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~~4th~~ 4th order effective Hamiltonian

$$T_n = \sum_{\alpha_1, \dots, \alpha_{n-1}} \frac{\langle f | V | \alpha_1 \rangle \langle \alpha_1 | V | \alpha_2 \rangle \dots \langle \alpha_{n-1} | V | i \rangle}{(E_{f\alpha_1} + i\epsilon) \dots (E_{f\alpha_{n-1}} + i\epsilon)}$$

$$V_{\text{eff}} = H_1 + H_2 + \dots$$

$$\text{From these : } \langle f | T_1 | i \rangle = \langle f | V | i \rangle$$

To ensure the effective ~~potential~~ theory has a ~~Hamiltonian~~ Hamiltonian which matches the transition matrix :

$$\langle f | H_1 | i \rangle_{\text{eff}} = \langle f | V | i \rangle$$

For second order we have

$$\langle f | T_2 | i \rangle = \sum_{\alpha} \frac{\langle f | V | \alpha \rangle \langle \alpha | V | i \rangle}{E_{f\alpha}}$$

Lower energy states (α) are already accounted for by H_1 so for H_2

$$\langle f | H_2 | i \rangle_{\text{eff}} = \sum_{\alpha} \frac{\langle f | V | \alpha \rangle \langle \alpha | V | i \rangle}{E_{d\alpha}}$$

For third order we get a new term β

$$\langle f | T_3 | i \rangle = \sum_{\alpha\beta} \frac{\langle f | V | \alpha \rangle \langle \alpha | V | \beta \rangle \langle \beta | V | i \rangle}{E_{f\alpha} E_{f\beta}}$$

We do the same as we did previously eliminating terms that we've already accounted for but maintaining the high energy states we haven't

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4th order effective Hamiltonian
First term \rightarrow High energy states

$$\langle f | H_3 | i \rangle_{\text{eff}} = \sum_{\alpha, \beta} \frac{\langle f | V | \alpha \rangle \langle \alpha | V | \beta \rangle \langle \beta | V | i \rangle}{E_f - E_{f\beta}} - \sum_{\alpha} \sum_{\beta} \frac{\langle f | V | \alpha \rangle \langle \alpha | V | \beta \rangle \langle \beta | V | i \rangle}{E_{\alpha\beta} - E_{f\beta}}$$

Produced by the lower order effective operators so we remove them

~~note~~ The subtraction term isn't explicitly here for H_3 , it's here to remove terms introduced by H_1 and H_2 and prevent double counting

Table of combinations:

	α	β	
1	>	>	We want this
2	<	>	Appears through H_1 and H_2 so eliminate
3	>	<	$\beta <$ never appears in H_3 to begin with
4	<	<	Low energy (H_1) and not double cou

3 and 4 don't appear in H_3

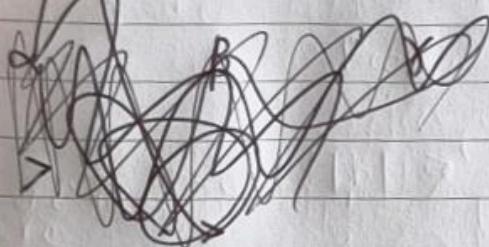
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4th order effective Hamiltonian

$$\langle f | T_4 | i \rangle = \sum_{\alpha, \beta, \gamma} \frac{\langle f | V | \alpha \rangle \langle \alpha | V | \beta \rangle \langle \beta | V | \gamma \rangle \langle \gamma | V | i \rangle}{E_{f\alpha} E_{f\beta} E_{f\gamma}}$$

~~$\langle f | T_4 | i \rangle = \sum_{\alpha, \beta, \gamma}$~~

$$= \sum_{\alpha, \beta, \gamma} \frac{V_{f\alpha} V_{\alpha\beta} V_{\beta\gamma} V_{\gamma i}}{E_{f\alpha} E_{f\beta} E_{f\gamma}} = \sum_{\alpha, \beta, \gamma} \frac{V_f}{E_f}$$



α	β	γ	α	β	γ
>	-	- x	>	<	-) x
-	>	- x	>	-	<) x
-	-	> x	-	>	<) x
>	>	- x	<	>	-) x
>	-	> x	<	-	>) x
-	>	> x	-	<	>) x
>	>	> ✓	✓	-	-)
-	-	- x	✓	-	-)
<	-	- x	✓	-	-) ✓
-	<	- x	✓	<	< ✓
-	-	< x	<	>	< ✓
<	<	- x	<	<	> ✓
<	-	< x	>	>	< ✓
-	<	< x	>	<	> ✓
<	<	< x	<	>	< ✓

Anything with - eliminated because don't want full range for any. I eliminated because all < so low energy handled already

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4th order effective Hamiltonian

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|---|---------|----------|---|
| > | β | γ | |
| > | > | > | 1 - Main high energy stuff |
| > | < | < | 2 - Already generated by H_1, H_2, H_3
as previous |
| < | > | < | |
| < | < | > | 3 - Parts removed by \uparrow but
we still need |
| > | > | < | |
| > | < | > | |
| < | > | > | |

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$$\langle f | H_4 | i \rangle = \sum_{\alpha, \beta, \gamma} \frac{V_{f\alpha} V_{\alpha\beta} V_{\beta\gamma} V_{\gamma i}}{E_{f\alpha} E_{\alpha\beta} E_{\beta\gamma} E_{\gamma i}} \quad \{ 1 \}$$

$$- \sum_{\alpha} \sum_{\beta < \gamma} \frac{V_4}{E_4} - \sum_{\beta} \sum_{\alpha < \gamma} \frac{V_4}{E_4} - \sum_{\gamma} \sum_{\alpha < \beta} \frac{V_4}{E_4} \quad \{ 2 \}$$

$$+ \sum_{\alpha < \beta} \sum_{\gamma < \delta} \frac{V_4}{E_4} + \sum_{\alpha < \gamma} \sum_{\beta < \delta} \frac{V_4}{E_4} + \sum_{\beta < \gamma} \sum_{\alpha < \delta} \frac{V_4}{E_4} \quad \{ 3 \}$$