PPCE

26/06/2014

Theoretical Question 1 (0.5 points if continuous assessment / 1 points if no continuous assessment)

Given a region of space with two magnetic fields B1 and B2, both in the same direction. B1 field is constant and its magnitude is 10 mT. B2 field is variable according to the expression 10·sin(ωt) mT. In the same region there are also two coils S1 and S2. The normal vector corresponding to S1 loop is constant and equal to 0.5 m^2 while the normal vector corresponding to S2 loop is variable according to the expression 0.5·sin (ωt) m^2.

Evaluate if the following statements are true or false. Justify your answer:

a) The S1 coil inside the B2 field produces Direct Current. (0.2 points continuous points and 0.1 points in non-continuous)

b) The S2 coil inside the B1 field produces Alternating Current. (0.2 points continuous points and 0.1 points in non-continuous)

c) The S2 inside the B2 field coil produces a Time Varying Current. (0.2 points continuous points and 0.1 points in non-continuous)

d) The S2 coil inside the B2 field produces Alternating Current. (0.2 points continuous points and 0.1 points in non-continuous)

e) The S1 coil inside the B1 field produces Direct Current. (0.2 points continuous points and 0.1 points in non-continuous)

Solution

a) False.

$$\phi = (0.5m^2)\cdot(10mT)\sin(\omega t)$$

$$\varepsilon = -\frac{d\phi}{dt} = -(0.5m^2)\cdot(10mT)\omega\cos(\omega t)$$

b) True.

$$\phi = (0.5m^2)\sin(\omega t)\cdot(10mT)$$

$$\varepsilon = -\frac{d\phi}{dt} = -(0.5m^2)\omega\cos(\omega t)\cdot(10mT)$$

c) True.

$$\phi = (0.5m^2)\sin(\omega t)\cdot(10mT)\cdot\sin(\omega t)$$

$$\varepsilon = -\frac{d\phi}{dt} = -(0.5m^2)\omega\cos(\omega t)\cdot(10mT)\sin(\omega t) - (0.5m^2)\cdot\sin(\omega t)(10mT)\cdot\omega\cos(\omega t) =$$

$$= -2\omega(0.5m^2)(10mT)\cdot\sin(\omega t)\cdot\cos(\omega t)$$

d) True.

$$\phi = (0.5m^2)\sin(\omega t)\cdot(10mT)\cdot\sin(\omega t)$$

$$\varepsilon = -\frac{d\phi}{dt} = -(0.5m^2)\omega\cos(\omega t)\cdot(10mT)\sin(\omega t) - (0.5m^2)\cdot\sin(\omega t)(10mT)\cdot\omega\cos(\omega t) =$$

$$= -2\omega(0.5m^2)(10mT)\cdot\sin(\omega t)\cdot\cos(\omega t)$$

e) False.

$$\phi = (0.5m^2)(10mT)$$

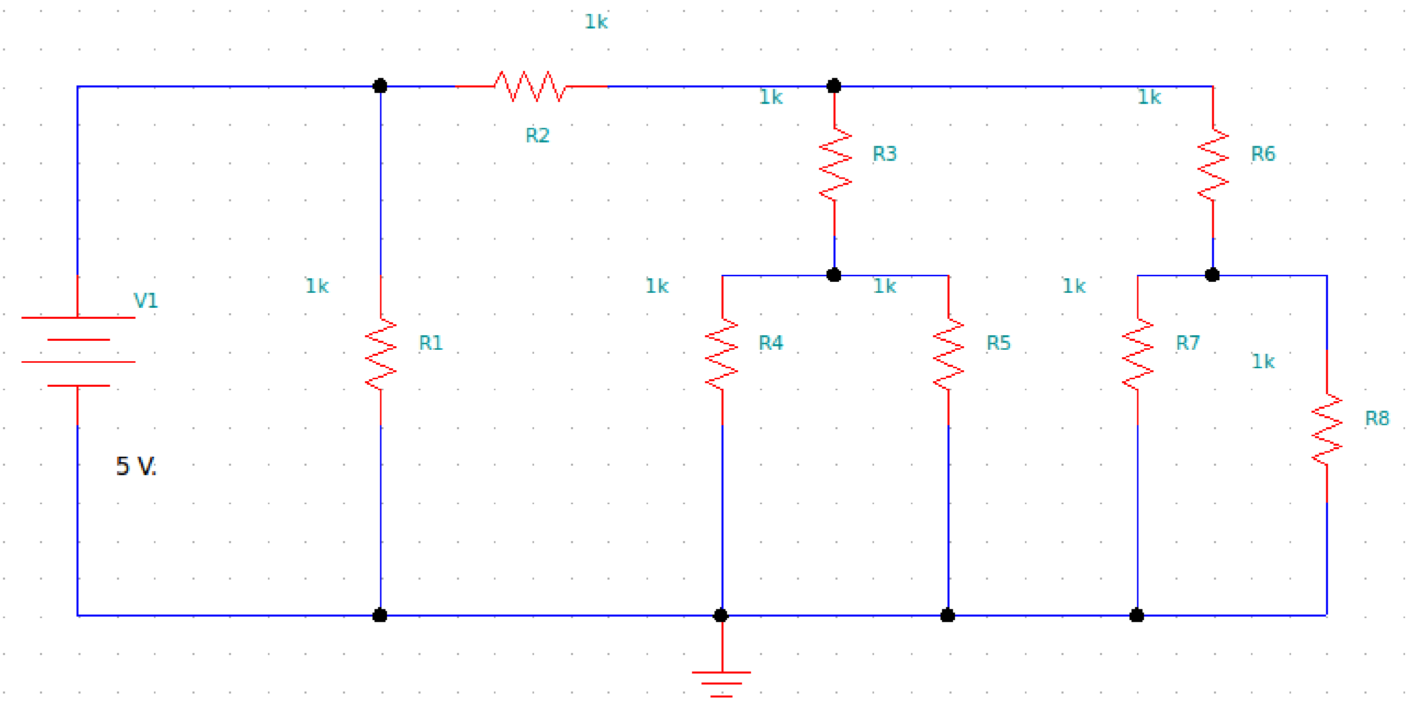
$$\varepsilon = -\frac{d\phi}{dt} = 0$$

Problem 1: (1.5 points if continuous assessment / 2 points if no continuous assessment)

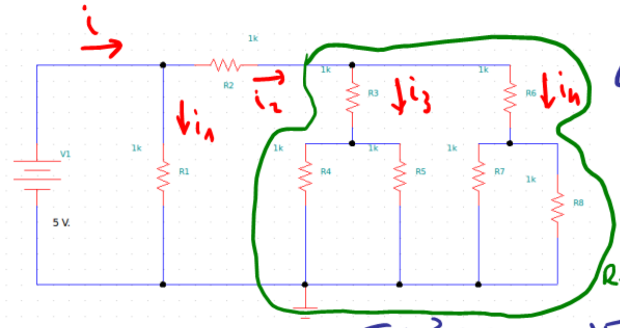
Given the circuit in figure (All resistors 1 KΩ).

(a) Use equivalent resistance and voltage division to find voltage across R2 terminals. (0.75 points if continuous, 1 point if not continuous)

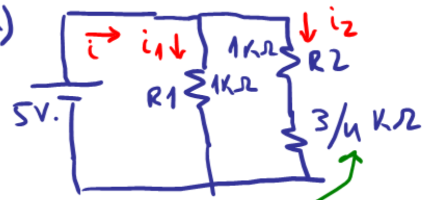
(b) Use equivalent resistance, voltage division and current division to find the power absorbed by R6. (0.75 points continuous. 1 points if not continuous)



Solution



(a)



$R_{eq} = \frac{3}{2}\,k\Omega \,||\, \frac{3}{2}\,k\Omega = \frac{3}{4}\,k\Omega$

$$V_{R2} = \frac{1\,k\Omega \cdot 5V}{\frac{3}{4}\,k\Omega + 1\,k\Omega} = \frac{20}{7}\,V.$$

(b)

$$V_{R6-R7||R8} = \frac{5V \cdot \frac{3}{4}\,k\Omega}{\frac{3}{4}\,k\Omega + 1\,k\Omega} = \frac{15}{7}\,V.$$

$$V_{R6} = \frac{\frac{15}{7}\,V \cdot 1\,k\Omega}{1\,k\Omega + \frac{1}{2}\,k\Omega} = \frac{10}{7}\,V.$$

$$i = \frac{5V.}{7/11\,k\Omega} = \frac{55}{7}\,mA$$

$$i_2 = \frac{i \cdot R_1}{R_1 + (R_2 + \frac{3}{4}\,k\Omega)}$$

$$i_2 = \frac{55/7 \cdot 1\,k\Omega}{2\,k\Omega + \frac{3}{4}\,k\Omega}\,mA = \frac{55/7}{11/4}\,mA = \frac{20}{7}\,mA$$

$$i_4 = i_{R6} = \frac{i_2 \cdot \frac{3}{2}\,k\Omega}{\frac{3}{2}\,k\Omega + \frac{3}{2}\,k\Omega} = \frac{20}{14}\,mA$$

$$P_{R6} = i_{R6} \cdot V_{R6} = \frac{10}{7}\,V \cdot \frac{20}{14}\,mA = \frac{200}{7 \cdot 14}\,mW$$

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Problem 2: (1.5 points if continuous assessment / 2 points if no continuous assessment)

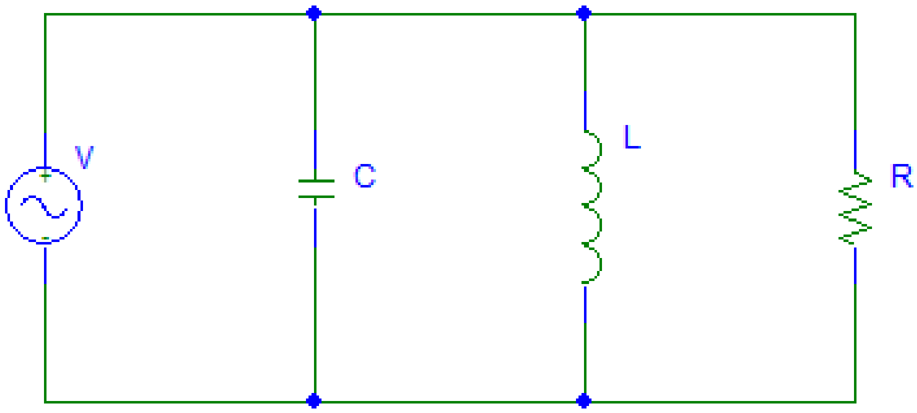
Given the circuit of the figure, composed of a resistor of 500 Ω, 80 mH inductance, a 0.5 uF capacitor and a AC source that provides an rms voltage of 12-j6 V. $\omega = 500$ rad/s. Determine:

a) The equivalent impedance. (0.375 points in continuous, 0.5 points if not continuous)

b) The complex expression of the rms current through impedance. (0.375 points in continuous, 0.5 points if not continuous)

c) The average power dissipated. (0.375 points in continuous 0.5 points if not continuous)

d) What impedance should be connected in series with the source for supplying the maximum power?. (0.375 points in continuous 0.5 points if not continuous)



Solution

a)

$$\frac{1}{\vec{Z}_{eq}} = \frac{1}{\vec{Z}_1} + \frac{1}{\vec{Z}_2} + \frac{1}{\vec{Z}_3}$$

$$\vec{Z}_1 = -j\frac{1}{C\omega};\ \vec{Z}_2 = jL\omega;\ \vec{Z}_2 = R$$

$$\vec{Y}_{eq} = \frac{1}{\vec{Z}_{eq}} = jC\omega - j\frac{1}{L\omega} + \frac{1}{R}$$

$$\vec{Y}_{eq} = \frac{1}{\vec{Z}_{eq}} = j0,5\cdot 10^{-6}\,\mathrm{F}\;500\;\mathrm{rad/s} - j\frac{1}{80\cdot 10^{-3}\,\mathrm{H}\;500\;\mathrm{rad/s}} + \frac{1}{500\;\Omega} = 0,002 - j0,02475\;\Omega^{-1}$$

$$\vec{Z}_{eq} = \frac{1}{\vec{Y}_{eq}} = \frac{1}{0,002 - j0,02475\;\Omega^{-1}} = 3,2438 + j40,1419\;\Omega$$

b)

$$\vec{I}_e = \frac{\vec{\varepsilon}_e}{\vec{Z}_{eq}} = \frac{12 - j6\;\mathrm{V}}{3,2438 + j40,1419\;\Omega} = -0,1245 - j0,309\;\mathrm{A} = 0,333\angle\text{-}114^{\circ}\;\mathrm{A}$$

c) $P = I_e^2 R$. $R_{eq} = \mathrm{Re}[\vec{Z}_{eq}] = 3,2438\;\Omega$.

$$P = I_e^2 R_{eq} = \left(0,333\;\mathrm{A}\right)^2 3,2438\;\Omega = 0,36\;\mathrm{W}$$

d) $X_L = \left|\mathrm{Im}[Z_{eq}]\right|$.

$$L = \frac{\left|\mathrm{Im}[Z_{eq}]\right|}{\omega} = \frac{0,309\;\;\Omega}{500\;\mathrm{rad/s}} = 6,18\cdot 10^{-4}\;\;\mathrm{H}$$

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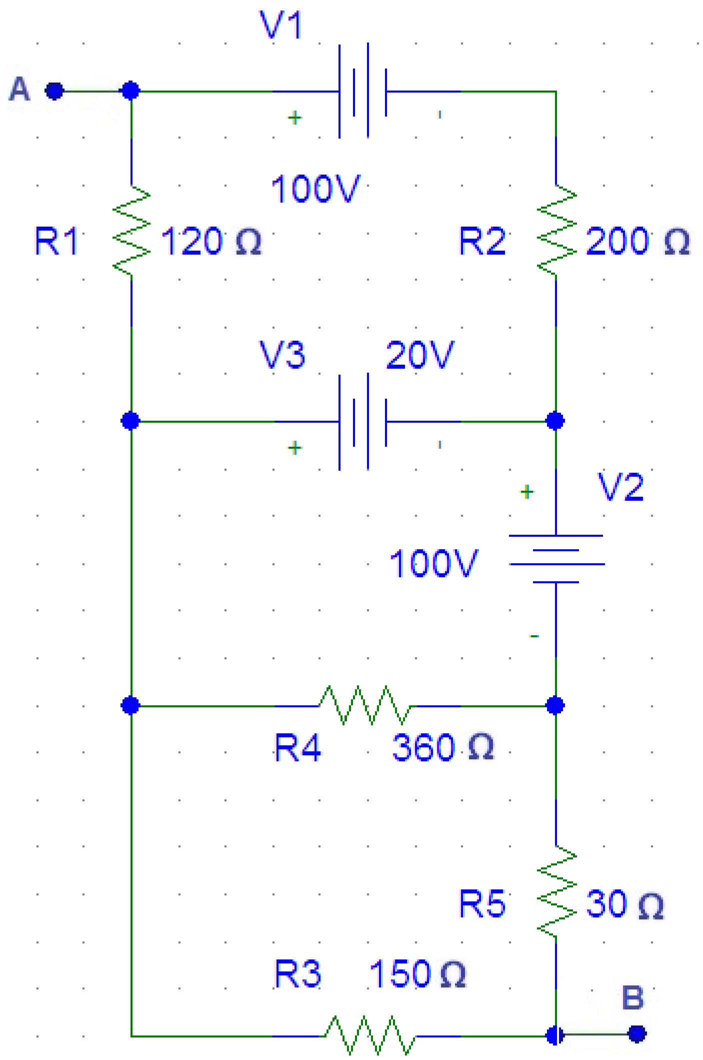
Problem 3: (1.5 points if continuous assessment / 2 points if no continuous assessment)

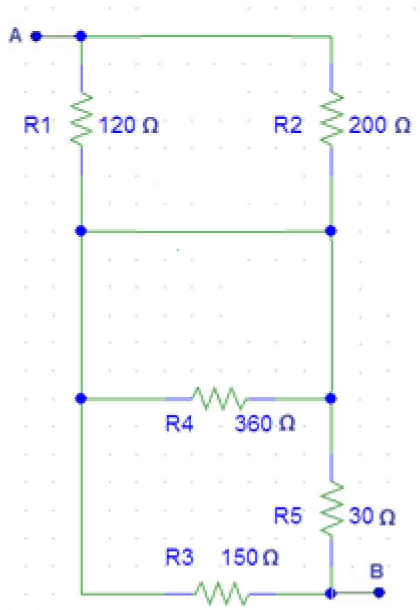
In the circuit of Figure:

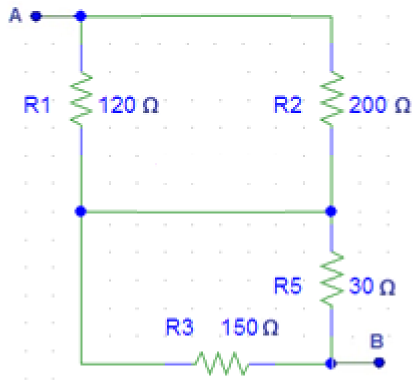
a) Calculate the Thevenin equivalent of the circuit across terminals A and B (No Continuous: 1 point, Continuous: 0.75 point)

b) Calculate the value and direction of the current flowing through the resistor R4 when the A and B terminals are open circuit (No Continuous: 1 point, Continuous: 0.75 point)



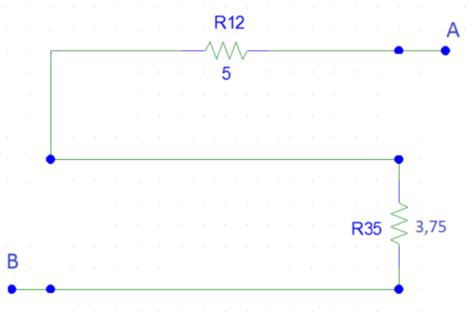
Solution





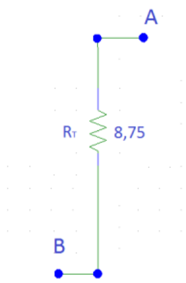
$$\frac{1}{R12}=\frac{1}{120}+\frac{1}{200}=\frac{1}{75}$$

R12 = 75 Ω



$$\frac{1}{R35} = \frac{1}{30} + \frac{1}{150} = \frac{1}{25}$$

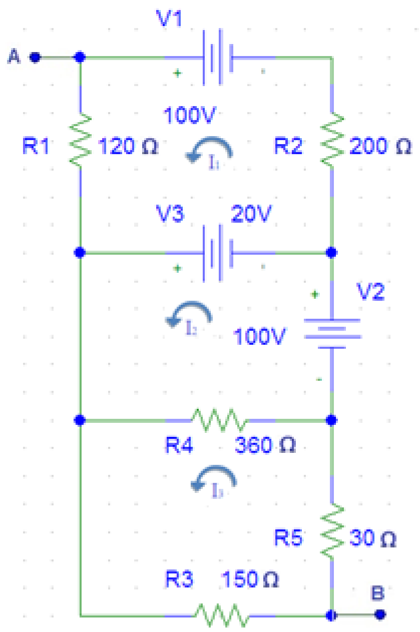
R35 = 25 Ω



R_T = R12+R35 = 75 + 25 = 100 Ω

Using Mesh Analisys:

$320\ I_1 = 80,\ I_1 = 0{,}25$ A



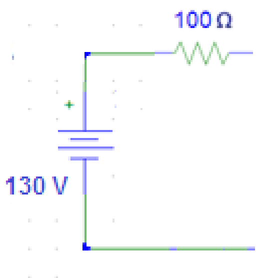
$$\left.\begin{aligned} 360\ (I_2 - I_3) &= 120 \\ 360\ I_2 = 540\ I_3) \end{aligned}\right\}$$

I_2 = 1 A
I_3 = 2/3 A

V(R2) = R2 · I_1 = 200 · 0,25 = 50 V
V(R5) = R5 · I_3 = 30 · 2/3 = 20 V

VAB= V_T = V1 – V(R2) + V2 – V(R5) = 100 – 50 + 100 – 30 = 130 V

Thevenin Eq:



I in R4 = $I_2 - I_3$, $I_{R4} = 1 - 2/3 = 1/3$ A

Question 2: (1 point if no continuous assessment)

An inductor L and a capacitor C are connected in series to an alternating voltage source of rms voltage 100 V. When ω is 106 rad/s, the rms current circulating in it is 0.250 A modulus. When ω is changed to 2.5·106 rad/s, then the modulus of the rms current is 0.637 A. What are the values of L and C?

Solution

$$\vec{Z}_{eq} = j\varpi L - \frac{j}{\varpi C}$$

Por tanto, trabajando sólo en módulo:

$$|\vec{\varepsilon}| = |\vec{I}| \cdot |\vec{Z}_{eq}| = |\vec{I}| \cdot \left| j\varpi L - \frac{j}{\varpi C} \right| = |\vec{I}| \cdot \left| \varpi L - \frac{1}{\varpi C} \right|$$

$$|\vec{\varepsilon}_1| = |\vec{I}_1| \cdot \left| \varpi_1 L - \frac{1}{\varpi_1 C} \right|$$

$$|\vec{\varepsilon}_2| = |\vec{I}_2| \cdot \left| \varpi_2 L - \frac{1}{\varpi_2 C} \right|$$

$$100 = 0.25 \cdot \left| 10^6 L - \frac{1}{10^6 C} \right|$$

$$100 = 0.637 \cdot \left| 2.5 \cdot 10^6 L - \frac{1}{2.5 \cdot 10^6 C} \right|$$

$$\frac{100}{0.25} = \left| 10^6 L - \frac{1}{10^6 C} \right|$$

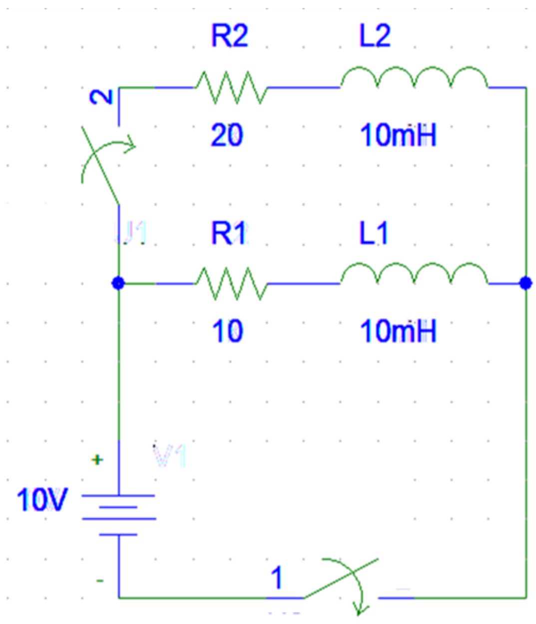
$$\frac{100}{0.637} = \left| 2.5 \cdot 10^6 L - \frac{1}{2.5 \cdot 10^6 C} \right|$$

L=1.3 mH

$1/C = 4 \cdot 10^8\ F^{-1}$ ➔ **C=2.5 nF**

Problem 4: (2 points if continuous assessment)

Given the circuit of Fig. For t <0 the two switches are open. At time t = 0 s to switch 1 is closed, and switch 2 remains open. At time t = 1.5 ms switch 2 is closed, and switch 1 remains closed.



a) What is time constant (tau) for R1-L1 branch? For the R2-L2 branch? (0.25 points)

b) What current flows through L1 at t = 1 ms? And through L2? Through the battery? (0.5 points)

c) What current flows through L1 at t = 2 ms? And through L2? Through the battery? (0.5 points)

d) What current flows through L1 at t = 10ms? And through L2? Through the battery? (0.25 points)

e) Draw a time graph showing the current through L1, L2 and battery from t = -10 to t = 10 s. (0.5 points)

Solution

a) τ=L/R

$\tau 1$=L1/R1=10 mH/10 Ω= 1 ms
$\tau 21$=L2/R2=10 mH/20 Ω= 0.5 ms

b)

$I_{L1}(t)=I_{01}(1-e^{-t/\tau 1})=(10V/10\ \Omega)(1-e^{-t/1ms})=1A(1-e^{-t/1ms})$

$I_{L1}(t=1\ ms)= =1A(1-e^{-1\ ms/1ms})=0.632\ A=I_V(t=1\ ms)$

$I_{L2}(t=1\ ms)=0$

$I_{Batt}(t=1\ ms)= 0.632\ A$

c)

$I_{L2}(t)=I_{02}(1-e^{-(t-1.5\ ms)/\tau 2})=(10V/20\ \Omega)(1-e^{-(t-1.5\ ms)/0.5ms})=$
$=0.5\ A(1-e^{-(t-1.5\ ms)/0.5ms})$

$I_{L2}(t=2\ ms)=0.5\ A(1-e^{-(2\ ms-1.5\ ms)/0.5ms})=0.316\ A$

$I_{L1}(t=2\ ms)= =1A(1-e^{-2\ ms/1ms})=0.865\ A$

$I_{Batt}(t=2\ ms)$=1.181 A.

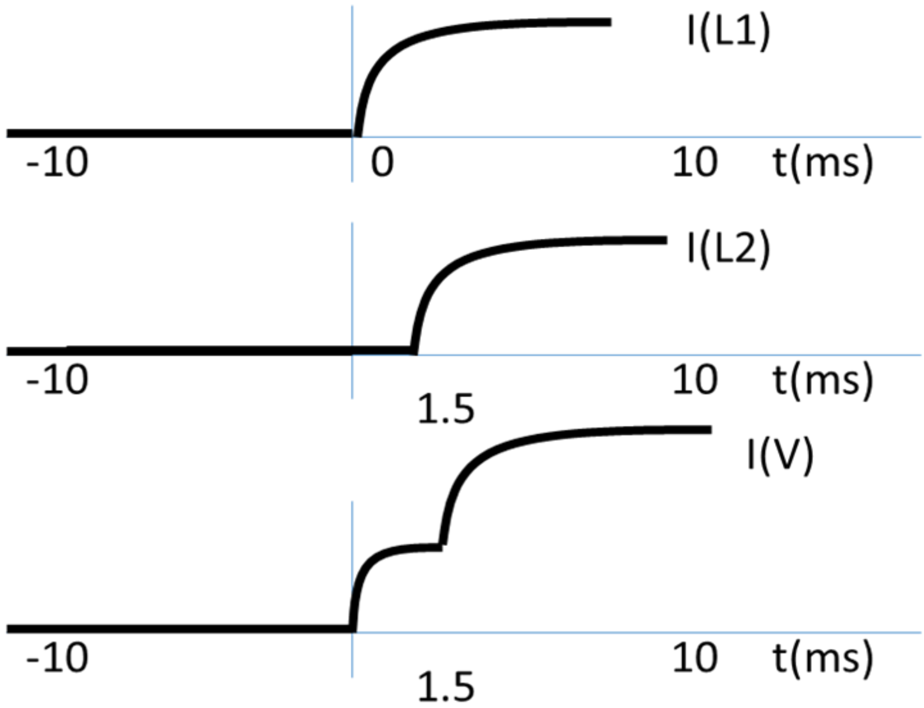
d)

$I_{L2}(t=10\ ms)=0.5\ A$

$I_{L1}(t=10\ ms)= =1A$

$I_{Batt}(t=10\ ms)$=1.5 A

e)


I(L1)
-10
0
10
t(ms)
I(L2)
-10
1.5
10
t(ms)
I(V)
-10
1.5
10
t(ms)