [3] 为机器学习内容, [4] 涵盖非寿险精算中的大部分知识, [5] 和 [6] 为 Casualty Actuarial Society Exam 5 的参考资料.

- [1] 本幻灯片
- [2] 孟生旺, 刘乐平, 肖争艳 (2015). 非寿险精算, 第三版. 北京: 中国人民大学出版社.
- [3] Wüthrich, M.V., Buser, C. (2019). Data analytics for non-life insurance pricing. Available at SSRN: <https://ssrn.com/abstract=2870308>.
- [4] Wüthrich, M.V. (2018). Non-Life Insurance: Mathematics & Statistics. Available at SSRN: <https://ssrn.com/abstract=2319328>.
- [5] Friedland, J. (2010). Estimating unpaid claims using basic techniques. 3rd edition. Casualty Actuarial Society study notes.
- [6] Werner, G., Modlin, C. (2016). Basic ratemaking. 5th edition. Casualty Actuarial Society study notes.