

First Code Block:

- builds HO basis
- construct free  $H$  ( $H_0$ ) and interaction  $V$  in full basis
- split basis into low and high energy ( $P$  and  $Q$ ),  $P$  is first  $n_P$  states
- computes full  $H$  effective for GS up to third order of  $V$  (can be used for excited states as well)
- effective energy given by sum of  $H_1$ ,  $H_2$  and  $H_3$
- computes relative error to reference value
- does sweep over  $\omega$  and plots relative error

Second code block:

- builds HO basis
- construct free  $H$  ( $H_0$ ) and interaction  $V$  in full basis
- produce an effective matrix acting only on low energy " $P$  space"
- effective put into a qubit space and then pads matrix with zeros to match qubit space size
- compares GS energy from digitized hamiltonian to known reference value
- plots percentage error vs  $\omega$