

# Labor Market Frictions and Entrepreneurship

Alexandre Gaillard\*      Sumudu Kankanamge†

February 15, 2017

## Abstract

We theoretically examine how downside risk insurance affects entrepreneurship decision. We develop a Bewley - Huggett - Aiyagari model with labor market friction incorporating entrepreneurship. In this context, entrepreneurs face the risk of losing part of their wealth, of being excluded from the financial market and falling in unemployment. We calibrate the model to match the observed transitions between and within occupations as well as the firm size distribution. We use the model to quantify the impact of relaxing entrepreneur's risk of bankrupt on the aggregate outcomes, prices and the pool of entrepreneurs.

**JEL Codes:** E24, E61, J21, J24

**Keywords:** Entrepreneurship, Downside Risk, Frictions, Heterogeneous Agents, Bankruptcy.

---

\*Toulouse School of Economics

†Toulouse School of Economics

# 1 Introduction

[TO BE DONE]

Our paper is organized as follows. We document in section 2 the stylized facts that lead our model specification. Section 3 describes the models used as well as the equilibrium conditions. We propose an extension of our benchmark model in section 4 to account for spillover effect in a model à la Mortensen and Pissarides (1994). We calibrate our benchmark model in section 5 and section 6 presents results. Section 7 runs policy experiments and section 8 concludes.

## 2 Model

### 2.1 Contribution

Our model contribution to the existing literature is twofold. First, We build a Bewley (1983) - Huggett (1993) - Aiyagari (1994) model with heterogenous agents in the spirit of Cagetti and De Nardi (2006) with labor market frictions incorporating three different types of occupations: workers, entrepreneurs and unemployed people. Donovan (2014) has also unemployed worker and entrepreneurs but he does not explicitly model the searching behavior of agents. Moreover, his model is build in order to identify the sources of capital misallocation. In our model instead, an entrepreneur can search on-the-business to find a job and a worker can search on-the-job to start a business. Unemployed people can do both at the same time. Visschers, Millan, and Kredler (2014) use a Mortensen and Pissarides (1994) framework including entrepreneurs to account for the observed transitions between occupations but their model does not incorporate wealth perspective. However, it appears crucial since small entrepreneurs are often financially constrained (Quadrini (2000)).

### 2.2 Demography, timing, endowments

There is a mass one of individuals in the economy. All of them live forever. There is neither entry nor exit in the model, so that the demography of the model is static.

Each period, an individual is in one of this three occupations ( $o$ ): unemployment ( $o_u$ ), entrepreneurship ( $o_e$ ) or worker ( $o_w$ ) with  $o \in \mathcal{O} \equiv \{o_u, o_e, o_w\}$ . Depending on the opportunities offer to him, the individual makes his occupational choice as described later. Each individual is endowed with an innate ability level  $\theta \in \Theta$ , a wealth level  $a \in \mathcal{A}$  and, except for the entrepreneur, a business idea  $\xi \in \Xi \equiv \{0, 1\}$ . An agent being in a worker situation also face an idiosyncratic shock over his productivity level  $y \in \mathcal{Y}$ . An entrepreneur is either in a good or in a bad state  $j \in \{bad, good\}$ . Innate ability is determined early in life, before the individual enter the model.

In order to reduce the size of the following value functions, we do not incorporate the innate ability since it does not evolve over time (for instance,  $EV(a', y', \xi)$  means  $EV(a', y', \theta, \xi)$ ).

### 2.3 Preferences and technologies

The preferences over consumption are described by a CRRA utility, which is the usual assumption in most of the quantitative macroeconomics literature. We also introduce disutility of efforts to

search for a job ( $s_w$ ) and to search for getting started the business idea ( $s_e$ ) such that

$$u(c, s_w, s_e) = \frac{c^{1-\sigma}}{1-\sigma} - s_w^{\psi_w} - s_e^{\psi_e}$$

Where  $c$  specifies the consumption of the household and  $\psi_w$  and  $\psi_e$  are the elasticity of the respective searching efforts.

As in Cagetti and De Nardi (2006), we adopt a two sectors economy: a corporate sector only populated by workers and an entrepreneurial sector only populated by entrepreneurs. The corporate sector displays a standard Cobb-Douglas technology with total factor productivity  $A$ , capital level  $K$  and labor employed  $L$ .

$$Y(K, L) = AK^\alpha L^{1-\alpha}$$

The entrepreneur's technology has decreasing return to scale with parameter  $0 < \nu < 1$ . The returns to entrepreneurship depend on the innate ability<sup>1</sup>  $\theta$  through a function  $g : \theta \mapsto \mathcal{R}$  and the entrepreneurial state  $j$  through  $a_j$ , such that

$$f(k, \theta, j) = a_j g(\theta) k^\nu$$

When getting an idea ( $\xi = 1$ ), a worker or an unemployed individual can choose how many effort to allocate in order to get the opportunity to open the business idea<sup>2</sup>. The probability of getting the opportunity of opening his business arrives at a poisson rate, function of the searching effort  $s_e$ . In the same way, an entrepreneur or an unemployed individual can choose how many effort to allocate in order to find a job opportunity. The probability of getting a job opportunity arrives at a poisson rate function of the searching intensity  $s_w$ . We have

$$\pi_e(s_e) = 1 - e^{-\kappa_e s_e}$$

$$\pi_w(s_w) = 1 - e^{-\kappa_w s_w}$$

Under our specification, we get seven types of different individuals in our economy: entrepreneurs, workers endowed with a business idea and those without idea as well as short-run and long-run unemployed individuals without and with a business idea.

---

<sup>1</sup>As we will see later, the worker's labor income also depends on his innate ability. Thus, there exists a correlation between the return to entrepreneurship and the worker's labor income.

<sup>2</sup>This effort describe the agency costs, the time needed to fill administrative forms, to respect product norms or to run the business until achieving its maturity.

## 2.4 Worker

A worker without idea receives a wage  $wy\theta$  depending on his innate ability level  $\theta$  and his idiosyncratic productivity level  $y$ . He has a probability  $\eta(\theta)$  to be laid off which depends on his innate ability and a probability  $\xi$  of receiving a business idea next period. A worker has to pay a tax  $\tau_w$  on his labor income. A worker without idea can not set up any business. Finally, the probability of switching from a productivity level  $y$  to another productivity level  $y'$  is given by a first-order markov chain  $\pi(y'|y)$ . We have

$$V^w(a, y, 0) = \max_{c, a'} \left\{ u(c, 0, 0) + \beta \left[ \xi \left( (1 - \eta(\theta)) EV^w(a', y', 1) + \eta(\theta) V^{us}(a', 1) \right) + (1 - \xi) \left( \eta(\theta) V^{us}(a', 0) + (1 - \eta(\theta)) EV^w(a', y', 0) \right) \right] \right\}$$

$$\text{Subject to: } c = (1 - \tau_w)wy\theta + (1 + r)a - a' \quad (1)$$

$$a' \geq 0 \quad c > 0 \quad (2)$$

When getting a business idea, a worker can choose how many effort  $s_e$  to allocate in order to run his business. He gets the opportunity with probability  $\pi_e(s_e)$  as described before. If the worker succeed, he chooses between remaining worker or starting his business. If his job is destroyed at the same time of receiving this opportunity (with probability  $\eta(\theta)\pi_e(s_e)$ ) he chooses between starting the business or falling in a short-run unemployment situation. A worker loses his business idea with probability  $\zeta$ .

When a worker chooses to start a business, he has to pay a fixed cost  $c$ . Moreover, he does not know the state of his future activity which can be either in a bad or in a good state. We assume that the probability of each state is given by the invariant distribution  $\hat{\Pi}_j$ . We have

$$V^w(a, y, 1) = \max_{c, a', s_e} \left\{ u(c, 0, s_e) + \beta \left[ (1 - \zeta) \left( (1 - \eta(\theta)) \left[ \pi_e(s_e) EV_{w,e}^w(a', y', 1) + (1 - \pi_e(s_e)) EV^w(a', y', 1) \right] + \eta(\theta) \left[ (1 - \pi_e(s_e)) V^{us}(a', 1) + \pi_e(s_e) V_{us,e}^w(a', 1) \right] \right) + \zeta \left( \eta(\theta) V^{us}(a', 0) + (1 - \eta(\theta)) EV^w(a', y', 0) \right) \right] \right\}$$

$$\text{Subject to: } (1), (2) \quad (3)$$

$$s_e \geq 0$$

$$V_{w,e}^w(a, y, 1) = \max \{ V^w(a, y, 1), E_j V^e(a - c_e, j) \} \quad (4)$$

$$V_{us,e}^w(a, 1) = \max \{ V^{us}(a, 1), E_j V^e(a - c_e, j) \} \quad (5)$$

Finally, the optimal searching effort  $s_e$  to start his business idea is given by the solution of

$$\psi_e(s_e^*)^{\psi_e} = \beta \kappa_e e^{-\kappa_e s_e} \left( (1 - \eta(\theta)) EV_{w,e}^w(a', y', 1) + \eta(\theta) V_{u,s,e}^w(a', 1) \right) \quad (6)$$

## 2.5 Unemployed worker

An unemployed worker can be either short-run or long-run unemployed. As the worker, an unemployed individual is endowed with an innate ability  $\theta$  and has a business idea or not. We assume that a short-run unemployed worker fall in long-run unemployment with probability  $p_l$ . A short-run unemployed worker ( $\epsilon = s$ ) receive an unemployment insurance (UI) proportional to his innate ability level  $w\theta$  with replacement rate  $z$  whereas the long-run unemployed worker ( $\epsilon = l$ ) gets a fixed amount  $m$  such that  $m \leq wz\theta_0$ , that is,  $m$  is lower than the lowest UI given to a short-run unemployed worker in the economy<sup>3</sup>. In both situation, he can search for a job with effort intensity  $s_w$  and he has a probability to find equal to  $\pi_w(s_w)$ . When finding a job, we assume that the future worker's productivity shock level  $y$  is drawn from the invariant probability distribution  $\hat{\Pi}_y$ .

$$V^{u_\epsilon}(a, 0) = \max_{c, a', s_w} \left\{ u(c, s_w, 0) + \beta \left[ \xi \left( (1 - \pi_w(s_w)) V_{u'_\epsilon}^{u_\epsilon}(a', 1) + \pi_w(s_w) E_{y'} V^w(a', y', 1) \right) + (1 - \xi) \left( (1 - \pi_w(s_w)) V_{u'_\epsilon}^{u_\epsilon}(a', 0) + \pi_w(s_w) E_{y'} V^w(a', y', 0) \right) \right] \right\}$$

$$\text{Subject to: } (\epsilon = s) \quad c = (1 - \tau_w) w \theta z + (1 + r) a - a' \quad (7)$$

$$(\epsilon = l) \quad c = m + (1 + r) a - a' \quad (8)$$

$$a' \geq 0 \quad (9)$$

$$s_w \geq 0 \quad (10)$$

$$(\epsilon = s) \quad V_{u'_\epsilon}^{u_s}(a, \xi) = p_l V^{u_l}(a, \xi) + (1 - p_l) V^{u_l}(a, \xi) \quad (11)$$

$$(\epsilon = l) \quad V_{u'_\epsilon}^{u_l}(a, \xi) = V^{u_l}(a, \xi) \quad (12)$$

An unemployed worker with a business idea chooses how many effort  $s_e$  he wants to allocate in order to get the opportunity to open his business which arrived at a probability equals to  $\pi_e(s_e)$ , as defined above. When receiving this opportunity, an unemployed individual can choose either to remain in unemployment or to start a business by paying a cost  $c$ . An unemployed worker does not know the future business state (good or bad), that he can create, which is drawn according to the invariant probability distribution  $\hat{\Pi}_j$  and therefore makes his decision according to the expected

---

<sup>3</sup>This is mainly to ensure that an individual can always consume and that long-run unemployment is more costly than short-run unemployment.

future value of being an entrepreneur. He also has a probability  $\zeta$  to lose his idea. We have

$$V^{u_e}(a, 1) = \max_{c, a', s_w, s_e} \left\{ u(c, s_w, s_e) + \beta \left[ (1 - \zeta) \left( \pi_w(s_w) [(1 - \pi_e(s_e)) E_{y'} V^w(a', y', 1) + \pi_e(s_e) V_{w,e}^{u_e}(a', 1)] \right. \right. \right. \\ \left. \left. + (1 - \pi_w(s_w)) [\pi_e(s_e) V_{e,u_e}^{u_e}(a', 1) + (1 - \pi_e(s_e)) V_{u_e}^{u_e}(a', 1)] \right) \right. \\ \left. \left. + \zeta \left( (1 - \pi_w(s_w)) V_{u_e}^{u_e}(a', 0) + \pi_w(s_w) E_{y'} V^w(a', y', 0) \right) \right] \right\}$$

$$\text{Subject to: } (7), (8), (9), (10), (11), (12) \quad (13)$$

$$s_e \geq 0$$

$$(\epsilon = s) \quad V_{e,u_e}^{u_s}(a, 1) = (1 - p_l) \max\{E_j V^e(a - c_e, j), V^{u_s}(a, 1)\} \quad (14)$$

$$+ p_l V^{u_l}(a, 1) \max\{E_j V^e(a - c_e, j), V^{u_l}(a, 1)\} \quad (15)$$

$$(\epsilon = l) \quad V_{e,u_e}^{u_l}(a, 1) = \max\{E_j V^e(a - c_e, j), V^{u_l}(a, 1)\} \quad (16)$$

$$V_{w,e}^{u_e}(a, 1) = \max\{E_y V^w(a, y, 1), E_j V^e(a - c_e, j)\} \quad (17)$$

Note that an unemployed worker can search at the same time for starting his business and for finding a job. The optimal solution of the searching effort intensity to start his business for a given effort  $s_w$  to find a job is given by the solution of the following equation, as computed in appendix 7.4.

$$1 - \frac{\psi_e(s_e^*)^{\psi_e}}{\beta \kappa_e(1 - \zeta) \mathcal{P}(s_w)} = \pi_e(s_e^*) \quad (18)$$

$$\mathcal{P}(s_w) = \pi(s_w)(V_{w,e}(a', 1) - E_{y'} V_w(a', y', 1)) + (1 - \pi(s_w))(V_{e,u_e}^{u_e}(a', 1) - V_{u_e}^{u_e}(a', 1))$$

## 2.6 Entrepreneur

An entrepreneur rises revenue from his business. His benefits are equal to  $f(k, j, \theta)$  as defined before and his cost is the interest paid over the amount that he borrowed equal to  $k - a$ . As in Buera, Kaboski, and Shin (2011), the entrepreneur is borrowing constrained by an external creditor such that the lender will accept to lend to him only a given amount such that he agrees on repaying his debt<sup>4</sup>. In this sense, we assume that the fraction of benefits that the entrepreneur can kept if he runs away from the contract with the creditor is equal to  $f^5$ .

An entrepreneur with state  $j$  has future stochastic returns given by  $j'$  through  $a_{j'}$ . The probability to switch from  $j$  to  $j'$  is given by a first-order markov chain  $\pi(j'|j)$ .

<sup>4</sup>We also tried to implement a Cagetti and De Nardi (2006) borrowing constraint type, such that the entrepreneur can borrow only if  $V^e > V^{u_l}$ . Supposing that the entrepreneur already refuse all job offer before deciding whether or not he wants to leave the contract with the creditor. However, this did not change the fundamental results of the calibration.

<sup>5</sup>In our specification, the higher is  $f$  the higher is the tightness of the borrowing constraint.

He can search a job on-the-business and find with probability  $\pi_w(s_w)$ <sup>6</sup>. In such case, he decides whether to remain entrepreneur or to switch to a worker situation. When he bankrupts with an exogenous probability  $\mu$ , he is assumed to fall in a long-run unemployment and can not get UI. We assume that the entrepreneur keep his business idea when he voluntary exits entrepreneurship. When the entrepreneur choose to be worker, his productivity level is assumed to be drawn from the invariant distribution  $\hat{\Pi}_y$ . Finally, he has to pay a corporate tax  $\tau_c$ . Of course, an entrepreneur does not repay the starting cost each period. We then have:

$$V^e(a, j) = \max_{c, a', s_w, k} \left\{ u(c, s_w, 0) + \beta \left[ (1 - \mu) \left( \pi_w(s_w) EV_{w,e}^e(a') + (1 - \pi_w(s_w)) EV_{u_l,e}^e(a', j') \right) + \mu \left( (1 - \pi_w(s_w)) V^{u_l}(a', 1) + \pi_w(s_w) E_y V^w(a', y', 0) \right) \right] \right\}$$

$$\text{Subject to: } c = (1 - \tau_c)(a_j g(\theta) k^\nu - (1 + r)(k - a) + (1 - \delta)k) - a' \quad (19)$$

$$a' \geq 0 \quad k \geq 0 \quad s_w \geq 0$$

$$(IC) \quad f[a_j g(\theta) k^\nu + (1 - \delta)k] - (1 + r)(k - a) \geq 0 \quad (20)$$

$$V_{w,e}^e(a, j) = \max\{E_y V^w(a, y, 1), V^e(a, j)\} \quad (21)$$

$$V_{u_l,e}^e(a, j) = \max\{V^{u_l}(a), V^e(a, j)\} \quad (22)$$

Where the optimal level of capital in the Buera, Kaboski, and Shin (2011) borrowing constraint type is defined as

$$k = \begin{cases} \text{solve (IC)} & \text{if } k \leq k^* \\ k = k^* & \text{if } k > k^* \end{cases} \quad k^* = \left( \frac{\nu g(y)}{r + \delta} \right)^{\frac{1}{1-\nu}} \quad (23)$$

The optimal searching effort  $s_w$  to find a job for an entrepreneur is given by

$$\psi_w(s_w^*)^{\psi_w} = \beta \kappa_w e^{-\kappa_w s_w} \left( (1 - \mu) EV_{w,e}^e(a', j') + E_y V^w(a', y', 0) \right) \quad (24)$$

## 2.7 Government

We assume that the only spending of the government is through the unemployment insurance offer to any short-run unemployed worker, such that

$$G = \sum_{\theta \in \Theta} \sum_{\xi \in \Xi} \int_{\underline{a}}^{\bar{a}} w \theta z \Gamma(u_s, \theta, \xi, da) \quad (25)$$

Where  $\Gamma(u_s, \theta, \xi, da)$  is the measure over short-run unemployed workers. For the following, let us denote  $\Gamma(s)$  the measure over individuals with occupation  $s$ .

---

<sup>6</sup>We allow the parameter  $\kappa_w$  to be different between an unemployed worker and an entrepreneur. This capture the fact that entrepreneurs could have more or less job opportunities than unemployed people.



Government rises two taxes. A corporate tax ( $\tau_c$ ) paid by all entrepreneurs and a proportional labor income tax ( $\tau_w$ ) paid by workers which is used to balance the government budget.

Government revenues are then given by

$$T = \sum_{\theta \in \Theta} \sum_{y \in \mathcal{Y}} \sum_{\xi \in \Xi} \int_{\underline{a}}^{\bar{a}} wy\tau_w \Gamma(\{w, u_s\}, y, \theta, \xi, da) \\ + \sum_{\theta \in \Theta} \sum_{j \in \mathcal{J}} \int_{\underline{a}}^{\bar{a}} \tau_c \left( a_j g(\theta) k^\nu - (1+r)(k-a) + (1-\delta)k \right) \Gamma(e, j, \theta, \xi, da) \quad (26)$$

Finally, we assume that government can issue debt in order to finance his budget. We then have the flow of government debt given by:

$$B_{t+1} = (1+r_t)B_t + T_t + G_t \quad (27)$$

## 2.8 Stationary Recursive Equilibrium

Defining the state vector  $\mathbf{x} = (s, y, \theta, j, a)$  with  $a \in A$ ,  $y \in \bar{Y}$ ,  $j \in \mathcal{J}$ ,  $\xi \in \Xi$ ,  $\theta \in \Theta$ ,  $s \in S$ , where  $s$  is the status of the household: entrepreneur, worker, short-run or long-run unemployed worker ( $S \hat{=} \{e, w, u_s, u_l\}$ ); a stationary recursive equilibrium in this economy consists of a set of value functions  $V_w(a, \theta, y, \xi)$ ,  $V_e(a, \theta, j)$ ,  $V_{u_s}(a, \theta, \xi)$ ,  $V_{u_l}(a, \theta, \xi)$ , decision rules with asset holding  $a'(\mathbf{x})$ , consumption  $c(\mathbf{x})$ , job search effort  $s_w(\mathbf{x})$ , business search effort  $s_e(\mathbf{x})$ , investment level  $k(\mathbf{x})$ , occupational choices  $s$ , prices ( $r, w \in \mathbb{R}$ ), tax parameter ( $\tau_w \in \mathbb{R}$ ) and a stationary measure over individuals  $\Gamma(\mathbf{x})$  such that

- the allocation choices  $c(\mathbf{x})$ ,  $a'(\mathbf{x})$ ,  $k(\mathbf{x})$ ,  $s_w(\mathbf{x})$  and  $s_e(\mathbf{x})$  maximize the household problem given the prices  $r$ ,  $w$  and the fixed tax parameter  $\tau_w$ .
- $\Gamma(\mathbf{x})$  is the stationary probability measure induced by  $a'(\mathbf{x})$  and the first-order Markov chains  $\Pi_y$ ,  $\Pi_j$  and  $\Pi_\xi$ .
- the asset and labor markets clear. So that capital used in the non-entrepreneurial sector is equal to the difference between total asset level in the economy minus total investment in the entrepreneurial sector and government debt.

$$K_c = \sum_{s \in S} \sum_{y \in \bar{Y}} \sum_{\xi \in \Xi} \sum_{\theta \in \Theta} \sum_{j \in \mathcal{J}} \int_{\underline{a}}^{\bar{a}} a(\mathbf{x}) \Gamma(s, da) - \sum_{s \in e} \sum_{\theta \in \Theta} \sum_{j \in \mathcal{J}} \int_{\underline{k}}^{\bar{k}} k(\mathbf{x}) \Gamma(e, dk) - B \quad (28)$$

For the labor market, total labor supply ( $L$ ) is given by

$$L = \sum_{s \in S} \sum_{y \in \bar{Y}} \sum_{\xi \in \Xi} \sum_{\theta \in \Theta} \int_{\underline{a}}^{\bar{a}} y \Gamma(w, da) \quad (29)$$

- the wage and the interest rate are equal to the marginal products of each factor of production. No arbitrage condition implies that the rate of return in the non-entrepreneurial sector must equate the risk-free rate that equates savings and investment in the entrepreneurial sector.
- the government budget constraint is balanced, so that at the steady state the government has to finance interest on debt plus government spendings.

$$T = G + rB \quad (30)$$

- The distribution  $\Gamma$  is time-invariant. The law of motion for the distribution of agents over the state space  $\mathbf{x}$  satisfies

$$\Gamma = R_\Gamma(\Gamma) \quad (31)$$

Where  $R_\Gamma$  is a one-period transition operator on the distribution  $\Gamma$  such that  $\Gamma_{t+1} = R_\Gamma(\Gamma_t)$ .

In addition, we define the measure of the total gross domestic product ( $\Upsilon$ ) in this economy as the sum of the outputs in the entrepreneurial and the non-entrepreneurial sectors.

$$\Upsilon = AK_c^\alpha L_c^{1-\alpha} + \sum_{\theta \in \Theta} \sum_{j \in \mathcal{J}} \int_{\underline{k}}^{\bar{k}} a_j g(\theta) k(\mathbf{x})^\nu \Gamma(e, dk) \quad (32)$$

The algorithm used to solve the model is described in the appendix ??.

### 3 Calibration

In this section, we describe the calibration strategy that we use. We apply a mix of external calibration combining existing parameter estimates and normalizations and internal parameter minimizing the distance between equilibrium moments and their data counterparts.

#### 3.1 Fixed parameters

Some fixed parameters are set to the observed data (e.g. labor share in the corporate sector, corporate tax rate), normalized to restrict the number of estimated parameters in the model (e.g. the probability to get an idea and to lose an idea, the total factor productivity and the domestic production benefit) or taken from their estimates in the literature (e.g. the depreciation rate, the replacement rate the intertemporal elasticity).

As a model is a year, we set the probability of falling in a long-run unemployment situation  $p_l$  to one, which corresponds approximately to the US unemployment benefit policy before 2014.

**Labor productivity process** In our model, an individual's labor productivity depends on two components: a deterministic innate ability level  $\theta$  and an idiosyncratic shock  $y$ . The natural logarithm of wages of a worker is given by

$$\log(w_t) + \log(\theta) + \log(y) \tag{1}$$

We consider three innate ability types  $\{\theta_1, \theta_2, \theta_3\}$  with population mass equal to the mass of people without diploma (<HS)  $p_1 = 0.275$ , with HS degree only  $p_2 = 0.45$  and with at least college level (complete or not)  $p_3 = 0.275$ . The ability values are chosen so that  $E(\log(\theta)) = 0$  meaning that  $\theta_1 = e^{-\sigma_\theta}$ ,  $\theta_2 = 1.0$  and  $\theta_3 = e^{\sigma_\theta}$  with  $Var(\log(\theta)) = \sigma_\theta$ .

We specify the stochastic process for the idiosyncratic part of log-wages as a discretized version, with five states, of a simple AR(1) process with persistence  $\rho$  and unconditional variance  $\sigma_y^2$ . This leaves us three parameters ( $\sigma_\theta^2, \rho, \sigma_y^2$ ) that we calibration according to the CPS data. We use three data moments which measure the cross-sectional labor income dispersion: (i). the average earning dispersion accross education level for a given age (of about 0.0718), (ii). the average earning dispersion along the life for given education level<sup>7</sup> (around 0.0165), (iii). the almost linear earning process along the life (with a persistency of 0.96 as it is common in the literature). Figure 11 in appendix displays the average earning profil at different educational attainment and different age.

---

<sup>7</sup>We will use PSID data in future calibration since CPS data is not the best dataset that should be use to estimate the dispersion of the earning process along the life. Indeed, CPS does not follow individuals over time whereas PSID does.

**Layoff rate** We calibrate the layoff process  $\eta(\theta)$  according to the CPS transition between worker to unemployed worker. The relationship is roughly linear and we run a simple linear regression to estimate the probability to be laid off, such that

$$\eta(\theta) = \alpha_\eta + \beta_\eta \theta \quad (2)$$

Table 1 summarizes the set of fixed parameters used.

Table 1: Fixed Parameters: benchmark model

Parameter		Value	Source
Corportation tax rate	$\tau_c$	0.134	OECD
Return to scale (corp.)	$\alpha$	0.36	Zeileis (2015)
Inter. Elasticity	$\sigma$	1.39	Gandelman and Hernandez-Murillo (2015)
Depreciation rate	$\delta$	0.06	Cagetti and De Nardi (2006)
Replacement rate	$z$	0.4	Shimer (2005)
UI duration	$p_l$	1.0	own calculation
TFP	$A$	1.0	Normalization
Entrepreneurship cost of entry	$c_e$	0.0	Normalization
Domestic production	$m$	0.05	Normalization
Getting an entrepreneurial idea	$\xi$	1.0	Normalization
Losing an entrepreneurial idea	$\zeta$	0.0	Normalization
Layoff rate: intercept	$\alpha_\eta$	0.05	own calculation
Layoff rate: slope	$\beta_\eta$	-0.015	own calculation

### 3.2 Estimated parameters

**Entrepreneur' states** We divide the business state into two values whose one is normalized to one (e.g.  $a_{good} = 1.0$ ). This leaves us only  $a_{bad}$  to estimate. We also assume that the probability to switch from  $a_j$  to  $a_{j'}$ , given by  $\pi(j|j')$ , is symmetric.

**Job finding rate.** We allow to differentiate between the probability of findind a job when the is entrepreneur or unemployed worker. This allows us to capture the fact that individuals in entrepreneurial situation are more likely to find a job than unemployed workers. The job finding rate for an entrepreneur is then given by  $\kappa_w^e$ . For the unemployed workers, we use a parametric form which depends on the educational attainment to compute the rate  $\kappa_w$ . In this sense, we use a linear relationship (which fits approximately the transition from unemployed worker to a worker

situation so that

$$\kappa_w(\theta) = \alpha_w + \beta_w \theta \quad (3)$$

Our calibration strategy leaves us 15 parameters to pin down 16 moments relative to the transitions between and within occupation, wealth and income distribution, unemployment rate, fraction of entrepreneurs, capital-output ratio. Results of the internal calibration are given in table 2.

Table 2: Estimated Parameters

Parameter		Value
Discount rate	$\beta$	0.869
Job search effort elasticity	$\psi_w$	1.854
Business search effort elasticity	$\psi_e$	2.064
Business idea matching par.	$\kappa_e$	0.56
Job matching par. (when E)	$\kappa_w^e$	6.49
Job matching par. (when U)	$\alpha_w$	1.2
Job matching par. (when U)	$\beta_w$	0.2
Bankruptcy rate	$\mu$	0.025
Borrowing constraint par.	$f$	0.847
Entrepreneur's return to scale	$\nu$	0.86
Entrepreneurial ability	$g(\theta)$	[0.481, 0.527, 0.715]
Entrepreneurial state	$[a_{bad} \quad a_{good}]$	[0.415 1.0]
Probability to switch	$\pi_{a_j, a_{j'}}$	0.185

### 3.3 Calibration targets and generated moments

Table 3 display some of the targets of our calibration.

Table 3: Observed and generated moments

Moment	Model	Value (U.S.)	Source
K/Y Ratio*	3.12	3.04	Zeileis (2015)
Entrepreneurs*	10.9%	10.9%	Own calculation
Ratio of med. worth E/W	8.73	8	Cagetti and De Nardi (2006)
Labor Market			
Unemployment rate*	5.32%	4 - 6 %	Bank (2016)
Worker exit rate*	5.83%	3 - 6.5%	Shimer (2012) and Rissman (2007a)
Unemployment exit rate*	56%	40 - 65 %	Ibid.

Entrepreneur exit rate*	24.5%	24 - 25 %	Own calculation
Necessity share*	12.4%	20%	Fairlie (2015)
Wealth and Income distributions			
1%wealth*	22	30	Cagetti and De Nardi (2006).
5% wealth	52	54	Ibid.
10% wealth	68	67	Ibid.
Wealth Gini*	0.76	0.8	Ibid.
Income Gini*	0.39	0.36	Kenworthy and Smeeding (2013)

(\*) Targeted.

Using the CPS data from 1994 to 2014, we compute an average fraction of entrepreneurs in the US economy of 10.9% which is in the range of the existing literature. For instance, Quadrini (2000) finds 12% and Gentry et al. (2004) report a fraction of entrepreneurs (households who reported owning active business assets without restriction) of 11.5% using various definition using SCF in 1989.

For the exit rate of entrepreneurs, we estimate it using the CPS data as well. We find a value between 22% to 25% depending on the time period considered. This is close to the finding of Cagetti and De Nardi (2006) and Rissman (2007b).

Transitions by productivity level within transition from worker to entrepreneur (W - E) and from entrepreneur to worker (E - W) occupations are also targeted (we show the resulting transitions in the next section). As described later, low-educated and high-educated individuals are less likely to come back to a worker situation when they are in business. We thus target transition probabilities by educational attainment in (E - W) match respectively 0.22, 0.24 and 0.22 depending if the individual has an education level lower than high-school, a high-school degree or has attended to college. We do the same for the transition (W - E) and we match respectively 0.03, 0.022 and 0.03 according to the same educational attainment groups. Using this, we indirectly infer the entrepreneurial abilities using these transitions.

Results show that our overidentified simulated method of moments provide results close to the observed data. The flow of individuals between occupation is either in the considered range or a little bit outside. We also match the mass of people in each occupation as well as the wealth and the income distributions. The top 1% is a bit far from the observed value but the number of entrepreneurs in this bracket is relatively small and does not change our fundamental results. We obtain an interest rate of 5.5% and a tax rate parameter  $\tau_w$  equal to 1.1%.

## 4 Results: benchmark economy

In this section we compare our model outcomes to the data. We use three sources of data, First, we compute the transition rates between and within occupations as well as the share of entrepreneurs by educational attainment using the Current Population Survey (CPS) from 1994 to 2014. We average all the transitions over the time periods considered so that our steady state values are comparable to the true data. Second, we use the Survey of Consumer Finance (SCF) and the Survey of Business Owners (SBO) to compare our model distributions (firm size, wealth and income) to the actual one.

### 4.1 Entrepreneurship and educational attainment

As seen before, the model generates a fraction of entrepreneurs (10.5%) close to the value of the data (10.9%). By looking at the rate of entrepreneurship by educational attainment (table 4), our benchmark model is able to reproduce the observed U-shaped curve between education level (or innate ability in our context) and the fraction of entrepreneurs within the group. For instance, 9.69% of the individuals with a high-school education level or less are entrepreneurs. As mentioned in Poschke (2013), this fact is observable in most of developed countries and seems to be consistent regardless of the entrepreneur definition used.

Table 4: Entrepreneurship rates by education category

	data source	E	Educational attainment				
			<HS	HS	<C	C	>C
Own calculations	U.S., 1994 - 2014 CPS	-		6.73	6.4	8.71	9.78
Hamilton (2000)	U.S., 1984 SIPP	-		12.6	11.1	12.6	15
Hipple (2004) (unincorporated)	U.S., 2003 CPS	-	9.1	8.7	8.2	7.5	9.1
Hipple (2004) (incorporated)	U.S., 2003 CPS	-	2.1	3.1	3.9	5.5	6.6
Lin, Picot and Compton (2000)	Canada, 1994	18.4	13.5	11.4	10.1	11.1	13.2
Schjerning and Le Maire (2007)	Denmark, 1980-96	10.9	-	10.9	7.4	3.6	12.9
Borjas and Bronars (1989)	U.S., 1980 Census	-		4.8	4.2	4.6	6.5
Benchmark	-	10.71	-	11.02	10.04	11.75	-

Source: Poschke (2013) and own calculations. The definition of an entrepreneur is someone who is self-employed in the sense of Cagetti and De Nardi (2006). That is, someone who does not hold always his business but actively manage it. Self-employed can also have employees, so that the term entrepreneur seems more adapted.

From an empirical side, this U-shaped relationship seems also to be pretty robust. For instance, using 19 OECD countries, Blanchflower (2000) find that even if we control for many demographics covariates (age, education, gender, household size, specific country unemployment rate, the number

of children as well as their age and their educational attainment) this relationship persists in discrete choice models. Schjerning and Le Maire (2007), using Danish data with fine education level and a number of controls, also find the statistical significance of this U-shaped curve.

## 4.2 Transition between and within occupations

### 4.2.1 Transition between occupations.

Using again the CPS data, averaged from 1994 to 2014, we compute the year-to-year transition probability between our three occupations (worker (W), entrepreneur (E) and unemployed worker (U)) in table 5 and we report our model results. Regarding our calibration strategy, our model does a pretty good job in reproducing the transitions between occupations.

Table 5: Year to year transition matrix between occupations

	W	E	U
W	0.941 (0.9417)	0.027 (0.0263)	0.032 (0.0320)
E	0.758 (0.7559)	0.226 (0.2107)	0.016 (0.0335)
U	0.562 (0.5039)	0.060 (0.0584)	0.377 (0.4376)

Source: CPS data and own calculations.

Benchmark model results are in parenthesis.

As compared to the existing literature, our model does a better job than Rissman (2007b) who was not able to reproduce well the E - W transition. Our model fits well this transition since a fraction of existing entrepreneurs, who failed in a bad state, search a job on-the-business in order to get an higher labor income than their current return to entrepreneurship.

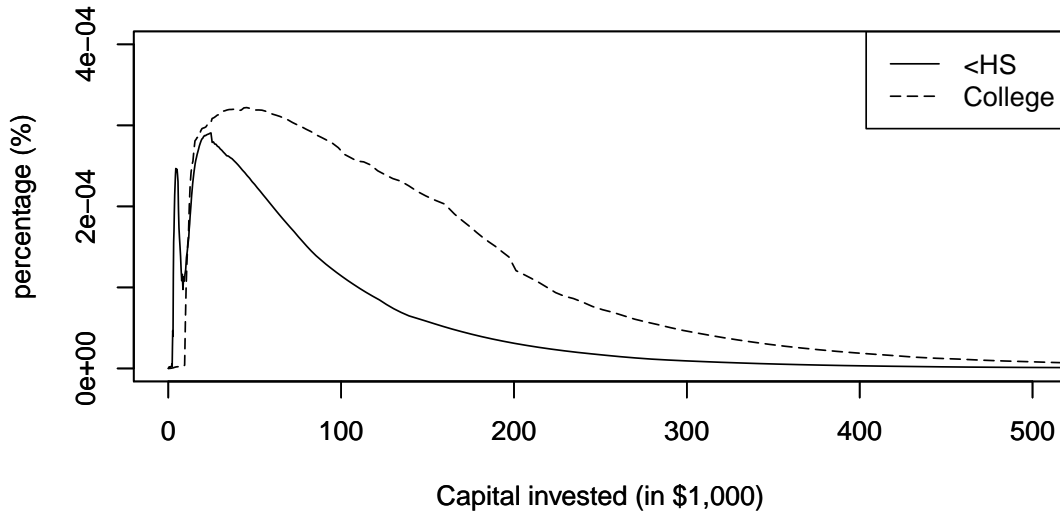
### 4.2.2 Transition within occupations by productivity level.

The model also reproduces the transition by productivity level for a given transition between occupations. To our knowledge, we are the first reporting the transition probability within occupations and therefore the first to create a model able to match them. A common issue in the literature with entrepreneurship is the absence of clear proxy in order to identify the link between the entrepreneurial ability and the individual's characteristics. In our calibration strategy, we indirectly estimate how the entrepreneurial ability is mapped from educational attainment using the transition W - E and E - W by productivity level. Figure 9 reports the transition probabilities between occupations for each productivity level. From the data, we use three different proxies: educational attainment (or year of schooling), wage level (which is only available when the individual is previously a worker) and the family income.



**W - E transition.** From the data, a U-shaped curve is found to be quite robust regardless on the proxy used within the transition W - E. A feature that our model is able to replicate for two reasons. First, high-qualified workers tend to be on average richer than the other categories. Therefore, they can accumulate more wealth (and thus more collateralized) than middle or low-educated individuals and are in average less financially constrained by the external creditor. They can run larger firm with higher returns to entrepreneurship. At the opposite, low-educated individuals can borrow less as entrepreneur but their labor income is also lower than other categories. Though, the wealth threshold at which they switch to entrepreneurship is below the one of the two other education categories in the model. Hence, low-educated and high-educated people are more likely to switch to entrepreneurship than the category at the middle, but as shown in figure 1, low-educated workers tend to open relatively small business as compared to high-educated workers<sup>8</sup>.

Figure 1: Firm size



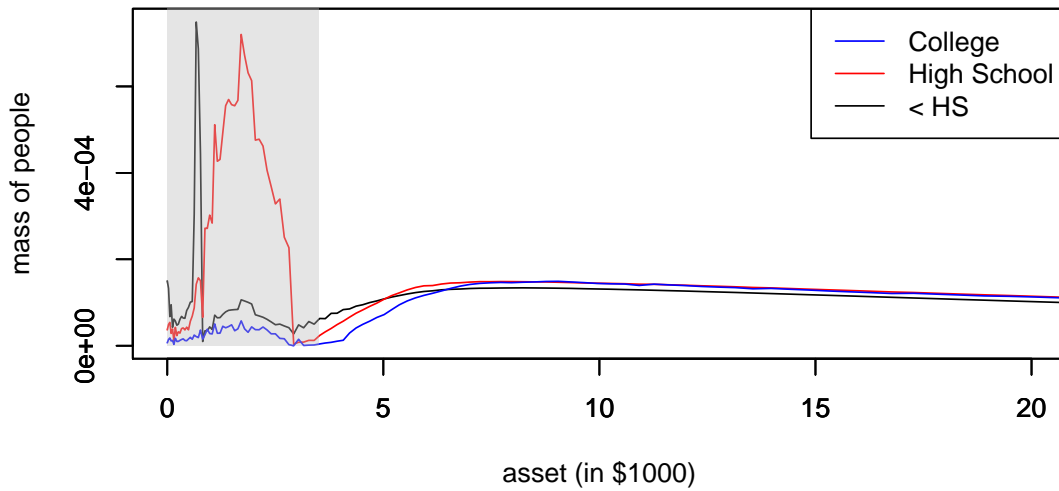
The U-shaped curve observed in the worker to entrepreneur transition is also found by Campbell and De Nardi (2009) using the Panel Study of Entrepreneur Dynamics (PSED) using year of schooling as proxy for productivity level. In appendix 7.3, we estimate this relationship using discrete choice model controlling for age, education, gender, marital status, unemployment situation and time fixed effect. Our statistical results are significant.

**E - W transition.** The inverted U-shaped curve within the E - W transition is also matched by the model. This is the case since low-educated and high-educated entrepreneurs find more profitable to stay at business than switching to a worker situation, at least when the business state is favorable. This point is obviously related to the previous one. However, since middle-educated

<sup>8</sup>Previous models are unlikely able to generate this feature since either they introduce only one of these dimensions as in Visschers, Millan, and Kredler (2014) who introduces only abilities.

workers would want to switch more often to entrepreneurship due to higher return at work, we should expect that only those with large returns to entrepreneurship would want to start their business. However, most of the middle-educated entrepreneurs who search for a job are those who enter entrepreneurship out-of-necessity when they were unemployed worker due to a lack of job opportunity. Figure 2 shows the distribution of entrepreneurs switching to a worker situation by educational attainment. Entrepreneurs located at the left hand side (in the grey polygon) are necessity entrepreneurs in the sense that choose to become entrepreneur during an unemployment situation while their return at work was greater<sup>9</sup> (e.g. they did not find a job opportunity).

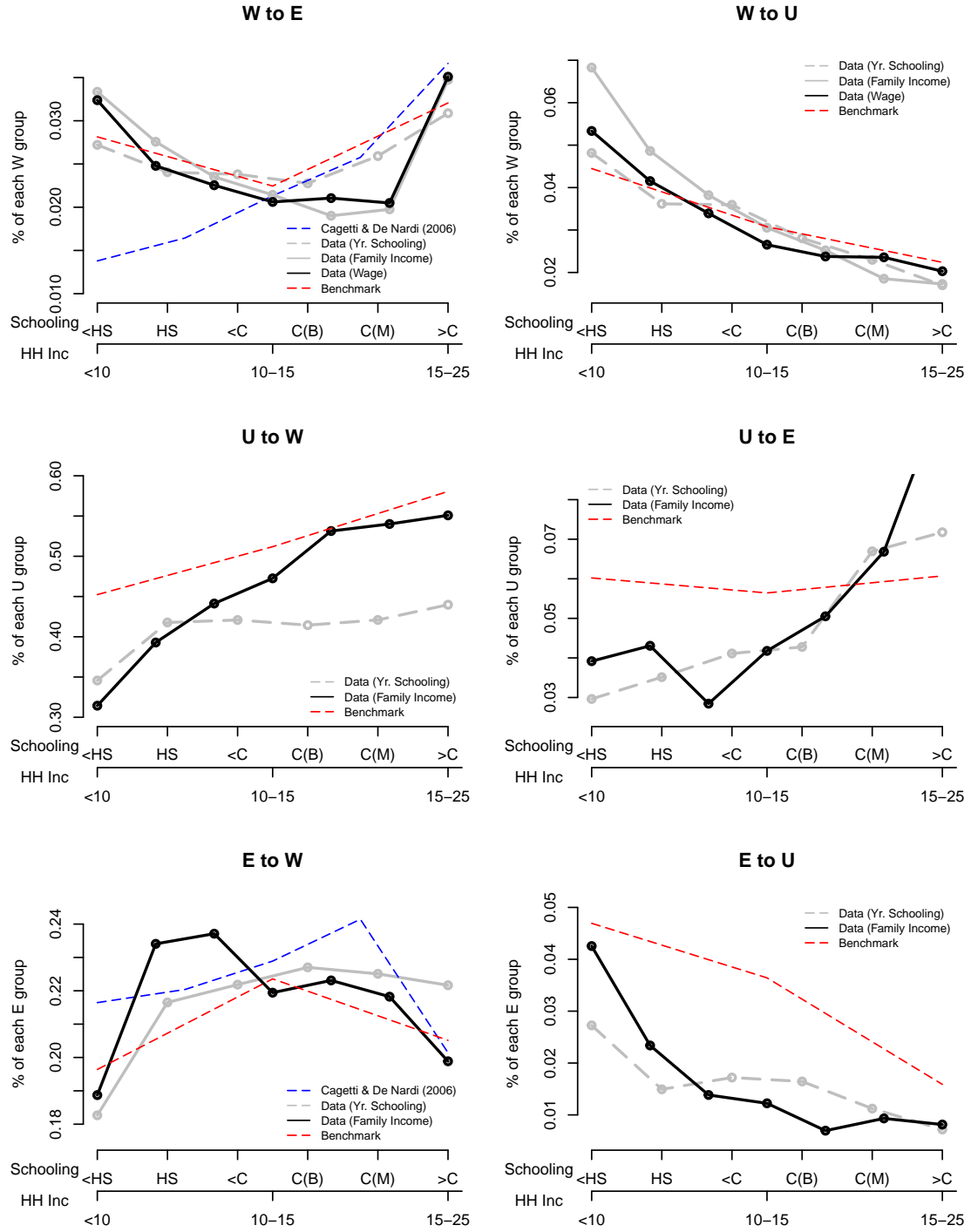
Figure 2: Transition E - W relative to the mass of each education category in entrepreneurship.



**Other within transitions.** The U - W and the W - E transitions are targetted with our calibration specification in the fixed part and are therefore quite close to the true data. The key thing, which is largely known, is that layoff probability is decreasing in education and whereas the probability of finding a job is increasing with education. Transition U - E is a bit overstated for low-educated unemployed worker in the model as compared to the data. This is because some entrepreneurship cost of entry exist in reality and affect more low-collateralized individuals. We add this cost later in a sensitive analysis, to better fit this transition.

<sup>9</sup>Formally, they choose to be entrepreneur such that  $V^w > V^e > V^u$ .

Figure 3: Transition probability by productivity level



Source: transitions are computed using averaged transition between occupation over twenty years from 1994 to 2014. Similar patterns can be found for quarterly and monthly data. We report the transitions generated by a Cagetti and De Nardi (2006) model which does not account for correlation between returns to entrepreneurship and labor income.

### 4.3 Wealth, income and firm size distributions

It is well known now that the upper tail of the wealth distribution can be well replicated using entrepreneur as in Quadrini (2000) and Cagetti and De Nardi (2006). The model generates a considerable degree of inequality that matches the data pretty well. Table 6 displays statistics concerning wealth inequalities and entrepreneurship.

Table 6: Entrepreneurship and wealth distribution

top	US data		Benchmark model	
	% of wealth held	% of entrep.	% of wealth held	% of entrep.
1%	30	62	22	49
5%	54	47	52	47
10%	67	38	69	46
20%	81	26	84	42
100%	100	10.9	100	10.7

Source: US data are from Cagetti and De Nardi (2006).

The income distribution is also matched to the true data. Table ?? reveals that the upper tail is quite well reproduced in the model. In fact, this is due to two things: first the presence of entrepreneurs generate substantial high income. The two dimensions of income heterogeneity  $\theta$  and  $y$  contribute to generate volatility in income and therefore help in generate a correct income distribution.

[NEED TO COMPUTE INCOME DISTRIBUTION]

Finally, let us point out the firm size distribution in the model. There are substantial differences between education category as well as considering the amount of capital invested in the firms. Table 7 describes the entrepreneurial activities by ability level. As said before, we fit the U-shaped that is found in the data when we look at the rate of entrepreneurship by education level. Three features contribute to the model's capability to generate such nondegenerate distribution of the firms' size. First, entrepreneurs are borrowing constraint depending on their amount of collaterals (e.g. their wealth level). Second, the decreasing returns to scale technology in the entrepreneurial sector, which endogenously determined the optimal firms' size. Lastly, the education level of the entrepreneur which contributes to generate cross-sectional differences between entrepreneur with the same level of collaterals.

Table 7: Benchmark model: entrepreneurial activities by education level

$\theta$	$\theta$	%	%	%	avg. invest.	avg. assets	avg. lev. ratio
grid	value	in pop.	in educ. grp	in entrep.	in \$1,000	in \$1,000	%
$\theta_1$	0.481	2.79	11.15	25.59	345	233	44.8
$\theta_2$	0.527	5.11	10.22	46.91	481	298	48.3
$\theta_3$	0.715	3	11.99	27.5	769	341	58.5
total	-	10.7	-	100	-	-	-
average	-	-	-	-	53	29	50.2

Source: US data are from Cagetti and De Nardi (2006).

## 5 Policy Experiments

In this section, we conduct two sets of policy experiments to study the incidence of downside risk insurance in our model economy. In the first one, we study the effect of introducing an unemployment insurance (UI) to unemployed workers who start a business. In the case where their returns to entrepreneurship is lower than what they could get by staying unemployed, the government offers them a complement to equalize the returns to entrepreneur to the UI benefits until they lose their right. From an empirical side, this policy was introduced in France in 2002 and its effect is studied in Hombert et al. (2014). We compare the general equilibrium effects in our model to their results. The two reforms are revenue neutral, in the sense that the government expenditures is fixed to be the same as in the benchmark model (that is, 0 except for the UI).

For each experiment, we conduct two sets of analysis. Firstly, we compute and focus on the steady state implied by the reform. Secondly, we compute the transition dynamics. We assume that period 0 is characterized by the steady state of our benchmark economy. In period 1, the reform is implemented while being not anticipated by any agent in the economy. Then, the economy converges to the new steady state. Throughout the transition, we allow for two government budget balance policies. First, the reform is financed using the proportional labor income tax  $\tau_w$  and then we use government debt. The computation method of the transition path is described in 7.2.

In order to compare the welfare effect of different reforms, we compute the consumption equivalent variation (CEV) and a pure utilitarian comparison. The former measures the constant increment in percentage of consumption in every state that has to be given to each agent so that he is indifferent between remaining in the benchmark economy and moving to another economy that makes a transition to a new steady state implied by the reform. The latter measures the difference in utility that affects the individuals in the economy.

### 5.1 Self-Employment Assistance (SEA) program

We implement a large Self-Employment Assistance (SEA) program in the spirit of the policy introduced in France in 2002. In order to do it, we compute a new value function for an entrepreneur entering in the downside risk insurance program. When he bankrupts, this entrepreneur falls in a short-run unemployed worker situation. When he does not bankrupt, the government provides him a complement if his current returns to entrepreneurship is lower than what he could get as an unemployed worker (i.e. his UI benefits). Such insurance is temporary and the entrepreneur has a probability  $p_l$  to fall in a normal entrepreneurial regime. We thus have

$$V^{eUI}(a, j) = \max_{c, a', s_w, k} \left\{ u(c, s_w, 0) + \beta \left[ (1 - \mu) \left( \pi_w(s_w) EV_{w, e'}^e(a', j') + (1 - \pi_w(s_w)) EV_{u, e'}^e(a', j') \right) + \mu \left( (1 - \pi_w(s_w)) V^{u_s}(a', 1) + \pi_w(s_w) E_y V^w(a', y', 0) \right) \right] \right\}$$

Subject to: (22)

$$c = \max\{(1 - \tau_c)(a_j g(\theta) k^\nu - (1 + r)(k - a) + (1 - \delta)k), w\theta z\} - a'$$

$$a' \geq 0 \quad k \geq 0 \quad s_w \geq 0$$

$$(IC) \quad f[a_j g(\theta) k^\nu + (1 - \delta)k] - (1 + r)(k - a) \geq 0$$

$$V_{w, e'}^e(a, j) = p_l \max\{E_y V^w(a, y, 1), V^e(a, j)\} + (1 - p_l) \max\{E_y V^w(a, y, 1), V^{eUI}(a, j)\}$$

$$V_{u, e'}^e(a, j) = p_l \max\{V^e(a, j), V^{u_l}(a)\} + (1 - p_l) \max\{V^{eUI}(a, j), V^{u_s}(a)\}$$

And the optimal searching effort  $s_w$  to find a job for such entrepreneur is given by

$$\psi_w(s_w^*)^{\psi_w} = \beta \kappa_w e^{-\kappa_w s_w} \left( (1 - \mu) EV_{w, e'}^e(a', j') + E_y V^w(a', y', 0) \right)$$

**Steady states.** Results from the steady states are summarized in table 8. We do not distinguish between debt or tax adjustment since the results are very close. In the two cases, a self-employment assistance program (SEA) that insure only unemployed workers in their entrepreneurial activity has relatively small effects on macroeconomic variables. Production and capital in both sectors are a little bit higher than before the reform but this is neglectable. In fact, this is because the reform impacts only a small fraction of the population (unemployed workers who wants to start a business). From an utilitarian welfare perspective, the change is also quite small in average.

Table 8: Large scale Self-Employment Assistance (SEA) program

	Benchmark	With SEA	Variation
Fraction of entrep. (%)	10.897	10.912	0.138%
Unemployment rate (%)	5.316	5.315	< 0.05%
Fraction of workers (%)	83.787	83.773	< 0.05%
Avg Welfare (tot.)	-18.025	-18.034	< 0.05%
Avg Welfare (Work.)	-18.699	-18.698	< 0.05%
Avg Welfare (Ent.)	-12.38	-12.385	< 0.05%
Avg Welfare (Unemploy.)	-21.390	-21.395	< 0.05%
Total Production	2.645	2.647	0.08%
Entrep. Production	1.101	1.102	0.09%
Corp. Capital	4.536	4.538	< 0.05%
Entrep. Capital	3.714	3.717	0.08%

As displayed in figure 4, this policy encourages unemployed worker to search for a business idea and therefore affect the incentive to search for a job. Now, unemployed workers decrease their search intensity for a job and increase their search for an idea. The result is an increase by 0.138% of the total fraction of entrepreneurs in the economy but almost no change in the unemployment rate level. This is because by reducing their searching intensity for a job, an unemployed worker is less likely to find a job. Thus, all reduction in unemployment due to the policy is almost offset by the increased discentive to search for a job.

Figure 4: Search effort intensity for a business (left), for a job (right): short-run unemployed worker

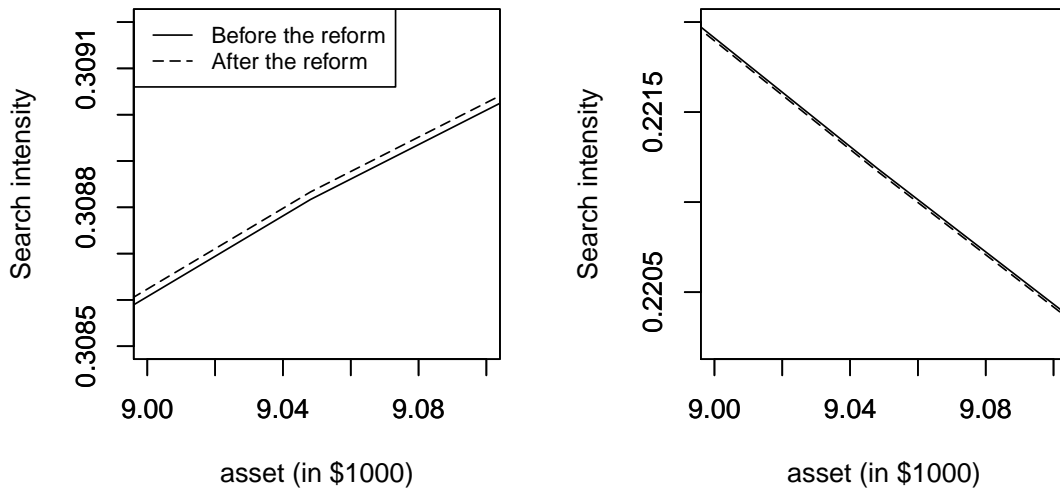


Table 9 reports the change in the pool of entrepreneur after the reform implementation. The increase in entrepreneurship rate is uniform among all education groups. This is consistent with the finding of Hombert et al. (2014) in France. The average firm size and asset level of the entrepreneurs increase after the reform as compared to before. This suggests that the reform encourages unemployed workers with relatively high level of asset to enter entrepreneurship.

Table 9: Downside risk insurance in a Self-Employment Assistance (SEA) program

	benchmark			with SEA (tax adj.)		
	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_1$	$\theta_2$	$\theta_3$
% of entrep.	11.157	10.219	11.978	11.169	10.231	11.992
Var. (rlv to bench.)	-	-	-	0.11%	0.12%	0.12%
Avg.firm	267.21	339.65	410.65	267.27	339.67	410.69
Avg.asset	233.37	297.66	341.29	233.42	297.69	341.33

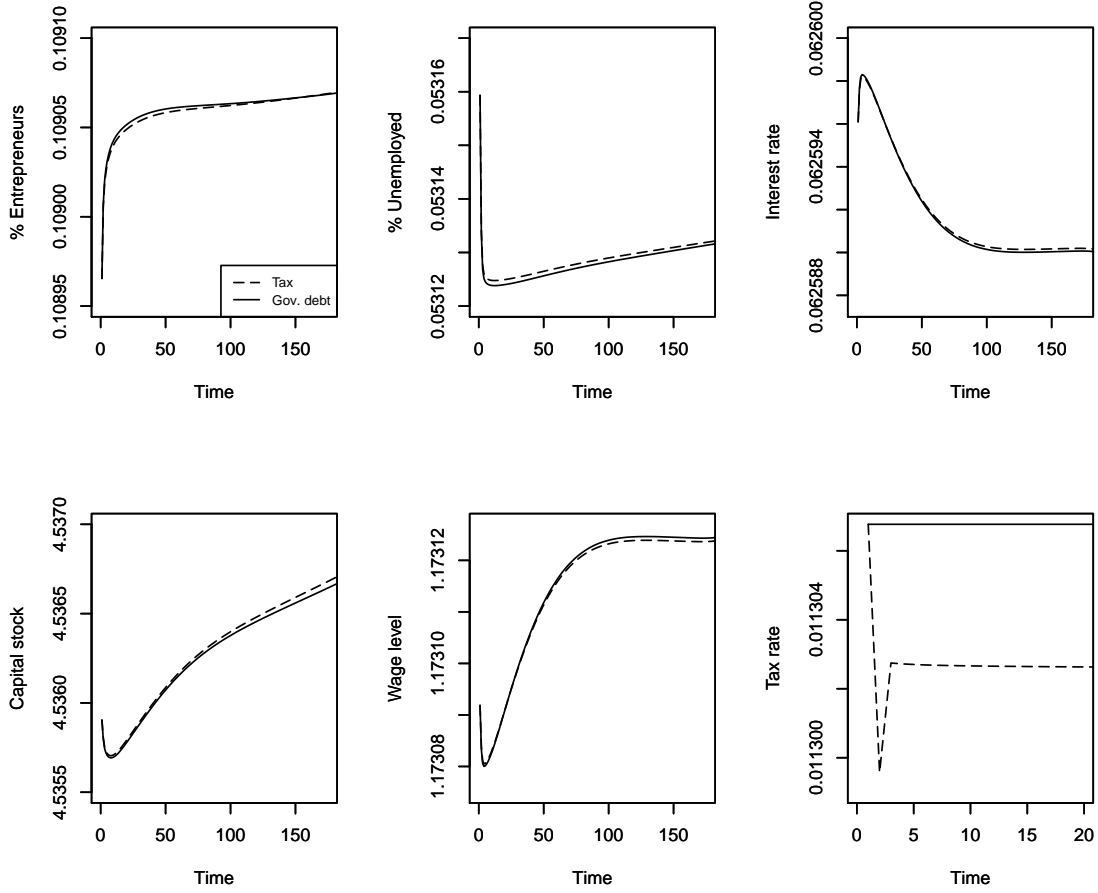
**Transition dynamics.** As shown in figure 5, when the policy is implemented in period 0, unemployment rate falls because some unemployed workers find it interesting to start a business.

First, this induces a rise in tax rate or a rise in debt level. In the two cases, agents dissave a bit so that aggregate capital stock increase inducing a lower wage rate and a higher interest rate. As time goes by, new entrepreneurs accumulate more capital and invest in their firm, capital stock increase, rising the wage level and decreasing the interest rate.

Because wage level is higher, some unemployed people reduce their effort to search a business idea and increase their effort in searching a job opportunity, this has the effect of increasing the unemployment rate steadily along the transition.



Figure 5: Evolution of aggregate quantities



**Welfare analysis of the reform.** Table 10 shows that the policy is beneficial for all categories at the steady state and this does not depend on the instrument used to adjust the government budget. This is due to at least three reasons. First, with the reform, unemployment rate is lower, which in turn reduce government spendings through UI. It follows a lower tax rate on workers and short-run unemployed workers. Second, the SEA provides additional insurance to short-run unemployed workers so that all workers are better off in case where they are laid off. Lastly, wage is higher after the reform, increasing the return at work whereas the interest rate, so that the cost at which entrepreneurs borrow is lower than before the reform was implemented.

During the transition, workers bear a slight cost due to sudden change in wages. Entrepreneurs are better off, at least the top 50%, because interest rate tends to fall. Finally, bottom 50% unemployed workers, who are the most borrowing constrained and therefore those who often prefer to work, face the cost of the workers due to a lower wage level.

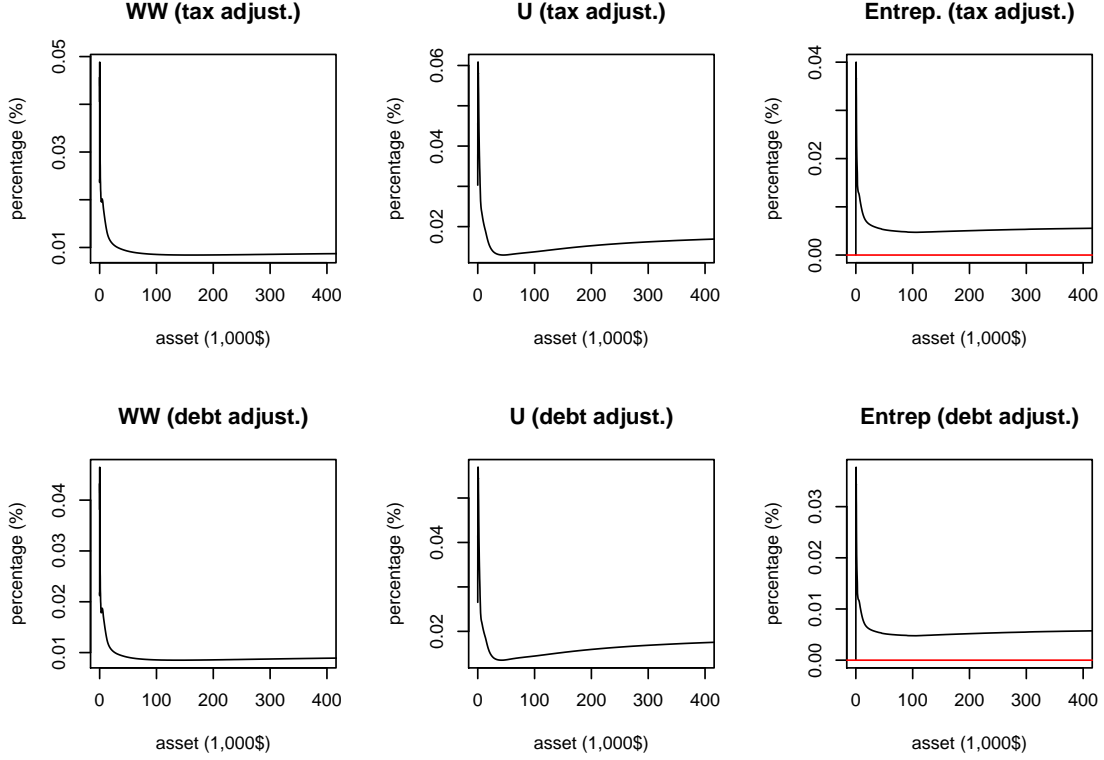
Figure 6 summarize the welfare consequences of the reform for the two instruments and the three occupations. In evaluating welfare, we take into account the costs associated with the transition.

As the benefits from the new reform largely offset the cost of the transition, all categories are now better off. The reform provides large welfare gains to poor people as compared to rich one. This is again because poor people are more collateralized and therefore directly benefit from the policy through the complement distributed by the government in the case where the returns to entrepreneurship is lower than the UI when they start a business as unemployed worker. This hold for the two types of instrument used.

Table 10: Consumption-equivalent variation with SEA

	CEV (debt adjust.)		CEV (tax adjust.)	
	Trans.	SS	Trans.	SS
Entrep.	0.002%	0.006%	0.002%	0.007%
bottom 50%	-( $<0.001\%$ )	0.011%	-( $<0.001\%$ )	0.012%
top 50%	0.005%	0.001%	0.004%	0.002%
Workers	-0.003%	0.025%	-0.003%	0.027%
bottom 50%	-0.005%	0.033%	-0.005%	0.035%
top 50%	-( $<0.001\%$ )	0.017%	-( $<0.001\%$ )	0.019%
Unemp. Workers	0.001%	0.034%	0.001%	0.037%
bottom 50%	-0.009%	0.058%	-0.009%	0.061%
top 50%	0.011%	0.011%	0.0106%	0.0128%

Figure 6: Consumption Equivalent Variation



## 5.2 Entrepreneurs' downside risk insurance

In this second experiment, we introduce a downside risk insurance to all entrepreneurs who bankrupt. In this sense, we assume that the entrepreneur get an UI proportional to his education level in the case where his firm is destroyed. That is, now, when the entrepreneur fall in unemployment with probability  $\mu$  and does not find any job opportunity, he then falls in a short-run unemployed worker situation.

**Steady states.** When the downside risk insurance is implemented, the fraction of entrepreneurs in the economy rise by more than 0.1 ( $>1\%$ ) as shown in table 11. Total production and capital in the entrepreneurial sector are higher than before the reform because entrepreneurs are more numerous.

Unemployment rate increase by a very little amount, suggesting that the increase in unemployment share (due to the discentive to search for a job) is higher than the increase in new entrepreneurial activities started by unemployed workers. Moreover, unemployed workers tend to be on average poorer than other occupations and are then more borrowing constrained. This could explain why many unemployed workers could not switch to entrepreneurship. Most of the change in the fraction of entrepreneurs comes from individuals in a worker situation who start their business. The latter

fact also explain the rise in savings, since now workers save a bit more in order to accumulate in the perspective of starting a business later.

Comparison between an adjustment of the government budget through either a proportional income tax or a rise in debt show that the policy is more effective if it financed by taxes. This is because when government uses debt, it artificially increases the interest rate and then the cost at which entrepreneurs borrow money.

From a pure utilitarian welfare perspective, on average, all occupations benefit from this insurance since it increases the value of starting a business. The reason why the average welfare of entrepreneurs is lower than before can be explain by the new entries. In fact, new entrepreneurs are on average poorer and reduce the average asset level hold by entrepreneurs as shown in the next table.

Table 11: Downside risk insurance (DRI) to all entrepreneurs

	Benchmark	Tax adjustment		Debt adjustment	
		With DRI	Variation	With DRI	Variation
Fraction of entrep. (%)	10.897	11.031	1.23%	11.024	1.165%
Unemployment rate (%)	5.316	5.318	< 0.05%	5.318	< 0.05%
Fraction of workers (%)	83.787	83.651	-0.162%	83.658	-0.154%
Avg Welfare (tot.)	-18.025	-18.010	0.083%	-18.009	0.089%
Avg Welfare (Work.)	-18.699	-18.691	< 0.05%	-18.688	0.059%
Avg Welfare (Ent.)	-12.38	-12.39	-0.08%	-12.389	-0.073%
Avg Welfare (Unemploy.)	-21.390	-21.177	1.01%	-21.174	1.01%
Total Production	2.645	2.654	0.34%	2.653	0.302%
Entrep. Production	1.101	1.110	0.817%	1.109	0.727%
Corp. Capital	4.536	4.544	0.176%	4.543	0.154%
Entrep. Capital	3.714	3.744	0.808%	3.741	0.727%

Table 12 displays the pool of entrepreneurs after the reform in the case where the policy is financed through taxes or through government debts. Individuals with an higher level of education tend to be more likely to enter after the reform. Again, this is because low-qualified workers are on average poorer and does not have enough collateral to run a beneficial business. In contrast, the policy lower the average firms' size since the new entrepreneurs are on average poorer than the incumbents. Indeed, the asset (or wealth) threshold at which an individual find it interesting to start a business is lower than before the reform. Hence, introducing a downside risk insurance to all entrepreneurs affect the pool of entrepreneur with a larger fraction of middle and high educated entrepreneurs and a decrease in the average firms' size.

Table 12: Downside risk insurance to all entrepreneurs

	benchmark			with DRI (tax adj.)			with DRI (debt adj.)		
	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_1$	$\theta_2$	$\theta_3$
% of entrep.	11.15	10.22	11.99	11.27	10.35	12.14	11.27	10.35	12.13
% change (rlv to bench.)	-	-	-	1.08%	1.27%	1.25%	1.08%	1.27%	1.17%
Avg.firm	267.21	339.65	410.65	266.63	338.09	409.04	266.60	338.08	409.01
Avg.asset	233.37	297.66	341.29	231.99	296.38	340.21	231.96	296.36	340.19

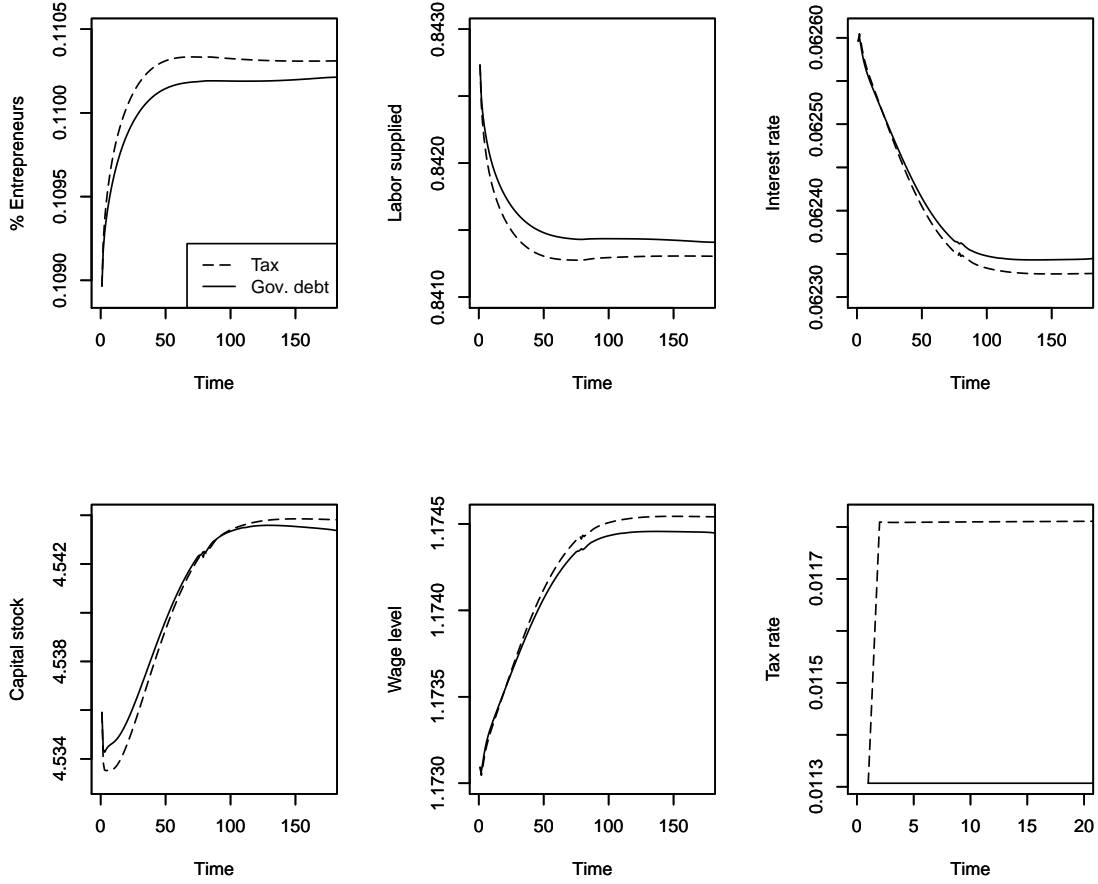
**Transition dynamics.** We present in figure 7 the evolution of macroeconomic variables along the transition path for the two instruments to close the government budget: a propotional labor income tax and debt issued.

When the government uses taxes, this immediately rises the expenditures and thus the tax rate and lower the savings in the economy. This rise is due to the fact that now, all entrepreneurs who bankrupt get the right to claim an UI. As time goes by, entrepreneurship becomes more valuable and a rising fraction of workers chooses to start a business. The total capital stock increases in the economy through the entrepreneurial sector and the rise in savings.

When the government uses debt, the amount of capital in the economy rise and this increases the interest rate relative to the case where the gouvernement uses taxes. This affect the cost at which the entrepreneurs finance their investment and lower their fraction.

Finally, as the number of workers decreases in the economy and that the capital level increases, the wage level tends to increase. This improves the value of being a worker and lower the effect of the downside risk insurance to favor entrepreneurship. In total, this general equilibrium effect is too small to counterbalance the effect of the DRI on the rise in the fraction of entrepreneurs in the economy.

Figure 7: Evolution of aggregate quantities



**Welfare consequences of the reform.** We compute the welfare effect through the consumption equivalent variation (CEV) of the reform along the transition and between the two steady states. Table 13 summarizes the welfare impact of introducing a DRI when it is financed either by government debt or by taxes.

At the steady state, for the two instruments, all agents are better off except wealthy unemployed worker in the case where the tax rate is used to balance the budget. This is because in that case, these individuals suffer from the lower interest rate as well as the higher tax rate. The increased tax rate lower the welfare gains of the workers as compared to a situation where the government use debt. In constrast, when the government uses debt, the effect of taxes does not reduce the welfare and the gain are greater than 1% consumption-equivalent change for the bottom 50% unemployed workers.

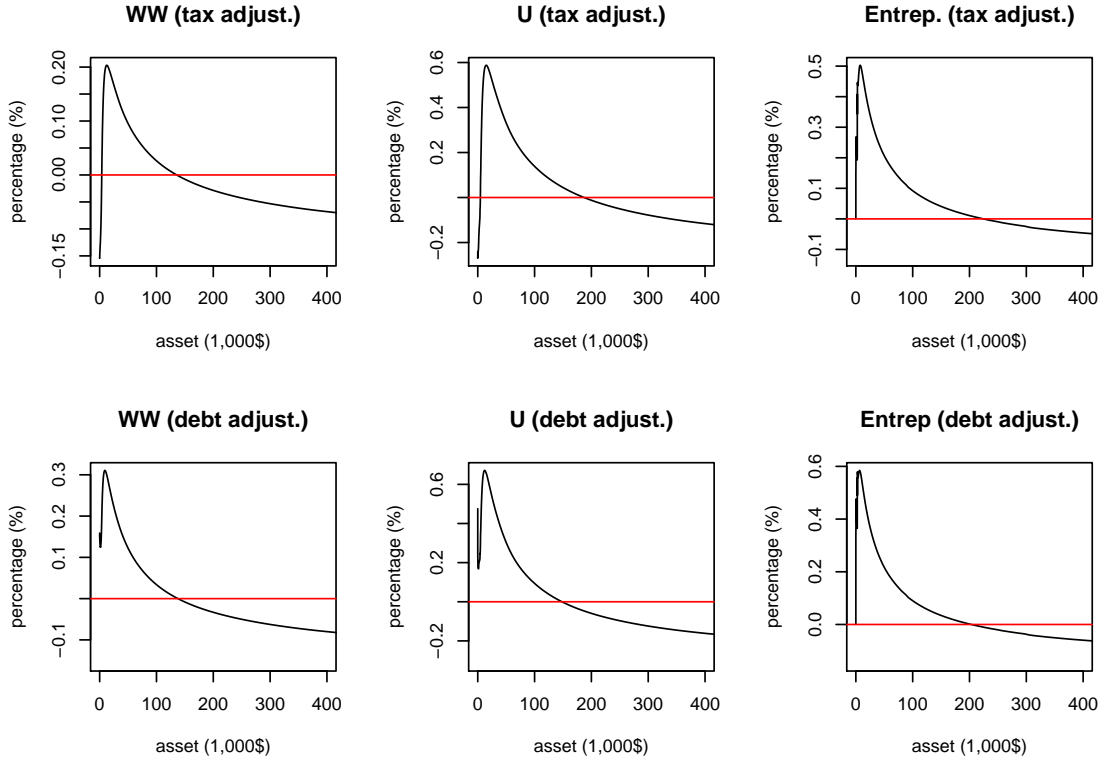
Along the transition,

Table 13: Consumption-equivalent Variation

	CEV (debt adjust.)		CEV (tax adjust.)	
	Trans.	SS	Trans.	SS
Entrep.	<0.05%	0.419%	0.176%	0.185%
bottom 50%	-0.219%	0.763%	<0.05%	0.454%
top 50%	0.209%	0.079%	0.347%	-0.079%
Workers	-0.591%	0.752%	-0.204%	0.142%
bottom 50%	-0.805%	0.939%	-0.369%	0.234%
top 50%	-0.386%	0.573%	-0.046%	0.055%
Unemp. Workers	-0.522%	0.777%	0.087%	-0.114%
bottom 50%	-1.339%	1.545%	-0.555%	0.32%
top 50%	0.261%	0.038%	0.702%	-0.531%

Figure 8 displays the welfare gain by education and asset level. As wealth increase, the policy becomes costly for each category since wealthy people rise income through interest on their asset. As the interest rate is now lower, these guys are now worse off. Except for the very poor in the case where the policy is financed through taxes, people with relatively low level of asset are better off. This is because the rise in the entrepreneurial value improves the welfare of all categories.

Figure 8: Consumption Equivalent Variation



### 5.3 Discussion

The main message is that downside risk insurance affect a little unemployment in our model economy, but we do not take into account spillover effect. Unemployed workers are in average poorer, and are then less likely to open a business.

The main message is that this does not modify the pool of entrepreneurs

The main message is that the rise in the fraction of entrepreneurs is mainly due to worker who wants to start their business.

## 6 Conclusion

For future research, one should account for a fine bankruptcy framework. Also, it would be interesting to explicitly account for endogenous labor, so that entrepreneurs could hire workers. This could substantially increase the effect of such policies.

## 7 Appendix

### 7.1 Data sources

[CPS description here]: to get transition data.

[SCF description here]: to compare wealth distribution.

[SBO description here]: to compare firms' size distribution.

### 7.2 Algorithms

**Steady state.** The algorithm is organized as follows.

1. Initialize a full dimension grid space composed of all different possible asset values ( $a$ ), productivity level ( $y$ ), innate ability ( $\theta$ ) and entrepreneurial state ( $j$ ). The maximum asset level is chosen sufficiently large to get ergodicity of the policy functions, so that the household's saving decision is not binding.
2. Guess an initial tax parameter  $\tau_w$  in addition to prices ( $w$  and  $r$ ).
3. Given these parameters, solve the value functions using value function iteration following the method in Stokey and Lucas (1996). Appendix 7.4 show how we solve for the optimal searching effort in the unemployed worker case.
4. Construct the transition matrix  $\mathbf{M}$  generated by  $\Pi_y$ ,  $\Pi_j$ , the saving decision and the search effort intensity decisions. Compute the associated stationary measure of individuals  $\Gamma(\mathbf{x})$ , by



first guessing an initial mass of one of households with zero asset (for instance,  $\Gamma(yw, y_0, \theta_0, a) = 1$ ) and then by iterating on  $\Gamma'(\mathbf{x}) = \mathbf{M}\Gamma(\mathbf{x})$  until  $|\Gamma'(\mathbf{x}) - \Gamma(\mathbf{x})| < \mu$ , with  $\mu$  very small.

5. Compute the resulting total asset level and total labor supplied in the economy and total investment level and total labor hired in the entrepreneurial sector. Total capital invested and in the non-entrepreneurial sector is given as the difference between total savings net of government debt and total capital invested in the entrepreneurial sector. Total labor used in the non-entrepreneurial sector is given by total labor supplied by workers.
6. Update prices ( $r, w$ ) using the marginal productivities in the non-entrepreneurial sector and the tax rate ( $\tau_w$ ) to close the government budget up to a relaxation.

**Transition dynamic.** We assume that the economy is in the initial steady state in period 0 and the reform is announced and implemented in period 1. The economy makes a transition to reach the final steady state in period T. We choose T large enough so that the resulting stationary distribution in period T is close enough to the steady state distribution after the reform.

1. Guess a path for the level of capital, labor as well as  $\tau_w$  (or  $B$  in case where we close the budget using government debt). Compute the resulting prices  $r$  and  $w$ .
2. Use the value function of the final steady state after the reform and solve the households' problem backwards starting from T until period 1.
3. Use the distribution of the initial steady state before the reform and the policy functions computed in step 2 and compute the path of the distribution.
4. Given these distributions, compute a new path for capital, labor and the tax rate (or government debt), and the resulting prices. Iterate from step 2 until the difference between the initial path is close enough to the final path.
5. When converge is achieved, check if the final distribution in period T is close enough to what should be the steady state in period T after the reform up to a criterion. If the two distributions are similar, then stop, else, increase the number of periods T.

### 7.3 Estimation of the transitions between occupations

Wage level is a better proxy for working skills, we use it when available (when the individual is currently in employment). Otherwise, we use the family income. These are CPS basic data.

Below are reported discrete choice model results (OLS and Probit) between the probability to switch and various explanatory variables.

[ADD NON-PARAMETRIC ANALYSIS]

Table 14: OLS regression on the probability to switch from a given state to another

	E - WW	E - U	WW - E	WW - U	U - E	U - WW
Sexe*	−.034*** (.004)	.003*** (.001)	.013*** (.001)	.006*** (.001)	.006*** (.002)	.001 (.007)
Age	−.002*** (.0002)	−.0004*** (.00004)	.001*** (.00002)	−.0004*** (.00002)	.001*** (.0001)	−.002*** (.0002)
Family inc.	.022* (.011)	−.002*** (.0002)			.002*** (.0003)	.014*** (.001)
Family inc. <sup>2</sup>	−.003** (.001)					
Family inc. <sup>3</sup>	.0001** (.0001)					
Wage			−.003*** (.0003)	−.001*** (.0001)		
Wage <sup>2</sup>			.0001*** (.00004)			
Wage <sup>3</sup>			.000001 (.000001)			
Unemp. dur.					−.0003*** (.00005)	−.002*** (.0001)
Agg. Unemp.					.006*** (.002)	−.005 (.006)
Constant	.317*** (.027)	.047*** (.003)	−.005*** (.001)	.039*** (.001)	−.057*** (.010)	.527*** (.032)
Observations	40,051	40,051	381,443	381,443	23,150	23,150

Note:

\*p&lt;0.1; \*\*p&lt;0.05; \*\*\*p&lt;0.01

Source: estimation using CPS data from 1994 to 2015.

(\*) dummy variables

Table 15: Probit regression on the probability to switch from a given state to another

	E - WW	E - U	WW - E	WW - U	U - E	U - WW
Sexe*	-.114*** (.015)	.082** (.038)	.184*** (.009)	.163*** (.008)	.183*** (.039)	-.038** (.017)
Age	-.010*** (.001)	-.009*** (.001)	.010*** (.0003)	-.008*** (.0003)	.013*** (.001)	-.006*** (.001)
Family inc.	.003 (.034)	-.037*** (.005)			.023*** (.005)	.029*** (.002)
Family inc. <sup>2</sup>	-.004 (.004)					
Family inc. <sup>3</sup>	.0003* (.0002)					
Wage			-.061*** (.005)	-.012*** (.001)		
Wage <sup>2</sup>			.004*** (.001)			
Wage <sup>3</sup>			-.0001*** (.00002)			
Unemp. dur.					-.002*** (.001)	-.006*** (.0003)
Agg. Unemp.					-.015 (.010)	-.057*** (.005)
Constant	-.186** (.082)	-1.473*** (.090)	-2.488*** (.017)	-1.675*** (.014)	-2.726*** (.094)	.389*** (.040)
Observations	40,051	40,051	381,443	381,443	23,150	23,150

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Source: estimation using CPS data from 1994 to 2015.

(\*) dummy variables

#### 7.4 Optimal searching effort: unemployed worker case

The derivative of the unemployed individual program with respect to the searching effort types are given by these two equations

$$\begin{aligned} \frac{\partial V_\epsilon^u(y, a, 1)}{\partial s_w} = & -\psi_w(s_w)^{\psi_w} + \beta\kappa_w e^{-\kappa_w s_w} \left[ (1 - \zeta) [(1 - \pi_e(s_e))(EV_w(1) - EV_u(1)) \right. \\ & \left. + \pi_e(s_e)(EV_{w,e}(1) - EV_{e,u}(1))] + \zeta(EV_w(0) - EV_u(0)) \right] = 0 \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{\partial V_\epsilon^u(y, a, 1)}{\partial s_e} = & -\psi_e(s_e)^{\psi_e} + \beta\kappa_e e^{-\kappa_e s_e} (1 - \zeta) \left[ (\pi_w(s_w)(EV_{w,e}(1) - EV_w(1)) \right. \\ & \left. + (1 - \pi_w(s_w))(EV_{e,u}(1) - EV_u(1))) \right] = 0 \end{aligned} \quad (2)$$

Using the notation  $\mathcal{P}(s_w) = \pi(s_w)(EV_{w,e}(1) - EV_w(1)) + (1 - \pi(s_w))(EV_{e,u}(1) - EV_u(1))$  then we have for fixed  $s_w$

$$1 - \frac{\psi_e(s_e^*)^{\psi_e}}{\beta\kappa_e(1 - \zeta)\mathcal{P}(s_w)} = \pi_e(s_e^*) \quad (3)$$

Several cases can be study to analyse the effect of increasing  $s_w$  on the optimal searching effort  $s_e^*$ .

- The case where the unemployed individual is too poor that he always find optimal to not invest to start a business (i.e  $EV_w(1) > EV_u(1) > EV_e(1)$ ), we have  $\mathcal{P}(s_w) = 0 \quad \forall s_w$ . In that case  $\pi(s_e^*) = 0$  and  $s_e^* = 0$ .

Given that the individual receive an idea, he never find it optimal to search for opening the business. The idea is completely loss.

- When the unemployed individual is in the necessity share, so that  $EV_w(1) > EV_e(1) > EV_u(1)$ , then we obtain  $\mathcal{P}(s_w) = (1 - \pi(s_w))(EV_e(1) - EV_u(1)) > 0 \quad \forall s_w \geq 0$ .

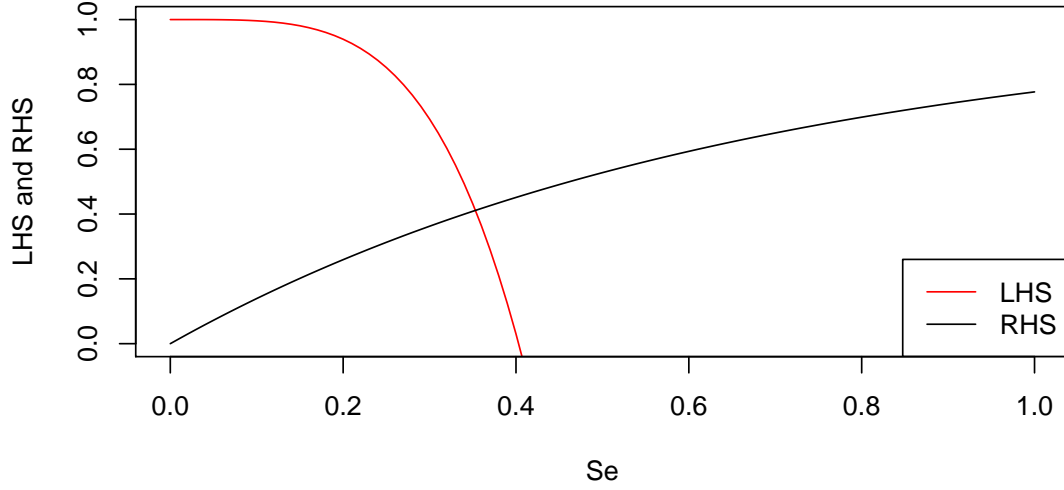
In that case,  $\mathcal{P}(s_w)$  is decreasing with  $s_w$  so that when  $s_w$  increases then  $\pi_e(s_e^*)$  decreases. There is a tradeoff between searching a job and searching to get the opportunity to open the business.

- When the unemployed individual is in the opportunity share, so that  $EV_e(1) > EV_w(1) > EV_u(1)$ , then we obtain  $\mathcal{P}(s_w) = \pi(s_w)[EV_u(1) - EV_w(1)] + EV_e(1) - EV_u(1) > 0 \quad \forall s_w \geq 0$ .

Moreover,  $\mathcal{P}(s_w)$  is decreasing with  $s_w$  and thus  $\pi_e(s_e^*)$  is decreasing with an increase of  $s_w$ . There is again a tradeoff between searching a job and searching for opening a business.

The figure below report the three cases depending on the case considered above.

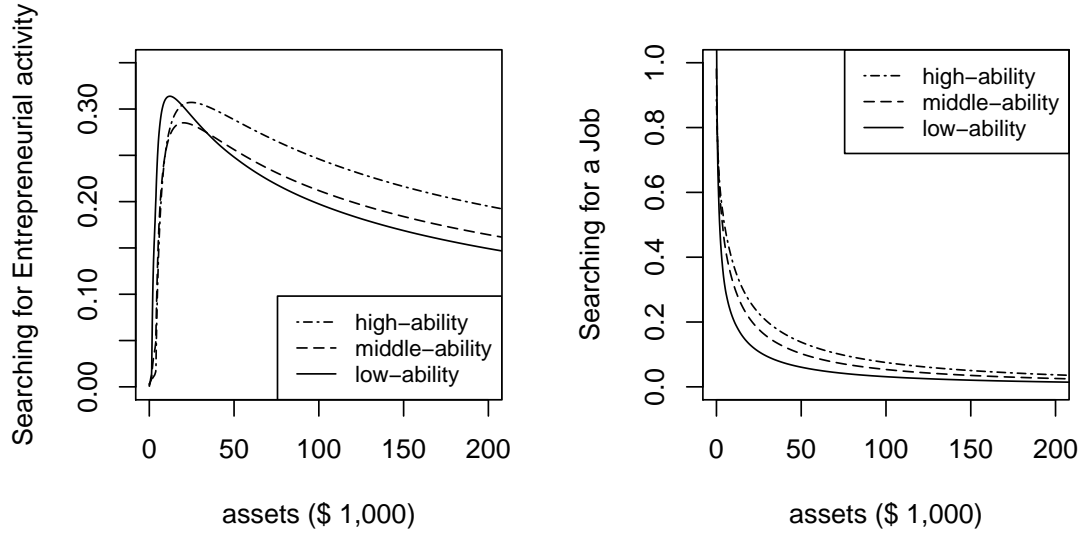
Figure 9: Optimal searching effort for getting the opportunity to open a business.



To resolve the problem of the unemployed individual, we then use sequential 1D solver. That is, for any value  $s_w$  we compute the associated optimal level  $s_e^*$  determined by equation (3).

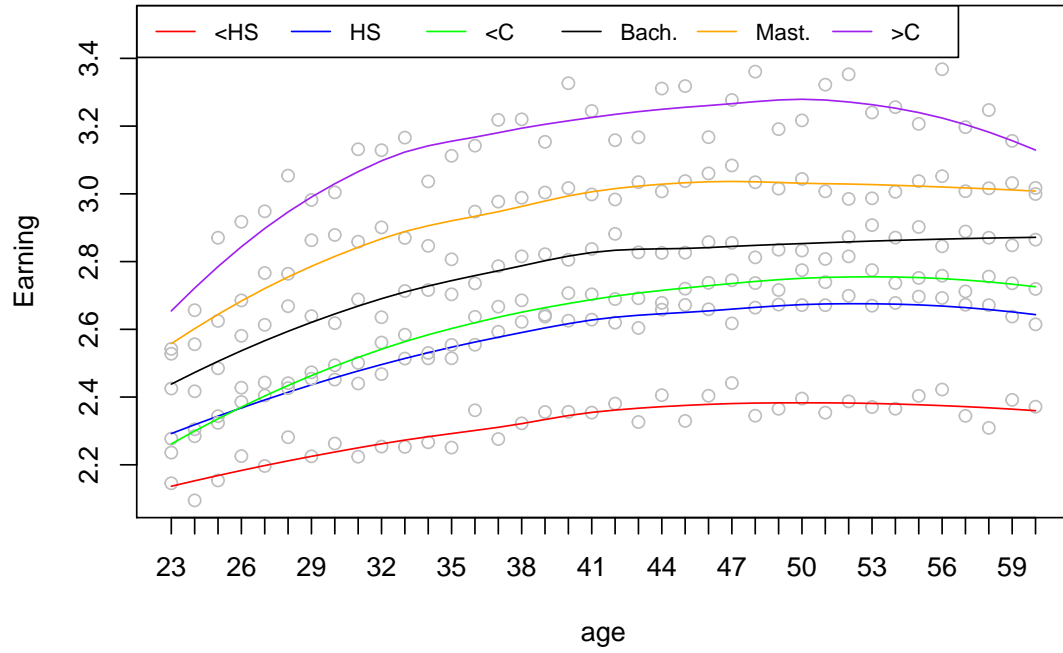
Figure 10 below reports the solutions of the algorithm.

Figure 10: Optimal searching effort: unemployed workers



## 7.5 Earning profil

Figure 11: Earning profil between educational attainment



## References

- [1] S. Aiyagari. “Uninsured Idiosyncratic Risk and Aggregate Saving”. In: *The Quarterly Journal of Economics* 109.3 (1994), pp. 659–684.
- [2] World Bank. *Unemployment rate*. 2016. URL: <http://data.worldbank.org/indicator/SL.UEM.TOTL.ZS>.
- [3] Truman Bewley. “A Difficulty with the Optimum Quantity of Money”. In: *Econometrica* 51.5 (1983), pp. 1485–1504. ISSN: 00129682, 14680262.
- [4] Francisco Buera, Joseph Kaboski, and Yongseok Shin. “Finance and Development: A Tale of Two Sectors”. In: *American Economic Review* 101.5 (2011), pp. 1964–2002.
- [5] Marco Cagetti and Mariacristina De Nardi. “Entrepreneurship, Frictions, and Wealth”. In: *Journal of Political Economy* 114.5 (2006), pp. 835–870.
- [6] Jeffrey Campbell and Mariacristina De Nardi. “A conversation with 590 Nascent Entrepreneurs”. In: *Annals of Finance* 5.3 (2009), pp. 313–340. URL: <http://EconPapers.repec.org/RePEc:kap:annfin:v:5:y:2009:i:3:p:313-340>.
- [7] Kevin Donovan. *Subsistence Entrepreneurs and Misallocation*. 2014 Meeting Papers 771. Society for Economic Dynamics, 2014.
- [8] Robert Fairlie. *The Kauffman Index: Startup Activity — National Trends*. Santa Cruz Department of Economics, Working Paper Series. Department of Economics, UC Santa Cruz, 2015.
- [9] Nestor Gandelman and Ruben Hernandez-Murillo. “Risk Aversion at the Country Level”. In: *Review* 97.1 (2015), pp. 53–66.
- [10] William M. Gentry et al. “Entrepreneurship and Household Saving”. In: *Advances in Economic Analysis and Policy*. 2004.
- [11] Johan Hombert et al. *Can Unemployment Insurance Spur Entrepreneurial Activity?* Working Paper 20717. National Bureau of Economic Research, 2014.
- [12] Mark Huggett. “The risk-free rate in heterogeneous-agent incomplete-insurance economies”. In: *Journal of Economic Dynamics and Control* 17.5-6 (1993), pp. 953–969. URL: <https://ideas.repec.org/a/eee/dyncon/v17y1993i5-6p953-969.html>.
- [13] Lane Kenworthy and Timothy Smeeding. *GINI Country Report: Growing Inequalities and their Impacts in the United States*. GINI Country Reports. AIAS, Amsterdam Institute for Advanced Labour Studies, 2013.
- [14] Dale Mortensen and Christopher A. Pissarides. “Job Creation and Job Destruction in the Theory of Unemployment”. In: *Review of Economic Studies* 61.3 (1994), pp. 397–415.

- [15] Markus Poschke. “Who becomes an entrepreneur? Labor market prospects and occupational choice”. In: *Journal of Economic Dynamics and Control* 37.3 (2013), pp. 693–710.
- [16] Vincenzo Quadrini. “Entrepreneurship, Saving and Social Mobility”. In: *Review of Economic Dynamics* 3.1 (2000), pp. 1–40.
- [17] Ellen R. Rissman. *Labor market transitions and self-employment*. Working Paper Series WP-07-14. Federal Reserve Bank of Chicago, 2007.
- [18] Ellen R. Rissman. *Labor market transitions and self-employment*. Working Paper Series WP-07-14. Federal Reserve Bank of Chicago, 2007. URL: <https://ideas.repec.org/p/fip/fedhwp/wp-07-14.html>.
- [19] Robert Shimer. “The Cyclical Behavior of Equilibrium Unemployment and Vacancies”. In: *American Economic Review* 95.1 (2005), pp. 25–49.
- [20] Robert Shimer. “Reassessing the ins and outs of unemployment”. In: *Review of Economic Dynamics* 15.2 (2012), pp. 127–148. ISSN: 1094-2025.
- [21] Nancy L. Stokey and Robert E. Lucas. *Recursive methods in economic dynamics*. 4. print. Cambridge, Mass. [u.a.]: Harvard Univ. Press, 1996. XVIII, 588. ISBN: 0674750969.
- [22] Ludo Visschers, Ana Millan, and Matthias Kredler. “Great opportunities or poor alternatives: self-employment, unemployment and paid employment over the business cycle”. In: 597 (2014).
- [23] Achim Zeileis. *pwt8: Penn World Table (Version 8.0)*. 2015.