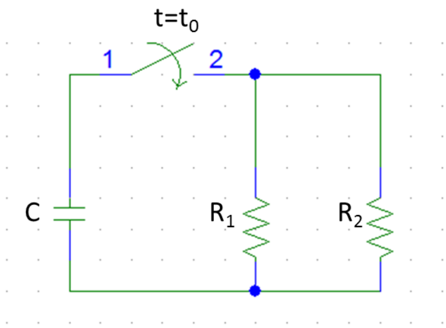
Question 1: (1 point)

Given a capacitor C charged with an initial charge Q_0 connected to two resistors R_1 and R_2 as shown in Fig. At time $t = t_0$ s the switch is closed so that the capacitor is discharged through the resistors.



Calculate the next variables, as a function of the parameters Q_0, C, R_1, R_2 and t_0:

a) The voltage across the capacitor as a function of time.
b) Maximun current.
c) The current through each resistor as a function of time.

Solution:

a) $$V(t) = \frac{Q_0}{C} e^{-\frac{t-t_0}{R_1 R_2 C}(R_1+R_2)}$$

b) $$I_{MAX} = \frac{Q_0}{\tau} = \frac{Q_0(R_1+R_2)}{R_1 R_2 C}$$

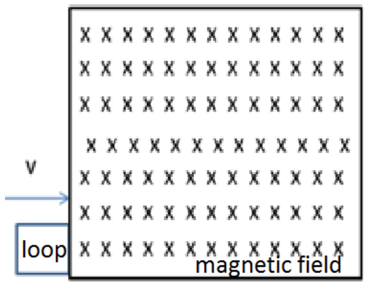
c)

$$I_1(t) = \frac{V(t)}{R_1} = \frac{Q_0}{R_1 C} e^{-\frac{t-t_0}{R_1 R_2 C}(R_1+R_2)}$$
$$I_2(t) = \frac{V(t)}{R_2} = \frac{Q_0}{R_2 C} e^{-\frac{t-t_0}{R_1 R_2 C}(R_1+R_2)}$$

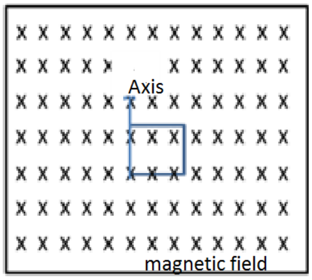
Problem 1 (3 points):

Given a finite magnetic field of 0,6T in a rectangular space of dimensions 20 m x 10 m, direction perpendicular to the paper, and a square loop of 10 cm side and resistance 10 Ω. Given the next cases:

- Case 1: The loop moves horizontally to the right along +X axis in the magnetic field from the initial position of the next figure with a constant speed of 2 m/s.



- Case 2: The loop moves vertically along +Y direction from the initial position of the previous figure with a constant speed of 1 m/s.
- Case 3: The loop is introduced into the magnetic field and rotates inside around its vertical left axis with an angular velocity of 2 rad/s, as shown in next figure.



Calculate:

a) The induced emf in the loop for each case (2 points)
b) The magnitude and direction of the induced current in the loop (1 point)

Solution:

a)

- Case 1
  - t=0, t= 0,1/2 s

$$\varepsilon = -B * l * v = -0{,}6 * 0{,}1 * 20 = -0{,}12\ V$$

  - t=0,05 s, t= (20-0,2)/2=9,9s → ε=0V
  - t=9,9s→ t=9,9+ 0,05

$$\varepsilon = B * l * v = 0{,}6 * 0{,}1 * 20 = 0{,}12\ V$$

  - t=9,95→∞ ε=0V
- Case 2 ε=0V
- Case 3 t=4,95s→∞ $\varepsilon = \omega BS\ Sen\ (\omega t) = 2 * 0{,}6 * 0{,}01\ sen\ 2t$ = 0,012 sin (2t) v

b)

X:

t=0→0,05s ; I=-0,012A counterclockwise
t=0,05→ 9,9 s; I=0A

t=9,9 s → 9,95 I=0,012A clockwise
t=9,95→ ∞ I=0

Y:
t=0→ ∞ I=0

Rotating: I= = 0,0012 sin (2t) A , alternating

Problem 2: (3 points)

Given the circuit in next figure where the voltage source supplies $v(t)=8\,\mathrm{sen}\left(t+\pi/4\right)\ \mathrm{V}$.

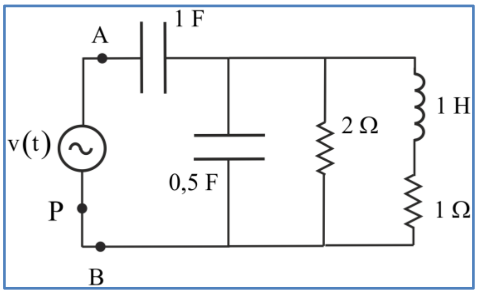
Calculate:

a) The equivalent impedance across nodes A and B. (1 point)

b) The current in the circuit (0.75 points).

c) The average power (0.75 points)

d) What element has to be connected across nodes P and B replacing the previous short-circuit so that the voltage source supply the maximum current? (0.5 points)



Solution:

a)

$$\vec{Z}_T = \vec{Z}_{C^*} + 1/\vec{Y}_P = 1 - j \;= \sqrt{2}\angle -45^\circ\,\Omega$$

b)

$$I_T = V/Z_T = (8_{\,-45º})/\sqrt{2}_{\,-45º} = 4\sqrt{2}_{\,0º}\ \mathrm{A}$$

c)

$$P_a = \left(\frac{1}{2}\right) I_0 V_0 \cos\varphi = {I_0}^2 R = \left(\frac{1}{2}\right) 4\cdot\sqrt{2}\cdot 8\cdot\frac{\sqrt{2}}{2} = \left(\frac{1}{2}\right)(4\cdot\sqrt{2})^2\cdot 1 = 16\,W$$

$$\cos\varphi = \cos 45^0 = \sqrt{2}/2$$

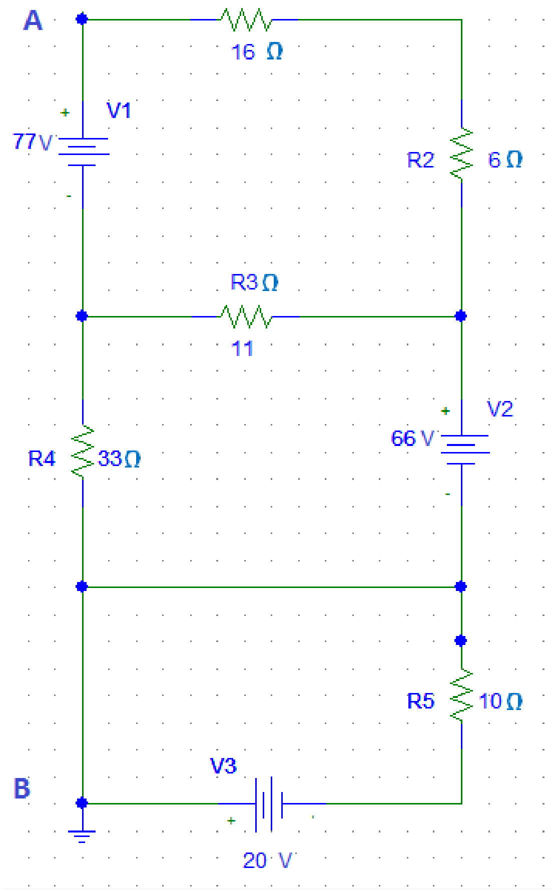
d)

$\mathrm{Im}[\vec{Z}_T] = 0$ $L = 1H$ or Z=1+j

Problem 3: (3 points).

Given the circuit in next figure.

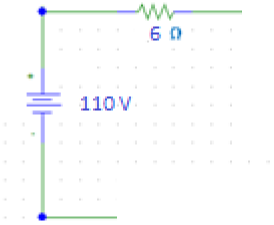
1. Calculate and draw the Thévenin equivalent across nodes A and B (2p)
2. Calculate the current through R2 and R5 (0,5 p)
3. Calculate and draw the Norton equivalent (0,5 p)



Solution:

1. $$V_{AB} = V_1 + R_4 \cdot I_2 = 77 + 33 \cdot 1 = 110\,V$$

$$R_{AB} = 6\,\Omega$$



2.

$$I_{R2} = I_1 = 2\ A$$
$$I_{R5} = I_3 = 2\ A$$

3.

$I_N = \frac{V_{Th}}{R_{Th}} = \frac{110}{6} = 18{,}3\ A$, R_N = 6 ohm