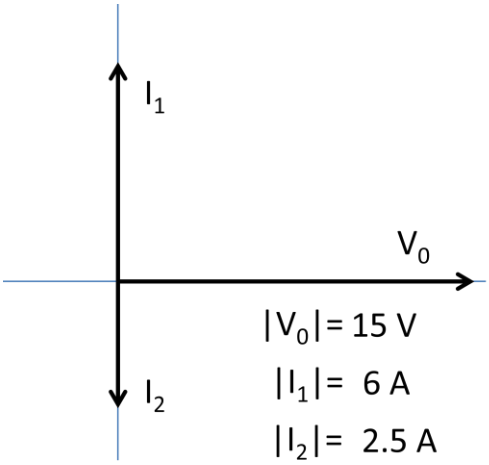
Theoretical Question 1: (1 point)

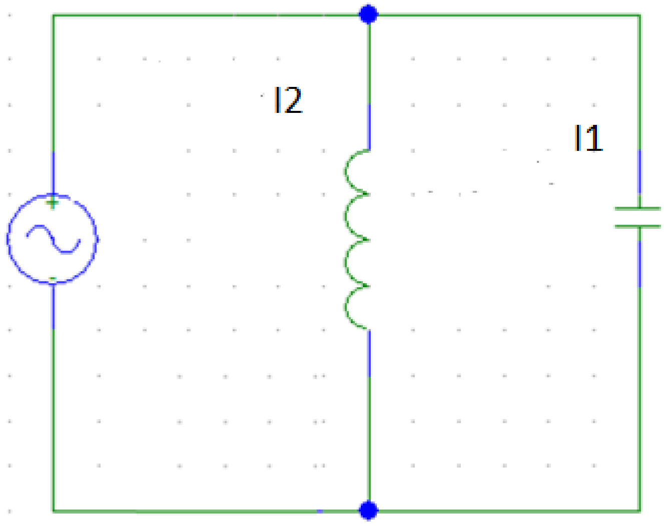
Given the following phasor diagram corresponding to an AC circuit with two parallel branches:

a) Draw its corresponding circuit diagram, show the current in every branch. (0.25 points)
b) Calculate the magnitude of the total current in the circuit and draw the associated phasor diagram (Use as reference V0). (0.25 points)
c) Draw the time-dependent graphs for V0, I1, I2 and Total current (ITOT). (0.5 Points)
NOTE: The values given for current and voltage are maximum values.



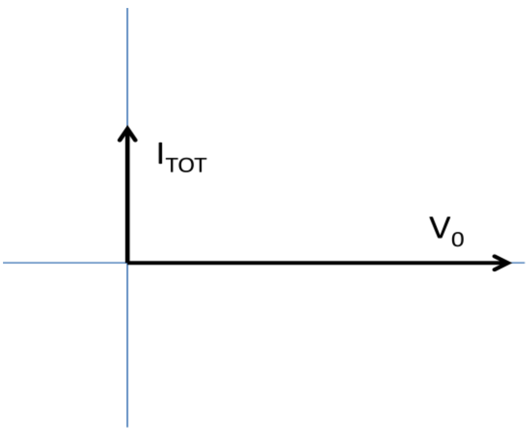
Solution:

a)

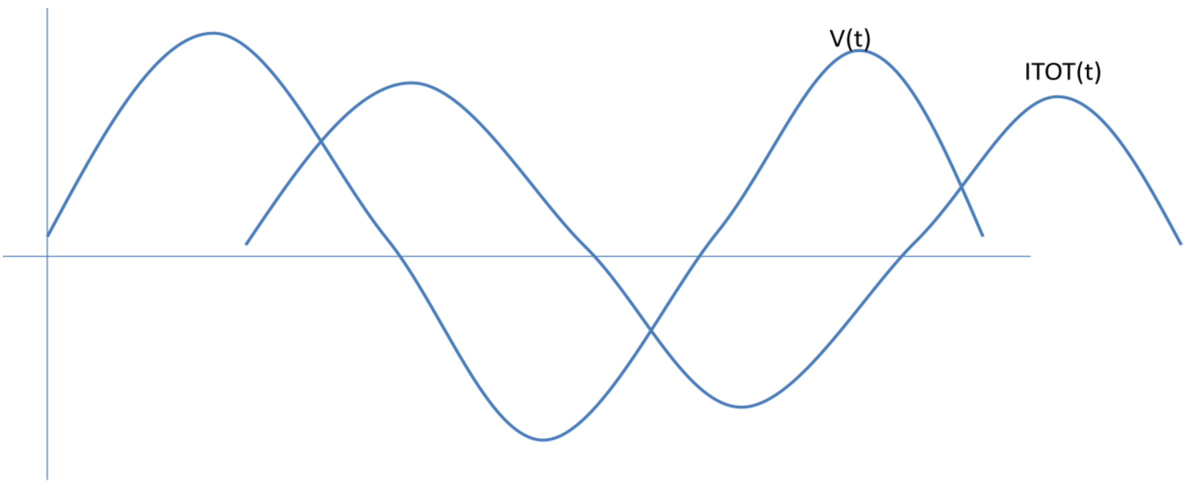


b)

$I_{TOT} = \vec{I_1} + \vec{I_2} = 6A < 90^o + 2.5A < -90^o = 3.5A < 90^o$

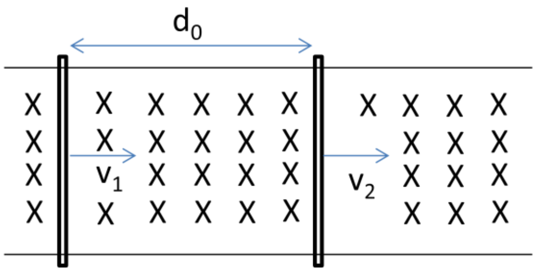


c)



Problem 1 (3 points):

Two parallel wires of negligible resistance are connected with two rods of length 10 cm and are moving with speeds v1 = 0.1 m / s and v2 = 0.2 m / s. In the loop there is a uniform and homogeneous magnetic field B = 10 mT as shown in the figure, calculate:



a) The magnetic flux through the loop defined by the rods and cables over time if they are initially separated by a distance d0 = 10 cm. (1 point)
b) The induced electromotive force over time. (1 Point)
c) If the resistivity of the rods is $\rho=1.7\cdot10^{-8}$ Ωm y its section $S=10^{-6}$ m^2, calculate the current in the loop over time. (1 Point)

Solution:

a)

$$\phi = BS = Bl(d_0 + vt) = 10 \cdot 10^{-3} \times 0.1 \times (0.1 + 0.1t) = 10^{-4} + 10^{-4} t \text{ Wb}$$

b)

$$\varepsilon = -\frac{d}{dt}\phi = -\frac{d}{dt}(10^{-4} + 10^{-4} t) = -10^{-4} V$$

c)

$$R = \rho \frac{L}{S} = 1.7 \cdot 10^{-8} \frac{0.1}{10^{-6}} = 1.7 \cdot 10^{-3} \Omega$$

R_{TOT}=Rx2=$3.4 \cdot 10^{-3}\Omega$.

$$\varepsilon = IR_{TOT} \Rightarrow I = \frac{|\varepsilon|}{R_{TOT}} = \frac{10^{-4}}{3.4 \cdot 10^{-3}} = 0.0294A = 29.4mA$$, counterclockwise

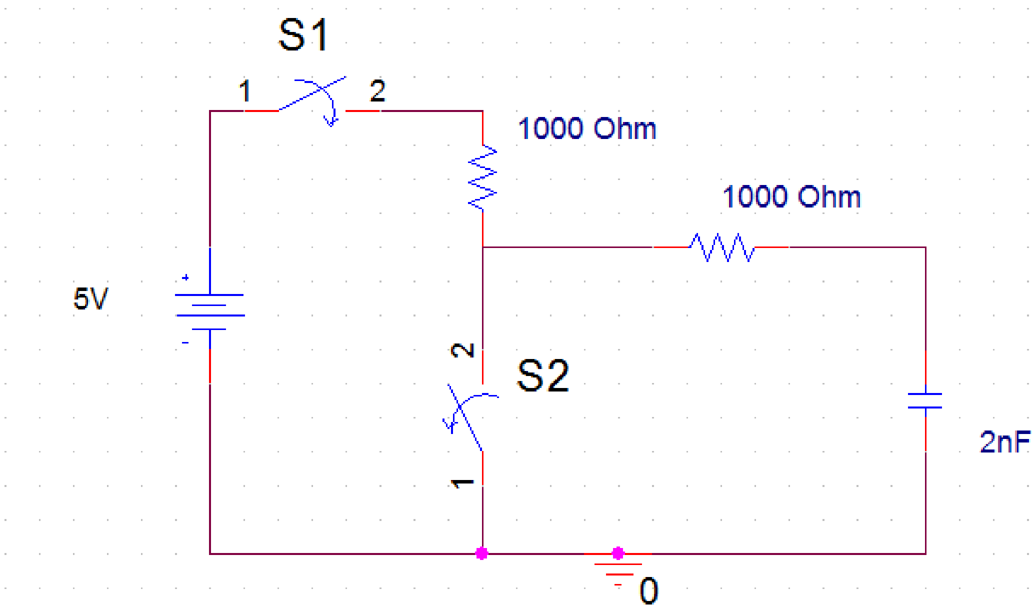
Problem 2: (3 points)

Given the circuit of figure where the following events and conditions occurs:

t <= 0	The 2 nF capacitor is uncharged, Switch S1 is open, Switch S2 is closed.
t = 0	Switch S1 is closed, S2 continued closed.
t = 1000 ns	Switch S2 opens, S1 continued closed.
t = 10000 ns	Switch S2 is closed, S1 continued closed.

(a) Study the behavior of the voltage (equations for V_C (t)) across the terminals of the 2 nF capacitor along time interval [ti = 0 ns, tf = 24000 ns]. (2 points)

(b) Find the numeric value of this voltage at t = 0 ns, t = 1000 ns, t = 10000 ns, and t = 24000 ns (1 Point)



Solution:

(a)

t <= 0	$V_C(t) = 0$ V.
t = 0 – 1000_- nsec	$V_C(t) = 0$ V.
t = 1000_+ nsec – 10000_- nsec	$\tau_1 = (1000 + 1000) \cdot 2 \cdot 10^{-9}$ s $V_c(t) = 5\left(1 - e^{-\frac{1}{\tau_1}(t - 1000 \cdot 10^{-9})}\right)$ V.
t = 10000_+ nsec – 24000 nsec	$\tau_2 = 1000 \cdot 2 \cdot 10^{-9}$ s $V_c(t) = 4.47\, e^{-\frac{1}{\tau_2}(t - 10000 \cdot 10^{-9})}$ V.

(b)

t = 0 nec , V_C = 0 V.

t = 1000 nsec, V_C = 0 V.

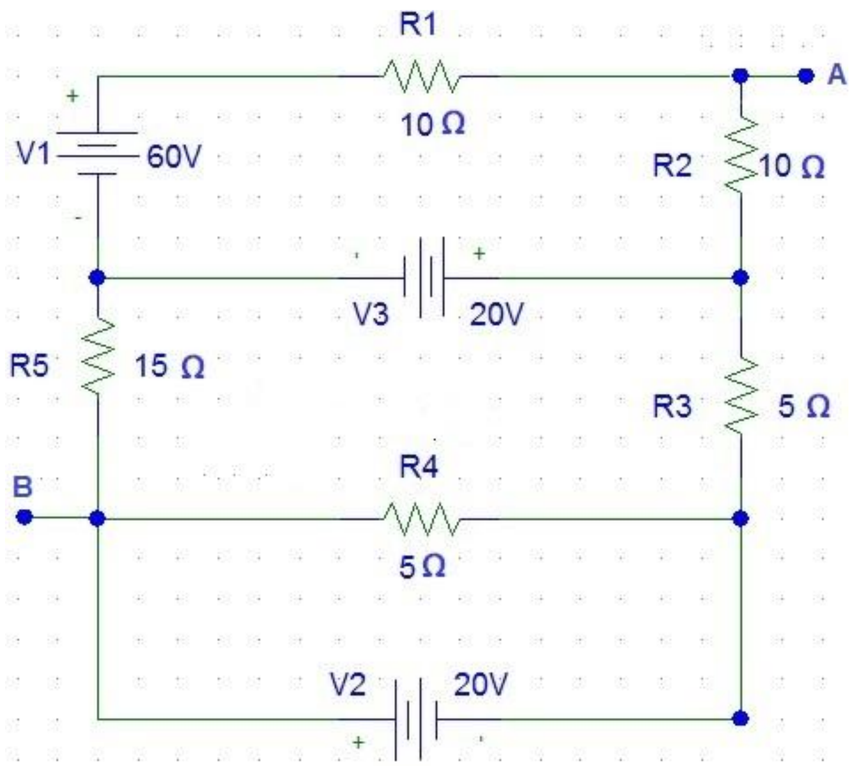
t = 10000 nsec, V_C = 4,47 V.

t = 24000 nsec , V_C = 0,0046 V.

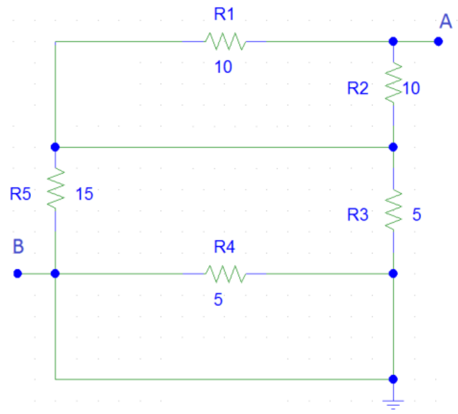
Problem 3: (3 points)

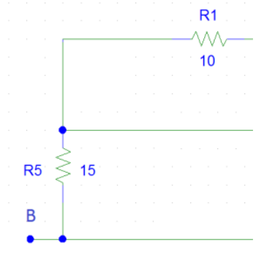
Given the circuit of the figure:

a) Calculate the Thevenin`s equivalent of this circuit across terminals A and B (1.5 points)
b) Calculate also the current through the resistor R4. (1.5 points)



Solution:



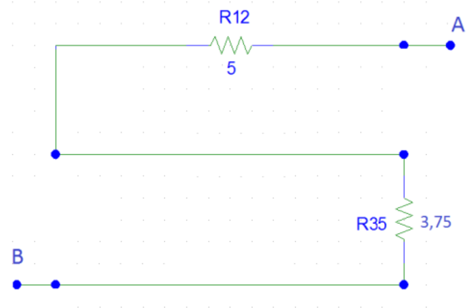


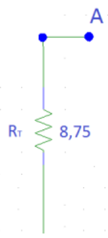
$$\frac{1}{R12} = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$$

R12 = 5 Ω

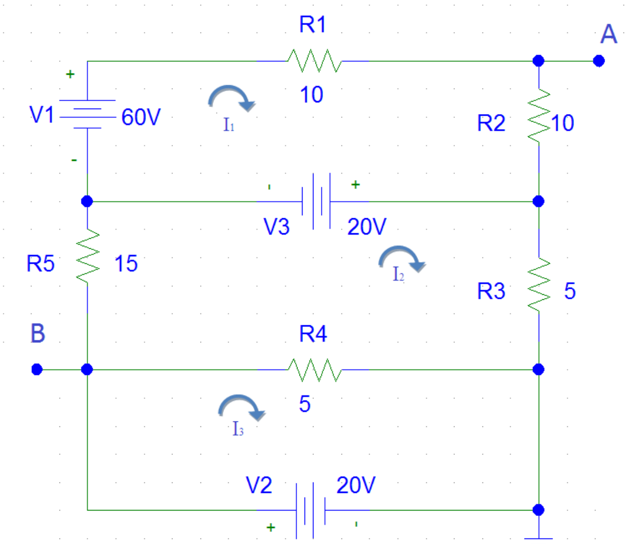
$$\frac{1}{R35} = \frac{1}{5} + \frac{1}{15} = \frac{4}{15}$$

R35 = 3,75Ω





R_{Th} = R12+R35 = 5 + 3,75 = 8,75 Ω



$20\ I_1 = 40,\ I_1 = 2\ A$

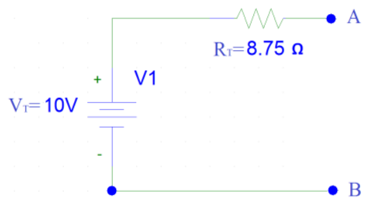
$$\left.\begin{array}{r} 25\ I_2 - 5I_3 = 20 \\ 5(I_3 - I_2) = 20 \end{array}\right\}$$

I_2 = 2A

I_3 = 6A

$V(R5) = R5 \cdot I_2 = 15 \cdot 2 = 30$ V
$V(R1) = R1 \cdot I_1 = 10 \cdot 2 = 20$ V

$V_{Th} = V1 - V(R1) - V(R5) = 60 - 20 - 30 = 10$ V



$I_{R4} = I_3 - I_2 = 2A$