Value-Added and Productivity Linkages Across Countries*

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Abstract

Traditional international real business cycle models produce a weak relationship between trade and cross-country real GDP correlations, contradicting widespread empirical findings. This puzzle can be resolved by defining real GDP in the model using double deflation, exactly mirroring how the data is constructed. Whenever the base period price used for imported input valuation does not reflect their marginal revenue product, real GDP movements become mechanically linked to fluctuations in imported inputs. Focusing on the cases of markups and love of variety, we show theoretically and confirm empirically that input trade is associated with the synchronization of real GDPs, measured productivities and profits. A quantitative model with these elements delivers a strong link between input trade and business cycle comovement, with a magnitude in line with empirical estimates. Conversely, our model-based simulations show that trade is much less associated with the synchronization of a theory-consistent measure of real value added.

Keywords: International Trade, International Business Cycle Comovement, Networks, Input-Output Linkages, Solow Residual.

JEL Classification: F12, F44, F62

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1 Introduction

Since the seminal contribution of Frankel and Rose (1998), the role of trade in propagating shocks across countries has been the subject of substantial empirical attention. Two countries with stronger trade linkages tend to experience more synchronized business cycles.[1] Yet, Kose and Yi (2001, 2006) show that standard international real business cycle (IRBC) models are unable to quantitatively account for the empirical relationship by an order of magnitude. This failure of standard models is referred to as the *Trade Comovement Puzzle* (TCP) and constitutes an important open question in international macroeconomics.[2]

In this paper, we take a step back and argue that the mismatch between the observed empirical association and standard models' predictions can be resolved by improving the mapping between macroeconomic models and the data. Once real GDP is defined in the model with the same procedure used by statistical agencies, we show that it becomes more sensitive to foreign shocks and the puzzle vanishes. Moreover, our simulations show that stronger trade linkages are not associated with higher synchronization of a theory-consistent index of real value added. Our results highlight that, when comparing a macroeconomic model to the data, it is key to define aggregate variables in a way that is consistent with statistical agencies' procedures.

What is real value added in an economy that imports intermediate inputs? In a seminal contribution, Fabricant (1940) described real value added as an ideal index of the *net physical output* of an industry. This concept has then been refined by Sims (1969) and Arrow (1974), arguing that if the production function for gross output is separable between primary factors and intermediate inputs, real value added can be defined implicitly from the production function itself. Taking a simple example, if a country produces gross output by combining a domestic input A and an imported input B, the Sims (1969)-inspired definition of real value added corresponds to the quantity of input A only. We call this object the "physical value added". By construction, changes in the quantity of imported input B impact physical value added only insofar as it changes the quantity of input A produced in the economy.

In practice, statistical agencies do not observe physical value added and construct a measure of "statistical value added" using double deflation, a method consisting of taking the difference between gross output and intermediate inputs, both valued using base period prices. This statis-

[1] For empirical studies supporting the association between international and business cycle synchronization, see Clark and van Wincoop (2001), Imbs (2004), Baxter and Kouparitsas (2005), Kose and Yi (2006), Calderon et al. (2007), Inklaar et al. (2008), Di Giovanni and Levchenko (2010), Ng (2010), Liao and Santacreu (2015), Duval et al. (2015), and Di Giovanni et al. (2018).

[2] For quantitative studies of the TCP, see for instance Kose and Yi (2001, 2006), Burstein et al. (2008), Arkolakis and Ramanarayanan (2009), Johnson (2014), and Liao and Santacreu (2015).

tical measure is what is called real GDP in most national accounts database. Our core argument is that real GDP measured by statistical agencies is, in general, equal to physical value added only if prices used in the construction of real GDP are un-distorted – that is, if the base period price used for intermediate input reflects both its marginal cost and the marginal revenue that can be derived from its usage. If instead the base period price used in the valuation of imported inputs is lower (higher) than the marginal revenue product of these inputs, then an increase in foreign input usage is associated with higher (lower) statistical value added even for fixed domestic factors and fixed technology.

We focus on two sources of such a discrepancy between imported inputs' base period price and their marginal revenue product, markups and love of variety, which are increasingly used in the macro and trade literature. With markups, which imply non-zero profits in the domestic economy, intermediate inputs generate more revenues than their cost. Hence, using more imported inputs results in higher statistical value added, even when domestic factors and technology are unchanged. Importantly, our argument does not rely on variable markups: the sheer presence of constant markups in the base-period prices used in the construction of real GDP creates a mechanical link between domestic real GDP and fluctuations in imported input usage.

Our second source for the wedge between measured real GDP and physical value added is the presence of love of variety. With love of variety, accessing a larger range of foreign inputs is associated with efficiency gains that are not priced in input costs. Again, using more imported inputs leads to higher statistical value added, over and beyond any change in domestic labor, capital or technology. As a result, our framework highlights a disconnect between technology and measured productivity that arises with markups and extensive margin adjustments. Since changes in imported inputs impact real GDP above and beyond changes in technology and factor supply, it follows that measured productivity is directly affected by foreign shocks. Within a simple accounting framework, we show that accounting for real GDP fluctuations that arise from changes in imports is key to generating a strong link between trade linkages and business cycle synchronization.

We then make a set of empirical contributions that support our theoretical predictions. Constructing a panel dataset which allows for dyadic and time windows fixed effects, we document that the positive relationship between trade and GDP comovement is mostly driven by *trade in intermediate inputs*, whereas trade in final goods is found to be insignificant or negative. This finding is in line with our argument that fluctuations in imported inputs have a strong impact on real GDP movement. Furthermore, country pairs that increase trade in inputs also experience a surge in the correlation of their measured productivity as well as aggregate profit, as measured by their aggregate *Net Operating Surplus*. This supports our view that real GDP comovement across countries is not solely driven by correlated factor supply. Additionally, and consistent with the cross-sectional analysis of Liao and Santacreu (2015), we find that higher business cycle synchronization is associated with movements in the number of traded varieties, with both the range and the variance of extensive margin fluctuations being associated with a surge in GDP correlation.

Finally, we argue that measuring real GDP in the model in a way that is consistent with the data helps reconcile theory and empirical findings, thereby solving the Trade-Comovement Puzzle. To do so, we propose an IRBC model with 15 countries, monopolistic competition and entry and exit of firms. Keeping technology shocks unchanged, we calibrate the model to different levels of trade flows and perform fixed effect regressions on a simulated dataset. When real GDP is constructed using double deflation, the model is able to reproduce almost exactly the *trade-comovement (TC) slope* observed in the data, hence offering a resolution to the Trade-Comovement Puzzle. To put things into perspective: both in the data and in our simulations, our point estimates imply that the observed increase in input trade between the 1970s and the 2000s is associated with an 11 percentage points increase in international real GDP correlation. As expected, the association between input trade and the synchronization of physical value added is significantly lower. In our simulations, physical value added correlation increases by less than half a percentage point for the same expansion of input trade. In our model as well as in the data, higher input trade is also associated with an increase in the synchronization of aggregate profits and measured productivity.

Relationship to the literature. Our work builds on a number of previous papers that helped refine our understanding of the relationship between bilateral trade and GDP comovement. Starting with Frankel and Rose (1998), many studies confirmed the positive association between trade and real GDP synchronization in the cross-section. The empirical part of this paper is most closely related to two recent contributions. First, Liao and Santacreu (2015) is the first to study the link between the extensive margin of trade and both GDP and TFP synchronization. We follow their lead but use a panel estimation. We notably add country-pair fixed effects which allows us to control for the average correlation of shocks for each pair of countries and only use within-pair variations in the estimation. Second, Di Giovanni et al. (2018) uses sector-level Input-Output tables together with firm-level information to show that firms that buy inputs from a country are more correlated with that country. Our findings are in line with their evidence and supports the role of Global Value Chains in the synchronization of GDP fluctuations.[3]

[3]Relatedly, Ng (2010) uses cross-country data and shows that bilateral production fragmentation increases business

If the empirical link between trade and real GDP comovement has long been known, the underlying economic mechanism of this relationship is still unclear. Using the workhorse IRBC model with three countries, Kose and Yi (2006) have shown that their model can explain at most 10% of the *slope* between trade and business cycle synchronization, leading to what they call the *Trade Comovement Puzzle* (TCP). Since then, many papers have refined the puzzle, highlighting ingredients that could bridge the gap between theory and data.

Burstein et al. (2008) show that allowing for international production sharing can deliver tighter business cycle correlation if the elasticity of substitution between home and foreign intermediate inputs is extremely low. Arkolakis and Ramanarayanan (2009) analyze the impact of vertical specialization on the relationship between trade and business cycle synchronization. Their model with perfect competition does not generate significant dependence of business cycle synchronization on trade intensity, but they show that the introduction of time-varying price distortions improves the model's fit with the data. Incorporating trade in inputs in an otherwise standard multi-country IRBC model, Johnson (2014) shows that adding international input-output linkages *alone* is not sufficient to solve the trade-comovement puzzle, but points that such production linkages do synchronize input usage. Our paper takes this insight on-board and further shows that input synchronization can also lead to real GDP synchronization when one adds markups and extensive margin. The three papers above feature perfectly competitive models where real GDP is measured as "physical value added". Compared to them, we add firms' entry/exit and monopolistic competition and argue that, once real GDP is measured using double deflation, those ingredients help reconcile models' predictions and the data.

The role of markups in generating a link between intermediate input and measured productivity has been discussed in several papers such as Hall (1988) and Basu and Fernald (2002), and more recently in Gopinath and Neiman (2014). With markups, intermediate inputs generate more revenues than their cost. Hence, statistical real value added can be created by simply using more inputs, even with fixed domestic factors and technology. The importance of love of variety and fluctuations in the number of imported varieties has been pioneered by Feenstra (1994) while the consequences on our understanding of the trade comovement puzzle and the endogenous synchronization of productivity has been investigated by Liao and Santacreu (2015), which builds on Ghironi and Melitz (2005) and Alessandria and Choi (2007) to develop a two-country IRBC model with trade in differentiated products. Compared to this paper, our analysis adds multinational production in a multi-country setup. Importantly, we highlight the importance of real GDP measurement in macroeconomic models and show that while the inclusion of price dis-

cycle comovement. The concept of bilateral production fragmentation used is different from us as it takes into account only imported inputs that are then further embodied in exports.

tortions and extensive margins significantly alters real GDP fluctuations, it does not materially change the model's propagation property if one looks at physical value added. Finally, a complementary approach has been developed by Drozd et al. (2020), which introduces a dynamic trade elasticity in the presence of convex capital adjustment costs. This approach sheds lights on the mechanisms that enable a model to generate a link between international trade and factor supply synchronization.

2 On the definition and measurement of real GDP

This section illustrates how real GDP, as measured by statistical agencies, is disconnected from the ideal theoretical definition of real value added. In particular: when a country imports intermediate inputs and pays a price that does not reflect the marginal revenue product of those inputs, the usual statistical definition of real GDP is not equal to the net physical production in the economy. When marginal revenue derived from input is above their marginal cost, real GDP captures not only the physical value added but also accounts for the "accounting value added" derived from imported input usage. In turn, this measurement issue creates a wedge between measured productivity and actual technology.

2.1 A Simple Accounting Framework

Consider an economy that produces a gross output (GO) using domestic factors (K, L) and imported inputs (X). According to Sims (1969) and Arrow (1974), if the production function for gross output is separable between primary factors and imported inputs, real value added can be defined implicitly from the production function itself. In particular, if $GO = Q(K, L, X)$ can be re-written as $GO = Q(V(K, L), X)$, then we can "*imagine capital and labor cooperating to produce an intermediate good called real value added (V), which in turn cooperates with materials to produce the final product*".[4] Using this definition, real value added can be thought of as "physical value added" and its fluctuations are only attributed to changes of the value added bundle $V(.)$.

In practice, real value added is measured using double-deflation as the difference between gross output and intermediate inputs, both valued using base period prices. As a result, we will see in this section that real GDP equals physical value added only if the base period price used for intermediate inputs reflects both their cost and the marginal revenue that can be derived from their usage.

[4] This quote is taken from Arrow (1974), pp 4-5.

We focus on two widely-used ingredients that create a wedge between imported inputs' base period price and their marginal revenue product: markups and love of variety. In presence of either or both of these features, we show that real GDP fluctuations, as measured in the data, are not limited to changes of the theory-consistent physical value added, but are also the direct result of changes in the quantity and variety of imported input. By creating a mechanical link between real GDP and imports, those features allow for a quantitative resolution of the TCP.

Setup. Consider an economy with N countries. Gross output in country n is produced using domestic factors (L_{nt} and K_{nt}) and imported inputs (X_{nt}) according to:

$$GO_{nt} = \Big[\underbrace{Z_{nt} L_{nt}^{\alpha} K_{nt}^{1-\alpha}}_{\text{Physical Value Added}} \Big]^{\gamma} \cdot \Big[\underbrace{X_{nt}}_{\text{Imported Inputs}} \Big]^{1-\gamma} \tag{1}$$

where γ is the value added share of gross output and Z_{nt} is value-added TFP.

Markup. Let μ_{nt} be the ratio between sales (i.e. Gross Output valued at current price) and total input cost in country n, defined as:

$$\mu_{nt} = \frac{P_{nt} GO_{nt}}{TC_{nt}} \tag{2}$$

There are many reasons why μ_{nt} could be above one. For example, monopoly power could allow gross output price to be above its marginal cost. Alternatively, any tax collected on the value added created in the production of gross output and passed on prices would also imply $\mu_{nt} > 1$.

Extensive margin. We introduce love of variety in gross output production in the form of a Dixit-Stiglitz aggregation of many varieties of imported inputs. Let the imported input bundle X_{nt} be a CES aggregate of $\mathcal{M}_{nt}$ varieties, such that:

$$X_{nt} = \left(\int_0^{\mathcal{M}_{nt}} x_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

Assuming foreign producers are symmetric and denoting by x_{nt} their (common) production level, X_{nt} reduces to $X_{nt} = \mathcal{M}_{nt}^{\sigma/(\sigma-1)} x_{nt} = \mathcal{M}_{nt}^{1/(\sigma-1)} \cdot \mathcal{M}_{nt} x_{nt}$. Moreover, denoting by p_{nt}^{x} the (common) price of a given variety, the ideal price index dual to the CES aggregation is given by $\mathcal{P}_{nt} = \mathcal{M}_{nt}^{1/(\sigma-1)} \cdot p_{nt}^{x}$. Denoting $\widehat{Y}_t = \frac{\Delta Y_t}{Y_{t-1}} \approx d\ln(Y_t)$ the proportional change of any variable Y, changes in the imported input bundle can be expressed as:

$$\widehat{X}_{nt} = \underbrace{\widehat{\mathcal{M}_{nt}x_{nt}}}_{\text{Change in total imports}} + \underbrace{\frac{1}{\sigma-1}\widehat{\mathcal{M}}_{nt}}_{\text{Entry/Exit Effect}} \tag{4}$$

In equation (4), the first term is simply the change in total imported inputs. It is completed by a second term that measures the additional variation in X_{nt} associated with changes in the number of available varieties. This is a well known results which has been discussed in Feenstra (1994) and Broda et al. (2006): when the production function exhibits love of variety, any increase in the mass of input suppliers leads to a surge in efficiency. As we will see, this channel amplifies the quantitative impact of imported input movements on (measured) real GDP fluctuations.

Value Added. Finally, we define two concepts of real value added in this economy. First, in line with Sims (1969) and others, we define *Physical Value Added* (PVA) implicitly from the production function as $PVA_{nt} = Z_{nt}L_{nt}^{\alpha}K_{nt}^{1-\alpha}$. Second, we follow the procedure used by statistical agencies and construct real GDP (RGDP) using double deflation. More precisely, real GDP growth between $t-1$ and t, is constructed by using $t-1$ prices to value changes in quantities.

Proportional changes of the two real value added indices can be expressed as:[5,6]

$$\text{Physical Value Added}: \quad \widehat{PVA}_{nt} = \widehat{Z}_{nt} + \alpha\widehat{L}_{nt} + (1-\alpha)\widehat{K}_{nt} \tag{5}$$

$$\text{Real GDP}: \quad \widehat{RGDP}_{nt} = \frac{P_{nt-1}\Delta GO_{nt} - p^{x}_{nt-1}\Delta(\mathcal{M}_{nt}x_{nt})}{P_{nt-1}GO_{nt-1} - p^{x}_{nt-1}(\mathcal{M}_{nt-1}x_{nt-1})} \tag{6}$$

Proposition 1 *Consider a production economy described by (1) to (3) and two definitions of real value added described by (5) and (6). Real GDP and Physical Value Added are related by:*

$$\widehat{RGDP}_{nt} = \omega_{nt-1}\left[\gamma\Big(\underbrace{\widehat{PVA}_{nt}}_{\textit{Physical Value Added}}\Big) + \frac{1-\gamma}{\mu_{nt-1}}\Big(\underbrace{(\mu_{nt-1}-1)\cdot\widehat{X}_{nt}}_{\textit{Markup Effect}} + \underbrace{\frac{1}{\sigma-1}\widehat{\mathcal{M}}_{nt}}_{\textit{Variety Effect}}\Big)\right] \tag{7}$$

[5]As discussed in Burstein and Cravino (2015), the BEA does not use $t-1$ prices to construct real GDP, but rather a Fisher chain-weighted price index, according to:

$$\widehat{RGDP}_{nt} = \left(\frac{P_{t-1}GO_t - P^X_{t-1}X_t}{P_{t-1}GO_{t-1} - P^X_{t-1}X_{t-1}}\right)^{0.5}\left(\frac{P_tGO_t - P^X_tX_t}{P_tGO_{t-1} - P^X_tX_{t-1}}\right)^{0.5}$$

Intuitively, the Fisher index is a geometric average between two base period pricing methods, alternatively using $t-1$ and t prices. We simplify the discussion and use $t-1$ prices, also known as the Laspeyres index.

[6]Equations (5) and (6) are expressed in terms of growth rate (and not in *levels*), which is consistent with all our quantitative results in section 6 where real GDP is HP-filtered. In practice, the *level* of RGDP at time t is constructed iteratively using the level at $t-1$ and the the growth rate as defined in (6).

Proof: Equation (6) can be written as:

$$\begin{aligned}\widehat{RGDP}_{nt} &= \underbrace{\frac{P_{nt-1}GO_{nt-1}}{P_{nt-1}GO_{nt-1} - p^x_{nt-1}(\mathcal{M}_{nt-1}x_{nt-1})}}_{=\,\omega_{nt-1}} \left[\frac{\Delta GO_{nt}}{GO_{nt-1}} - \frac{p^x_{nt-1}\Delta(\mathcal{M}_{nt}x_{nt})}{P_{nt-1}GO_{nt-1}}\right] \\ &= \omega_{nt-1}\left[\gamma\left(\widehat{Z}_{nt} + \alpha\widehat{L}_{nt} + (1-\alpha)\widehat{K}_{nt}\right) + (1-\gamma)\widehat{X}_{nt} - \frac{p^x_{nt-1}(\mathcal{M}_{nt-1}x_{nt-1})}{P_{nt-1}GO_{nt-1}}\widehat{\mathcal{M}_{nt}x_{nt}}\right]\end{aligned}$$

Using the definition of PVA as well as equation (4) then leads to:

$$\begin{aligned}\widehat{RGDP}_{nt} &= \omega_{nt-1}\Bigg[\gamma\left(\widehat{PVA}_{nt}\right) + \left((1-\gamma) - \frac{p^x_{nt-1}(\mathcal{M}_{nt-1}x_{nt-1})}{P_{nt-1}GO_{nt-1}}\right)\widehat{X}_{nt} \\ &\quad + \frac{p^x_{nt-1}(\mathcal{M}_{nt-1}x_{nt-1})}{P_{nt-1}GO_{nt-1}}\frac{1}{\sigma-1}\widehat{\mathcal{M}}_{nt}\Bigg]\end{aligned} \tag{8}$$

The definition of μ_{nt} in (2), provides a relationship between the intermediate input share of total cost, $(1-\gamma)$, and the share of total sales, $\frac{p^x_{nt-1}(\mathcal{M}_{nt-1}x_{nt-1})}{P_{nt-1}GO_{nt-1}}$, such that:

$$\frac{p^x_{nt-1}(\mathcal{M}_{nt-1}x_{nt-1})}{P_{nt-1}GO_{nt-1}} = \frac{1-\gamma}{\mu_{nt-1}} \tag{9}$$

Finally, using (9) in (8) delivers equation (7). ■

Discussion. Two features are worth noting in proposition 1. First, the scaling term ω_{nt-1} is the ratio of sales over nominal value added, also called a Domar weight.[7] From an accounting perspective, one can decompose total sales into total cost and profits (Π_{nt}), such that: $P_{nt-1}GO_{nt-1} = w_{nt-1}L_{nt-1} + r_{nt-1}K_{nt-1} + p^x_{nt-1}(\mathcal{M}_{nt-1}x_{nt-1}) + \Pi_{nt-1}$. When total sales equal total cost, $\Pi_{nt-1} = 0$ and the Domar weight is simply equal to the inverse of the value added share in gross output, $\omega_{nt-1} = \frac{P_{nt-1}GO_{nt-1}}{w_{nt-1}L_{nt-1}+r_{nt-1}K_{nt-1}} = 1/\gamma$. However, in the presence of a wedge between sales and cost, we can use equations (2) and (9) to rewrite the Domar weight as:

$$\omega_{nt-1} = \frac{P_{nt-1}GO_{nt-1}}{P_{nt-1}GO_{nt-1} - p^x_{nt-1}(\mathcal{M}_{nt-1}x_{nt-1})} = \frac{1}{1 - \frac{p^x_{nt-1}(\mathcal{M}_{nt-1}x_{nt-1})}{P_{nt-1}GO_{nt-1}}} = \frac{\mu_{nt-1}}{\gamma + \mu_{nt-1} - 1}$$

As long as μ_{nt-1} is close to one, the Domar weight ω_{nt-1} is close to $1/\gamma$. However, when profits are large and $\mu_{nt-1} > 1$, we have $\gamma\omega_{nt-1} < 1$ and equation (7) implies that $RGDP$ reacts less than one-for-one with PVA. Second, Real GDP as measured by statistical agencies and Physical Value Added are identical if there is no price distortions ($\mu_{nt} = 1$) and there is no love of variety in the

[7]These weight are common in models with input-output linkages and also appear in recent papers such as Baqaee and Farhi (2020), Huo et al. (2020) or in the seminal Hulten (1978).

production function ($\sigma = +\infty$).

Practical Implications. The difference between physical value added and measured real GDP as expressed in (7) has important implications for the interpretation of real GDP fluctuations and for our understanding of international business cycle synchronization. With $\mu_{nt} > 1$ and $\sigma < +\infty$, real GDP as measured in the data is not only tied to movements in technology or factor supply, but also reflects changes in the quantity and variety of imported inputs. If both $\widehat{X}_{nt}$ and $\widehat{\mathcal{M}}_{nt}$ fluctuate with foreign technology shocks, equation (7) implies that an increase in the share of imported input in domestic production (i.e. a decrease in γ) raises the association between foreign shocks and domestic real GDP. International macroeconomic models that identify real GDP to physical value added cannot account for this relationship.

To better see this point, consider first a model with perfect competition and no love of variety, meaning that $\mu_{nt} = 1$ and $\sigma = +\infty$. In this case, equation (7) shows that real GDP and physical value added are identical and real GDP fluctuations can only arise from changes in factor supplies and changes in technology. When simulating such a model and assuming exogenous technology shocks, a researcher would find that foreign shocks could impact domestic real GDP only to the extent that it affects factor supply. This simple observation is at the heart of the negative result presented in Kehoe and Ruhl (2008): in a model where firms take prices as given, profit maximization insures that the marginal benefit of using an additional unit of imported input is equal to its marginal cost. Hence, foreign shocks can only affect real GDP to the extent that it triggers a change in domestic factor supply. This result, which is a consequence of the envelope theorem applied to a profit maximization problem of a competitive firm, lies at the heart of the trade co-movement puzzle and explains why trade is not a powerful channel of propagation in standard IRBC models. In frameworks where real GDP is equal to physical value added, real GDP changes in response to a foreign shock can only arise from variations in factors supply which, in turn, is disciplined by (i) the elasticity of domestic factor supply and (ii) the complementarity between domestic factors and foreign inputs.[8] As shown in Johnson (2014), complementarity in production factors *alone* is not sufficient to solve quantitatively the TCP.

We now consider a situation where $\mu_n > 1$ and $\sigma < +\infty$. In such a case, equation (7) reveals that changes in intermediate input usage have a first order impact on real GDP fluctuations beyond the movements of domestic factors or technology. In the presence of markups or love of variety, imported inputs yield more gains than what is reflected in their price. Using more foreign inputs is associated with profits (when $\mu_n > 1$) or with efficiency gains (when $\sigma < +\infty$).[9] Hence,

[8]The role of input complementarity is discussed at length in Burstein et al. (2008) or in Boehm et al. (2015).

[9]In practice, many models feature a single parameter governing both the size of the markup and the degree of Love of Variety – this is obviously the case with CES aggregation and monopolistic competition. For clarity, equation

constructing real GDP using "base period prices" that do not reflect imported inputs' marginal revenue product creates a wedge between physical value added and real GDP fluctuations.

Importantly, the disconnect between real GDP and physical value added does not rely on the cyclicality of markups: even with constant markups, a wedge between the marginal cost and marginal revenue product of imported inputs leads to a first order impact of intermediate input usage on measured real value added. In a sense, the wedge is a purely measurement issue: when constructing real GDP, statistical agencies do not simply measure the quantity of goods produced, but use observed prices (fixed at a base period) to assign a value to measured quantities. If base period prices used in valuing quantities contain a markup, it creates a wedge between the marginal revenue generated by an additional unit of imported input x_n and its marginal cost p_n^X.

Finally, our results bears important implications for the calibration of macroeconomic models. As revealed by proposition 1, a researcher who builds a model with distorted prices cannot equate a model-based measure of physical value added to real GDP data in the calibration process: doing so would misleadingly attribute the markup and variety effects in (7) to changes in PVA, either through technology shocks ($\widehat{Z}_{nt}$) or through factor supply ($\widehat{L}_{nt}$ or $\widehat{K}_{nt}$).[10]

2.2 Productivity and technology

The real GDP decomposition presented in equation (7) also bears important implications for the measure of productivity based on the Solow Residual (SR). As standard, we define SR so that it captures fluctuations in real GDP that are not explained by changes in domestic factors:

$$\widehat{SR}_{nt} = \widehat{RGDP}_{nt} - \alpha \widehat{L}_{nt} - (1-\alpha)\widehat{K}_{nt} \tag{10}$$

Proposition 2 *When real GDP is constructed as in* (6)*, the relationship between SR and technology is given by:*

$$\begin{aligned} \widehat{SR}_{nt} \quad &= \quad \gamma\omega_{nt-1} \cdot \widehat{Z}_{nt} + \underbrace{(\gamma\omega_{nt-1} - 1) \cdot \left(\alpha \widehat{L}_{nt} + (1-\alpha)\widehat{K}_{nt}\right)}_{\textit{Scale Effect}} \\ &+ \frac{\omega_{nt-1} \cdot (1-\gamma)}{\mu_{nt-1}} \left(\underbrace{(\mu_{nt-1} - 1) \cdot \widehat{X}_{nt}}_{\textit{Imported Input Effect}} + \underbrace{\frac{1}{\sigma - 1} \widehat{\mathcal{M}}_{nt}}_{\textit{Variety Effect}} \right) \end{aligned} \tag{11}$$

(7) shows a specification in which the two channels are perfectly distinguishable.

[10]In the present paper, we focus on the implication of precise real GDP definition for the resolution of the Trade Comovement Puzzle and hence emphasize the role of *imported* inputs. However, our results can also be extended to a closed economy with multiple sectors and input-output linkages. When real GDP is computed at the sector level, proposition 1 holds and implies a disconnect between real GDP and PVA with markups and/or love of variety creates.

Proof: The result follows from replacing (7) in (10). ■

Discussion. When $\mu_n = 1$, we have $\omega_{nt-1} = 1/\gamma$ and both the scale effect and markup effect terms in equation (11) vanish. Additionally, in absence of love of variety ($\sigma = +\infty$), the variety effect term also disappears, implying that productivity is an accurate measure of technology.

In general, when $\mu_n > 1$ and $\sigma < +\infty$, equation (11) makes it clear that productivity, as measured by the Solow Residual, is disconnected from the true technology shock Z_{nt}. First, with positive markups, fluctuations in real GDP can result from movement in profits which are captured by Solow Residual fluctuations. Such profits movements can arise either from changes in domestic factors (the scale effect) or from changes in foreign input usage (the imported input effect). Second, an additional term captures the gains associated with accessing more variety from abroad whenever $\sigma < +\infty$. With love of variety, this change in efficiency is also reflected in measured productivity.

All told, the above decomposition highlights the disconnect between standard measures of productivity, such as the *Solow Residual*, and actual technology. The introduction of markups and love of variety creates new channels through which foreign shocks impact domestic productivity. As a result, two countries that trade intermediate inputs should have correlated Solow Residuals, a prediction we test in the data and which our quantitative model is able to reproduce. Finally, our results highlight the risk of identifying the Solow Residual to technology shocks in the calibration of a macroeconomic model.

3 The Empirical Trade-Comovement Slope

According to section 2, trade in intermediate inputs should be associated with an increase of both real GDP and measured productivity synchronization. Moreover, since this synchronization arises when the cost of imported inputs is lower than their marginal revenue product, we also expected that higher input trade increases the correlation of profits. We now verify those predictions in the data.

3.1 The Role of Intermediate Inputs

To investigate the role of intermediate inputs, we update the initial Frankel and Rose (1998) (henceforth, FR) analysis on the relationship between bilateral trade and GDP comovement. First, we use a decomposition of trade into trade in intermediate inputs and trade in final goods. Second, we make use of a panel estimation and exploit within country-pair variations to estimate the relation between changes in trade linkages and changes in GDP co-movement.

Our panel is composed of 40 countries, accounts for 90% of world GDP, and covers the period stretching from 1970 to 2009. We use real GDP measured at chained PPPs from the 9th Penn World Table, which is transformed in two ways: (i) HP filter with smoothing parameter 6.25 to capture the business cycle frequencies and (ii) log first difference. Bilateral trade flows are taken from Johnson and Noguera (2017), which allows us to separate between trade in final goods and trade in intermediate inputs.[11] As standard in the literature, we construct symmetric measures of bilateral trade intensity using the sum of total exports ($T^d_{i\to j}$) from country i to j in category $d \in \{\text{input}, \text{final}\}$ and total imports ($T^d_{j\to i}$) relative to GDP, such that: $\text{Trade}^d_{ij} = \frac{T^d_{i\to j} + T^d_{j\to i}}{GDP_i + GDP_j}$.

The extent to which countries have correlated GDP can be influenced by many factors beyond international trade, including correlated shocks, financial linkages, common monetary policies, etc. Because those other factors can themselves be correlated with the index of trade proximity in the cross section, using cross-section identification could yield biased results. Indeed, in their seminal paper, FR use cross-sectional variations to evaluate whether bilateral trade intensity correlates with business cycle synchronization, but their specification does not rule out omitted variable bias such as, for example, the fact that neighboring countries have at the same time more correlated shocks and larger trade flows. This limitation of cross sectional analysis has also been discussed by Imbs (2004), who notes that bilateral trade intensity can be a proxy of country-pair similarity, and thus of correlated shocks. In order to separate the effect of trade linkages from other unobservable elements, we construct a panel dataset by creating four periods of ten years each. By constructing a panel dataset and controlling for both country-pair and time windows fixed effects, this paper relates to recent studies that try to control for unobserved characteristics. Within each time window, we compute GDP correlation (Corr GDP) as well as the average trade intensities defined above.[12]

[11] We provide additional details on data sources and the list of countries in the Online Appendix A.1. The distinction between intermediate inputs and final goods is based on the classification of Broad Economic Categories (BEC). For example, intermediate inputs include industrial supplies not elsewhere specified, parts and accessories of capital goods, fuels and lubricants, food and beverages mainly for industry and industrial transport equipment, while final goods include food and beverages mainly for household consumption, non-industrial transport equipment and consumer goods not elsewhere specified.

[12] Frankel and Rose (1998) and other papers using cross-sectional analysis instruments trade variables using a combination of time invariant variables such as distance, common border, former colonial ties, etc. Since our empirical strategy consists of using within country pair variations, instruments that were used in cross-sectional analysis are not useful: any time invariant country-pair characteristics, in particular the *average* GDP correlation across all time windows, is absorbed by country-pair fixed effect. Moreover, adding time-windows fixed effect controls for the recent rise of world GDP correlation since the 90s, which could be unrelated to trade intensity. Our approach is related to Di Giovanni and Levchenko (2010), which includes country pair fixed effects in a large *cross-section* of industry-level data to investigate the relationship between sectoral trade and gross output comovement at the industry level.

Our empirical strategy relies on the estimation of the following specifications:

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(\text{Trade}_{ijt}^{\text{input}}) + \beta_2 \ln(\text{Trade}_{ijt}^{\text{final}}) + \mathbf{X}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{12}$$

where i and j denote the two countries and t the time window. CP_{ij} and TW_t stand for country-pair and time windows fixed effects. The set of controls $\mathbf{X}_{ijt}$ include dummy variables for countries among trade union: the different waves of the European Unions, the Euro Area, and countries among the USSR. We later include a set of additional controls that we discuss below.

Results are gathered in Table 1. Using within country-pair variations, trade in intermediate inputs is significantly and positively associated with higher GDP co-movement (columns (1) and (4)). This result confirms the cross-sectional estimates in Di Giovanni and Levchenko (2010), who investigate the role of vertical linkages in output synchronization using I/O matrices from the BEA. Columns (2) and (5) reveal that the relationship between trade in intermediate inputs and GDP co-movement is robust to the inclusion of time-windows fixed effect, Our estimates are also economically significant. Based on estimates in column (2) and noting that the median increase of the log trade intensity in intermediate goods between 1970-1979 and 2000-2009 is about 1.84, the slope coefficient implies a surge of GDP correlation of 11%, a non negligible increase. In contrast, trade in final goods is insignificant, or weakly negatively correlated.

Table. 1. Panel estimation: Trade proximity and GDP correlation [a]

	Corr $\text{GDP}^{\text{HP filter}}$			Corr ΔGDP		
	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\text{Trade}^{\text{input}})$	0.068^{***}	0.060^{**}	0.063^{***}	0.053^{**}	0.055^{**}	0.053^{**}
	(0.025)	(0.024)	(0.024)	(0.023)	(0.022)	(0.022)
$\ln(\text{Trade}^{\text{final}})$	-0.020	-0.038	-0.048^{**}	-0.024	-0.038	-0.036
	(0.023)	(0.024)	(0.025)	(0.020)	(0.023)	(0.023)
Country-pair fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Time windows fixed effects	No	Yes	Yes	No	Yes	Yes
Additional controls	No	No	Yes	No	No	Yes
N	2,900	2,900	2,900	2,900	2,900	2,900
Within R^2	0.076	0.169	0.172	0.082	0.162	0.169

Notes: *p<0.1; **p<0.05; ***p<0.01. SE clustered on country-pairs.

Our results are also robust to additional controls. We first construct two additional measures that capture the similarity of *trade networks* (which measures common exposure to third countries) as well as the *sectoral composition* of trade. Our "network proximity" index is motivated by the fact that two countries with similar trade partners could co-move because of their common exposure to other countries. We define $network_{prox}(i,j) = 1 - \frac{1}{2}\sum_{k\neq i,j}\left|\frac{T^{\text{total}}_{i\to k} + T^{\text{total}}_{k\to i}}{\sum_s T^{\text{total}}_{i\to s} + T^{\text{total}}_{s\to i}} - \frac{T^{\text{total}}_{j\to k} + T^{\text{total}}_{k\to j}}{\sum_s T^{\text{total}}_{j\to s} + T^{\text{total}}_{s\to j}}\right|$.

It measures the degree of similarity in the geographical distribution of trade shares between country i and country j, and is equal to 0 if countries i and j have completely separated trade partners while it is equal to 1 if all trade shares are equal. The "sectoral proximity" index controls for changes in specialization and is defined as: $sector_{prox}(i,j) = 1 - \frac{1}{2}\sum_{s\in\mathcal{S}}\left|\frac{T_i(s)}{\sum_{s\in\mathcal{S}} T_i(s)} - \frac{T_j(s)}{\sum_{s\in\mathcal{S}} T_j(s)}\right|$, with $T_i(s)$ the total export of country i in the specific sector s in the set of all sectors $\mathcal{S}$. If two countries export exactly the same share of each product, then the index is equal to 1. If shocks have a sectoral component, then two countries that tend to specialize over time in the same sectors could have an increase in business cycle comovements over and beyond any trade effects. For those two indexes, we use bilateral trade data (SITC4 REV. 2) from the Observatory of Economic Complexity. As shown in Table 1, the results are robust to the inclusion of sector proximity and network proximity (columns (3) and (6)). We additionally test alternative specifications, different sample selection and variable definitions as well as additional controls as discussed in section A.2 of the Appendix and summarized in Table 10. The online Appendix provides additional analysis and robustness.

3.2 Measured Productivity co-movements and Net Operating Surplus

In section 2, we showed that measured productivity is not an accurate measure of technology and changes in productivity can arise from a change in imported input usage. Moreover, this imported input channel is at play when the cost of imported inputs is lower than their marginal product: in other words, movements in productivity reflect movements in the profits derived from imports. It follows from this observation that an increase in intermediate input trade should be associated with an increase in the co-movement of both measured productivity and aggregate profits. We now test those two predictions.[13]

We first study the relation between international trade and measured productivity. We calculate productivity as the Solow Residual in a standard Cobb-Douglas production function, that we then transform using, alternatively, HP-filter and log difference. As before, we separate bilateral trade intensity into intermediate inputs and final goods and test:

$$\text{Corr SR}_{ijt} = \beta_1 \ln(\text{Trade}_{ijt}^{\text{input}}) + \beta_2 \ln(\text{Trade}_{ijt}^{\text{final}}) + \mathbf{X}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{13}$$

Second, we study the relation between international trade and profits, as measured by the Net Operating Surplus (NOS). We use the NOS measured by the OECD at quarterly frequency,

[13] Note that Kose and Yi (2006) and Liao and Santacreu (2015) study the relationship between measured TFP and bilateral trade in the cross-section. We add to their analysis by separating bilateral trade into intermediate inputs and final goods and use a panel estimation.

that we then transform using HP-filter and log-difference. To avoid any bias due to inflation synchronization, we also deflate these nominal value by the consumer price index. We then test the following specification:

$$\text{Corr NOS}_{ijt} \quad = \beta_1 \ln(\text{Trade}_{ijt}^{\text{input}}) + \beta_2 \ln(\text{Trade}_{ijt}^{\text{final}}) + \mathbf{X}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{14}$$

Results are depicted in Table 2. As expected, we find that higher trade intensity in intermediate inputs tend to have a positive effect on the co-movement of measured aggregate productivity and net operating surplus. The estimates are statistically significant when including time windows fixed effects. Trade in final goods has a negative statistically insignificant effect.

Moreover, according to our theoretical results in section 2, controlling for SR correlation in specification (12) should capture both the markup and love of variety effects highlighted in proposition 1. In table 10 in appendix, we confirm this intuition and show that once we control for SR correlation in (12), we obtain a lower and insignificant point estimate for trade in intermediate inputs, in line with support our theoretical predictions.[14]

Table. 2. Trade, Measured Productivity and Net Operating Surplus

	Corr Productivity$^{\text{HP}a}$		Corr ΔProductivity^{a}		Corr NOS$^{\text{HP}b}$		Corr ΔNOS^{b}	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\ln(\text{Trade}^{\text{input}})$	.055**	.065**	.045*	.051**	.371***	.175*	.288***	.250***
	(.027)	(.026)	(.025)	(.024)	(.104)	(.095)	(.083)	(.086)
$\ln(\text{Trade}^{\text{final}})$	.002	−.032	.001	−.022	−.085	−.105	−.100	−.157**
	(.025)	(.025)	(.022)	(.022)	(.079)	(.073)	(.062)	(.079)
Country-pair fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time windows fixed effects	No	Yes	No	Yes	No	Yes	No	Yes
N	2,340	2,340	2,340	2,340	364	364	364	364
R^2	.113	.217	.130	.200	.104	.263	.121	.273

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs. Controls include third indexes and trade unions. Results are robust without third indexes.
^{a} Data for measured productivity are constructed using the PWT 9.1 using total employment and measured capital. We use 2/3 for the labor share.
^{b} Data for Corr NOS are measured at quarterly frequency, for 16 consecutive quarters in time-windows of 4 years. Due to data limitation, our sample covers 1999Q1-2014Q4. Data for trade flows are taken from OEC data. Result are robust without the Great Recession time-window (2007-2010) and additional controls: dummies for trade unions and similarity indexes.

[14] We acknowledge that, as discussed in Huo et al. (2020), unobserved factor utilization can create a measurement error for SR. In the online appendix, we introduce an unobserved component in labor input leading to measurement errors in SR. Using simulated data from our model presented in section 4, we show that such unobserved component leads to a downward bias in the slope between trade and $corr(SR)$. This finding implies that the positive and significant results obtained in the data using specification (13) is a lower bound of the slope's true value. Moreover, measurement errors in SR also means that there is an attenuation bias when adding $corr(SR)$ as a control in (12), which means that the estimated slope between $corr(SR)$ and $corr(GDP)$ is lower than its true value. This, in turn, can explain why the slope trade-$RGDP$ coefficient remains non-zero (although it is statistically insignificant) when adding $corr(SR)$ as a control in (13).

3.3 Extensive Margin fluctuations and Comovement

We finally investigate the role of extensive margin adjustments in generating the observed association between trade and real GDP comovement. We use the Hummels and Klenow (2005) (HK) decomposition and investigate the relation between the average and the volatility *within* each time window of the Extensive Margin (EM) and Intensive Margin (IM) of bilateral trade intensities in intermediate inputs and real GDP comovement. In line with section 3.1, we use a panel estimation and focus on trade in intermediate goods.[15] We use data from the NBER United Nations Trade Data covering the period from 1962 to 2000 and the UN COMTRADE data for the period from 2001 to 2014. We use the bilateral trade flows as categorized under the SITC (rev. 2, 4-digits) classification. We classify intermediate inputs as Johnson and Noguera (2017).

Using the HK decomposition, we construct the EM and IM of trade in intermediate inputs for each directed pair of country $(i \to j)$. The *Rest-of-the-World*, indexed k, is taken as a reference country. The EM is defined as a weighted count of varieties of intermediate inputs exported from j to m relative to those exported from k to m, i.e. $EM_{jm} = \sum_{i \in I_{jm}} T^{\text{input}}_{k \to m}(i) / \sum_{i \in I} T^{\text{input}}_{k \to m}(i)$, where I_{jm} is the set of observable categories in which j has a positive shipment to m and I is the set of all categories of intermediate inputs exported by the reference country. If all categories are of equal importance and the reference country k exports all categories to m, then the extensive margin is simply the fraction of categories in which j exports to m. The corresponding IM is the ratio of nominal shipments from j to m and from k to m in a common set of intermediate goods, i.e. $IM_{jm} = \sum_{i \in I_{jm}} T^{\text{input}}_{j \to m}(i) / \sum_{i \in I_{jm}} T^{\text{input}}_{k \to m}(i)$. Note that the product of the two measures provide a measure of the overall trade from j to m relative to the overall trade from k to m. Finally, since those measures are not symmetric within a country-pair we sum, for each country pair (i, j), the IM and EM from i to j and from j to i, and normalize this by the sum of GDP. We then compute the average and the standard deviation of those measures *within* each time windows and test:

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(\text{EM}_{ijt}) + \beta_2 \ln(\text{IM}_{ijt}) + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{15}$$

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(std(\text{EM})_{ijt}) + \beta_2 \ln(std(\text{IM})_{ijt}) + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \tag{16}$$

Results are reported in Table 3. In line with results in Liao and Santacreu (2015) using an IV estimator, our results show that the correlation between the extensive margin of trade in intermediate inputs and real GDP comovement is positive and significant for the two specifications

[15] Concerning the sample, in their IV estimation Liao and Santacreu (2015) uses a set of 30 countries over the period from 1980-Q1 to 2009-Q4 while we use 38 countries from 1971 to 2010 (we drop Czechoslovakia, Estonia, Russia, Slovenia and Slovakia due to lack of observations).

(using HP-filter and log difference). In contrast, the intensive margin of trade is found not significantly related with GDP comovement. In the online appendix, we further investigate the role of the extensive and intensive margins of trade using a new dataset using a direct measure of the number of firms and find that the extensive margin of trade is positive and significant.

Table. 3. Real GDP correlations and the margins of intermediate inputs trade

	Corr $\text{GDP}_{ijt}^{\text{HP filter}}$		Corr ΔGDP_{ijt}	
	(avg measure)	(std measure)	(avg measure)	(std measure)
EM	0.050**	0.049**	0.045***	0.068***
	(0.022)	(0.021)	(0.020)	(0.021)
IM	−0.010	−0.013	−0.003	−0.015
	(0.017)	(0.011)	(0.016)	(0.043)
Country-pair fixed effects	Yes	Yes	Yes	Yes
Time-windows fixed effects	Yes	Yes	Yes	Yes
N	2,347	2,347	2,347	2,347
Within R^2	0.083	0.084	0.102	0.107

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. dev. SE clustered on country-pairs.

Taken together, the empirical results reported in this section indicate that countries trading more intermediate inputs experience an increase in the synchronization of their GDP, measured productivity, and aggregate profit. Moreover, this relationship is largely driven by fluctuations along the extensive margin of input trade.

4 A Model of International Trade with Cross-Border Input Linkages

In this section, we depart from the standard IRBC model and develop a many-country international business cycle model that features trade in both final and intermediate goods, imperfect competition and extensive margin adjustments. We then quantitatively assess how those mechanisms generate a disconnect between measured real GDP, measured productivity and profits.

The model is related to Ghironi and Melitz (2005) and Alessandria and Choi (2007) extended to multiple asymmetric countries with two main differences. First, we follow Krugman (1980) and assume that firms are homogeneous and all firms are able to export. Second, firms are engaged in both importing and exporting, which implies that intermediate goods cross borders multiple times.[16] Importantly, we distinguish between physical value added (PVA) and real GDP ($RGDP$) as defined earlier. We show that our model completely replicates the association observed in the data between international trade in inputs and real GDP synchronization when the definition of

[16] Alternatively, the model presented here can be thought of as an extension of the IRBC model presented in Johnson (2014) with two new elements: markups and extensive margin adjustments. It is also related to the static small open economy model in Gopinath and Neiman (2014)

real GDP is consistent with measures used by statistical agencies. In this case, the combination of international I/O linkages, price distortions and extensive margin adjustments provides a quantitative solution to the TCP.

4.1 Consumption and Labor Supply

We consider a multi-period world economy with many countries ($i, j \in \{1, ..., N\}$). Each country is populated by a representative consumer who consumes final goods and supplies labor $L_{i,t}$ for production. Consumers' preferences are described by the following utility function:

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log \left(C^F_{i,t} \right) - \frac{\chi_i}{1+\nu} L^{1+\nu}_{i,t} \right], \tag{17}$$

$$\text{with} \quad C^F_{i,t} = \left(\sum_{j=1}^{N} \omega^F_i(j)^{\frac{1}{\rho^F}} \cdot C_{j,i,t}^{\frac{\rho^F - 1}{\rho^F}} \right)^{\frac{\rho^F}{\rho^F - 1}}, \quad \text{and} \quad C_{j,i,t} = \left(\int\limits_{s \in \Omega^F_{j,i,t}} c_{j,i,t}(s)^{\frac{\sigma_j - 1}{\sigma_j}} ds \right)^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{18}$$

where χ_i is a scaling parameter, β is the rate of time preference, ν the inverse of the Frisch elasticity of labor supply and σ_i the elasticity of substitution between different varieties of final goods originating from country i. $\omega^F_i(j)$ measures the share of country j in the *consumption* bundle of country i, and $\Omega^F_{j,i,t}$ is the endogenous set of firms from country j that serve the *final good* market in country i. Finally, ρ^F is the *final goods* Armington elasticity of substitution. Final good price indices are defined as:

$$\mathcal{P}^F_{i,t} = \left(\sum_{j=1}^{N} \omega^F_i(j) \cdot \left(\widetilde{\mathcal{P}}^F_{j,i,t} \right)^{1-\rho^F} \right)^{\frac{1}{1-\rho^F}}, \quad \text{and} \quad \widetilde{\mathcal{P}}^F_{j,i,t} = \left(\int\limits_{s \in \Omega^F_{j,i,t}} p^F_{j,i,t}(s)^{1-\sigma_i} ds \right)^{\frac{1}{1-\sigma_i}}, \tag{19}$$

where $p^F_{j,i,t}(s)$ is the price charged by firm s in the set $\Omega^F_{j,i,t}$ when selling in the *final good* market in country i. As we will see below, given our assumptions, firms charge the same price in both final and intermediate good markets in a given country.

Asset markets. Our benchmark economy assumes financial autarky between countries, so that agents choose consumption, investment and labor, subject to:[17,18]

$$\mathcal{P}^F_{i,t}\left(C_{i,t}+\mathcal{I}_{i,t}\right)=w_{i,t}L_{i,t}+r_{i,t}K_{i,t}-\mathcal{T}_i\,, \tag{20}$$

$$K_{i,t+1}=(1-\delta)K_{i,t}+\mathcal{I}_{i,t}\left[1-\Psi\left(\frac{\mathcal{I}_t}{\mathcal{I}_{t-1}}\right)\right]\,, \tag{21}$$

where the term $\mathcal{T}_i$ captures potential trade imbalance in country i, i.e. $\mathcal{T}_i<0$, corresponds to a trade deficit meaning that country i consumes more than the value of its production. Following Christiano et al. (2005), we introduce investment adjustment costs that satisfy the following properties: $\Psi(1)=\Psi'(1)=0$ and $\Psi''(1)>0$. We use the following function form: $\Psi\left(\frac{I_t}{I_{t-1}}\right)=\frac{\psi}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2$, where ψ governs the degree of the adjustment costs. Given prices, consumers choose $\{C_{i,t},L_{i,t},K_{i,t+1}\}$ to maximize (17) subject to (20) and (21).

4.2 Production side

In country i, production is performed by a continuum of homogeneous firms with productivity $Z_{i,t}$. Firms produce with a Cobb-Douglas technology using labor $\ell_{i,t}$, capital $k_{i,t}$ and intermediate inputs $I_{i,t}$ bought from both home and foreign firms. The intermediate input index in country i, $I_{i,t}$, is a CES aggregation of country specific bundles $M_{j,i,t}$, with an *intermediate goods* Armington elasticity ρ^I. Each country specific bundle is itself a CES aggregation of many varieties, with an elasticity of substitution σ_j.[19] The production function of an individual firm in country i is:

$$q_{i,t}=\left(Z_{i,t}\ell^{\alpha}_{i,t}k^{1-\alpha}_{i,t}\right)^{\gamma_i}I^{1-\gamma_i}_{i,t}\,, \tag{22}$$

$$\text{with}\quad I_{i,t}=\left(\sum_{j=1}^{N}\omega_i(j)^{\frac{1}{\rho^I}}M_{j,i,t}^{\frac{\rho^I-1}{\rho^I}}\right)^{\frac{\rho^I}{\rho^I-1}},\quad\text{and}\quad M_{j,i,t}=\left(\int\limits_{s\in\Omega_{j,i,t}}m_{j,i,t}(s)^{\frac{\sigma_i-1}{\sigma_i}}ds\right)^{\frac{\sigma_i}{\sigma_i-1}}, \tag{23}$$

where γ_i is the share of value added in gross output, $\omega^I_i(j)$ measures the share of country j in the *production process* of country i, and $\Omega^I_{j,i,t}$ is the endogenous set of firms based in j and serving

[17] Heathcote and Perri (2002) have shown that IRBC models with financial autarky yield a closer fit to some key business cycle moments than a version under complete markets. In Appendix B, we present a version with complete financial markets. In such case, the magnitude of the Trade Comovement slope is only slightly lower and the main message is qualitatively similar.

[18] Note that the right hand side of equation (20) includes firms' profits since, as explained below, firms pay entry costs using domestic labor. It should then be understood that $L_{i,t}$ includes both production and "entry cost" workers.

[19] This parameter governs both the markup charge by firm from country j and the degree of love of variety.

the *intermediate input* market in country i.[20] Similarly to the *final good* market, we have

$$\mathcal{P}^I_{i,t} = \left(\sum_{j=1}^{N} \omega^I_i(j) \left(\widetilde{\mathcal{P}}^I_{j,i,t} \right)^{1-\rho^I} \right)^{\frac{1}{1-\rho^I}}, \quad \text{and} \quad \widetilde{\mathcal{P}}^I_{j,i,t} = \left(\int\limits_{s \in \Omega^I_{j,i,t}} p^I_{j,i,t}(s)^{1-\sigma_i} ds \right)^{\frac{1}{1-\sigma_i}}, \tag{24}$$

$$\text{and } \mathcal{P}^{IB}_{i,t} = \left(\frac{w_{i,t}}{\alpha \gamma_i} \right)^{\alpha \gamma_i} \left(\frac{r_{i,t}}{(1-\alpha)\gamma_i} \right)^{(1-\alpha)\gamma_i} \left(\frac{\mathcal{P}^I_{i,t}}{1-\gamma_i} \right)^{1-\gamma_i}, \tag{25}$$

where $\mathcal{P}_{j,i,t}$ denotes the price of the country-pair specific bundle $M_{j,i,t}$ and $\mathcal{P}^{IB}_{i,t}$ is the unit cost of the Cobb Douglas bundle aggregating $I_{i,t}$, $k_{i,t}$ and $\ell_{i,t}$ (called the *input bundle*) and represents the price of the basic production factor in country i. $p^I_{j,i,t}(s)$ is the price charged by any firm s in the set $\Omega^I_{j,i,t}$ when selling in the *intermediate input* market in country i.

Finally, there is an overhead entry cost f^E_i, sunk at the production stage. Based on their expected profit in all markets, firms enter the economy until the expected value of doing so equals the overhead entry cost. This process determines the mass of firms $M_{i,t}$.

4.3 Equilibrium

We specify the equilibrium conditions of the model by introducing $X_{i,t}$ the aggregate consumers' revenue and $S_{i,t}$ the total firms' spendings in country i. Given prices, total demand faced by a firm in country i is given by the sum of demand stemming from both the *final good* and the *intermediate input* markets in all countries:

$$q_{i,t} = \underbrace{\sum_j \left(\frac{p^F_{i,j,t}}{\widetilde{\mathcal{P}}^F_{i,j,t}} \right)^{-\sigma_i} \left(\frac{\widetilde{\mathcal{P}}^F_{i,j,t}}{\mathcal{P}^F_{j,t}} \right)^{-\rho^F} \frac{\omega^F_j(i) X_{j,t}}{\mathcal{P}^F_{j,t}}}_{\text{Final goods demand}} + \underbrace{\sum_j \left(\frac{p^I_{i,j,t}}{\widetilde{\mathcal{P}}^I_{i,j,t}} \right)^{-\sigma_i} \left(\frac{\widetilde{\mathcal{P}}^I_{i,j,t}}{\mathcal{P}^I_{j,t}} \right)^{-\rho^I} \frac{\omega^I_j(i)(1-\gamma_j) S_{j,t}}{\mathcal{P}^I_{j,t}}}_{\text{Intermediate goods demand}}. \tag{26}$$

Firms choose their price to maximize profits. Since the price elasticity of demand is constant, they charge a constant markup over marginal cost. For a firm from country i, the only elasticity that is relevant for pricing is σ_i, capturing the fact that firms compete primarily with other firms coming from their home country since their individual pricing decision has no impact on the country-specific price index in every market.[21] As a result, firms charge the same markup in the

[20] In an earlier version of this paper, we also introduced heterogeneity with firm's idiosyncratic productivity as in Ghironi and Melitz (2005) or Fattal Jaef and Lopez (2014). The results are quantitatively similar to those obtained in this version and we therefore dropped this layer of complexity.

[21] With a finite number of firms, elasticities σ_i, ρ^I and ρ^F would all appear in the pricing strategy. In such a case, every firm would take into account the fact that its own price has an impact on the unit cost of the corresponding country-specific bundle.

final and intermediate good markets, and we have: $p^F_{i,j,t} = p^I_{i,j,t} = p_{i,j,t}$ and $\widetilde{\mathcal{P}}^F_{i,j,t} = \widetilde{\mathcal{P}}^I_{i,j,t} = \widetilde{\mathcal{P}}_{i,j,t}$. The marginal cost of a firm in country i is $\mathcal{P}^{IB}_{i,t}/(Z^{\gamma_i}_{i,t})$ and its optimal price in country j is:

$$p_{i,j,t} = \tau_{ij}\frac{\sigma_i}{\sigma_i - 1}\frac{\mathcal{P}^{IB}_{i,t}}{Z^{\gamma_i}_{i,t}}\,. \tag{27}$$

Unlike Krugman (1980) or Ghironi and Melitz (2005), one needs to jointly solve for all prices in the economy. Through $\mathcal{P}^{IB}_{i,t}$, the price charged by a firm in country i depends on the prices charged by all firms supplying country i which in turn depend on the prices charged by their suppliers and so on and so forth. Determining prices requires solving jointly for all country-pair specific price indexes $\widetilde{\mathcal{P}}_{i,j,t}$. Using the fact that $\widetilde{\mathcal{P}}_{i,j,t} = \tau_{ij}\widetilde{\mathcal{P}}_{i,i,t}$, the definition of price indexes in every country yields a system of N equations which jointly defines all *inner* price indexes:

$$(\widetilde{\mathcal{P}}_{i,i,t})^{1-\rho^I} = \mu_i\left(\sum_{j=1}^{N}\omega^I_i(j)\left(\tau_{ji}\widetilde{\mathcal{P}}_{j,j,t}\right)^{1-\rho^I}\right)^{1-\gamma_i} \quad , \tag{28}$$

with μ_i depending on the mass of firms and parameters.[22] For given mass of firms, this system admits a unique non-negative solution.[23]

Finally, the mass of firms is determined by the free entry condition defined as:

$$\Pi_{i,t} = M_{i,t}\frac{w_{i,t}}{Z^{\gamma_i}_{i,t}}.f^E_i \quad \text{for all } i\,, \tag{29}$$

where the sunk cost f^E_i is labeled in labor units and $\Pi_{i,t}$ dis aggregate profits in country i.

Closing the model involves standard market clearing conditions for capital, labor and goods. Total revenues of all firms from country i can be written as:

$$R_{i,t} = \sum_{j=1}^{N}\left[\left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{j,t}}\right)^{1-\rho^F}\omega^F_j(i)X_{j,t} + \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^I_{j,t}}\right)^{1-\rho^I}\omega^I_j(i)(1-\gamma_j)S_{j,t}\right]\,. \tag{30}$$

Total exports are the sum of final goods and intermediate inputs exports, defined as:

$$T_{i\to j} = \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{j,t}}\right)^{1-\rho^F}\omega^F_j(i)X_{j,t} + \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^I_{j,t}}\right)^{1-\rho^I}\omega^I_j(i)(1-\gamma_j)S_j\,. \tag{31}$$

[22]From equation (24) and (27), we have: $\mu_i^{\frac{1-\sigma_i}{1-\rho^I}} = M_{i,t}\left(\frac{\sigma_i}{\sigma_i-1}\left(\frac{w_{i,t}}{\alpha\gamma_i}\right)^{\alpha\gamma_i}\left(\frac{r_{i,t}}{(1-\alpha)\gamma_i}\right)^{(1-\alpha)\gamma_i}\left(\frac{1}{1-\gamma_i}\right)^{1-\gamma_i}\frac{1}{Z^{\gamma_i}_{i,t}}\right)^{1-\sigma_i}$.

[23]Following Kennan (2001) and denoting $G_k = (\widetilde{\mathcal{P}}_{i,i,t})^{1-\rho^I}$ and G the associated $N\times 1$ vector, it suffices to show that the system is of the form $G = f(G)$ with $f\ :\ \mathbb{R}^N \to \mathbb{R}^N$ a vector function which is strictly concave with respect to each argument, which is obvious as long as $0 < \gamma_i < 1$ for all countries.

With Cobb-Douglas production, consumer's revenues $X_{i,t}$ are equal to the sum of the payment to production workers $\alpha\gamma_i S_{i,t}$, rent from capital $(1-\alpha)\gamma_i S_{i,t}$, total firms' profits $\Pi_{i,t}$ (which, at the free entry equilibrium, is completely used to pay the entry cost f_i^E), and potential trade imbalances $-\mathcal{T}_{i,t}$. Moreover, in absence of fixed cost of exporting, note that both profits and spending can be expressed as a function of revenues with $\Pi_{i,t} = \frac{1}{\sigma_i} R_{i,t}$ and $S_{i,t} = \frac{\sigma_i - 1}{\sigma_i} R_{i,t}$. Using $X_{i,t} = w_{i,t} L_{i,t} + r_{i,t} K_{i,t} - \mathcal{T}_{i,t} = \gamma_i S_{i,t} + \Pi_{i,t} - \mathcal{T}_{i,t}$, equation (30) can be written as:

$$R_{i,t} = \sum_{j=1}^{N} \left[\left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{j,t}} \right)^{1-\rho^F} \omega_j^F(i) \left[\frac{\gamma_j(\sigma_j - 1) + 1}{\sigma_j} R_{j,t} - \mathcal{T}_{j,t} \right] + \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^I_{j,t}} \right)^{1-\rho^I} \omega_j^I(i)(1-\gamma_j) \left(\frac{\sigma_j - 1}{\sigma_j} \right) R_{j,t} \right]. \tag{32}$$

Which can be expressed in compact form as:

$$\mathbf{G} \cdot \mathbf{R}' = -\left[\mathbf{W}_F' \circ \mathbf{P}_F \right] \boldsymbol{\mathcal{T}}', \tag{33}$$

where $\boldsymbol{\mathcal{T}}$ and $\mathbf{R}$ are vectors that stack respectively trade imbalances and total revenues of all firms. $\mathbf{W}_F$ s the weighting matrix associated with final good aggregation and whose elements are defined as $W_{ij}^F = \omega_i^F(j)$, $\mathbf{P}_F$ is a matrix defined by elements $P^F_{i,j,t} = \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{i,t}} \right)^{1-\rho^F}$, and $\circ$ is the element-wise (Hadamard) product. The matrix $\mathbf{G}$ is defined at any time t as:

$$G_{i,j,t} = \mathcal{I}_{i,j} - \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{j,t}} \right)^{1-\rho^F} \omega_j^F(i) \frac{\gamma_j \cdot (\sigma_j - 1) + 1}{\sigma_j} - \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^I_{j,t}} \right)^{1-\rho^I} \omega_j^I(i)(1-\gamma_j) \left(\frac{\sigma_j - 1}{\sigma_j} \right) \tag{34}$$

Finally, labor can be used either for production ($L^p_{i,t}$) or for the entry cost ($L^e_{i,t}$) so that $L_{i,t} = L^p_{i,t} + L^e_{i,t}$. Setting $w_1 = 1$, implying $S_1 = L^p_1 / (\alpha_1 \gamma_1)$, provides a unique solution for all variables by solving together the consumer problem (17), the price system (28), the Free Entry system (29) and the Revenue system (33).

4.4 Real Value Added definitions

Before turning to the calibration details, we need to define real GDP in a way that is consistent with the measure used by statistical agencies, which will constitute the empirical counter-part of our model. As in section 2, we introduce two distinct measures of real value added: a *model-based* measure of physical value added (PVA) and a *statistical* measure of real GDP ($RGDP$).

Physical Value Added. Thanks to separability between domestic factor and inputs in the firm-level production function (22), physical value added can be implicitly defined for each firm as $pva_{i,t} = Z_{i,t}\ell_{i,t}^{\alpha}k_{i,t}^{1-\alpha}$. Aggregating across all firms yields a simple definition of aggregate physical value added as:

$$PVA_{i,t} = Z_{i,t}L_{i,t}^{\alpha}K_{i,t}^{1-\alpha} \tag{35}$$

This measure of real value added is unit-less and, following Arrow (1974), one can interpret it as a measure of a purely theoretical bundle that is used, in combination with intermediate input, to produce the gross output.

Real GDP. Real GDP as constructed by statistical agencies is not unit-less but uses prices at their base period level to express each component in a commonly accepted unit of account.[24]

Since our main goal is to reconcile theory and empirical observations regarding the association between international trade and real GDP synchronization, it is important that we define real GDP in our model in a way that is consistent with statistical agencies. In most database, real GDP is defined using the Fisher ideal quantity index which is a geometric mean of the Laspeyres and Paasche indices. Hence, for any period t, the base period price used in the construction of real GDP growth from $t-1$ to t is a geometric mean between period t and period $t-1$ prices. In order to be as close as possible to the method used in the construction of the data used in section 3 while simplifying the analysis, we define real GDP (RGDP) using steady state prices as base-period prices. Real GDP is obtained by deflating nominal spending using price indices that are corrected from product variety effects to measure "quantity indices", and then by valuing these "quantity indices" using steady-state prices. More precisely:

$$RGDP_{i,t} = \underbrace{\underbrace{\widehat{\mathcal{P}_i^{F,ss}}\frac{X_{i,t}}{\widehat{\mathcal{P}_{i,t}^{F}}}}_{\text{Consumption + Investment}} + \underbrace{\sum_j \widehat{\mathcal{P}_{i,j}^{ss}}\frac{T_{i\to j,t}}{\widehat{\mathcal{P}_{i,j,t}}}}_{\text{Total exports (final+ inputs)}}}_{\text{= Gross Output + Imported Final Goods}} - \underbrace{\sum_j \widehat{\mathcal{P}_{j,i}^{ss}}\frac{T_{j\to i,t}}{\widehat{\mathcal{P}_{j,i,t}}}}_{\text{Total imports (final + inputs)}} \tag{36}$$

where, in order to be consistent with the way actual data are collected, we defined variety-corrected price indices as $\widehat{\mathcal{P}_{i,j,t}} = (M_{i,t})^{1/(\sigma_i-1)}\,\widetilde{\mathcal{P}}_{i,j,t}$ and $\widehat{\mathcal{P}_{i,t}^{F}} = \left(\sum_j \omega_i^F(j)\cdot\left(\widehat{\mathcal{P}_{j,i,t}}\right)^{1-\rho^F}\right)^{\frac{1}{1-\rho^F}}$. Since both consumers' utility and production functions have a CES component, it is well known that the associated price indexes can be decomposed into components reflecting average prices (captured by statistical agencies) and product variety (which is not taken into account in national

[24]In most cases, real GDP is constructed using chain weighted prices, as discussed here. Some database report real GDP in "constant prices" where prices used in the construction of real value added are fixed at a reference year. Obviously, no database reports real GDP by "counting the number of goods" produced in a country.

statistics).[25]

In equation (36), we defined RGDP from the expenditure side. One could also define RGDP from the production side, by summing gross domestic output sold in all markets and subtracting imported inputs. All our results are unchanged using such alternative measure.

Taking the Model to Data. We conclude the model section by discussing two broad points about the quantitative exercise that deserve more attention. First, it is important to note that IRBC models where the production function is expressed in value-added terms use physical value-added as a measure of real GDP. In the context of the TCP literature, papers such as Kose and Yi (2006), Burstein et al. (2008) and Johnson (2014) evaluate the Trade-Comovement slope using PVA. In these perfectly competitive models, our discussion in section 2 shows that $RGDP$ and PVA are equal. A notable exception is Liao and Santacreu (2015) which finds that a statistical measure including an extensive margin component can account for 20% of the Trade-Comovement slope. In this paper, we formally compare both real value added measures and investigate the difference in their properties. This distinction clarifies how, in the presence of imperfect competition and extensive margin adjustments, $RGDP$ measured by statistical agencies is disconnected from a theory-consistent measure of physical value-added.

Second, our proposition 2 in section 2.2 shows that one cannot use the standard Solow Residual to calibrate the technology process. Indeed, the Solow Residual also captures changes in profits and changes in efficiency in response to movements of the number of foreign varieties. We circumvent this issue by calibrating the technology shocks in our model using real GDP targets: we set the shocks so that real GDP in our simulation matches moments observed in the real GDP data. In particular, our simulations deliver an average correlation of real GDP over all country-pairs equal to the co-movement observed in our sample. Linking back to our empirical analysis above which includes country-pair fixed effects that control for the average *level* of GDP comovement, recall that our analysis is focused on the Trade-Comovement *slope*: the change in GDP correlation when countries are more integrated through trade.

5 Calibration

We solve the model with 14 countries and a composite *Rest-Of-the-World* for the time period 1980-1990 using standard linearization techniques. As compared to our empirical sample, it represents around 78% of total trade flows, 79% of total trade in final goods and 77% of total trade flows in

[25]See for example the illuminating discussion in Feenstra (1994) or Ghironi and Melitz (2005).

intermediate goods.[26] In what follows, we describe the parameterization of the model. For each country-pair (i,j), we set values for $\{\omega_i^F(j), \omega_i^I(j), \tau_{ij}\}$. For each country i we set $\{\chi_i, f_i^E, \mathcal{T}_i, \sigma_i, \gamma_i\}$ and the set of common parameters is given by $\{\alpha, \beta, \nu, \rho^I, \rho^F\}$. We finally need to set the volatility $\sigma_Z(i)$, covariance $\sigma_Z(i,j)$ and auto-correlation ρ_Z of the technology shocks.

We first set parameter values for several structural parameters. Table 5 reports those fixed parameters. Starting with preferences, we set $\beta = 0.99$ and $\nu = 0.5$, leading to a Frisch elasticity of 2. We set parameters χ_i in each country to replicate the *relative difference* of working age population with a normalization ensuring an average capital-output ratio of 13 in the model. Regarding the technology, we set $\alpha = 0.67$ and $\delta = 0.025$. The degree of investment adjustment costs, ψ, is chosen to obtain a volatility of investment with respect to real GDP consistent with the data. We set the value of the macro (Armington) elasticity ρ^I and ρ^F to be equal to unity, which is in the range of the literature. For comparison, Saito (2004) provides estimations from 0.24 to 3.5 for the Armington elasticity.[27] There is also a theoretical convenience to use $\rho^I = \rho^F = 1$, as it allows the model to take the same form as other network models such as Acemoglu et al. (2012).

Table. 4. Fixed parameters of the model.

	Parameter	Value	Moment / Source
Discount factor	β	.99	Annual discount rate of 4%
Labor curvature	ν	0.5	Frisch elasticity of 2.0
Investment adjustment cost	ψ	1.25	Volatility of σ_I/σ_{RGDP} between 3-4.5
Labor Supply Scaling	χ_i	$[5.4e{-5}, 0.16]$	Relative working age population
Labor share	α	0.67	Standard value
Argminton elasticities	ρ^I, ρ^F	1.0	Saito (2004), Feenstra et al. (2014)
Micro elasticity of substitution	$\sigma_i, \forall i$	4.35	Markup of 30%, De Loecker and Eeckhout (2018)
Sunk entry cost	f_i^E/f_{US}^E	[0.4 - 3.9]	*Doing Business Database* - World Bank
Fixed trade cost	f_{ij}^c	[3.3 - 18]	*Doing Business Database* - World Bank
Iceberg trade cost	τ_{ij}	[1 - 2.8]	ESCAP - World Bank

We set a value of $\sigma_i = \sigma = 4.3, \forall i$ for the micro elasticity of substitution in the baseline simulation. Anderson and van Wincoop (2004) reports estimates in the range of 3 to 10. Following Bernard et al. (2003), Ghironi and Melitz (2005) choose a micro elasticity of 3.8 and recently, papers such as Barrot and Sauvagnat (2016) or Boehm et al. (2015) argue that firms' ability to substitute between their suppliers can be very low. This choice leads to markups of 30%, which is consistent with recent estimates in De Loecker and Eeckhout (2018). As a robustness, we also

[26]The set of countries is: Australia, Austria, Canada, Denmark, France, Germany, Ireland, Italy, Japan, Mexico, Netherlands, RoW, Spain, United Kingdom and United States.
[27]Studying macro and micro elasticities for final goods, Feenstra et al. (2014) finds that, for the majority of goods, the macro elasticity is lower than the micro elasticity.

consider alternative elasticities for σ_i in section 6.4, and defer discussion of those cases till then.[28]

The value added shares, γ_i, are calibrated using cost of intermediates and total sales as observed in the WIOD database at the 2-digits sector level. Specifically, $(1-\gamma_{i,s}) = \frac{cost\_intermediates_s}{total\_sales_s}$, represents the share of intermediate inputs in total costs in a given sector. We use the fact that $total\ sales_s = \mu_i \times total\ cost_s$ with μ_i the markups in country i. Therefore, we fix $\gamma_{i,s} = 1 - \frac{cost\_intermediates_s}{total\_sales_s}\frac{\sigma_i}{\sigma_i - 1}$. The implied mean values of γ_i, weighted by the sector importance in total sales, range from 0.31 to 0.45 for the considered countries (we set the value for *RoW* to the mean value), which is consistent with values reported in Halpern et al. (2015).

The sunk entry costs f_i^E in each country are computed from the *Doing Business Indicators*.[29] We measure the relative entry fixed costs by using the information on the amount of time required to set up a business in the country relative to the US, where we normalize f_{US}^E in order to generate a ratio of total number of firms divided by the working population, $\frac{M}{L}$, of about 12%. This is motivated by the fact that there are about 22-24 millions of non-employer businesses and 5.5 millions of employer businesses in the US, while the working age population represents around 180 millions of individuals during the considered period.[30] The variable (iceberg) trade costs for each country-pairs, $\tau_{ij} > \tau_{ii}$, are taken from the ESCAP World Bank: *International Trade Costs Database*, where we normalize $\tau_{ii} = 1$. This database features symmetric bilateral trade costs in its wider sense, including not only international transport costs and tariffs but also other trade cost components discussed in Anderson and van Wincoop (2004).

Data on bilateral trade flows, $\{T^I_{j\to i}/RGDP_i, T^F_{j\to i}/RGDP_i\}$, are sufficient to identify the shares $\omega_i^I(j)$ and $\omega_i^F(j)$. Similar to our empirical part, we use trade data from Johnson and Noguera (2017) dis-aggregated into final and intermediate goods. Moreover, since complete financial autarky is inconsistent with the trade balances observed in the data, we calibrate the model trade imbalances $\{\mathcal{T}_1, ..., \mathcal{T}_N\}$ to match steady-state trade imbalances relative to real GDP, and then hold those nominal imbalances constant during the simulation. Finally, to be as close as possible to the data used in the empirical analysis, we model-based estimates that are mapped to the data are constructed by deflating nominal values by a price index that do not take into account love

[28]In particular, we re-calibrate the model with heterogeneous elasticities σ_i based on two measures: the De Loecker and Eeckhout (2018) markup estimates and a Price Cost Margin approach. As shown in section 6.4, the introduction of heterogenous markups makes it possible to study the role of market power in shaping the correlation between terms of trade and real GDP, in line with empirical evidence.

[29]The World Bank's Doing Business Initiative collected data on regulations regarding obtaining licenses, registering property, hiring workers, getting credit, and more. See `http://doingbusiness.org/data/exploretopics/trading-across-borders` and `http://doingbusiness.org/data/exploretopics/starting-a-business`. As shown later, f_i^E plays a little role in the correlation between trade and real GDP comovement.

[30]This is also close to the 12% self-employment rate usually reported for the US between 1990 and 2000 (BLS). Results are not sensitive to this assumption. We provide a comparison of this rate and the self-employment rate in each economy in the online appendix C.

of variety, as described in section 4.3. By construction, the model's steady state matches relative bilateral trade flows and relative trade imbalances exactly.

Aggregate Technology Process. The *level* of real GDP comovement in our simulations is driven both by correlated technology shocks and by the transmission of those shocks across countries via trade linkages. As discussed earlier, productivity measures available in usual macro database can not be used as a proxy for the country-specific technology process $Z_{i,t}$. To generate an international correlation of GDP consistent with the data, we pin down the off diagonal elements of the covariance matrix so that the average correlation of real GDP in the model matches exactly the one observed in the data, which is 0.27. We also calibrate the variance $\sigma_Z(i)$ and persistence ρ_Z of technology shocks to match the (de-trended using HP-filter) real GDP volatility and an average auto-correlation of 0.84. This allows us to generate real GDP fluctuations in the simulated economy that are similar to those observed in the data.[31] Table 5 reports the list of calibrated parameters. All our targeted moments are perfectly matched.[32]

Table. 5. Calibrated parameters of the model.

	Parameter	Value	Main target
Inputs spending weights	$\omega_i^I(j)$	in sup. app.	Import shares in inputs
Final goods spending weights	$\omega_i^F(j)$	in sup. app.	Import shares in final goods
Trade imbalance	$\{\mathcal{T}_i, ..., \mathcal{T}_N\}$	in sup. app.	Trade imbalance over GDP
Persistency of Techno. shocks	ρ_Z	.71	Avg. RGDP auto-correlation
Std. of Techno. shocks	$\sigma_Z(i)$	[.0012, .0050]	RGDP volatility (de-trended)
Covariance of Techno. shocks	$\sigma_Z(i,j), \forall i \neq j$	.22	Avg. RGDP correlation of 0.27

Interestingly, in order to match an observed international real GDP correlation ($RGDP$) of 0.27, the correlation of technology shocks is only 0.22. This implies that, according to our framework, trade linkages explain 20% of the international real GDP correlations ($= 0.27 - 0.22 = 0.05$), with shock correlation explaining the remaining 80%. In contrast, the international correlation of physical value added (PVA) is only 0.24, implying that the correlation of shocks explains more than 92% (=0.22/0.24) of PVA comovement. The rest of the paper clarifies the channels through which trade linkages propagates international shocks and synchronizes business cycle, by studying and decomposing the trade comovement slope.

[31] Again, recall that the goal of this exercise is not to explain the ***level*** of comovement across countries, but its ***slope*** following a change in trade intensities.

[32] We report standard business cycle statistics of our model in the online appendix.

6 Results

We now present our main quantitative results. We frame this section by focusing on three questions: (i). Is the model able to generate a TC slope consistent with empirical estimates of section 3? (ii) What role do markups and extensive margin adjustments play in generating the TC slope? (iii) What role does real GDP measurement play to rationalize these findings?

6.1 A Quantitative Resolution of the Trade Comovement Puzzle

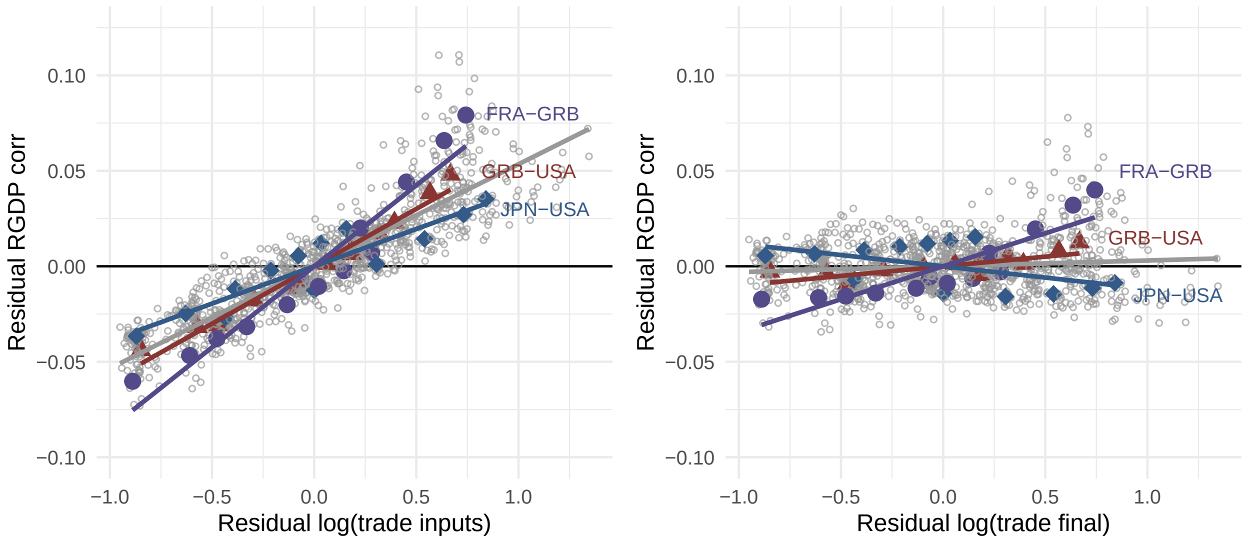
To assess the model's ability to replicate the link between trade and real GDP-comovement, we simulate the exact same sequence of 5,000 shocks in different configurations in which we vary bilateral trade intensities. Starting with our baseline calibration of trade flows as described above, we simulate our model under 12 alternative calibration targets, using varying levels of final goods and intermediate goods bilateral trade intensities, each time resulting in new values for the shares $\omega_i^F(j)$ and $\omega_i^I(j)$. We use the data to discipline the variations of bilateral trade intensities across all configurations. Relative to the first time window (1970:1979) of our sample, the median bilateral trade intensity in intermediate goods and final goods increased by 5-6 and 8-9 in the last time-windows (2000:2009), respectively. For each configuration, we feed in the *exact same* technology shocks from our baseline calibration and record the pairwise correlation of logged and HP-filtered real GDP.

All told, this procedure gives rise to a panel dataset of $14 \times 13/2 = 91$ country-pairs (excluding *RoW*) for each of the 13 configurations, hence a total of 1,183 observations. Similar to what we did in the empirical exercise in specification (12), we use within country-pair variation to gauge how changes in trade intensity impact bilateral real GDP comovement. Since only country-pairs trade intensities are modified, each configuration can be thought of as a different time-window.

We first focus on the relationship between measured real GDP correlations (corr $RGDP$) and bilateral trade intensities in intermediate inputs ($\text{Trade}_{ij}^{\text{input}}$) and final goods ($\text{Trade}_{ij}^{\text{final}}$). Similar to the exercise performed in the empirical section 3, we test the specification (12) within the model. We plot in Figure 1 the residual relationship between bilateral trade in final and intermediate inputs and $RGDP$, after controlling for country-pair fixed effects and change in the other good bilateral trade intensity (either final or intermediate goods). We find that the model captures well the positive and high correlation between bilateral trade in intermediate goods and real GDP, as well as the low correlation between trade in final goods and real GDP.[33]

[33] In appendix B, we show that a higher Armington elasticity for final goods aggregation can account for the insignificant and negative slope between cross-country GDP correlations and trade in final goods.

Figure 1. Model-based association between RGDP correlation and Trade intensities. Left chart shows Intermediate Inputs trade, right chart shows Final goods trade.



Note: residual relationship in the model after controlling for other covariates, i.e. country-pair fixed effects and the other trade intensities (either final or intermediate goods). The grey solid line reports the Trade Comovement slope including all country-pairs.

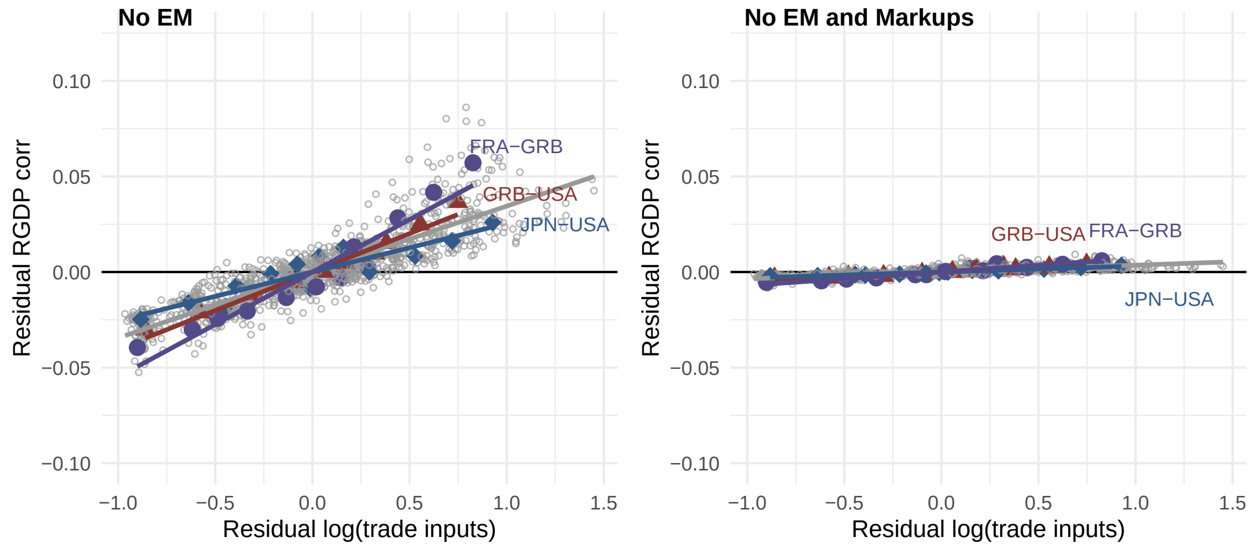
We report the implied average slope in Table 6 (the 1^{st} row, columns (1)-(2)). The model generates a significant and positive trade comovement slope of 0.056, which is quantitatively in line with our findings in section 3.[34] In terms of magnitude: our point estimates suggest that an increase in the log input trade index by 1.84, which is the average increase between 1970-1979 and 2000-2009, generates an increase of real GDP correlation in our simulations of about 10.3%. To put this into perspective, in our sample, the observed increase in cross-country GDP correlation between 1980-1990 and 2000-2009 in the data was about 25 percentage points. In our model, increasing trade intensities in the same proportion as what was observed during this period would lead to a 6 percentage points increase in cross-country GDP correlation. Therefore, trade linkages would explain roughly 24% of the observed increase in cross-country GDP correlations.

To understand the quantitative success of the model in replicating the large trade comovement slope observed in the data, we turn off one by one the key elements of the model: (i) movements along the extensive margin, and (ii) monopolistic competition. Figure 2 depicts the implied relationship between *RGDP* and bilateral trade intensity in intermediate inputs under these two alternatives, with points estimates in Table 6 With markups but no extensive margin (EM) adjustments, the trade comovement slope in intermediate inputs is 0.035. In a model with neither markups nor extensive margin adjustments (right panel of Figure 2 and the 3^{rd} row of

[34]Using total bilateral trade intensity which captures the sum of input and final good trade, we find a slope of 0.07, comparable to the range of values $[4.8\% - 11\%]$ reported in the literature.

Table 6), the model delivers a meager TC-slope of 0.004, in line with results obtained using a standard IRBC model with input-output (I/O) linkages. Table 6 allows for a decomposition of our result: Input-Output links alone explain only 7.0% of our TC-slope (=0.004/0.056) while the markup and extensive margin channels contribute 55.5% and 37.5% respectively.

Figure 2. Decomposition of the Input Trade-comovement slope using alternative model specifications. Left chart shows a model without Extensive Margin adjustments, right chart shows a model with neither extensive margin nor markups.



Note: residual relationship in the model after controlling for other covariates, i.e. country-pair fixed effects and the other trade intensities (either final or intermediate goods).

Table. 6. Quantitative assessment of the Trade Comovement Slope: Data versus Model.

Trade – comovement slope [a]	Based on *RGDP*		Based on *PVA*	
	Input	*Final*	*Input*	*Final*
	(1)	(2)	(3)	(4)
Data: CP & TW fixed effects	.060**	−.038		
Model:				
1. IO link. + Markups + EM	.056***	.006***	.001***	.004***
2. IO link. + Markups	.035***	.001*	.001***	.002***
3. IO link.	.004***	.004***	.001***	.005***

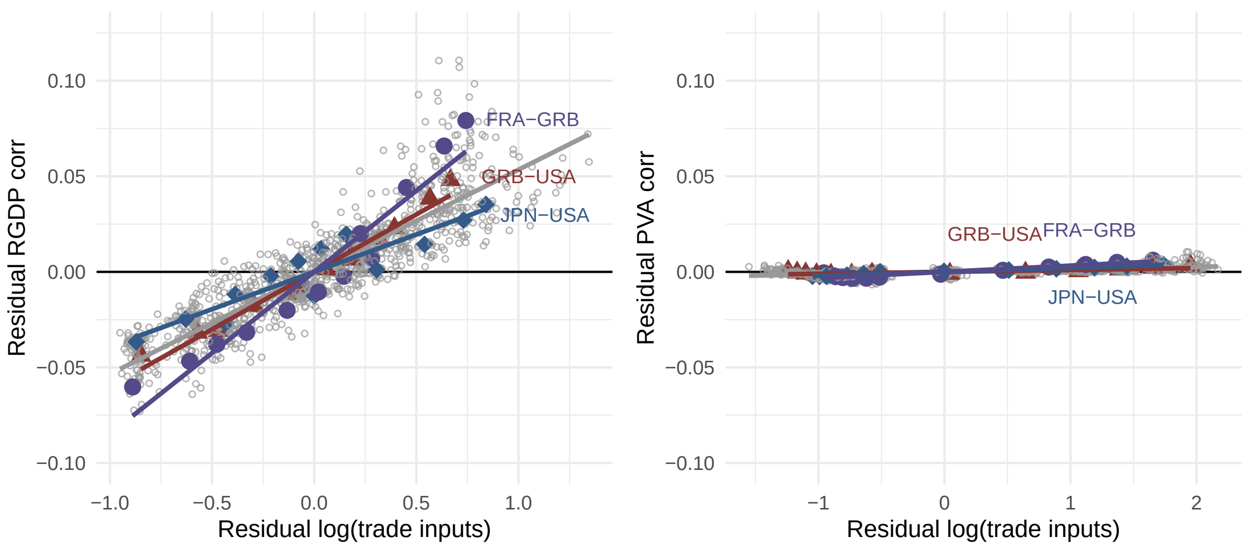
*p<0.1; **p<0.05; ***p<0.01. SE clustered on country-pairs. Results are robust to the inclusion of our $network_{prox}$ index.
[a]The trade indexes used in those experiments are $(T_{i\to j} + T_{j\to i})/(GDP_i + GDP_j)$.

6.2 The Importance of Measurement

As discussed in section 2, the introduction of price distortions and extensive margin adjustments generate a disconnect between measured real GDP (*RGDP*) and physical value added (*PVA*).

Figure 3 confirms this prediction in our quantitative model. In our simulations, the association between trade and the synchronization of physical value added is low, consistent with earlier findings in the literature. In columns (3)-(4) of Table 6, we show that for each model specification, excluding first extensive margin adjustments and then also price distortion, using the *PVA* measure consistently results in a low estimated trade comovement slope. These results highlight that it is important to define real GDP in the model in a way that is consistent with data construction procedure: a researcher using *PVA* as a proxy for real GDP would mistakenly conclude that the model is not consistent with the data.

Figure 3. Model-based Trade Comovement Slopes: Comparing Real GDP and Physical Value Added. Left chart shows the TC-slope using RGDP, the right chart shows the TC slope using PVA.



Note: residual relationship in the model after controlling for other covariates, i.e. country-pair fixed effects and the other trade intensities (either final or intermediate goods).

To sum up, our results show that adding intermediate inputs crossing multiple borders to an otherwise standard IRBC model is not sufficient to solve the TCP, as shown in Johnson (2014). This is because in these models, RGDP is equal to PVA, which reacts only modestly to foreign shocks. However, a model combining I/O linkages with markups and extensive adjustments provides a quantitative solution to the TCP, once we measure real GDP in a way that is consistent with the procedures used by statistical agencies.

6.3 Measured Productivity and International Trade

As discussed in the simple framework layed out in section 2, the presence of price distortions and love of variety creates a link between changes in measured productivity and changes in imported input useage. This prediction was confirmed in the data in section 3, with trade in input being associated with synchronized measured productivity. We now verify that our quantitative framework is able to replicate this observed relationship.

In the model, we measure productivity (SR) using the Solow Residual such that: $SR_{it} = \log(RGDP_{it}) - \alpha \log(L_{it}) - (1-\alpha)\log(K_{it})$. In Table 7, we present the results of regressing changes in productivity on changes in trade proximity for different versions of our models. The first row reveals that, in our model, measured productivity correlation increases when country-pairs trade more intermediate inputs, and the magnitude of this association is in line with the data. In line with the insights emerging from equation (10), this association dissipates in a version of the model without extensive margin adjustments or without markup: without these ingredients, productivity is simply equal to technology shocks, which are kept constant in all simulations. Moreover, if one constructs productivity as the Solow Residual of PVA (defined as: $SR_{it}^{PVA-based} = \log(PVA_{it}) - \alpha \log(L_{it}) - (1-\alpha)\log(K_{it})$, in columns (3)-(4)), none of the additional ingredients generate a disconnect between technology and measured productivity.

Table. 7. Association between trade and SR comovement: Data versus Model.

Trade – Productivity comovement slope:[a]	SR based on *RGDP*		SR based on *PVA*	
	Input	*Final*	*Input*	*Final*
	(1)	(2)	(3)	(4)
Data: CP & TW fixed effects	.065**	−.032		
Model:				
1. IO link. + Markups + EM	.051***	.003***	−.000***	.000***
2. IO link. + Markups	.033***	−.000	.000*	.000***
3. IO link.	.000**	.000***	.000	.000***

*p<0.1; **p<0.05; ***p<0.01. SE clustered on country-pairs. Results are robust to the inclusion of our $network_{prox}$ index.
[a]The trade indexes used in those experiments are $(T_{i\to j} + T_{j\to i})/(GDP_i + GDP_j)$. Productivity is measured as the Solow Residual using either $RGDP$ or PVA as a measure for real value added.

Again, this discussion underscores the importance of measuring real GDP in our model in a way that is consistent with actual data construction procedures. Indeed, the international correlation of measured productivity in our simulations is about 0.264, while the correlation of technology shocks, captured by Z_{it}, is only 0.22. This illustrates that using measured productivity in the data as a calibration target for technology would overstate the actual correlation of shocks between countries.

6.4 A Focus on the Role of Markups and Profits

In the baseline calibration, we assumed a common lower-tier elasticity of substitution $\sigma = 4.35$ in all countries, leading to 30% markups and 20% profit rates once accounting for fixed cost payments. In this section, we perform numerical exercises and compare the effects of varying markups. Then, we further validate the role of markups in generating a link between terms of trade and real GDP fluctuations.

6.4.1 Variations on Markups

We present in table 8 various exercises regarding the role of price distorsions in the model. A first set of analysis investigates the sensitivity of the TC-slope to changes in the value of σ. We test the implication of higher ($\sigma = 3.5$) and lower ($\sigma = 6.0$) markups in the 2^{th} and 3^{rd} row, respectively. As expected, an increase in markups leads to a higher association between trade and real GDP comovement. Increasing markups from 20% to 33% almost doubles the association between input trade and real GDP synchronization. Unsurprisingly, we find that physical value added *PVA* does not respond more to foreign shocks when we change the value of the (constant) markup, as shown in the last two columns of the table.

A second set of exercises aims to relax the homogeneous assumption and introduce heterogenous market power across countries. We evaluate whether plausible heterogeneity in observed markups – for the set of countries we consider – have important consequences for the implied TC-slope. Specifically, we simulate the model with heterogenous σ_i estimated from the data using two different sources. We first use Price Cost Margin (PCM) (4^{th} row) as an estimate of markups within each industry, which measures the difference between revenue and variable cost. Second, we use direct markup estimates from De Loecker and Eeckhout (2018) (DLE) (5^{th} row).[35] In each experiment, we center the heterogenous markups $\{\sigma_1, ..., \sigma_N\}$ around the baseline value. The implied standard deviation of σ are .63 and .73 for the PCM and the DLE experiments respectively. Interestingly, adding heterogeneous markups centered around our benchmark value does not substantially affect the trade comovement slope, which suggests that accounting for cross-country heterogenous markups does not change the aggregate strength of international propagation in our model.

Those results are especially interesting because they help rationalize the large heterogeneity in the estimated trade comovement slopes found in the literature, with estimates ranging from 4.8% to 11%. For example, Kose and Yi (2006) find a log slope of total trade of 0.091, significantly higher than our estimate. Moreover, using the period 1970-1990, we find a TC-slope of about 8.1% for trade in inputs, as shown in the online appendix A.3.1. Using 20-years time windows, we find a slope of 7.4%. Consequently, a key takeaway here is that our model can generate such an heterogeneity through different levels of market power over time and between countries.

[35] We provide details on the two measures in appendix A.

Table. 8. The role of price distorsions and heterogenous markups [a]

	Elasticity	Trade - GDP comovement slope coefficients: based on *RGDP* *Input*	*Final*	based on *PVA* *Input*	*Final*
Data: CP & TW fixed effects	-	.060**	-.038		
Model:					
1. Baseline	$\sigma = 4.35$	.056***	.006***	.001***	.004***
2. Low markups	$\sigma = 6.0$	.037***	.005***	.001***	.004***
3. High markups	$\sigma = 3.5$	.075***	.007***	.001***	.003***
4. Heterogenous markups, PCM	$\sigma_i \in [3.20, 5.65]$	.054***	.006***	.001***	.004***
5. Heterogenous markups, DLE	$\sigma_i \in [3.68, 6.07]$	.056***	.006***	.001***	.004***

Notes: *p<0.1; **p<0.05; ***p<0.01. Results are robust to the inclusion of our $network_{prox}$ index.
[a]The simulations are based on the exact same sequence of shocks, under the five variations of trade indexes used in the benchmark.

6.4.2 Markups, Terms of Trade and real GDP fluctuations

Our model predicts that markups play an important role to make GDP react to foreign shocks, as shown in the decomposition in Table 7. We now present additional empirical support and external validation for the role of markups in generating a link between terms of trade and real GDP fluctuations. We test the following hypothesis: countries where markups are high experience a larger decrease in real GDP when experiencing an increase in their terms-of-trade. For this, we compute the correlation of real GDP with the terms of trade and regress this correlation on markups estimates, such that:

$$\text{Corr(GDP , ToT)}_{it} = \beta \,\text{Markup.Index}_{it} + \text{Country}_i + \text{TW}_t + \epsilon_{it} \tag{37}$$

Table 9 gathers our findings.[36] We first show the results of pooled cross-section analysis and then perform fixed effect regression and add time dummies to control for time-window specific factors that might affect the correlation of GDP and terms-of-trade. We also run the exact same regression with the model generated data using $\frac{\sigma_i}{\sigma_i - 1}$ as markup index and using variations in σ_i. Regressions performed on model-generated data show that countries with higher markups also experience a larger decrease in their GDP when the relative price of their imports rises, consistent with what we observe in the data.

[36]Data on real GDP and terms of trade at the annual frequency are both taken from the OECD database and are HP filtered to capture business cycle frequencies. We also use first difference data and results are consistent with our findings using HP-filter, as shown in the online appendix A.5.2.

Table. 9. Markups and GDP-Terms of Trade correlation

	Corr(ln(GDP_i^{HP}),ln(ToT_i^{HP}))				
	Data				Model
Markup measure	PCM [a]		De Loecker and Eeckhout (2018) [b]		
Markup index	-1.151	-2.650**	-0.756***	-0.495*	-0.262***
	(0.967)	(0.911)	(0.187)	(0.289)	(0.019)
Country FE	Yes	Yes	Yes	Yes	Yes
Time windows FE	No	Yes	No	Yes	-
N	43	43	80	80	112
R^2	0.066	0.322	0.132	0.232	0.790

*p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs.
[a]We use two time-windows from 1971-2010 over 22 countries reported in appendix.
[b]We use three time-windows from 1980-2009 for 29 countries reported in appendix.

7 Conclusion

This paper analyzes the relationship between international trade and business cycle synchronization across countries, with a focus on improving the mapping between real GDP in the data and its counterpart in standard macroeconomic models. We show that real GDP, as constructed by statistical agencies, is not equal to the theory-consistent "physical value added". This disconnect appears when the base period price used to value imported input does not reflect their marginal product, a situation that arises for example in presence of markup and love of variety. With those ingredients, real GDP fluctuations are not only tied to movements of technology and factor supply, but can also fluctuate as a result of changes in imported input usage.

We then derive several predictions that are tested in the data. First, higher trade in intermediate inputs is associated with an increase in the bilateral correlation of both real GDP and productivity. Second, higher trade in intermediate input is also associated with synchronized profits. Third, real GDP is sensitive to fluctuations in the number of varieties imported, implying that the extensive margin of trade plays an important role in business cycle synchronization.

We finally show that a quantitative model with markups and love of variety delivers a strong link between input trade and business cycle comovement, with a magnitude in line with empirical estimates, offering the first quantitative solution for the *Trade Comovement Puzzle*. Conversely, model-based simulations show that trade is much less associated with the synchronization of physical value added.

To conclude, this paper seeks to draw attention to real GDP measurement in macroeconomic models and how it should be compared with its data counterpart. In the context of the *Trade-Comovement Puzzle*, we showed that IRBC models that generate weak cross-country propagation properties in terms of physical value-added can actually feature strong propagation in terms of real GDP. More generally, recognizing that real GDP fluctuations are not only tied to physical value added movements could be a promising research avenue for business cycle analysis.

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A Empirical Appendix

A.1 Markup measures

Markups. We used two different markup index estimates. We first used aggregated micro markups estimated by De Loecker and Eeckhout (2018). They use micro data of 70,000 firms in 134 countries from 1980 to 2016 and estimate aggregate average markups using a cost-based approach. This method defines markups as the ratio of the output price to the marginal costs, and therefore relies solely on information from the financial statements of firms (sales value and cost of goods sold). Aggregating all firms specific markups for each country, De Loecker and Eeckhout (2018) provide a detailed and comparable measure of market power between countries. The sample that we use from their estimates includes 29 countries from 1980 to 2016.[37]

Second, we use Price Cost Margin (PCM) as an estimate of markups within each industry using data from 22 countries from 1971 to 2010.[38] Introduced by Collins and Preston (1969) and widely used in the literature, PCM is the difference between revenue and variable cost (the sum of labor and material expenditures, over revenue): $PCM = \frac{\text{Sales}-\text{Labor exp.}-\text{Material exp.}}{\text{Sales}}$. Data at the industry level come from the OECD STAN database, an unbalanced panel covering 107 sectors for 34 countries between 1970 and 2010. Due to missing data for many countries in the earliest years, we restrict the analysis for 22 countries.[39] We compute PCM for each industry-country-year and then construct an average of PCM within each country-year by taking the sales-weighted average of PCM over each industry. Finally, the average PCM for a given time window is simply the mean of country-year PCM over all time periods.

[37]The list of countries is: Austria, Belgium, Canada, Colombia, Denmark, Finland, France, Germany, Greece, Ireland, Iceland, Indonesia India, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Portugal, South Africa, Spain, Sweden, Switzerland, Turkey, the United-Kingdom and the United-States.

[38]The list of countries is: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Iceland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, Portugal, Spain, Sweden, the United-Kingdom and the United-States.

[39]For Germany, data are available only from 1991 onward (after the reunification), which is why the total number of observation in the regressions is 43.

A.2 Sensitive analysis of the Trade Comovement Slope

Table 10 provides various sensitivity analysis of our main empirical results regarding the trade-comovement slope. More details of those results are provided in the online appendix and a short summary of our investigation is reported here.

In the first series of robustness analysis, we ask how our results change when we restrict our sample in different ways. The first row simply restates our baseline results for reference. In the second row, we use 20 years time windows when computing GDP correlation. In the third and fourth row, we reduce our sample by excluding coutry pairs in the European Union and the USSR respectively. In the fifth row, we restrict our sample to the first three time windows (from 1970 to 1999) so that our time coverage is in line with original Kose and Yi (2006) analysis. In all these cases, our main results are virtually unchanged.

A second series of analysis relates to the inclusion of alternative controls for sectoral decomposition. Recall that our regressions include controls for similarity in specialization patterns which could impact GDP synchronization over and beyond any trade effect. In rows six, seven and eight, we use different sector categorization and dis-aggregation to compute our sectoral proximity measure. These variations do not affect our empirical results.

A third series of robustness exercises relate to the definition of our trade proximity indices. In our main results, we construct, for each country-pair and time window, the average over all years in the time window of the log of bilateral trade divided by the sum of GDP. In row nine, we do not take the log of the trade over (bilateral) GDP ratios and instead use simply the levels. In row ten, we do not construct the average of log ratio, but instead use the log of the average ratio. Row eleven presents results where instead of using the ratio of bilateral trade over sum of GDP, we use the maximum of bilateral trade divided by each country in the pair. Finally, row twelve presents results where we change our database and use the Structural Analysis (STAN) database for the trade flows. Again, our main message are not altered by these variations.

The last set of sensitivity investigations relates to the definition of GDP as well as other robustness exercises. In our main results, we use annual data on real GDP at chained PPPs from the 9th Penn World Table. Rows thirteen and fourteen show the results when using the variable called "RGDPNA" which measures real GDP at constant 2011 national prices, when we use HP filter and first difference respectively. In rows fifteen and sixteen, we perform a weighted regression where each observation is weighted by the sum of GDP of both countries in the pair – hence giving relatively more weights to pairs containing at least one big country such as the US. As previously, our results are not materially affected by such changes. In row eighteen and nineteen, we come back to the role of measured productivity. In eighteen, we estimate the effect of trade on SR comovement using the Baxter nad King (BK) filter. Finally, we test in row nineteen whether adding bilateral Solow Residual correlation in specification (12) has a significant impact on the relation between real GDP co-movement and intermediate inputs trade. Once controlling for the cross-country correlation in measured productivity (SR), the coefficient associated to trade in intermediate inputs becomes insignificant, while still positive.

Table. 10. Robustness exercises on estimated Trade Comovement-slope in the data

	Coefficient on trade in inputs	Coefficient on trade in final goods	GDP Filter	Countries | Obs.	Period	TW	CP
Sample selection							
1. Whole Sample	0.060**	−0.038	HP	40 | 2,900	1970-2009	Yes	Yes
2. 20 years TW	0.074**	−0.054	HP	40 | 1,450	1970-2009	Yes	Yes
3. Excluding EU CP	0.056**	0.005	HP	40 | 2,280	1970-2009	Yes	Yes
4. Excluding USSR	0.064**	−0.006	HP	34 | 2,244	1970-2009	Yes	Yes
5. Alternative TW	0.081***	0.014	HP	34 | 2,244	1970-1999	Yes	Yes
Alternative controls for sectoral composition							
6. 4Digits SITC	0.058**	−0.045*	HP	36 | 2,520	1970-2009	Yes	Yes
7. ISIC classification	0.059**	−0.045*	HP	36 | 2,520	1970-2009	Yes	Yes
8. 1Digit Agg. sectors	0.088	−0.044	HP	38 | 1,291	1970-2009	Yes	Yes
Alternative indexes							
9. $level(trade)^a$	34,9*	−36.8	HP	40 | 2,900	1970-2009	Yes	Yes
10. $\log(mean(trade))$	0.053**	−0.034	HP	40 | 2,900	1970-2009	Yes	Yes
11. $\max\left(\frac{T_{i\leftrightarrow j}}{GDP_i}, \frac{T_{i\leftrightarrow j}}{GDP_j}\right)$	0.060**	−0.040*	HP	40 | 2,900	1970-2009	Yes	Yes
12. STAN data	0.209**	−0.107	HP	20 | 760	1995-2014	Yes	Yes
Other robustnesses							
13. RGDPNA measure	0.110***	−0.096***	HP	40 | 2,900	1970-2009	Yes	Yes
14. RGDPNA measure	0.069***	−0.058***	FD	40 | 2,900	1970-2009	Yes	Yes
15. Weighted GDP	0.048**	−0.046*	HP	40 | 2,900	1970-2009	Yes	Yes
16. Weighted GDP	0.061**	−0.047*	FD	40 | 2,900	1970-2009	Yes	Yes
17. Whole sample	0.036*	−0.020	BK	40 | 2,900	1970-2009	Yes	Yes
18. SR-slope	0.055*	−0.033	BK	40 | 2,367	1970-2009	Yes	Yes
19. With SR-corr. as control	0.023	−0.004	HP	40 | 2,367	1970-2009	Yes	Yes

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs. TW stands for Time Windows Fixed Effects while CP stands Coutry-Pairs Fixed Effects, which are inluded in all our analysis.

B Theoretical and Quantitative appendix

Our results are robust to a number of alternative specifications, as presented in table 11.

Parameters. We restate our baseline result in the 1^{st} row for reference. As shown in rows 2 and 3, the Frisch elasticity (which is equal to $1/\nu$) has a more significant impact on the magnitude of the overall trade comovement slope, while preserving the relative importance of final goods versus intermediate inputs. The level of trade frictions in the calibrated steady state $\{\tau_{ij}, f^c_{ij}, f^E_{ij}\}$ does not affect the implied TC *slope*, as indicated by the result in rows 4 and 5. Regardless of the initial level of those trade frictions, increasing trade proximity is associated with the same reaction for real GDP comovement. In row 6 we artificially remove all trade imbalances observed in the data. Under all those alternative specifications, our main results hold. In rows 7 and 8 we vary ψ and find that lower adjustment costs results in more volatile real value added and investments $\mathcal{I}$, which magnifies the Trade Comovement slope. In row 9, we set a higher Armington elasticity in the CES final good aggregation, $\rho_F = 1.5$, and show that it can rationalize the negative and insignificant slope with respect to trade in final goods.

Productivity process. We also conduct robustness on the properties of technology processes $\{Z_i\}$ in the 10^{th} and 11^{th} rows. We first use the observed estimated covariance matrix of standard TFP data computed as the Solow residual in Penn World Tables ($\widetilde{\sum}$). While this approach is sometimes used in the literature, it leads to overshooting the *level* of cross-country real GDP correlation. Results regarding the TC slope remain similar to the benchmark calibration. Furthermore, we also simulate the model under the counterfactual assumption that technology shocks are uncorrelated across countries and set the off-diagonal elements of the covariance-variance matrix to zero (i.e. $cov(Z_{i,t}, Z_{j,t}) = 0, \forall i \neq j$), in row 11. In this case, the TC slope decreases to 0.035. This finding echoes the discussion about the consequences of using correlated vs. uncorrelated shocks in Johnson (2014), namely that the TC slope decreases with uncorrelated shocks.[40] However, it is important to recall that our exercise are very different: while Johnson (2014) uses a cross-section regression, our panel estimation includes country-pair FE implying that differences in average shock correlation across different country-pair are controlled for.

Reference period. In rows 12 to 13 we use a different reference period for the calibration of the CES weights ω^I and ω^F, a variation that does not materially alter our key messages.

Alternative Financial Markets. Our benchmark specification assume financial autarky. We verify if the results of our quantitative model hold under complete financial markets, which can be thought as the other extreme modelling assumption. We assume that there are complete contingent claims dominated in units of one of the countries' tradable final good. Let s_t denote the state of an economy in period t, with transition probability density $f(s_{t+1}, s_t)$. We denote $B_i(s_{t+1})$ denote the country i's holdings of a one-period state-contingent bonds, paying off one unit of the

[40] Looking at panel B in table 1 of Johnson (2014), the TC slope generated by a model with correlated shocks is about 15% of its empirical counterpart. Results with uncorrelated shocks (in panel C) are only about 3% of the empirical one. In contrast, while we also find a small decrease when using uncorrelated shocks, the magnitude of the drop is significantly smaller.

numeraire good in state s_{t+1}, and let $b(s_{t+1}, s_t)$ be the price of that security in state s_t at date t. Furthermore, these state-contingent bonds are in zero net supply in all states: $\sum_i B_i(s_{t+1}) = 0$. In this case, the consumer budget constraint is given by:

$$P^F_{i,t}(C_{i,t} + \mathcal{I}_{i,t}) + \int b(s_{t+1}, s_t) B_i(s_{t+1}) ds_{t+1} = r_{i,t} K_{i,t} + w_{i,t} L_{i,t} + B_i(s_t) \tag{38}$$

The consumer's problem is then to choose $\{C_{i,t}, L_{i,t}, K_{i,t+1}\}$ and asset holdings $\{B_i(s_{t+1})\}$ given prices and initial asset endowments $\{B_i(s_0)\}$ to maximize equation (17). The results of this alternative specification is provided in Table 11 (in rows 14 and 15). We find that the trade comovement slope persist even under complete markets and the main message of the paper remains unchanged.

Table. 11. Model-based simulations: sensitivity analysis [a]

Robustness/Alternative specification	Parameter change	Trade - RGDP slope	
		Input	*Final*
1. Baseline	-	.056***	.006***
A. Model parameter & trade frictions			
2. High Frisch elasticity	$\nu = .25$	.064***	.010***
3. Low Frisch elasticity	$\nu = .75$	.052***	.004***
4. Iceberg costs	+10%	.056***	.006***
5. Fixed costs	+10%	.056***	.006***
6. No trade imbalance	$\mathcal{T}_i = 0,\ \forall i$	.056***	.006***
7. Low adjustment cost	$\psi = 1.0$	.061***	.009***
8. High adjustment cost	$\psi = 1.5$	.052***	.004***
9. Alternative CES elasticity	$\rho^F = 1.5$	.055***	$-.001$
B. Productivity process			
10. PWT-estimated TFP shocks	$\widetilde{\Sigma}$	.046***	.006***
11. Uncorrelated techno. shocks	$cov(Z_{i,t}, Z_{j,t}) = 0, \forall i \neq j$	.030***	.005***
C. Reference period for $\{\omega^I, \omega^F\}$			
12. Baseline	1990-2000	.084***	.015***
13. Baseline, no EM/markups	1990-2000	.007***	.009***
D. Asset Market			
14. Complete Markets	–	.055***	.010***
15. Complete Markets, no EM/markups	–	.005***	.005***

[a] The simulations are based on the exact same sequence of shocks, under the five variations of trade indexes used in the benchmark.