Value Added and Productivity Linkages Across Countries[*]

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Abstract

What is the relationship between international trade and business cycle synchronization? Using data from 40 countries, we find that GDP comovement is significantly associated with trade in intermediate inputs but not with trade in final goods. Motivated by this new fact, we build a model of international trade that is able to replicate the empirical trade-comovement slope, offering the first quantitative solution for the *Trade Comovement Puzzle*. The model relies on (i) global value chains, (ii) price distortions due to monopolistic competition and (iii) fluctuations in the mass of firms serving each country. The combination of these ingredients creates a link between domestic measured productivity and foreign shocks through trade linkages, generating a disconnect between technology and measured productivity. Finally, we provide empirical evidence for the importance of these elements in generating a link between foreign shocks and domestic GDP.

Keywords: International Trade, International Business Cycle Comovement, Networks, Input-Output Linkages, Solow Residual.

JEL Classification: F12, F44, F62

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1 Introduction

The *Trade Comovement Puzzle* (TCP), uncovered by Kose and Yi (2001, 2006), refers to the inability of international business cycle models to quantitatively account for the positive empirical relationship between international trade and GDP comovement.[1] Using international real business cycle (IRBC) models, several authors have succeeded to qualitatively replicate the positive link between trade and GDP comovement but fall short of the quantitative relationship by an order of magnitude.[2]

This paper has three main contributions. First, it contributes to empirical investigations of the association between bilateral trade and GDP comovement and shows that trade in intermediate inputs is significantly associated with synchronized GDP fluctuations. Second, it proposes a model of trade in both inputs and final goods with monopolistic pricing and firms entry/exit which is able to replicate the observed trade-comovement slope, offering the first quantitative solution of the TCP. Finally, the paper documents the disconnect between technology and measured productivity in presence of markups and extensive margin adjustments and shows that our model generates a trade-*Solow Residual* slope in line with the data.

Empirics. Since the seminal paper by Frankel and Rose (1998), a large empirical literature has studied cross countries' GDP synchronization, showing that pairs of countries with stronger trade linkages tend to have more highly correlated business cycles. The paper refines previous analysis by constructing a panel dataset of 40 countries consisting of four 10-years windows ranging from 1970 to 2009, which allows for dyadic as well as time windows fixed effects. In this setting, we document that the positive relationship between trade and GDP-comovement is mostly driven by *trade in intermediate inputs*, whereas trade in final good is found insignificant or negative. Those new findings suggest a possible link between global value chains (GVC) and the rising synchronization of GDP across countries.

Theory. As discussed in Kehoe and Ruhl (2008), international production linkages alone do not generate a link between domestic GDP and foreign shocks. With perfect competition and constant returns to scale, firms equalize marginal cost and marginal revenues of imported input, so that changes in the quantity of imported input yields exactly as much benefit as it brings costs. Hence, foreign shocks have an impact on domestic value added *only* to the extent that they impact the supply of domestic factors. This "negative result" is at the heart of the TCP. We incorporate two ingredients that create an endogenous relationship between domestic productivity and foreign shock through trade linkages.

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[1] For empirical studies, among many others, see Frankel and Rose (1998), Clark and van Wincoop (2001), Imbs (2004), Baxter and Kouparitsas (2005), Kose and Yi (2006), Calderon et al. (2007), Inklaar et al. (2008), Di Giovanni and Levchenko (2010), Ng (2010), Liao and Santacreu (2015), Duval et al. (2015) and Di Giovanni et al. (2016).

[2] For quantitative studies, see for instance Kose and Yi (2001, 2006), Burstein et al. (2008), Arkolakis and Ramanarayanan (2009), Johnson (2014) or Liao and Santacreu (2015).

First, when firms choose their price, they do not equalize the marginal cost and the marginal revenue product of their inputs. As noted previously by Hall (1988) and discussed in Basu and Fernald (2002), Gopinath and Neiman (2014) or Llosa (2014), this wedge between marginal cost and marginal product of inputs implies that any change in intermediate inputs usage is associated with a first order change in value added, over and beyond changes in domestic factors.[3] Second, fluctuations along the extensive margin have the potential to create an additional amplification mechanism between domestic productivity and foreign shocks. With love of variety, any variation in the mass of suppliers leads to a first order productivity change. Love of variety is a form of increasing returns to scale: a firm with more suppliers is more efficient at transforming inputs into output, which leads to an increases of value added over and beyond variations in domestic factor supply. Those ingredients create a link between foreign shocks and measured domestic productivity.

Quantitative analysis. Motivated by the discussion above, we propose a multi-country dynamic general equilibrium model of international trade in *final goods* and in *intermediate inputs* that relies on (i) monopolistic competition and (ii) fluctuations in the mass of firms serving each country. We calibrate the model to 14 countries and a composite *Rest-Of-the-World* and assess its ability to replicate the strong correlation between trade in intermediate inputs and GDP synchronization. Fixed effect regressions on this simulated dataset shows that the model is able to account for the trade-comovement (TC) *slope* observed in the data mainly through trade in intermediate inputs, a significant improvement compared to previous studies. Decomposing the role of each ingredient, we show that trade in intermediates alone is not sufficient to replicate the trade-comovement relationship. The addition of monopolistic pricing and extensive margin adjustments increase the simulated TC slope by a factor seven and improve the model fit.

Further empirical evidence. Finally, we provide evidence supporting our modeling assumptions. First, using different measures of monopoly power, we find that countries with higher markups have a GDP that is more systematically negatively correlated with terms-of-trade movements, meaning that they experience a larger GDP decrease when the price of their imports rises. Second, we empirically test the correlation between the extensive and intensive margins of trade with country-pair GDP correlations. A higher degree of business cycle synchronization is associated with an increase in the range of goods traded and is not associated with an increase in the quantity traded for a given set of goods.[4]

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[3]Related to this point, Burstein et al. (2008) show that if all firms take prices as given, a change in trade costs can affect aggregate productivity only to the extent that it changes the production possibility frontier at constant prices. This can be interpreted as saying that shocks to the *foreign* trading technology have no impact on aggregate *domestic* productivity if all firms have constant returns to scale and take prices as given.

[4]This result is in line with the analysis in Liao and Santacreu (2015) which emphasizes the role of the extensive margin. Compared to them, we are adding the panel dimension by performing fixed effect regression which allows us to control for country-pair fixed effects that can be correlated with trade intensity. Moreover, we also relate GDP

Relationship to the literature. Starting with Frankel and Rose (1998), a number of papers have studied and confirmed the positive association between trade and comovement in the cross-section.[5] The empirical part of this paper is mostly related to two recent contributions. First, Liao and Santacreu (2015) is the first to study the link between the extensive margin and GDP and TFP synchronization. Second, Di Giovanni et al. (2016) uses a *cross-section* of French firms and presents evidence that international I/O linkages at the micro level are an important driver of the value added comovement observed at the macro level. Their evidence is in line with the findings of this paper.[6]

If the empirical link between bilateral trade and GDP comovement has long been known, the underlying economic mechanism of this relationship is still unclear. Using the workhorse IRBC with three countries, Kose and Yi (2006) have shown that the model can explain at most 10% of the *slope* between trade and business cycle synchronization, leading to what they called the *Trade Comovement Puzzle* (TCP). Since then, many papers have refined the puzzle, highlighting ingredients that could bridge the gap between the data and the predictions of classic models.

Burstein et al. (2008) show that allowing for production sharing among countries can deliver tighter business cycle synchronization if the elasticity of substitution between home and foreign intermediate inputs is extremely low.[7] Arkolakis and Ramanarayanan (2009) analyze the impact of vertical specialization on the relationship between trade and business cycle synchronization. Their model with perfect competition does not generate significant dependence of business cycle synchronization on trade intensity, but they show that the introduction of price distortions that react to foreign economic conditions allows their model to better fit the data. Incorporating trade in inputs in an otherwise standard many-countries IRBC model, Johnson (2014) shows that adding international input-output (I/O) linkages *alone* is not sufficient to solve the trade-comovement puzzle, but the paper points that such production linkages do synchronize input usage. Compared to those papers, we add firms entry/exit and monopolistic competition and argue that those are key ingredients for the model to deliver quantitative results in line with the data. Liao and Santacreu (2015) build on Ghironi and Melitz (2005) and Alessandria and Choi (2007) to develop a two-country IRBC model with trade in differentiated varieties. Compared to this paper, our analysis adds multinational production in a many-country setup which creates a strong interdependency in firms' pricing and export decisions. We also highlight both

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comovement to the standard deviation of each margin and show that an increase of the variance of extensive margin fluctuations is associated with higher GDP correlation.

[5]See papers cited for instance in footnote 2.

[6]Relatedly, Ng (2010) uses cross-country data from 30 countries and shows that bilateral production fragmentation has a positive effect on business cycle comovement. The concept of bilateral production fragmentation used is different from this paper as it takes into account only a subset of trade in intermediates, namely imported inputs that are then further embodied in exports. Moreover, the cross section nature of the analysis does not allow neither for dyadic nor time windows fixed effects.

[7]In their benchmark simulations, the authors take the value of 0.05 for this elasticity.

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quantitatively and empirically the role of markups and extensive margin fluctuations.[8] Finally, a complementary approach has been developed by Drozd et al. (2019) which model the dynamics of trade elasticity in final goods and use GHH preferences. Building on Drozd and Nosal (2012), their quantitative 3-country model features customers accumulation with matching frictions between producers and retailers.

2 Empirical Evidence

In this section, we update the initial Frankel and Rose (1998) (henceforth, FR) analysis on the relationship between bilateral trade and GDP comovement and we also provide empirical support for the specific role of *trade in intermediate inputs* in this relationship. Our sample is composed of 40 countries , which account for around 90% of world GDP, and cover the period stretching from 1970 to 2009. We use annual data on real GDP at chained PPPs from the 9th Penn World Table, which is transformed in two ways: (i) HP filter with smoothing parameter 6.25 to capture the business cycle frequencies and (ii) log first difference. Trade data come from Johnson and Noguera (2017) who combine data on exports, imports, production, and inputs use to construct bilateral trade flows from 1970 to 2009 separating between trade in final good and trade in intermediate inputs within main sectors: agriculture, service, non-manufacturing and manufacturing.[9] We construct a symmetric measure of bilateral trade intensity (hereafter "trade intensity") using the sum of total exports ($T_{i \to j}$) from country i to j and total imports ($T_{j \to i}$), such as: $\text{Trade}_{ij} = \frac{T_{i \to j} + T_{j \to i}}{GDP_i + GDP_j}$, and measures the importance of the trade relationship relative to total GDP.[10] In a similar way, we disentangle trade intensity in inputs and final goods by constructing indexes $\text{Trade}_{ij}^{\text{final}} = \frac{T_{i \to j}^F + T_{j \to i}^F}{GDP_i + GDP_j}$ and $\text{Trade}_{ij}^{\text{input}} = \frac{T_{i \to j}^I + T_{j \to i}^I}{GDP_i + GDP_j}$ by taking into account only the exports and imports in final and intermediate goods respectively. In practice, as standard in the literature, we take the natural logarithm of both ratios.[11]

The extent to which countries have correlated GDP can be influenced by many factors beyond international trade, including correlated shocks, financial linkages, common monetary policies,

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[8]In their model, no firm is both an importer *and* an exporter. The absence of production linkages makes it essentially a model of trade in final good only in which domestic and foreign goods are substitutes. This, in turn, creates forces toward negative GDP correlation as is illustrated by the *negative* association between trade and GDP comovement when the elasticity of substitution is equal to 3.1.

[9]We provide additional details on data sources and the list of countries in the online appendix A.1.

[10]We also used an index defined as $\text{Total}_{ij} = \max\left(\frac{Total\ Trade_{ij}}{GDP_i}, \frac{Total\ Trade_{ij}}{GDP_j} \right)$. This measure has the advantage to take a high value whenever one of the two countries depends heavily on the other for its imports or exports. Both our empirical and simulated results hold when we use this index.

[11]To be more precise, we first apply the log transformation on trade intensities and then we average over the time windows. This is motivated by the fact that the original trade data grow exponentially from 1970 to 2009. We also report the results of the regressions using the log transformation on the mean trade intensities in the supplementary appendix A.3.3. Results are quite similar. In the supplementary appendix A.3.4, we also report the results using the *level* of trade intensities and show that our findings are robust to this specification. Finally, notice that the log specification has a larger explanatory power (measured by the R^2) compared to regressions in levels.

etc. Because those other factors can themselves be correlated with the index of trade proximity in the cross section, using cross-section identification could yield biased results. Indeed, in their seminal paper, FR use cross-sectional variations to evaluate whether bilateral trade intensity correlates with business cycle synchronization, but their specification does not rule out omitted variable bias such as, for example, the fact that neighboring countries have at the same time more correlated shocks and larger trade flows. By constructing a panel dataset and controlling for both country-pair and time windows fixed effects, this paper relates to recent studies that try to control for unobserved characteristics.[12] Therefore, in order to separate the effect of trade linkages from other unobservable elements, we construct a panel dataset by creating four periods of ten years each.[13] Within each time window, we compute GDP correlation (Corr GDP) as well as the average trade intensities defined above.

We then estimate two panel data regressions. In the first we follow the existing literature by running linear regression estimation of Corr GDP_{ijt} on the log of trade intensity Trade_{ijt}:

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(\text{Trade}_{ijt}) + \text{controls}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{1}$$

where i and j denote the two countries and t the time window. In the second, we run the regression on the log of trade intensity disaggregated into final goods and inputs:

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(\text{Trade}_{ijt}^{\text{input}}) + \beta_2 \ln(\text{Trade}_{ijt}^{\text{final}}) + \text{controls}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{2}$$

We finally specify the additional controls that we include (one-by-one) in the analysis. First, we include dummy variables for countries among the European Union (each wave are entitled a different dummy variables) and the Euro Area. Second, we construct two additional measures that capture the effect of *trade network* (third country effect) and the *sectoral composition* of trade.[14] Our "third country" index is motivated by the fact that two countries with similar partners could co-move because of their link with common partners. Moreover, our "sectoral" index controls for changes in specialization. If shocks have a sectoral component, then two countries that tend to specialize over time in the same sectors could have an increase in business cycle comovements over and beyond any direct trade effects. The third index is specified as $third_{country}(i,j) = 1 - \frac{1}{2} \sum_{k \neq i,j} \left| \frac{T_{i \to k} + T_{k \to i}}{\sum_k T_{i \to k} + T_{k \to i}} - \frac{T_{j \to k} + T_{k \to j}}{\sum_k T_{j \to k} + T_{k \to j}} \right|$. It measures the degree of similarity in the geographical distribution of trade shares between country i and country j, and is

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[12]Di Giovanni and Levchenko (2010) includes country pair fixed effects in a large cross-section of industry-level data with 55 countries from 1970 to 1999 in order to test for the relationship between sectoral trade and *output* (not *value-added*) comovement at the industry level. Duval et al. (2015) includes country pair fixed and year effects in a panel of 63 countries from 1995 to 2013 and test the importance of *value added* trade in GDP comovement.

[13]Adding time windows fixed effect controls for the recent rise of world GDP correlation since the 90s, which could be unrelated to trade intensity.

[14]In the supplementary appendix A.3.2, we also control for sectoral composition of total value added. However, due to missing data, the sample is much smaller.

equal to 0 if countries i and j have completely separated trade partners while it is equal to 1 if all trade shares are equal. The sectoral composition index is constructed based on 2-digit SITC trade data as $sector_{proximity}(i,j) = 1 - \frac{1}{2}\sum_{s \in \mathcal{S}} \left| \frac{T_i(s)}{\sum_{s \in \mathcal{S}} T_i(s)} - \frac{T_j(s)}{\sum_{s \in \mathcal{S}} T_j(s)} \right|$, with $T_i(s)$ the total export of country i in the specific sector (or products) s in the set of sectors $\mathcal{S}$. This index controls for the composition of trade and can be thought of as measuring common sectoral specialization within each country-pair: if two countries export exactly the same share of each products, then the index is equal to 1. For those two indexes, we use bilateral trade data (SITC4 REV. 2) from the Observatory of Economic Complexity.

In columns (1) and (5) of table 1, we first report results using only *within* country-pair variations without time window fixed effects (FE). Our estimates are significant and consistent with those in the empirical literature (ranging from a trade-comovement slope of about 4.8% and 11% in log), and show a positive relationship between bilateral trade and GDP correlation.[15] Then, in columns (2) and (6), we run the same regression controlling for aggregate time windows fixed effects. When controlling for both country-pair and time FE, the positive relationship between trade and GDP correlation still holds for HP filter and first differences, but effects are significantly dampened and about half as large as what is implied without time FE.[16] To have a sense of the magnitude involved, notice that the median change of the log trade intensity between 2000-2009 and 1970-1979, across all country-pairs, is an increase of factor 2. According to the point estimates, this corresponds to an increase in GDP correlation of 0.044.

In columns (3), (4), (7) and (8) of table 1, we separate trade in intermediate inputs from trade in final goods. Results highlight a significant positive relationship between GDP correlation and trade in inputs, while trade in final goods is found insignificant.[17] Interestingly, adding time window FE only slightly reduces the correlation between trade proximity in inputs and GDP comovement. Provided that the median increase of the log trade intensity in intermediate goods between 2000-2009 and 1970-1979 is about 1.84, the slope coefficient implies an associated increase of GDP correlation of 0.098 , a non negligible increase.[18]

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[15]Frankel and Rose (1998) (FR), estimate an elasticity of nominal GDP comovement to trade intensity of about 4.8%, using a different set of 21 countries, time period (1957 to 1997) and three instrumental variables (IV) for trade intensity: (i) log of distance between countries, (ii), dummy for common border, (iii) dummy for common language. With a specification similar to FR, Kose and Yi (2006) use 21 countries from 1970-2000 and find an elasticity of trade intensity and GDP of 9.1% using HP-filtered GDP and 7.8% using log-difference. Finally, using the same measure for trade intensity as in this paper without time window fixed effects, they estimate a coefficient β of about 0.115. In a similar way, Liao and Santacreu (2015) use IV estimation over a sample of 30 countries covering the period between 1980 and 2009 and find estimates between 0.112 (HP filter) and 0.066 (FD). In appendix A.3, we also provides estimates using the 1970-1990 period.

[16]See also footnote 1 for papers finding a high and robust association between total trade and business cycle comovement using cross-sectional settings.

[17]Di Giovanni and Levchenko (2010) investigate the role of vertical linkages in *output* synchronization (not *value added*) using I/O matrices from the BEA. Their estimates imply that vertical production linkages account for some 30 percent of the total impact of bilateral trade on the business cycle correlation

[18]Notice that the estimate using the log of the mean trade intensity in intermediate inputs within time windows implies an associated increase of GDP correlation of 0.091.

Table. 1. Trade proximity and GDP correlation [a]

	Corr GDP$^{\text{HP filter}}$				Corr ΔGDP			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ln(Trade)	0.055***	0.022**			0.044***	0.027**		
	(0.007)	(0.011)			(0.006)	(0.011)		
ln(Trade$^{\text{input}}$)			0.054**	0.053**			0.055**	0.042*
			(0.025)	(0.024)			(0.023)	(0.023)
ln(Trade$^{\text{final}}$)			0.003	−0.030			−0.008	−0.016
			(0.022)	(0.024)			(0.020)	(0.023)
Country-pair	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time window	No	Yes	No	Yes	No	Yes	No	Yes
N	2,900	2,900	2,900	2,900	2,900	2,900	2,900	2,900
R^2	0.035	0.153	0.037	0.155	0.024	0.141	0.024	0.142

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation.
[a]We use four time windows of 10 years each from 1970 to 2009.

We then add our controls in table 2, where columns (1) and (5) report the regression results without the additional controls. In columns (2) and (6) we show the results with EU and USSR dummies while in columns (3) and (7) we include the third country index. Finally, columns (4) and (8) include our index controlling for the sectoral composition of trade. In all specifications, trade in intermediate inputs is shown to be significant at 5% while trade in final goods is insignificant (or weakly negatively correlated). Notice also that the effect of *trade network* is also significant and high; implying that there is a relationship between GDP comovement and the fact that two countries have similar trade partners.[19,20]

3 A simple model

For the sake of exposition, we consider here a static small open economy. In such a world, Kehoe and Ruhl (2008) (henceforth KR) show that a change in the price of imported inputs has no impact, up to a first order approximation, on measured productivity. Therefore, any change in GDP is due to variations in domestic factors supply. We start by briefly reviewing this result.

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[19]Results presented here use a fixed effect specification. To discriminate between fixed or random effects, we run a Hausman test which display a significant difference ($p < 0.001$), and we therefore reject the random effect model. We also test the need for time-windows effects against the alternative without time-windows FE. The results of a Lagrange multipliers test provide strong support for the model with time-windows fixed effects ($p < 0.001$). The supplemental appendix provides many other robustness tests with alternative datasets and time windows.

[20]The results are also robust to a number of alternative specifications, time periods, time windows, different set of countries (excluding Euro area or European countries), world GDP correlation and an alternative dataset and method of separating intermediate from final goods. We provide an overview of those results in table 14 in appendix. We also provide in table 12 in appendix results with financial controls. We finally disaggregated further the role of intermediate inputs by main sectors in the supplementary appendix A.3.7, and find that the manufacturing and non-manufacturing industrial sectors play a key role in the positive relationship between trade proximity and GDP correlation. These additional results are provided.

	Corr GDP$^{\text{HP filter}}$				Corr ΔGDP			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ln(Trade$^{\text{input}}$)	0.053**	0.059**	0.060**	0.061**	0.042*	0.050**	0.052**	0.049**
	(0.024)	(0.024)	(0.024)	(0.024)	(0.023)	(0.023)	(0.023)	(0.023)
ln(Trade$^{\text{final}}$)	−0.030	−0.038	−0.047*	−0.048*	−0.016	−0.024	−0.035	−0.033
	(0.024)	(0.024)	(0.025)	(0.025)	(0.023)	(0.023)	(0.023)	(0.023)
$sector_{prox}$				0.088				−0.247*
				(0.146)				(0.138)
$third_{country}$			0.307**	0.305**			0.400***	0.407***
			(0.149)	(0.150)			(0.141)	(0.141)
Country-Pair FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time Window FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
USSR + EU dum.	No	Yes	Yes	Yes	No	Yes	Yes	Yes
N	2,900	2,900	2,900	2,900	2,900	2,900	2,900	2,900
R^2	0.155	0.167	0.170	0.170	0.142	0.155	0.159	0.160

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation.

3.1 The Kehoe and Ruhl (2008) negative result

The economy produces a final good y, used for consumption and exports, which is produced by combining imported inputs x and domestic factors of production ℓ (possibly a vector), according to $y = F(\ell, x)$, where $F(.,.)$ has constant returns to scale and is concave with respect to each of its arguments. The final good producer chooses domestic factors and imported inputs to maximize profit, taking all prices as given. Optimality requires that factors are paid their marginal product and we have $p_y F_\ell(\ell, x) = w$ and $p_y F_x(\ell, x) = p_x$, with p_y the final good price, p_x the price of imported inputs x and w the price of domestic factors.

Gross Domestic Product is the sum of value added in the country, which is simply the value of final goods minus the value of imported inputs. Importantly, many statistical agencies use *base period prices* when valuing estimated quantities in the construction of GDP.[21] Since prices are kept constant at their base value, we denote them with the superscript b to emphasize the fact that they are treated as parameter and not as endogenous objects:

$$GDP = p_y^b F(\ell, x) - p_x^b . x \tag{3}$$

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[21]The Penn World Tables used in our empirical section uses base period prices. The Bureau of Economic Analysis uses a Fisher chain-weighted price index to construct GDP at time t relative to GDP at time $t-1$ according to:

$$\frac{GDP_t}{GDP_{t-1}} = \left(\frac{\sum_k p_{t-1}^k q_t^k}{\sum_k p_{t-1}^k q_{t-1}^k} \right)^{0.5} \left(\frac{\sum_k p_t^k q_t^k}{\sum_k p_t^k q_{t-1}^k} \right)^{0.5}$$

where k indexes all components of GDP. Intuitively, the Fisher index is a geometric average between two base period pricing methods where the base price is alternatively the price at $t-1$ and at t.

Let us now compute the first order change in GDP when the Terms-of-Trade ($\equiv p_x$) change:

$$\frac{dGDP}{dp_x} = \underbrace{p_y^b F_\ell(\ell, x)\frac{\partial \ell}{\partial p_x}}_{\text{Factor Supply Effect}} + \underbrace{\frac{\partial x}{\partial p_x}(p_y^b F_x(\ell, x) - p_x^b)}_{\text{Input-Output Effect}} \qquad (4)$$

The first term in equation (4) captures the value added change due to variations in factor supply. Quantitatively, this terms depends on the degree of complementarity between foreign and domestic inputs as well as on the elasticity of factor supply.[22] The second term captures the direct impact that changes of imported input usage have on GDP. With perfect competition, profit maximization insures that $p_y F_x(\ell, x) = p_x$ when using current prices. When base period prices p_y^b and p_x^b are close to their current value,[23] this term vanishes. In such a model, any first order change in GDP following a terms of trade shock is solely driven by variations in domestic factor supply. This is the negative result presented in KR: when firms take prices as given, profit maximization insures that the marginal benefit of using an additional unit of imported input x ($p_y F_x(\ell, x)$) is equal to its marginal cost (p_x). Up to a first order approximation, foreign technological shocks affect real GDP only through a change in factor supply. In other words, the measured productivity is not affected by foreign shocks.[24]

Equation (4) encapsulates in a simple way the reasons why standard IRBC models cannot generate a quantitatively important link between trade linkages and GDP comovement. In models with perfect competition and constant returns to scale, the change in GDP after a foreign shock is solely driven by variations in domestic factors supply. Such a change, in turn, is disciplined by (i) the elasticity of labor supply and (ii) the complementarity between domestic and foreign inputs.[25]

3.2 Markups and love of variety

Consider now a variant of the economy described above with an additional production step: inputs are imported by a continuum of intermediate producers with a linear production function $m = x$. Critically, we now add two new elements: (1) a *price wedge* for intermediate producers $\mu > 1$ so that the price of intermediates m is given by $p_m = \mu \times p_x$, and (2) love of variety in the

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[22]The role of complementarity is discussed at length in Burstein et al. (2008) or in Boehm et al. (2015).

[23]With a Fisher chain-weighted price index in the construction GDP, base period prices are always close to current prices.

[24]Note that an important part of the reasoning rests upon the fact that GDP is constructed using constant base prices. If the prices used to value final goods and imported inputs were to change due to the shock, one would have an additional term in equation (4).

[25]If domestic and foreign inputs are strongly complement, any shock that increases foreign input usage also rises demand for domestic inputs, which increases GDP. However, as shown in Johnson (2014), complementarity in production factors *alone* is not sufficient to solve the TCP. More precisely, Johnson (2014) shows that cross country production synchronizes input usage and our paper takes this insight on-board and further shows that input synchronization can also lead to GDP synchronization when one adds markups and extensive margin adjustments.

final good production technology in the form of a Dixit-Stiglitz aggregation of intermediates.[26]
The production function in the final good sector is:

$$y = F(\ell, \mathcal{I}) \qquad \text{with } \mathcal{I} = \left(\int_0^{\mathcal{M}} m_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{5}$$

This production function displays love fof variety: for a given amount of total imports, the larger
the mass of input suppliers $\mathcal{M}$, the higher the amount of final production obtainable.

For each variety m_i, there is a producer with a linear technology using imports only:

$$\forall i \in [0, \mathcal{M}], \quad m_i = x_i \tag{6}$$

All intermediate producers are completely symmetric and we denote by m their (common) pro-
duction and by x their (common) import levels. The bundle $\mathcal{I}$ can then be simply expressed as
$\mathcal{I} = \mathcal{M}^{\sigma/(\sigma-1)} m$ and the price index dual to the definition of the bundle is $\mathcal{P} = \mathcal{M}^{1/(1-\sigma)} p_m$,
which is also equal to $F_{\mathcal{I}}(\ell, \mathcal{I})$, the marginal productivity of the input bundle in final good pro-
duction. Finally, taking the derivative of GDP with respect to p_x while keeping prices constant
at their base period value, we obtain:

$$\frac{dGDP}{dp_x} = \underbrace{p_y^b F_\ell(\ell, \mathcal{I}) \frac{\partial \ell}{\partial p_x}}_{\text{Factor Supply Effect}} + \underbrace{\left(\mathcal{M} \frac{\partial m}{\partial p_x} + \frac{\partial \mathcal{M}}{\partial p_x} m \right) \cdot (\mu - 1) p_x^b}_{\text{Markup Effect}} + \underbrace{\frac{1}{\sigma-1} p_m^b m \frac{\partial \mathcal{M}}{\partial p_x}}_{\text{Entry/Exit Effect}} \tag{7}$$

Equation (7) is the counterpart of (4) in a model with extensive margin adjustments and
where some domestic firms are not price takers. These two elements create a link between foreign
shocks and domestic real GDP variations, over and beyond any change in domestic factor supply.

First, the existence of a price wedge $\mu > 1$ means that the first term does not vanish. With
$m'(p_x) < 0$, a decrease in the price of imported inputs leads to an increase in GDP. When firms
are price setters and earn a positive profit, the marginal revenue generated by an additional unit
of imported input x is larger than its marginal cost p_x. Hence, cheaper inputs means more sales,
more profit and more value added.

Moreover, any change in the mass of firms $\mathcal{M}$ also impacts domestic value added. One can
model many reasons why the mass of producing firms would change, including a free entry
condition as in the quantitative model in section 4. A change in the number of price setting firms
gives a time varying element to the effect described above, triggering a greater reaction of GDP

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[26]In many models, the elasticity of substitution in the CES aggregation governs at the same time the markup
charged by monopolistic competitors and the degree of love of variety. In order to clearly differentiate the sheer effect
of markup from the love of variety, we assume here that the markup μ can take any value, including the case where
$\mu = \sigma/(\sigma-1)$.

after a foreign shock, independently of the love of variety which is captured by the parameter σ. Overall, the key idea governing this term can be expressed as follows: firms that charge a markup have a disconnect between the marginal cost and the marginal revenue product of their inputs. The difference between these two is accounted as value added in the form of profits. Any change in input usage leading to a change in profits triggers a change in value added.

Second, when $\sigma < +\infty$, another effect arises. When the production function exhibits love of variety, any change in the mass of suppliers implies an additional reaction for the input bundle $\mathcal{I}$. If the decrease of p_x is accompanied by an increase in the mass of producing firm,[27] the bundle $\mathcal{I}$ increases not only because each intermediate producer produces more, but also because an increase in the mass of firms mechanically increases $\mathcal{I}$ even for a fixed amount of intermediates.

With love of variety, a producer that has access to more suppliers can produce more output for the same level of input. In other words, the set of feasible combinations of output $\mathcal{I}$, and inputs $\int_0^{\mathcal{M}} m_i di = X$ is not independent of the mass of producers $\mathcal{M}$: a change of $\mathcal{M}$ shifts the production possibility frontier. Interestingly, this channel is at work independently of the price distortion channel discussed previously. Even in the absence of monopoly pricing, the sheer fluctuation in the mass of producing firms coupled with a love of variety creates a link between import price and GDP fluctuation.

Finally, note that the introduction of markups and love of variety allows GDP to change over and beyond changes in the domestic factors of production. Using a *growth accounting* perspective, this means that the introduction of these two elements makes measured domestic productivity change after a foreign shock, even though technology is unchanged. Two countries that have important trade flows in intermediate inputs should then have correlated measured TFP (i.e. *the Solow Residual*), a prediction we test in the data in section 6.3 and which our quantitative model is able to reproduce.

4   A Model of International Trade with Cross-Border Input Linkages

We develop a many-country international business cycle model with trade in final and intermediate goods. The model is related to Ghironi and Melitz (2005) and Alessandria and Choi (2007), extended to multiple asymmetric countries and intermediate goods crossing borders multiple times. In contrast to a standard IRBC framework, the model features *monopolistic competition* and *firms entry/exit*.[28]   As we will show, the combination of international I/O linkages, price distortions and extensive margin adjustments provide a quantitative solution to the TCP.

---

[27]If the mass of firms is pinned down by a free entry condition, the increase in profits of each intermediate producer when the price of imported input goes up leads to a increase in the mass of firms.

[28]Alternatively, the model presented here can be thought of as an extension of the IRBC model presented in Johnson (2014) with two new elements: markups and extensive margin adjustments. It is also related to the static small open economy model in Gopinath and Neiman (2014)

4.1 Consumption and Labor Supply

Consider a multi-period world economy with many countries ($i, j \in \{1, ..., N\}$). In each country, there is a representative consumer who consumes final goods and supplies labor $L_{i,t}$ for production. Consumers' utility function is:

$$U_0 = \mathbb{E}_0 \left[\sum_{t=0}^{+\infty} \beta^t \left(\log \left(C_{i,t}^F \right) - \psi_i \frac{L_{i,t}^{1+\nu}}{1+\nu} \right) \right] \tag{8}$$

$$\text{with} \quad C_{i,t}^F = \left(\sum_j \omega_i^F(j)^{\frac{1}{\rho^F}} \cdot C_{j,i,t}^{\frac{\rho^F-1}{\rho^F}} \right)^{\frac{\rho^F}{\rho^F-1}} \quad \text{and} \quad C_{j,i,t} = \left(\int_{s \in \Omega_{j,i,t}^F} c_{j,i,t}(s)^{\frac{\sigma_j-1}{\sigma_j}} ds \right)^{\frac{\sigma_i}{\sigma_i-1}} \tag{9}$$

where ψ_i is a scaling parameter, ν the inverse of the Frisch elasticity of labor supply and σ_i the elasticity of substitution between different varieties of final goods originating from country i. $\omega_i^F(j)$ is the share of country j in the *consumption* bundle of country i, with $\sum_j \omega_i^F(j) = 1$, and $\Omega_{j,i,t}^F$ is the endogenous set of firms from country j that serve the *final good* market in country i.[29] Finally, ρ^F is the *final goods* Armington elasticity of substitution. Final good price indices are defined as:

$$\mathcal{P}_{i,t}^F = \left(\sum_j \omega_i^F(j) \cdot \left(\widetilde{\mathcal{P}}_{j,i,t}^F \right)^{\frac{\rho^F-1}{\rho^F}} \right)^{\frac{\rho^F}{\rho^F-1}} \quad \text{and} \quad \widetilde{\mathcal{P}}_{j,i,t}^F = \left(\int_{s \in \Omega_{j,i,t}^F} p_{j,i,t}^F(s)^{\frac{\sigma_i-1}{\sigma_i}} ds \right)^{\frac{\sigma_i}{\sigma_i-1}} \tag{10}$$

where $p_{j,i,t}^F(s)$ is the price charged by firm s in the set $\Omega_{j,i,t}^F$ when selling in the *final good* market in country i. As we will see below, given our assumptions, firms charge the same price in both final and intermediate good markets in a given country.

The agent chooses consumption, investment and labor, subject to the budget constraint:[30]

$$\mathcal{P}_{i,t}^F \left(C_{i,t} + K_{i,t+1} - (1-\delta)K_{i,t} \right) = w_{i,t}L_{i,t} + r_{i,t}K_{i,t} - \mathcal{T}_i \tag{11}$$

where we introduced the term $\mathcal{T}_i$ which captures potential trade imbalance in country i ($\mathcal{T}_i < 0$, corresponds to a trade deficit meaning that country i consumes more than the value of its

---

[29]As we will see below, given our assumptions, the set of firms serving the final good and the intermediate input market in any country will be identical.

[30]Note that the right hand side of this equation include firms' profits since, as explained below, firms pay entry costs using domestic labor. It should then be understood that $L_{i,t}$ includes both production and "entry cost" workers. Moreover, an implicit assumption of the budget constraint above is that investment in the capital stock is done using the aggregated consumption good.

production). Optimality yields the standard Euler equation and labor supply:

$$\frac{1}{C_{i,t}} = \beta\mathbb{E}_t\left[\frac{1}{C_{i,t+1}} \times \left(\frac{r_{i,t+1}}{\mathcal{P}_{i,t}^F} + (1-\delta)\right)\right] \tag{12}$$

$$\psi_i L_{i,t}^v = \frac{w_{i,t}}{\mathcal{P}_{i,t}^F}\frac{1}{C_{i,t}} \tag{13}$$

4.2   Production

In any country i, production is performed by a continuum of firms with heterogeneous productivity, defined as the product of an idiosyncratic component φ and a country specific component $Z_{i,t}$. In all countries, productivity φ follows a Pareto distribution with shape parameter γ. Firms produce with a Cobb-Douglas technology using labor $\ell_{i,t}(\varphi)$, capital $k_{i,t}(\varphi)$ and intermediate inputs $I_{i,t}(\varphi)$ bought from other firms from their home country as well as from abroad. The intermediate input index in country i, $I_{i,t}$, is a CES aggregation of country specific bundles $M_{j,i,t}$, with an *intermediate goods* Armington elasticity ρ^I. To introduce a rationale for markups and for love of variety, each country specific bundle is itself a CES aggregation of many varieties, with the elasticity of substitution σ_j.[31] The production function is:

$$Q_{i,t}(\varphi) = Z_{i,t}.\varphi \, . \, I_{i,t}(\varphi)^{1-\eta_i-\chi_i} \, . \, \ell_{i,t}(\varphi)^{\chi_i} \, . \, k_{i,t}(\varphi)^{\eta_i} \tag{14}$$

$$\text{with}\qquad I_{i,t}(\varphi) = \left(\sum_i \omega_i(j)^{\frac{1}{\rho^I}}M_{j,i,t}^{\frac{\rho^I-1}{\rho^I}}\right)^{\frac{\rho^I}{\rho^I-1}} \qquad \text{and}\qquad M_{j,i,t} = \left(\int_{s\in\Omega_{j,i,t}} m_{j,i,t}(s)^{\frac{\sigma_i-1}{\sigma_i}}ds\right)^{\frac{\sigma_i}{\sigma_i-1}} \tag{15}$$

where $\omega_i^I(j)$ is the share of country j in the *production process* of country i, with $\sum_j \omega_i^I(j) = 1$, and $\Omega_{j,i,t}^I$ is the endogenous set of firms based in j and serving the *intermediate input* market in country i. Similarly to the *final good* market, we have

$$\mathcal{P}_{i,t}^I = \left(\sum_j \omega_i^I(j)\cdot\left(\widetilde{\mathcal{P}}_{j,i,t}^I\right)^{\frac{\rho^I-1}{\rho^I}}\right)^{\frac{\rho^I}{\rho^I-1}} \qquad \text{and}\qquad \widetilde{\mathcal{P}}_{j,i,t}^I = \left(\int_{s\in\Omega_{j,i,t}^I} p_{j,i,t}^I(s)^{\frac{\sigma_i-1}{\sigma_i}}ds\right)^{\frac{\sigma_i}{\sigma_i-1}} \tag{16}$$

$$\text{and } \mathcal{P}_i^{IB} = \chi_i^{-\chi_i} \times \eta_i^{-\eta_i} \times (1-\eta_i-\chi_i)^{(\eta_i+\chi_i-1)} \times \left(\mathcal{P}_{i,t}^I\right)^{1-\eta_i-\chi_i} \times w_{i,t}^{\chi_i} \times r_{i,t}^{\eta_i} \tag{17}$$

where $\mathcal{P}_{j,i,t}$ denotes the price of the country-pair specific bundle $M_{j,i,t}$ and $\mathcal{P}_{i,t}^{IB}$ is the unit cost of the Cobb Douglas bundle aggregating $I_{i,t}$, $k_{i,t}$ and $\ell_{i,t}$ (called the *input bundle*) and represents the price of the basic production factor in country i. $p_{j,i,t}^I(s)$ is the price charged by any firm s in the

---

[31] This parameter governs both the markup charge by firm from country j and the degree of love of variety.

set $\Omega^I_{j,i,t}$ when selling in the *intermediate input* market in country i.[32]

To be allowed to sell its variety to a country j, a firm from country i must pay a fixed cost f^c_{ij} (labeled in unit of the *input bundle*) as well as a variable (iceberg) cost τ_{ij}. Firms choose which countries they enter (if any), affecting both the level of competition and the marginal cost of all firms in the country. As will be clear below, profits are strictly increasing with productivity φ so that equilibrium export decisions are defined by country-pair specific thresholds $\overline{\varphi}_{i,j,t}$ above which firms from i find it profitable to pay the fixed cost f^c_{ij} and serve the *final good* and *intermediate inputs* markets in country j. Finally there is an overhead entry cost f^E_i, sunk at the production stage, to be paid before firms know their actual productivity. Based on their expected profit in all markets, firms enter the economy until the expected value of doing so equals the overhead entry cost. This process determines the mass of firms $M_{i,t}$.

4.3 Equilibrium

We specify the equilibrium conditions of the model by introducing $X_{i,t}$ the aggregate consumers' revenue and $S_{i,t}$ the total firms' spendings (including bilateral fixed costs payments to access all markets) in country i. Given prices, total demand faced by firm φ is given by the sum of demand stemming from both the *final good* and the *intermediate input* markets:

$$
\begin{aligned}
q_{i,t}(\varphi) = &\sum_j \left(\frac{p^F_{i,j,t}(\varphi)}{\widetilde{\mathcal{P}}^F_{i,j,t}} \right)^{-\sigma_i} \left(\frac{\widetilde{\mathcal{P}}^F_{i,j,t}}{\mathcal{P}^F_{j,t}} \right)^{-\rho^F} \frac{\omega^F_j(i) X_{j,t}}{\mathcal{P}^F_{j,t}} \\
&+ \sum_j \left(\frac{p^I_{i,j,t}(\varphi)}{\widetilde{\mathcal{P}}^I_{i,j,t}} \right)^{-\sigma_i} \left(\frac{\widetilde{\mathcal{P}}^I_{i,j,t}}{\mathcal{P}^I_{j,t}} \right)^{-\rho^I} \frac{\omega^I_j(i)(1 - \eta_j - \chi_j) S_{j,t}}{\mathcal{P}^I_{j,t}}
\end{aligned}
\tag{18}
$$

where the summation is done over all markets that are served by a firm with productivity φ.

Firms choose their price to maximize profits. Since the price elasticity of demand is constant, they charge a constant markup over marginal cost. For a firm from country i, the only elasticity that is relevant for pricing is σ_i, capturing the fact that firms compete primarily with other firms coming from their home country since their individual pricing decision has no impact on the country-specific price index in every market.[33] As a result, firms charge the same markup in the final and intermediate good markets, and we have: $p^F_{i,j,t}(\varphi) = p^I_{i,j,t}(\varphi) = p_{i,j,t}(\varphi)$ and $\widetilde{\mathcal{P}}^F_{i,j,t} = \widetilde{\mathcal{P}}^I_{i,j,t} = \widetilde{\mathcal{P}}_{i,j,t}$. The marginal cost of a firm with productivity φ in country i is $\mathcal{P}^{IB}_{i,t}/(Z_{i,t}\varphi)$

---

[32]The exact expressions of these objects are standard and can be found in the supplementary appendix B.

[33]With a finite number of firms, elasticities σ_i, ρ^I and ρ^F would all appear in the pricing strategy. In such a case, every firm would take into account the fact that its own price has an impact on the unit cost of the corresponding country-specific bundle. Therefore, when decreasing its price, a firm would attract more demand compared to firms from its own country but also increase the share of total demand that goes to every other firms from its country.

and its optimal price in country j is:

$$p_{i,j,t}(\varphi) = \tau_{ij} \frac{\sigma_i}{\sigma_i - 1} \frac{\mathcal{P}_{i,t}^{IB}}{Z_{i,t}\varphi} \tag{19}$$

Unlike in the canonical Krugman (1980), Melitz (2003) or Ghironi and Melitz (2005) models, one needs to jointly solve for all prices in the economy. Through $\mathcal{P}_{i,t}^{IB}$, the price charged by firm φ in country i depends on the prices charged by all firms supplying country i (both domestic and foreign) which in turn depend on the prices charged by their suppliers and so on and so forth. Determining prices requires solving jointly for all country-pair specific price indexes $\widetilde{\mathcal{P}}_{i,j,t}$.

The definitions of price indexes give rise to a simple relationship between the price of the country i specific bundle at home, $\widetilde{\mathcal{P}}_{i,i,t}$, and its counterpart in country j, $\widetilde{\mathcal{P}}_{j,i,t}$:

$$\widetilde{\mathcal{P}}_{j,i,t} = \tau_{ij} \left(\frac{\overline{\varphi}_{i,j,t}}{\overline{\varphi}_{i,i,t}} \right)^{\frac{\sigma_i - \gamma_i - 1}{1 - \sigma_i}} \times \widetilde{\mathcal{P}}_{i,i,t} \tag{20}$$

where $\overline{\varphi}_{i,j,t}$ defines the threshold of idiosyncratic productivity φ above which firms from i serve country j. Intuitively, the ratio between the price of a country specific bundle in two different markets depends on the relative iceberg costs as well as the relative entry thresholds. Using this relation in the definition of price indexes in every country yields a system of N equations which jointly defines all *inner* price indexes:

$$(\widetilde{\mathcal{P}}_{i,i,t})^{1-\rho^I} = \mu_i \left(\sum_j \omega_i^I(j) \left(\tau_{ji} \left(\frac{\overline{\varphi}_{j,i,t}}{\overline{\varphi}_{j,j,t}} \right)^{\frac{\sigma_j - \gamma_j - 1}{1 - \sigma_j}} \widetilde{\mathcal{P}}_{j,j,t} \right)^{1-\rho^I} \right)^{1-\eta_i-\chi_i} \tag{21}$$

with μ_i depending on entry thresholds, the mass of firms and parameters.[34] For given thresholds and mass of firms, this system admits a unique non-negative solution.[35]

Turning to export strategies, the productivity thresholds above which firms from country i serve market j are implicitly defined by:

$$\pi_{i,j,t}(\overline{\varphi}_{i,j,t}) = \frac{\mathcal{P}_{i,t}^{IB}}{Z_{i,t}}.f_{ij}^c \quad \text{for all } i,j \in \{1,...,N\} \tag{22}$$

where $\pi_{i,j,t}(\varphi)$ is the variable profit earned by a firm with productivity φ in market j. Similar to Ghironi and Melitz (2005), the fixed cost f_{ij}^c is paid in units of the basic production factor in

---

[34] $\mu_i^{\frac{1-\sigma_i}{1-\rho^I}} = \frac{\gamma \overline{\varphi}_{i,i,t}^{\sigma_i - \gamma_i - 1}}{\gamma_i - (\sigma_i - 1)} M_{i,t} \left(\frac{\sigma_i}{\sigma_i - 1} \frac{w_{i,t}^{\chi_i} \times r_{i,t}^{\eta_i}}{\chi_i^{\chi_i} \times \eta_i^{\eta_i} \times (1 - \eta_i - \chi_i)^{1 - \eta_i - \chi_i}} \frac{1}{Z_{i,t}} \right)^{1-\sigma_i}$

[35] Following Kennan (2001) and denoting $G_k = (\widetilde{\mathcal{P}}_{i,i,t})^{1-\rho^I}$ and G the associated $N \times 1$ vector, it suffices to show that the system is of the form $G = f(G)$ with $f : \mathbb{R}^N \to \mathbb{R}^N$ a vector function which is strictly concave with respect to each argument, which is obvious as long as $0 < \eta_k + \chi_k < 1$.

country i deflated by aggregate technology Z.[36]

Finally, the mass of firms is determined by the free entry condition defined as:

$$\Pi_{i,t} = M_{i,t} \frac{w_{i,t}}{Z_{i,t}} . f_i^E \quad \text{for all } i \tag{23}$$

where f_i^E is labeled in units of labor and $\Pi_{i,t}$ denotes aggregate profits of all firms in country i. Following Eaton and Kortum (2005), we can show that $\Pi_{i,t}$ is proportional to total revenues. Defining $R_{i,t}$ the total sales of all firms from country i, we have:

Lemma 1. : Total profits in country i are proportional to total revenues:

$$\Pi_{i,t} = \frac{\sigma_i - 1}{\gamma_i \sigma_i} R_{i,t} \tag{24}$$

Proof: see Appendix B.

Closing the model involves standard market clearing conditions for capital, labor and goods. Labor can be used either for production ($L_{i,t}^p$) or for the entry cost ($L_{i,t}^e$) so that $L_{i,t} = L_{i,t}^p + L_{i,t}^e$. With Cobb-Douglas production, consumer's revenues $X_{i,t}$ are equal to the sum of the payment to production workers $\chi_i S_{i,t}$, rent from capital $\eta_i S_{i,t}$, total firms' profits $\Pi_{i,t}$ (which, at the free entry equilibrium, is completely used to pay the entry cost f_i^E), and potential trade imbalances $-\mathcal{T}_{i,t}$. Total revenues of all firms from i are:

$$R_{i,t} = \sum_j \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_j^F}\right)^{1-\rho^F} \omega_j^F(i) X_{j,t} + \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_j^I}\right)^{1-\rho^I} \omega_j^I(i)(1 - \eta_j - \chi_j) S_{j,t} \tag{25}$$

And total exports (the sum of *final goods* and *intermediate inputs* exports) from i to j is defined as

$$T_{i \to j} = \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_{j,t}^F}\right)^{1-\rho^F} \omega_j^F(i) X_{j,t} + \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_{j,t}^I}\right)^{1-\rho^I} \omega_j^I(i)(1 - \eta_j - \chi_j) S_j$$

Using $X_{i,t} = w_{i,t} L_{i,t} + r_{i,t} K_{i,t} - \mathcal{T}_{i,t} = (\eta_i + \chi_i) S_{i,t} + \Pi_{i,t} - \mathcal{T}_{i,t}$, the good market clearing condition writes:

$$R_{i,t} = \sum_j \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_{j,t}^F}\right)^{1-\rho^F} \omega_j^F(i) \left[(\eta_j + \chi_j) S_j + \Pi_j - \mathcal{T}_j\right] + \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}_j^I}\right)^{1-\rho^I} \omega_j^I(i)(1 - \eta_j - \chi_j) S_j \tag{26}$$

---

[36]In every market, entry occurs until the profit of the least productive firms is equal to the fixed cost of accessing the market. Denoting by $X_{i,t}$ total final good spending by consumers ($X_{i,t} = w_{i,t} L_{i,t} + r_{i,t} K_{i,t} - \mathcal{T}_{i,t}$), we have for any i

and j: $\overline{\varphi}_{i,j,t} = \left(\tau_{ij} \frac{\sigma_i}{\sigma_i - 1} \frac{\mathcal{P}_{i,t}^{IB}}{Z_{i,t}} \frac{1}{\widetilde{\mathcal{P}}_{i,j,t}}\right) \times \left(\frac{\sigma_i f_{ij}^c (\mathcal{P}_{i,t}^{IB}/Z_{i,t})}{\left(\widetilde{\mathcal{P}}_{i,j,t}/\mathcal{P}_{j,t}^I\right)^{1-\rho^I} \omega_j^I(i)(1-\eta_j-\chi_j) S_j + \left(\mathcal{P}_{i,j,t}/\mathcal{P}_{j,t}^F\right)^{1-\rho^F} \omega_j^F(i) X_{j,t}}\right)^{\frac{1}{\sigma_i - 1}}$.

Furthermore, using lemma 1 above and the fact that $R_{i,t} = S_{i,t} + \Pi_{i,t}$, we get:

$$S_{i,t} = \left(\frac{\sigma_i \gamma_i - \sigma_i + 1}{\sigma_i \gamma_i} \right) R_{i,t}$$

Replacing $\Pi_{i,t}$ and $S_{i,t}$ as a function of $R_{i,t}$, equation (26) can be written as:

$$
\begin{aligned}
R_{i,t} = & \sum_j \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{j,t}} \right)^{1-\rho^F} \omega^F_j(i) \left[\frac{(\eta_j + \chi_j) \cdot (\sigma_j \gamma_j - \sigma_j + 1) + \sigma_j - 1}{\sigma_j \gamma_j} R_{j,t} - \mathcal{T}_{j,t} \right] \\
& + \sum_j \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^I_{j,t}} \right)^{1-\rho^I} \omega^I_j(i)(1 - \eta_j - \chi_j) \left(\frac{\sigma_j \gamma_j - \sigma_j + 1}{\sigma_j \gamma_j} \right) R_{j,t}
\end{aligned}
\tag{27}
$$

Which can be expressed in compact form as:

$$
M \cdot \begin{pmatrix} R_1 \\ \vdots \\ R_N \end{pmatrix} = - \left((W^F)' \circ \mathbb{P}^F \right) \begin{pmatrix} \mathcal{T}_1 \\ \vdots \\ \mathcal{T}_N \end{pmatrix}
\tag{28}
$$

Where $\circ$ is the element-wise (Hadamard) product and W^F is the weighting matrix associated with final good aggregation and is defined as $W^F_{ij} = \omega^F_i(j)$. $\mathbb{P}^F$ is a matrix defined by $\mathbb{P}^F_{i,j,t} = \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{i,t}} \right)^{1-\rho^F}$. Moreover, the matrix M is defined at any time t as:

$$
\begin{aligned}
M_{i,j,t} = & \mathcal{I}_{i,j} - \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{j,t}} \right)^{1-\rho^F} \omega^F_j(i) \frac{(\eta_j + \chi_j)(\sigma_j \gamma_j - \sigma_j + 1) + \sigma_j - 1}{\sigma_j \gamma_j} \\
& - \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^I_{j,t}} \right)^{1-\rho^I} \omega^I_j(i)(1 - \eta_j - \chi_j) \left(\frac{\sigma_j \gamma_j - \sigma_j + 1}{\sigma_j \gamma_j} \right)
\end{aligned}
\tag{29}
$$

Setting $w_1 = 1$, implying $S_1 = L^p_1 / \chi_1$, provides a unique solution for all variables by solving together the investment Euler equation (12), the labor supply equation (13), the price system (21), the threshold system (22), the Revenue system (28) and the Free Entry system (23).

GDP definition. In the data, GDP is constructed using base prices and quantity estimates. In order to be as close as possible to the method used in the construction of the data used in the empirical analysis, we define GDP using steady state prices as base prices.[37] GDP is obtained by deflating nominal spending using steady-state price indices that are corrected from product

---

[37]In the data, GDP is defined using the Fisher ideal quantity index which is a geometric mean of the Laspeyres and Paasche indices. Hence, for all periods t, the base period price is a geometric mean between period t and period $t + 1$.

variety effects, such that:

$$GDP_{i,t} = \underbrace{\widehat{\mathcal{P}_i^{F,ss}} \frac{X_{i,t}}{\widehat{\mathcal{P}_{i,t}^F}}}_{\text{Consumption + Investment}} + \underbrace{\sum_j \widehat{\mathcal{P}_{i,j}^{ss}} \frac{T_{i \to j,t}}{\widehat{\mathcal{P}_{i,j,t}}}}_{\text{Total exports (final+ inputs)}} - \underbrace{\sum_j \widehat{\mathcal{P}_{j,i}^{ss}} \frac{T_{j \to i,t}}{\widehat{\mathcal{P}_{j,i,t}}}}_{\text{Total imports (final + inputs)}} \quad (30)$$

where we defined $\widehat{\mathcal{P}_{i,j,t}} = \left(M_{i,t} . (\overline{\varphi}_{i,j,t})^{-\gamma_i} \right)^{1/(\sigma_i - 1)} \widetilde{\mathcal{P}}_{i,j,t}$ and $\widehat{\mathcal{P}_{i,t}^F} = \left(\sum_j \omega_i^F(j) \cdot \left(\widehat{\mathcal{P}_{j,i,t}} \right)^{\frac{\rho^F - 1}{\rho^F}} \right)^{\frac{\rho^F}{\rho^F - 1}}$ in

order to be consistent with the way actual data are collected.[38]

5 Calibration

The model is calibrated to 14 countries and a composite *Rest-Of-the-World* for the time period 1980-1990. As compared to our empirical sample, it represents around 78% of total trade flows, 79% of total trade in final goods and 77% of total trade flows in intermediate goods.[39] With N countries, there are $4 \times N^2 + 4N + 5$ parameters to determine, to which one must add parameters relative to the technological shocks.[40]

5.1 Parameterization

We set $\beta = 0.99$ and we choose $\nu = 1$, leading to a Frisch elasticity of 1. We set the value of the macro (Armington) elasticity ρ^I and ρ^F to be equal to unity, which is in the range of the literature. For instance, Saito (2004) provides estimations from 0.24 to 3.5 for the Armington elasticity.[41] There is also a theoretical convenience to use $\rho^I = \rho^F = 1$, as it allows the model to take the same form as other network models such as Acemoglu et al. (2012). We set parameters ψ_i in each country to replicate the *relative difference* of working age population with a normalization ensuring an average capital-output ratio of 13 in the model.[42]

Markups and Value Added Shares. Concerning the micro elasticities, we set a value of $\sigma_i =$

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[38]Since both consumers' utility and production functions have a CES component, it is well known that the associated price indexes can be decomposed into components reflecting average prices (captured by statistical agencies) and product variety (which is not taken into account in national statistics). See Feenstra and Markusen (1994) or Ghironi and Melitz (2005) for a discussion of this.

[39]The set of countries is: Australia, Austria, Canada, Denmark, France, Germany, Ireland, Italy, Japan, Mexico, Netherlands, RoW, Spain, United Kingdom and United States.

[40]For each country-pair (i, j), we specify values for $\omega_i^F(j), \omega_i^I(j)$ (each with $N \times (N-1)$ values), τ_{ij} and f_{ij}^c. For every country i we have $(\eta_i + \chi_i)$, ψ_i, f_i^E, $\mathcal{T}_i$, σ_i and γ_i. The set of common parameters is given by $\chi_i / (\chi_i + \eta_i)$, ν, β, ρ^I and ρ^F. On top of these, we also need to set the volatility, covariance and auto-correlation of the technology shocks in all countries.

[41]Feenstra et al. (2014) studies the macro and micro elasticities for final goods and reports estimates between -0.29 and 4.08 for the Armington elasticity. They find that for half of goods the macro elasticity is significantly lower than the micro elasticity, even when they are estimated at the same level of disaggregation.

[42]This normalization has no influence on the results because FOCs are independent from this parameter.

$\sigma = 5$, $\forall i$ in the baseline simulation. Anderson and van Wincoop (2004) report available estimates for the micro elasticity in the range of 3 to 10. Following Bernard et al. (2003), Ghironi and Melitz (2005) choose a micro elasticity of 3.8 and recently, papers such as Barrot and Sauvagnat (2016) or Boehm et al. (2015) argue that firms' ability to substitute between their suppliers can be very low. This choice leads to markups of 25%. The aggregate profit rate, however, is only of 17.4% since firms have to pay fixed cost in order to access any market. In Section 7, we consider alternative elasticities for σ_i, and defer discussion of those cases till then.[43] We set the Pareto Shape of the firm-specific productivity distribution to $\gamma_i = \sigma_i - 0.4$, as in Fattal Jaef and Lopez (2014).

The value added share, $\eta_i + \chi_i$, for a given country i, are calibrated using cost of intermediates and total sales as observed in the WIOD database at the 2-digits sector level. Specifically, $(1 - \eta_{i,s} - \chi_{i,s}) = \frac{cost\_intermediates_s}{total\_sales_s}$, represents the share of intermediate inputs in total costs in a given sector. We use the fact that $total\ sales_s = \mu_i \times total\ cost_s$ with μ_i the markups in country i. Therefore, we fix $(\eta_{i,s} + \chi_{i,s}) = 1 - \frac{cost\_intermediates_s}{total\_sales_s} \frac{\sigma_i}{\sigma_i - 1}$. With $\sigma_i = 5$, the implied mean values of $\eta_i + \chi_i$, weighted by the sector importance in total sales, range from 0.31 to 0.45 for the considered countries (we set the value for *RoW* to the mean value), which seems to be consistent with values reported in Halpern et al. (2015). Finally, the capital and labor shares in value added are fixed at 2/3 and 1/3 respectively.

Entry costs. The sunk entry cost f_i^E in each country are computed from the *Doing Business Indicators*.[44] We measure the relative entry fixed costs by using the information on the amount of time required to set up a business in the country relative to the US, where we normalize f_{US}^E in order to generate a ratio of total number of firms divided by the working population, $\frac{M}{L}$, of about 12%. This is motivated by the fact that there are about 22-24 millions of non-employer businesses and 5.5 millions of employer businesses in the US, while the working age population represents around 180 millions of individuals during the considered period.[45] As shown later, the results are not sensitive to this specification.

Trade frictions. The variable (iceberg) trade costs for each country-pairs, $\tau_{ij} > \tau_{ii}$, are taken from the ESCAP World Bank: *International Trade Costs Database*, where we normalize $\tau_{ii} =$

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[43]We for instance recalibrate the model with heterogeneous elasticities of substitution across varieties, σ_i, based on two measures: the De Loecker and Eeckhout (2018) markup estimates and a Price Cost Margin approach. As shown in section 7, the introduction of heterogenous markups makes it possible to study the role of market power in shaping the correlation between terms of trade and GDP, in line with empirical evidence.

[44]The World Bank's Doing Business Initiative collected data on regulations regarding obtaining licenses, registering property, hiring workers, getting credit, and more. See http://doingbusiness.org/data/exploretopics/trading-across-borders and http://doingbusiness.org/data/exploretopics/starting-a-business. Unfortunately, due to data limitations, we use observation available in 2015. However, as shown later, f_i^E plays a little role in the correlation between trade and GDP comovement.

[45]This is also close to the 12% self-employment rate usually reported for the US between 1990 and 2000 (BLS). Results are not sensitive to this assumption. We provide a comparison of this rate and the self-employment rate in each economy in the supplementary appendix C.

1.[46] This database features symmetric bilateral trade costs in its wider sense, including not only international transport costs and tariffs but also other trade cost components discussed in Anderson and van Wincoop (2004). Similar to Helpman et al. (2008), we assume domestic fixed costs $f_{ii}^c = 1$ for every country i. We set the values for the fixed costs of exporting from country i to country j, $f_{ij}^c > 1$ for $i \neq j$, in line with Di Giovanni and Levchenko (2013) using the Trading Across Borders module of the *Doing Business Indicators*. Specifically, we choose the number of days it takes to export to a specific country relative to the number of days it takes to supply in the home country (normalized to 1 in the model).[47]

Steady-State Trade Flows and Imbalance. Data relative to bilateral flows in final goods and intermediate inputs, $\{T_{j \to i}^I / GDP_i, T_{j \to i}^F / GDP_i\}$, are sufficient to identify the shares $\omega_i^I(j)$ and $\omega_i^F(j)$. Similar to our empirical part, we use trade data from Johnson and Noguera (2017) dis-aggregated into final and intermediate goods. Moreover, since complete financial autarky is inconsistent with the trade balances observed in the data, we calibrate the model trade imbalance $\{\mathcal{T}_1, ..., \mathcal{T}_N\}$ to match steady-state trade imbalances relative to GDP, and then hold those nominal imbalances constant during the simulation. Finally, to be as close as possible to the data used in the empirical analysis, we construct estimates by deflating the nominal spending by the price index that do not take into account love of variety, as described in section 4.3. By taking all of this information, the model steady state matches relative bilateral trade flows and relative trade imbalances exactly.

Aggregate Technology Process. The *level* of GDP comovement in our simulations is driven both by correlated technology shocks and by the transmission of those shocks across countries via trade linkages. In the model, $Z_{i,t}$ is the country-specific technology process which is not properly measured in the data by the Solow Residual (see section 6.3 for a discussion on this). We take a different route and set the cyclical properties of $(Z_i)_{i=1,...,N}$ to replicate observed GDP properties. To calibrate the variance-covariance matrix and the persistence of those technology shocks, we set the off diagonal elements (the covariance terms) so that the average correlation of GDP in the model matches exactly the one observed in the data, which is 0.27 for the selected countries in 1980-1999. We then calibrate the volatility (the diagonal elements of the covariance matrix) so that the model replicates exactly the observed GDP volatility (de-trended using HP-filter) in every country. This allows us to generate GDP fluctuations in the simulated economy that are similar to those observed in the data.[48] It is informative to note that, in order to match an observed international GDP correlation of 0.27, the correlation of technology shocks is only

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[46]See at http://artnet.unescap.org/.

[47]This approach means that the fixed cost associated with trade from France to the US is the same as the one from Germany to the US. One must keep in mind, however, that the iceberg variable cost will differ.

[48]Recall that the goal of this exercise is not to explain the *level* of comovement across countries, but its *slope* following a change in trade intensities.

0.189, with cross-country propagation through trade making up for the gap between technology and GDP correlations.[49] In this sense, through the lens of our model, propagation through trade explains about a third of international comovement.

Finally, we set a common value for auto-correlation of shocks so that the GDP series generated by the model is exactly 0.8, which is the average GDP auto-correlation in the data. One last detail regarding the simulation is that we parameterize the variance of the shocks to the *Rest-of-the-World* based on median GDP value in the data and the RoW covariance terms are set to 0. Table 3 reports the list of parameters. All targeted moments are perfectly matched.

Table. 3. Parameters of the model.

Parameter	Symbol	Value	Moment / Source
A. Fixed parameters			
Discount factor	β	.99	Annual discount rate of 4%
Labor curvature	ν	1.0	Frisch elasticity of 1.0
Labor Supply Scaling	ψ_i	$[5.4e-5, 0.16]$	Relative working age population
Labor share	$\chi_i/(\chi_i + \eta_i)$	2/3	67% of domestic value added
Argminton elasticities	ρ^I, ρ^F	1.0	Saito (2004), Feenstra et al. (2014)
Micro elasticity of substitution	$\sigma_i, \forall i$	5.0	Markup of 25%, profit rate of 17.4%
Sunk entry cost	f_i^E/f_{US}^E	[0.4 - 3.9]	*Doing Business Database* - World Bank
Fixed trade cost	f_{ij}^c	[3.3 - 18]	*Doing Business Database* - World Bank
Iceberg trade cost	τ_{ij}	[1 - 2.8]	ESCAP - World Bank
Pareto shape	γ_i	$\sigma_i - 0.4$	Fattal Jaef and Lopez (2014)

Parameter	Symbol	Value	Main target
A. Steady states			
Inputs spending weights	$\omega_i^I(j)$	in sup. app.	Import shares in inputs
Final goods spending weights	$\omega_i^F(j)$	in sup. app.	Import shares in final goods
Trade imbalance	$\{\mathcal{T}_i, ..., \mathcal{T}_N\}$	in sup. app.	Trade imbalance over GDP
B. Simulation: Technology process			
Persistency of Techno. shocks	ρ_Z	.77	Avg. GDP auto-correlation
Std. of Techno. shocks	$\sigma_Z(i)$	[.0012, .0050]	GDP volatility (de-trended)
Covariance of Techno. shocks	$\sigma_Z(i,j), \forall i \neq j$	.189	Avg. GDP correlation of 0.27

6 Results

Following our empirical findings in section 2, we examine the model's ability to match the aggregate TC slope. The analysis focus on three questions: (i). Is the model able to generate a TC slope of the same magnitude as in the data? (ii). What are the role of price distorsions and extensive margin in generating this TC slope? (iii). What is the quantitative importance of trade and TFP correlation in generating the observed *level* of GDP co-movement?

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[49] Indeed, when we calibrate the model with zero trade flows for all country pairs, then GDP correlation is very close to the correlation of technology shocks, as shown in table 5. This is not surprising since in such a case, the world is essentially a collection of island that do not interact with one another.

6.1 Baseline Experiment

To assess the model ability to replicate the correlation between trade and GDP-comovement, we simulate a sequence of 5,000 shocks (identical across each configuration) and record the correlation of logged and HP-filtered GDP as well as the average index of logged trade proximity in intermediate inputs and final goods. As the objective is to use within country-pair variations, we then recalibrate the spending shares $\omega_i^I(j)$ and $\omega_i^I(j)$ for all country-pairs i and j with different targets for trade proximity across countries, decreasing and increasing the targeted imports in intermediate inputs relative to GDP by 10% and then decreasing and increasing the targeted imports in final goods relative to GDP by 10% (this amounts to 5 experiments, including the baseline simulation).[50] This gives rise to a panel dataset of $14 \times 13/2 = 91$ country-pairs (excluding *RoW*) for each of the 5 configurations, hence a total of 455 observations.[51]. We then use this simulated dataset to estimate the model-implied TC slope, controlling for country-pair fixed effects, as we did in the empirical analysis. Table 4 shows the results.

Table. 4. Trade Comovement Slope: Data versus Model

Dependent variable: Corr GDP$^{\text{HP}}$		Data		Model	
Trade index measure		(1)	(2)	(3)	(4)
Measure	Variable			Bench. $\rho^F = 1$	$\rho^F = 1.05$
$\left(\frac{T_{i\leftrightarrow j}}{GDP_i + GDP_j}\right)$	$\ln(Trade_{Input})$	0.054**	0.053**	0.051***	0.065***
	$\ln(Trade_{Final})$	0.003	−0.030	0.017***	0.007***
$\max\left(\frac{T_{i\leftrightarrow j}}{GDP_i}, \frac{T_{i\leftrightarrow j}}{GDP_j}\right)$	$\ln(Trade_{Input})$	0.050***	0.052**	0.052***	0.065***
	$\ln(Trade_{Final})$	0.004	−0.032	0.016***	0.005**
Country FE		Yes	Yes	Yes	Yes
Time windows FE		No	Yes	-	-

Note: *p<0.1; **p<0.05; ***p<0.01.

The benchmark model generates a realistic trade comovement slope of about 7.0%, comparable to the range of values $[4.8\% - 11\%]$ reported in the literature for different set of countries, time periods and specifications. Turning to the relative importance of intermediate inputs relative to final goods, we find that trade in inputs has 3 times the explanatory power of that of final goods and account for 70% of the total trade comovement slope. Our simulated slope with trade in intermediate inputs is close to one estimated from the data, and this hold for the two measures of trade proximity usually considered in the literature. In the data, the trade comovement slope associated with trade in final goods is found small and insignificant, a feature that can be obtained in our model by using a higher final goods Armington elasticity of $\rho^F = 1.05$, as shown in table 4, column (4).

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[50]The model results are not very sensitive to the percent increase of trade flows between experiment, suggesting that the impact of trade on GDP-comovement is fairly linear in the model.

[51]Each configuration can be thought as a different time-window, except that we do not need to add controls for each configuration in the model, as we only change trade intensities between experiments.

6.2 Decomposition - Role of the ingredients

To assess the role of each ingredient in the quantitative results, we then turn off one by one the key elements of the model, namely movements in the number of firms supplying each market and price distorsions. Results are gathered in table 5.

Table. 5. Decomposition of the roles of price distorsions and the extensive margin.

Model	Benchmark $\rho^F = 1$			High elasticity $\rho^F = 1.05$		
	TC - Slope [a]		GDP corr [b]	TC - Slope [a]		GDP corr [b]
	Input	Final		Input	Final	
Data (with CP & TW FE)	0.053**	-0.030	0.270	0.053**	-0.030	0.270
I/O linkages + Markups + EM	0.051***	0.017***	0.270	0.065***	0.007***	0.258
I/O linkages + Markups	0.024***	0.005***	0.229	0.026***	0.002***	0.225
I/O linkages	0.007***	0.005***	0.212	0.007***	0.004***	0.210

*p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation.
[a]The trade indexes used in those experiments are $(T_{i \to j} + T_{j \to i})/(GDP_i + GDP_j)$.
[b]Average logged and HP filtered GDP correlation for the selected sample.

The sole addition of price distorsions to an otherwise standard IRBC model with I/O linkages increases the trade comovement slope for inputs from 0.007 to 0.024, while the amplification coming through the fluctuation in the mass of firms increases the slope further to 0.051.[52] Turning to the implied GDP correlation, adding price distorsions and adjustments along the extensive margins imply an overall non negligeable increase in the average GDP correlation by 5.8 percentage points relative to the model featuring only I/O linkages. Notice that those findings are robust to a higher Armington elasticity of $\rho^F = 1.05$. Overall, the key insight emerging from this analysis is that adding GVC to an otherwise standard IRBC model is not sufficient to solve the TCP, as shown in Johnson (2014). However, a model combining GVC with markups and extensive adjustments can provide the first quantitative solution to the TCP.

6.3 Solow Residual and Technology

In section 3, we showed how the introduction of two elements, extensive margin adjustments and market power, creates a link between foreign shocks and domestic productivity (usually measured as the *the Solow Residual*), even with fixed technology. More precisely, our model predicts that an increase in trade flows in input synchronizes *Solow residual* (SR) fluctuations across countries. Defining SR as: $SR_{it} = log(GDP_{it}) - \left(\frac{\eta_i}{\eta_i + \chi_i}\right) log(K_{it}) - \left(\frac{\chi_i}{\eta_i + \chi_i}\right) log(L_{it})$ we present in table 6 the relationship between SR and trade intensity in inputs and final goods as

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[52]This is in contrast with empirical findings in Gopinath and Neiman (2014), who argue that the extensive margin plays a small role in explaining the Argentine trade collapse. We provide new evidence of the role of extensive margin in section 7, where our empirical analysis embeds a number of different countries and assess adjustments of the extensive margin over a much longer time horizon (10 years in our empirical specification) as well as within each time windows.

estimated from the data and in our simulations. Both in the data and in our baseline calibration, an increase of trade in inputs is associated with an increase in *SR* comovement by a factor of 6 relative to trade in final goods. Quantitatively, our model is able to reproduce a realistic trade-SR slope. As expected, this association is absent in a model without markups and extensive margin adjustments and is decreasing in σ.

Table. 6. Model: trade-TFP comovement slope [a]

	Exact Techno. (Z)		Solow Residual (SR) [c]			Trade-SR slope [c]	
	corr [b]	ACF	corr [b]	ACF	$\frac{\sigma_{SR}}{\sigma_Z}$	Inter. inputs	final
Data (with TW + CP FE)	-	-	0.228	0.639	-	0.055**	-0.044*
Baseline ($\sigma = 5.0$)	0.189	0.77	0.246	0.736	2.709	0.037***	0.007***
I/O linkages + Markups	0.189	0.77	0.213	0.775	1.906	0.016***	-0.002***
I/O linkages only	0.189	0.77	0.196	0.774	2.008	-0.001***	-0.001***
High markup ($\sigma = 4.0$)	0.189	0.77	0.282	0.726	3.765	0.063***	0.013***
Low markup ($\sigma = 6.0$)	0.189	0.77	0.230	0.743	2.552	0.025***	0.004***
High elasticity ($\rho^F = 1.05$)	0.189	0.77	0.237	0.737	2.716	0.044***	0.000

[a]Simulations are based on the exact same sequence of shocks Z.
[b]corr correspond to the average cross-country correlation.
[c]Data on SR are constructed using Penn World Tables as $SR_{ij} = log(rgdpo) - \alpha log(rnna) - (1-\alpha)log(emp * hc)$, with *emp*, *hc* and *rnna* variables corresponding to employment, human capital and capital stock and $\alpha = 1/3$. Correlation and ACF are computed using the same sample of countries as in the model from 1970 to 1999. Results of the Trade-SR slope are robust using first difference and when adding trade dummies as shown in appendix A.4.

Using our simulations, we can also compare the cyclical properties of Z and SR. When measuring productivity as the change in GDP that is not explained by movements of capital and labor, fluctuations in SR do not only capture changes in technology, but also capture fluctuations of profits and adjustments along the extensive margin. As a result, the baseline implied average SR correlation of about 0.246 is much larger than the one implied by the underlying technology process (Z) (0.189 in all experiments). The difference simply reflects the endogenous synchronization of SR through trade, due to profits and extensive margin movements. SR is also much more volatile and less auto-correlated than Z, with a ratio of standard deviations larger than 3, showing that SR is potentially a poor proxy for calibrating technology shocks.

6.4 Robustness checks and business cycle properties

Our results are robust to a number of alternative specifications. Changing the value of parameters γ_i does not change the implied slope. In contrast, ν has a more significant impact on the magnitude of the overall trade comovement slope, while preserving the relative importance of final goods versus intermediate inputs. We also test the model with $\rho_F = 0.95$. As expected, in that case, trade in final goods generates more GDP comovements as compared to the benchmark. The level of trade frictions in the calibrated steady state $\{\tau_{ij}, f_{ij}^c, f_{ij}^E\}$ do not affect the

implied TC *slope*. Regardless of the initial level of those trade frictions, increasing trade proximity is associated with the same reaction for GDP comovement. We also conduct robustness on the specified technology processes $\{Z_i\}$. We first use the observed estimated covariance matrix of standard TFP data computed as the Solow residual in Penn World Tables ($\widetilde{\Sigma}$). While this approach is sometimes used in the literature, it leads to overshooting the *level* of cross-country GDP correlation. Results regarding the TC slope remain similar to the benchmark calibration. In order to better assess the implications of the trade channel, we also simulate the model under the counterfactual assumption that technology shocks are uncorrelated across countries and set the off-diagonal elements of the covariance-variance matrix to zero (i.e. $cov(Z_{i,t}, Z_{j,t}) = 0, \forall i \neq j$). Under all those alternative specifications, the implied TC slope is large and significant, with a much larger association with trade in intermediate inputs. We provide all the detailed results in table 7.

Finally, we report business cycle properties in table 8. By comparing the first three columns, we find that adding extensive margin and price distortions leads to an increase in investment and consumption volatility relative to GDP, while keeping other properties unchanged. Interestingly, our baseline calibration features a higher cross-country correlation of GDP than consumption, implying that the model is not subject to the Backus et al. (1992)'s consumption correlation puzzle.[53] Another dimension worth looking at is the volatility of extensive margin adjustments as measured by the standard deviation of the (log) number of exporters. Compared to the data, our model tends to be conservative as it slightly under-predicts the volatility of this margin.[54] The last column of the table shows the Business Cycle properties when the covariance matrix of technology shocks are calibrated using the Solow Residual from Penn World Tables. Such a calibration leads to strong overshooting in terms of GDP, consumption and investment volatility as well as all cross-country correlations.

7 Model Mechanisms and Empirical Relevance

In this section, we further investigate the role of firms' entry/exit and markups in the model and test their empirical relevance.

7.1 The Role of Extensive Margin of Trade

We first study the role of extensive margin (EM) and intensive margin (IM) fluctuations on the correlation between trade and GDP comovement. We conduct two empirical tests. First, in line

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[53]The so called "BKK consumption correlations puzzle" refers to the fact in standard models, consumption is more correlated across countries than output, which is at odds with the data.

[54]Note that introducing life cycle properties in firms' behavior, such as "long term fixed costs" instead of per-period fixed costs, would only widen the gap with the data as such elements tend to give more persistence to exporting decisions.

Table. 7. Robustness check [a]

Experiment	Parameter change	GDP corr	TC - slope	
			Inter. inputs	*Final goods*
Baseline	-	0.270	0.051***	0.017***
A. Model parameter				
High pareto shape	$\gamma_i = \sigma_i - 0.3$	0.271	0.052***	0.017***
Low pareto shape	$\gamma_i = \sigma_i - 0.5$	0.270	0.051***	0.017***
Low Frisch elasticity	$\nu = 0.75$	0.295	0.067***	0.025***
High Frisch elasticity	$\nu = 1.25$	0.257	0.043***	0.013***
Low CES Elasticity	$\rho^F = 0.95$	0.276	0.065***	0.022***
B. Trade frictions				
Iceberg costs	+10%	0.270	0.051***	0.017***
Fixed costs	+10%	0.270	0.051***	0.017***
C. Productivity process				
Estimated TFP shocks	$\widetilde{\Sigma}$	0.347	0.038***	0.013***
Uncorrelated techno. shocks	$cov(Z_{i,t}, Z_{j,t}) = 0, \forall i \neq j$	0.030	0.025***	0.008***
D. Trade imbalance				
No trade imbalance	$\mathcal{T}_i = 0, \ \forall i$	0.273	0.050***	0.017***
E. Reference period for $\{\omega^I, \omega^F\}$				
Cobb-Douglas $\rho^F = 1.0$	1990-2000	0.340	0.088***	0.034***
CES specification $\rho^F = 1.05$	1990-2000	0.316	0.119***	0.012**

[a]The simulations are based on the exact same sequence of shocks, under the five variations of trade indexes used in the benchmark.

with Liao and Santacreu (2015), we use the Hummels and Klenow (2005) (HK) decomposition and investigate the relation between the average and the volatility *within* each time window of the EM and IM of trade intensities between different time windows and GDP comovement. Compared to Liao and Santacreu (2015), we use a different identification strategy and a broader set of countries over a longer period.[55] Second, we use the recent Exporter Dynamics Database (EDD) from the World Bank with measures for the EM and IM that directly report the number of active exporters as well as the average trade value per exporter. This allows us to directly test if the average and the volatility of the number of exporters is associated with higher GDP synchronization. As in section 2, we aim to handle the heterogeneity between countries that are closer each other, and who experience common macro policies using fixed effects.

EM-IM HK decomposition. Building on Feenstra and Markusen (1994) and Hummels and Klenow (2005) (HK), we use data from the NBER United Nations Trade Data covering the period from 1962 to 2000 and the UN COMTRADE data for the period from 2001 to 2014. We use the

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[55]Liao and Santacreu (2015) use a set of 30 countries over the period from 1980-Q1 to 2009-Q4 while we use 38 countries from 1971 to 2010 (we drop Czechoslovakia, Estonia, Russia, Slovenia and Slovakia due to lack of observations).

Table. 8. Business Cycle Statistics: Data and Models.[a]

Statistics	Data [b,c]	No Markup/EM	No EM	Baseline	SR as Techno. Shocks
Average standard deviation (%)					
GDP	1.38	0.90	0.84	1.38	6.49
Nb. Exp. (annual)	2.44	-	-	1.54	6.43
Standard deviation relative to GDP					
Consumption	1.03	0.18	0.26	1.19	1.29
Investment	3.21	3.66	5.65	7.72	8.22
Nb. Exp.	1.61	-	-	1.09	1.09
International contemporaneous cross correlations					
GDP	0.27	0.21	0.23	0.27	0.35
Consumption	0.16	0.27	0.29	0.26	0.41
Investment	0.25	0.20	0.21	0.25	0.33
Contemporaneous correlations with GDP					
GDP(-1)	0.80	0.80	0.82	0.80	0.80
Consumption	0.69	0.66	0.77	0.82	0.82
Investment	0.77	0.99	0.98	0.98	0.94

[a] All statistics are computed using log transformation and HP-filter. Recall that the baseline model targets an international contemporaneous cross correlations of about 0.27 and a GDP auto-correlation of 0.80.

[b] All statistics refer to the mean values in the data from 1980Q1 - 1999Q4, except for the log number of exporters which is computed using the EDD from 1997 to 2014.

[c] We use the EDD to estimate the standard deviation of the HP-filtered (with $\lambda = 6.25$) and logged annual number of firms exporting from a country i to a country j between 1997 to 2014. Notice that among the 91 country-pairs in the model, our estimates are based on 30 country-pairs present in the EDD.

bilateral trade flows as categorized under the SITC (rev. 2, 4-digits) classification. This choice is made because of the longer period covered by this classification.[56] Using the HK decomposition, we construct the Extensive and Intensive margins of trade for each directed pair of country $(i \rightarrow j)$.[57] Since those measures are not symmetric within every country-pair we sum, for each country pair (i, j), the margins from i to j and from j to i. We then compute the average and the standard deviation of those measures *within* each time window and run:

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(\text{EM}_{ijt}^{HK}) + \beta_2 \ln(\text{IM}_{ijt}^{HK}) + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \qquad (31)$$

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(std(\text{EM}^{HK})_{ijt}) + \beta_2 \ln(std(\text{IM}^{HK})_{ijt}) + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \qquad (32)$$

Results are gathered in table 9 (columns (1) and (2)) and show that the correlation between the extensive margin of trade and GDP comovement is positive and significant for the two specifications. This result is particularly striking given that most of the variation in trade is explained by variations along the intensive margin.[58,59]

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[56] In the online appendix A.4.5, we also show that our results are consistent with a finer HS (6-digits) classification.

[57] See in Appendix for more details on the HK decomposition.

[58] Performing a Shapley value decomposition of total trade on the intensive and extensive margins, one finds that only one fourth of the total variance is explained by the variation of the extensive margin. Those results are in line with the similar analysis in Liao and Santacreu (2015).

[59] The results are robust when adding dummies for countries in the 2000 Euro Area or within the different waves of

EM-IM decomposition using firms data. As an additional experiment, we use the recent Exporter Dynamics Database (EDD) from the World Bank in order to test whether a change in the number of exporters (EM) and a change in the average value added per exporter (IM) *within* different time windows are correlated with changes in GDP comovement. This database provides measures of micro-characteristics of the export sector; number of exporters (their size and growth), their dynamics in terms of entry, exit and survival, and the average unit prices of the products they trade, across 70 countries from 1997 to 2014. In order to study the correlation between the extensive and intensive margins on GDP correlations, we average the GDP (transformed with log and HP-filter) correlations between country-pairs at quarterly frequency over 3 time-windows of 5 years, starting in 1997-Q1.[60] Due to the lack of coverage of the EDD relative to our sample of countries, we use the only reported information of a reference country within a country-pair as direct measure for the EM and the IM.[61]

We first estimate the role of the EM using as indicator the number of new exporters net of exiting firms between country i and country j, normalized by the total number of exporters. For the IM, we use the natural logarithm of the average value added per exporter.

$$\text{Corr GDP}_{ijt} = \beta_1 \left[\frac{\text{Entry - Exit}}{\text{Nb Exp}} \right]_{ijt} + \beta_2 \ln \left(\left[\frac{\text{value}}{\text{exporter}} \right]_{ijt} \right) + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \qquad (33)$$

Table 9, column (3), summarizes the results. Point estimates imply that an increase of 1% of the number of new net exporters is associated with an increase in GDP correlation of about 3.5%. On the contrary, we find that the IM correlates negatively with GDP correlation.[62] We then investigate in column (4) whether more variability along the extensive and intensive margins are associated with more GDP correlation within the considered time-windows. We regress:

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(\text{std nb exp}_{ijt}) + \beta_2 \ln \left(\left[\text{std } \frac{\text{value}}{\text{exporter}} \right]_{ijt} \right) + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \qquad (34)$$

Results feature a positive and significant relationship between variations in the number of exporters and GDP correlation, while variations along the intensive margin is negatively correlated with GDP comovement. This again suggests a potential key role of the extensive margin in generating GDP comovement as opposed to the variation along the intensive margin.

The role of the EM and IM in the model. In order to capture the respective role of EM and

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the European Union (1970, 1980, 1990).

[60]OECD GDP at quarterly frequency is not available for all the countries. We therefore reduce the sample. Further details are provided in the online appendix A.1 and robustnesses are conducted in the online appendix A.4.

[61]For instance, the database contains information about exports from Belgium to many destinations, but there is no information about Belgium's imports. It is therefore not possible to compute symmetric measures.

[62]We point out that those results are robust to the period excluding the crisis (1997 - 2006) and to alternative measures, such as the number of new entrants surviving at longer horizons (one, two or three years) and using FD GDP correlations as shown in a supplementary appendix A.4.

IM in the model, we perform similar analysis on our simulated dataset. The extensive margin is constructed as the number of firms producing goods in a specific international submarket (i, j), while the intensive margin is computed as the average production by exporter, such as:

$$EM_{ijt} = M_i \overline{\phi}_{ij}^{-\gamma_i} + M_j \overline{\phi}_{ji}^{-\gamma_j} \qquad \text{and} \qquad IM_{ijt} = \frac{1}{2} \frac{T_{j \to i}}{M_j \overline{\phi}_{ji}^{-\gamma_j}} + \frac{1}{2} \frac{T_{i \to j}}{M_i \overline{\phi}_{ij}^{-\gamma_i}} \qquad (35)$$

where the index t refers to different configurations (i.e. to different steady-states where only trade proximity as been changed).[63] We then estimate in column (5) of table 9 the relative correlation of those measures (averaged and log transformed as in (32)) with changes in GDP comovement in the five configurations described in section 6. Moreover, to investigate the importance of each margin's volatility , we also compute the standard deviation of those measures as $\ln(std(EM)_{ijt})$ and $\ln(std(IM)_{ijt})$ in each configuration and regress those measures on GDP correlation for each country-pairs in column (6) of table 9. Consistent with our empirical findings, both average EM and the volatility of EM fluctuations are associated with a significant increase in GDP correlation. Finally, we also show in columns (7) and (8) the relationship between measures using the HK decomposition and SR comovement and we report the corresponding results using the model in columns (9) and (10).[64] Consistent with previous findings, SR comovement is positively correlated with the extensive margin in the data and in the model.

Table. 9. GDP and Solow Residual (SR) correlations and the margins of trade [a]

	Corr GDP$_{ijt}^{\text{HP filter}}$						Corr SR$_{ijt}^{\text{HP filter}}$			
	HK indexes		EDD measures		Model (base.)		HK indexes		Model (base.)	
	Avg.	Std.	Avg.	Std.	Avg.	Std.	Avg.	Std.	Avg.	Std.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
EM measure	0.046*	0.060***	3.480***	0.109*	0.070***	0.089***	0.075***	0.014	0.045***	0.053***
	(0.026)	(0.021)	(1.285)	(0.065)	(0.012)	(0.004)	(0.028)	(0.020)	(0.009)	(0.002)
IM measure	−0.019	−0.020*	−0.038	−0.022	0.002	0.033***	0.010	−0.005	0.007	0.025***
	(0.011)	(0.012)	(0.163)	(0.043)	(0.010)	(0.001)	(0.020)	(0.012)	(0.007)	(0.001)
CP FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
TW FE	Yes	Yes	Yes	Yes	-	-	Yes	Yes	-	-
N	2,357	2,356	135	135	455	455	2,235	2,231	455	455
R^2	0.083	0.086	0.586	0.558	0.253	0.630	0.204	0.201	0.169	0.730

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. dev.
Avg. refers to specifications where we assess the link between GDP/SR comovement and the *average* of each margin in different configuration. *Std.* refers to specifications where we assess the link between GDP/SR comovement and the *volatility* (standard deviation) of each margin in each configuration.

[a]We use EDD data from 1997 to 2014 for EDD measures, UN COMTRADE data from 2001-2010 and NBER United Nations Trade Data from 1971 to 2000 for the HK decomposition.

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[63]This exercice can be compared to our empirical experiments where we compute the average extensive and intensive margins in a given time-windows and the associated GDP comovement in this same time-windows.
[64]We do not use the EDD here because there is not sufficient data to construct the SR at quarterly frequency for most countries. With 5 years time-windows, using annual data to compute bilateral correlation is not reliable.

7.2 The Role of Markups

In the benchmark model, we made the assumption of homogeneous micro elasticities of substitution between goods across countries with $\sigma_i = 5$, $\forall i$. In this section, we first test the implication of higher (i.e. lower σ_i) and lower (i.e. higher σ_i) price distorsions on the trade comovement slope. We then relax the homogeneous assumption and introduce heterogenous market power across countries. This allows us to test the direct implications of price distortions on GDP comovement through trade. To do so, we simulate the model with heterogenous σ_i estimated from the data using two different estimates. We first use Price Cost Margin (PCM) as an estimate of markups within each industry, which measures the difference between revenue and variable cost. Second, we use direct markup estimates from De Loecker and Eeckhout (2018).[65] In each experiment, we center the heterogenous markups $\{\sigma_1, ..., \sigma_N\}$ around the baseline value $\sigma = 5$. Table 10 presents the results when we implement the two different estimates.

Table. 10. The role of price distorsions [a]

Experiment	Elasticity	Markup	GDP corr	TC - slope	
				Inter. inputs	*Final goods*
Data (with CP & TW FE)	-	-	0.270	0.053**	-0.030
Baseline	$\sigma = 5.0$	25%	0.270	0.050***	0.017***
High markups	$\sigma = 4.0$	33%	0.311	0.080***	0.025***
Low markups	$\sigma = 6.0$	20%	0.253	0.038***	0.014***
Heterogenous markups, PCM	$\sigma_i \in [3.20, 5.65]$	[22%, 45%]	0.269	0.050***	0.017***
Heterogenous markups, De Loecker and Eeckhout (2018)	$\sigma_i \in [3.68, 6.07]$	[20%, 37%]	0.277	0.055***	0.018***

[a]The simulations are based on the exact same sequence of shocks, under the five variations of trade indexes used in the benchmark.

As expected, an increase in markups leads to a higher TC slope for intermediate and a lower TC slope for final goods. Quite surprisingly, adding heterogeneous markups centered around the value of $\sigma = 5$ does not change substantially the implied trade comovement slope, which suggests that accounting for cross-country heterogenous markups does not change the aggregate strength of international propagation. Moreover, it should be noticed that the estimated trade comovement slopes in the literature vary quite a lot depending on the sample and the time period, ranging from 4.8% to 11%.[66] A feature that our model can rationalize through different market power over time and across countries.

Finally, we assess the role of markups in generating a link between terms of trade and GDP fluctuations. Our model predicts that markups play an important role to make GDP react to foreign shocks, as shown in the decomposition in table 5. To find empirical support for the role

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[65]We provide details on the two measures in appendix A.

[66]For instance, we find a TC-slope of about 8.1% using the period 1970-1990 for trade in inputs, as shown in the supplemental appendix A.3. Using 20-years time windows, we find a slope of 7.4%.

of markup, we depart from a direct test of the model and test the following hypothesis: countries where markups are high experience a larger decrease in GDP when experiencing an increase in their terms-of-trade. For this, we compute the correlation of GDP with the terms of trade and regress this correlation on markups estimates, such that:

$$\text{Corr(GDP , ToT)}_{it} = \beta_1 \text{Markup.Index}_{it} + \text{Country}_i + \text{TW}_t + \epsilon_{it} \tag{36}$$

Table 11 gathers the results for the two measures of markup estimates and the implied slope in the model.[67] We first show the results of pooled cross-section analysis and then perform fixed effect regression and add time dummies to control for time-window specific factors that might affect the correlation of GDP and terms-of-trade. We also run the exact same regression with the model generated data using $\frac{\sigma_i}{\sigma_i-1}$ as markup index and using variations in σ_i. Results using the model generated data show that countries with higher markups also experience a larger decrease in their GDP when the relative price of their import rises, consistent with observed data.

Table. 11. Markups and GDP-ToT correlation

	$Corr(ln(GDP_i^{HP}), ln(ToT_i^{HP}))$				
	Data				Model
Markup measure	PCM [a]		De Loecker and Eeckhout (2018) [b]		
Markup index	-1.151	-2.650**	-0.756***	-0.495*	-0.527***
	(0.967)	(0.911)	(0.187)	(0.289)	(0.090)
Country FE	Yes	Yes	Yes	Yes	Yes
Time windows FE	No	Yes	No	Yes	-
N	43	43	80	80	112
R^2	0.066	0.322	0.132	0.232	0.260

*p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation.
[a]We use two time-windows from 1971-2010 over 22 countries reported in appendix.
[b]We use three time-windows from 1980-2009 for 29 countries reported in appendix.

8 Conclusion

This paper analyzes the relationship between international trade and business cycle synchronization across countries. We start by refining previous empirical studies and show that higher trade in intermediate input is associated with an increase in GDP comovement, while trade in final good is found insignificant. Motivated by this new fact, we propose a model of trade and business cycle with (i) global value chains, (2) monopolistic pricing and (3) firms' entry/exit. All elements are necessary for foreign shocks to have a first order impact on domestic productivity

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[67]Data on real GDP and terms of trade at the annual frequency are both taken from the OECD database and are HP filtered to capture business cycle frequencies. We also use first difference data and results are consistent with our findings using HP-filter, as shown in the supplementary appendix A.5.2.

through trade linkages. The propagation of technological shocks across countries depends on the worldwide network of input-output linkages, which emphasize the importance of going beyond two-country models to understand international GDP comovement.

We calibrate this model to 14 countries and assess its ability to replicate the empirical findings. Overall, the quantitative exercise suggests that the model is able to generate a realistic trade comovement slope, offering the first quantitative solution for the *Trade Comovement Puzzle*. Consistent with new data, both adjustments along the extensive margin and price distortions explain this result. Together, those elements give rise to a disconnect between aggregate technology and the Solow Residual.

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A   Empirical Appendix

A.1   Extensive Margin: Hummels and Klenow (2005) decomposition

We construct the extensive margin (EM) and intensive margin (IM) between countries j and m using the *Rest-of-the-World* as a reference country k. The EM is defined as a weighted count of varieties exported from j to m relative to those exported from k to m. If all categories are of equal importance and the reference country k exports all categories to m, then the extensive margin is simply the fraction of categories in which j exports to m. More generally, categories are weighted by their importance in k's exports to m. The corresponding IM is the ratio of nominal shipments from j to m and from k to m in a common set of goods. Formally, the margins are defined as:

$$\text{Extensive Margin} \quad EM_{jm}^{HK} = \frac{\sum\limits_{i \in I_{jm}} p_{kmi} q_{kmi}}{\sum\limits_{i \in I} p_{kmi} q_{kmi}} \qquad\qquad \text{Intensive Margin} \quad IM_{jm}^{HK} = \frac{\sum\limits_{i \in I_{jm}} p_{jmi} q_{jmi}}{\sum\limits_{i \in I_{jm}} p_{kmi} q_{kmi}}$$

Where I_{jm} is the set of observable categories in which j has a positive shipment to m and I is the set of all categories exported by the reference country. We normalize both measures by the sum of GDP of the two countries.

A.2 Markup measures

Markups. In section 7, we used two different markup index estimates. We first used aggregated micro markups estimated by De Loecker and Eeckhout (2018). They use micro data of 70,000 firms in 134 countries from 1980 to 2016 and estimate aggregate average markups using a cost-based approach. This method defines markups as the ratio of the output price to the marginal costs, and therefore relies solely on information from the financial statements of firms (sales value and cost of goods sold). Aggregating all firms specific markups for each country, De Loecker and Eeckhout (2018) provide a detailed and comparable measure of market power between countries. The sample that we use from their estimates includes 29 countries from 1980 to 2016.[68]

Second, we use Price Cost Margin (PCM) as an estimate of markups within each industry using data from 22 countries from 1971 to 2010.[69] Introduced by Collins and Preston (1969) and widely used in the literature, PCM is the difference between revenue and variable cost (the sum of labor and material expenditures, over revenue): $PCM = \frac{\text{Sales}-\text{Labor exp.}-\text{Material exp.}}{\text{Sales}}$

Data at the industry level come from the OECD STAN database, an unbalanced panel covering 107 sectors for 34 countries between 1970 and 2010. Due to missing data for many countries in the earliest years, we restrict the analysis for 22 countries.[70] We compute PCM for each industry-country-year and then construct an average of PCM within each country-year by taking the sales-weighted average of PCM over each industry. Finally, the average PCM for a given time window is simply the mean of country-year PCM over all time periods.

A.3 Trade comovement slope with financial controls

We provide additional robustness of the trade comovement slope using financial controls. To do this, we construct two additional indexes capturing the financial interconnection of two countries. First, we construct an index of financial integration (FI) using Foreign Direct Investment (FDI) data, as follows: $FI_{ijt} = \frac{FDI_{i \to j,t}+FDI_{j \to i,t}}{GDP_{it}+GDP_{jt}}$. Second, we use the total bilateral cross-border claims (including bank and non-bank sectors for all maturities) from the consolidated banking statistics

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[68]The list of countries is: Austria, Belgium, Canada, Colombia, Denmark, Finland, France, Germany, Greece, Ireland, Iceland, Indonesia India, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Portugal, South Africa, Spain, Sweden, Switzerland, Turkey, the United-Kingdom and the United-States.

[69]The list of countries is: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Iceland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, Portugal, Spain, Sweden, the United-Kingdom and the United-States.

[70]For Germany, data are available only from 1991 onward (after the reunification), which is why the total number of observation in the regressions is 43.

from the Bank for International Settlement to construct an index of financial proximity (FP) between a country i and j: $FP_{ijt} = \frac{C_{i \to j,t} + C_{j \to i,t}}{GDP_{it} + GDP_{jt}}$, where here $C_{i \to j,t}$ refers to total cross-border claims from country i to country j.

Table 12 summarizes the results with financial controls. Except for the specification using correlation of first difference GDP together with financial proximity index, the results are shown to be robust to the inclusion of financial controls. Using a larger sample including high and low income countries, World Bank (2019) show consistent findings.

Table. 12. Trade - GDP correlation, Disaggregated trade, controls with financial variables

	Corr GDP$^{\text{HP filter}}$				Corr ΔGDP			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\ln(\text{Trade}^{\text{input}})$	0.170***	0.177***	0.298***	0.312***	0.067	0.074	0.202*	0.186*
	(0.065)	(0.063)	(0.097)	(0.095)	(0.075)	(0.074)	(0.104)	(0.098)
$\ln(\text{Trade}^{\text{final}})$	−0.006	−0.048	−0.367***	−0.351***	0.074	0.036	−0.340***	−0.316***
	(0.057)	(0.057)	(0.092)	(0.094)	(0.063)	(0.067)	(0.093)	(0.095)
$\ln(\text{FP})$		0.039**				0.027		
		(0.016)				(0.019)		
$\ln(\text{FI})$				−0.022				−0.036*
				(0.020)				(0.021)
$third_{country}$		0.322		−0.319		0.400		0.429
		(0.301)		(0.502)		(0.330)		(0.612)
Country-Pair FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time Window FE	No	Yes	Yes	Yes	No	Yes	Yes	Yes
EU + USSR dum.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	1,030	1,030	728	728	1,030	1,030	728	728
R^2	0.425	0.432	0.440	0.443	0.343	0.347	0.350	0.355

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation.

A.4 Trade comovement slope using Solow Residual correlation

The Solow Residual in the data is constructed using the PWT9.1 using the variables of real GDP (*rgdpo*), real capital stock (*rnna*), total employment (*emp*) and the index of human capital per employee (*hc*), such that: $SR_{ij} = log(rgdpo) - \alpha log(rnna) - (1 - \alpha)log(emp * hc)$, with $\alpha = 1/3$. With this method, we can compute the SR for up to 592 country-pairs over 4 time-windows. Complete results of the trade-SR comovement slope are shown in table 13, where point estimates are positive and significant for intermediate inputs. Results hold for both HP-filter and first difference.

A.5 Sensitive analysis of main empirical results

We provide in table 14 sensitive analysis concerning our main results of the trade-comovement slope. Details of those results are provided in the supplementary appendix.

Table. 13. Trade and Solow Residual correlation with 10 years time windows

	Corr SR$^{\text{HP filter}}$			Corr ΔSR		
	(1)	(2)	(3)	(4)	(5)	(6)
ln(Trade$^{\text{total}}$)	0.010			0.013		
	(0.012)			(0.012)		
ln(Trade$^{\text{input}}$)		0.055**	0.066***		0.054**	0.064***
		(0.025)	(0.025)		(0.025)	(0.024)
ln(Trade$^{\text{final}}$)		-0.044^*	-0.044^*		-0.040^*	-0.040^*
		(0.024)	(0.024)		(0.024)	(0.024)
Country-Pair FE	Yes	Yes	Yes	Yes	Yes	Yes
Time Window FE	Yes	Yes	Yes	Yes	Yes	Yes
URSS + EU dum.	No	No	Yes	No	No	Yes
N	2,367	2,367	2,367	2,367	2,367	2,367
R^2	0.213	0.215	0.235	0.208	0.210	0.228

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation.

Table. 14. Sensitive analysis: TC-slope

	Coefficient on trade in inputs	Coefficient on trade in final goods	GDP Filter	Countries ∣ Obs.	Period	TW	CP
Sample selection							
Whole Sample	0.053**	-0.030	HP	40 ∣ 2,900	1970-2009	Yes	Yes
20 years TW	0.074**	-0.054	HP	40 ∣ 1,450	1970-2009	Yes	Yes
Excluding EU CP	0.056**	0.005	HP	40 ∣ 2,280	1970-2009	Yes	Yes
Excluding USSR	0.064**	-0.006	HP	34 ∣ 2,244	1970-2009	Yes	Yes
Alternative TW	0.081***	0.014	HP	34 ∣ 2,244	1970-1999	Yes	Yes
Alternative controls for sectoral composition							
4Digits SITC	0.058**	-0.045^*	HP	36 ∣ 2,520	1970-2009	Yes	Yes
ISIC classification	0.059**	-0.045^*	HP	36 ∣ 2,520	1970-2009	Yes	Yes
1Digit Agg. sectors	0.088	-0.044	HP	38 ∣ 1,291	1970-2009	Yes	Yes
Alternative indexes							
$level(trade)^a$	33,96*	-34.92	HP	40 ∣ 2,900	1970-2009	Yes	Yes
$log(mean(trade))$	0.044*	-0.027	HP	40 ∣ 2,900	1970-2009	Yes	Yes
$\max\left(\frac{T_{i\leftrightarrow j}}{GDP_i}, \frac{T_{i\leftrightarrow j}}{GDP_j}\right)$	0.052**	-0.032	HP	40 ∣ 2,900	1970-2009	Yes	Yes
STAN data	0.209**	-0.107	HP	20 ∣ 760	1995-2014	Yes	Yes

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation.
^{a} We provide the results using EU and USSR dummies since adding those controls substantially reduce the significance of trade in final goods.

B Theoretical appendix – proof of Lemma 1

Reminder of Lemma 1. : Total profits in country i are proportional to total revenues:

$$\Pi_i = \frac{\sigma_i - 1}{\gamma_i \sigma_i} R_i$$

Proof: For simplicity, we write the proof in the Cobb-Douglas case with $\sigma_i = \sigma$ and $\gamma_i = \gamma$, although it extends immediately to a more general CES case, and we omit the time subscript. First, since firms charge a constant markup $\sigma/(\sigma - 1)$, variable profits are a fraction $1/\sigma$ of total revenues and total profits net of fixed costs for all firms in i are $\Pi_i = \frac{R_i}{\sigma} - \sum_j FC_{i \to j}$, where $FC_{i \to j}$ is the sum of fixed cost payment from all firms from country i serving market j. Then, note that total fixed cost payment for all firms in country i is:

$$FC_{i \to j} = M_i \int_{\overline{\varphi_{i,j}}}^{+\infty} f_{ij}^c \times \frac{PB_i}{Z_i} \times \gamma \varphi^{-\gamma - 1} \times d\varphi = M_i f_{ij} \frac{PB_i}{Z_i} \times \overline{\varphi_{i,j}}^{-\gamma}$$

For all i, j, total revenues (sales) from i to j can be written as:

$$R_{i,j} = M_i \int_{\overline{\varphi_{i,j}}}^{+\infty} \left(\tau_{ij} \frac{\sigma}{\sigma - 1} \frac{PB_i}{Z_i} \frac{1}{\widetilde{\mathcal{P}}_{i,j}} \right)^{1 - \sigma} \times \left[\frac{\omega_j^I(i) S_j}{\mathcal{P}_j^I} + \frac{\omega_j^F(i) X_j}{\mathcal{P}_j^F} \right] \varphi^{\sigma - 1} g(\varphi) d\varphi$$

$$= \frac{\gamma M_i}{\gamma - (\sigma - 1)} \times \left(\tau_{ij} \frac{\sigma}{\sigma - 1} \frac{PB_i}{Z_i} \right)^{1 - \sigma} \times \left[\frac{\omega_j^I(i) S_j}{\mathcal{P}_j^I} + \frac{\omega_j^F(i) X_j}{\mathcal{P}_j^F} \right] \overline{\varphi_{i,j}}^{\sigma - \gamma - 1}$$

Next, using the expression for $\overline{\varphi}_{i,j}$, we get

$$R_{i,j} = \frac{\gamma M_i}{\gamma - (\sigma - 1)} \times \sigma f_{i,j}^c \frac{PB_i}{Z_i} \overline{\varphi_{i,j}}^{-\gamma} = \frac{\gamma}{\gamma - (\sigma - 1)} \times \sigma FC_{i \to j}$$

Combining those expressions, we get

$$\sum_j FC_{i \to j} = \frac{\gamma - (\sigma - 1)}{\gamma \sigma} \times \left(\sum_j R_{i,j} \right) = \frac{\gamma - (\sigma - 1)}{\gamma \sigma} \times R_i$$

Using this expression of $\sum_j FC_{i \to j}$ in the definition of profits completes the proof.