Value-Added and Productivity Linkages Across Countries*

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October 20, 2020

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Abstract

Using data from 40 countries, we show that an increase in bilateral input trade is associated with higher GDP correlation, largely driven by a surge in Solow residual comovement. The association with final good trade is statistically insignificant or negative. Motivated by these new facts, we build a model of international trade that is able to replicate the empirical trade-comovement slope, offering the first quantitative solution for the *Trade Comovement Puzzle*. The model relies on (i) global value chains, (ii) price distortions due to monopolistic competition, and (iii) fluctuations in the mass of firms serving each country. The combination of these ingredients creates a link between domestic measured productivity and foreign shocks through trade linkages, generating a disconnect between technology and measured productivity. Finally, we present evidence that these elements generate a link between foreign shocks and domestic GDP and that an increase in trade is associated with higher correlation in profits.

Keywords: International Trade, International Business Cycle Comovement, Networks, Input-Output Linkages, Solow Residual.

JEL Classification: F12, F44, F62

*The views in this paper are solely the responsibility of the authors and should not necessarily be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. We are indebted to Thomas Chaney for his invaluable guidance. We are also grateful for the comments by Manuel Amador, Ariel Burstein, Patrick Fève, Simon Fuchs, Christian Hellwig, Rob Johnson, Tim Kehoe, Marti Mestieri, Alban Moura, Franck Portier, Ana-Maria Santacreu, Constance de Soyres, Shekhar Tomar, Robert Ulbricht, Kei-Mu Yi, and seminar/workshop participants from various venues. We thank Henry Young for his research assistance. Finally, we thank the Federal Reserve Bank of Minneapolis, where part of this research has been conducted, for their hospitality and the ERC grant N°337272–FiNet for financial support. All errors are our own.

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1 Introduction

The *Trade Comovement Puzzle* (TCP), uncovered by Kose and Yi (2001, 2006), refers to the inability of international business cycle models to quantitatively account for the positive empirical relationship between international trade and GDP comovement.[1] Using international real business cycle (IRBC) models, several authors have succeeded to qualitatively replicate the positive link between trade and GDP comovement but fall short of the quantitative relationship by an order of magnitude.[2]

This paper makes three main contributions. First, it contributes to empirical investigations of the association between bilateral trade and GDP comovement and shows that an increase in bilateral input trade is associated with higher GDP correlation, largely driven by a surge in Solow residual comovement. The association with final good trade is statistically insignificant or negative. Second, it proposes a model of trade in both inputs and final goods with monopolistic pricing and firms entry/exit. In the benchmark calibration, the model is able to replicate the observed trade-comovement slope, offering the first quantitative solution of the TCP. Finally, the paper documents the disconnect between technology and measured productivity in the presence of markups and extensive margin adjustments. Our model generates a trade-*Solow residual* slope in line with the data, largely driven by a surge in profit synchronization.

Empirics. Since the seminal paper by Frankel and Rose (1998), a large empirical literature has studied cross countries' GDP synchronization, showing that pairs of countries with stronger trade linkages tend to have more correlated business cycles. The paper refines previous analyses by constructing a panel dataset of 40 countries consisting of four 10-year windows ranging from 1970 to 2009, which allows for dyadic as well as time windows fixed effects. In this setting, we document that the positive relationship between trade and GDP comovement is mostly driven by *trade in intermediate inputs*, whereas trade in final goods is found insignificant or negative. These new findings suggest a possible link between global value chains (GVC) and the rising synchronization of GDP across countries. Moreover, we take an additional step and show that trade integration is not associated with an increase in the synchronization of labor or capital supply, while it is correlated with a surge in Solow residual comovement. Such a result is useful because it helps discipline our theoretical investigation and directs us towards propagation channels that generate a link between trade and measured productivity comovement.

Theory. As discussed in Kehoe and Ruhl (2008), international production linkages alone do not generate a link between domestic GDP and foreign shocks. With perfect competition

[1] For empirical studies, among many others, see Frankel and Rose (1998), Clark and van Wincoop (2001), Imbs (2004), Baxter and Kouparitsas (2005), Kose and Yi (2006), Calderon et al. (2007), Inklaar et al. (2008), Di Giovanni and Levchenko (2010), Ng (2010), Liao and Santacreu (2015), Duval et al. (2015), and Di Giovanni et al. (2016).

[2] For quantitative studies, see for instance Kose and Yi (2001, 2006), Burstein et al. (2008), Arkolakis and Ramanarayanan (2009), Johnson (2014), and Liao and Santacreu (2015).

and constant returns to scale, firms equalize marginal costs and marginal revenues of imported input, so that changes in the quantity of imported input yield exactly as much benefit as they bring costs. Hence, foreign shocks have an impact on domestic value added *only* to the extent that they impact the supply of domestic factors. This "negative result" is at the heart of the TCP. We incorporate two ingredients that create an endogenous relationship between domestic productivity and foreign shocks through trade linkages.

First, when firms choose their price, they do not equalize the marginal cost and marginal revenue product of their inputs. As noted previously by Hall (1988) and discussed in Basu and Fernald (2002), Gopinath and Neiman (2014), and Llosa (2014), this wedge between marginal cost and marginal product of inputs implies that any change in intermediate input usage is associated with a first order change in value added, over and beyond changes in domestic factors.[3] This result can also be interpreted as saying that when domestic firms charge a markup, real domestic GDP includes aggregate profits. Hence, fluctuations in profits play a role in real GDP fluctuations. In the last part of the paper, we present empirical support for this prediction.

Second, fluctuations along the extensive margin have the potential to create an additional amplification mechanism between domestic productivity and foreign shocks. With "love of variety," any variation in the mass of suppliers leads to a first order productivity change. Love of variety is a form of increasing returns to scale: a firm with more suppliers is more efficient at transforming inputs into output, which leads to an increase of value added over and beyond variations in domestic factor supply. In line with our empirical findings, both ingredients create a link between foreign shocks and domestic productivity measured by the Solow residual.

Quantitative analysis. Motivated by the discussion above, we propose a multi-country dynamic general equilibrium model of international trade in *final goods* and in *intermediate inputs* that relies on (i) monopolistic competition and (ii) fluctuations in the mass of firms serving each country. We calibrate the model to 14 countries and a composite *Rest-Of-The-World* and assess its ability to replicate the strong correlation between trade in intermediate inputs and GDP synchronization. Fixed effect regressions on this simulated dataset show that the model is able to account for the *trade-comovement (TC) slope* observed in the data mainly through trade in intermediate inputs, which is a significant improvement compared to previous studies. Decomposing the role of each ingredient, we show that trade in intermediates alone is not sufficient to replicate the trade-comovement relationship. The addition of monopolistic pricing and extensive margin adjustments increase the simulated TC slope by a factor of seven and improve the model fit. In our model as well as in the data, the surge in GDP comovement associated with higher input

[3] Related to this point, Burstein et al. (2008) show that if all firms take prices as given, a change in trade costs can affect aggregate productivity only to the extent that it changes the production possibility frontier at constant prices. This can be interpreted as saying that shocks to the *foreign* trading technology have no impact on aggregate *domestic* productivity if all firms have constant returns to scale and take prices as given.

trade is largely due to an increase in the synchronization of aggregate profits, which then leads to higher Solow residual comovement. Finally, we show how changes in key parameters, in particular markups and in the Armington elasticity of final goods, allow the model to account for the range of TC-slope estimates found in the literature as well as the negligible role of trade in final goods.

Further empirical evidence. Finally, we provide evidence supporting our modeling assumptions. First, using different measures of monopoly power, we find that countries with higher markups have a GDP that is more systematically negatively correlated with terms-of-trade movements, meaning that they experience a larger GDP decrease when the prices of their imports rise. Furthermore, we show that country pairs that increase bilateral trade also experience a surge in the correlation of their profit, as measured by the *Net Surplus*. Second, we empirically test the correlation between the extensive and intensive margins of trade with country-pair GDP correlations. A higher degree of business cycle synchronization is associated with an increase in the range of goods traded and is not associated with an increase in the quantity traded for a given set of goods.[4]

Relationship to the literature. Starting with Frankel and Rose (1998), a number of papers have studied and confirmed the positive association between trade and comovement in the cross-section. The empirical part of this paper is mostly related to two recent contributions. First, Liao and Santacreu (2015) is the first to study the link between the extensive margin and GDP and TFP synchronization. Second, Di Giovanni et al. (2016) uses a cross-section of French firms and presents evidence that international I/O linkages at the micro level are an important driver of the value added comovement observed at the macro level. Their evidence is in line with the findings of this paper.[5]

If the empirical link between bilateral trade and GDP comovement has long been known, the underlying economic mechanism of this relationship is still unclear. Using the workhorse IRBC model with three countries, Kose and Yi (2006) have shown that their model can explain at most 10% of the *slope* between trade and business cycle synchronization, leading to what they call the *Trade Comovement Puzzle* (TCP). Since then, many papers have refined the puzzle, highlighting ingredients that could bridge the gap between the data and the predictions of classic models.

[4] This result is in line with the analysis in Liao and Santacreu (2015), which emphasizes the role of the extensive margin. Compared to them, we add a panel dimension and perform fixed effect regressions, which allow us to control for country-pair fixed effects that can be correlated with trade intensity. Moreover, we further the analysis and relate GDP comovement to the standard deviation of each margin, showing that an increase in the variance of extensive margin fluctuations is associated with higher GDP correlation.

[5] Relatedly, Ng (2010) uses cross-country data from 30 countries and shows that bilateral production fragmentation has a positive effect on business cycle comovement. The concept of bilateral production fragmentation used is different from this paper as it takes into account only a subset of trade in intermediates, namely imported inputs that are then further embodied in exports. Moreover, the cross-section nature of the analysis does not allow for either dyadic or time windows fixed effects.

Burstein et al. (2008) show that allowing for production sharing among countries can deliver tighter business cycle synchronization if the elasticity of substitution between home and foreign intermediate inputs is extremely low. Arkolakis and Ramanarayanan (2009) analyze the impact of vertical specialization on the relationship between trade and business cycle synchronization. Their model with perfect competition does not generate significant dependence of business cycle synchronization on trade intensity, but they show that the introduction of price distortions that react to foreign economic conditions allows their model to better fit the data. Incorporating trade in inputs in an otherwise standard multi-country IRBC model, Johnson (2014) shows that adding international input-output (I/O) linkages *alone* is not sufficient to solve the trade-comovement puzzle, but the paper points that such production linkages do synchronize input usage. Compared to those studies, we add firms' entry/exit and monopolistic competition and argue that those are key ingredients for the model to deliver quantitative results in line with the data. Liao and Santacreu (2015) builds on Ghironi and Melitz (2005) and Alessandria and Choi (2007) to develop a two-country IRBC model with trade in differentiated products. Compared to this paper, our analysis adds multinational production in a multi-country setup. Our framework features "global value chains" in the sense that firms are engaged in both importing and exporting activities, and goods cross multiple borders. This creates interdependencies in firms' pricing and export decisions. We also highlight both quantitatively and empirically the role of markups and extensive margin fluctuations in the synchronization of GDP and measured productivity.[6] Finally, a complementary approach has been developed by Drozd et al. (2020), which models the dynamics of trade elasticity in final goods and uses GHH preferences. In contrast, our empirical evidence suggest that trade in final goods is not associated with higher GDP correlation, and underlines that the increase in GDP comovement associated with input trade is largely driven by Solow residual synchronization. Our model is in line with these observations.

2 Empirical Investigation

We start by updating the initial Frankel and Rose (1998) (henceforth, FR) analysis on the relationship between bilateral trade and GDP comovement. We then provide empirical support for the specific role of *trade in intermediate inputs* in this relationship. Finally, decomposing GDP fluctuations into changes in factor supply and the Solow Residual, we show that while trade is associated with an increase in GDP synchronization, it is not associated with higher synchronization of labor or capital supply – implying that the surge in synchronization is driven by comovement of the Solow Residual.

[6] In the model proposed in Liao and Santacreu (2015), no firm is both an importer *and* an exporter. The absence of production linkages makes it essentially a model of trade in final goods where domestic and foreign goods are substitutes. This, in turn, creates forces toward negative GDP correlation as is illustrated by the *negative* association between trade and GDP comovement in their Table 9, when the elasticity of substitution is equal to 3.1.

2.1 Trade and GDP comovement

Our sample is composed of 40 countries, which account for around 90% of world GDP, and cover the period stretching from 1970 to 2009. We use annual data on real GDP at chained PPPs from the 9th Penn World Table, which is transformed in two ways: (i) HP filter with smoothing parameter 6.25 to capture the business cycle frequencies and (ii) log first difference. Trade data come from Johnson and Noguera (2017) who combine data on exports, imports, production, and inputs use to construct bilateral trade flows from 1970 to 2009 separating between trade in final goods and trade in intermediate inputs within main sectors: agriculture, service, non-manufacturing and manufacturing.[7] We construct a symmetric measure of bilateral trade intensity (hereafter "trade intensity") using the sum of total exports ($T_{i \to j}$) from country i to j and total imports ($T_{j \to i}$), such as: $\text{Trade}_{ij} = \frac{T_{i \to j} + T_{j \to i}}{GDP_i + GDP_j}$, and measure the importance of the trade relationship relative to total GDP.[8] Furthermore, we disentangle trade intensity in inputs and final goods by constructing indexes $\text{Trade}_{ij}^{\text{final}} = \frac{T^F_{i \to j} + T^F_{j \to i}}{GDP_i + GDP_j}$ and $\text{Trade}_{ij}^{\text{input}} = \frac{T^I_{i \to j} + T^I_{j \to i}}{GDP_i + GDP_j}$ by taking into account only the exports and imports in final and intermediate goods respectively. In practice, as standard in the literature, we take the natural logarithm of both ratios.[9]

The extent to which countries have correlated GDP can be influenced by many factors beyond international trade, including correlated shocks, financial linkages, common monetary policies, etc. Because those other factors can themselves be correlated with the index of trade proximity in the cross section, using cross-section identification could yield biased results. Indeed, in their seminal paper, FR use cross-sectional variations to evaluate whether bilateral trade intensity correlates with business cycle synchronization, but their specification does not rule out omitted variable bias such as, for example, the fact that neighboring countries have at the same time more correlated shocks and larger trade flows. By constructing a panel dataset and controlling for both country-pair and time windows fixed effects, this paper relates to recent studies that try to control for unobserved characteristics.[10] Therefore, in order to separate the effect of trade linkages from other unobservable elements, we construct a panel dataset by creating four periods

[7] We provide additional details on data sources and the list of countries in the online appendix A.1.

[8] We also used an index defined as $\text{Total}_{ij} = \max\left(\frac{\textit{Total Trade}_{ij}}{GDP_i}, \frac{\textit{Total Trade}_{ij}}{GDP_j}\right)$. This measure has the advantage to take a high value whenever one of the two countries depends heavily on the other for its imports or exports. Both our empirical and simulated results hold when we use this index.

[9] To be more precise, we first apply the log transformation on trade intensities and then we average over the time windows. This is motivated by the fact that the original trade data grow exponentially from 1970 to 2009. We also report the results of the regressions using the log transformation on the mean trade intensities in the online appendix A.3.3. Results are quite similar. In section A.3.4 of the online appendix, we also report the results using the *level* of trade intensities and show that our findings are robust to this specification. Finally, notice that the log specification has a larger explanatory power (measured by the R^2) compared to regressions in levels.

[10] Di Giovanni and Levchenko (2010) includes country pair fixed effects in a large cross-section of industry-level data with 55 countries from 1970 to 1999 in order to test for the relationship between sectoral trade and *output* (not *value-added*) comovement at the industry level. Duval et al. (2015) includes country pair fixed and year effects in a panel of 63 countries from 1995 to 2013 and test the importance of *value added* trade in GDP comovement.

of ten years each.[11] Within each time window, we compute GDP correlation (Corr GDP) as well as the average trade intensities defined above.

We then estimate two panel data regressions. In a first exercise, we follow the existing literature by running linear regression estimation of Corr GDP_{ijt} on the log of trade intensity Trade_{ijt}:

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(\text{Trade}_{ijt}) + \text{controls}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{1}$$

where i and j denote the two countries and t the time window. CP_{ij} and TW_t stand for country-pair and time windows fixed effects. In a second exercise, we run the regression on the log of trade intensity disaggregated into final goods and inputs:

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(\text{Trade}_{ijt}^{\text{input}}) + \beta_2 \ln(\text{Trade}_{ijt}^{\text{final}}) + \text{controls}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{2}$$

We finally specify the additional controls that we include (one-by-one) in the analysis. First, we include dummy variables for countries among the European Union (each wave is entitled a different dummy variable) and the Euro Area. Second, we construct two additional measures that capture the effect of *trade network* (third country effect) and the *sectoral composition* of trade.[12] Our "third country" index is motivated by the fact that two countries with similar partners could co-move because of their link with common partners. Moreover, our "sectoral" index controls for changes in specialization. If shocks have a sectoral component, then two countries that tend to specialize over time in the same sectors could have an increase in business cycle comovements over and beyond any direct trade effects. The third index is specified as $third_{country}(i,j) = 1 - \frac{1}{2}\sum_{k \neq i,j} \left| \frac{T_{i \to k} + T_{k \to i}}{\sum_k T_{i \to k} + T_{k \to i}} - \frac{T_{j \to k} + T_{k \to j}}{\sum_k T_{j \to k} + T_{k \to j}} \right|$. It measures the degree of similarity in the geographical distribution of trade shares between country i and country j, and is equal to 0 if countries i and j have completely separated trade partners while it is equal to 1 if all trade shares are equal. The sectoral composition index is constructed based on 2-digit SITC trade data as $sector_{proximity}(i,j) = 1 - \frac{1}{2}\sum_{s \in \mathcal{S}} \left| \frac{T_i(s)}{\sum_{s \in \mathcal{S}} T_i(s)} - \frac{T_j(s)}{\sum_{s \in \mathcal{S}} T_j(s)} \right|$, with $T_i(s)$ the total export of country i in the specific sector (or products) s in the set of sectors $\mathcal{S}$. This index controls for the composition of trade and can be thought of as measuring common sectoral specialization within each country-pair: if two countries export exactly the same share of each product, then the index is equal to 1. For those two indexes, we use bilateral trade data (SITC4 REV. 2) from the Observatory of Economic Complexity.

In columns (1) and (5) of table 1, we first report results using only *within* country-pair variations without time window fixed effects (FE). Our estimates are significant and consistent with

[11] Adding time windows fixed effect controls for the recent rise of world GDP correlation since the 90s, which could be unrelated to trade intensity.

[12] In the online appendix A.3.2, we also control for sectoral composition of total value added. However, due to missing data, the sample is much smaller.

those in the empirical literature (ranging from a trade-comovement slope of about 4.8% and 11% in log), and show a positive relationship between bilateral trade and GDP correlation.[13] Then, in columns (2) and (6), we run the same regression controlling for aggregate time windows fixed effects. When controlling for both country-pair and time windows FE, the positive relationship between trade and GDP correlation still holds for HP filter and first differences, but effects are significantly dampened and about half as large as what is implied without time windows FE. By including time windows FE, our aim is to control for changes in the average correlation of shocks across all countries between different time windows. However, we note that if the average correlation of shocks is constant over time, the inclusion of these fixed effects also captures the effect of the observed worldwide increase in trade proximity. As such, including these FE could create a negative bias in our estimates, a prediction that seems in line with the reduction of the measured slopes.

In columns (3), (4), (7) and (8) of table 1, we separate trade in intermediate inputs from trade in final goods. Results highlight a significant positive relationship between GDP correlation and trade in inputs, while trade in final goods is found insignificant.[14] Interestingly, adding time window FE only slightly reduces the correlation between trade proximity in inputs and GDP comovement. Provided that the median increase of the log trade intensity in intermediate goods between 2000-2009 and 1970-1979 is about 1.84, the slope coefficient implies an associated increase of GDP correlation of 9.8%, a non negligible increase.

We then add our controls in table 2, where columns (1) and (5) report the regression results without the additional controls. In columns (2) and (6) we show the results with EU and USSR dummies while in columns (3) and (7) we include the third country index. Finally, columns (4) and (8) include our index controlling for the sectoral composition of trade. In all specifications, trade in intermediate inputs is shown to be significant at 5% while trade in final goods is insignificant (or weakly negatively correlated). Notice also that the effect of *trade network* is high and significant; implying that there is a relationship between GDP comovement and the fact that two countries have similar trade partners.[15]

[13] Frankel and Rose (1998) (FR), estimate an elasticity of nominal GDP comovement to trade intensity of about 4.8%, using a different set of 21 countries, time period (1957 to 1997) and three instrumental variables (IV) for trade intensity: (i) log of distance between countries, (ii), dummy for common border, (iii) dummy for common language. With a specification similar to FR, Kose and Yi (2006) use 21 countries from 1970-2000 and find an elasticity of trade intensity and GDP of 9.1% using HP-filtered GDP and 7.8% using log-difference. Finally, using the same measure for trade intensity as in this paper without time window fixed effects, they estimate a coefficient β of about 0.115. In a similar way, Liao and Santacreu (2015) use IV estimation over a sample of 30 countries covering the period between 1980 and 2009 and find estimates between 0.112 (HP filter) and 0.066 (FD). In appendix A.3, we also provide estimates using the 1970-1999 period in the fifth row of table 16.

[14] Di Giovanni and Levchenko (2010) investigates the role of vertical linkages in *output* synchronization (not *value added*) using I/O matrices from the BEA. Their estimates imply that vertical production linkages account for some 30 percent of the total impact of bilateral trade on the business cycle correlation.

[15] Results presented here use a fixed effect specification. To discriminate between fixed or random effects, we run a Hausman test which display a significant difference ($p < 0.001$), and we therefore reject the random effect model. We also test the need for time-windows effects against the alternative without time-windows FE. The results of a

Table. 1. Trade proximity and GDP correlation [a]

	Corr GDP$^{\text{HP filter}}$				Corr ΔGDP			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ln(Trade)	0.055***	0.022**			0.044***	0.027**		
	(0.007)	(0.011)			(0.006)	(0.011)		
ln(Trade$^{\text{input}}$)			0.054**	0.053**			0.055**	0.042*
			(0.025)	(0.024)			(0.023)	(0.023)
ln(Trade$^{\text{final}}$)			0.003	−0.030			−0.008	−0.016
			(0.022)	(0.024)			(0.020)	(0.023)
Country-pair	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time window	No	Yes	No	Yes	No	Yes	No	Yes
N	2,900	2,900	2,900	2,900	2,900	2,900	2,900	2,900
R^2	0.035	0.153	0.037	0.155	0.024	0.141	0.024	0.142

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs.
[a]We use four time windows of 10 years each from 1970 to 2009.

Our results are also robust to a number of alternative specifications, time periods, time windows, different sets of countries (excluding Euro area or European countries), world GDP correlation and an alternative dataset and method of separating intermediate from final goods. We provide an overview of those results in table 16 in appendix. We also provide in table 13 in appendix results with financial controls. We finally disaggregate further the role of intermediate inputs by main sectors in the supplementary appendix A.3.7, and find that the manufacturing and non-manufacturing industrial sectors play a key role in the positive relationship between trade proximity and GDP correlation.

2.2 Trade and GDP components

To better understand the source of the positive association between GDP comovement and trade, we now propose a simple refinement to the empirical analysis performed so far.

We decompose GDP fluctuations into changes in factor supply (labor and capital) and variations of the Solow Residual (SR). A natural question to ask is: if trade is associated with higher GDP correlation, is it also associated with higher synchronization of labor and/or capital movements? This is an important question because its answer should guide our theoretical construction. For example, Johnson (2014) notes that, in a perfectly competitive framework with constant returns to scale, "*real value added depends on productivity and factor inputs alone*". In his framework, when the correlation of technology shocks across countries is independent of trade, then the trade-GDP comovement slope can only be generated by an increased correlation of factor supply.

However, in a framework where the Solow Residual does not only measure technology, one

Lagrange multipliers test provide strong support for the model with time-windows fixed effects ($p < 0.001$). The supplemental appendix provides many other robustness tests with alternative datasets and time windows.

Table. 2. Trade proximity and GDP correlation with controls

	Corr $GDP^{HP\ filter}$				Corr ΔGDP			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\ln(\text{Trade}^{input})$	0.053^{**}	0.059^{**}	0.060^{**}	0.061^{**}	0.042^{*}	0.050^{**}	0.052^{**}	0.049^{**}
	(0.024)	(0.024)	(0.024)	(0.024)	(0.023)	(0.023)	(0.023)	(0.023)
$\ln(\text{Trade}^{final})$	-0.030	-0.038	-0.047^{*}	-0.048^{*}	-0.016	-0.024	-0.035	-0.033
	(0.024)	(0.024)	(0.025)	(0.025)	(0.023)	(0.023)	(0.023)	(0.023)
$sector_{prox}$				0.088				-0.247^{*}
				(0.146)				(0.138)
$third_{country}$			0.307^{**}	0.305^{**}			0.400^{***}	0.407^{***}
			(0.149)	(0.150)			(0.141)	(0.141)
Country-Pair FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time Window FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
USSR + EU dum.	No	Yes	Yes	Yes	No	Yes	Yes	Yes
N	2,900	2,900	2,900	2,900	2,900	2,900	2,900	2,900
R^2	0.155	0.167	0.170	0.170	0.142	0.155	0.159	0.160

Notes: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$. In parenthesis: std. deviation. SE clustered on country-pairs.

should not focus on the comovement in factor supply. As will be clear below, we argue that a key element in solving the Trade Comovement Puzzle lies in our recognition that real GDP fluctuations are not restricted to movements in technology, labor and capital.

We examine this issue by running two sets of regressions. We first investigate the relationship between trade and factor supply synchronization. Denoting Corr L_{ijt} and Corr I_{ijt} the correlation of labor and investment between countries i and j at time t, we estimate:

$$\text{Corr } L_{ijt} = \beta_1 \ln(\text{Trade}_{ijt}^{\text{input}}) + \beta_2 \ln(\text{Trade}_{ijt}^{\text{final}}) + \text{controls}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \quad (3)$$

$$\text{Corr } I_{ijt} = \beta_1 \ln(\text{Trade}_{ijt}^{\text{input}}) + \beta_2 \ln(\text{Trade}_{ijt}^{\text{final}}) + \text{controls}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \quad (4)$$

Second, we construct the Solow Residual for each country as: $SR_{it} = log(GDP_{it}) - \alpha log(K_{it}) - (1-\alpha)\, log(L_{it})$, with $\alpha = 1/3$. We then compute all country-pair correlations of Solow Residuals for each time window.[16] In line with previous specifications, we estimate:

$$\text{Corr } SR_{ijt} = \beta_1 \ln(\text{Trade}_{ijt}^{\text{input}}) + \beta_2 \ln(\text{Trade}_{ijt}^{\text{final}}) + \text{controls}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \quad (5)$$

Results in table 3 reveal two key insights for the relationship between trade and GDP synchronization. First, looking at columns (5) and (6), we note that higher trade integration is not associated with an increase in labor comovement. In terms of modelling, this implies that a model where GDP comovement is achieved by inducing a strong labor supply reaction to a foreign shock is likely to be at odds with the data. Second, looking at columns (1) to (4), we note

[16] Capital stock and labor are measured using the variable *rkna* and *emp* in the PWT 9.1.

Table. 3. Trade and SR, K and L correlation with 10 years time windows

	Corr SR$^{\text{HP filter}}$		Corr I$^{\text{HP filter}}$		Corr L$^{\text{HP filter}}$	
	(1)	(2)	(3)	(4)	(5)	(6)
ln(Trade$^{\text{total}}$)	0.025*		-0.055^{***}		-0.032^{**}	
	(0.013)		(0.014)		(0.016)	
ln(Trade$^{\text{input}}$)		0.059**		0.057**		0.022
		(0.026)		(0.026)		(0.026)
ln(Trade$^{\text{final}}$)		-0.034		-0.114^{***}		-0.056^{**}
		(0.025)		(0.027)		(0.027)
CP + TW FE	Yes	Yes	Yes	Yes	Yes	Yes
Third country	Yes	Yes	Yes	Yes	Yes	Yes
URSS + EU dum.	Yes	Yes	Yes	Yes	Yes	Yes
N	2,367	2,367	2,367	2,367	2,367	2,367
R^2	0.211	0.212	0.257	0.262	0.112	0.114

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs. SR is computed using PWT 9.1, with $SR = log(rgdpna) - \alpha log(rkna) - (1-\alpha)log(emp)$ and $\alpha = 0.33$. Results are robust to the use of *rnna* and various values of α. Compared to table 2, sample is reduced due to lack of observations. Investment I is computed using PWT 9.1 using capital investment in structure, machinery, transport and other investment. We deflate investments using their corresponding price index.

that while total trade proximity is only significantly associated with the synchronization Solow Residual, capital stocks are more correlated for when countries increase bilateral input trade.

Recall that our specifications are not intended to use information about the *level* of GDP comovement across countries, but rather to account for the *change* in GDP comovement when countries are more integrated through trade. Hence, although the *level* of factor supply synchronization is high in the data (as discussed in appendix B.2), this synchronization does not seem to systematically increase with trade proximity. As a result, we argue that an important part of the high value of the observed trade-comovement slope comes from an increase in the synchronization of the Solow Residual. In turn, we highlight below that Solow Residual comovement can arise from a synchronization of aggregate profits, which could increase with production linkages as measured by trade in inputs – a prediction supported in the data in section 6.3.

3 A simple model

For the sake of exposition, we consider here a static small open economy. In such a world, Kehoe and Ruhl (2008) (henceforth KR) show that a change in the price of imported inputs has no impact, up to a first order approximation, on measured productivity. Therefore, any change in GDP is due to variations in domestic factors supply. We start by briefly reviewing this result.

3.1 The Kehoe and Ruhl (2008) negative result

The economy produces a final good y, used for consumption and exports, which is produced by combining imported inputs x and domestic factors of production ℓ (possibly a vector), according to $y = F(\ell, x)$, where $F(.,.)$ has constant returns to scale and is concave with respect to each of its arguments. The final good producer chooses domestic factors and imported inputs to maximize profit, taking all prices as given. Optimality requires that factors are paid their marginal product and we have $p_y F_\ell(\ell, x) = w$ and $p_y F_x(\ell, x) = p_x$, with p_y the final good price, p_x the price of imported inputs x and w the price of domestic factors.

Gross Domestic Product is the sum of value added in the country and can be computed as the value of final goods minus the value of imported inputs. Importantly, many statistical agencies use *base period prices* when valuing estimated quantities in the construction of GDP.[17] Since prices are kept constant at their base value, we denote them with the superscript b to emphasize the fact that they are treated as parameters and not as endogenous objects:

$$GDP = p_y^b F(\ell, x) - p_x^b . x \tag{6}$$

Let us now compute the first order change in GDP resulting from a variation of the Terms-of-Trade p_x:

$$\frac{dGDP}{dp_x} = \underbrace{p_y^b F_\ell(\ell, x) \frac{\partial \ell}{\partial p_x}}_{\text{Factor Supply Effect}} + \underbrace{\frac{\partial x}{\partial p_x}(p_y^b F_x(\ell, x) - p_x^b)}_{\text{Input-Output Effect}} \tag{7}$$

The first term in equation (7) captures the value added change due to variations in factor supply. Quantitatively, this terms depends on the degree of complementarity between foreign and domestic inputs as well as on the elasticity of factor supply.[18] The second term captures the direct impact that changes of imported input usage have on GDP. With perfect competition, profit maximization insures that $p_y F_x(\ell, x) = p_x$ when using current prices. When base period prices p_y^b and p_x^b are close to their current value,[19] this term vanishes. In such a model, any first order change in GDP following a terms of trade shock is solely driven by variations in domestic factor supply. This is the negative result presented in KR: when firms take prices as given, profit maximization insures that the marginal benefit of using an additional unit of imported

[17] The Penn World Tables used in our empirical section uses base period prices. The Bureau of Economic Analysis uses a Fisher chain-weighted price index to construct GDP at time t relative to GDP at time $t-1$ according to:

$$\frac{GDP_t}{GDP_{t-1}} = \left(\frac{\sum_k p_{t-1}^k q_t^k}{\sum_k p_{t-1}^k q_{t-1}^k}\right)^{0.5} \left(\frac{\sum_k p_t^k q_t^k}{\sum_k p_t^k q_{t-1}^k}\right)^{0.5}$$

where k indexes all components of GDP. Intuitively, the Fisher index is a geometric average between two base period pricing methods where the base price is alternatively the price at $t-1$ and at t.

[18] The role of complementarity is discussed at length in Burstein et al. (2008) or in Boehm et al. (2015).

[19] With a Fisher chain-weighted price index in the construction GDP, base period prices are always close to current prices.

input x ($p_y F_x(\ell, x)$) is equal to its marginal cost (p_x). Up to a first order approximation, foreign technological shocks affect real GDP only through a change in factor supply. In other words, the measured productivity is not affected by foreign shocks. Note that an important part of the reasoning rests upon the fact that GDP is constructed using constant base prices. If the prices used to value final goods and imported inputs were to change due to the shock, one would have an additional term in equation (7).

Equation (7) encapsulates in a simple way the reasons why trade is not a powerful channel of propagation in standard IRBC models. In models with perfect competition and constant returns to scale, the change in domestic GDP in response to a foreign shock is driven by variations in domestic factors supply. Such a change, in turn, is disciplined by (i) the elasticity of labor supply and (ii) the complementarity between domestic and foreign inputs. If domestic and foreign inputs are complement, any shock that increases foreign input usage also rises demand for domestic inputs, which then increases GDP to the extent that factors are elastic. However, as shown in Johnson (2014), complementarity in production factors *alone* is not sufficient to solve quantitatively the TCP.[20]

3.2 Markups and love of variety

Consider now a variant of the economy described above with an additional production step: inputs are imported by a continuum of intermediate producers with a linear production function $m = x$. Critically, we now add two new elements: (1) a *price wedge* for intermediate producers $\mu > 1$ so that the price of intermediates m is given by $p_m = \mu \times p_x$, and (2) love of variety in the final good production technology in the form of a Dixit-Stiglitz aggregation of intermediates.[21] The production function in the final good sector is:

$$y = F(\ell, \mathcal{I}) \qquad \text{with } \mathcal{I} = \left(\int_0^{\mathcal{M}} m_i^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}} \tag{8}$$

This production function displays love of variety: for a given amount of total imports, the larger the mass of input suppliers $\mathcal{M}$, the higher the amount of final production obtainable.

For each variety m_i, there is a producer with a linear technology using imports only:

$$\forall\, i \in [0, \mathcal{M}], \quad m_i = x_i \tag{9}$$

[20] More precisely, Johnson (2014) shows that cross country production synchronizes input usage. Our paper takes this insight on-board and further shows that input synchronization can also lead to GDP synchronization when one adds markups and extensive margin adjustments.

[21] In many models, the elasticity of substitution in the CES aggregation governs at the same time the markup charged by monopolistic competitors and the degree of love of variety. In order to clearly differentiate the sheer effect of markup from the love of variety, we assume here that the markup μ can take any value, including the case where $\mu = \sigma/(\sigma - 1)$.

All intermediate producers are completely symmetric and we denote by m their (common) production and by x their (common) import levels. The bundle $\mathcal{I}$ can then be simply expressed as $\mathcal{I} = \mathcal{M}^{\sigma/(\sigma-1)}m$ and the price index dual to the definition of the bundle is $\mathcal{P} = \mathcal{M}^{1/(1-\sigma)}p_m$, which is also equal to $F_{\mathcal{I}}(\ell, \mathcal{I})$, the marginal productivity of the input bundle in final good production. Finally, taking the derivative of GDP with respect to p_x while keeping prices constant at their base period value, we obtain:

$$\frac{dGDP}{dp_x} = \underbrace{p_y^b F_\ell(\ell, \mathcal{I})\frac{\partial \ell}{\partial p_x}}_{\text{Factor Supply Effect}} + \underbrace{\left(\mathcal{M}\frac{\partial m}{\partial p_x} + \frac{\partial \mathcal{M}}{\partial p_x}m\right).(\mu-1)\,p_x^b}_{\text{Markup Effect}} + \underbrace{\frac{1}{\sigma-1}p_m^b m\frac{\partial \mathcal{M}}{\partial p_x}}_{\text{Entry/Exit Effect}} \tag{10}$$

Equation (10) is the counterpart of (7) in a model with extensive margin adjustments and where some domestic firms are not price takers. These two elements create a link between foreign shocks and domestic real GDP variations, over and beyond any change in domestic factor supply.

First, the existence of a price wedge $\mu > 1$ means that the first term does not vanish. With $m'(p_x) < 0$, a decrease in the price of imported inputs leads to an increase in GDP. When firms are price setters and earn a positive profit, the marginal revenue generated by an additional unit of imported input x is larger than its marginal cost p_x. Hence, cheaper inputs means more sales, more profit and more value added.

Moreover, any change in the mass of firms $\mathcal{M}$ also impacts domestic value added. One can model many reasons why the mass of producing firms would change, including a free entry condition as in the quantitative model in section 4. A change in the number of price setting firms gives a time varying element to the effect described above, triggering a greater reaction of GDP after a foreign shock, independently of the love of variety which is captured by the parameter σ. Overall, the key idea governing this term can be expressed as follows: monopoly power creates a disconnect between the marginal cost and the marginal revenue product of imported inputs. The difference between these two is accounted in aggregate value added in the form of profits. Hence, any change in input usage leading to a change in profits triggers a change in value added.

Second, when $\sigma < +\infty$, another effect arises. When the production function exhibits love of variety, any change in the mass of suppliers implies an additional reaction for the input bundle $\mathcal{I}$. If the decrease of p_x is accompanied by an increase in the mass of producing firms,[22] the bundle $\mathcal{I}$ increases not only because each intermediate producer produces more, but also because an increase in the mass of firms mechanically increases $\mathcal{I}$ even for a fixed amount of intermediates.

With love of variety, a producer who has access to more suppliers can produce more output for the same level of input. In other words, the set of feasible combinations of output $\mathcal{I}$, and

[22] If the mass of firms is pinned down by a free entry condition, the increase in profits of each intermediate producer when the price of imported input goes up leads to a increase in the mass of firms.

inputs $\int_0^{\mathcal{M}} m_i di = X$ is not independent of the mass of producers $\mathcal{M}$: a change of $\mathcal{M}$ shifts the production possibility frontier. Interestingly, this channel is at work independently of the price distortion channel discussed previously. Even in the absence of monopoly pricing, the sheer fluctuation in the mass of producing firms coupled with a love of variety creates a link between import price and GDP fluctuation.

Finally, note that the introduction of markups and love of variety allows GDP to change over and beyond changes in the domestic factors of production. Using a *growth accounting* perspective, the introduction of profits and extensive margin adjustments makes measured domestic productivity change after a foreign shock, even though technology is unchanged. In other words, one cannot decompose GDP fluctuations into changes in factor supply and technology. Our analysis shows that two countries trading intermediate inputs should have correlated measured productivity (i.e. *the Solow Residual*), a prediction we test in the data in section 6.4 and which our quantitative model is able to reproduce.

4 A Model of International Trade with Cross-Border Input Linkages

We develop a many-country international business cycle model with trade in final and intermediate goods. The model is related to Ghironi and Melitz (2005) and Alessandria and Choi (2007), extended to multiple asymmetric countries and intermediate goods crossing borders multiple times. In contrast to a standard IRBC framework, the model features *monopolistic competition* and *firms entry/exit*.[23] As we will show, the combination of international I/O linkages, price distortions and extensive margin adjustments provides a quantitative solution to the TCP.

4.1 Consumption and Labor Supply

Consider a multi-period world economy with many countries $(i, j \in \{1, ..., N\})$. In each country, there is a representative consumer who consumes final goods and supplies labor $L_{i,t}$ for production. Consumers' utility function is:

$$U_0 = \mathbb{E}_0 \left[\sum_{t=0}^{+\infty} \beta^t \left(\log\left(C^F_{i,t}\right) - \psi_i \frac{L_{i,t}^{1+\nu}}{1+\nu} \right) \right] \tag{11}$$

$$\text{with} \qquad C^F_{i,t} = \left(\sum_j \omega^F_i(j)^{\frac{1}{\rho^F}} \cdot C_{j,i,t}^{\frac{\rho^F-1}{\rho^F}} \right)^{\frac{\rho^F}{\rho^F-1}} \quad \text{and} \quad C_{j,i,t} = \left(\int_{s\in\Omega^F_{j,i,t}} c_{j,i,t}(s)^{\frac{\sigma_j-1}{\sigma_j}} ds \right)^{\frac{\sigma_i}{\sigma_i-1}} \tag{12}$$

[23] Alternatively, the model presented here can be thought of as an extension of the IRBC model presented in Johnson (2014) with two new elements: markups and extensive margin adjustments. It is also related to the static small open economy model in Gopinath and Neiman (2014)

where ψ_i is a scaling parameter, ν the inverse of the Frisch elasticity of labor supply and σ_i the elasticity of substitution between different varieties of final goods originating from country i. $\omega_i^F(j)$ measures the share of country j in the *consumption* bundle of country i, and $\Omega_{j,i,t}^F$ is the endogenous set of firms from country j that serve the *final good* market in country i.[24] Finally, ρ^F is the *final goods* Armington elasticity of substitution. Final good price indices are defined as:

$$\mathcal{P}_{i,t}^F = \left(\sum_j \omega_i^F(j) \cdot \left(\widetilde{\mathcal{P}}_{j,i,t}^F\right)^{1-\rho^F}\right)^{\frac{1}{1-\rho^F}} \quad \text{and} \quad \widetilde{\mathcal{P}}_{j,i,t}^F = \left(\int_{s \in \Omega_{j,i,t}^F} p_{j,i,t}^F(s)^{1-\sigma_i} ds\right)^{\frac{1}{1-\sigma_i}} \tag{13}$$

where $p_{j,i,t}^F(s)$ is the price charged by firm s in the set $\Omega_{j,i,t}^F$ when selling in the *final good* market in country i. As we will see below, given our assumptions, firms charge the same price in both final and intermediate good markets in a given country.

As a baseline, we first assume financial autarky between countries, so that agents choose consumption, investment and labor, subject to the budget constraint:[25]

$$\mathcal{P}_{i,t}^F \left(C_{i,t} + K_{i,t+1} - (1-\delta)K_{i,t}\right) = w_{i,t}L_{i,t} + r_{i,t}K_{i,t} - \mathcal{T}_i \tag{14}$$

where we introduced the term $\mathcal{T}_i$ which captures potential trade imbalance in country i ($\mathcal{T}_i < 0$, corresponds to a trade deficit meaning that country i consumes more than the value of its production). In section 6, we also present our results when assuming a complete international asset market. Optimality yields the standard Euler equation and labor supply:

$$\frac{1}{C_{i,t}} = \beta \mathbb{E}_t \left[\frac{1}{C_{i,t+1}} \times \left(\frac{r_{i,t+1}}{\mathcal{P}_{i,t}^F} + (1-\delta)\right)\right] \tag{15}$$

$$\psi_i L_{i,t}^{\nu} = \frac{w_{i,t}}{\mathcal{P}_{i,t}^F} \frac{1}{C_{i,t}} \tag{16}$$

4.2 Production

In any country i, production is performed by a continuum of firms with heterogeneous productivity, defined as the product of an idiosyncratic component φ and a country specific component $Z_{i,t}$. In all countries, productivity φ follows a Pareto distribution with shape parameter γ. Firms produce with a Cobb-Douglas technology using labor $\ell_{i,t}(\varphi)$, capital $k_{i,t}(\varphi)$ and intermediate inputs $I_{i,t}(\varphi)$ bought from other firms from their home country as well as from abroad. The

[24]As we will see below, given our assumptions, the set of firms serving the final good and the intermediate input market in any country will be identical.

[25]Note that the right hand side of this equation include firms' profits since, as explained below, firms pay entry costs using domestic labor. It should then be understood that $L_{i,t}$ includes both production and "entry cost" workers. Moreover, an implicit assumption of the budget constraint above is that investment in the capital stock is done using the aggregated consumption good.

intermediate input index in country i, $I_{i,t}$, is a CES aggregation of country specific bundles $M_{j,i,t}$, with an *intermediate goods* Armington elasticity ρ^I. To introduce a rationale for markups and for love of variety, each country specific bundle is itself a CES aggregation of many varieties, with the elasticity of substitution σ_j.[26] The production function is:

$$Q_{i,t}(\varphi) = Z_{i,t}.\varphi \;.\; I_{i,t}(\varphi)^{1-\eta_i-\chi_i} \;.\; \ell_{i,t}(\varphi)^{\chi_i} \;.\; k_{i,t}(\varphi)^{\eta_i} \tag{17}$$

$$\text{with} \qquad I_{i,t}(\varphi) = \left(\sum_i \omega_i(j)^{\frac{1}{\rho^I}} M_{j,i,t}^{\frac{\rho^I-1}{\rho^I}}\right)^{\frac{\rho^I}{\rho^I-1}} \quad \text{and} \quad M_{j,i,t} = \left(\int\limits_{s\in\Omega_{j,i,t}} m_{j,i,t}(s)^{\frac{\sigma_i-1}{\sigma_i}} ds\right)^{\frac{\sigma_i}{\sigma_i-1}} \tag{18}$$

where $\omega_i^I(j)$ measures the share of country j in the *production process* of country i, and $\Omega_{j,i,t}^I$ is the endogenous set of firms based in j and serving the *intermediate input* market in country i. Similarly to the *final good* market, we have

$$\mathcal{P}_{i,t}^I = \left(\sum_j \omega_i^I(j) \cdot \left(\widetilde{\mathcal{P}}_{j,i,t}^I\right)^{1-\rho^I}\right)^{\frac{1}{1-\rho^I}} \quad \text{and} \quad \widetilde{\mathcal{P}}_{j,i,t}^I = \left(\int\limits_{s\in\Omega_{j,i,t}^I} p_{j,i,t}^I(s)^{1-\sigma_i} ds\right)^{\frac{1}{1-\sigma_i}} \tag{19}$$

$$\text{and } \mathcal{P}_i^{IB} = \chi_i^{-\chi_i} \times \eta_i^{-\eta_i} \times (1-\eta_i-\chi_i)^{(\eta_i+\chi_i-1)} \times \left(\mathcal{P}_{i,t}^I\right)^{1-\eta_i-\chi_i} \times w_{i,t}^{\chi_i} \times r_{i,t}^{\eta_i} \tag{20}$$

where $\mathcal{P}_{j,i,t}$ denotes the price of the country-pair specific bundle $M_{j,i,t}$ and $\mathcal{P}_{i,t}^{IB}$ is the unit cost of the Cobb Douglas bundle aggregating $I_{i,t}$, $k_{i,t}$ and $\ell_{i,t}$ (called the *input bundle*) and represents the price of the basic production factor in country i. $p_{j,i,t}^I(s)$ is the price charged by any firm s in the set $\Omega_{j,i,t}^I$ when selling in the *intermediate input* market in country i.[27]

To be allowed to sell its variety to a country j, a firm from country i must pay a fixed cost f_{ij}^c (labeled in unit of the *input bundle*) as well as a variable (iceberg) cost τ_{ij}. Firms choose which countries they enter (if any), affecting both the level of competition and the marginal cost of all firms in the country. As will be clear below, profits are strictly increasing with productivity φ so that equilibrium export decisions are defined by country-pair specific thresholds $\overline{\varphi}_{i,j,t}$ above which firms from country i find it profitable to pay the fixed cost f_{ij}^c and serve the *final good* and *intermediate inputs* markets in country j. Finally there is an overhead entry cost f_i^E, sunk at the production stage, to be paid before firms know their actual productivity. Based on their expected profit in all markets, firms enter the economy until the expected value of doing so equals the overhead entry cost. This process determines the mass of firms $M_{i,t}$.

[26] This parameter governs both the markup charge by firm from country j and the degree of love of variety.

[27] The exact expressions of these objects are standard and can be found in the online appendix B.

4.3 Equilibrium

We specify the equilibrium conditions of the model by introducing $X_{i,t}$ the aggregate consumers' revenue and $S_{i,t}$ the total firms' spendings (including bilateral fixed costs payments to access all markets) in country i. Given prices, total demand faced by firm φ is given by the sum of demand stemming from both the *final good* and the *intermediate input* markets:

$$
\begin{aligned}
q_{i,t}(\varphi) = & \sum_j \left(\frac{p^F_{i,j,t}(\varphi)}{\widetilde{\mathcal{P}}^F_{i,j,t}}\right)^{-\sigma_i} \left(\frac{\widetilde{\mathcal{P}}^F_{i,j,t}}{\mathcal{P}^F_{j,t}}\right)^{-\rho^F} \frac{\omega^F_j(i) X_{j,t}}{\mathcal{P}^F_{j,t}} \\
& + \sum_j \left(\frac{p^I_{i,j,t}(\varphi)}{\widetilde{\mathcal{P}}^I_{i,j,t}}\right)^{-\sigma_i} \left(\frac{\widetilde{\mathcal{P}}^I_{i,j,t}}{\mathcal{P}^I_{j,t}}\right)^{-\rho^I} \frac{\omega^I_j(i)(1-\eta_j-\chi_j) S_{j,t}}{\mathcal{P}^I_{j,t}}
\end{aligned} \tag{21}
$$

where the summation is done over all markets that are served by a firm with productivity φ.

Firms choose their price to maximize profits. Since the price elasticity of demand is constant, they charge a constant markup over marginal cost. For a firm from country i, the only elasticity that is relevant for pricing is σ_i, capturing the fact that firms compete primarily with other firms coming from their home country since their individual pricing decision has no impact on the country-specific price index in every market.[28] As a result, firms charge the same markup in the final and intermediate good markets, and we have: $p^F_{i,j,t}(\varphi) = p^I_{i,j,t}(\varphi) = p_{i,j,t}(\varphi)$ and $\widetilde{\mathcal{P}}^F_{i,j,t} = \widetilde{\mathcal{P}}^I_{i,j,t} = \widetilde{\mathcal{P}}_{i,j,t}$. The marginal cost of a firm with productivity φ in country i is $\mathcal{P}^{IB}_{i,t}/(Z_{i,t}\varphi)$ and its optimal price in country j is:

$$
p_{i,j,t}(\varphi) = \tau_{ij} \frac{\sigma_i}{\sigma_i - 1} \frac{\mathcal{P}^{IB}_{i,t}}{Z_{i,t}\varphi} \tag{22}
$$

Unlike in the canonical Krugman (1980), Melitz (2003) or Ghironi and Melitz (2005) models, one needs to jointly solve for all prices in the economy. Through $\mathcal{P}^{IB}_{i,t}$, the price charged by firm φ in country i depends on the prices charged by all firms supplying country i (both domestic and foreign) which in turn depend on the prices charged by their suppliers and so on and so forth. Determining prices requires solving jointly for all country-pair specific price indexes $\widetilde{\mathcal{P}}_{i,j,t}$.

The definitions of price indexes give rise to a simple relationship between the price of the country i specific bundle at home, $\widetilde{\mathcal{P}}_{i,i,t}$, and its counterpart in country j, $\widetilde{\mathcal{P}}_{j,i,t}$:

$$
\widetilde{\mathcal{P}}_{j,i,t} = \tau_{ij} \left(\frac{\overline{\varphi}_{i,j,t}}{\overline{\varphi}_{i,i,t}}\right)^{\frac{\sigma_i - \gamma_i - 1}{1-\sigma_i}} \times \widetilde{\mathcal{P}}_{i,i,t} \tag{23}
$$

[28] With a finite number of firms, elasticities σ_i, ρ^I and ρ^F would all appear in the pricing strategy. In such a case, every firm would take into account the fact that its own price has an impact on the unit cost of the corresponding country-specific bundle. Therefore, when decreasing its price, a firm would attract more demand compared to firms from its own country but also increase the share of total demand that goes to every other firms from its country.

where $\overline{\varphi}_{i,j,t}$ defines the threshold of idiosyncratic productivity φ above which firms from i serve country j. Intuitively, the ratio between the price of a country specific bundle in two different markets depends on the relative iceberg costs as well as the relative entry thresholds. Using this relation in the definition of price indexes in every country yields a system of N equations which jointly defines all *inner* price indexes:

$$(\widetilde{\mathcal{P}}_{i,i,t})^{1-\rho^I} = \mu_i \left(\sum_j \omega_i^I(j) \left(\tau_{ji} \left(\frac{\overline{\varphi}_{j,i,t}}{\overline{\varphi}_{j,j,t}} \right)^{\frac{\sigma_j-\gamma_j-1}{1-\sigma_j}} \widetilde{\mathcal{P}}_{j,j,t} \right)^{1-\rho^I} \right)^{1-\eta_i-\chi_i} \tag{24}$$

with μ_i depending on entry thresholds, the mass of firms and parameters.[29] For given thresholds and mass of firms, this system admits a unique non-negative solution.[30]

Turning to export strategies, the productivity thresholds above which firms from country i serve market j are implicitly defined by:

$$\pi_{i,j,t}(\overline{\varphi}_{i,j,t}) = \frac{\mathcal{P}^{IB}_{i,t}}{Z_{i,t}}.f^c_{ij} \quad \text{for all } i,j \in \{1,...,N\} \tag{25}$$

where $\pi_{i,j,t}(\varphi)$ is the variable profit earned by a firm with productivity φ in market j. Similar to Ghironi and Melitz (2005), the fixed cost f^c_{ij} is paid in units of the basic production factor in country i deflated by aggregate technology Z.[31]

Finally, the mass of firms is determined by the free entry condition defined as:

$$\Pi_{i,t} = M_{i,t}\frac{w_{i,t}}{Z_{i,t}}.f^E_i \quad \text{for all } i \tag{26}$$

where f^E_i is labeled in units of labor and $\Pi_{i,t}$ denotes aggregate profits of all firms in country i. Following Eaton and Kortum (2005), we can show that $\Pi_{i,t}$ is proportional to total revenues. Defining $R_{i,t}$ the total sales of all firms from country i, we have:

Lemma 1. : Total profits in country i are proportional to total revenues:

$$\Pi_{i,t} = \frac{\sigma_i - 1}{\gamma_i\sigma_i} R_{i,t} \tag{27}$$

[29] $\mu_i^{\frac{1-\sigma_i}{1-\rho^I}} = \frac{\gamma\overline{\varphi}_{i,i,t}^{\sigma_i-\gamma_i-1}}{\gamma_i-(\sigma_i-1)} M_{i,t} \left(\frac{\sigma_i}{\sigma_i-1} \frac{w_{i,t}^{\chi_i} \times r_{i,t}^{\eta_i}}{\chi_i^{\chi_i} \times \eta_i^{\eta_i} \times (1-\eta_i-\chi_i)^{1-\eta_i-\chi_i}} \frac{1}{Z_{i,t}} \right)^{1-\sigma_i}$

[30] Following Kennan (2001) and denoting $G_k = (\widetilde{\mathcal{P}}_{i,i,t})^{1-\rho^I}$ and G the associated $N \times 1$ vector, it suffices to show that the system is of the form $G = f(G)$ with $f : \mathbb{R}^N \to \mathbb{R}^N$ a vector function which is strictly concave with respect to each argument, which is obvious as long as $0 < \eta_k + \chi_k < 1$.

[31] In every market, entry occurs until the profit of the least productive firms is equal to the fixed cost of accessing the market. Denoting by $X_{i,t}$ total final good spending by consumers ($X_{i,t} = w_{i,t}L_{i,t} + r_{i,t}K_{i,t} - \mathcal{T}_{i,t}$), we have for any i and j: $\overline{\varphi}_{i,j,t} = \left(\tau_{ij} \frac{\sigma_i}{\sigma_i-1} \frac{\mathcal{P}^{IB}_{i,t}}{Z_{i,t}} \frac{1}{\widetilde{\mathcal{P}}_{i,j,t}} \right) \times \left(\frac{\sigma_i f^c_{ij}(\mathcal{P}^{IB}_{i,t}/Z_{i,t})}{\left(\widetilde{\mathcal{P}}_{i,j,t}/\mathcal{P}^I_{j,t}\right)^{1-\rho^I} \omega^I_j(i)(1-\eta_j-\chi_j)S_j + \left(\mathcal{P}_{i,j,t}/\mathcal{P}^F_{j,t}\right)^{1-\rho^F} \omega^F_j(i)X_{j,t}} \right)^{\frac{1}{\sigma_i-1}}$.

Proof: see Appendix B.

Closing the model involves standard market clearing conditions for capital, labor and goods. Total revenues of all firms from country i can be written as:

$$R_{i,t} = \sum_j \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_j^F} \right)^{1-\rho^F} \omega_j^F(i) X_{j,t} + \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_j^I} \right)^{1-\rho^I} \omega_j^I(i)(1-\eta_j-\chi_j)S_{j,t} \tag{28}$$

Total exports are the sum of final goods and intermediate inputs exports, defined as:

$$T_{i\to j} = \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_{j,t}^F} \right)^{1-\rho^F} \omega_j^F(i) X_{j,t} + \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_{j,t}^I} \right)^{1-\rho^I} \omega_j^I(i)(1-\eta_j-\chi_j)S_j$$

With Cobb-Douglas production, consumer's revenues $X_{i,t}$ are equal to the sum of the payment to production workers $\chi_i S_{i,t}$, rent from capital $\eta_i S_{i,t}$, total firms' profits $\Pi_{i,t}$ (which, at the free entry equilibrium, is completely used to pay the entry cost f_i^E), and potential trade imbalances $-\mathcal{T}_{i,t}$. Using $X_{i,t} = w_{i,t}L_{i,t} + r_{i,t}K_{i,t} - \mathcal{T}_{i,t} = (\eta_i+\chi_i)S_{i,t} + \Pi_{i,t} - \mathcal{T}_{i,t}$, the good market clearing condition writes:

$$R_{i,t} = \sum_j \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_{j,t}^F} \right)^{1-\rho^F} \omega_j^F(i) \Big[(\eta_j+\chi_j)S_j + \Pi_j - \mathcal{T}_j \Big] + \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_j^I} \right)^{1-\rho^I} \omega_j^I(i)(1-\eta_j-\chi_j)S_j \tag{29}$$

Furthermore, using lemma 1 above and the fact that $R_{i,t} = S_{i,t} + \Pi_{i,t}$, we get:

$$S_{i,t} = \left(\frac{\sigma_i\gamma_i - \sigma_i + 1}{\sigma_i\gamma_i} \right) R_{i,t}$$

Replacing $\Pi_{i,t}$ and $S_{i,t}$ as a function of $R_{i,t}$, equation (29) can be written as:

$$\begin{aligned} R_{i,t} = &\sum_j \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_{j,t}^F} \right)^{1-\rho^F} \omega_j^F(i) \left[\frac{(\eta_j+\chi_j)\cdot(\sigma_j\gamma_j - \sigma_j + 1) + \sigma_j - 1}{\sigma_j\gamma_j} R_{j,t} - \mathcal{T}_{j,t} \right] \\ &+ \sum_j \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_{j,t}^I} \right)^{1-\rho^I} \omega_j^I(i)(1-\eta_j-\chi_j) \left(\frac{\sigma_j\gamma_j - \sigma_j + 1}{\sigma_j\gamma_j} \right) R_{j,t} \end{aligned} \tag{30}$$

Which can be expressed in compact form as:

$$M \cdot \begin{pmatrix} R_1 \\ \vdots \\ R_N \end{pmatrix} = -\left((W^F)' \circ \mathbb{P}^F \right) \begin{pmatrix} \mathcal{T}_1 \\ \vdots \\ \mathcal{T}_N \end{pmatrix} \tag{31}$$

Where $\circ$ is the element-wise (Hadamard) product and W^F is the weighting matrix associated with final good aggregation and is defined as $W^F_{ij} = \omega^F_i(j)$. $\mathbb{P}^F$ is a matrix defined by $\mathbb{P}^F_{i,j,t} = \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{i,t}}\right)^{1-\rho^F}$. The matrix M is defined at any time t as:

$$\begin{aligned} M_{i,j,t} =& \mathcal{I}_{i,j} - \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{j,t}}\right)^{1-\rho^F} \omega^F_j(i) \frac{(\eta_j+\chi_j)(\sigma_j\gamma_j-\sigma_j+1)+\sigma_j-1}{\sigma_j\gamma_j} \\ & - \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^I_{j,t}}\right)^{1-\rho^I} \omega^I_j(i)(1-\eta_j-\chi_j)\left(\frac{\sigma_j\gamma_j-\sigma_j+1}{\sigma_j\gamma_j}\right) \end{aligned} \tag{32}$$

Labor can be used either for production ($L^p_{i,t}$) or for the entry cost ($L^e_{i,t}$) so that $L_{i,t} = L^p_{i,t} + L^e_{i,t}$. Setting $w_1 = 1$, implying $S_1 = L^p_1/\chi_1$, provides a unique solution for all variables by solving together the investment Euler equation (15), the labor supply equation (16), the price system (24), the threshold system (25), the Free Entry system (26) and the Revenue system (31).

GDP definition. In the data, GDP is constructed using base prices and quantity estimates. In order to be as close as possible to the method used in the construction of the data used in the empirical analysis, we define GDP using steady state prices as base prices.[32] GDP is obtained by deflating nominal spending using price indices that are corrected from product variety effects to measure "quantity indices", and then by valuing these "quantity indices" using steady-state prices. More precisely:

$$GDP_{i,t} = \underbrace{\widehat{\mathcal{P}}^{F,ss}_i \frac{X_{i,t}}{\widehat{\mathcal{P}}^F_{i,t}}}_{\text{Consumption + Investment}} + \underbrace{\sum_j \widehat{\mathcal{P}}^{ss}_{i,j} \frac{T_{i\to j,t}}{\widehat{\mathcal{P}}_{i,j,t}}}_{\text{Total exports (final+ inputs)}} - \underbrace{\sum_j \widehat{\mathcal{P}}^{ss}_{j,i} \frac{T_{j\to i,t}}{\widehat{\mathcal{P}}_{j,i,t}}}_{\text{Total imports (final + inputs)}} \tag{33}$$

where we defined $\widehat{\mathcal{P}}_{i,j,t} = \left(M_{i,t}.(\overline{\varphi}_{i,j,t})^{-\gamma_i}\right)^{1/(\sigma_i-1)} \widetilde{\mathcal{P}}_{i,j,t}$ and $\widehat{\mathcal{P}}^F_{i,t} = \left(\sum_j \omega^F_i(j)\cdot\left(\widehat{\mathcal{P}}_{j,i,t}\right)^{1-\rho^F}\right)^{\frac{1}{1-\rho^F}}$ in order to be consistent with the way actual data are collected. Since both consumers' utility and production functions have a CES component, it is well known that the associated price indexes can be decomposed into components reflecting average prices (captured by statistical agencies) and product variety (which is not taken into account in national statistics). See Feenstra and Markusen (1994) or Ghironi and Melitz (2005) for a discussion of this.

In equation (33), we defined GDP from the expenditure side. One could also define GDP from the production side, by summing gross domestic output sold in all markets and subtracting imported inputs. All our results are unchanged using this alternative measure.

[32] In the data, GDP is defined using the Fisher ideal quantity index which is a geometric mean of the Laspeyres and Paasche indices. Hence, for all periods t, the base period price is a geometric mean between period t and period $t+1$.

5 Calibration

The model is calibrated to 14 countries and a composite *Rest-Of-the-World* for the time period 1980-1990. As compared to our empirical sample, it represents around 78% of total trade flows, 79% of total trade in final goods and 77% of total trade flows in intermediate goods.[33] With N countries, there are $4 \times N^2 + 4N + 5$ parameters to determine, to which one must add parameters relative to the technological shocks.[34]

5.1 Parameterization

We set $\beta = 0.99$ and we choose $\nu = 1$, leading to a Frisch elasticity of 1. We set the value of the macro (Armington) elasticity ρ^I and ρ^F to be equal to unity, which is in the range of the literature. For instance, Saito (2004) provides estimations from 0.24 to 3.5 for the Armington elasticity.[35] There is also a theoretical convenience to use $\rho^I = \rho^F = 1$, as it allows the model to take the same form as other network models such as Acemoglu et al. (2012). We set parameters ψ_i in each country to replicate the *relative difference* of working age population with a normalization ensuring an average capital-output ratio of 13 in the model.

Markups and Value Added Shares. Concerning the micro elasticities, we set a value of $\sigma_i = \sigma = 5$, $\forall i$ in the baseline simulation. Anderson and van Wincoop (2004) report available estimates for the micro elasticity in the range of 3 to 10. Following Bernard et al. (2003), Ghironi and Melitz (2005) choose a micro elasticity of 3.8 and recently, papers such as Barrot and Sauvagnat (2016) or Boehm et al. (2015) argue that firms' ability to substitute between their suppliers can be very low. This choice leads to markups of 25%. The aggregate profit rate, however, is only of 17.4% since firms have to pay fixed cost in order to access any market. In Section 7, we consider alternative elasticities for σ_i, and defer discussion of those cases till then.[36] We set the Pareto Shape of the firm-specific productivity distribution to $\gamma_i = \sigma_i - 0.4$, as in Fattal Jaef and Lopez (2014).

The value added share, $\eta_i + \chi_i$, for a given country i, are calibrated using cost of intermediates and total sales as observed in the WIOD database at the 2-digits sector level. Specifically, $(1 -$

[33]The set of countries is: Australia, Austria, Canada, Denmark, France, Germany, Ireland, Italy, Japan, Mexico, Netherlands, RoW, Spain, United Kingdom and United States.

[34]For each country-pair (i, j), we specify values for $\omega_i^F(j)$, $\omega_i^I(j)$ (each with $N \times (N-1)$ values), τ_{ij} and f_{ij}^c. For every country i we have $(\eta_i + \chi_i)$, ψ_i, f_i^E, $\mathcal{T}_i$,σ_i and γ_i. The set of common parameters is given by $\chi_i/(\chi_i + \eta_i)$, ν, β, ρ^I and ρ^F. On top of these, we also need to set the volatility, covariance and auto-correlation of the technology shocks in all countries.

[35]Feenstra et al. (2014) studies the macro and micro elasticities for final goods and reports estimates between -0.29 and 4.08 for the Armington elasticity. They find that for half of goods the macro elasticity is significantly lower than the micro elasticity, even when they are estimated at the same level of disaggregation.

[36]We for instance recalibrate the model with heterogeneous elasticities of substitution across varieties, σ_i, based on two measures: the De Loecker and Eeckhout (2018) markup estimates and a Price Cost Margin approach. As shown in section 7, the introduction of heterogenous markups makes it possible to study the role of market power in shaping the correlation between terms of trade and GDP, in line with empirical evidence.

$\eta_{i,s} - \chi_{i,s}) = \frac{cost\_intermediates_s}{total\_sales_s}$, represents the share of intermediate inputs in total costs in a given sector. We use the fact that $total\ sales_s = \mu_i \times total\ cost_s$ with μ_i the markups in country i. Therefore, we fix $(\eta_{i,s} + \chi_{i,s}) = 1 - \frac{cost\_intermediates_s}{total\_sales_s}\frac{\sigma_i}{\sigma_i - 1}$. With $\sigma_i = 5$, the implied mean values of $\eta_i + \chi_i$, weighted by the sector importance in total sales, range from 0.31 to 0.45 for the considered countries (we set the value for *RoW* to the mean value), which seems to be consistent with values reported in Halpern et al. (2015). Finally, the capital and labor shares in value added are fixed at 2/3 and 1/3 respectively.

Entry costs. The sunk entry costs f_i^E in each country are computed from the *Doing Business Indicators*.[37] We measure the relative entry fixed costs by using the information on the amount of time required to set up a business in the country relative to the US, where we normalize f_{US}^E in order to generate a ratio of total number of firms divided by the working population, $\frac{M}{L}$, of about 12%. This is motivated by the fact that there are about 22-24 millions of non-employer businesses and 5.5 millions of employer businesses in the US, while the working age population represents around 180 millions of individuals during the considered period.[38] As shown later, the results are not sensitive to this specification.

Trade frictions. The variable (iceberg) trade costs for each country-pairs, $\tau_{ij} > \tau_{ii}$, are taken from the ESCAP World Bank: *International Trade Costs Database*, where we normalize $\tau_{ii} = 1$. This database features symmetric bilateral trade costs in its wider sense, including not only international transport costs and tariffs but also other trade cost components discussed in Anderson and van Wincoop (2004). Similar to Helpman et al. (2008), we assume domestic fixed costs $f_{ii}^c = 1$ for every country i. We set the values for the fixed costs of exporting from country i to country j, $f_{ij}^c > 1$ for $i \neq j$, in line with Di Giovanni and Levchenko (2013) using the Trading Across Borders module of the *Doing Business Indicators*. Specifically, we choose the number of days it takes to export to a specific country relative to the number of days it takes to supply in the home country (normalized to 1 in the model).[39]

Steady-State Trade Flows and Imbalance. Data relative to bilateral flows in final goods and intermediate inputs, $\{T^I_{j\to i}/GDP_i, T^F_{j\to i}/GDP_i\}$, are sufficient to identify the shares $\omega_i^I(j)$ and $\omega_i^F(j)$. Similar to our empirical part, we use trade data from Johnson and Noguera (2017) dis-aggregated into final and intermediate goods. Moreover, since complete financial autarky is

[37]The World Bank's Doing Business Initiative collected data on regulations regarding obtaining licenses, registering property, hiring workers, getting credit, and more. See `http://doingbusiness.org/data/exploretopics/trading-across-borders` and `http://doingbusiness.org/data/exploretopics/starting-a-business`. Unfortunately, due to data limitations, we use observation available in 2015. However, as shown later, f_i^E plays a little role in the correlation between trade and GDP comovement.

[38]This is also close to the 12% self-employment rate usually reported for the US between 1990 and 2000 (BLS). Results are not sensitive to this assumption. We provide a comparison of this rate and the self-employment rate in each economy in the online appendix C.

[39]This approach means that the fixed cost associated with trade from France to the US is the same as the one from Germany to the US. One must keep in mind, however, that the iceberg variable cost will differ.

inconsistent with the trade balances observed in the data, we calibrate the model trade imbalances $\{\mathcal{T}_1, ..., \mathcal{T}_N\}$ to match steady-state trade imbalances relative to GDP, and then hold those nominal imbalances constant during the simulation. Finally, to be as close as possible to the data used in the empirical analysis, we construct estimates by deflating the nominal spending by the price index that do not take into account love of variety, as described in section 4.3. By taking all of this information, the model steady state matches relative bilateral trade flows and relative trade imbalances exactly.

Aggregate Technology Process. The *level* of GDP comovement in our simulations is driven both by correlated technology shocks and by the transmission of those shocks across countries via trade linkages. In the model, $Z_{i,t}$ is the country-specific technology process which is not properly measured in the data by the Solow Residual (see section 6.4 for a discussion on this). We take a different route and set the cyclical properties of $(Z_i)_{i=1,...,N}$ to replicate observed GDP properties. To calibrate the variance-covariance matrix and the persistence of those technology shocks, we set the off diagonal elements (the covariance terms) so that the average correlation of GDP in the model matches exactly the one observed in the data, which is 0.27 for the selected countries in 1980-1999. We then calibrate the volatility (the diagonal elements of the covariance matrix) so that the model replicates exactly the observed GDP volatility (de-trended using HP-filter) in every country. This allows us to generate GDP fluctuations in the simulated economy that are similar to those observed in the data.[40] It is informative to note that, in order to match an observed international GDP correlation of 0.27, the correlation of technology shocks is only 0.189, with cross-country propagation through trade making up for the gap between technology and GDP correlations.[41] In this sense, through the lens of our model, propagation through trade explains about a third of international comovement.

Finally, we set a common value for auto-correlation of shocks so that the GDP series generated by the model is exactly 0.8, which is the average GDP auto-correlation in the data. One last detail regarding the simulation is that we parameterize the variance of the shocks to the *Rest-of-the-World* based on median GDP value in the data and the RoW covariance terms are set to 0. Table 5 reports the list of parameters. All targeted moments are perfectly matched.

6 Results

Following our empirical findings in section 2, we examine the model's ability to match the aggregate TC slope. The analysis focus on three questions: (i). Is the model able to generate a TC

[40] Recall that the goal of this exercise is not to explain the *level* of comovement across countries, but its *slope* following a change in trade intensities.

[41] Indeed, when we calibrate the model with zero trade flows for all country pairs, then GDP correlation is very close to the correlation of technology shocks, as shown in table 6. This is not surprising since in such a case, the world is essentially a collection of island that do not interact with one another.

Table. 4. Fixed parameters of the model.

Parameter	Symbol	Value	Moment / Source
Discount factor	β	.99	Annual discount rate of 4%
Labor curvature	ν	1.0	Frisch elasticity of 1.0
Labor Supply Scaling	ψ_i	$[5.4e{-}5, 0.16]$	Relative working age population
Labor share	$\chi_i/(\chi_i+\eta_i)$	2/3	67% of domestic value added
Argminton elasticities	ρ^I, ρ^F	1.0	Saito (2004), Feenstra et al. (2014)
Micro elasticity of substitution	$\sigma_i, \forall i$	5.0	Markup of 25%, profit rate of 17.4%
Sunk entry cost	f_i^E/f_{US}^E	[0.4 - 3.9]	*Doing Business Database* - World Bank
Fixed trade cost	f_{ij}^c	[3.3 - 18]	*Doing Business Database* - World Bank
Iceberg trade cost	τ_{ij}	[1 - 2.8]	ESCAP - World Bank
Pareto shape	γ_i	$\sigma_i - 0.4$	Fattal Jaef and Lopez (2014)

Table. 5. Calibrated parameters of the model.

Parameter	Symbol	Value	Main target
Inputs spending weights	$\omega_i^I(j)$	in sup. app.	Import shares in inputs
Final goods spending weights	$\omega_i^F(j)$	in sup. app.	Import shares in final goods
Trade imbalance	$\{\mathcal{T}_i, ..., \mathcal{T}_N\}$	in sup. app.	Trade imbalance over GDP
Persistency of Techno. shocks	ρ_Z	.77	Avg. GDP auto-correlation
Std. of Techno. shocks	$\sigma_Z(i)$	[.0012, .0050]	GDP volatility (de-trended)
Covariance of Techno. shocks	$\sigma_Z(i,j), \forall i \neq j$	.189	Avg. GDP correlation of 0.27

slope of the same magnitude as in the data? (ii). What are the role of markups and extensive margin adjustments in generating this TC slope? (iii). What is the quantitative importance of trade and TFP correlation in generating the observed *level* of GDP co-movement?

6.1 A Quantitative Resolution of the Trade Comovement Puzzle

To assess the model's ability to replicate the link between trade and GDP-comovement, we simulate the exact same sequence of 5,000 shocks in different configurations, where we vary trade proximity.

First, we use our baseline calibration described above, feed in the technology shocks, and record the correlation of logged and HP-filtered GDP as well as the average index of logged trade proximity in intermediate inputs and final goods. Then, we recalibrate the spending shares $\omega_i^I(j)$ and $\omega_i^I(j)$ for all country-pairs i and j with different targets for trade proximity across countries: decreasing and increasing the targeted imports in intermediate inputs relative to GDP by 10% and then decreasing and increasing the targeted imports in final goods relative to GDP by 10%. This amounts to 5 experiments, including the baseline simulation and allows us to use within country-pair variation to gauge how changes in trade intensity impact bilateral GDP comovement.[42] This gives rise to a panel dataset of $14 \times 13/2 = 91$ country-pairs (excluding

[42]The Trade-Comovement slope in our simulated dataset is not very sensitive to the percent increase of trade flows

RoW) for each of the 5 configurations, hence a total of 455 observations. Compared to our empirical exercise, each configuration can be thought of as a different time-window. We then use this simulated dataset to estimate the model-implied TC slope, controlling for country-pair fixed effects, as we did in the empirical analysis. Note that, with this method, we do not need to include "configuration fixed effects" in our regressions, since the only change between different configuration are the country-pairs trade intensities.

Table 6 shows our main quantitative results regarding the resolution of the trade comovement puzzle. As shown in row 3, our baseline model generates a realistic trade comovement slope of 7.2%, comparable to the range of values $[4.8\% - 11\%]$ reported in the literature for different set of countries, time periods and specifications and somewhat *higher* than our own empirical findings. Moreover, we find that the trade-comovement slope for trade in input is 3 times larger than the slope for final good trade. Our simulated slope with trade in intermediate inputs is close to one estimated from the data. In terms of magnitude: our point estimates suggest that doubling the input trade index generates an increase of GDP correlation of about 3.6%, while doubling the trade index in final goods generates an increase in GDP correlation of about 1.1%.

To better understand our quantitative success and investigate the role of each ingredient, we turn off one by one the key elements: (i) movements along the extensive margin, and (ii) monopolistic competition. The 5^{th} row presents the result of a standard IRBC model with input-output (I/O) linkages – that is, without markups and extensive margin adjustments. In this case, the trade comovement slopes for total trade, input and final goods are all low. From this reference point, the 4^{th} row adds price distortions which lead to the inclusion of profits movements in GDP fluctuations. This ingredient alone increases the trade-comovement slope for total trade to 3.1% and for inputs to 2.4%. The addition of fluctuations in the mass of firms (baseline case, 3^{rd} row) then implies an input trade-comovement slope of 5.1%. As such, our simulations show a sizeable role for extensive margin adjustments, which is in contrast with empirical findings in Gopinath and Neiman (2014), who argue that the extensive margin plays a small role in explaining the Argentine trade collapse. We provide additional and more direct evidence of the role of extensive margin in section 7.

In simulations presented in rows 3, 4 and 5 of table 6, we used the exact same sequence of technology shocks. This allows us to also investigate the role of our ingredient for level of GDP correlation (and not only the slope as discussed above). We note that adding price distortions and adjustments along the extensive margins increases GDP correlation by 5.8 percentage points relative to the model featuring only I/O linkages. This amounts to around 21% of the overall GDP correlation observed in the data.

The key insight emerging from this analysis is that adding intermediate inputs crossing multiple borders (also called Global Value Chains) to an otherwise standard IRBC model is not

between experiments, suggesting that the impact of trade on GDP-comovement is fairly linear in the model.

Table. 6. Quantitative assessment of the Trade Comovement Slope: data versus baseline model.

	corr^{GDP}	TC - Slope [a]			
		Total	*Input*	*Final*	slope input / slope final
1. Data (with CP FE)	.270	.055***	.054**	.003	-
2. Data (with CP & TW FE)	.270	.022**	.053**	−.030	-
3. Baseline: IO link. + Markups + EM	.270	.072***	.051***	.017***	3.0
4. IO link. + Markups	.229	.031***	.024***	.005***	4.8
5. IO link.	.212	.013***	.007***	.005***	1.4

*p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs. CP FE and TW FE mean country-pair and time-window fixed effects respectively. The column corr^{GDP} shows the average logged and HP filtered GDP correlation for the selected sample.
[a]The trade indexes used in those experiments are $(T_{i\to j} + T_{j\to i})/(GDP_i + GDP_j)$. Results are similar when using $\max\left(\frac{T_{i\leftrightarrow j}}{GDP_i}, \frac{T_{i\leftrightarrow j}}{GDP_j}\right)$.

sufficient to solve the TCP, as shown in Johnson (2014). However, a model combining GVC with markups and extensive adjustments provides the first quantitative solution to the TCP.

6.2 Accounting for the Low Final Goods TC-slope

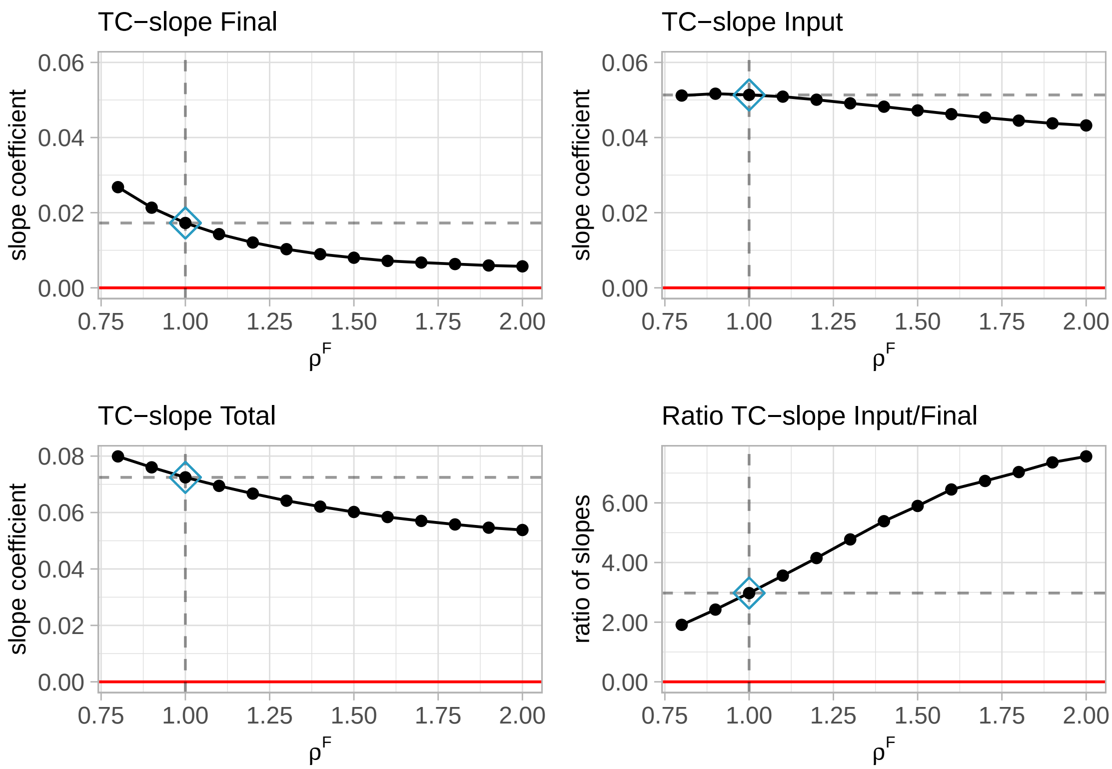
The baseline calibration generates a ratio between the slopes implied by trade in intermediate inputs and trade in final goods of 3.0, with both trade intensities significantly associated with higher GDP comovement. This contrasts with our empirical investigation, where we found that the trade comovement slope associated with trade in final goods is small and insignificant. In this section, we show that our model can reproduce this observation by using a higher final goods Armington elasticity.

Intuitively, when households can more easily substitute between different country-specific bundles, a higher trade integration in final goods does not lead to more value added comovement since a decrease in the price of foreign goods triggers a decrease in spending on local goods. Figure 1 shows the implied TC-slopes as a function of the value of final good Armington elasticity ρ^F. For this sensitivity analysis, we simulated our model and replicated our quantitative exercise for each value of ρ^F shown in the graphs. For instance, with $\rho^F = 1.5$, the association between GDP comovement and input trade is 6 times larger than the association with final good trade. With this parametrization, the final good TC-slope amounts to a mere 0.007, in line with the data.

6.3 A Focus on the Role of Markups and Profits

In the baseline calibration, we assumed a common lower-tier elasticity of substitution $\sigma = 5$ in all countries, leading to 25% markups and 17.4% profit rates once accounting for fixed cost payments. In this section, we show that this parameter has a key impact on the simulated

Figure 1. TC-slope and final good Armington elasticity ρ^F.



Note: the crossing of dashed lines refer to benchmark ρ^F and estimates.

TC-slope and that different calibrations can rationalize the range of TC-slope estimates found in the literature. Furthermore, we empirically confirm that higher bilateral trade in input is associated with a surge in the synchronization of aggregate profits. Overall, our results reveal the importance of accounting for aggregate profits in an analysis of business cycle comovement.

6.3.1 Variations on Markups

We present in table 7 various analysis regarding the role of price distorsions in the model. A first set of analysis investigates the sensitivity of the TC-slope to changes in the value of σ. We test the implication of higher ($\sigma = 4.0$) and lower ($\sigma = 6.0$) markups in the 4^{th} and 5^{th} row, respectively. As expected, an increase in markups leads to a higher association between trade and GDP comovement. Increasing markups from 20% to 33% almost doubles the association between total trade and GDP comovement, with both intermediate inputs and final goods playing a larger role. Moreover, in line with the discussion above, results in the 6^{th} row show that a higher elasticity of substitution for foreign goods $\rho^F = 2$ can generate a higher TC-slope while reducing the role of final good trade.

A second set of analysis aims to relax the homogeneous assumption and introduce heterogenous market power across countries. We therefore evaluate whether plausible heterogeneity in observed markups – for the set of countries we consider – have important consequences on the implied TC-slope. Specifically, we simulate the model with heterogenous σ_i estimated from the data using two different estimates. We first use Price Cost Margin (PCM) (7^{th} row) as an estimate of markups within each industry, which measures the difference between revenue and variable cost. Second, we use direct markup estimates from De Loecker and Eeckhout (2018) (DLE) (8^{th} row).[43] In each experiment, we center the heterogenous markups $\{\sigma_1, ..., \sigma_N\}$ around the baseline value. The implied standard deviation of σ are respectively .63 and .73 for the PCM and the DLE experiments. Quite surprisingly, adding heterogeneous markups centered around the value of $\sigma = 5$ does not change substantially the trade comovement slope, which suggests that accounting for cross-country heterogenous markups does not change the aggregate strength of international propagation.

Table. 7. The role of price distorsions and heterogenous markups [a]

Experiment	Elasticity	Markup	corr^{GDP}	TC - slope *Total*	TC - slope *Input*	TC - slope *Final*
1. Data (with CP FE)	-	-	.270	.055***	.054**	.003
2. Data (with CP & TW FE)	-	-	.270	.022**	.053**	-.030
3. Baseline	$\sigma = 5$	25%	.270	.072***	.051***	.017***
4. Low markups	$\sigma = 6$	20%	.253	.058***	.038***	.014***
5. High markups	$\sigma = 4$	33%	.311	.112***	.080***	.025***
6. High markups, High elasticity	$\sigma = 4, \rho^F = 2$	33%	.290	.084***	.070***	.005***
7. Heterogenous markups, PCM	$\sigma_i \in [3.20, 5.65]$	[22%, 45%]	.269	.071***	.050***	.017***
8. Heterogenous markups, DLE	$\sigma_i \in [3.68, 6.07]$	[20%, 37%]	.277	.078***	.055***	.018***

Notes: *p<0.1; **p<0.05; ***p<0.01.
[a] The simulations are based on the exact same sequence of shocks, under the five variations of trade indexes used in the benchmark.

Those results are especially interesting because they help rationalize the large heterogeneity in the estimated trade comovement slopes found in the literature, with estimates ranging from 4.8% to 11%. For example, Kose and Yi (2006) find a log slope of total trade of 0.091 and Liao and Santacreu (2015) of 0.112, significantly higher than our estimate. Moreover, using the period 1970-1990, we find a TC-slope of about 8.1% for trade in inputs, as shown in the online appendix A.3.1. Using 20-years time windows, we find a slope of 7.4%. Consequently, a key takeaway here is that our model can generate such an heterogeneity through different levels of market power over time and between countries.

6.3.2 On Profits and GDP Synchronization

Profits play an important part of our quantitative success in generating a strong reaction of domestic GDP to foreign shocks. In section 2, we showed that the trade comovement slope was

[43] We provide details on the two measures in appendix A.

in large part driven by an increase in the synchronization of the Solow Redisual. We then showed that this residual captured, among other things, fluctuations in aggregate profits.

We now show that, in the data as well as in our model, an increase in trade is associated with an increase of aggregate profits synchronization. For this exercise, we use data from the OECD on quarterly *Net Surplus*. According to the BEA, this variable captures "business income after subtracting the costs of compensation of employees, taxes on production and imports less subsidies, and consumption of fixed capital from value added". To avoid any bias due to inflation synchronization, we also deflate these nominal value by the consumer price index. We then run the following specification:

$$\text{Corr Net Surplus}_{ijt} \quad = \beta_1 \ln(\text{Trade}^{\text{input}}_{ijt}) + \beta_2 \ln(\text{Trade}^{\text{final}}_{ijt}) + \text{controls}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \quad (34)$$

Results are displayed in table 8. As expected, an increase in production linkages between two countries, as proxied by trade in intermediate inputs, is strongly associated with a surge in the correlation of their *Net Surplus*. As shown in columns 9 and 10, our model successfully replicates this empirical association.

This result points towards the importance of profit movements when interpreting GDP fluctuations. This insight is particularly relevant for the design of business cycle accounting exercises, because it implies that not including profits in a measurement framework can be misleading. Indeed, since profits are a non-negligible part of GDP, changes in profits mechanically impact GDP fluctuations. By construction, when interpreting GDP fluctuations through the lenses of a model with perfect competition, these profit movements must be mistakenly captured by another margin.

6.4 Solow Residual and Technology

As discussed above, the empirical association between trade and GDP comovement is mostly driven by an increased synchronization of the Solow residual. In our model, the introduction of extensive margin adjustments and market power creates a link between foreign shocks and domestic Solow residual, even with fixed technology. As a result, an increase in trade flows synchronizes *Solow residual* (SR) fluctuations across countries.

We formally compare our model with this our empirical findings. Defining SR as: $SR_{it} = log(GDP_{it}) - \left(\frac{\eta_i}{\eta_i+\chi_i}\right) log(K_{it}) - \left(\frac{\chi_i}{\eta_i+\chi_i}\right) log(L_{it})$, we present in table 9 the relationship between SR comovement and trade intensity as estimated from the data (in the first row) and in our simulations (rows two to seven, depending on model specification and calibration). Both in the data and in our baseline calibration, an increase of trade in inputs is associated with an increase in *SR* comovement by a factor of 6 relative to trade in final goods. Quantitatively, our model is able to reproduce a realistic trade-SR slope. This result is particularly telling since it is obtained

Table. 8. Trade and Real Net Surplus correlation

	Data								Model	
	Corr Real Net Surplus^{HP}				Corr ΔReal Net Surplus				Corr Π^{HP}	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
ln(Trade^{total})	.284***	.076			.183**	.101			.102***	
	(.075)	(.084)			(.075)	(.094)			(.003)	
ln(Trade^{input})			.371***	.175*			.288***	.250***		.077***
			(.104)	(.095)			(.083)	(.086)		(.002)
ln(Trade^{final})			$-$.085	$-$.105			$-$.100	$-$.157**		.017***
			(.079)	(.073)			(.062)	(.079)		(.002)
CP FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
TW FE	No	Yes	No	Yes	No	Yes	No	Yes	-	-
N	364	364	364	364	364	364	364	364	455	455
R^2	.104	.263	.121	.273	.028	.043	.047	.069	.714	.822

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs.
Data for trade flows are taken from OEC data. We add URSS and EU dummy variables. Data for correlation of Real Net Surplus are measured at quarterly frequency, for 16 consecutive quarters in time-windows of 4 years. Due to data limitation, our sample covers 1999Q1-2014Q4. Result are robust without the Great Recession time-window (2007-2010).

by feeding the model with the *exact same* technology shocks but different trade flows. Therefore, such experiment shows that SR comovement can significantly vary even absent of any change in technology. As expected, this association is absent in a model without markups and extensive margin adjustments and is decreasing with σ.

As discussed previously, this finding highlights the importance of profits movements and extensive margin adjustments in understanding GDP fluctuations. From this result, it follows that using a perfectly competitive framework with constant returns to scale to perform an accounting decomposition of GDP fluctuation could be misleading since, for example, movements in aggregate profits in the data would have to be attributed to either the Solow Residual or could be interpreted as evidence of increasing returns to scale.

To emphasize the differences between technology (or aggregate productivity) and the Solow Residual in our framework, table 9 also compares the cyclical properties of Z and SR in our simulations. Because SR measures the change in GDP that is not explained by movements of capital and labor, fluctuations in SR do not only capture changes in technology, but also capture fluctuations of profits and adjustments along the extensive margin. As a result, the baseline average SR correlation of about 0.246 (third row) is much larger than the one implied by the underlying technology process (Z) (0.189 in all experiments). The difference simply reflects the endogenous synchronization of SR through trade, due to profits and extensive margin movements. SR is also much more volatile and less auto-correlated than Z, with a ratio of standard deviations larger than 3, showing that SR is potentially a poor proxy for calibrating technology shocks.

Table. 9. Model: trade-TFP comovement slope [a]

	Exact Techno. (Z)		*Solow Residual (SR)* [c]			*Trade-SR slope* [c]		
	corr [b]	ACF	corr [b]	ACF	$\frac{\sigma_{SR}}{\sigma_Z}$	*Total*	*Input*	*Final*
1. Data (with CP FE)	-	-	.234	-	-	.051***	.047*	.005
2. Data (with TW + CP FE)	-	-	.234	-	-	.025*	.059**	-.034
3. Baseline ($\sigma = 5.0$)	.189	.77	.246	.736	2.709	.047***	.037***	.007***
4. IO link. + Markups	.189	.77	.213	.775	1.906	.016***	.016***	-.002***
5. IO link.	.189	.77	.196	.774	2.008	-.002***	-.001***	-.001***
6. High markups ($\sigma = 4.0$)	.189	.77	.282	.726	3.765	.083***	.063***	.013***
7. Low markup ($\sigma = 6.0$)	.189	.77	.230	.743	2.552	.034***	.025***	.004***
8. High elasticity ($\rho^F = 1.5$)	.189	.77	.237	.737	2.716	.043***	.034***	.006***

[a]Simulations are based on the exact same sequence of shocks Z.
[b]Due to data limitation at quarterly frequency, annual international SR correlation from 1970:1999 is reported.
[c]Data on SR are constructed using Penn World Tables as $SR_{ij} = log(rgdpna) - \alpha log(rkna) - (1-\alpha)log(emp)$, with emp, and $rkna$ variables corresponding to employment and capital stock and $\alpha = 1/3$. Results of the Trade-SR slope are robust with first difference as shown in appendix.

6.5 Robustness Checks

Our results are robust to a number of alternative specifications, as presented in table 10. We restate our baseline result in the 1^{st} row for reference with implied trade slopes of 5.1% and 1.7% for input and final goods respectively. As presented in rows 2 and 3, changing the value of parameters γ_i does not materially change the implied slopes. In contrast, as shown in rows 4 and 5, the Frisch elasticity (which is equal to $1/\nu$) has a more significant impact on the magnitude of the overall trade comovement slope, while preserving the relative importance of final goods versus intermediate inputs. We change two parameters in row 6 and illustrate how an alternative calibration could lead to results that are close to our benchmark. More precisely: a strong reduction of the Frish elasticity together with higher markups lead to an implied input trade slope of 4.3%.

The level of trade frictions in the calibrated steady state $\{\tau_{ij}, f^c_{ij}, f^E_{ij}\}$ does not affect the implied TC *slope*, as indicated by the result in rows 7 and 8. Regardless of the initial level of those trade frictions, increasing trade proximity is associated with the same reaction for GDP comovement. In row 9 we artificially remove all trade imbalances observed in the data. Under all those alternative specifications, our main results hold. We also conduct robustness on the properties of technology processes $\{Z_i\}$ in the 10^{th} and 11^{th} rows. We first use the observed estimated covariance matrix of standard TFP data computed as the Solow residual in Penn World Tables ($\widetilde{\sum}$). While this approach is sometimes used in the literature, it leads to overshooting the *level* of cross-country GDP correlation. Results regarding the TC slope remain similar to the benchmark calibration. Furthermore, we also simulate the model under the counterfactual assumption that technology shocks are uncorrelated across countries and set the off-diagonal elements of the covariance-variance matrix to zero (i.e. $cov(Z_{i,t}, Z_{j,t}) = 0, \forall i \neq j$). In this case, the TC slope decreases to 0.035. In rows 12 to 13 we use a different reference period for the calibration of the CES weights ω^I and ω^F, a variation that does not materially alter our key messages.

Finally, we present in rows 14 to 17 results obtained with small changes to the fundamental structure of our model. Keeping technology shocks unchanged, a framework with no wealth effect in labor supply (row 14) yields smaller trade slopes, which simply comes from a mechanical decrease in the volatility of GDP and most other aggregates, including profits and extensive margin adjustments. Inelastic labor supply reduces the trade slope more significantly, as shows in row 15. This result is interesting because it underlines that, while fluctuations in labor supply alone are not sufficient to create a strong trade-comovement slope, those fluctuations are quantitatively important in a model with markups and extensive margin adjustments. The last variation we present pertains to the structure of the international asset market. While our benchmark model features financial autarky, we now perform a set of simulations under complete markets. As discussed in Heathcote and Perri (2002), financial autarky usually does a better job at replicating observed data on cross-country output, consumption, investment and employment correlations. Indeed, since Backus et al. (1992), it is well known that complete market setups lead to a negative correlation in investments: in these models, as the authors state, capital has a tendency to "*make hay where the sun shines*", which is at odds with most empirical observations. In the 16^{th} and 17^{th} rows, we assess the relationship between asset market and trade-comovement slope and find that using a complete market setup leads to a very large reduction in the trade-comovement slope. This finding is consistent with Kose and Yi (2006), where authors argue that, under complete market, "*greater trade linkages lead to more resource-shifting, in which capital and other resources shift to the country receiving the favorable productivity shock*."

7 Model Mechanisms and Empirical Relevance

In this section, we present additional empirical evidence for the role of firms' entry/exit and markups in the dynamics of GDP comovement.

7.1 The Role of Extensive Margin of Trade

We first study the role of extensive margin (EM) and intensive margin (IM) fluctuations in generating the observed association between trade and GDP comovement. We conduct two empirical tests. First, in line with Liao and Santacreu (2015), we use the Hummels and Klenow (2005) (HK) decomposition and investigate the relation between the average and the volatility *within* each time window of the EM and IM of trade intensities between different time windows and GDP comovement. Compared to Liao and Santacreu (2015), we use a different identification strategy

Table. 10. Sensitive analysis [a]

Robustness/Alternative specification	Parameter change	corr$^{\text{GDP}}$	TC - slope		
			Total	*Input*	*Final*
1. Baseline	-	.270	.072***	.051***	.017***
A. Model parameter & trade frictions					
2. High pareto shape	$\gamma_i = \sigma_i - .3$	.271	.073***	.052***	.017***
3. Low pareto shape	$\gamma_i = \sigma_i - .5$	.270	.072***	.051***	.017***
4. High Frisch elasticity	$\nu = .75$	.295	.097***	.067***	.025***
5. Low Frisch elasticity	$\nu = 1.25$	.257	.059***	.043***	.013***
6. Low Frisch, high markup	$\nu = 2.0, \sigma_i = 4.0$	.254	.056***	.043***	.010***
7. Iceberg costs	+10%	.270	.072***	.051***	.017***
8. Fixed costs	+10%	.270	.072***	.051***	.017***
9. No trade imbalance	$\mathcal{T}_i = 0,\ \forall i$	.273	.071***	.050***	.017***
B. Productivity process					
10. Estimated TFP shocks	$\widetilde{\Sigma}$	.285	.055***	.040***	.014***
11. Uncorrelated techno. shocks	$cov(Z_{i,t}, Z_{j,t}) = 0, \forall i \neq j$	.030	.035***	.025***	.008***
C. Reference period for $\{\omega^I, \omega^F\}$					
12. baseline	1990-2000	.339	.125***	.086***	.035***
13. baseline, no EM/markups	1990-2000	.225	.024***	.012***	.012***
D. Alternative specification					
14. GHH preference	$\nu = 2$	.253	.053***	.039***	.011***
15. Fixed Labor	$L_t = \overline{L}$	.213	.014***	.013***	-.000
16. Complete markets	-	.202	.004***	.003***	.001***
17. Complete markets, no EM/markups	-	.201	.003***	.002***	.001***

[a]The simulations are based on the exact same sequence of shocks, under the five variations of trade indexes used in the benchmark.

and a broader set of countries over a longer period.[44] Second, we use the recent Exporter Dynamics Database (EDD) from the World Bank, which contains information about the number of active exporters as well as the average trade value per exporting firm. This allows us to directly test if the average value as well as the volatility of both EM and IM are associated with higher GDP synchronization.

EM-IM Hummels and Klenow decomposition. Building on Feenstra and Markusen (1994) and Hummels and Klenow (2005) (HK), we use data from the NBER United Nations Trade Data covering the period from 1962 to 2000 and the UN COMTRADE data for the period from 2001 to 2014. We use the bilateral trade flows as categorized under the SITC (rev. 2, 4-digits) classification. This choice is made because of the longer period covered by this classification.[45]

[44]Liao and Santacreu (2015) use a set of 30 countries over the period from 1980-Q1 to 2009-Q4 while we use 38 countries from 1971 to 2010 (we drop Czechoslovakia, Estonia, Russia, Slovenia and Slovakia due to lack of observations).

[45]To get a sense of how dis-aggregated SITC 4 categories are, consider the fact that code 6871 represents "*Tin and tin alloys, unwrought*" while code 6872 is "*Tin and tin alloys worked*". In the online appendix A.4.5, we also show that our results are consistent with a finer HS (6-digits) classification.

Using the HK decomposition, we construct the Extensive and Intensive margins of trade for each directed pair of country $(i \to j)$.[46] Since those measures are not symmetric within every country-pair we sum, for each country pair (i, j), the margins from i to j and from j to i. We then compute the average and the standard deviation of those measures *within* each time window and run:

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(\text{EM}^{HK}_{ijt}) + \beta_2 \ln(\text{IM}^{HK}_{ijt}) + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{35}$$

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(std(\text{EM}^{HK})_{ijt}) + \beta_2 \ln(std(\text{IM}^{HK})_{ijt}) + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{36}$$

Results are gathered in table 11 (columns (1) and (2)) and show that the correlation between the extensive margin of trade and GDP comovement is positive and significant for the two specifications. This result is particularly striking given that most of the variation in trade is explained by variations along the intensive margin.[47,48]

EM-IM decomposition using firms data. As an additional experiment, we use the recent Exporter Dynamics Database (EDD) from the World Bank in order to test whether a change in the number of exporters (EM) and a change in the average value added per exporter (IM) *within* different time windows are correlated with changes in GDP comovement. This database provides measures of micro-characteristics of the export sector; number of exporters (their size and growth), their dynamics in terms of entry, exit and survival, and the average unit prices of the products they trade, across 70 countries from 1997 to 2014. In order to study the correlation between the extensive and intensive margins on GDP correlations, we average the GDP (transformed with log and HP-filter) correlations between country-pairs at quarterly frequency over 3 time-windows of 5 years, starting in 1997-Q1.[49] Due to the lack of coverage of the EDD relative to our sample of countries, we use the only reported information of a reference country within a country-pair as direct measure for the EM and the IM.[50]

We first estimate the role of the EM using as indicator the number of new exporters net of exiting firms between country i and country j, normalized by the total number of exporters. For the IM, we use the natural logarithm of the average value added per exporter.

$$\text{Corr GDP}_{ijt} = \beta_1 \left[\frac{\text{Entry - Exit}}{\text{Nb Exp}}\right]_{ijt} + \beta_2 \ln\left(\left[\frac{\text{value}}{\text{exporter}}\right]_{ijt}\right) + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{37}$$

[46]See in Appendix for more details on the HK decomposition.

[47]Performing a Shapley value decomposition of total trade on the intensive and extensive margins, one finds that only one fourth of the total variance is explained by the variation of the extensive margin. Those results are in line with the similar analysis in Liao and Santacreu (2015).

[48]The results are robust when adding dummies for countries in the 2000 Euro Area or within the different waves of the European Union (1970, 1980, 1990).

[49]OECD GDP at quarterly frequency is not available for all the countries. We therefore reduce the sample. Further details regarding our sample are provided in the online appendix A.1 and robustness exercises are conducted in the online appendix A.4.

[50]For instance, the database contains information about exports from Belgium to many destinations, but there is no information about Belgium's imports. It is therefore not possible to compute symmetric measures.

Table 11, column (3), summarizes the results. Point estimates imply that an increase of 1% of the number of new net exporters is associated with an increase in GDP correlation of about 3.5%. On the contrary, we find that the IM correlates negatively with GDP correlation.[51] We then investigate in column (4) whether more variability along the extensive and intensive margins are associated with more GDP correlation within the considered time-windows. We regress:

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(\text{std nb exp}_{ijt}) + \beta_2 \ln\left(\left[\text{std } \frac{\text{value}}{\text{exporter}}\right]_{ijt}\right) + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{38}$$

Results feature a positive and significant relationship between variations in the number of exporters and GDP correlation, while variations along the intensive margin is negatively correlated with GDP comovement. This again suggests a potential key role of the extensive margin in generating GDP comovement as opposed to the variation along the intensive margin.

The role of the EM and IM in the model. In order to capture the respective role of EM and IM in the model, we perform similar analysis on our simulated dataset. The extensive margin is constructed as the number of firms producing goods in a specific international submarket (i, j), while the intensive margin is computed as the average production by exporter, such as:

$$EM_{ijt} = M_i \overline{\phi}_{ij}^{-\gamma_i} + M_j \overline{\phi}_{ji}^{-\gamma_j} \qquad \text{and} \qquad IM_{ijt} = \frac{1}{2} \frac{T_{j \to i}}{M_j \overline{\phi}_{ji}^{-\gamma_j}} + \frac{1}{2} \frac{T_{i \to j}}{M_i \overline{\phi}_{ij}^{-\gamma_i}} \tag{39}$$

where the index t refers to different configurations (i.e. to different steady-states where only trade proximity has been changed).[52] We then estimate in column (5) of table 11 the relative correlation of those measures (averaged and log transformed as in (36)) with changes in GDP comovement in the five configurations described in section 6. Moreover, to investigate the importance of each margin's volatility, we also compute the standard deviation of those measures as $\ln(std(EM)_{ijt})$ and $\ln(std(IM)_{ijt})$ in each configuration and regress those measures on GDP correlation for each country-pairs in column (6) of table 11. Consistent with our empirical findings, both average EM and the volatility of EM fluctuations are associated with a significant increase in GDP correlation. Finally, we also show in columns (7) and (8) the relationship between measures using the HK decomposition and SR comovement and we report the corresponding results using the model in columns (9) and (10). Consistent with previous findings, SR comovement is positively correlated with the extensive margin in the data and in the model.

[51] We point out that those results are robust to the period excluding the crisis (1997 - 2006) and to alternative measures, such as the number of new entrants surviving at longer horizons (one, two or three years) and using FD GDP correlations as shown in a online appendix A.4.

[52] This exercice can be compared to our empirical experiments where we compute the average extensive and intensive margins in a given time-windows and the associated GDP comovement in this same time-windows.

Table. 11. GDP and Solow Residual (SR) correlations and the margins of trade [a]

	Corr $\text{GDP}_{ijt}^{\text{HP filter}}$						Corr $\text{SR}_{ijt}^{\text{HP filter}}$			
	HK indexes		EDD measures		Model (base.)		HK indexes		Model (base.)	
	Avg.	Std.	Avg.	Std.	Avg.	Std.	Avg.	Std.	Avg.	Std.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
EM measure	0.046*	0.060***	3.480***	0.109*	0.070***	0.089***	0.075***	0.014	0.045***	0.053***
	(0.026)	(0.021)	(1.285)	(0.065)	(0.012)	(0.004)	(0.028)	(0.020)	(0.009)	(0.002)
IM measure	−0.019	−0.020*	−0.038	−0.022	0.002	0.033***	0.010	−0.005	0.007	0.025***
	(0.011)	(0.012)	(0.163)	(0.043)	(0.010)	(0.001)	(0.020)	(0.012)	(0.007)	(0.001)
CP FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
TW FE	Yes	Yes	Yes	Yes	-	-	Yes	Yes	-	-
N	2,357	2,356	135	135	455	455	2,235	2,231	455	455
R^2	0.083	0.086	0.586	0.558	0.253	0.630	0.204	0.201	0.169	0.730

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. dev. SE clustered on country-pairs.
Avg. refers to specifications where we assess the link between GDP/SR comovement and the *average* of each margin in different configuration. *Std.* refers to specifications where we assess the link between GDP/SR comovement and the *volatility* (standard deviation) of each margin in each configuration.

[a] We use EDD data from 1997 to 2014 for EDD measures, UN COMTRADE data from 2001-2010 and NBER United Nations Trade Data from 1971 to 2000 for the HK decomposition.

7.2 The Role of Markups

Finally, we present empirical support for the role of markups in generating a link between terms of trade and GDP fluctuations. Our model predicts that markups play an important role to make GDP react to foreign shocks, as shown in the decomposition in table 6. To find empirical support for the role of markup, we depart from a direct test of the model and test the following hypothesis: countries where markups are high experience a larger decrease in GDP when experiencing an increase in their terms-of-trade. For this, we compute the correlation of GDP with the terms of trade and regress this correlation on markups estimates, such that:

$$\text{Corr(GDP , ToT)}_{it} = \beta_1 \text{Markup.Index}_{it} + \text{Country}_i + \text{TW}_t + \epsilon_{it} \tag{40}$$

Table 12 gathers the results for the two measures of markup estimates and the implied slope in the model.[53] We first show the results of pooled cross-section analysis and then perform fixed effect regression and add time dummies to control for time-window specific factors that might affect the correlation of GDP and terms-of-trade. We also run the exact same regression with the model generated data using $\frac{\sigma_i}{\sigma_i - 1}$ as markup index and using variations in σ_i. Results using the model generated data show that countries with higher markups also experience a larger decrease in their GDP when the relative price of their imports rises, consistent with what we observe in the data.

[53] Data on real GDP and terms of trade at the annual frequency are both taken from the OECD database and are HP filtered to capture business cycle frequencies. We also use first difference data and results are consistent with our findings using HP-filter, as shown in the online appendix A.5.2.

Table. 12. Markups and GDP-ToT correlation

	$Corr(ln(GDP_i^{HP}), ln(ToT_i^{HP}))$				
	Data				Model
Markup measure	PCM [a]		De Loecker and Eeckhout (2018) [b]		
Markup index	-1.151	-2.650**	-0.756***	-0.495*	-0.527***
	(0.967)	(0.911)	(0.187)	(0.289)	(0.090)
Country FE	Yes	Yes	Yes	Yes	Yes
Time windows FE	No	Yes	No	Yes	-
N	43	43	80	80	112
R^2	0.066	0.322	0.132	0.232	0.260

*p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs.
[a]We use two time-windows from 1971-2010 over 22 countries reported in appendix.
[b]We use three time-windows from 1980-2009 for 29 countries reported in appendix.

8 Conclusion

This paper analyzes the relationship between international trade and business cycle synchronization across countries. We start by refining previous empirical studies and show that higher trade in intermediate inputs is associated with an increase in GDP comovement, while trade in final goods is found insignificant. Motivated by this new fact, we propose a model of trade and business cycles with (i) global value chains, (ii) monopolistic pricing, and (iii) firms' entry/exit. All elements are necessary for foreign shocks to have a first order impact on domestic productivity through trade linkages. The propagation of technological shocks across countries depends on the worldwide network of input-output linkages, which emphasizes the importance of going beyond two-country models to understand international GDP comovement.

We calibrate this model to 14 countries and assess its ability to replicate the empirical findings. Overall, the quantitative exercise suggests that the model is able to generate a realistic trade comovement slope, offering the first quantitative solution for the *Trade Comovement Puzzle*. Consistent with new data, both adjustments along the extensive margin and price distortions explain this result. Together, those elements give rise to a disconnect between aggregate technology and the Solow residual. Overall, our analysis points to the significant role of aggregate profit movements in understanding GDP fluctuations.

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A Empirical Appendix

A.1 Extensive Margin: Hummels and Klenow (2005) decomposition

We construct the extensive margin (EM) and intensive margin (IM) between countries j and m using the *Rest-of-the-World* as a reference country k. The EM is defined as a weighted count of varieties exported from j to m relative to those exported from k to m. If all categories are of equal importance and the reference country k exports all categories to m, then the extensive margin is simply the fraction of categories in which j exports to m. More generally, categories are weighted by their importance in k's exports to m. The corresponding IM is the ratio of nominal shipments from j to m and from k to m in a common set of goods. Formally, the margins are defined as:

$$\text{Extensive Margin} \quad EM^{HK}_{jm} = \frac{\sum_{i \in I_{jm}} p_{kmi} q_{kmi}}{\sum_{i \in I} p_{kmi} q_{kmi}} \qquad \text{Intensive Margin} \quad IM^{HK}_{jm} = \frac{\sum_{i \in I_{jm}} p_{jmi} q_{jmi}}{\sum_{i \in I_{jm}} p_{kmi} q_{kmi}}$$

Where I_{jm} is the set of observable categories in which j has a positive shipment to m and I is the set of all categories exported by the reference country. We normalize both measures by the sum of GDP of the two countries.

A.2 Markup measures

Markups. In section 7, we used two different markup index estimates. We first used aggregated micro markups estimated by De Loecker and Eeckhout (2018). They use micro data of 70,000 firms in 134 countries from 1980 to 2016 and estimate aggregate average markups using a cost-based approach. This method defines markups as the ratio of the output price to the marginal costs, and therefore relies solely on information from the financial statements of firms (sales value and cost of goods sold). Aggregating all firms specific markups for each country, De Loecker and Eeckhout (2018) provide a detailed and comparable measure of market power between countries. The sample that we use from their estimates includes 29 countries from 1980 to 2016.[54]

Second, we use Price Cost Margin (PCM) as an estimate of markups within each industry using data from 22 countries from 1971 to 2010.[55] Introduced by Collins and Preston (1969) and widely used in the literature, PCM is the difference between revenue and variable cost (the sum of labor and material expenditures, over revenue): $PCM = \frac{\text{Sales}-\text{Labor exp.}-\text{Material exp.}}{\text{Sales}}$

Data at the industry level come from the OECD STAN database, an unbalanced panel covering

[54] The list of countries is: Austria, Belgium, Canada, Colombia, Denmark, Finland, France, Germany, Greece, Ireland, Iceland, Indonesia India, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Portugal, South Africa, Spain, Sweden, Switzerland, Turkey, the United-Kingdom and the United-States.

[55] The list of countries is: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Iceland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, Portugal, Spain, Sweden, the United-Kingdom and the United-States.

107 sectors for 34 countries between 1970 and 2010. Due to missing data for many countries in the earliest years, we restrict the analysis for 22 countries.[56] We compute PCM for each industry-country-year and then construct an average of PCM within each country-year by taking the sales-weighted average of PCM over each industry. Finally, the average PCM for a given time window is simply the mean of country-year PCM over all time periods.

A.3 Trade comovement slope with financial controls

We provide additional robustness of the trade comovement slope using financial controls. To this effect, we construct two additional variables capturing the financial interconnection between every country-pairs. First, we construct an index of financial integration (FI) using Foreign Direct Investment (FDI) data, as follows: $FI_{ijt} = \frac{FDI_{i\to j,t}+FDI_{j\to i,t}}{GDP_{it}+GDP_{jt}}$. Second, we use the total bilateral cross-border claims (including bank and non-bank sectors for all maturities) from the consolidated banking statistics from the Bank for International Settlement to construct an index of financial proximity (FP) between a country i and j: $FP_{ijt} = \frac{C_{i\to j,t}+C_{j\to i,t}}{GDP_{it}+GDP_{jt}}$, where here $C_{i\to j,t}$ refers to total cross-border claims from country i to country j.

Table 13 summarizes the results with financial controls. Except for the specification using correlation of first difference GDP together with financial proximity index, the results are shown to be robust to the inclusion of financial controls. Using a larger sample including high and low income countries, World Bank (2019) show consistent findings.

Table. 13. Trade - GDP correlation, Disaggregated trade, controls with financial variables

	Corr GDP$^{\text{HP filter}}$				Corr ΔGDP			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ln(Trade$^{\text{input}}$)	0.170***	0.177***	0.298***	0.312***	0.067	0.074	0.202*	0.186*
	(0.065)	(0.063)	(0.097)	(0.095)	(0.075)	(0.074)	(0.104)	(0.098)
ln(Trade$^{\text{final}}$)	−0.006	−0.048	−0.367***	−0.351***	0.074	0.036	−0.340***	−0.316***
	(0.057)	(0.057)	(0.092)	(0.094)	(0.063)	(0.067)	(0.093)	(0.095)
ln(FP)		0.039**				0.027		
		(0.016)				(0.019)		
ln(FI)				−0.022				−0.036*
				(0.020)				(0.021)
$third_{country}$		0.322		−0.319		0.400		0.429
		(0.301)		(0.502)		(0.330)		(0.612)
Country-Pair FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time Window FE	No	Yes	Yes	Yes	No	Yes	Yes	Yes
EU + USSR dum.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	1,030	1,030	728	728	1,030	1,030	728	728
R^2	0.425	0.432	0.440	0.443	0.343	0.347	0.350	0.355

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs.

[56] For Germany, data are available only from 1991 onward (after the reunification), which is why the total number of observation in the regressions is 43.

A.4 Alternative measure of SR correlation

In this robustness, the Solow Residual in the data is constructed using the PWT9.1 using the variables of real GDP (*rgdpo*), real capital stock (*rnna*), total employment (*emp*) and the index of human capital per employee (*hc*), such that: $SR_{ij} = log(rgdpo) - \alpha log(rnna) - (1-\alpha) log(emp * hc)$, with $\alpha = 1/3$. With this method, we can compute the SR for up to 592 country-pairs over 4 time-windows. Complete results of the trade-SR comovement slope are shown in table 14, where point estimates are positive and significant for intermediate inputs. Results hold for both HP-filter and first difference.

Table. 14. Trade and SR correlation with 10 years time windows, alternative measure

	Corr $SR^{\text{HP filter}}$			Corr ΔSR		
	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\text{Trade}^{\text{total}})$	0.010			0.013		
	(0.012)			(0.012)		
$\ln(\text{Trade}^{\text{input}})$		0.055^{**}	0.066^{***}		0.054^{**}	0.064^{***}
		(0.025)	(0.025)		(0.025)	(0.024)
$\ln(\text{Trade}^{\text{final}})$		-0.044^{*}	-0.044^{*}		-0.040^{*}	-0.040^{*}
		(0.024)	(0.024)		(0.024)	(0.024)
Country-Pair FE	Yes	Yes	Yes	Yes	Yes	Yes
Time Window FE	Yes	Yes	Yes	Yes	Yes	Yes
URSS + EU dum.	No	No	Yes	No	No	Yes
N	2,367	2,367	2,367	2,367	2,367	2,367
R^2	0.213	0.215	0.235	0.208	0.210	0.228

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs.

A.5 Sensitive analysis of main empirical results

Table 15 shows that our results concerning the slope with respect to SR, I and L are somewhat robust to the use of first difference and the use of BK filter. We point out however that the slope with respect to investment and trade in intermediate inputs turns to be not significant using those two filtering procedures.

Table 16 provides various sensitivity analysis of our main empirical results regarding the trade-comovement slope. More details of those results are provided in the online appendix and a short summary of our investigation is reported here.

In the first series of robustness analysis, we ask how our results change when we restrict our sample in different ways. The first row simply restates our baseline results for reference. In the second row, we use 20 years time windows when computing GDP correlation. In the third and fourth row, we reduce our sample by excluding coutry pairs in the European Union and the USSR respectively. In the fifth row, we restrict our sample to the first three time windows (from 1970 to 1999) so that our time coverage is in line with original Kose and Yi (2006) analysis. In all these cases, our main results are virtually unchanged.

Table. 15. Trade and SR, I and L correlation with 10 years time windows - Disaggregated trade

	Corr ΔSR	Corr ΔI	Corr ΔL	Corr $SR^{\text{BK filter}}$	Corr $I^{\text{BK filter}}$	Corr $L^{\text{BK filter}}$
	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\text{Trade}^{\text{input}})$	0.044^{*}	0.024	0.001	0.049^{*}	0.009	0.010
	(0.024)	(0.028)	(0.024)	(0.027)	(0.034)	(0.025)
$\ln(\text{Trade}^{\text{final}})$	-0.025	-0.140^{***}	-0.011	-0.034	-0.101^{***}	-0.039
	(0.022)	(0.029)	(0.025)	(0.025)	(0.033)	(0.027)
Country-Pair FE	Yes	Yes	Yes	Yes	Yes	Yes
Time Window FE	Yes	Yes	Yes	Yes	Yes	Yes
Third country	Yes	Yes	Yes	Yes	Yes	Yes
URSS + EU dum.	Yes	Yes	Yes	Yes	Yes	Yes
N	2,367	2,367	2,367	2,367	2,367	2,367
R^2	0.196	0.159	0.132	0.213	0.187	0.125

Notes: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$. In parenthesis: std. deviation. SE clustered on country-pairs.

A second series of analysis relates to the inclusion of alternative controls for sectoral decomposition. Recall that our regressions include controls for similarity in specialization patterns which could impact GDP synchronization over and beyond any trade effect. In rows six, seven and eight, we use different sector categorization and dis-aggregation to compute our sectoral proximity measure. These variations do not affect our empirical results.

A third series of robustness exercises relate to the definition of our trade proximity indices. In our main results, we construct, for each country-pair and time window, the average over all years in the time window of the log of bilateral trade divided by the sum of GDP. In row nine, we do not take the log of the trade over (bilateral) GDP ratios and instead use simply the levels. In row ten, we do not construct the average of log ratio, but instead use the log of the average ratio. Row eleven presents results where instead of using the ratio of bilateral trade over sun of GDP, we use the maximum of bilateral trade divided by each country in the pair. Finally, row twelve presents results where we change our database and use the Structural Analysis (STAN) database for the trade flows. Again, our main message are not altered by these variations.

The last set of sensitivity investigations relate to the definition of GDP. In our main results, we use annual data on real GDP at chained PPPs from the 9th Pen nWorld Table. Rows thirteen and fourteen show the results when using the variable called "RGDPNA" which measures real GDP at constant 2011 national prices, when we use HP filter and first difference respectively. In rows fifteen and sixteen, we perform a weighted regression where each observation is weighted by the sum of GDP of both countries in the pair – hence giving relatively more weights to pairs containing at least one big country such as the US. As previously, our results are not materially affected by such changes.

Table. 16. Sensitive analysis: TC-slope

	Coefficient on trade in inputs	Coefficient on trade in final goods	GDP Filter	Countries | Obs.	Period	TW	CP
Sample selection							
1. Whole Sample	0.053^{**}	-0.030	HP	40 | 2,900	1970-2009	Yes	Yes
2. 20 years TW	0.074^{**}	-0.054	HP	40 | 1,450	1970-2009	Yes	Yes
3. Excluding EU CP	0.056^{**}	0.005	HP	40 | 2,280	1970-2009	Yes	Yes
4. Excluding USSR	0.064^{**}	-0.006	HP	34 | 2,244	1970-2009	Yes	Yes
5. Alternative TW	0.081^{***}	0.014	HP	34 | 2,244	1970-1999	Yes	Yes
Alternative controls for sectoral composition							
6. 4Digits SITC	0.058^{**}	-0.045^{*}	HP	36 | 2,520	1970-2009	Yes	Yes
7. ISIC classification	0.059^{**}	-0.045^{*}	HP	36 | 2,520	1970-2009	Yes	Yes
8. 1Digit Agg. sectors	0.088	-0.044	HP	38 | 1,291	1970-2009	Yes	Yes
Alternative indexes							
9. $level(trade)^a$	$33{,}96^{*}$	-34.92	HP	40 | 2,900	1970-2009	Yes	Yes
10. $\log(mean(trade))$	0.044^{*}	-0.027	HP	40 | 2,900	1970-2009	Yes	Yes
11. $\max\left(\frac{T_{i\leftrightarrow j}}{GDP_i}, \frac{T_{i\leftrightarrow j}}{GDP_j}\right)$	0.052^{**}	-0.032	HP	40 | 2,900	1970-2009	Yes	Yes
12. STAN data	0.209^{**}	-0.107	HP	20 | 760	1995-2014	Yes	Yes
Other robustnesses							
13. RGDPNA measure	0.106^{***}	-0.102^{***}	HP	40 | 2,900	1970-2009	Yes	Yes
14. RGDPNA measure	0.064^{***}	-0.063^{***}	FD	40 | 2,900	1970-2009	Yes	Yes
15. Weighted GDP	0.048^{**}	-0.046^{*}	HP	40 | 2,900	1970-2009	Yes	Yes
16. Weighted GDP	0.061^{**}	-0.047^{*}	FD	40 | 2,900	1970-2009	Yes	Yes
17. Whole sample	0.034^{*}	-0.025	BK	40 | 2,900	1970-2009	Yes	Yes
Slope w.r.t factor supply and SR							
18. SR slope	0.049^{*}	-0.034	BK	40 | 2,367	1970-2009	Yes	Yes
19. I slope	0.009	-0.101^{***}	BK	40 | 2,367	1970-2009	Yes	Yes
20. L slope	-0.022	-0.051^{*}	BK	40 | 2,367	1970-2009	Yes	Yes

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs. TW stands for Time Windows Fixed Effects while CP stands Coutry-Pairs Fixed Effects, which are inluded in all our analysis.
[a] We provide the results using EU and USSR dummies since adding those controls substantially reduce the significance of trade in final goods.

B Theoretical and Quantitative appendix

B.1 proof of Lemma 1

Reminder of Lemma 1. : Total profits in country i are proportional to total revenues:

$$\Pi_i = \frac{\sigma_i - 1}{\gamma_i \sigma_i} R_i$$

Proof: For simplicity, we write the proof in the Cobb-Douglas case with $\sigma_i = \sigma$ and $\gamma_i = \gamma$, although it extends immediately to a more general CES case, and we omit the time subscript. First, since firms charge a constant markup $\sigma/(\sigma-1)$, variable profits are a fraction $1/\sigma$ of total revenues and total profits net of fixed costs for all firms in i are $\Pi_i = \frac{R_i}{\sigma} - \sum_j FC_{i\to j}$, where $FC_{i\to j}$ is the sum of fixed cost payment from all firms from country i serving market j. Then, note that total fixed cost payment for all firms in country i is:

$$FC_{i\to j} = M_i \int_{\overline{\varphi_{i,j}}}^{+\infty} f_{ij}^c \times \frac{PB_i}{Z_i} \times \gamma \varphi^{-\gamma-1} \times d\varphi = M_i f_{ij} \frac{PB_i}{Z_i} \times \overline{\varphi_{i,j}}^{-\gamma}$$

For all i, j, total revenues (sales) from i to j can be written as:

$$\begin{aligned} R_{i,j} &= M_i \int_{\overline{\varphi}_{i,j}}^{+\infty} \left(\tau_{ij} \frac{\sigma}{\sigma-1} \frac{PB_i}{Z_i} \frac{1}{\widetilde{\mathcal{P}}_{i,j}} \right)^{1-\sigma} \times \Big[\frac{\omega_j^I(i) S_j}{\mathcal{P}_j^I} + \frac{\omega_j^F(i) X_j}{\mathcal{P}_j^F} \Big] \varphi^{\sigma-1} g(\varphi) d\varphi \\ &= \frac{\gamma M_i}{\gamma - (\sigma-1)} \times \left(\tau_{ij} \frac{\sigma}{\sigma-1} \frac{PB_i}{Z_i} \right)^{1-\sigma} \times \Big[\frac{\omega_j^I(i) S_j}{\mathcal{P}_j^I} + \frac{\omega_j^F(i) X_j}{\mathcal{P}_j^F} \Big] \overline{\varphi_{i,j}}^{\sigma-\gamma-1} \end{aligned}$$

Next, using the expression for $\overline{\varphi}_{i,j}$, we get

$$R_{i,j} = \frac{\gamma M_i}{\gamma - (\sigma-1)} \times \sigma f_{i,j}^c \frac{PB_i}{Z_i} \overline{\varphi_{i,j}}^{-\gamma} = \frac{\gamma}{\gamma - (\sigma-1)} \times \sigma FC_{i\to j}$$

Combining those expressions, we get

$$\sum_j FC_{i\to j} = \frac{\gamma - (\sigma-1)}{\gamma\sigma} \times \left(\sum_j R_{i,j} \right) = \frac{\gamma - (\sigma-1)}{\gamma\sigma} \times R_i$$

Using this expression of $\sum_j FC_{i\to j}$ in the definition of profits completes the proof.

B.2 Business cycle properties

In an effort to locate our framework in the broader literature on international business cycles, we report business cycle properties in table 17. By comparing the data in column (1) to different versions of our model in columns (2) to (4), we find that adding extensive margin and price distortions leads to an increase in investment and consumption volatility relative to GDP, while keeping other properties unchanged.[57]

Interestingly, our baseline calibration features a higher cross-country correlation of GDP than consumption, implying that the model is not subject to the Backus et al. (1992)'s consumption correlation puzzle.[58] Another dimension worth looking at is the volatility of extensive margin adjustments as measured by the standard deviation of the (log) number of exporters. Compared to the data, our model tends to be conservative as it slightly under-predicts the volatility of this margin.[59] Columns (5) of table 17 shows the Business Cycle properties when the covariance matrix of technology shocks are calibrated using the Solow Residual from Penn World Tables. Such a calibration leads to strong overshooting in terms of GDP, consumption and investment volatility as well as all cross-country correlations. Finally, column (6) displays the properties of a version with complete financial markets. As expected, consumption becomes less correlated with GDP but more synchronized. Interestingly, there is an increase in extensive margin fluctuations, as measured by the standard deviation of the number of exporters.

[57]One could decrease investment volatility in the simulation by adding adjustment frictions. In terms of TC-slope, adding adjustment frictions would further reduce the investment reaction after a foreign shock, which would reduce the role of factor supply in GDP comovement. Note, however, that the quantitative role of factor supply is already very limited in our simulation, as illustrated by the fact the very low TC-slope in a model without markups and extensive margin adjustments, see table 6.

[58]The so called "BKK consumption correlations puzzle" refers to the fact in standard models, consumption is more correlated across countries than output, which is at odds with the data.

[59]Note that introducing life cycle properties in firms' behavior, such as "long term fixed costs" instead of per-period fixed costs, would only widen the gap with the data as such elements tend to give more persistence to exporting decisions.

Table. 17. Business Cycle Statistics: Data and Models.[a]

Statistics	(1) Data [b,c]	(2) No EM/ No markups	(3) No EM	(4) Baseline	(5) SR as techno. Shocks	(6) complete markets	(7) GHH
A. Average standard deviation (%)							
GDP	1.38	0.90	0.84	1.38	6.58	1.43	1.12
Nb. Exp. (annual)	2.44	-	-	1.54	6.48	1.76	1.31
B. Standard deviation relative to GDP							
Consumption	1.03	0.18	0.26	1.19	1.27	0.80	1.43
Investment	3.21	3.66	5.65	7.72	8.14	7.75	5.84
Hours [d]	0.93	0.42	0.38	0.50	0.52	0.50	0.40
Nb. Exp.	1.61	-	-	1.09	1.09	1.23	1.09
RER [e]	3.90	2.24	2.22	2.14	2.40	0.50	2.16
C. International contemporaneous cross correlations							
GDP	0.27	0.21	0.23	0.27	0.29	0.20	0.25
Consumption	0.16	0.27	0.29	0.26	0.33	0.72	0.21
Investment	0.25	0.25	0.26	0.27	0.27	0.21	0.25
Hours [d]	0.29	0.25	0.26	0.31	0.32	0.21	0.29
RER [e]	-0.19	-0.30	-0.28	-0.23	-0.33	0.06	-0.25
D. Contemporaneous correlations with GDP							
GDP(-1)	0.80	0.80	0.82	0.80	0.80	0.80	0.89
Consumption	0.69	0.66	0.77	0.82	0.78	0.43	0.93
Investment	0.77	0.99	0.98	0.98	0.94	0.98	0.99
Hours [d]	0.60	0.98	0.97	0.98	0.98	0.97	1.0

[a] All statistics are computed using log transformation and HP-filter after being deflated by their corresponding price index. Recall that the baseline model targets an international contemporaneous cross correlations of about 0.27 and a GDP auto-correlation of 0.80.
[b] All statistics refer to the mean values in the data from 1980Q1 - 1999Q4, except for the log number of exporters which is computed using the Exporter Dynamics Database (EDD) from 1997 to 2014.
[c] We use the EDD to estimate the standard deviation of the HP-filtered (with $\lambda = 6.25$) and logged annual number of firms exporting from a country i to a country j between 1997 to 2014. Notice that among the 91 country-pairs in the model, our estimates are based on 30 country-pairs present in the EDD.
[d] Data from Ohanian and Raffo (2012). We select countries appearing in our model sample.
[e] Real Exchange Rate (RER) in the model are computed using RoW price index $RER_{i,t} = \mathcal{P}^F_{i,t}/\mathcal{P}^F_{RoW,t}$. Data reports statistics for the US relative to OECD countries, from Corsetti et al. (2008) for the period 1970–2001, using consumer price indices.