Finite difference Operator derivation 1

$\bar{\pi} = -i \frac{d}{d\bar{\phi}}$ $\quad \pi^2 = d$

$\bar{\pi} = -i \frac{\partial}{\partial \bar{\phi}}$ $\quad \pi^2 = -\frac{\partial^2}{\partial \bar{\phi}^2}$

Need to replace this with an ~~operator~~ matrix
first need to get expression of
2^{nd} order derivative: taylor expand

$\psi(\phi + \delta\phi)$ and $\psi(\phi - \delta\phi)$

$$\psi(\phi - \delta\phi) = \psi(\phi) - \delta\phi\, \psi'(\phi) + \frac{\delta\phi^2}{2!}\psi''(\phi) - \frac{\delta\phi^3}{3!}\psi'''(\phi)$$
$$\psi(\phi + \delta\phi) = \psi(\phi) + \delta\phi\, \psi'(\phi) + \frac{\delta\phi^2}{2!}\psi''(\phi) + \frac{\delta\phi^3}{3!}\psi^3(\phi)$$

$$\psi(\phi - \delta\phi) + \psi(\phi + \delta\phi) = 2\psi(\phi) + \delta\phi^2\, \psi''(\phi) + O(\delta\phi^2)$$

$$\psi''(\phi) = \frac{\psi(\phi - \delta\phi) + \psi(\phi + \delta\phi) - 2\psi(\phi)}{\delta\phi^2}$$

$$\psi''(\phi_n) = \left(\psi(\phi_{n-1}) + \psi(\phi_{n+1}) - 2\psi(\phi_n)\right)/\delta\phi^2$$

$$\therefore \quad \langle \psi_n | \pi^2 | \psi_m \rangle$$

$$= \langle \psi_n | -\psi_{m-1} - \psi_{m+1} + 2\psi_m \rangle = M_{nm}$$

for $n = m-1 \Rightarrow M_{nm} = -1$
$n = m+1 \Rightarrow M_{nm} = -1$
$n = m \Rightarrow M_{nm} = 2$
$n \neq m, m+1, m-1 \Rightarrow M_{nm} = 0$

$$\therefore \langle \psi | \pi^2 | \cancel{\phi} \psi \rangle = \frac{1}{\delta\phi^2} \begin{pmatrix} 2 & -1 & 0 & 0 & \cdots & -1 \\ -1 & 2 & -1 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -1 & 2 & -1 \\ -1 & 0 & 0 & \cdots & -1 & 2 \end{pmatrix}$$