How the HO basis works?

The Harmonic Oscillator basis represents a quantum field using eigenstates of a simple harmonic oscillator rather than discrete field values. To do this the field operator and momentum are expressed using ladder operators whose commutator equals 1. These operators define number states which are orthonormal and correspond to energy levels of the oscillator with different frequencies. The Hamiltonian can then be constructed from these operators.

For omega = 1 we just end up with a unit HO and so the error vanishes