Optimal ϕ_{max} derivation 1

$\beta_\phi = 0, 1, \ldots n_s - 1$

Have: $\phi_i = -\phi_{max} + \beta_\phi \delta\phi$ $\delta\phi = \dfrac{2\phi_{max}}{n_s - 1}$

\# basis states — $n_s = 2^{n_Q}$ $n_Q = \#$ Qubits

Find eq for ϕ_{max} minimises error by maximising supported regions in field and momentum space

$K_{max} = \dfrac{\pi}{\delta\phi}$ $K_i = -K_{max} + \beta_K \delta K$ $\delta_K = \dfrac{2\pi}{\delta_\phi n_s}$

$\beta_K = 1, 2, \ldots n_s$

~~For each site~~

For a free scalar field: (position space)

HO

$$H = \frac{1}{2}\sum_i \pi_i^2 + (\nabla\phi_i)^2$$

(momentum space) $\tilde{\phi}_K = \frac{1}{\sqrt{N}}\sum_{j=0}^{N-1} \phi_j e^{\ldots}$ $\phi_j = \frac{1}{\sqrt{N}}\sum_K \tilde{\phi}_K e^{\ldots}$

$$H = \frac{\delta x}{2}\sum_K \left(|\pi_K|^2 + \frac{\omega_K^2}{\delta x^2}|\phi_K|^2\right)$$

$$\omega_K^2 = \frac{4}{\delta x^2}\sin^2\frac{K_i \delta x}{2}$$

$$\omega_K = \frac{2}{\delta x}\left|\sin\left(\frac{K_i \delta x}{2}\right)\right|$$

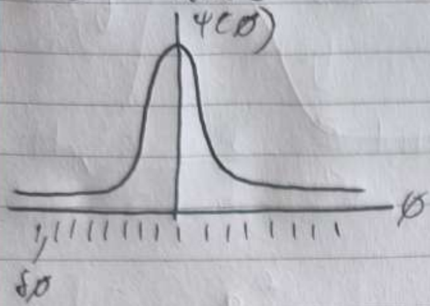
$$\bar{\omega} = \frac{1}{N}\sum_i \omega_i$$

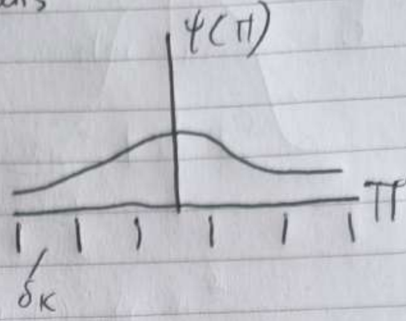
really long fourier transform done by expressing $(\nabla\phi_i)^2 = \dfrac{(\phi_{j+1} - \phi_j)^2}{\delta x^2}$ in the sum and then you end up with a factor $|1 - e^{iK\delta x}|^2$ which gives the sin term in ω_K

$\pi_i^2 \to |\pi_K|^2$ so easy.

Optimal $\varnothing_{max}$ Calc 2

We have 2 Gaussians





We want to find an eqn for $\varnothing_{max}$ that optimises support for both graphs. To do this we find an equation that relates $\delta_\varnothing$ and δ_K. This can be done by transforming both Gaussians into a normal distribution and setting them equal:

$$Z = \frac{x-\mu}{\sigma} \Rightarrow \frac{\delta_\varnothing}{\sigma_\varnothing} = \frac{\delta_K}{\sigma_K}$$

we know: $\delta\varnothing = \frac{2\varnothing_{max}}{n_s - 1}$ $\quad \delta_K = \frac{\pi(n_s - 1)}{n_s \varnothing_{max}}$

To determine $\sigma_\varnothing$ and σ_K we look at $\psi_0(x)$ for a HO: $\psi_0(x) \propto e^{-\frac{m\omega x^2}{2}}$

$$\sigma_x^2 = \langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi_0(x)|^2 dx = \frac{1}{2m\omega}$$

$$\sigma_K^2 = \langle p^2 \rangle = \int_{-\infty}^{\infty} \left(-i\frac{d}{dx}\right)^2 |\psi_0(x)|^2 dx = \frac{m\omega}{2}$$

Optimal $\varnothing_{max}$ Calc 3

Using the earlier Hamiltonian in K space and comparing to a standard HO:

$$H_K = \frac{\delta x}{2} \sum_K |\pi_K|^2 - \frac{\omega_K^2}{\delta x^2} |\varnothing_K|^2$$

$$H = \frac{p^2}{2 m_{eff}} + \frac{1}{2} m_{eff} \omega_{eff}^2 x^2$$

We can see $m_{eff} = \frac{1}{\delta x}$

$$\therefore \quad \sigma_x = \frac{1}{\sqrt{2 \delta x \bar{\omega}}} \qquad \sigma_p = \sqrt{\frac{\delta x \bar{\omega}}{2}}$$

then

$$\frac{\delta \varnothing}{\sigma_\varnothing} = \frac{\delta K}{\sigma_K}$$

$$\cancel{\frac{1}{\sqrt{2 \delta x \bar{\omega}}} \frac{2\varnothing_{max}}{n_s - 1} = \sqrt{\quad}}$$

$$\sqrt{2 \delta x \bar{\omega}} \; \frac{2 \varnothing_{max}}{n_s - 1} = \sqrt{\frac{2}{\delta x \bar{\omega}}} \; \pi \; \frac{n_s - 1}{n_s \varnothing_{max}}$$

$$\varnothing_{max}^2 = \frac{1}{\delta x \bar{\omega}} \frac{\pi}{2} \frac{(n_s - 1)^2}{n_s}$$

$$\varnothing_{max} = \frac{1}{\sqrt{\delta x \bar{\omega}}} \sqrt{\frac{\pi}{2} \frac{(n_s - 1)^2}{n_s}}$$