Question 1 (1 point not continuous evaluation, 0,5 points continuous evaluation):

Given a conductive loop that rotates within a magnetic field producing an electromotive force $\varepsilon(t) = 4\pi \cdot \sin(2\pi t)$, all magnitudes are expressed in international system units. This electromotive force is connected as the source of a circuit with impedance $Z = 2\pi \angle 45° \ \Omega$.

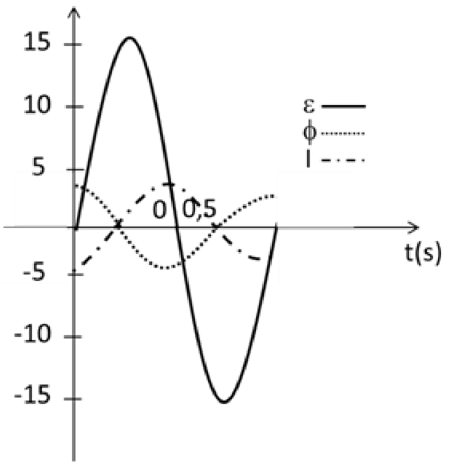
a) Obtain the expression of the magnetic flux in the loop as a function of time,

$$\phi = -\int 4\pi \cdot \sin(2\pi t)\, dt = 2 \cdot \cos(2\pi t) \ \ (S.I.)$$

b) Obtain the expression of the current flowing through the circuit as a function of time,

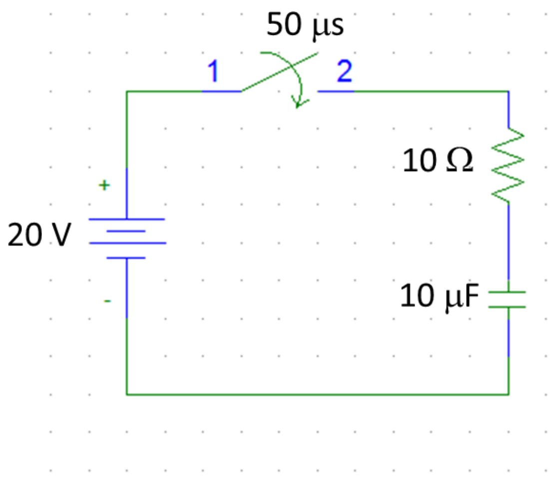
$I(t) = 2 \cdot \sin(2\pi t - \pi/4)$ A.

c) Draw a graph the magnetic flux, the current and the electromotive force in the interval t=0 s y t=1 s.



Problem 1:(2 point not continuous evaluation, 1,5 points continuous evaluation)

Given the circuit in next figure, where the capacitor is initially uncharged, and the switch is closed at t=50 μs.



a) What is the maximum charge stored in the capacitor

Qmax=CV= $10\cdot10^{-6}\cdot20$=0.2 mC=200 µC

b) What is the value of the time constant of the circuit and what is its meaning?

τ=RC=$10\cdot10\cdot10^{-6}$ = 0.1 ms=100 µs

c) What is the value of the charge stored in the capacitor at instants: t=0 µs, t=75 µs, t=150 µs y t=200 µs?

$$Q(t) = 200\ \mu C\left(1 - e^{-\frac{t-50\mu s}{100\mu s}}\right)$$

Q(0 µs)= 0 C (ya que no ha empezado el proceso de carga)

$$Q(75\ \mu s) = 200\ \mu C\left(1 - e^{-\frac{(75-50)\mu s}{100\mu s}}\right) = 44.24\ \mu C$$

$$Q(150\ \mu s) = 200\ \mu C\left(1 - e^{-\frac{(150-50)\mu s}{100\mu s}}\right) = 126\ \mu C$$

$$Q(200\ \mu s) = 200\ \mu C\left(1 - e^{-\frac{(200-50)\mu s}{100\mu s}}\right) = 155\ \mu C$$

d) Obtain the expression of the current flowing through the circuit as a function of time.

$$I(t) = \frac{dQ(t)}{dt} = \frac{200\ \mu C}{100\mu s}\left(e^{-\frac{t-50\mu s}{100\mu s}}\right) = 2A\cdot e^{-\frac{t-50\mu s}{100\mu s}}$$

Problem2 2: (2 point not continuous evaluation, 1,5 points continuous evaluation)

Given the circuit in next figure, where V_L = 16 + 0j V.

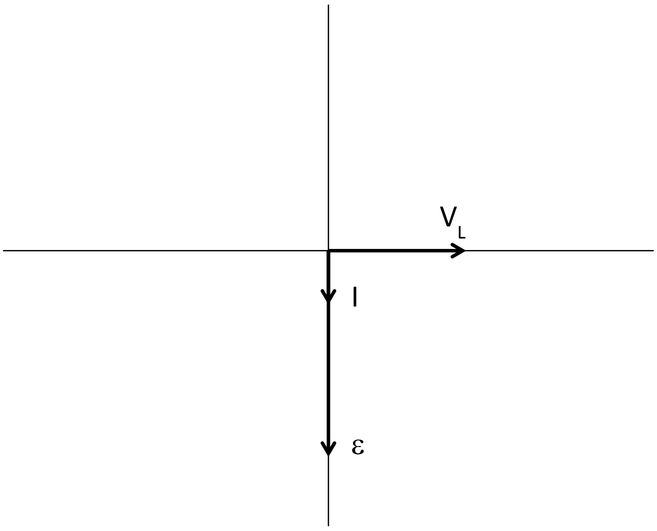
(a) Calculate the current I.

$$V_L = IZ_L$$
$$I = \frac{V_L}{Z_L} = \frac{16 < 0°\ V}{8 < 90°\ \Omega} = 2 < -90°\ A$$

(b) Calculate the impedance Z.

Z=-8j Ω

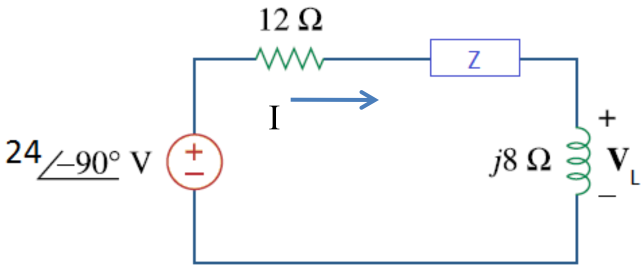
(c) Draw the phasor diagram for voltage at the source, the current I and the voltage V_L.



(d) Calculate the power factor and average power of the whole circuit and the power factor and average power at the inductance.

Power factor in inductance = 0, Av. Power Inductance = 0 W

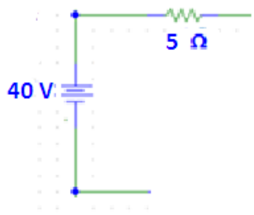
Power factor in circuit = 1, Av Power in circuit = 24 W



Problem 3:(2 point not continuous evaluation, 1,5 points continuous evaluation)

Given the circuit in next figure. Calculate:

1. Thevenin equivalent circuit across nodes A and B



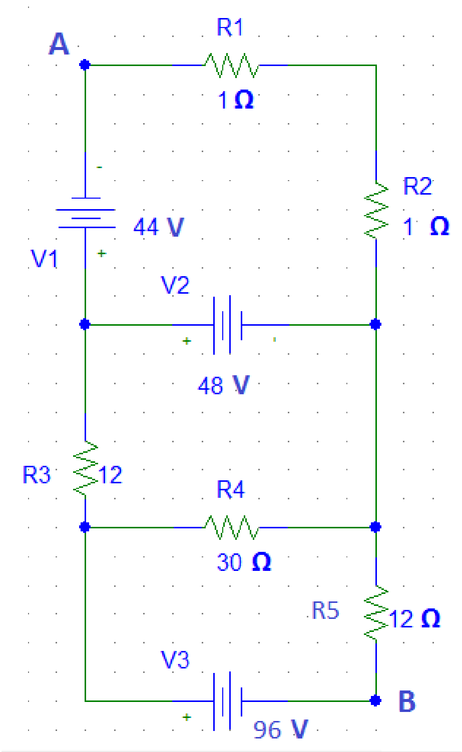
2. The current through resistances R2 and R5.

$$I_{R2} = I_1 = 2\ A$$
$$I_{R5} = I_3 = 3\ A$$

3. Norton equivalent circuit through nodes A and B.

I_N= 8 A

R_N= 5 ohms


A
R1
1 Ω
R2
1 Ω
44 V
V1
V2
48 V
R3
12
R4
30 Ω
R5
12 Ω
V3
96 V
B

Only for Not Continuous Evaluation Route

Question 2: (1 point not continuous evaluation)

Given an AC circuit composed of a source and an impedance. The source is V(t)= 20 sin 3t. The average power P is 200W and the reactive power Q is 40 W, as shown in next figure. Calculate:

a) The value of the power factor. Explain the meaning of the value of this power factor.

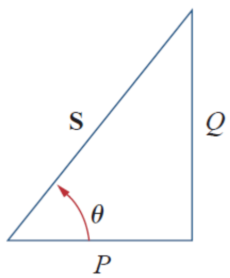
$$\frac{P(react)}{P(act)} = \frac{40}{200} = 0.2 = tg\phi$$

$$tg\phi = 0.2 \qquad \phi = 11.31°$$

cosϕ=0.98

b) The apparent power (S).

$$S = \sqrt{P^2 + Q^2} = 204\ W$$



Problem 4: (2 point not continuous evaluation)

Given a series RLC circuit with an AC voltage source. The current through the circuit is I(t)=(10 mA)·cos(10t+π/4). The inductor L of the circuit is a very long solenoid with radius R = 2 cm, and n=10 turns/m. Inside L there is a coaxial circular coil with N =10 turns of r = 1 cm radius, and 10 Ω resistance. Calculate:

a) The magnetic flux in the inside coaxial coil due to the magnetic field produced by the solenoid L.

$\Phi=N\pi r^2\mu_0 n I_0\cos(\omega t+\varphi_I)=10\cdot\pi\cdot(0.01)^2\cdot4\cdot\pi\cdot10^{-7}\cdot10\cdot10\cdot10^{-3}\cdot\cos(10t+\pi/4)=$

$= 3.95\cdot10^{-10}\cdot\cos(10t+\pi/4)$ Wb

b) The induced emf in the inside coaxial coil.

$\varepsilon=-d\Phi/dt=N\pi r^2\mu_0 n I_0\omega\sin(\omega t+\varphi_I)=10\cdot\pi\cdot(0.01)^2\cdot4\cdot\pi\cdot10^{-7}\cdot10\cdot10\cdot10^{-3}\cdot10\cdot\sin(10t+\pi/4)=$
$=3.95\cdot10^{-9}\cdot\sin(10t+\pi/4)$ V

c) The magnitude and direction of the current in the inside coaxial coil.

$I=\varepsilon/R=\quad N\pi r^2\mu_0 n I_0\omega\sin(\omega t+\varphi_I)/R=\quad 10\cdot\pi\cdot(0.01)^2\cdot4\cdot\pi\cdot10^{-7}\cdot10\cdot10\cdot10^{-3}\cdot10\cdot\sin(10t+\pi/4)/10=3.95\cdot10^{-10}\cdot\sin(10t+\pi/4)$ A

d) The capacitance C given that the resistance R is 5 Ω and the solenoid L has an inductance of 1 H, taking the reference phase of voltage the voltage source.

$\phi_Z=\mathrm{arctg}((\omega L-1/\omega C)/R)=\mathrm{arctg}((10\cdot1-1/10\cdot C)/5)=-45º$

$(10\cdot1-1/10\cdot C)/5=-1 \Rightarrow 10-1/10C=-5 \Rightarrow 15=1/10C \Rightarrow C=1/150\ F= 6.67\cdot10^{-3}\ F$

e) The voltage of the source as a function of time.

$Z=5+j(10-15)=5-5j\ \Omega$

$\boldsymbol{\varepsilon}=\mathbf{I}\cdot\mathbf{Z}= (10\cdot10^{-3}\ A)<45° \cdot ((50)^{1/2}\ \Omega)<-45° =0.0707 < 0°\ V$

$\varepsilon(t)=0.0707\cos(10t)$ V

Data: $\mu_0=4\pi\cdot10^{-7}$ Tm