Optimal ϕ_{max} derivation 1

$\beta_\phi = 0, 1, \dots n_s - 1$

Have: $\phi_i = -\phi_{max} + \beta_\phi \delta\phi$ $\delta\phi = \frac{2\phi_{max}}{n_s - 1}$

basis states — $n_s = 2^{n_Q}$ n_Q = # Qubits

Find eq for ϕ_{max} minimises error by maximising supported regions in field and momentum space

$K_{max} = \frac{\pi}{\delta\phi}$ $K_i = -K_{max} + \beta_K \delta K$ $\delta_K = \frac{2\pi}{\delta_\phi n_s}$

$\beta_K = 1, 2, \dots n_s$

~~For each site~~

For a free scalar field: (position space)

HO

$H = \frac{1}{2}\sum_i \pi_i^2 + (\nabla\phi_i)^2$

(momentum space) $\tilde{\phi}_K = \frac{1}{\sqrt{N}}\sum_{j=0}^{N-1} \phi_j e^{-iKx}$ $\phi_j = \frac{1}{\sqrt{N}}\sum_K \tilde{\phi}_K e^{iKx}$

$H = \frac{1}{2}\sum_K (|\pi_K|^2 + \omega_K^2 |\phi_K|^2)$

really long fourier transform done by expressing $(\nabla\phi_i)^2 = \frac{(\phi_{j+1} - \phi_j)^2}{\delta x^2}$ in the sum and then you end up with a factor $|1 - e^{iK\delta x}|^2$ which gives the sin term in ω_K $\pi_i^2 \to |\pi_K|^2$ so easy.

$\omega_K^2 = \frac{4}{\delta x^2}\sin^2\frac{K_i \delta x}{2}$

$\therefore$

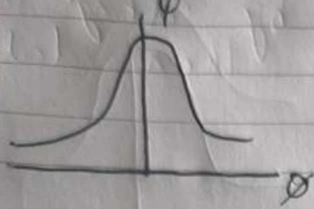
$\omega_K = \frac{2}{\delta x}\left|\sin\left(\frac{K_i \delta x}{2}\right)\right|$

$\bar{\omega} = \frac{1}{N}\sum_i \omega_i$

Optimal $\emptyset_{max}$ derivation 12

Relation between $\emptyset_{max}$, $\bar{\omega}$

For SHO :



Gaussian : $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

$$\Psi(\emptyset) = \left(\frac{\omega}{\pi}\right)^{1/4} \exp\left(-\frac{\omega \emptyset^2}{2}\right)$$

(standard result for SHO)

$\therefore$

$$\sigma^2 = \frac{1}{\omega} \quad \therefore \quad \langle \emptyset^2 \rangle = \frac{1}{\omega}$$

$$\langle \emptyset^2 \rangle^{1/2} = \frac{1}{\sqrt{\omega}}$$

I think $\omega = \bar{\omega}\,\delta x$ I don't know if this is true but definitely feels like it should be

Presumeably we can do $\emptyset_{max} \propto \langle \emptyset^2 \rangle^{\frac{1}{2}}$

$\emptyset_{max} = \frac{\alpha}{\sqrt{\bar{\omega}\,\delta x}}$ so now why does

$$\alpha = \cancel{A} \sqrt{\frac{\pi}{2} \frac{(n_\emptyset - 1)^2}{n_\emptyset}}$$

probably just some scaling thing

$$\overbrace{\frac{n_s - 1}{\sqrt{n_s}}}^{\emptyset_{max}?}$$

?

almost reminscent of variance eqn in statistics?

$$\sigma^2 = \frac{\sum (x-\bar{x})^2}{N}$$

could be this or found online about people using erfunc cut offs so this could be a certain higher energy level past which its redundant to include

Just realised my wave function is for the GS so this is probably a generalisation for higher energy levels

— NS Theorem ???

Nyquist-Shannon - If $\tilde{f}(k)$ has no support beyond a max k the $f(x)$ can be exactly reconstructed by some discrete sample δx

~~This is where $\delta\phi = 2\phi_{max}$~~

$\therefore \quad \delta x < \frac{\pi}{k_{max}} \rightarrow \delta\phi < \frac{\pi}{\pi_{max}}$

~~To~~ then sets a value on $\delta\phi$ by using ϕ_{max} $\quad \delta\phi = \frac{2\phi_{max}}{N_s - 1}$, this is why we see a big dip in the error because where that dip is is where the bound is saturated

$\therefore$ find where the bound is saturated for a given wavefunction and we find the optimal ϕ_{max}

Target: $\phi_{max} = \frac{1}{\sqrt{\delta x \bar{\omega}}} \sqrt{\frac{\pi}{2} \frac{(n_\phi - 1)^2}{n_\phi}}$

$\cancel{\frac{2\phi_{max}}{N_s - 1} = \delta\phi < \frac{\pi}{\pi_{max}} \Rightarrow \phi_{max} < \frac{\pi}{2} \frac{N_s - 1}{\pi_{max}}}$

where $\bar{\omega} = \frac{1}{N} \sum_i \omega_i$, $\omega_i = \frac{2}{\delta x} \left| \sin\left(\frac{p_i \delta x}{2}\right) \right.$

$\emptyset_{max}$ 4

Some cut off for Π? some multiple of variance:

$$\Pi_{cut} \approx s\,\sigma_\Pi \qquad \delta\emptyset < \frac{\Pi}{s\,\sigma_\Pi}$$

$$\emptyset_{max} = \alpha\,\sigma_\emptyset \qquad \delta\emptyset = \frac{2\emptyset_{max}}{N_s - 1} < \frac{\Pi}{\Pi_{max}}$$

$$\frac{2\,\overset{\emptyset_{max}}{\cancel{\alpha\sigma_\emptyset}}}{N_s - 1} < \frac{\Pi}{s\,\sigma_\Pi}$$

$$\emptyset_{max} < \frac{\Pi}{2}\,\frac{N_s - 1}{s\,\sigma_\Pi}$$

because F of Gaussian is another Gaussian we still have $\sigma_\Pi^2 = \frac{1}{\omega_\Pi}$ Not helpful though

Also $N_s = 2^n$?

How are you meant to get to this thing???