Wealth, Returns, and Taxation: A Tale of Two Dependencies

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JM seminar

Toulouse, November 29, 2021

Introduction

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- some discuss its implementation, ex: Ultra Millionaire Tax in US,
- academic: Guvenen et al. (2019), Saez and Zucman (2019), Scheuer and Slemrod (2021).

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2. build a **dynamic general equilibrium framework** of the US economy.
3. quantify **optimal wealth taxation** in the US.

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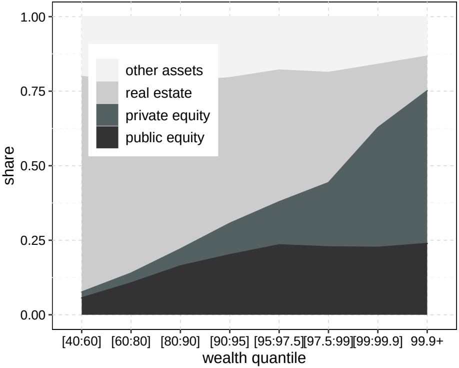
Data: returns to wealth are ***large* at the top** and very ***persistent***:

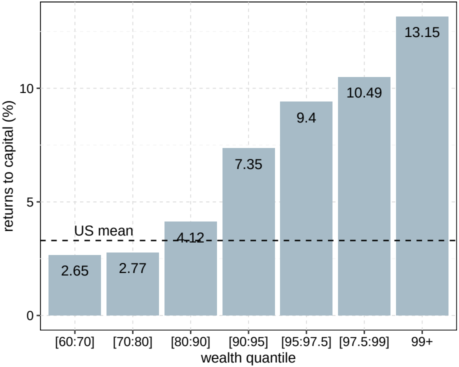
- systematic differences in **investment decisions**: private/public equity investments display high expected returns.

Portfolio shares and wealth returns

(a) portfolio shares, SCF (1998-2019)



(b) wealth returns, PSID (2000-2018), more



- **Risky portfolio/wealth returns** positively **correlated** with **wealth**.

What is driving this observed correlation?

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Empirically: supported by recent evidence

- Fagereng et al. (2020, ECMA): both account for 50% of persistent observed heterogeneity btw top/bottom in Norway.
- Bach et al. (2020, AER) use a sample of twins and find role for type/scale dependence in Sweden.

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ex: *rent-extraction*: bargaining/market power, unequal access to market...

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Important interaction: (1) *and* (2) determine whether and how to tax wealth.

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If rents increase in returns: above forces *offset each other* ($\rightarrow$ B)

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  → to generate high **wealth inequality** (and its dynamics),

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  $\Rightarrow$ sign/magnitude: whether type/scale *and* returns are MPK/rent.

Roadmap

We substantiate our quantitative results in **three steps**.

1. **Simple analytical model** to lay out the main concepts.
   - focus on portfolio/return heterogeneity and capital channel,
   - isolate ***four key statistics*** for wealth taxation.
2. A full-blown quantitative **dynamic model**,
   - joint distribution of wealth/skill type is *endogenous*,
   - calibration focus on the key statistics isolated in the simple model,
3. Characterize wealth taxation in the US.

A simple model

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- They invest in:
  - **risky assets**, used in entrepreneurial/innov sector with return R^{risky}.
  - **safe assets**, used in traditional sector with return R^{safe}.

$$\text{(risky asset share)} \quad \omega(a, \vartheta) \underset{\text{tractability}}{\propto} \vartheta a^{\gamma}$$

microfoundation

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microfoundation

$\rightarrow$ Heterogeneity in returns: generated by **correlation** between risky portfolio and wealth/types.

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Risky return R^{risky} may not reflect the marginal product of capital MPK^{risky}.

- Data: expected returns $R^{risky} > R^{safe}$.

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$\rightarrow$ not informative on whether $MPK^{risky} > MPK^{safe}$.

$\rightarrow$ introduce a wedge μ to control the extent that R^{risky} reflect MPK^{risky}.

$$MPK^{risky} = \mu R^{risky} + (1 - \mu) MPK^{safe} \leq R^{risky}$$

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$$Y = F\left(\int_{(a,\vartheta)} \text{Capital}^{\mathit{eff}} \left(\underbrace{\omega(a,\vartheta)}_{\substack{\text{investment}\\ \text{portfolio}}}, \mu \right) \underbrace{d\mathcal{G}(a,\vartheta)}_{\text{joint density}}, \text{Labor} \right)$$

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η: shape of the wealth distribution, assuming $a \sim Pareto(\eta)$,

ϱ: **sorting of skilled-type** along the distribution, i.e. $cov(\vartheta, a)$,

γ: wealth-dependent **risk taking elasticity**, i.e. $\frac{\partial \ln(\omega(a,\vartheta))}{\partial \ln(a)}$,

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Efficiency-Inequality Representation

Proposition: *the marginal effect of a change in inequality η on output is*

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③ amplification if same sign.

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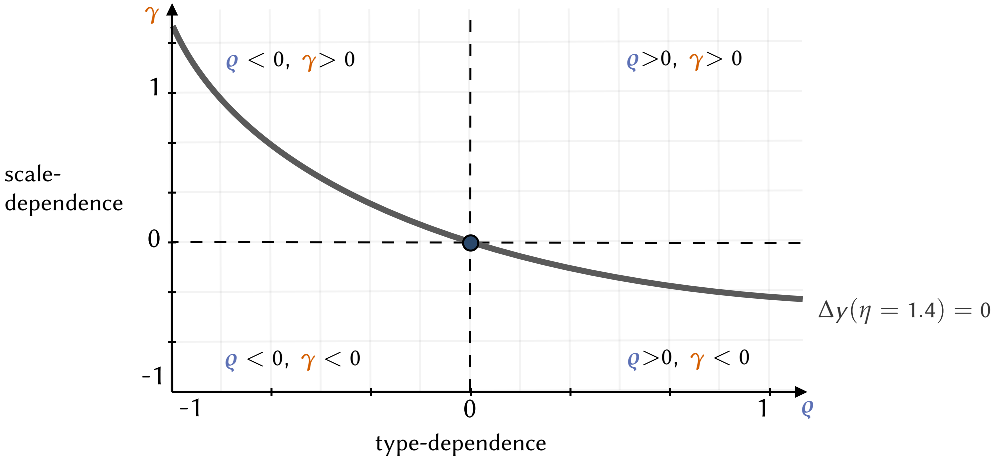

existence, properties, macro identification, location of models, example

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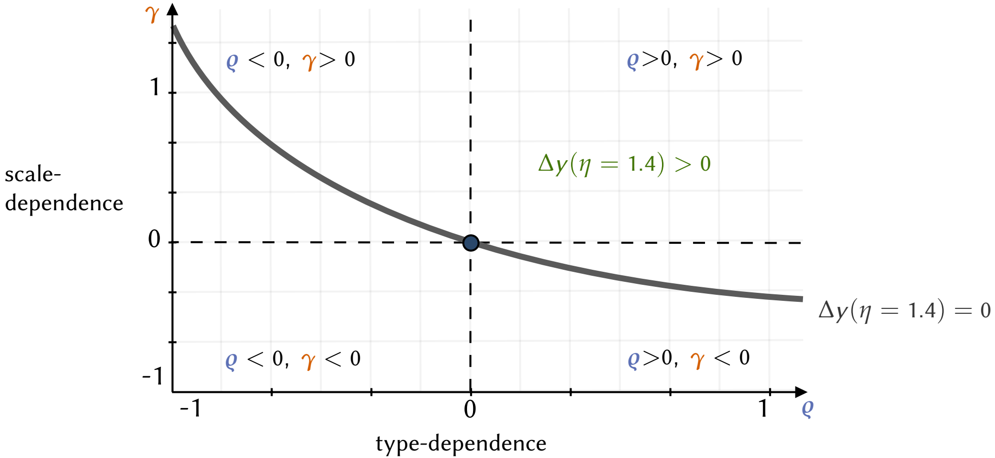

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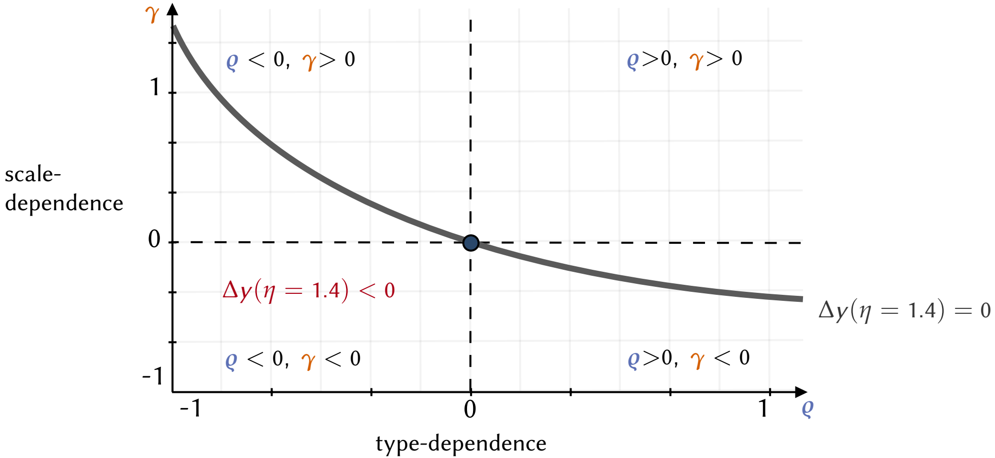

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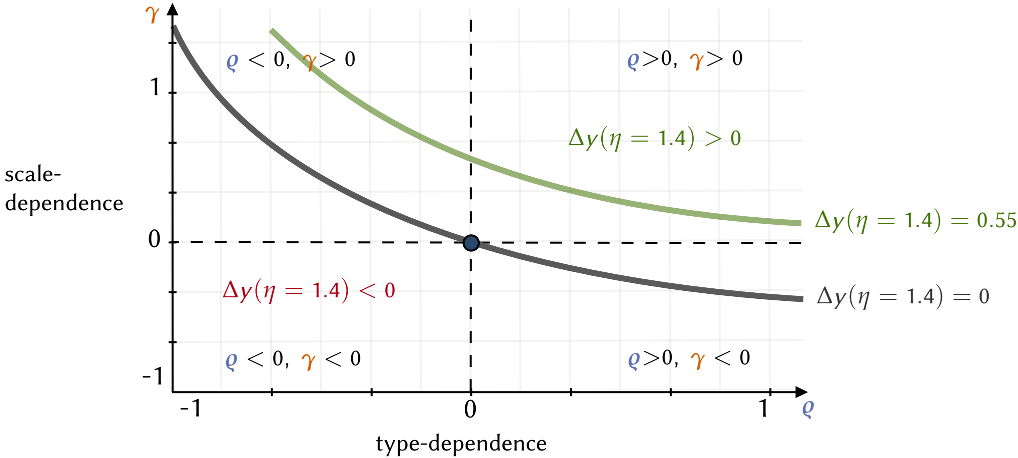

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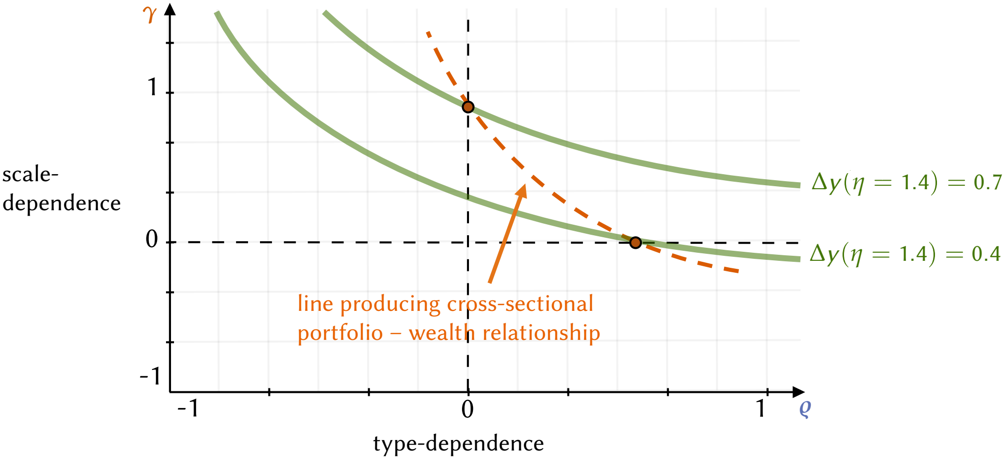

From efficiency to welfare

What is the effect of wealth taxation on welfare?

- Measured by equivalent consumption making indifferent pre/post reform,
- Wealth redistribution affects welfare through:

1. **inequality-efficiency trade-off**, Δy, which affects wages.
2. **size of rents in returns** (captured by μ).

$\rightarrow$ the larger the rent extrated, the lower the returns to ensure that the total income distributed = total income generated.

3. **standard equity concern**: redistribute from low marginal utility of consumption (wealthy) to high marginal utility of consumption (poor).

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Theory:

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Next:

1. build a **dynamic GE incomplete markets** model.
   → extended Aiyagari – Bewley – Huggett á la Conesa et al. (2019)
   → *objective:* joint distribution of skill-types and wealth **endogenous**.
2. calibrate using Survey of Consumer Finance (SCF) and Panel Survey of Income Dynamics (PSID),
3. check properties of wealth/return distributions under type/scale,
4. characterize optimal wealth taxation in the US.

Environment

Households

- Life-cycle earning component and retirement, persistent and transitory labor income component,

$\Rightarrow$ matches Pareto shape of earning distribution ($<$ Pareto wealth).

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- revenue: taxes on bequest, wealth, consumption, capital and labor income,
- expenditure: exogenous G, social security pensions.

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Government

- revenue: taxes on bequest, wealth, consumption, capital and labor income,
- expenditure: exogenous G, social security pensions.

Production

- use risky/safe assets of households to produce efficiency units of capital.

Households

- labor productivity z depends on age j, persistent/transitory component,
- have heterogeneous investment skill type ϑ and wealth a,
  - $\rightarrow$ drive investments in high return assets (i.e. $\omega(a, \vartheta)$).
  - $\rightarrow$ subject to idiosyncratic return risk κ,
- decide how much to consume, work, and save,
- take $\{w, r_F, r_R, \underline{r}\}$, taxes and transfers as given.

$$V(a, \vartheta, \kappa, z, j) = \max_{c, \ell, a' \geq 0} u(c, \ell) + \beta(1 - d_j)\mathbb{E}\Big[V(a', \vartheta, \kappa', z', j')\Big]$$

subject to

$$c + a' = w\ell z(1 - \tau_w) + (1 + r(a, \vartheta)(1 - \tau_k))a - t_a(a)$$

$$(return) \qquad r(a, \vartheta) = \underline{r} + r_F \cdot (1 - \omega(a, \vartheta)) + r_R\kappa \cdot \omega(a, \vartheta)$$

Closing the model

Production:

- final: $Y = F(X, L)$, rents x intermediates and labor at prices p, w.
- intermediate sells units at price p, $X = \int_i x_i di$, produce with:
  $x(a, \vartheta) = \Big[(1 - \omega(a, \vartheta))A + \omega(a, \vartheta)(\mu\phi + A(1 - \mu))\Big] a$

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Stationary equilibrium: definition

- w and p equalize their marginal product.
- $r_F = \underbrace{AF_X(X, L)}_{MPK_F}, \quad r_R = \phi F_X(X, L) \geq \underbrace{(\mu\phi + A(1 - \mu))F_X(X, L)}_{MPK_R(\mu)}$
- $\underline{r}$ adjusts if $\mu < 1$ to ensure capital income = capital product.
- government: $T_w + T_b + T_c + T_k + T_a = transfers + G$

Calibration

Objective: calibrate the model to match the average US economy (2000-2019).

Three sets:

1. Parameters set to values in literature/data,
   - $u(c, \ell) = \frac{c^{1-1/\sigma}}{1-1/\sigma} - \chi \frac{\ell^{1+1/\lambda}}{1+1/\lambda}$, which $\sigma = 0.5$; $\lambda = 0.6$.
   - baseline tax rates set to US values, no wealth tax,
   - $F(X, l) = X^{\alpha} L^{1-\alpha}$, with $\alpha = 0.33$; depreciation $\delta = 4.5\%$.
2. Parameters set to match moments,
   - β, excess risky return $\phi - A$, match $\frac{K}{Y}$ and top 1% wealth share.
   - disutility of labor χ matches 1/3 time on market work,
   - params governing portfolio match investment decisions in SCF/PSID.
3. Assumption: no rent-extraction, i.e. $\mu = 1$.

$\rightarrow$ later $\mu = 0.8$, calibrated to values used in (Rothschild et Scheuer (2016))

Portfolio allocation/returns

- Two types: investor ($\vartheta = 1$) and non-investor ($\vartheta = 0$).
- **Transition** to equity investor increases with wealth:

$$\pi_\vartheta(\vartheta'|\vartheta, a) = \begin{bmatrix} 1 - \underline{\pi}_\vartheta - \lambda(a) & \underline{\pi}_\vartheta + \lambda(a) \\ \overline{\pi}_\vartheta & 1 - \overline{\pi}_\vartheta \end{bmatrix}$$

  - $\lambda(a)$ matches relation investor transition/wealth in PSID, show
  - $\underline{\pi}$ and $\overline{\pi}$ match fraction investors and transition out.

- Portfolio **share** increases with wealth:

$$\omega(a, \vartheta) = \vartheta(\underline{\omega} + \varpi(a))$$

  - $\varpi(a)$ matches increases in *net* private equity investments at the top SCF,
  - $\underline{\omega}$ matches average equity portfolio for $\vartheta = 1$.

Wealth inequality

Properties of the model and alternatives with different degree of type/scale dependence.

Table 1: Wealth distribution in the data and models.[a]

	Gini [c]	Share of wealth (in %) held by the top x%					
		20	10	5	1	0.1	0.01
US data (adjusted SCF)	0.82	86.4	72.7	59.7	37.2	17.8	7.3
benchmark model	0.80	84.2	71.9	59.3	35.4	18.2	8.9
pure scale model – recalibrated	0.82	85.7	73.6	60.3	35.2	20.7	11.7
pure type model – recalibrated	0.78	82.0	67.1	56.2	35.7	20.2	10.9
typical type–DRS entrep. model	0.78	82.0	68.9	56.6	35.9	14.7	4.9

- Model generate a consistent wealth Pareto tail (η).
- **Return heterogeneity key to generate high inequality**.

Distribution of returns

Table 2: Mean returns to wealth (in %): data and model.

Wealth group	Data[a] PSID	Data[a] SCF	Data[a] Norway	Data[a] Sweden	Model benchmark total	Model benchmark type	Model benchmark scale	Model pure scale	Model pure type	Model entrepreneur model total	Model entrepreneur model scale effect
P40-P50	REF	REF	REF	REF	REF	REF	REF	REF	REF	REF	REF
P80-P90	**0.5**	0.2		0.5	**1.7**	1.7	1.6	1.4	2.1	2.7	−1.0
P90-P95	**3.8**	1.4	~ 4.0	0.8	**3.6**	1.5	2.0	3.8	2.7	3.4	−1.4
P95-P97.5	**5.8**	2.6	~ 6.0	1.1	**4.6**	1.3	3.3	5.9	2.9	5.6	−1.6
P97.5-P99	**6.9**	3.8		1.5	**7.9**	3.9	4.0	7.8	7.4	9.9	−1.9
Top 1%	**9.6**	4.6	~ 10.0	2.5	**12.5**	7.0	5.5	12.2	9.8	7.0	−2.4

[a] Estimates are our own for the PSID. They are taken from Xavier (2019) for the SCF, from Bach et al. (2020) for Sweden and from Fagereng et al. (2020) and Halvorsen et al. (2021) for Norway.

- In the cross-section: benchmark, pure type, pure scale are all generating a **positive correlation** between returns and wealth.

  → endogenous positive type dependence $cov(\vartheta, a) > 0$.

- coexistence of type/scale is consistent with evidence in Norway/Sweden.

Aggregate response to top wealth tax

- Experiment: 1% wealth tax on the top 1% wealthiest households.

Statistics	*Scale*	*Benchmark*	*Type*	no portfolio heterogeneity
Δ*GDP* (in %)	$-\mathbf{1.19}$	-0.77	-0.64	$-\mathbf{0.10}$
Δtop 1% share (in pp)	$-\mathbf{3.50}$	-1.82	-1.77	-0.80

- Underlying force behind wealth inequality is crucial!
  - $\rightarrow$ scale: *dynamic self-enforcing behavioral multiplier* as high-return investment is function of wealth.
  - $\rightarrow$ the benchmark model falls in between a pure scale/type model.
  - $\rightarrow$ benchmark, pure type, pure scale cannot be used interchangeably.

Quantitative exercise

Objective: characterize optimal wealth taxation in the US.

$$t_a(a; \tau_a, \underline{a}_{max}) = \mathbb{1}_{a \geq \underline{a}_{max}} \tau_a (a - \underline{a}_{max})$$

$\rightarrow$ maximizes over τ_a and $\underline{a}_{max}$ to max welfare.

$\rightarrow$ progressive if $\underline{a}_{max} > 0$.

Welfare measure:

- based on a utilitarian consumption equivalent variation (Δ^{CEV}) criterion:

$$\int_{\mathbf{s}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \widetilde{\beta}^t u\Big(c_t^{post}(\mathbf{s}), \ell_t^{post}(\mathbf{s})\Big) \right] d\mathcal{G}^{post}(\mathbf{s}) = \int_{\mathbf{s}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \widetilde{\beta}^t u\Big(\big(1 + \Delta^{CEV}\big) c_t^{pre}(\mathbf{s}), \ell_t^{pre}(\mathbf{s})\Big) \right] d\mathcal{G}^{pre}(\mathbf{s})$$

Two results

In the long-run:

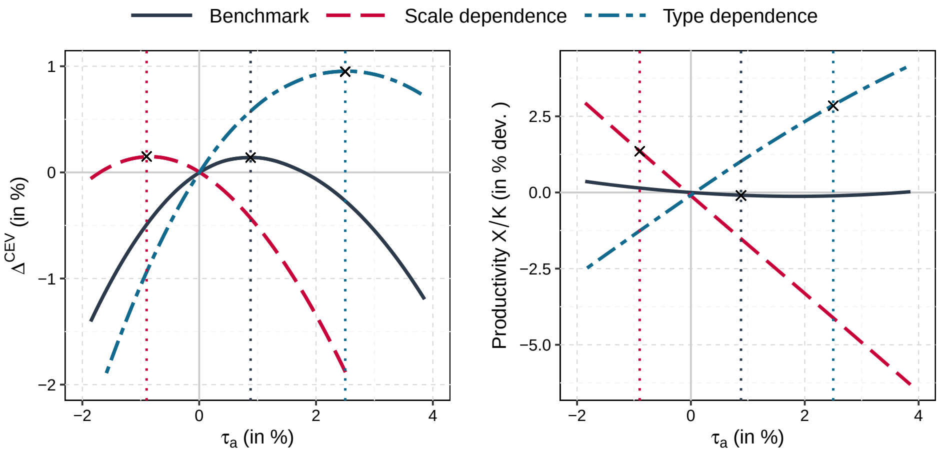
A. welfare-max wealth tax rate is **positive**: $\sim 0.8\%$ above \$550K wealth,

B. size of **rents** in returns (rltv to MPK) **only slightly increases** the tax.

How to understand those two results?

1. dissect underlying force; keeping the exemption level fix (for simplicity).
2. adjust the return wedge $\mu = 0.8$ in line with evidence in Lockwood et al. (2017) and Rothschild et al. (2016), and recompute tax rate.

Result A: positive wealth tax



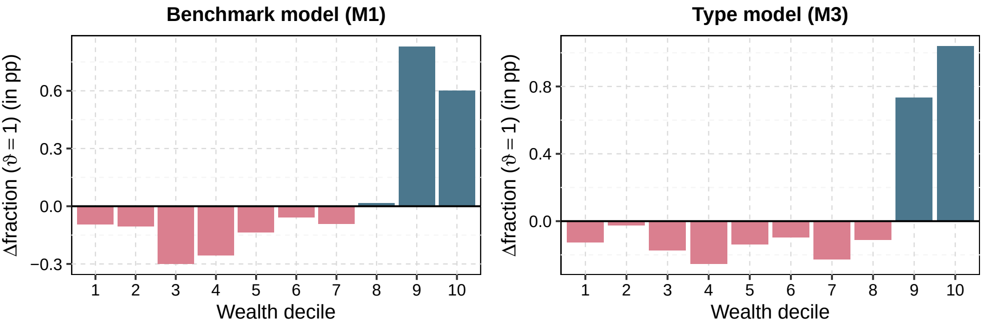
- scale dep: *snowball effect* which amplifies productivity response to tax.
  $\rightarrow$ GDP response is large.
- type dep: *only the fittest survive at the top* $\rightarrow$ productivity increases.
  $\rightarrow$ GDP falls but less due to better allocation.

Result A: positive wealth tax

Under type-dependence: a wealth tax changes the sorting of skilled investors along the wealth distribution.

→ At the top: more Elon Musk ("new" money), less Albert de Monaco ("old" money).

Figure 2: Change in the fraction of skilled investors along the wealth distribution.

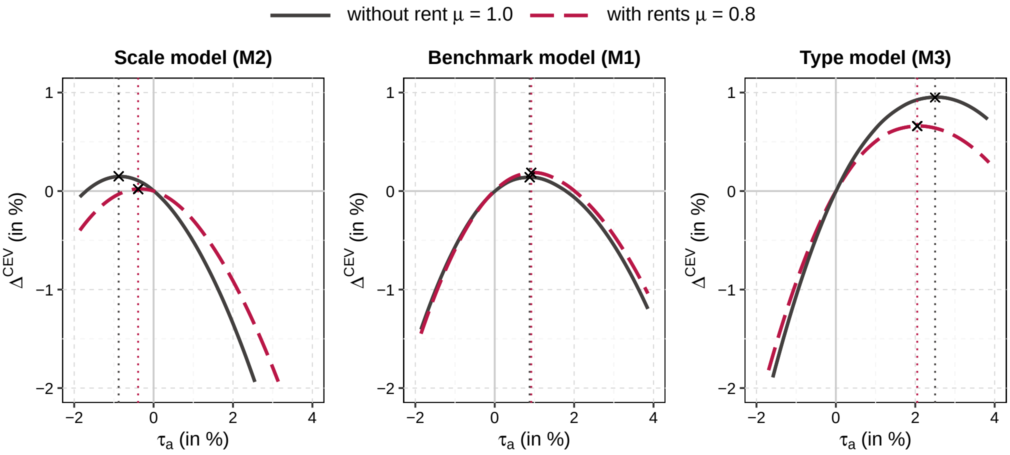


- Benchmark: fall in between type/scale dependence results.

Result B: effects of rents

What if returns reflect rents instead of MPK?

- scale dependence: wealth tax rate *increases* with rents.
- type dependence: wealth tax rate *decreases* with rents.
- benchmark: both effects offset $\rightarrow$ *only slightly* increases with rents.



Conclusion

Study macro implications of wealth redistribution with return heterogeneity.

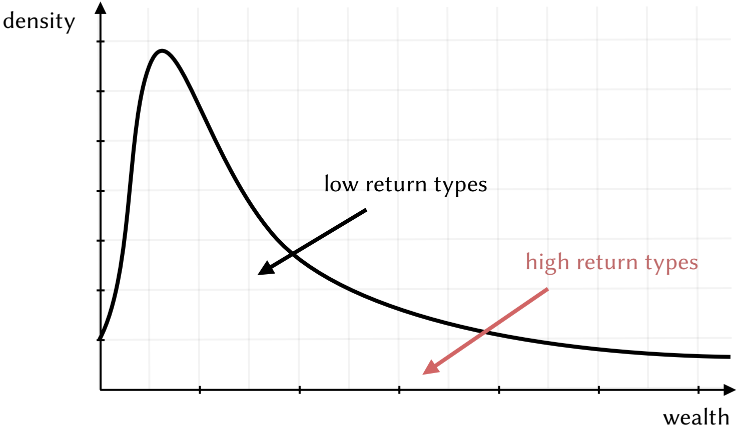
Unravel relevant economic forces:

- **type** and **scale** dependence and extent that returns reflect **MPK/rents**,
- implies different responses to taxes.

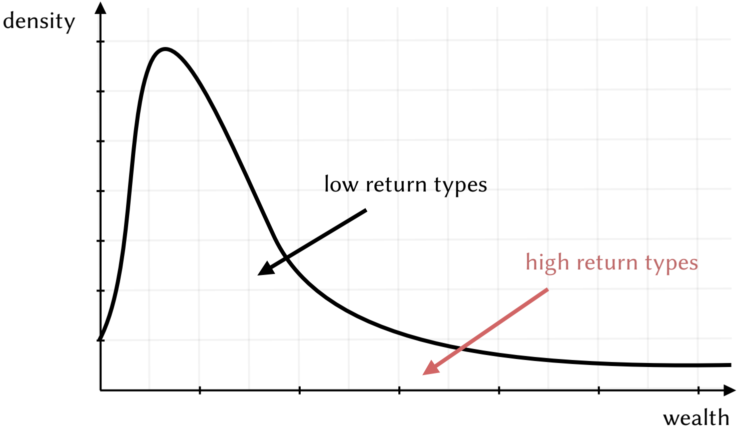
Quantify the optimal wealth tax in the US:

- given the degree of type/scale, optimal to **tax positively wealth**.
- the almost **unresponsive wealth tax** rate to the size of **rents** is due to **opposing forces** between type/scale dependence.

Selection of types: before tax



Selection of types: after tax



Parameterization

	Symbol	Value	Source
A. External parameters			
preferences $u(c,\ell) = \frac{c^{1-\sigma_1}}{1-\sigma_1} - \chi\frac{\ell^{1+\sigma_2}}{1+\sigma_2}$	$\{\sigma_1, \sigma_2\}$	$\{2.5, 1.7\}$	Brüggemann (2021)
persistent process h with Pareto tail	$\{\sigma_h, \rho_h, \eta_h, q_h\}$	$\{0.22, 0.95, 2.1, 0.9\}$	Hubmer et al. (2021)
stochastic aging part for h process	*in paper*	*in paper*	Sommer et Sullivan (2018)
inheritance of h skills	ρ_h	0.65	Chetty (2014)
transitory process labor y	σ_y	0.15	Hubmer et al. (2021)
production	$\{\alpha, \delta, A\}$	$\{0.33, 0.05, 1.0\}$	standard values
tax rates	$\{\tau_w, \tau_k, \tau_b\}$	$\{0.22, 0.25, 0.4\}$	standard values
B. K-heterogeneity parameters			
riskiness of equity investment	σ_κ	0.51	estimates PSID table
return wedge	μ	1.0	benchmark value
inheritance of ϑ skills	ρ_ϑ	0.15	Fagereng et al. (2020)

- internally calibrated: $\{\beta, \phi\}$, match $\frac{K}{Y}$ and top 1% wealth share.

Equivalence capital tax/wealth tax back to intro

$$
\begin{aligned}
&\text{(capital tax)} \qquad conso_i + asset'_i = (1 + \boldsymbol{r}(1 - \tau_k))asset_i + wage_i \\
&\text{(wealth tax)} \qquad conso_i + asset'_i = (1 - \tau_a)asset_i + \boldsymbol{r}(1 - \tau_a)asset_i + wage_i
\end{aligned}
$$

- If returns $\boldsymbol{r}$ are homogeneous, $\tau_a \equiv \frac{\boldsymbol{r}\tau_k}{(1+\boldsymbol{r})}$
- Data: wealth returns are individual specific: $\boldsymbol{r} \to \boldsymbol{r}_i$,

Returns to wealth back

Table 3: Average returns to wealth, PSID (1999-2018).

	Mean	Std.	P20	P80
Networth (before-tax)	0.033	0.158	−0.035	0.089
Risk equity (private & public equity)	0.112	**0.51**	−0.232	0.425
Safe asset	0.004	0.009	0.000	0.003

Literature portfolio back

- scale dependence mechanism to fit portfolio shares/returns: Kaplan et al. (2018), Hubmer et al. (2020)...
- type dependence mechanism to fit portfolio shares/returns: Moll (2014), Benhabib et al. (2019), Xavier (2021)...

Maximisation problem back toy

Final producer is competitive, with technology $Y = XL^{\varphi}$

$$\max_{L, \{x_s^j\}_{j,s}} XL^{\varphi} - wL - \sum_s \int_j p_s^j \, x_s^j \, dj \tag{1}$$

Solving for L and normalizing $L = 1$, we get:

$$\max_{\{x_s^j\}_{j,s}} (1 - \varphi) X - \sum_s \int_j p_s^j \, x_s^j \, dj \tag{2}$$

The price $p_s^j = (1 - \varphi)$ given the CRS on X.

Simple numerical example

back to vegetables

Experiment: a 1% marginal wealth tax on the top 1% wealthiest household.

Suppose two different models:

- type-dependence only: $\gamma = 0$, and $\varrho > 0$ to match increasing average risky portfolio share in data.
  - Response of output (all else equal) is $-\mathbf{0.42\%}$.
- wealth-dependence only: $\varrho = 0$, and $\gamma > 0$ to match increasing average risky portfolio share in data.
  - Response of output (all else equal) is $-\mathbf{0.77\%}$.

$\rightarrow$ **wealth-dependence implies a strong behavioral effect.**

- the multiplier $\Lambda^w(\eta) > \Lambda^s(\eta)$.

Utility: micro foundation back

Households:

- initial **wealth** a_i, and innate **type** ϑ_i
- CARA utility $u_i = -\frac{\mathbb{E}[e^{-\alpha_i c_i}]}{\alpha_i}$
- with innate risk-aversion correlates with ***type***/***wealth***: $\boldsymbol{\alpha_i} = \frac{\bar{\vartheta}}{\vartheta_i a_i^{\gamma}}$

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Budget constraint:

$$conso_i = wage_i + k_i R_f + (a_i - k_i) R_r^i a_i \qquad \text{with} \quad R_r^i \sim \mathcal{N}(\mathbb{E}[R_r^i], \sigma_r^2)$$

Utility: micro foundation back

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Optimal risky asset demand k_i and portfolio share $\omega(a_i, \vartheta_i)$

$$k_i \propto \varpi \underbrace{\mathcal{T}(a_i, \vartheta_i)}_{\text{risk tolerance}} = \varpi \cdot \frac{\boldsymbol{\vartheta_i}}{\bar{\vartheta}} \cdot a_i^{\gamma}, \qquad \omega(a_i, \vartheta_i) \propto \varpi \cdot \frac{\boldsymbol{\vartheta_i}}{\bar{\vartheta}} \cdot a_i^{\gamma - 1}$$

Generally: demand for risky assets for arbitrary utility: $k_i \approx \frac{\mu_k^p}{var_k} \mathcal{T}(a_i \mathbb{E}[R_r])$

- we generalize the risk-tolerance shape with scale/type dependence.

Analytics: production side back

Final producer: $y = \left(\int_i x_i di\right) L^{\varphi}$, with x_i intermediate goods problem

Analytics: production side back

Final producer: $y = \left(\int_i x_i di\right) \; L^{\varphi}$, with x_i intermediate goods problem

Intermediate Producer: use hhs funds, produce x_i with two technologies

$$x_i = \Bigg[\underbrace{A}_{TFP_{tradi}} \underbrace{(1-\omega_i)}_{\text{HH'i riskless asset}} + \underbrace{(\phi\mu + A(1-\mu))}_{TFP_{innov}} \underbrace{\omega_i}_{\text{HH'i risky asset}} \Bigg] a_i$$

redistributed to hhs with returns rate (net of labor payment):

$$(risky) \qquad \mathbb{E}[R_r^i] = \phi \qquad (safe) \qquad R_f = A$$

Analytics: production side back

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redistributed to hhs with returns rate (net of labor payment):

$$(risky) \qquad \mathbb{E}[R_r^i] = \phi \qquad (safe) \qquad R_f = A$$

- $\mu = 1$, expected risky returns $=$ net MPK_r.
- $\mu < 1$, expected risky returns $>$ net MPK_r.

ex: rent extraction: bargaining power, market power, political connection etc.

$\rightarrow$ common return rate adjusts: total capital product = total capital income.

Returns in GE model

Back to vegetable

Final producer: $Y = \left(\int x_j dj\right)^{\alpha} L^{1-\alpha}$

Intermediates produce homogenous x with two technologies (risky/safe), sell it to final producer

$$\pi^{risky}(\vartheta, a) = \underbrace{p}_{=\partial Y/\partial x} \underbrace{(\mu\phi + (1-\mu)A)\omega(a, \vartheta)a}_{=x} - \delta a\omega(a, \vartheta) \tag{3}$$

$$\pi^{safe}(\vartheta, a) = \underbrace{p}_{=\partial Y/\partial x} \underbrace{A(1-\omega(a, \vartheta))}_{=x} a - \delta a(1-\omega(a, \vartheta)) \tag{4}$$

Redistribute to households, such that:

$$r_{risky} = p\phi - \delta \lesseqgtr \frac{\pi^{risky}(\vartheta, a)}{a\omega(a, \vartheta)} \tag{5}$$

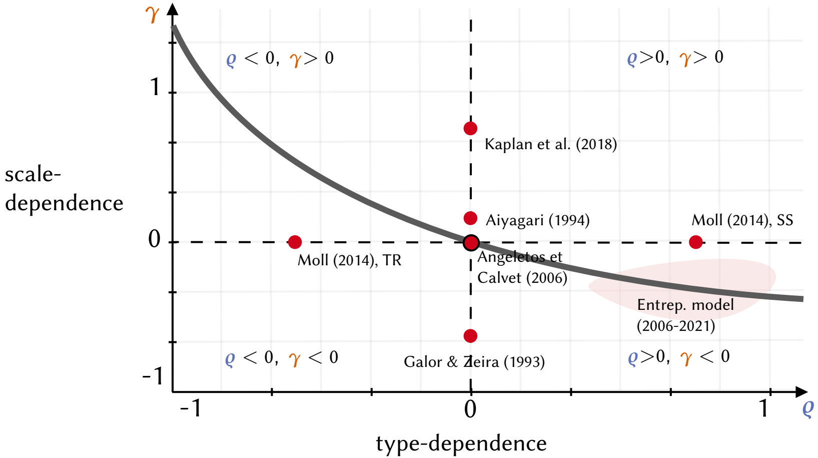
$$r_{safe} = \frac{\pi^{safe}(\vartheta, a)}{a(1-\omega(a, \vartheta))} = pA - \delta \tag{6}$$

- μ: disconnect risky returns from $MPK_r = \mu\phi + (1-\mu)A$.

The efficiency-inequality diagram

back

- large class of heterogeneous agent models with predictions between inequality/efficiency can be reinterpreted within our framework.



Portfolio shares: details

3. *hybrid model*: empirically realistic type + scale dependence
   - two types: investor ($\vartheta = 0$) and non-investor ($\vartheta = 1$).
   - extensive margin: entry in equity investor $\uparrow$ with wealth

     $$\pi^{hybrid}(\vartheta' | \vartheta, a) = \begin{bmatrix} 1 - \underline{\pi}_\vartheta - \phi(a) & \underline{\pi}_\vartheta + \phi(a) \\ \overline{\pi}_\vartheta & 1 - \overline{\pi}_\vartheta \end{bmatrix}$$

     $\phi(a)$ fit wealth-dependence in PSID Hurst et Lusardi (2005) show.
   - intensive margin:
     - share at top $\uparrow$ with wealth through diversification in new/recent priv. equity investments.
     - approximated by step-wise function $\omega^{exc.\ PE}(a)$ show

     $$\omega^{hybrid}(\vartheta = 1, a) = \overline{\omega} + \omega^{exc.\ PE}(a)$$

Properties of the GIF back to vegetables

Lemma 1 (Properties GIF)

The GIF is strictly decreasing on the defined set of Lemma 2. Moreover, a higher tail η *shifts the GIF such that* $\frac{d\gamma}{d\bar{g}}|_{d\varrho=0} < 0$, $\frac{d\gamma}{d\eta}|_{d\varrho=0} > 0$ *for* $\gamma > 1$ *and* $\frac{d\gamma}{d\eta}|_{d\varrho=0} \leq 0$ *for* $\gamma \leq 1$*. Finally, for a higher expected growth level* $\bar{g}$*,* $\mathcal{G}(\eta, \bar{g})$ *is the translation of* $\mathcal{G}(\eta, 0)$ *and* $\frac{d\gamma}{d\bar{g}}|_{d\varrho=0} > 0$*.*

Existence of the GIF back to vegetables

Lemma 2 (Existence GIF)

Firstly, let us consider the limiting case of a completely egalitarian or unegalitarian economy. If the economy approaches a completely unegalitarian wealth distribution, i.e. $\underline{\eta} \equiv \eta \searrow \max\{\gamma, 1\}$*, then* $GIF_{g_y}(\underline{\eta}, 0) = \{\varnothing\}$*. Contrary, if the economy approaches a completely egalitarian distribution, i.e.* $\overline{\eta} \equiv \eta \nearrow \infty$ *for some finite* $\gamma \ll \infty$*, then* $GIF_{g_y}(\overline{\eta}, 0) = \{\varrho(\gamma)\}$ *on* $[\underline{\gamma}(\epsilon), \overline{\gamma}(\epsilon)]$*. If simultaneously* $\epsilon \to \infty$*, the GIF is a singleton at* $GIF_{g_y}(\overline{\eta}, 0)|_{\epsilon\to\infty} = \{(\gamma, \varrho) = (1, 0)\}$*.*
Secondly, in the interior case of wealth inequality $1 < \eta < \infty$ *there exists a Pareto tail* η^* *which lies on the* $GIF_{g_y}(\eta^*, 0)$ *under the imposition of bounds on the strength of the wealth dependent risk taking effect* $\underline{\gamma} < \gamma < \overline{\gamma}$*, where* $\underline{\gamma} = \frac{2(2\epsilon-1)}{1+2(2\epsilon-1)}$ *and* $\overline{\gamma} = 2$*.*

(a) *The GIF exists for a unique* $\eta^* \in (\gamma, \infty)$ *if* $1 < \gamma < \overline{\gamma}$ *and* $-1 \leq \varrho \leq 2\frac{(2\epsilon-1)(1-\gamma)}{\gamma}$*.*

(b) *Any Pareto tail* $\eta^* \in (\gamma, \infty)$ *lies on the GIF if* $\gamma = 1$ *and* $\varrho = 0$*.*

(c) *The GIF exists for a unique* $\eta^* \in (\underline{\eta}^{dc}, \infty)$ *if* $\underline{\gamma} < \gamma < 1$ *and* $2\frac{(2\epsilon-1)(1-\gamma)}{\gamma} \leq \varrho \leq 1$*, where* $\underline{\eta}^{dc} > 1$*.*

Result I: return heterogeneity

back to vegetable

Table 4: Returns to wealth across the wealth distribution: data and model

Wealth group	Data[a]				*Scale*-model	*Type*-model			*benchmark*
	PSID	SCF	Norway	Sweden		CRS	DRS[b] base.	DRS[b] scale	
P40-P50	REF.	REF.	REF.	REF.	REF.	REF.	REF.	REF.	REF.
P50-P60	−0.6	|	1.5	0.2	0.0	0.4	0.9	−0.1	0.3
P60-P70	−0.9	−0.4		0.3	0.0	1.0	2.4	−0.2	0.6
P70-P80	−0.8	0.0	2.6	0.3	0.0	2.0	4.4	−0.4	1.0
P80-P90	0.5	0.2		0.5	0.7	3.1	6.2	−0.6	1.6
P90-P95	3.8	1.4	3.6	0.8	2.6	3.2	5.6	−0.9	1.6
P95-P97.5	5.8	2.6	5.2	1.1	3.8	3.1	8.1	−1.0	1.7
P97.5-P99	6.9	3.8		1.5	5.4	5.2	9.2	−1.2	3.2
Top 1%	9.6	4.6	8.3	2.5	7.3	6.7	8.1	−1.5	5.7
Top 0.1%	–	–	–	3.7	8.6	6.1	6.4	−1.8	6.0

[a] Estimates from Xavier (2021) (SCF), Bach et al. (2020) (Sweden), Fagereng et al. (2020) + Halvorsen et al (2021) (Norway).
[b] Type + DRS refers to a prototype entrepreneurship model with DRS on private equity. Cagetti et Denardi (2006), Guvenen et al. (2021)

SCF – portfolio increase at top

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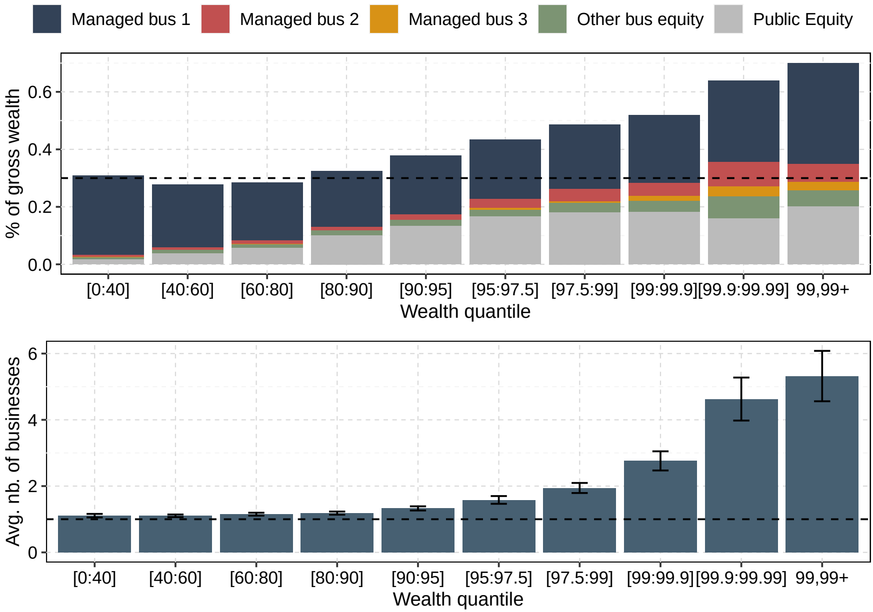
Figure 3: Decomposition into multiple priv. equity business investments, SCF



PSID – participation increase at top

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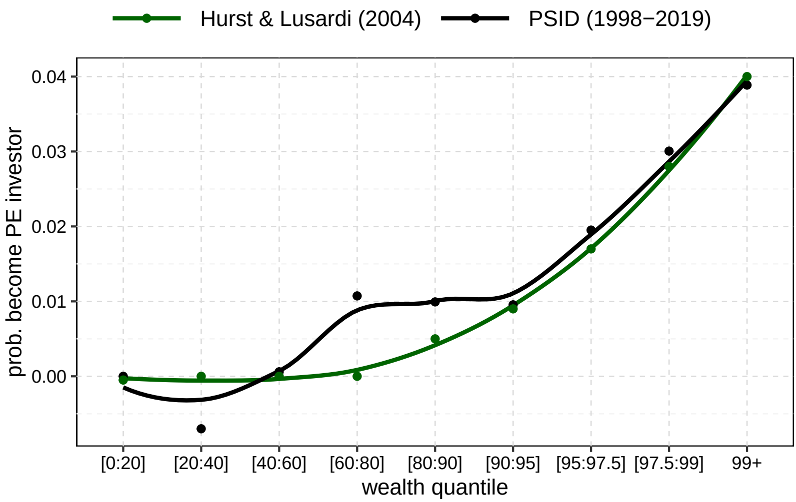

Calibration

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External parameters:

- earnings: mixture log-normal/Pareto + stochastic aging.
- labor share, preferences, depreciation: standard macro values.
- tax system: $\tau_w = 0.22$, $\tau_r = 0.25$, $\mathcal{T}(a) = 0$.
- portfolio shares: three alternatives (described later).

Internal parameters:

- fix benchmark $\mu = 1$, select discount factor β and labor share ϕ.
- match top 1% wealth share and K/Y ratio.

Related Literature Back

Empirics

- Inequality & Output: Benabou (1996, NBER), Barro (2000, JoEG), Forbes (2000, AER), Perotti (1994, EER), Voitchovsky (2005, JoEG) . . .
- Saving Rate, Returns & Wealth: Fagereng et al. (2019, ECMA + WP), Bach et al. (2020, AER), Xavier (2021) . . .
- Risky Firms & Financing: Haltiwanger et al. (2014 JoEP, 2016 EER, 2018 WP), Samila & Sorenson (2011, RES) . . .

Theory

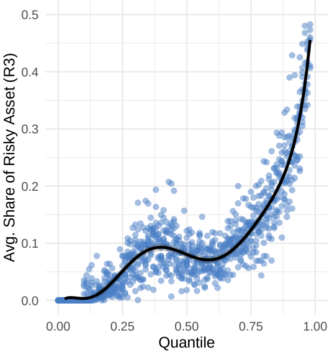
- Incomplete Market, K heterogeneity and Taxation: Conesa et al. (2009, AER), Stantcheva and Saez (2018, JoPE), Guvenen et al. (2019, WP), Straub and Werning (2019, AER), Benhabib and Szöke (2019, WP), Moll (2014, AER), Moll and Itskhokin (2019, ECMA). . .
- Rent-seeking: Piketty (2014), Rothschild et Scheuer (2016, ReStud). . .
- Portfolio Choice & Growth: Acemoglu and Zilibotti (1997, JPE), Bencivenga and Smith (1991, ReStud), Greenwod and Jovanovic (1990, JPE). . .
- Risk-Taking: Peress (2004, RFS), Spiritus and Boadway (2017, WP). . .

Wealth distribution and risk-taking illustration

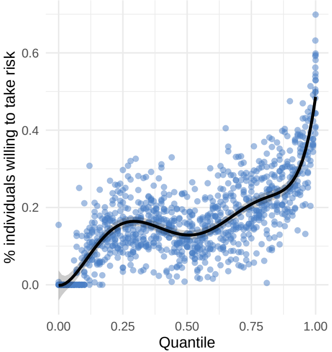
1. Wealth-rich **hold dis-proportionally more risky assets**.
2. Wealth-rich are more **inclined to take risk** Robinson (2019).

→ Appears in many developed economies with regularity Evidence.

→ Combination of **type dependence**+ **wealth dependence**.



(a) Share of risky assets (SCF)



(b) Risk taking index (SCF)

A unified framework to think about inequality/efficiency

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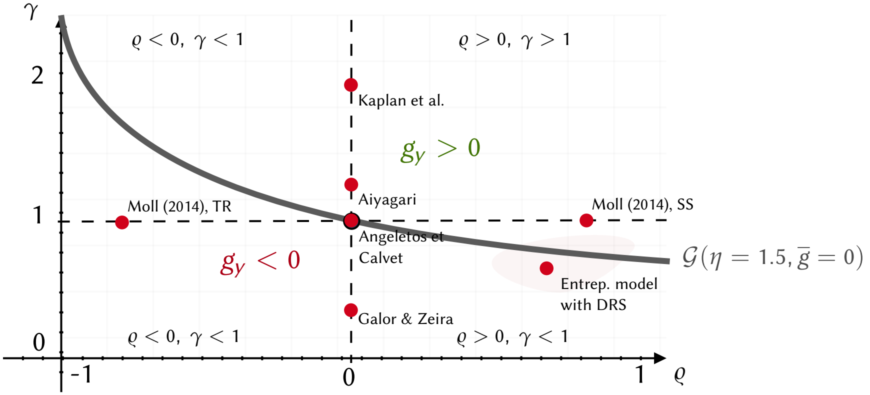
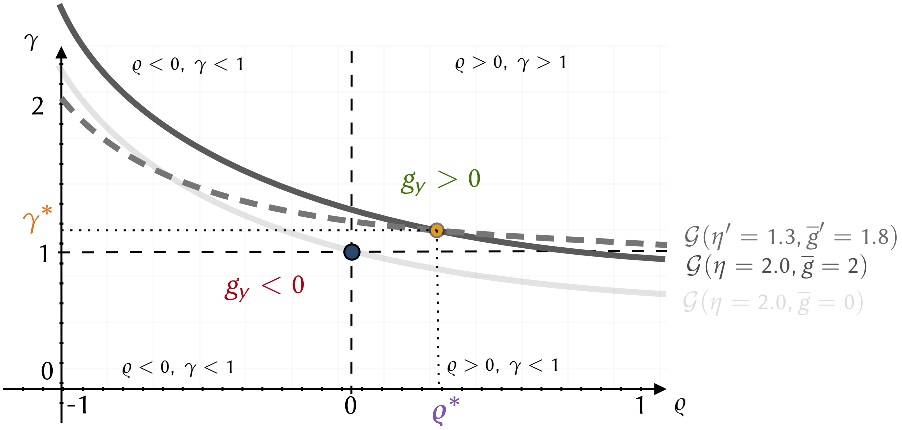
- model with predictions between inequality/efficiency can be understood within the framework.



Is type/wealth dependence identified? back to vegetables

Let $\bar{g} = \partial \mathbb{E}[g_y(\eta)]/\partial \eta$. As macroeconomist, suppose we observe two couples $(\bar{g}, \eta)$ and $(\bar{g}', \eta')$ and the model is correct, then (γ^*, ϱ^*) are identified:



- Requires to measure $\partial \mathbb{E}[g_y(\eta)]/\partial \eta$: Forbes, Banerjee & Duflo, Barro, IG-slope
- **Requires that γ and ϱ have differentiated effects when η moves.**

Returns in the PSID back to intro back to contrib

Recent waves 1999+, include information about:

- debt/asset for PE, stocks, IRA, housing, saving, other assets
- we compute net returns as:

$$r_{it} = \frac{y_{it}^K + y_{it}^I - y_{it}^D}{\left(w_{it-2}^g + w_{it}^g + F_{it}\right)/2},$$

  - w_{it}^g is amount of assets.
  - F_{it} are inflow minus outflow.
  - y_{it}^K, y_{it}^I, y_{it}^D: respectively capital gains, income, debt cost.

Running the following panel estimation:

$$r_{it} = \beta p(a_{it}) + X_{it} + F_i + F_t + u$$

- scale-dep: explains 45% of top 90% excess returns rltv to bottom 10%

Variance decomposition of returns

back to vegetables

Returns heterogeneity is the composition of three components:

- *luck*: captured by ε.
- *type-dependence*: skill/risk tolerance ϑ.
- *wealth-dependence*: through a.

Variance decomposition in the PSID: $\approx 80\%$ of return variance is due to "luck".

- consistent with Fagereng et al. (ECMA,2020) with 27%.

Model-based variance decomposition: $\approx 67\%$ of return variance is due to "luck".

Appendix

A General Equation

- Consider a mean preserving redistribution of wealth s.t. all agents face da_i.
- Normalize productivity in the risk free sector to unity.

The change in efficiency units of aggregate capital can be written as

$$d\mathcal{K}^e = \underbrace{\Delta\mathcal{Z}}_{\text{productivity gap}} \times \underbrace{\text{cov}\left(\text{MPS}_i \times \text{MPR}_i, da_i\right)}_{\text{MPR-MPS interaction}} + \underbrace{\text{cov}\left(\text{MPS}_i, da_i\right)}_{\text{pure MPS}}$$

$$= \Delta\mathcal{Z} \times \text{cov}\left(\epsilon_i \times \phi_i^*, da_i\right) + \text{cov}\left(\text{MPS}_i, da_i\right) \; ,$$

with the ***sufficient statistic*** ϵ_i as risk taking elasticity of wealth and ϕ_i^* as risky asset share prior to redistribution.

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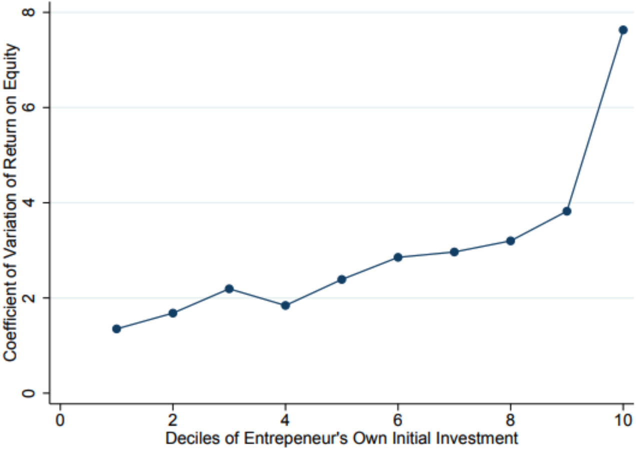
Riskiness and private equity profitability, Robinson (2019)



This figure shows the coefficient of variation of the firm's average return on equity over the 8 yea
sample for the ten deciles of the entrepreneur's own initial investments.

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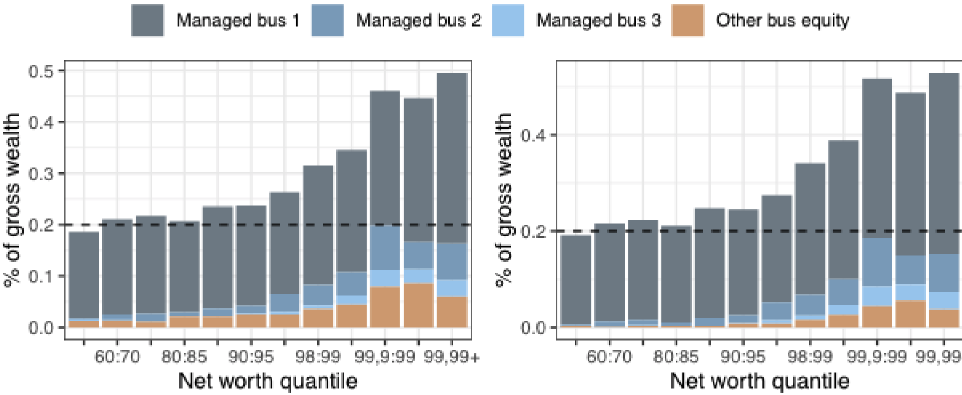
Wealth dependence in private equity

back to vegetables

Figure 1. Average share held in private equity for business owners (left panel) and business owners manage their businesses (right panel).



Source: Author's computation using SCF waves from 1989 to 2019. The dashed line indicates an ave share of private business equity.

Adjusted and Non-Adjusted Survey Estimates

We use up-to-date survey to measure wealth inequality, correcting for:

- Over-sampling: Missing values at the top, wealthy HHs not represented.
- Under-reporting: Survey aggregation $\neq$ national balance sheet.

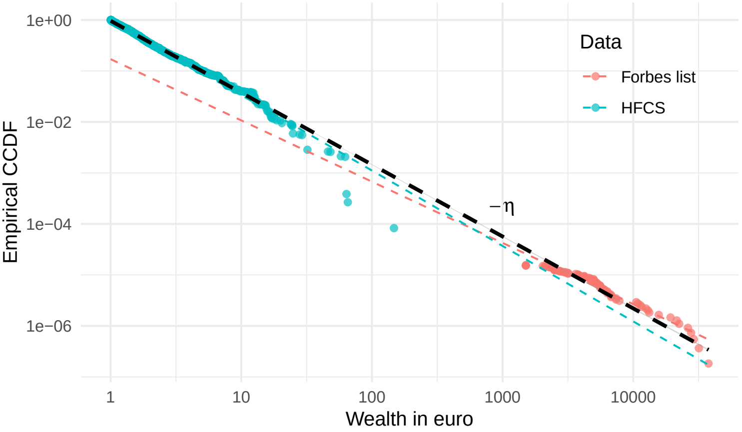
Correction procedure:

1. Wealth distribution well approximated by *Pareto law*.
   - reconstruct top shares using Pareto shape estimates.
2. National accounts to *rescale* financial/non-financial assets and liabilities.
   - harmonize estimates across countries.

$\rightarrow$ Results robust to using adjusted / non-adjusted series.

Over-sampling correction: Pareto Shape Estimation

- Estimate the Pareto shape η using the empirical CDF: $P(a > a) = \left(\frac{a_{min}}{a}\right)^{\eta}$
- $\sim \log(n(a)/n) = -\eta \log(\frac{a}{a_{min}})$, where a_{min} is fixed to 1 million.



(a) Germany

Figure 5: Pareto shape estimation for Germany, log-log scale.

IG-slope: income and wealth

back to vegetable

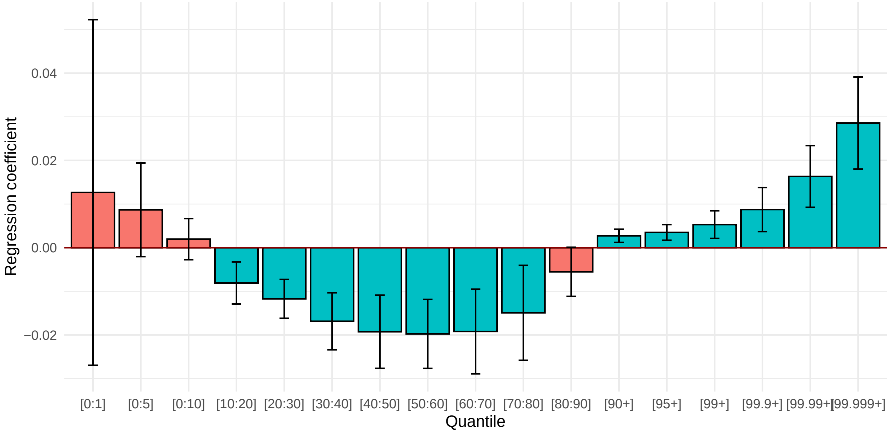


Figure 6: GDP growth and share of income held in different quantile and GDP growth.

IG-slope: income and wealth

back to vegetable

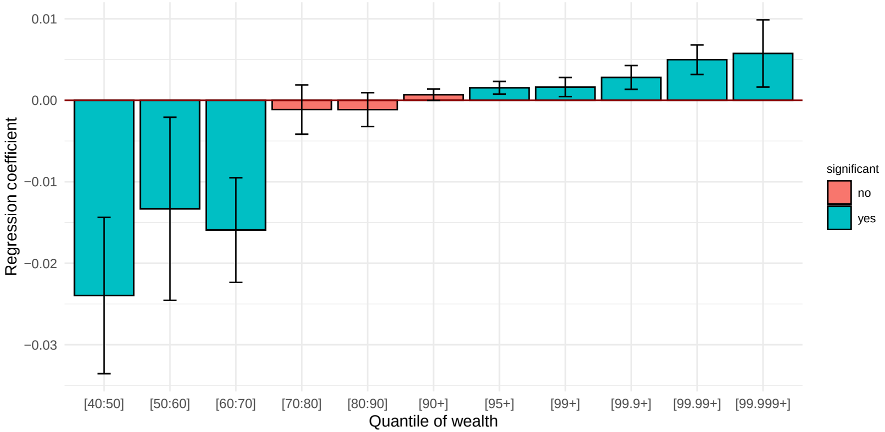


Figure 7: GDP growth and share of wealth held in different quantile and GDP growth.

The household's problem

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$$v(a,\vartheta,e) = \max_{a'} \left\{ u(c) + \beta(1-p_d)\mathbb{E}\left[v(a',\vartheta,e') | e \right] \right\} \tag{7}$$

$$\text{s.t.} \quad c + a' = we(1-\tau_w) + (1 + r(a,\vartheta,j))a - \tau_{cap}\Big[r(a,\vartheta,j)a \Big] - \tau_a(a), \tag{8}$$

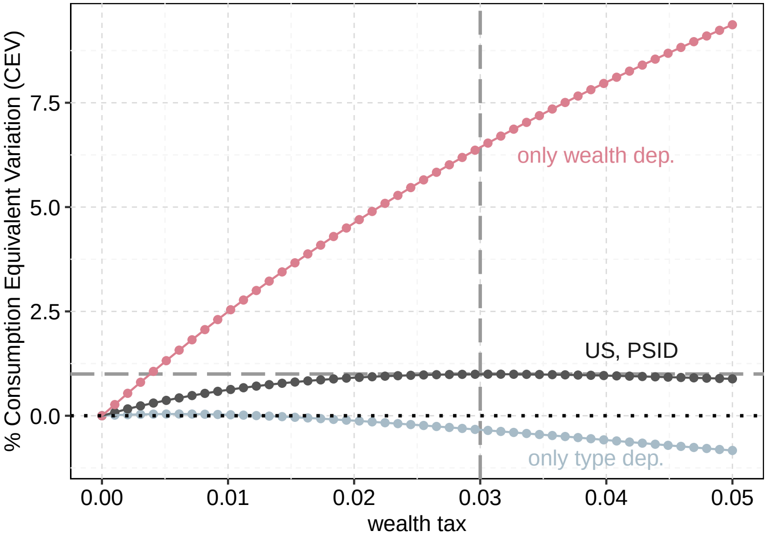
$$a' \geq \underline{a}\,, \tag{9}$$

- w: wage rate, e labor productivity,
- j production shock, ϑ: skill/risk tolerance,
- a': saving,
- $\tau_a(.)$, $\tau_{cap}(.)$, τ_w: wealth, capital and labor tax.

Optimum with full rent-seeking (case $\mu \to 0$) back to vegetables

In that case, returns heterogeneity reflects rent-seeking, and $A_{innov} = A_{tradi}$.

- $\underline{r}$ adjust to make sure that capital distributed = capital perceived,
- high returns from some households means less returns for others,
- wealth-dependence especially harmful, high wealth tax at the top is optimal.



Social welfare calculation

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We measure welfare using consumption-equivalent-variation Mcgrattan (1994).

- the % Δ^{CEV} by which every household's steady-state per-period consumption c has to be changed to make the household indifferent btw *pre* and *post* tax-reform.

$$\underbrace{\int_x \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u((1+\Delta^{CEV})c_t(x)) \right] d\Gamma(x)}_{\text{pre-tax reform}} = \underbrace{\int_x \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(\tilde{c}_t(x)) \right] d\Gamma(x)}_{\text{post-tax reform}}$$

With rent-seeking

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Under rent-seeking, $r_{risky} = MPK_{innov}/\mu$, returns do not reflect only MPK Piketty (2014),

- Let households returns given by:

$$r_i = \underline{r} + r_{safe} + \omega(a, \vartheta) \cdot ((r_{risky} + \varepsilon) - r_{safe}) \tag{10}$$

- if $\mu < 1$, then $\underline{r} < 0$ adjusts to equate returns distributed = returns perceived.

$\rightarrow$ difference in returns do not reflect difference in productivity;