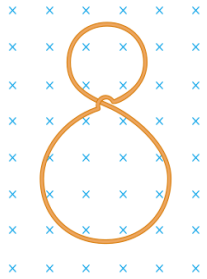
Problem 1 (5 points)

A piece of wire is shaped into an number "8", as shown in next figure. The radius of the upper circle is 2.00 cm and that of the lower circle is 3.00 cm. The wire has a uniform resistance per unit length of 2.00 ohms/m. A uniform magnetic field is applied perpendicular to the plane of the two circles, in the direction shown (into the page). The magnetic field is increasing at a constant rate of 1.00 T/s. (a) Find the induced e.m.f in the upper and lower loops, and (b) Find the magnitude and direction of the induced current in the wire.

	Length	Resistance	Area	Radius
Upper loop	L_u	R_u	A_u	0.02 m
Lower loop	L_L	R_L	A_L	0.03 m



$L_u = 2\pi \cdot 0.02\ m = 0.13\ m$ $\quad R_u = 0.13\ m \times 2\ \Omega/m = 0.26\ \Omega$

$L_L = 2\pi \cdot 0.03\ m = 0.19\ m$ $\quad R_L = 0.19\ m \times 2\ \Omega/m = 0.38\ \Omega$

$A_u = \pi \cdot (0.02)^2\ m^2 = 1.26 \cdot 10^{-3}\ m^2$ $\quad A_L = \pi (0.03)^2\ m^2 = 2.83 \cdot 10^{-3}\ m^2$

(a) $|e.m.f_{upper}| = \left| d\frac{\Phi_u}{dt} \right| = \left| \frac{d\ 1\ T/s \cdot t \cdot 1.26 \cdot 10^{-3}\ m^2}{dt} \right| = 1.26\ mV$

$|e.m.f_{Lower}| = \left| d\frac{\Phi_L}{dt} \right| = \left| \frac{d\ 1\ T/s \cdot t \cdot 2.83 \cdot 10^{-3}\ m^2}{dt} \right| = 2.83\ mV$

(b) $I_u = \frac{1.26\ mV}{0.26 + 0.38\ \Omega} = \frac{1.26\ mV}{0.64\ \Omega} = 1.97\ mA$ c-c. wise

$I_L = \frac{2.83\ mV}{0.64\ \Omega} = 4.42\ mA$ c. c. wise

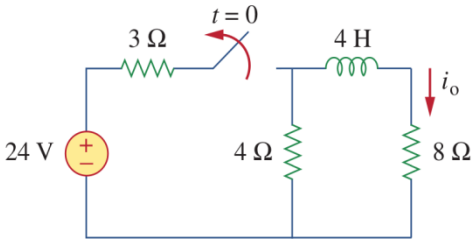
I_u, I_L } Induced Currents

Total current in the wire $= 4.42\ mA - 1.97\ mA = 2.45\ mA$

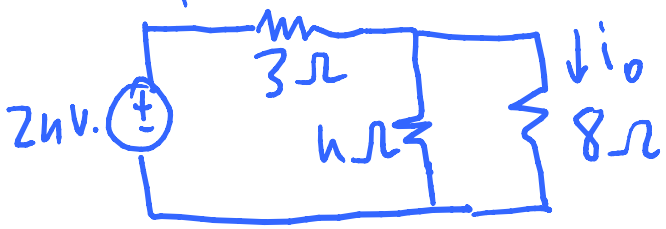
Clockwise in the upper loop which becomes Counter clockwise in the lower loop

Problem 2 (5 points)

Given the circuit in next figure where for t < 0 s the switch has been closed during 1 hour. At t =0 s the switch is open. (a) Find i_0 (t) for t >= 0 s, and (b) Find the voltage as function of time across the 4 ohm resistor.



(a) When the switch is closed the equivalent circuit is



$$i_{o(t<0)} = \frac{24\,V \cdot \frac{4\,\Omega \| 8\,\Omega}{3\,\Omega + (4\,\Omega \| 8\,\Omega)}}{8\,\Omega} = 1.41\,A$$

For $t>0$ we have a source-free RL circuit with $\tau = \frac{L}{R} = \frac{4\,H}{4\,\Omega + 8\,\Omega} = \frac{1}{3}\,s$

$$i_o(t) = i_{o(t<0)} \cdot e^{-3t}\,A = 1.41 \cdot e^{-3t}\,A$$

(b) $$V_{4\Omega}(t) = 4\,\Omega \cdot i_o(t) = 5.64 \cdot e^{-3t}\,V$$