What we've done so far (summarised):

What is a qubit?
-fundamental information, not just a 0 or 1, superposition of both
-in our case represent finite truncation of a QFT
-for n qubits there are 2^n states

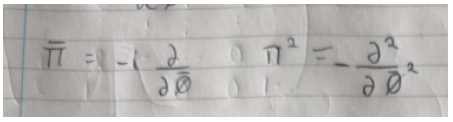
What is digitisation?
-digitisation is mapping a qft to a finite space so it can be simulated
-Truncate because of a fixed input of the wavefunction and the basis size 2^n

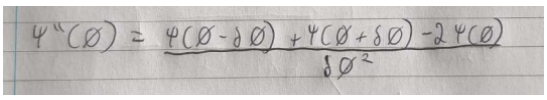
Methods of digitisation of Hamiltonian (JLP, FD, HO basis), introduction of interaction

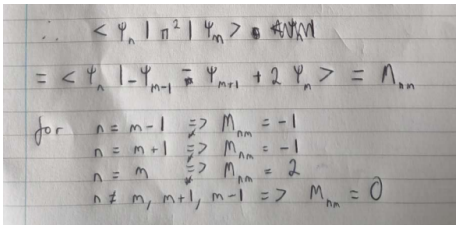
$H = 1/2\,(\pi^2 + \omega^2\phi^2)$ <- harmonic oscillator hamiltonian (For now we've assumed w = 1)
Phi and pi are the field values at a given point and the canonical momentum

JLP:
-Discretise field space, phi is diagonal in field space
-pi^2 is diagonal in momentum space so use qufotr to get it in field space
-we use delta phi and delta k (which are based on # of qubits and phi max) to get the diagonal matrix in each space
-uses periodic boundary conditions -> because we use discrete fourier transform
-mention existence of twisted boundary conditions that introduce a phase, not used yet, is suggested in summary of savage
- Then put that into hamiltonian
-can calculate an optimal phi max (as we have to enter this manually) by ensuring the wavefunctions in both momentum and phi space are supported -> error increase if one or other is bad

FD:
-uses same phi stuff as JLP
-we approximate pi^2 by using pi operator (derivative wrt phi)







-"3 point finite difference" -> using taylor series to get wavefunction and then from their the matrix
-uses hard wall boundary conditions at phi max wavefunction = 0 because we're dealing with a difference, making outside anything but 0 requires extra assumptions (would have to extrapolate outside)

Check twisted FD (minus on loop around)

HO Basis:
-We expand the wavefunction in the basis of the eigenfunction of the harmonic oscillator, Essentially expanding it as a sum of eigenstates of HO
-then truncation is based on states 0->nmax which is determined by number of qubits

$|\psi> = \sum_{n=0}^{n_{max}} c_n |n>$ boundary condition occurs in fock space (quantum states of a system)

-phi and pi are expressed using ladder operators and we end up with a Hamiltonian expressed in terms of these operators (for the code we reverse this process, define a and a dagger (they're the standard creation and annihilation operators for a harmonic oscillator, which we can use to make matrices out of) then use those to define phi and pi and stick that into the standard Hamiltonian we've been using

Evaluating the hamiltonian on a truncated basis

Interaction:
For the hamiltonian we can also add an interaction term to the hamiltonian which is given by lambda * pi^4 / 24

$$\bar{H} = \frac{1}{2}\bar{\Pi}^2 + \frac{1}{2}\bar{\phi}^2 + \frac{\bar{\lambda}_0}{4!}\bar{\phi}^4 \quad ,$$

Comes from QFT Lagrangian describing a system with self interaction

Errors:
-using each of the methods we compare them to known values
-without interaction we can use E=(n+½)w
-with interaction we can use found values for specific energy levels