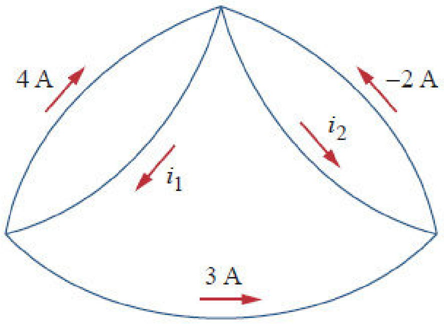
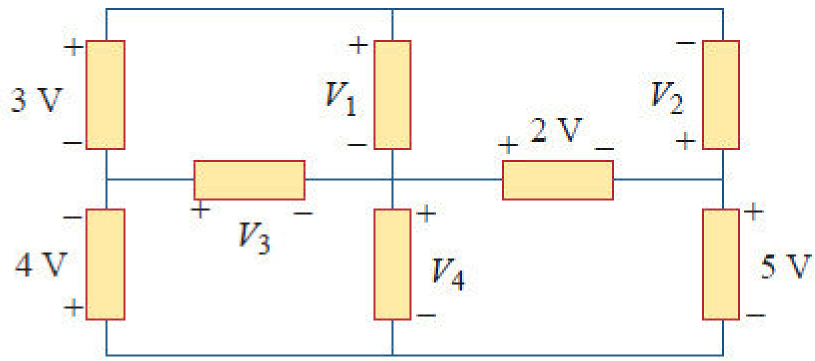
1. Use KCL to determine i1 and i2 in the circuit



$i_1 = 4A + 3A$ $\quad$ $i_1 = 7A$

$i_2 + 3A = -2A$ $\quad$ $i_2 = -5A$

2. Given the circuit in Fig, use KVL to find the branch voltages V1 to V4



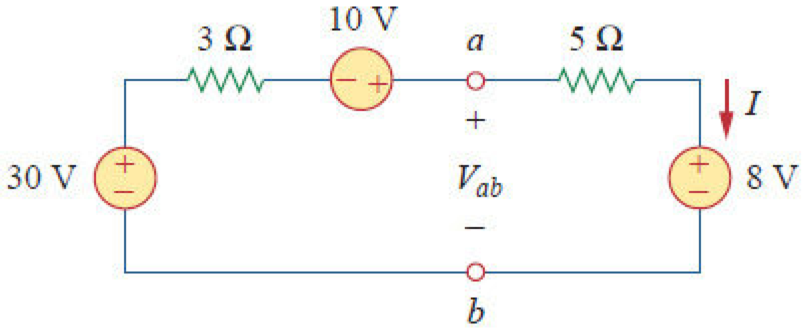
$2V + 5V = V_4 \rightarrow V_4 = 7V$

$V_3 + V_4 = 4V \rightarrow V_3 + 7V = -4V \rightarrow V_3 = -11\,V$

$V_1 + 11\,V = 3V \rightarrow V_1 = -8\,V$

$-V_2 - 2V = -8V \rightarrow V_2 = 6V$

3. Find I and Vab in the circuit of Fig



$$-10V + 8 \cdot I + 8V - 30V = 0$$

$$I = \frac{32V}{8\Omega} = 4A$$

$$V_{ab} = I \cdot 5\Omega + 8V = 4A \cdot 5\Omega + 8V = 28V$$

also $V_{ab} = 30V - 4A \cdot 3\Omega + 10V = 28V$

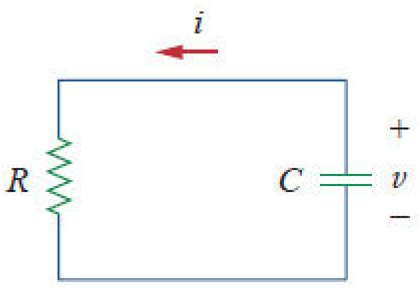
4. For the circuit in Fig , if

$$v(t) = 56e^{-200t}\ \text{V}, \quad t > 0$$
$$i(t) = 8e^{-200t}\ \text{mA}, \quad t > 0$$

(a) Find the values of R and C

(b) Calculate the time constant (tau)

(c) Determine the time required for the voltage to decay half its initial value (at t=0)



Part A:
use the equations $\tau = RC$ and V = I R,
so:
i = i(0) = 8mA derived from the give current equation.
v = v(0) = 56V derived from the given voltage equation.

then $R = \frac{V}{I}$

$$R = \frac{56}{8\times 10^{-3}}$$

$$R = 7k\Omega$$

plug this into

$$C = \frac{\tau}{R}$$

$$C = \frac{5\times 10^{-3}}{7\times 10^{3}}$$

$$C = 0.714\mu F$$

Part B: using equation $v(t) = v(0)e^{-\frac{t}{\tau}}$ and comparing it with the giving voltage equation,

$$-\frac{t}{\tau} = -200t$$

solving for the time constant gives $\tau = \frac{1}{200}$

$\tau = 5\text{ms}$

Part C: initial voltage is v(0) = 56exp$^{(-200*0)}$ = 56
so the half of that is 28V set that equal to the voltae equation: 28 = 56 exp$^{(-200t)}$

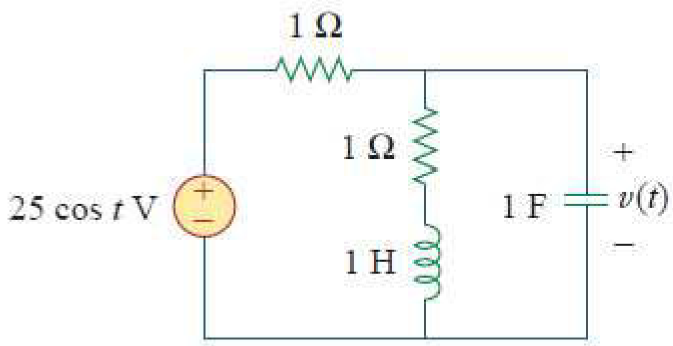
then solve for time: $e^{-200t} = \frac{28}{56}$

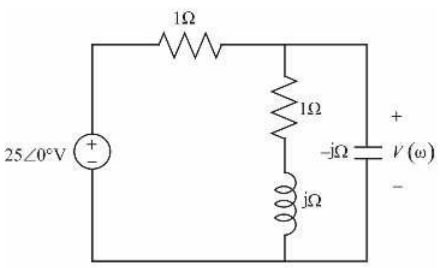
$-200t = \ln\left(\frac{1}{2}\right)$

$t = \frac{-0.693}{-200}$

$t = 3.465\text{ms}$

1. Find v(t) in the circuit.





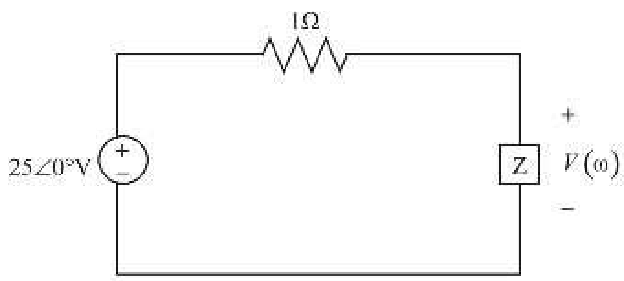
The parallel combination $-j\Omega$ impedance and $(1+j)\Omega$ impedance give,

$$Z = -j\Omega \,\|\, (1+j)\Omega$$

$$Z = \frac{-j(1+j)}{-j+1+j}$$

$$Z = 1 - j\Omega$$

Above circuit can be represented as,



By using voltage division principle to above circuit,

$$V(\omega) = 25\angle 0° \times \left[\frac{Z}{Z+1}\right]$$

Substitute Z in the above equation

$$V(\omega) = 25 \times \left[\frac{1-j}{1-j+1}\right]$$

$$V(\omega) = 25 \times \left[\frac{1-j}{2-j}\right]$$

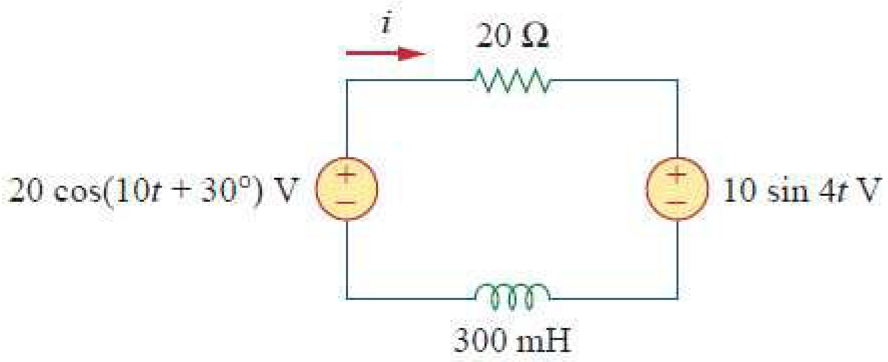
$$V(\omega) = 15 - j5\,\text{V}$$

$$V(\omega) = 15.81\angle -18.435°\text{V}$$

Convert this into time domain we get

$$\boxed{v(t) = 15.81\cos(t - 18.435°)\,\text{V}}$$

2. Use superposition to find i(t) in the circuit



Let

$$i(t) = i_1(t) + i_2(t)$$

Where $i_1(t)$ and $i_2(t)$ are due to $20\cos(10t + 30°)$ and $10\sin 4t$ sources respectively.

To find $i_1(t)$ consider

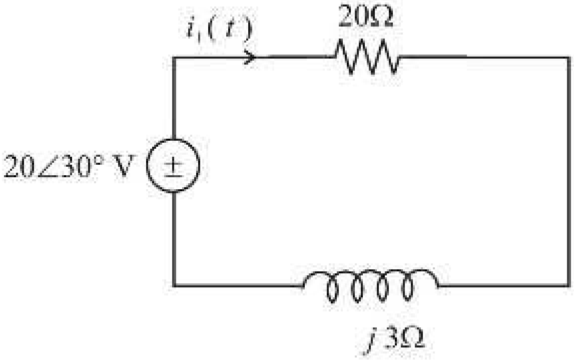
$$20\cos(10t + 30°) = 20\angle 30°$$

$$\omega = 10\,\text{rad/sec}$$

$$300\,\text{mH} \Rightarrow j \times 10 \times 300 \times 10^{-3}$$

$$300\,\text{mH} \Rightarrow j3\Omega$$

frequency domain circuit is



Applying KVL to above circuit ,

$$-20\angle30° + 20i_1(t) + j3i_1(t) = 0$$

$$(20 + j3)i_1(t) = 20\angle30°$$

$$i_1(t) = \frac{20\angle30°}{20 + j3}$$

$$i_1(t) = \frac{20\angle30°}{20.224\angle8.531°}$$

$$i_1(t) = 0.98892\angle21.47°\,\text{A}$$

$$i_1(t) = 989\angle21.47°\,\text{mA}$$

$$\therefore\ i_1(t) = 989\cos(10t + 21.47°)\,\text{A}$$

To find $i_2(t)$ consider

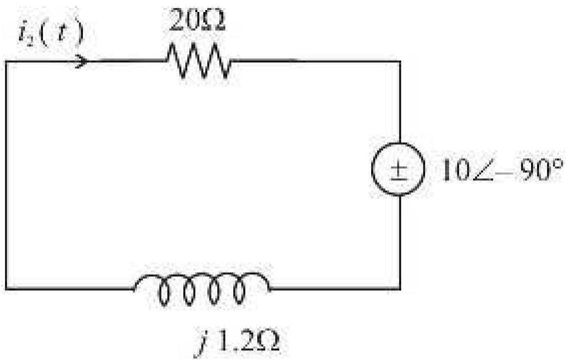
$$10\sin 4t = 10\cos(4t - 90°)$$

$$10\sin 4t = 10\angle -90°$$

$$\omega = 4 \text{ rad/sec}$$

$$300\,\text{mH} \Rightarrow j \times 4 \times 300 \times 10^{-3}$$

$$300\,\text{mH} \Rightarrow j1.2\Omega$$



Applying KVL to above circuit ,

$$j1.2i_2(t)+20i_2(t)+10\angle-90°=0$$

$$(20+j1.2)i_2(t)=j10$$

$$i_2(t)=\frac{j10}{20+j1.2}$$

$$i_2(t)=0.4991\angle 86.5655°\ \text{A}$$

$$i_2(t)=499.1\cos(4t+86.5655°)\ \text{mA}$$

$$i_2(t)=499.1\sin(4t+86.5655°+90°)\ \text{mA}$$

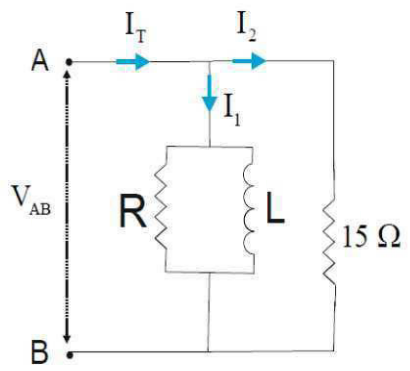
$$\therefore\ i_2(t)=499.1\sin(4t+176.57°)\ \text{mA}$$

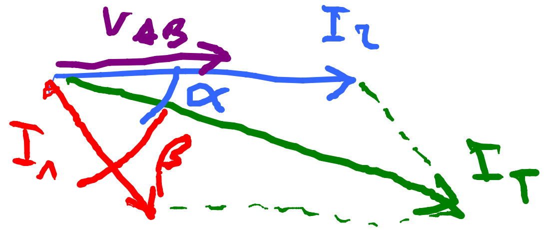
Substitute $i_1(t)$ and $i_2(t)$ in equation (1) ,

$$\therefore\ \boxed{i_0(t)=\left[989\cos(10t+21.47°)+499.1\sin(4t+176.57°)\right]\ \text{mA}}$$

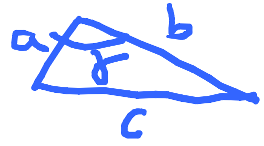
Given the circuit of figure in which the effective values of the currents are $I_T = 29{,}9$ A, $I_1 = 22{,}3$ A, and $I_2 = 8$ A. Solve the following questions:

a) Draw the phasor diagram for I_T, I_1, I_2 and V_{AB}. (Consider the origin of phases I_2),
b) Calculate the complex impedance of the RL branch,
c) Calculate the active power consumed by the entire circuit.





Cosine Theorem



$$c^2 = a^2 + b^2 - 2ab \cdot \cos\gamma$$

a) $I_1^2 = I_2^2 + I_T^2 - 2 I_2 \cdot I_T \cdot \cos\alpha \rightarrow \cos\alpha = \dfrac{I_2^2 + I_T^2 - I_1^2}{2 I_2 \cdot I_T} = \dfrac{8^2 + 29{,}9^2 - 22{,}3^2}{2 \cdot 8 \cdot 29{,}9} = 0{,}96304348$

$I_2^2 = I_1^2 + I_T^2 - 2 I_1 \cdot I_T \cdot \cos\beta \rightarrow \cos\beta = \dfrac{I_1^2 + I_T^2 - I_2^2}{2 I_1 \cdot I_T} = \dfrac{22{,}3^2 + 29{,}9^2 - 8^2}{2 \cdot 22{,}3 \cdot 29{,}9} = 0{,}99532073$

$\alpha = \cos^{-1} 0{,}96304348 = 15{,}6253528^\circ$ $\quad \alpha + \beta = 21{,}170287^\circ$

$\beta = \cos^{-1} 0{,}99532073 = 5{,}54493418^\circ$

b)

$$V_{AB} = I_2 R = 120 \text{ V} \Rightarrow \vec{V}_{AB} = 120\angle\, 0^\circ \text{ V}$$

$$\vec{I}_1 = 22{,}3\ \angle -21{,}17^\circ \text{ A}$$

then

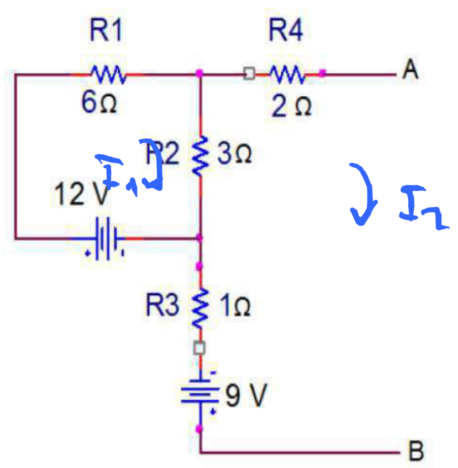
$$\vec{Z}_{RL} = \frac{\vec{V}_{AB}}{\vec{I}_1} = 5{,}38\ \angle\ 21{,}17^\circ\ \Omega$$

c) $\vec{I}_1 = 22{,}3\ \angle -21{,}17^\circ$ A y $\vec{I}_2 = 8\ \angle 0^\circ$ A

$$\vec{I}_T = \vec{I}_1 + \vec{I}_2 = 20{,}79 - 8{,}05\ j + 8 = 29{,}89\ \angle -15{,}62^\circ \text{ A}$$

$P = V_{AB} \cdot Re(I_T) = 120\text{V} \cdot 29{,}89 \cdot \cos(-15{,}62) = 3.451$ W

(1) Calculate and draw the Norton equivalent of the following DC circuit across the terminals A and B, (2) Calculate and draw the Thevenin equivalent circuit.



① Norton:

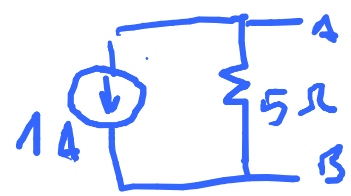
$$R_N = R_4 + \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} + R_3 = 2\Omega + \frac{1}{\frac{1}{6}+\frac{1}{3}}\Omega + 1\Omega =$$

$$= 2\Omega + 2\Omega + 1\Omega = 5\Omega$$

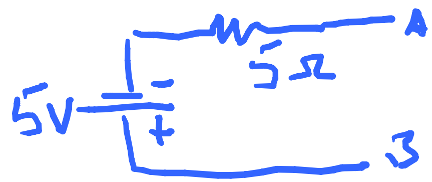
$I_N = I_2 \longrightarrow I_2 = \dfrac{\begin{vmatrix} 9 & 12 \\ -3 & -9 \end{vmatrix}}{\begin{vmatrix} 9 & -3 \\ -3 & 6 \end{vmatrix}} = \dfrac{-81+36}{54-9} = \dfrac{-45}{45} = -1A$

$9I_1 - 3I_2 = 12V$

$-3I_1 + 6I_2 = -9V$

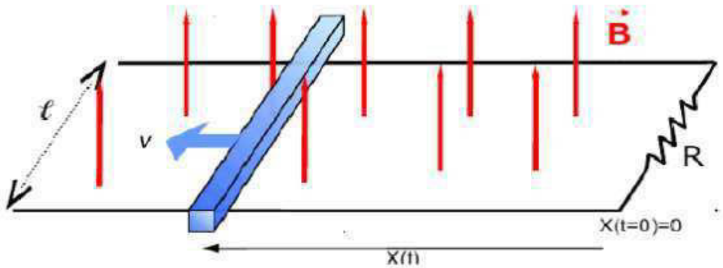


② Thevenin:



Over two rails with a resistance per unit length equal to 2Ω/m separated by a distance l =0,5m and closed at one end by a lead wire that connects to a resistance of 100Ω, as shown in the figure, a metal rod slides freely at a constant speed of v = 2,3m/s. In the space delimited by the rails and the rod there is a uniform magnetic field that varies with time B (t) = 0,8cos (3t) T, perpendicular to the plane the rod and the rails and in the direction indicated in figure. At t=0 the rod is located at x = 0 (near to the resistance) and thereafter moves with constant speed in the direction indicated by the figure. Calculate:

a) The induced electromotive force at any instant of time,
b) The current flowing through the resistor R at t = 10 s,
c) The power dissipated by the resistance R versus time.



30	4,77464829	30	0,15425145		
			-0,98803162	30	-29,6409487
					-29,7952002
					-27,4115842
					-0,18775058

a) $\phi(t) = \vec{B}\cdot\vec{s} = B(t)\cdot s(t)$

$s(t) = \ell \cdot v \cdot t$, $\phi(t) = 0{,}5 \cdot 2{,}3 \cdot t \cdot 0{,}8\cos(3t)$ Wb

$\phi(t) = 0{,}92\, t\cos(3t)$ Wb

$\varepsilon(t) = -\dfrac{d\phi(t)}{dt} = -0{,}92\left(\cos(3t) - 3t\cdot\sin(3t)\right) V =$

$= 0{,}92\left(3t\cdot\sin(3t) - \cos(3t)\right) V$

b) $i(t) = \dfrac{\varepsilon(t)}{R(t)} = \dfrac{0{,}92\left(3t\cdot\sin(3t) - \cos(3t)\right)}{100\Omega + 2\cdot v\, m/s \cdot t}\, A =$

$= \dfrac{0{,}92\left(3t\cdot\sin(3t) - \cos(3t)\right)}{100\Omega + 2\cdot 2{,}3\, m/s\cdot t} =$

$= \dfrac{0{,}92\left(3t\cdot\sin(3t) - \cos(3t)\right)}{100\Omega + 4{,}6\, m/s\cdot t} = [t = 10s] = \dfrac{-27{,}4115842\ V}{146\,\Omega}$

Suppossing the arguments in radians

$= -0{,}18775058\ A$

c) Power in R (only R) $P_R(t) = i^2(t)\cdot R = \left(\dfrac{0{,}92\left(3t\cdot\sin(3t) - \cos(3t)\right)}{100\Omega + 4{,}6\, m/s\cdot t}\right)^2 \cdot 100$ W

Power in the whole circuit

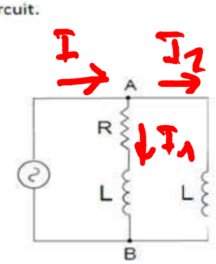
$P_W(t) = \dfrac{V^2(t)}{R(t)} = \dfrac{\left(0{,}92\left(3t\cdot\sin(3t) - \cos(3t)\right)\right)^2}{100\Omega + 4{,}6\, m/s\cdot t}$ W

Problem 1: (3,5 p.)

Given the AC circuit of the figure below, where the current through the (R and L) branch is 10 A. Determine:

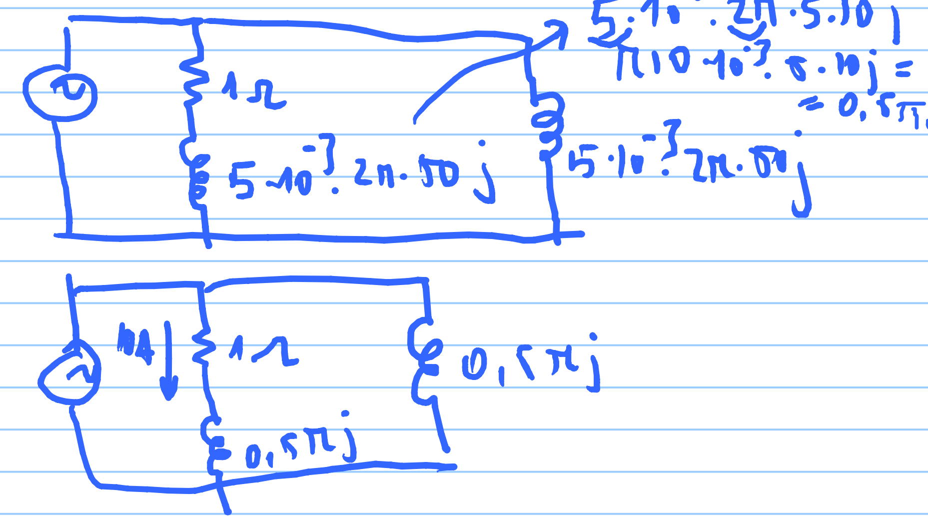
a) The potential difference $V_{AB}(t)$, and the currents (time domain) flowing in the other two branches,
b) Draw the phasor diagram of the currents calculated in the previous section, taking the current flowing through the resistor R as phase origin,
c) Calculate the active power of the circuit.

Datum: R= 1 Ω, L= 5 mH, f = 50 Hz



$\omega = 2\pi \cdot f\ s^{-1} = 2\pi \cdot 50\ s^{-1}$

(a)



(a)

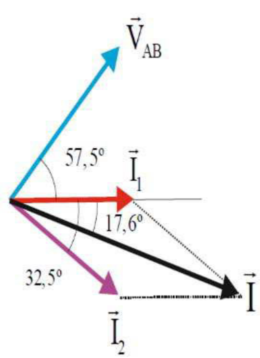
$V_{AB} = 10_{\lfloor 0°} A \cdot (1 + 0,5\pi j) = 10_{\lfloor 0°} A \cdot \sqrt{1^2 + (0,5\pi)^2}_{\lfloor \tan^{-1} 0,5\pi}\ \Omega = 10_{\lfloor 0°} A \cdot 1.8621_{\lfloor 57,51°}\ \Omega =$

$= 18,62_{\lfloor 57,5}\ V$

$I_2 = \dfrac{V_{ab}}{0,5\pi_{\lfloor 90°}\ \Omega} = \dfrac{18,62_{\lfloor 57,5°}\ V}{0,5\pi_{\lfloor 90°}\ \Omega} = 11,86_{\lfloor -32,5°}\ A$, $\quad I = I_1 + I_2 =$

$\bar{I} = \bar{I}_1 + \bar{I}_2 = 10 + (10 - 6,37\ j) = 20,99 \angle -17,6°\ A$

(b)



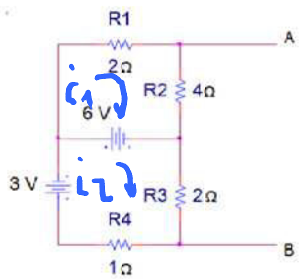
(c)

$\overline{P} = I_1^2\ R = (10\ A)^2\ 1\ \Omega = 100\ W$

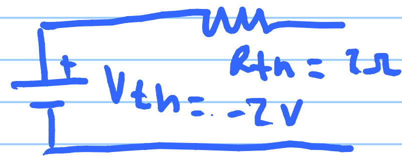
Problem 2: (3,5 p.)

A) Calculate and draw the Thevenin equivalent circuit across its terminals A to B of the DC circuit in the figure below,

B) Calculate and draw the equivalent Norton circuit through terminals A and B.



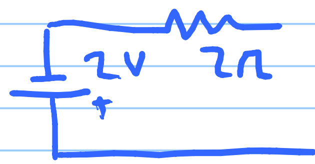
A) Thevenin eq.



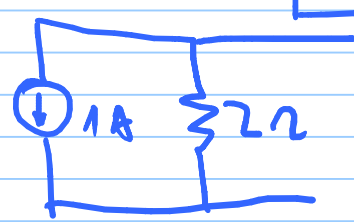
$$R_{th} = \frac{1}{\frac{1}{2}+\frac{1}{4}} + \frac{1}{\frac{1}{2}+1}\,\Omega = \frac{1}{\frac{6}{8}} + \frac{1}{\frac{3}{2}}\,\Omega = \frac{8}{6} + \frac{2}{3}\,\Omega = \frac{4}{3} + \frac{2}{3} = 2\,\Omega$$

$6 i_1 = 6V$, $i_1 = 1A$

$3 i_2 = -9V$, $i_2 = -3A$, $V_{th} = 1A \cdot 4\Omega - 3A \cdot 2\Omega = -2V$



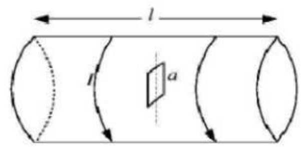
B)



(Norton)

Problem 3: (3,0 p.)

A square loop of side a = 10 cm, is placed inside a solenoid of length l = 20 cm and N = 1000 turns. The solenoid axis is perpendicular to the plane of the loop, as shown in figure below. The loop is completely inside the solenoid and has a resistance R = 1Ω.



a) Calculate the magnetic flux through the loop when through the solenoid flows a current $I_0 = 1$ A,
b) Suppose now that the current through the solenoid is given by $I(t) = I_0\cos(\omega t)$, with $\omega = 1{,}53 \cdot 10^3\ s^{-1}$ and I_0 = 1A. What will be the electromotive force and induced current through the loop?
c) Maintain the conditions used for (a) and suppose now that at t = 0, the loop begins to rotate with an angular velocity $\omega = 1{,}53 \cdot 10^3\ s^{-1}$ on the axis through its center and parallel to one side of the loop (as shown in figure). Calculate the electromotive force and the induced current through the loop. Compare with the result of (b).

$\mu_0 = 4\pi \cdot 10^{-7}\ Ns^2/C^2$

$B \cdot A \cdot \cos 0° = BA$

a) $B_a = \mu_0 I_0 n = 4\pi \cdot 10^{-7} \cdot 1A \cdot \frac{1000}{0{,}2} \ \Upsilon = \frac{4\pi \cdot 10^{-7} \cdot 10^4}{2} T = 2\pi \cdot 10^{-3} T$, $\phi_a = \int_S B \cdot dA = 2\pi \cdot 10^{-3} \cdot (0{,}1)^2$ wb

b) $B_b = \mu_0 \frac{1000}{0.2} \cos 1.53 \cdot 10^3 t\ T = 2\pi \cdot 10^{-3} \cos 1{,}53 \cdot 10^3 t$, $\phi_b = 2\pi \cdot 10^{-3} \cdot (0{,}1)^2 \cos 1.53 \cdot 10^3 t$ wb

$\mathcal{E}_b = -\frac{d\phi}{dt} = 2\pi \cancel{10^{-3}} (0{,}1)^2 \cdot 1{,}53 \cancel{10^3} \cdot \sin 1{,}53 \cdot 10^3 t$ V, $i_b = \frac{\mathcal{E}}{R} = 2\pi (0.1)^2 \cdot 1{,}53 \cdot \sin(1.53 \cdot 10^3 t)$ A

c) $B_c = 2\pi \cdot 10^{-3} t$, $\phi_c(t) = \int_S A\cancel{(t)} \cdot dA \cdot B \overset{wb}{=} 2\pi \cdot 10^{-3} \cos 1{,}53 \cdot 10^3 t\ (0{,}1)^2 \equiv \phi_b$

$\mathcal{E}_c \equiv \mathcal{E}_b$

$i_c = i_b$