A Characterization of the Underlying Mechanisms of Entry and Exit in the Small Business Sector*

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Abstract

This paper gathers new evidences on small business owners' entry and exit behaviors and emphasizes the importance of business transfers. It introduces a new theoretical framework designed to acount for the underlying mechanisms of entry and exit with endogenous option to buy, found, sell or liquidate business assets. The model embeds a business for sale market with asset transfers and an equilibirum price designed to capture both the intertemporal and the intangible value of a firm. We use the model to characterize the important mismatches occuring in the business for sale market and recount the current episode of aging entrepreneurs and its consequences.

Keywords: Entrepreneurship, Business Selling and Buying Frictions, Aging

JEL classification: E21, L26, J11

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1 Introduction

Entry and exit constitute a fundamental aspect of the entrepreneurial activity and as such has been the subject of a substantial literature. But surprisingly, not much has been said about the underlying mechanisms supporting entry and exit despite their importance in the entrepreneurial lifecycle: a small and medium firm is either acquired (founded or bought mainly) by the entering entrepreneur and either transferred or liquidated by the exiting one. Instead, the existing literature has either treated the entrepreneur as an individual, discarding the details of how business assets are acquired and gotten rid of or focused solely on the firm aspects discarding the business owner. However, the empirical evidences on how businesses are founded, bought and sold tend to show that accounting for both the entrepreneur and the business assets is necessary. One such evidence is the existence of important mismatches on the business for sale market, barring transfers. Depending on the survey, between 11% and 17% of owners declare selling their business upon exit. However, this number is small compared to the number of entrepreneurs who were planning on selling: in the Annual Survey of Entrepreneurs (ASE) 2016, between 50% to 60% declared planning to sell upon exit.[1] It is thus illusory to consider that, for instance, an entrepreneur can exit immediately and without much financial harm and misguided to believe that productive business assets might survive independently of the capacity of the owner to sell. At the other end, consistently across a number of US surveys, 1 in 5 entries into entrepreneurship is due to a business purchase. Thus accordingly, the transfer of existing productive assets is dependent on the ability of entrants to buy and on the trade-off between purchasing and founding a business.

The observations above are reinforced by well known demographic trends showing that over the next decades the peak of the retiring baby-boomers will be reached, with a pivotal implication: the aging of entrepreneurs. In 2016, 30% of entrepreneurs were aged 60 years and more compared to only 22% in 2004 and 18% in 1989. Evidences indicate quite predictably that the aging of the entrepreneurial population will only become more severe in the upcoming years.[2] This evolution might be quite significant, both for entrepreneurial entry and exit and since entrepreneurs hold very undiversified portfolios (**?**), which is of crucial importance for individuals planning to retire. Thus the underlying mechanisms of entry and exit, with the business for sale market as the main means of asset transfers, are key in understanding the recent transformations of the entrepreneurial sector. However, the agenda of assessing these underlying mechanisms presents a few challenges. First, data on small and medium-sized en-

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[1]An European Commission Report support the same phenomenon in Europe with up to 60% of businesses failing to transfer, including a large number unrelated to economic performances.

[2]See for instance **?** on the decrease of young entrepreneurs starting businesses.—> pas sur de la pertinence de cette référence ici...

terprises (SME) transfers is scarce. Second, there is no theoretical framework in the literature to properly consider transfers in a standard entrepreneurial setting. Thus, the main contribution of this paper is to provide such a theoretical framework with, notably, endogenous options to buy or found on the entry side and sell and liquidate on the exit side. The model embeds a business for sale market to properly consider business transfers and is designed to capture the mismatches appearing on this market. We further use the model to provide a quantitative assessment of entrepreneurial aging on entry and exit and business transfers. We support the above model by reporting a number of micro-data based new empirical facts on business creation and acquisition, the SMEs for sale market and on the aging and life-cycle aspects of entrepreneurship. In addition, we supplement the above empirical evidences by collecting and building our own dataset. To the best of our knowledge, our paper is the first to tackle this agenda.

Our baseline economy is a stylized life-cycle incomplete markets model with heterogenous agents and occupational choices. Households age probabilistically through nine different stages designed to concisely capture the phases of adult professional life and retirement as well as the key saving, occupational and entrepreneurial decisions. We introduce endogenous business buying, founding, selling and liquidating decisions into this framework as a decisive and novel mechanism for entrepreneurial entry and exit. Each period an incumbent entrepreneur might need to sell (voluntarily or involuntarily) her business and will face an endogenous selling price as well as a selling probability. Absent a selling opportunity, the incumbent will be forced to either dismantle the business and liquidate the entrepreneurial assets or continue the activity. Conversely, a non-entrepreneur might enter an entrepreneurial activity by endogenously choosing a firm size and either finding an available business to buy or founding a new business, these decisions being subject to credit constraints. A small and medium-sized enterprises for sale market (SMESM) aggregates selling and buying decisions. Its equilibrium price is designed as an abstract object to account for both the intertemporal and intangible value of a business. Outside this market, illiquid capital adjustments are subject to adjustment costs. We argue that all the above ingredients are key in reproducing the business selling, buying/founding and age related frictions appearing in the empirical data.

We mainly use data from the Annual Survey of Entrepreneurs (ASE), the National Longitudinal Survey of Youth 1979 (NLYS79), the Survey of Small Business Finance (SSBF) and the Panel Study of Income Dynamics (PSID) to document entrepreneurial acquisition and ceasing characteristics. We complement the available data by collecting our own dataset from transactions on the online business selling marketplace *Bizbuysell.com* (BBS) while comparing our sample to private business valuation datasets BizComps and ValuSource. We find that in the US, founding a business is the dominant means of entry into entrepreneurship despite the fact that, as mentioned above, 20% of entries are the result of the purchase of an existing business.

We stress the importance of the age profile in the decision to become an entrepreneur: younger and older individuals are more likely to found businesses while middle-aged individuals are more likely to purchase. On the other hand, we show that business assets selling peaks at two age brackets: the 45-50 and the 60-65 brackets mirroring the proximity of the retirement threshold. Our collected BBS dataset confirms the magnitude of the frictions on the business selling market: depending on the period, we find only 10% to 20% successful business selling transactions within a year. This corroborates the ASE evidence on the gap between intentions to sell and actual transactions. We also discuss evidences on the aging of entrepreneurs: we find that, unlike the age distribution of workers, that of entrepreneurs displays a clear shift to the right and that the phenomenon is accelerating. This is corroborated in the leading reasons to cease a business with the *retirement motive* showing a substantial increase between 2007 and 2016.

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We simulate a calibrated version of our benchmark economy. Prior to calibration, we test the exact effect of the type of business acquisition (buying versus founding) on the interest rate charged on the businesses as well as on the credit limit using SSBF data. Our estimation results find a significative relationship between the type of acquisition and the interest rate. Recent owners (with less than 5 years in operation) having purchased an existing business are charged, on average, an interest rate 1.59% lower than those who founded their businesses. Our simulation results show that our model and calibration are able to reproduce the key elements that we pointed out in our empirical findings. We match the distribution of founders and purchasers as well as the corresponding life-cycle dynamics. Moreover, individuals beyond the legal retirement age hold large portions of illiquid entrepreneurial capital and are an important fraction of the business sale market. We also find that business sale market mismatches, together with the life-cycle setup are key in reproducing the empirical data. In particular, we find that the common assumption of illiquid capital adjustment costs, although important, is not sufficient to reproduce the actual entrepreneurial entry and exit dynamics appearing in the data.

We detail the impact of key ingredients of our model on the entrepreneurial pool, aggregate production and capital and the characteristics of the business sale market. To this end, we first quantify the effects of improving the matching between business buyers and sellers. We find that the fraction of businesses sold and bought increases in equilibrium, lowering the overall entrepreneurial capital destruction through liquidation. As a result, entrepreneurial capital significantly increases as well as production. Second, we explore the impact of lowering the interest rate on business loans. Measuring this channel is of some importance for a number of active labor market policies aiming to help self-employment. Theses policies often target the borrowing costs, specially to promote early stage entrepreneurship. We find that the effects of this channel are large on the equilibrium. The share of entrepreneurs significantly increases

and especially the ratio of existing businesses to new comers change in favor of the latter. The business sale market is also significantly impacted with a simultaneous decrease of selling and buying, as the tradeoff between purchasing and founding a business changes in favor the latter. Third we also measure the impact of illiquid capital adjustment costs by symmetrically increasing the cost of both capital increases and decreases. We find that the impact of this channel is very significant on the business sale market as the share of business buyers and specially the share of sellers increase remarkably. This is a clear sign that the business sale market is a preferred alternative to costly illiquid capital liquidation, a feature that models with only capital adjustment costs can not capture. Finally, we change the population structure by artificially increasing the fraction of agents of age $[51, 65[$ to favor on average richer individuals in the life-cycle. This change has a significant impact on entrepreneurial and total capital, that both increase along with production. This also leads to an increase of business buyers and sellers: because a larger fraction are older, more of them switch to retirement, increasing the fraction of sellers while the fraction of buyer increases endogenously increase with age. Overall, we find our setup to be both appropriate and tractable to investigate a rich entrepreneurial environment. On tractability, despite the fact that the model includes a number of computationally challenging elements, we provide an algorithm that tackle both the kinks in the value function and allows for a fast computation time by combining the insight in recent papers by **?** and **?**.

Finally, we intend to use the model to simulate a demographic change similar to the consequences of the baby-boom generation reaching the peak of retirement. To this end, we modify the stationary assumption of our model and simulate a specific transition. This contribution is still in progress.

Related Literature Our paper is related to the literature that study small and middle sized businesses and entrepreneurship. Many papers, both quantitative and theoretical, are interested in understanding entrepreneur's behavior and the selection into entrepreneurship. Many of those papers model entrepreneurs as agents utilizing physical capital and hiring employees subject to idiosyncratic business shocks, entrepreneurial abilities or unexpected capital destruction, subject to financial frictions. See the seminal papers by **?** and **?** on the modeling of entrepreneurship and the link with wealth inequality. Using this framework, a number of studies focus on issues related to misallocation (**?** or **?**), business tax policies (for instance, **?** on capital taxation and **?** on extate taxation), or policies that aim to promote entrepreneurship (**???**). Also related to our study are **?** and **?** showing the link between entry into entrepreneurship and age. Relative to this literature, our paper adds an empirically realistic model that account for the life-cycle property of entrepreneurship, as well as explicitly modeling the market frictions arising upon the transfer of business capital. As in **?**, our model business capital distinguishes between intangible assets and tangible assets. In particular, liquidating a business allows en-

trepreneurs to recover business tangible assets, while selling a (or part of a) business means selling both intangible and tangible assets.

Our paper also contributes to the growing literature trying to understand the implications of illiquid assets versus liquid assets. In our model, entrepreneurs face adjustment costs when investing and disinvesting in business assets, which is crucial in generating the opportunity to buy and sell an existing business. We use the recent state-of-the-art *DC-EGM* computational techniques in **??** that extend the Endogenous Grid Method (EGM) initially introduced by **?** to the combination of multiple assets with discrete and continuous endogenous variables.[3] In addition to achieve some notable computational advantages, this method also allows for *IID extreme value taste shocks* which is of particular interest since, as argued **??**, unobservable heterogeneity in non-pecuniary benefits is of first order importance for the decision to become an entrepreneur.[4] Finally, on the macroeconomic effects of the demographic change, **?** argue that the lower population growth rate and the aging of the population has led to a lower start-up rate in the US. Since entrepreneurship and demographic are closely linked, our calibration takes into account the demographic change that occured in the 80s by targeting the (possibly) non-stationary 2007 US economy.

The rest of the paper proceeds as follows. Section 2 documents facts related to entrepreneurship, life-cycle and the business sale market. Section 3 lays out our framework and Section 4 describes how we take the model to the data. We discuss the results of the model in Section 5 In Section 6, we simulate the implications of a demographic shock on occupational choice and aggregate outcomes. Finally, section 7 concludes.

2   Entrepreneurship, business transfers and the life-cycle

In this section, we first document the life-cycle of entrepreneurs with a particular focus on how they enter and exit entrepreneurship. We then document that transferring businesses is subject to important mismatches, with a number of entrepreneurs unable to sell their company. We finally describe the recent trend on the exit side from entrepreneurship observed in the data with the ongoing aging of entrepreneurs.

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[3]This is particularly needed to make the model tractable. The two assets structure, the presence of occupational choices in addition to occasionally binding constraints make the problem particularly difficult to solve. It generates kinks and as a result, first order conditions are no longer sufficient, while still necessary.

[4]In particular, they have shown that a significant fraction of entrepreneurs choose to own businesses due to non-pecuniary reasons and never intend to grow their firms. In line with this, our model allow for non-pecuniary self-employment benefits which is reflected by direct utility gain and idiosyncratic taste shocks over household's occupational choice.

2.1 Data

We use various US survey data to identify business sellers and buyers together with their individual demographic as well as wealth and income related characteristics. We gather information from different datasets: the Current Population Survey (CPS), the National Longitudinal Survey of Youth 1979 (NLYS79), the Survey of Consumer Finance (SCF), the 2003 Survey of Small Business Finance (SSBF), the Panel Study of Income Dynamics (PSID) and the 2007 Survey of Business Owners (SBO). We also use aggregated data from the Annual Survey of Entrepreneurs (ASE) in order to document the recent evolution of the entrepreneurial exit strategy and ceasing reasons.

In order to obtain information on the small and middle sized business sale market, we also build a new dataset from transactions on the online business selling platform *Bizbuysell.com* (hereafter: BBS).[5] We also compare the resulting sample statistics to other private business valuation datasets such as BizComps and ValuSource (VS) data.[6] Each survey provides specific information (as well as additional estimates) of the main empirical findings, and can be classified as follows:[7]

- Business acquisition and buyer side: NLYS79, SSBF, SBO, ASE and SCF.

- Business exit, seller side and business valuation: NLYS79, PSID, SBO, ASE, BBS, VS and BizComps.

- Retiring entrepreneurs: CPS, PSID, BBS.

Finally, for consistency reasons among the various datasets and the model specification, we define an entrepreneur as an active self-employed business owner whenever it is possible and as business owners otherwise.[8] The Appendix B provides further details on the datasets and the variables that we use.

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[5]This platform is among the oldest and largest platform dedicated to facilitate business selling transactions in the US either directly or through brokers. The data correspond to 92900 observations of for sale and sold businesses records obtained from continuously gathering public data available on this website over a number of years. The Pratt's data as in **?** is also a candidate dataset, however the number of observations is much lower and we cannot identify the reasons for selling.

[6]Those two datasets report the sales of a number of SMEs from 1990 onward including: the time on market, the price, amount of fixed assets/intangibles, profit, ebitda etc.

[7]Specifically, the PSID and NLYS provide information about the selling behavior of entrepreneurs and business owners along the life-cycle. The SCF, which is recognized to be more representative of the top of the wealth distribution, provide information on the type of acquisition. Finally, the SSBF provide information about the type of acquisition and the amount invested at the start of the business.

[8]This is possible for all the surveys that we use. Unfortunately, we cannot control for active self-employment on the *Bizbuysell.com* platform. We therefore capture some business owners not actually entrepreneurs.

2.2 Business acquisition and exit

2.2.1 How do entrepreneurs acquire businesses?

The literature on entrepreneurship has long been interested in the behavior of incumbent entrepreneurs but has been somewhat silent on how businesses came to be in the first place. Throughout this paper, we argue that purchasing and selling a business are important components of entrepreneurship, accounting for a non negligible fractions of entrepreneurs in the data. Survey questions often define as *acquisition* the principal way the entrepreneur became the owner of the business: founding a new business or purchasing an existing business are two common alternative types of acquisition. Table 1 provides estimates of the type of acquisition. If we exclude inheritance and gifts, we find that around 20% of entries into entrepreneurship are the result of the purchase of an existing business. This number is consistent across a number of surveys in the US.[9]

Table. 1. Type of business acquisition

Survey	Year	Sample [a]	Founded	Purchased	Inherited [b]	Other/Gift [b]
SCF	2016	All entrepreneurs	74.4	18.2	3.5	3.9
ACF	2016	Businesses with employee(s)	68.1	20.8	4.0	7.1
SSBF	2003	All entrepreneurs	79.8	16.7	– 3.4 –	
SSBF	2003	Early-stage entrepreneurs	77.4	20.8	– 1.8 –	
SBO	2007	All entrepreneurs	74.8	18.2	4.8	2.2
SBO	2007	Early-stage entrepreneurs	74.0	19.3	5.5	1.2

[a] The estimates are based on entrepreneurs defined as business owners who have a self-employment role in their businesses. Recent entrepreneurs are those who acquired their businesses within the last 5 years.

[b] when possible, we separate the acquisition type between gift/other and inheritance.

The individual decision to become an entrepreneurs is very much linked to the age profile as it has already been shown in the literature. On the one hand, young individuals (less than 30 years old) tend to be more financially constrained. On the other hand, older individuals (more than 55 years old) can find it less beneficial to start a business due to the associated loss of accumulated worker advantages (experience on the job, firm specific promotions, responsibility status, etc.) and the fewer years to benefit from an entrepreneurial investment despite potentially better financing conditions. Focusing on the question of purchasing versus founding a business, Table 2 shows that younger and older individuals are more likely to start businesses

[9]We also provide estimates for other advanced economies, such as France and Spain, in the Appendix–> préciser laquelle. Results remain quite consistent, despite the fact that business transfers through inheritance seems more frequent in Europe.

whereas middle-aged individuals are more likely to purchase existing businesses.[10]

Table. 2. Business acquisition conditional on entrepreneurs' age

Age	Founded/started	Inherited	Purchased	Gift/Received
under 25	81.51	1.88	10.25	6.36
25 to 34	76.62	1.12	16.36	5.90
35 to 44	72.01	0.98	21.43	5.59
45 to 54	71.33	1.26	22.20	5.20
55 to 64	74.82	1.45	18.71	5.03
65 or over	75.88	4.80	15.49	3.84

Source: SBO 2007 using only early-stage businesses (< 3 years in operation).

Finally, it is worth noting that individuals who are purchasing an existing business are not more likely to have experienced a previous self-employment situation than those starting a new business: 28.6% of purchasers already experienced a self-employment situation against 38.6% for founders (SBO, 2007). We find similar evidence using the NLSY79.

2.2.2 Reasons to cease a business and business selling

On the exit side, using the SBO and the NLSY79, we first document the main reasons explaining why self-employed business owners cease their activity. We find that a non-negligible fraction of entrepreneurs sell their businesses upon exit: 7.5% according to the SBO (2007) (11% if we take into account business owners with paid employee(s)), 15% in the NLSY79 (2002-2016) and 17.1% in the 2016 ASE for business owners with paid employee(s).[11] This selling behavior is largely related to the entrepreneurial age profile. Using the PSID, Figure 1 shows that the selling of business assets peaks at two age brackets: the 45-50 and the 60-65, close to the US average retirement age. This is corroborated by the SBO, where reasons to sell linked to business being sold are particularly high for age bracket 55-64 and 65 or over-70.

2.3 Small and middle sized businesses for sale market

We find evidence of substantial difficulties for transferring businesses. According to the ASE 2016, among the business owners with paid employee(s) reporting how they plan to exit entrepreneurship, 50% plan to sell their businesses. This number increases to 60% if we take into

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[10]The nature of an entrepreneur's borrowing constraint has long been the subject of debates in the academic literature (see, among others, **?**). Here, we suggest that younger entrepreneurs are on average poorer than middle aged individuals, which may limit the amount of collateral that they engage in their businesses. For older entrepreneurs, however, banks might be less likely to lend due to the higher mortality rate.

[11]Note that we collect this information from different years using the NLSY79, due to the lack of observations.

Figure 1. Distribution of businesses sold by age of the entrepreneur. *Source:* PSID 1990 - 2015.
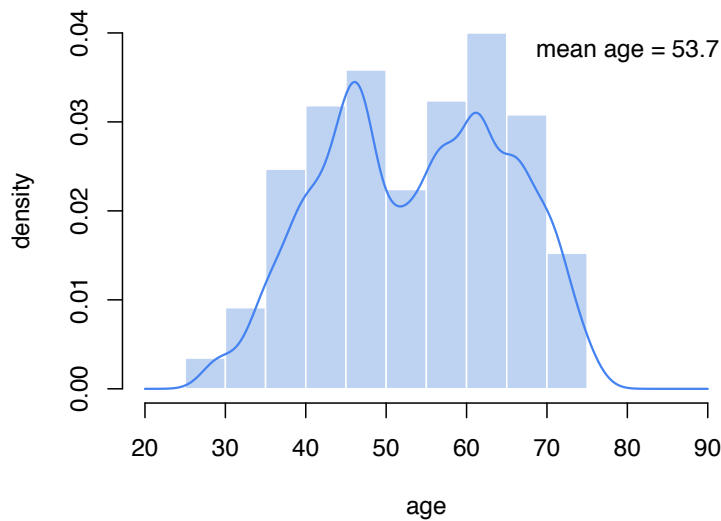
account those planning to sell their business to a member of their family. Given the remarkably low number of entrepreneurs reporting having sold their business, it seems that there are potentially important failures when trying to sell one: this is indicative of a gap between what entrepreneurs plan to, and what they are actually able to do. In particular, we are interested in whether there are mismatches in the businesses for sale market, with some SME owners not able make any deal, resulting in the liquidation of assets (i.e. selling at a very low price and usually restricted to the tangible assets). First, a substantial number of transactions are terminated without a successful deal according to the International Business Broker Association (IBBA), a private source aggregating transaction information from business brokers. In their estimates, between 18% to 30% of businesses for sale were successfully transferred to a new owner. The remaining was terminated.[12] In Europe, 690000 of the 1.7 millions EU-27 SMEs closing every year attempt transfer, of which 60% fail to transfer. Among those business transfer failures, 25% failed for reasons unrelated to the performance but due to inefficiencies in business transfers, representing 9% of total business closures (European Commission Report–> date).

To understand the characteristics of sold businesses, business sellers and the small businesses for sale market in general, we use the new BBS dataset. Our dataset contains information on SMEs sold and for sale in the US from 2011 to 2018.[13] On the representativity of this dataset as compared to other surveys, we estimate that among sold businesses, 23% to 25% were sold because the owner(s) retired, against 19% in ASE - 2016 (and close to 40%-50% in IBBA reports). Concerning the distribution of selling prices in BBS, we find a mean value of

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[12]This survey is available here: https://www.ibba.org/resource-center/industry-research/.

[13]Available variables are selling/listing prices, cash-flows, gross-revenues, ebitda, number of employees, inventory, date of the first listing, date when the business was sold and main reason to sell. The oldest data had to be recovered through archives and brokers.

579K USD and a median value of 190K USD against respectively 682K USD and 95K USD in the PSID.[14] From this, we conclude that the BBS dataset displays a distribution of middle-sized SMEs, which are comparable to those observed in the data—> comprend pas cette remarque, quelle data ? PSID ?.[15]

We find important frictions on the business sale market with only 10% to 20% successful transactions within a year. Compared to housing, another common illiquid asset, the ratio of sold houses over houses for sale is usually X% within a year, according to Y. [Describe maybe other related data, for other countries using European Commission, etc..]

Whether it is related to the underlying business performance (non-sufficient profitability, cash-flows, etc.) or to potential mismatches (impossibility to find a buyer), not transferring a business is a first-order concern for entrepreneurs planning to retire, and, to a larger extend, it influences employment and payrolls, leading to potential misallocations. By not transferring businesses, some tangible (machinery, property, equipment, furniture, inventory, cash) and intangible assets (customer lists, good credit conditions, business methods, patents and copyrights, trademarks, etc.) will be lost, a fact that has so far attracted only very little attention in the academic literature.

2.4   The Aging of Entrepreneurs

Over the last decades, the entrepreneurship rate has been gradually declining in the US. According to **?**, one explanation is the lower start-up rate of the younger generations, most likely to select into entrepreneurship, due to the changes in the growth rate of the working age population. Moreover, following the recent work of **?**, selection into entrepreneurship follows a hump-shaped pattern by age and peaks around 35 years old. With this explanation, the entrepreneurship rate would have declined due to a composition effect of the US age structure.

The lower start-up rate of younger generations together with the aging of entrepreneurs in the baby-boom cohort contribute to an overall aging of the entrepreneurial population. In figure 2, we display the mass of entrepreneurs (left panel) and workers (right panel) between 1989 to 2016 estimated from the SCF. As compared to the density per age of workers, that remains roughly stable during that period, the fraction of entrepreneurs of age 60 and more rise from 18% in 1989 to 22% in 2004 and 30% in 2016, with an acceleration of the phenomenon in the recent years.[16] Taking entrepreneurs of age 54 and more, those numbers drastically

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[14]Notice that we can not control for self-employment in the BBS dataset. We therefore compare with all the business owners that sold a business in the PSID. Results are similar if we count only entrepreneurs (self-employed business owners). Finally, the comparison with the PSID is indicative, since the number of observations concerning sold businesses is 357 in the PSID (1990 to 2015) against more than 60000 in BBS.

[15]The low number of very-small businesses in the BBS dataset can be explained by the fact that entrepreneurs have to pay a monthly premium membership to list their ad, which might be constraining for very small businesses.

[16]We also look at the mass of individuals not in the labor force (NILF). We find that the fraction of NILF at each

increase from 26% in 1989, to 34% in 2004 and 47% in 2016. That is, around half of the total population of entrepreneurs will retire in the next two decades.[17]

Figure 2. Density of entrepreneurs (left) and workers (right) using SCF (1989, 2004, 2016)
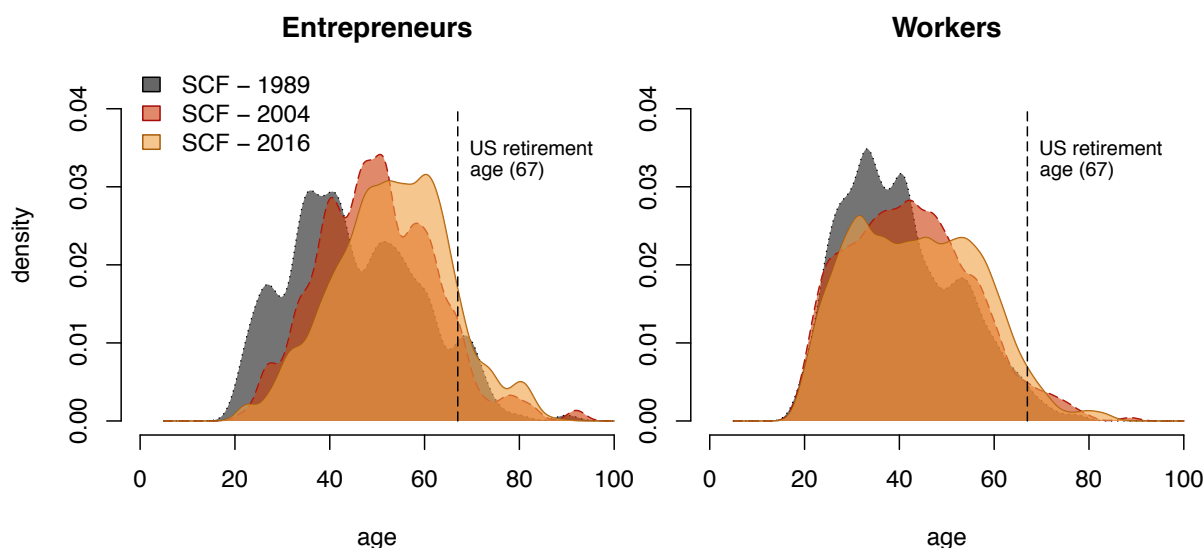
While it is clear that the entrepreneurs' age distribution shifts to the right over time, table 3 documents the importance of the aging of entrepreneurs in terms of business assets and show that approximatively 45% of all business assets is in the hand of older individuals.

Table. 3. Share of business assets and entrepreneurs' age

% of entrepreneurs	1989	2004	2016	source
Entrepreneur of age 60 and over	17	22	**32**	SCF
- share of total business assets held	37	30	**45**	SCF

Finally, the aging of entrepreneurs can also be observed in the leading reasons to cease an entrepreneurial activity documented in the SBO and the ASE. To keep the consistency between those two dataset, we compare the answers reported by the business owners of private businesses with at least one employee, since the ASE do not incorporate self-employment without paid-employee. Results are gathered in table 4. Two facts emerge. First, *business selling* and *retirement* reasons substantially increased between 2007 and 2016. Second, those statistics depend on the number of operating years: old businesses are more likely to cease for *business selling* and *retirement* reasons while younger firms cease more often due to business conditions.

In this section, we have shown that the entrepreneur's life-cycle and the purchase/found and business assets sales are important component of privately held businesses. We also

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age remains roughly stable between 1989, 2004 and 2016.

[17]We find very consistent evidence using the CPS, as shown in Appendix–> lequel.

Table. 4. Entrepreneurs' reasons for ceasing their activity check later the numbers [a]

	SBO (2007)	ASE (2016)		
	All	**All**	**< 2 years**	**> 5 years**
% of business owners ceasing per year (%)	5.5	6.4	8.6	5.5
business conditions (%)	44 .0	22.7	35.1	16.5
owner(s) retired (%)	9.9	18.9	3.5	27.3
owner(s) sold the business (%)	11	17.9	11.0	21.0
other reasons [b] (%)	35.1	34.2	50.4	35.2

[a] As the question related to the ceasing reasons allow for multiple choices. We normalize the number in terms of the total number of answers.

[b] Contains reasons linked to illness, injury or died, started another business and another reasons.

showed that the sale of businesses is a long process with lots of unsuccessful attempts. We now propose a framework to study the business sale market from the point of view of the entrepreneurs. –> j'aurais enlevé ca
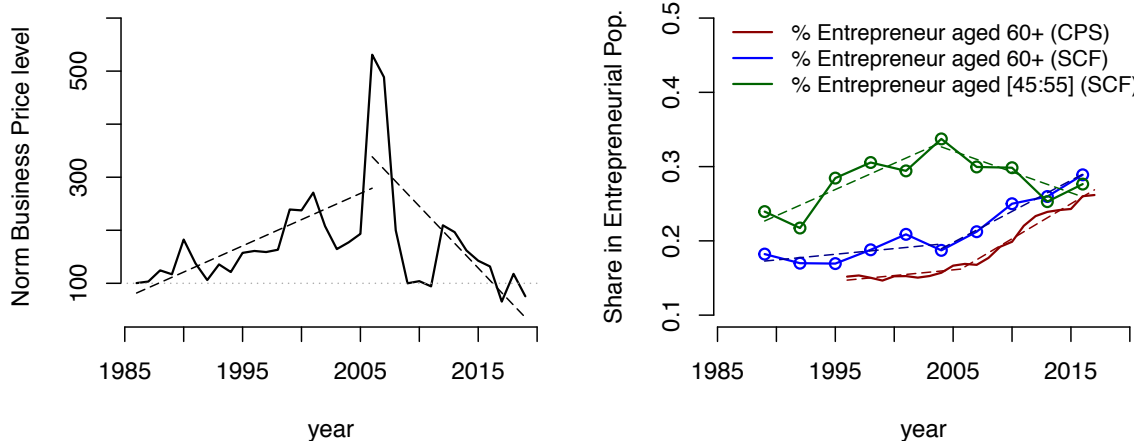
3   Model

The economy consists of a corporate sector and a unit measure of heterogenous agents. A fraction of the latter, called entrepreneurs, hold small and medium sized businesses while the remaining, called workers, occupy a wage paying job in the corporate sector. Entry and exit in and out of the small business sector is subject to specific conditions. On the one hand, individuals entering self-employment have to either found a new business or purchase an existing one. On the other hand, upon exit, entrepreneurs can either sell their business or liquidate the physical business assets. Therefore, a *small and medium-sized enterprises for sale* market (*SMESM*) constitutes a pivotal piece of our model. Finally, a government levies a menu of taxes to cover

for old age pensions and other public spendings.

3.1 Aggregate Variables and Corporate Sector

The corporate sector output Y is produced by a single competitive representative firm using a Cobb-Douglas technology with capital share $\alpha \in (0,1)$ and total factor productivity A, capital level K_c and labor L_c, such that: $Y = F(K_c, L_c) = AK_c^\alpha L_c^{1-\alpha}$. Capital depreciates at rate δ in both the corporate and the SME sectors.

The model is characterized by a distribution of agents Φ and a set of prices. Factor prices are the interest rate $r = F'_{K_c}(K_c, L_c)$ and the wage rate $w = F'_{L_c}(K_c, L_c)$. Finally q is the private business price clearing the $SMESM$ market. Because we are interested in the impact of demographic evolutions and thus will not employ a stationary calibration, model variables can be subject to specific aggregate changes. We regroup these changes in the set of aggregate variables Ω. Examples of aggregate variables include demographic changes or aggregate productivity changes. In essence, these aggregate variables have an effect of the distribution of agents and prices over time.[18] However, for the sake of clarity, we drop in our notations this dependence and write $\Phi = \Phi(\Omega)$, $r = r(\Phi, \Omega)$, $w = w(\Phi, \Omega)$ and $q = q(\Phi, \Omega)$. A stationary equilibrium is attained by assuming that the variables in Ω are constant.

3.2 Agents

We use a stylized life-cycle setup with multi-generational probabilistic aging. Households live through nine stages of life and the total population, of unit mass, is divided among 9 generations indexed with $j \in [1;9]$. Each generation retains striking features of their counterparts in an actual life cycle economy. For notations purposes, groups 1 through 8 put together are called Juniors and have access to the labor market. On the other hand, we call Seniors the 9th group comprised of individuals beyond the retirement age. Each period, Junior households face a constant probability $P(j'|j)$ of aging and reaching the next generation. At the same time, we assume that a fraction p_{die} of the Seniors decease and exit the model.[19] Over the life-cycle, households belong in an occupation $o \in \{o_e, o_w, o_r\}$. Junior households can be entrepreneurs (o_e) or occupied in the workforce (o_w) whereas Senior households are either retired (o_r) or are old age entrepreneurs.

Households have preferences over consumption c described by utility:

$$\mathcal{U}(c, j, o) = u(c) + \mathbb{1}_{o=o_e}(u_E - \mathbb{1}_{j=9}u_R)$$

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[18]Our calibration strategy consists in replicating the demographic transition starting in the 1970's and 1980's.

[19]This assumption is widely used in the literature, see **?** for an application in housing models or **?** in a related literature.

Senior households also face an additional utility cost u_R when operating a business, in order to translate the difficulty of still being active in old age. Finally, following **?**, we allow for non-pecuniary benefits on top of the business activity taking the form of an additional utility component u_E.[20]

Depending on its occupation, a household can possess liquid and/or illiquid assets. Liquid assets are akin to savings and are noted a. Illiquid (business) assets, noted k, are used to produce with the entrepreneurial technology. Both the corporate and the entrepreneurial sector produce a homogenous consumption good. The liquid asset can be freely used to purchase it but not the illiquid asset. Our setup explicitly defines conditions to convert illiquid capital into liquid assets and conversely. To obtain liquid assets from illiquid assets, individual have to either sell their firm contingent on finding a buyer or liquidate partly or totally subject to an adjustment cost. Conversely, acquiring illiquid capital using liquid capital is subject to an adjustment cost but can be also achieved by buying a firm with a specific illiquid capital amount.

The state space for an entrepreneur are savings a, business capital k, and $\mathbf{x}_e = \{j, m\}$, where $m = \{0, 1\}$ indicates whether the business is mature. Entrepreneurs are not permitted to possess multiple firms. Similarly, the state space for a worker is a, and $\mathbf{x}_w = \{j, y, \iota\}$, with y the worker's productivity and ι her entrepreneurial ability to manage a business. We note $\mathcal{Y}(j, y)$ the worker's income. The entrepreneurial income comes from entrepreneurial production using production function $f(k)$.

Note that for the sake of parsimony, we abstract from entrepreneurs hiring workers but it remains a straightforward extension. We also abstract from the explicit representation of unemployment dynamics in the economy, mainly for the sake of simplicity.

3.3 Dynamic Problem

We decompose an agent's intra-period decision process into a sequence of three subperiods. In the last subperiod, the consumption-saving and entrepreneurial investment problems are tackled. In the middle subperiod, the buying and selling problems are addressed contingent on occupational changes and the maturity of a business.[21] Finally, in the first subperiod occupational choices are made. Given that $\mathcal{W}(a, \mathbf{x}_w)$ and $E(a, k, \mathbf{x}_e)$ are respectively the general value function of a worker and an entrepreneur, Figure 3 summarizes this decomposition. Following this decomposition, the problem is solved backwards and we detail below the problem in each subperiods.

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[20]This component is used to match the data and based on the fact that individuals have a preference for being self-employed. In a related model, **?** also incorporate such preferences.

[21]Only mature firms ($m = 1$) can be sold on the *SMESM*. This is to reflects the fact that the average age of sold businesses is much higher than the average age of all firms.
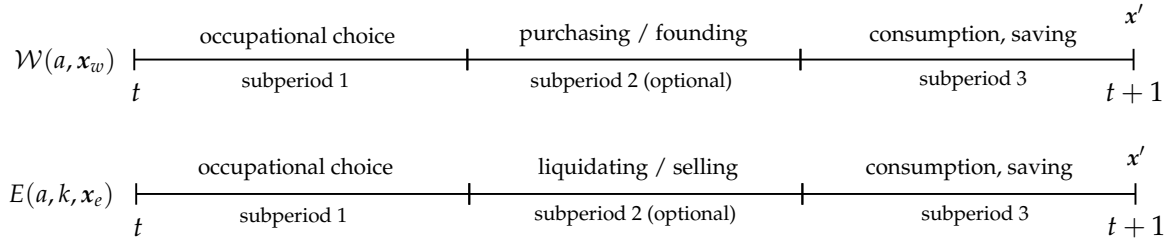
$\mathcal{W}(a, \boldsymbol{x}_w)$ — occupational choice (subperiod 1, t) — purchasing / founding (subperiod 2 (optional)) — consumption, saving (subperiod 3) — x' ($t+1$)

$E(a, k, \boldsymbol{x}_e)$ — occupational choice (subperiod 1, t) — liquidating / selling (subperiod 2 (optional)) — consumption, saving (subperiod 3) — x' ($t+1$)

Figure 3. Intra-period timing of an individual.

3.3.1 The last subperiod: Consumption-Saving problem

Depending on choices made in the previous subperiods, consumption and saving decisions in the last subperiod can be distinguished into those of workers either continuing or exiting their activity and those of entrepreneurs continuing or exiting theirs. For the sake of simplicity, continuing workers are subject to a no-borrowing constraint. Similarly to an incumbent entrepreneur, an exiting worker entering an entrepreneurial activity can borrow in order to invest in a level of business assets k, as long as a minimum amount is pledged using their own wealth. Thus those individuals are subject to the following borrowing constraint:[22]

$$a' \geq -\psi(k) \tag{1}$$

$$\psi(k) = (1-\theta)qf(k) \tag{2}$$

An indebted entrepreneur faces an interest $r_b(m)$ that depends on the maturity m of her business such that $\tilde{r}(m) = \mathbb{1}_{a' \geq 0}r - \mathbb{1}_{a' < 0}r_b(m)$.

Continuing entrepreneurs An incumbent entrepreneur continuing her activity chooses next period's illiquid capital k' and saving a' given her current income $f(k)$. Investing or disinvesting in the illiquid capital are subject to a capital adjustment cost noted $\mathcal{C}(k, k', \phi^0)$ with ϕ an adjustment cost parameter. The consumption-saving problem of this entrepreneur is thus:

$$E^c(a, k, \mathbf{x}_e) = \max_{c>0,\, a' \geq -\psi(k'),\, k' \geq 0} \left\{ \mathcal{U}(c, j, o_e) + \beta \mathbb{E}_{j', m'|j, m} E(a', k', \mathbf{x}'_e) \right\} \tag{3}$$

$$s.t. \quad c + a' + k' = (1 + \tilde{r}(m))a + f(k)(1-\tau_y) + k(1-\delta) - \mathcal{C}(k(1-\delta), k', \phi^0) \tag{4}$$

with E^c the subperiod specific value function of this entrepreneur and τ_y the income tax rate.

Exiting entrepreneurs When exiting, an entrepreneur has to choose savings a' subject to the no-borrowing constraint. Depending on whether the entrepreneur is exiting entrepreneurship

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[22]This kind of borrowing constraint specification is widely used in the literature. See for instance the papers by **?** or **?**.

by voluntarily liquidating ($z = 0$), is forced to liquidate because of business failure ($z = 1$) or is selling the business ($z = 2$), we have:

$$E^e(a, k, \mathbf{x}_e, z) = \max_{c>0,\, a'\geq 0} \left\{ \mathcal{U}(c, j, o_e) + \beta \mathbb{E}_{j',\iota'|j} \mathcal{W}(a', \widetilde{\mathbf{x}}'_w) \right\} \tag{5}$$

$$s.t. \quad c + a' = (1 + \tilde{r}(m))a + f(k)(1 - \tau_y) + \mathbb{1}_{z=\{0,1\}}(k(1 - \delta) - \mathcal{C}(k(1 - \delta), 0, \phi^z))$$

$$+ \mathbb{1}_{z=2}q(k(1 - \delta))(1 - \tau_s) \tag{6}$$

with E^e the subperiod specific value function of this entrepreneur and $\widetilde{\mathbf{x}}'_w$ the specific exogenous worker state of an exiting entrepreneur.[23] Liquidating is identical to adjusting the business capital to zero by fully paying the corresponding adjustment cost $\mathcal{C}(k(1 - \delta), 0, \phi^z)$. Alternatively, by successfully selling the business the entrepreneur recovers the total amount $q(k)$, subject to the sales tax τ_s.

Continuing workers Such a worker has to choose savings a' subject to the no-borrowing constraint and solves:

$$\mathcal{W}^c(a, \mathbf{x}_w) = \max_{c>0,\, a'\geq 0} \left\{ \mathcal{U}(c, j, o_w) + \beta \mathbb{E}_{j',y',\iota'|j,y,\iota} \mathcal{W}(a', \mathbf{x}_w) \right\} \tag{7}$$

$$s.t. \quad c + a' = (1 + r_s)a + (1 - \tau_y)\mathcal{Y}(j, y) \tag{8}$$

with $\mathcal{W}^c$ the subperiod specific value function of this worker.

Exiting workers An exiting worker enters entrepreneurship with business assets k' in the next period, either by purchasing an existing mature business ($d = 1$ and $m' = 1$) and paying the total amount $q(k')$ or by founding a new business ($d = 0$ and $m' = 0$) and paying the adjustment cost $\mathcal{C}(0, k', \phi^0)$ with d the index for the type of acquisition. This worker solves:

$$\mathcal{W}^e(a, k', \mathbf{x}_w, d) = \max_{c>0,\, a'\geq -(1-\theta)k'} \left\{ \mathcal{U}(c, j, o_w) + \beta \mathbb{E}_{j'|j} E(a', k', \mathbf{x}'_e) \right\} \tag{9}$$

$$s.t. \quad c + a' = (1 + \tilde{r}_s)a + (1 - \tau_w)\mathcal{Y}(j, y) - dq(k')(1 + \tau_b) - (1 - d)(k' + \mathcal{C}(0, k', \phi^0)) \tag{10}$$

with $\mathcal{W}^e$ the subperiod specific value function and τ_b the business purchase tax.

3.3.2 The middle subperiod: acquisition and selling problems

In the middle subperiod the buying/founding and selling/liquidating problems are solved.

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[23]The main specificity is the assumption that any new worker coming from the entrepreneurial sector starts with the lowest level of worker productivity. The argument is that the productivity state y is strongly related to a worker's experience in a specific corporate job. This seniority on a job can not be randomly obtained but has to be *earned*. Recall, however, that there is an age-component in the determination of the wage process. Moreover, in this case the entrepreneurial ability is determined using the invariant distribution of this process.

The selling problem An entrepreneur with a mature business ($m = 1$) can try to sell it on the *SMESM*. A buyer is found with probability $h_s(q)$. Otherwise, the entrepreneur chooses whether to liquidate or continue the business. In the end, when the entrepreneur exits endogenously, the following problem is solved :

$$\mathcal{S}(a, k, \mathbf{x}_e) = h_s(q) \underbrace{E^e(a, k, \mathbf{x}_e, 2)}_{Selling} + (1 - h_s(q)) \max \left\{ \underbrace{E^e(a, k, \mathbf{x}_e, 0)}_{Liquidating}, \underbrace{E^c(a, k, \mathbf{x}_e)}_{Continuing} \right\} \tag{11}$$

and the following alternative problem when the entrepreneur is forced to exit:

$$\widetilde{\mathcal{S}}(a, k, \mathbf{x}_e) = h_s(q) \underbrace{E^e(a, k, \mathbf{x}_e, 2)}_{Selling} + (1 - h_s(q)) \underbrace{E^e(a, k, \mathbf{x}_e, 1)}_{Liquidating(forced)} \tag{12}$$

where $\mathcal{S}$ and $\widetilde{\mathcal{S}}$ are the subperiod specific value functions for this problem.

The founding and purchasing problem Depending on whether a worker is currently buying ($d = 1$) or founding ($d = 0$) a business, the problem of the future entrepreneur is to choose the optimal capital size k':

$$\widetilde{\mathcal{W}}(a, \mathbf{x}_w, d = 0) = \max_{k'} \mathcal{W}^e(a, k', \mathbf{x}_w, d = 0) \tag{13}$$

$$\widetilde{\mathcal{W}}(a, \mathbf{x}_w, d = 1) = \max_{k' \geq \overline{k}} \mathcal{W}^e(a, k', \mathbf{x}_w, d = 1) \tag{14}$$

We assume that businesses have to be larger than a minimum size $\overline{k}$ in order to be bought and we parameterize this minimum size on the empirical data. A worker trying to buy a business has a probability $h_b(q)$ of finding a seller. Otherwise, she chooses whether to found a new business or to continue being a worker. In the end, the following problem is solved:

$$\mathcal{B}(a, \mathbf{x}_w) = h_b(q) \underbrace{\widetilde{\mathcal{W}}(a, \mathbf{x}_w, 1)}_{Buying} + (1 - h_b(q)) \max \left\{ \underbrace{\widetilde{\mathcal{W}}(a, \mathbf{x}_w, 0)}_{Founding}, \underbrace{\mathcal{W}^c(a, \mathbf{x}_w)}_{Continuing} \right\} \tag{15}$$

with $\mathcal{B}(a, \mathbf{x}_w)$ the subperiod specific value function for this problem.

3.3.3 The first subperiod: occupational choice and exit strategy

Worker A worker starts the period with states $\{a, \mathbf{x}_w\}$ and, provided she has an entrepreneurial ability (i.e. $\iota = 1$), chooses whether to try to purchase an existing business, to found a new business or to remain a worker.[24] In the end, the following problem is solved:

$$\mathcal{W}(a, \mathbf{x}_w) = (1 - \iota)\mathcal{W}^c(a, \mathbf{x}_w) + \iota \max \left\{ \mathcal{B}(a, \mathbf{x}_w) - u_b, \widetilde{\mathcal{W}}(a, \mathbf{x}_w, 0), \mathcal{W}^c(a, \mathbf{x}_w) \right\} \tag{16}$$

with u_b the implied small utility cost of searching for a seller.

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[24]The entrepreneurial ability follows a first order Markov process with two states: possessing ($\iota = 1$) or not possessing ($\iota = 0$) the ability. An alternative specification would be to set an exogenous probability of drawing an entrepreneurial ability each period. However, our specification allows for a persistent entrepreneurial ability that generates higher saving rates and brings the model closer to the data.

Entrepreneur An entrepreneur starts the period with states $\{a, k, \mathbf{x}_e\}$ and decides whether to sell, liquidate or continue her business endogenously unless she is forced to exit. In the end, the following problem is solved:

$$E(a, k, \mathbf{x}_e) = (1 - \chi(m)) \Big[\zeta \Big(m \max\{\mathcal{S}(a, k, \mathbf{x}_e) - u_s, E^e(a, k, \mathbf{x}_e, 0)\} + (1 - m) E^e(a, k, \mathbf{x}_e, 0) \Big)$$

$$+ (1 - \zeta) \Big(m \max\{\mathcal{S}(a, k, \mathbf{x}_e) - u_s, E^e(a, k, \mathbf{x}_e, 0), E^c(a, k, \mathbf{x}_e)\}$$

$$+ (1 - m) \max\{E^e(a, k, \mathbf{x}_e, 0), E^c(a, k, \mathbf{x}_e)\} \Big) \Big] + \chi(m) E^e(a, k, \mathbf{x}_e, 0) \qquad (17)$$

with u_s the implied small utility cost of searching for a buyer. ς is the unconditional probability of entrepreneurial exit whereas $\chi(m)$ is the conditional probability of entrepreneurial exit due to business failure, assuming that the maturity of the business is an important determinant of the latter probability. Only businesses that does not fail can be sold.

3.4 The small and medium-sized enterprises for sale market (SMESM)

On the *SMESM*, businesses sellers and buyers meet. For the sake of tractability, we make a number of assumptions:

(*i*) The market is intermediated by passive brokers on both the selling and the buying side. With this assumption, we avoid the complex problem of the direct matching between heterogenous buyers and sellers. Moreover, the brokers provide liquidity to the market.

(*ii*) We assume that firms are valued and exchanged in this market as units of production. A firm generating a production $f(k)$ will have each unit of production valued at the equilibrium price q (i.e. the price of a business of size k is $qf(k)$).Selling a firm means providing all the units of production owned by an entrepreneur at the same time to the market. But, as units of production are undistinguishable, buying a firm is equivalent to collecting available units of production until the endogenously decided production size is attained. This is a key assumption and we argue that a number of reasons supports it. First, it lets us recover in a stylized way that entire businesses are exchanged without loosing global production value.[25] Second, we find empirical evidence that in many instances SMEs are bought not by a single individual but by several individuals grouped together.—> je pense on a besoin de justifier par citation ou data cette affirmation Our assumption lets us recover this fact: a sold business can be bought by many individuals as bundles of units of production. In the aggregate, we thus have a seamless transfer of the value of a

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[25]If for instance units of illiquid capital were exchanged instead, the aggregate total production value, bought as a sum of smaller units, would diminish due to the (generally decreasing) returns to scale of the entrepreneurial production function.

business and when it disappears we do not need to track each of its profit units. Finally, it lets us capture that selling a business cannot be reduced to selling only its tangible assets. Instead, the value recovered after a transaction should cover the discounted value of future profits and as argued for instance by **?**, intangible assets constitute a large part of this value.[26] We convey this idea here through the fact that the price q is an abstract object. It is determined at the global equilibrium between the amount of units of production sold and bought translating at the same time the intertemporal and intangible value of a business.[27]

(iii) Sellers, buyers and brokers are price takers. We assume that a seller upon meeting a broker agrees on a selling price that is $q(1 - \mu_s) \leq q$. Symmetrically, a buyer upon meeting a broker agrees a buying price $q(1 + \mu_b) \geq q$. where μ_s and μ_b represents commissions that have to be paid by the seller and the buyer to brokers. Brokers exchange units of production at equilibrium price q among themselves. The equilibrium of the market also solves the matching problem as shown below.

We call sellers side tightness θ_s the brokers to sellers ratio. We use a standard matching technology with constant returns to scale with parameter γ. The probability that a broker matches a seller is $\alpha_s(\theta_s) = (\frac{1}{\theta_s})^{1-\gamma}$, computed as the ratio of matches to brokers. Conversely, the probability that a seller meets a broker is $h_s = \theta_s^{\gamma}$, computed as the ratio of matches to sellers. γ is a matching function parameter. The brokers' free entry condition states that they have to equalize the per unit entry cost κ_s with net revenues from buying a business given the probability of a match. The buyers' side is symmetric with tightness θ_b. The probability α_b that a broker matches a buyer is $\alpha_b = \alpha_b(\theta_b) = (\frac{1}{\theta_b})^{1-\gamma}$. The probability that a buyer meets a broker is $h_b = \theta_b^{\gamma}$. The free entry condition on the two sides of the market are:

$$\kappa_s \pi(k) = \underbrace{\alpha_s(\theta_s)}_{\text{Probability of a match}} \underbrace{q\mu_s f(k)}_{\text{Net revenue}}$$

$$\kappa_b \pi(k) = \underbrace{\alpha_b(\theta_b)}_{\text{Probability of a match}} \underbrace{q\mu_b f(k)}_{\text{Net revenue}}$$

From the above broker condition we can extract the tightness as $\theta_s = \left(\frac{q\mu_s}{\kappa_s}\right)^{\frac{1}{1-\gamma}}$ and $\theta_b = \left(\frac{q\mu_b}{\kappa_b}\right)^{\frac{1}{1-\gamma}}$. The seller's selling probability is thus: $h_s(q) = \left(\frac{q\mu_s}{\kappa_s}\right)^{\frac{\gamma}{1-\gamma}}$ and the buyer's buying probability is thus: $h_b(q) = \left(\frac{q\mu_b}{\kappa_b}\right)^{\frac{\gamma}{1-\gamma}}$. Note that there is an implicit assumption that $q >$

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[26]Using the VS data—> c koi VS data ?, we estimate a ratio of intangible assets over the business price of about 38% for the median and 54% for the mean.

[27]**?** find that there is little cross-sectional dispersion in intangible assets valuation, supporting our choice of a single price q for all production units.

1.[28] Moreover these conditions create a link between the price of a unit of private business production and the probabilities of finding a buyer and a seller.

3.5 Demography and bequest

We assume that individuals in the Senior group value leaving a bequest and that this valuation is obtained with a warm-glow utility $\widehat{u}\big((1-\tau_a)a\big)$, with τ_a the estate tax rate. Seniors have a probability p_{die} of dying such that their continuation value $V(\mathbf{x})$ is defined as:

$$V(\mathbf{x}) = (1 - p_{die})V(\mathbf{x'}) + p_{die}\Lambda\widehat{u}\big((1-\tau_a)a\big) \tag{18}$$

with Λ a parameter controlling their degree of altruism. We assume for each such death, a new worker with a financial wealth equal to the bequest minus the estate tax appears in the economy as a Junior from the 1st group.

3.6 Government

The government collects revenues from income taxes on labor and entrepreneurial earnings and pensions as well as from the product of estate taxation and revenues from taxes on the sale and the purchase of a business. Government expenditures comprises an exogenous government spending proportional to aggregate output, $G = \bar{G}Y$ and pensions. The government budget constraint is:

$$\int_{\mathbf{x}_w} \Big(\mathcal{Y}(y,j)\tau_y + \mathbb{1}_{d=1}qf(k)\tau_b\Big) d\Phi(\mathbf{x}_w) + \int_{\mathbf{x}_e} \Big(f(k)\tau_y + \mathbb{1}_{z=2}qf(k)\tau_s\Big) d\Phi(\mathbf{x}_e) +$$
$$\int_{\mathbf{x}} \mathbb{1}_{j=9}p_{die}\tau_a a\, d\Phi(\mathbf{x}) = \bar{G}Y + \int_{\mathbf{x}_w} \mathcal{Y}(y,9)\, d\Phi(\mathbf{x}_w) \tag{19}$$

with $\Phi(.)$ a measure over agents of the specified type.

3.7 Equilibrium

A Recursive Equilibrium in this economy consists of a set of agent's decision rules, a distribution $\Phi(\mathbf{x},\Omega)$ of agents, factor prices $\{w(\Omega,\Phi), r(\Omega,\Phi)\}$, a price $q(\Omega,\Phi)$ for a unit of business production and government spending G such that:

- The decision rules $a'(\mathbf{x},\Omega), k'(\mathbf{x},\Omega), d(\mathbf{x}_w,\Omega), z(\mathbf{x}_e,\Omega)$ solve the agent's problem.

- The distribution of agents $\Phi(\mathbf{x},\Omega)$ is induced by the transition matrix of the system $M(x',\Phi',\Omega'|x,\Phi,\Omega)$. A steady state is implied by a constant aggregate behavior $\Omega' \equiv \Omega$ such that $\Phi(\mathbf{x},\Omega)$ is stationary.

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[28]This is likely to be the case since a business can be kept several periods. Therefore, an entrepreneur enjoy the profit of a business of size k during the current period and the next ones. In a model with infinitely lived firms discounting the future at rate r and pricing the business according to the stream of future profits, with no exit probabilities and constant capital size, the price would be $q\pi(k)$, with $q = \frac{1-r}{r}$ reflecting the discount rate.

- The government budget constraint in (19) is balanced with $\bar{G}$.

- The labor market clears and total labor demand by the corporate sector equals household labor supply. The wage is determined by the marginal productivity of labor in the corporate sector, such that $L_c = \int_{\mathbf{x}} \mathbb{1}_{\{o=o_w\}} d\Phi(\mathbf{x}, \Omega)$.

- The capital market clears. Corporate capital and the total entrepreneurial capital equate total agent's net worth in the economy. The interest rate is determined by the marginal productivity of capital in the corporate sector.[29]

$$K_c + \int_{\mathbf{x}_e} k(\mathbf{x}_e) d\Phi(\mathbf{x}_e, \Omega) = \int_{\mathbf{x}} a(\mathbf{x}) d\Phi(\mathbf{x}, \Omega) \tag{20}$$

- The *SMESM* clears. The price q clears the market such that: $\int_x \mathbb{1}_{z(\mathbf{x})=2} h_s(q) f(\mathbf{x}) d\Phi(\mathbf{x}, \Omega) = \int_x \mathbb{1}_{d(\mathbf{x})=1} h_b(q) f(\mathbf{x}) d\Phi(\mathbf{x}, \Omega)$ and $\{\theta_b(q(\Omega, \Phi)), \theta_s(q(\Omega, \Phi))\}$ adjust accordingly.

This problem has no analytical solution and has to be solved numerically. Two major problems arise in our setup. First, the dimensionality of the problem with two-assets is itself a challenge and one would refer to fast optimization method in order to solve the model. Second, due to the presence of both discrete (occupational choice) and continuous choices, FOCs are no longer sufficient (while still necessary). To encompass these issues, we modified the recent implementation of *DC-EGM* as introduced in **?** and extended it to multiple assets as in **?**. While making the model tractable, this also substantially increases the speed of the whole algorithm.[30,] We provide details on the pertubated solution we use to solve the model using the DC-EGM procedure in appendix **??**.

4 Parameterization

We have three broad goals in setting the parameters of the model. First we need to replicate the occupational choice between the two alternatives of self-employment and paid-employment. Second, we seek a calibration of the exogenous stochastic process for labor earnings, which fits the lifetime pattern as observed in the data. Finally, in order to obtain quantitatively realistic business dynamics, our model must generate realistic exit and entry transitions from entrepreneurship. Of particular importance are the distributions of business buyers and business sellers, as well as the fraction of entrepreneurs stopping their activity each year.

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[29]Profit maximization implies: $r = A\alpha \left(\frac{L_c}{K_c}\right)^{1-\alpha} - \delta$ and $w = A(1-\alpha)\left(\frac{K_c}{L_c}\right)^{\alpha}$, with w and r the wage and interest rates, which by a no arbitrage condition are identical in the entrepreneurial sector.

[30]Our model equilibrium is solved on average in ten minutes for reasonably fine grids for both illiquid capital and liquid net worth using a reasonably fast desktop computer.

Preferences   We use the CRRA utility function $u(c) = \frac{(c^{1-\sigma} - 1)}{1-\sigma}$ with relative risk aversion $\sigma = 1.5$. The benchmark altruism parameter $\Lambda = 0$ while the preference parameter u_R.[31]

$$\mathcal{U}(c, j, o) = \frac{(c^{1-\sigma} - 1)}{1 - \sigma} - \mathbb{1}_{j=J, o=o_e} u_R \tag{21}$$

Labor income and labor frictions   In the model, labor income allows workers to accumulate savings at different rates, especially important for the decision to switch toward entrepreneurship.[32]

$$= wyh(j) \tag{22}$$

We define labor earnings as a function of an age-dependent component, the wage level and a persistent stochastic process for labor productivity, such that:

$$\log(\mathcal{Y}(j, y)) = \log(w_t) + \log(y_{i,t}) + \log(h(j)_{i,t}) \tag{23}$$

$$\log(y_{i,t}) = \rho_y \log(y_{i,t-1}) + \epsilon_{i,t}^y; \qquad \epsilon_{i,t}^y \sim \mathcal{N}(0, \sigma_y) \tag{24}$$

where $h(j)$ with $j = 9$ defines the retirement pension that we fix to 40% of the average income in the economy. Once retired, an agent keep the same y component forever and the offspring's working productivity is drawn from the invariant distribution $F(y)$. We model the age-dependent components $h(j)$ for $j \in \{1, ..., 40\}$ in order to replicate the average lifetime earning profile within each earning percentile as observed in the PSID between age 25 and age 65, such that $h(j) = \alpha_0^j + \alpha_1^j(j + 24) + \alpha_2^j(j + 24)^2$. We estimate $\alpha_0^j = 0.413$, $\alpha_1^j = 0.045$ and $\alpha_2^j = -0.001$.

The probability of aging from j to j' for any $j \neq R$ is set to 1 for $j \in 1, .., 40$ and the probability of dying p_{die} is set to 0.091 (corresponding to an expected retirement period of 11 years as in **?**). The logarithm of productivity y follows an AR(1) process with autocorrelation ρ_y and standard deviation σ_y. We discretize the process by fixing the value of $\rho_y = 0.96$ and adjusting the variance of σ_y to generate an earning Gini of 0.36.[33]

Business maturity   We have assumed two types of maturity within the model: mature and immature (early-stage) businesses. We fix the probability to switch from early stage business to a mature business to 20% (about 5 years in operation). On top of the possibility to be sold, a

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[31]We do not specify the functional form of the warm-glow utility function for the benchmark calibration, as $\Lambda = 0$.

[32]Three saving motives arise in the model. A precautionary one due to the inherent productivity risk, a life-cycle one and an accumulation motive in order to become an entrepreneur.

[33]Notice that this Gini is not necessarily what is obtained in the model, after the individuals have made their occupational choice. Moreover, this Gini is the one implied by the process y, which may differ from the earning Gini produced by the model once we take into account the age component. The results are not very sensitive to the process y, but it helps to obtain some moments related to wealth.

mature business is assumed to pay a lower interest rate, translating the higher amount of information that a creditor have access to (i.e. history of past transactions, a list of customers etc.). We therefore set $r_b(m) = r + v_s + v_m \mathbb{1}_{m=0}$, where v_s is a wedge common to all businesses, while v_m is the additional interest rate premium charged on immature business. We use the 2003 SSBF data in order to estimate the effect of the type of acquisition (purchased versus founded) on the interest rate charged on the businesses, with the underlying assumption that purchased business are, on average, well established since many years. We then run the following regression on the sample of early stage firms (acquired in the last 5 years) and the full sample of businesses:

$$int.rate_i = \alpha_i + \beta_i D(purchased)_i + X_i + u_i \tag{25}$$

where $D(purchased)_i$ is a dummy variable indicating whether a business as been purchased by the current owner or founded. X_i is a vector of controls for both the business (employment and capital size, sector etc.) and the owners (net worth, age, education, sex, entrepreneurial experience, past default history etc.) and u_i is an error term. Table 5 summarizes the results.

| | Interest rate (%) | |
| | All firms | Young firms (<5y) |
	(1)	(2)
Purchased	−0.58***	−1.59***
	(0.13)	(0.28)
Controls	Yes	Yes
N	8,919	1,811
R^2	0.12	0.31

Notes: p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation.

Table. 5. Interest rate, credit limit and type of acquisition.

We find a significative relationship between the type of acquisition and the interest rate. Recent owners having purchased an existing business are charged, on average, an interest rate 1.59% smaller than those who founded their business. Taking the sample of all firms, we consistently find that purchasers are charged an interest rate smaller than founders, but the effect is lower than for young firms. We interpret these findings in the following way: financial intermediaries lower the premium charged on their loans when they are able to infer business characteristics over a longer period. Following these findings, for the interest rate charged by creditors, we set $v_s = 2\%$ and, according to the point estimate, we set $v_m = 1.6\%$.[34]

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[34]In the baseline model, we do not take in account different credit limits. However, an extended version of the model accounting for this features could generate additional misallocation effects concomitantly to the destruction of older mature businesses.

Finally, we also assume that mature and immature businesses display different probability $\chi(m)$ to fail. We assume that the probability of failing is 21% for early stage firms, while it is 5% for mature businesses. Those two numbers are chosen in order to closely reproduce the observed business survival rates in function of the number of years of existence. We then adjust ζ, the unconditional probability of entrepreneurial exit (independently of the business stage), in order to match data moments relative to the share of businesses sold and the entrepreneurial exit rate.

Technology   Private businesses produce according to the technology $f(k) = k^\nu$ with $\nu < 1$. The capital share in the corporate sector is $\alpha = 0.33$ and the capital depreciation rate in both sectors is $\delta = 0.1$. whereas the corporate sector features a constant returns to scale Cobb-Douglas production function with capital share $\alpha = 0.33$, total factor productivity A, capital level K_c and labor L_c, such that: $Y = F(K_c, L_c) = AK_c^\alpha L_c^{1-\alpha}$. Capital depreciates at rate $\delta = 0.1$ in both sectors.

Adjustment costs and liquidation   When investing in an incumbent business or when founding a new business, owners of privately held businesses are assumed to pay a fixed cost τ_u on each unit of positive investment, such that $C(k, k', \tau) = \tau_u(k' - k)$. The parameter τ_u over the adjustment cost is calibrated endogenously such as to reproduce the aggregate fraction of private businesses investing a positive capital amount.

When liquidating the business, the entrepreneur is assumed to recover a fraction τ_d of the value of the private business. We fix the parameter τ_d to 70% of the business value. We perform sensitive analysis on the effect of such parameters in the Appendix–> dire ou exactement.On top of determining the tradeoff between buying or founding a new business, as well as liquidating the business assets or selling the business, together with the matching frictions imposed by P_s and P_b. For the sake of simplicity, we make the assumption of non-convex fixed costs in our baseline economy, for both investment and disinvestment.[35] We use the following cost structure:

where we fix the value $\tau_{up}^s = 0.04$ and we calibrate endogenously the value of τ_{down} —> a changer suite a l'introduction de proba de exit because failure .[36].

Matching probabilities   The parameters μ_b and μ_s are set to 0.1 of the price, meaning that brokers charge a commission of 10%, consistent with what is reported on *Bizbuysell*. The pa-

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[35]We extend our model to account for convex non linear adjustment costs in an alternative model specification.

[36]The non-symmetric cost structure that we choose is motivated by the observation made by **?** using the KFS data. In his paper, $\tau_{up} = 0.034\%$ and $\tau_{down} = 0.32$. However, his model does not account for matching frictions that can also limit the entry of new entrepreneurs. As we will fix the value of P_s, the value of τ_{down} helps to match the fraction of entrepreneurs who sold their business in equilibrium.

rameters $A_s = \kappa_s^{\frac{\gamma-1}{\gamma}}$, $A_b = \kappa_b^{\frac{\gamma-1}{\gamma}}$ are determined jointly to match the ratio of sellers to exiting entrepreneurs and the ratio of buyers to new entrants. γ is estimated by OLS from the data using the following underlying specification:

$$log(Proba_t^{sell}) = \alpha_s + \frac{\gamma}{1-\gamma}log(q_t) + \epsilon_t \tag{26}$$

where $log(Proba_t^{sell})$ is computed as the probability to sell the business during a quarter (i.e. as proxied by 91 days over the time needed to sell the business) and q_t is the price index, defined as the average ratio of the sale price over the EBITDA, such as:

$$q_t = \sum_i^N \frac{Price_{i,t}}{EBITDA_{i,t}} \frac{1}{N} \tag{27}$$

We find an estimated value for the elasticity γ of about 0.20.

of private businesses, after controlling for business characteristics in the BBS data, estimated as:

$$price_{i,t} = \tilde{q}_t + \boldsymbol{\beta X_{i,t}} + u_{i,t} \tag{28}$$

with $q_t = \tilde{q}_t - \tilde{q}_0$ the price index of private businesses at date t. We assume a simple matching structure on the business sale market which is summarized by two key probabilities: P_s, the probability that a business seller find a buyer, and P_b the probability that a buyer find a seller. Those probability could feature matching frictions arising during the transaction and the bargaining processes, or the frictions arising due to mismatch between buyer's entrepreneurial idea and seller's business activity.

Other parameters The estate taxation is set to 30%, consistent with the statutory tax rate in the US and the value used in **?**. Taxes related to the sale of a private business is set to 17.5%, which is between the middle statutory tax rate on capital gains when selling a business, which typically range from 15% and 20%. There are also taxes associated to the purchase of an existing business. We set this tax rate to 10% in the benchmark economy. Finally, we calibrate the transition probability of the entrepreneurial ability process ι: we endogenously determine $P(\iota' = 1|\iota = 0) = p_\iota$ to match the share of self-employed in the economy while we fix $P(\iota' = 0|\iota = 1) = \chi(0)$. The borrowing constraint parameter θ is also picked to match model moments to the data.

Joint parameterization Our calibration leaves us with 9 parameters that are picked to match data moments. We target a share of self-employed individuals relative to the working age population of 11%, a share of new entrepreneurs buying their business of 20%, a share of business sold relative to exiting entrepreneurs of 10%, a self-employment exit rate (per year) of 25% and a share of worker becoming self-employed of about 3%, as observed in the CPS. We target a

fraction of entrepreneur relative to the population in the last age brackets corresponding to retirement of 2.8%, a capital output ratio of 3.0, a ratio of median entrepreneurs' net worth to workers' net worth between 7 and 8, as computed in the SCF (averaged over the 2001, 2004 and 2007 waves) and a fraction of private businesses expanding of about 60% in the SBO. The target for the share of production coming from the SMEs is of about 50%.[37]. Our model is exactly identified, with 9 parameters used to pin down 9 moments. Table 6 displays the parameters.

Table. 6. Model Parameterization

Description	Symbol	Value	Target	Model	Source/Main moment*
		External Parameters			
Risk aversion	σ	1.5			Standard value
Degree of altruism	Λ	0			Benchmark normalization
Depreciation of capital	δ	0.7			Standard value
Returns to scale in corp.	α	0.33			Standard value
Proba of aging/die	$P(j'\|j)$	*in text*			5y age bracket, 11y retirement
Persistency/SD of ability y	ρ_y, σ_y	0.96, 0.2			?
Life-cycle earnings $h(j)$	$\alpha_1^j, \alpha_2^j, \alpha_3^j$	*in text*			Earnings Gini 0.38 (PSID)
Replacement rate	ρ	40%			?
Wedge mature/immature	v_s, v_m	2%, 1.6%			SSBF (2003)
Prob. switch mature	P_m	0.2			5 years in operation
Probability to fail	$\chi(m)$	$\{18\%, 5\%\}$			ASE (2016), ceasing reasons
Matching technology	γ	0.2			BBS data
Broker's commission	$\mu_s = \mu_b$	10%			*Bizbuysell*
Discount of business liqui.	τ_d	30%			60-80% of business value
Tax on selling/purch. a bus.	τ_s, τ_b	15%, 10%			IRS capital gains
Estate taxation	τ_a	30%			IRS statutory tax rates
		Jointly Determined Parameters			
Discount factor	β	0.911	3.0	2.8	K/Y
Returns to scale priv. bus.	v	0.813	48	48	% production priv. bus.
Buyer/seller's side broker cost	κ_b	0.1	25	31	% purchasing existing bus. (SCF)
Seller's side broker cost	κ_s	0.9	10	11	% selling their bus. (SBO)
Prob. to fail for exo reasons	ζ	0.17	25	25	% exiting self-employed (CPS)
Disutility of working (retired)	u_R	0.6	2.0	1.9	% retired entrepreneurs (SCF)
Adjustment cost	τ_u	0.1	60	55	% of SME expanding (SBO)
Borrowing constraint	θ	0.4	8-10	8.9	% self-employed (CPS)
Probability entrep. ability	p_ι	0.047	2.5	2.3	% of worker $\rightarrow$ entrep. (CPS)

* The main moments are indicative. Changing one endogenous parameter affects the whole equilibrium.

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[37]This share is taken from the Small Business Administration report *Small Business GDP: Update 2002-2010*.

5 Results

We first describe the characteristics of businesses in our baseline model and then report study our policy experiments in the next sections.

Wealth. While not the primarily focus of the paper, the resulting wealth inequality implies a wealth Gini of 0.75 in 2007, close to the estimates in the US data of 0.8. Additionally, the share of wealth hold by the top 10% is 26% and the median wealth ratio of self-employed individuals to that of workers is about 5 in the model, against respectively 30-40% and 6 in the SCF.

Business sale market. In Figure 4, we finally display the distribution of startup capital by type of acquisition in the model. In the data, the ratio of median startup capital between buyers and founders is about 6, against 4.5 in the model.
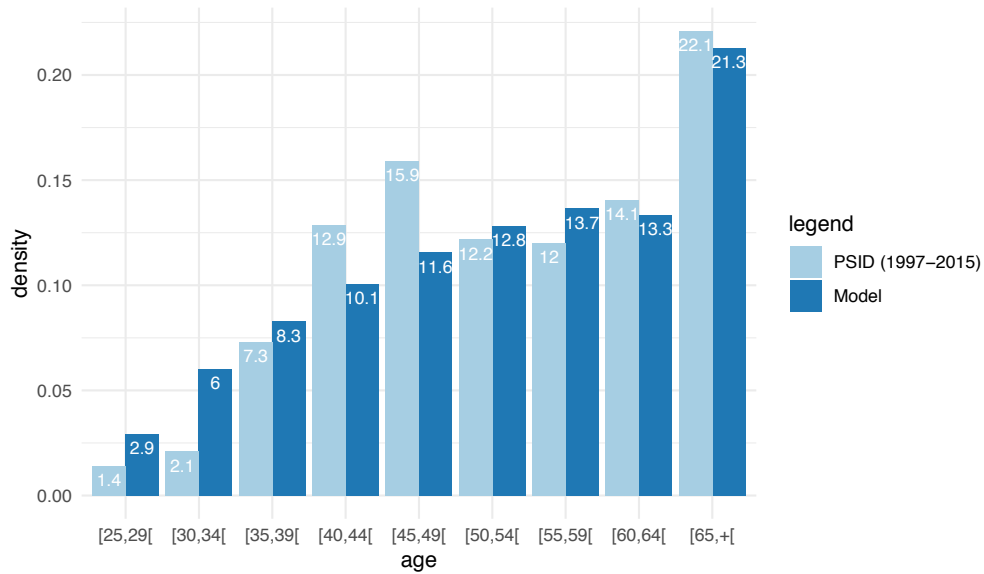

Figure 4. Distribution of sellers in the model and the data (PSID).

Dynamics of the model We display in figure **??** the dynamic of the model following the surge of new entrants in the labor force between 1970 and 1980.

Finally the model is consistent with the recent debate on the declining start-up rate highlighted by pugsley et al. (2019, WP). In particular, following the demographic change occurring between 1970-1980, the

6 Demographic change and the aging of entrepreneurs

Our quantitative analysis so far is predicated on the assumption that the age structure of the economy is stationary. In this section, we begin studying the implications of our model when
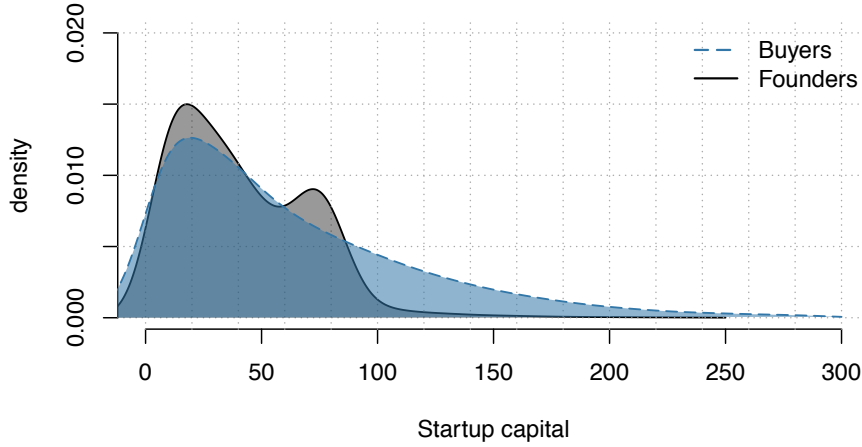
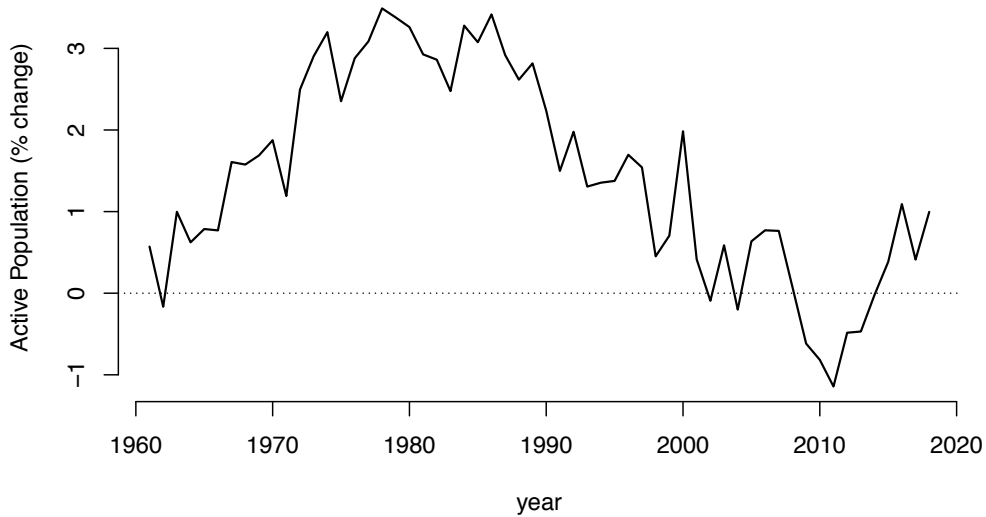**Figure 5.** Distribution of buyers and founders in the model.

Figure 6. Distribution of buyers and founders in the model.

we relax the stationary assumption and try to match the demographic change that started in the 1970s with the so-called boomers generation. We thus try to match the non-stationary distribution as observed in 2007.

More specifically, the exercice we perform is as follows. We choose a set of parameters corresponding to the observed stationary 1970 distributions of occupation, age structure and wealth as initial condition. We then shock the economy by increasing the number of new young individuals in the model during 10 periods. Such demographic shock propagates along the transition until date 2007 and, at such point in time, we compute the distance between the model generated moments and the data. We redo this experiment until finding the set of parameters that minimizes this distance. By doing so, we are able to answer the following question: "what was the effect of the demographic change started in the 1970s on the occupational choice?" and
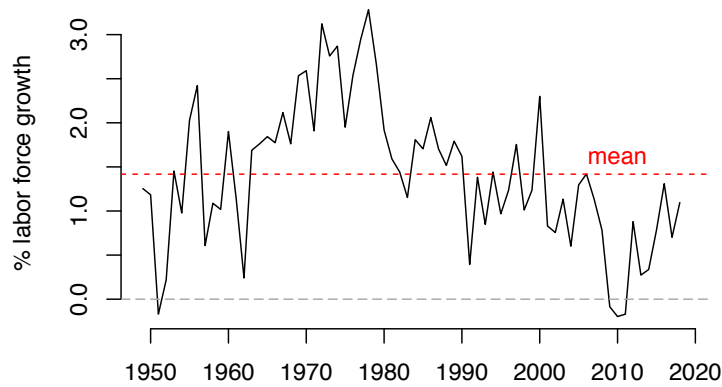
29

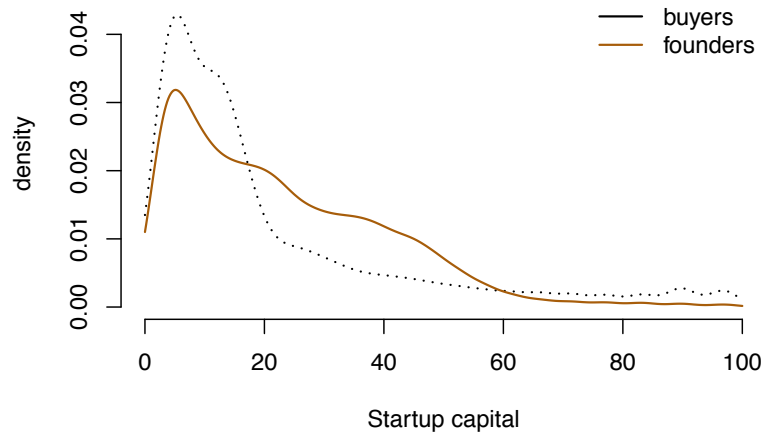**Figure 7.** Labor force growth rate.

Figure 8. Labor force growth rate.

"what would be the implications of the ongoing aging of entrepreneurs on the economy, and more specifically, on the transmission of businesses?".[38]

The fundamental feature of the change in the age structure from 1970 to 2007, in the data, is the substantial entry of baby-boomers following the significant increase of birth rate from 1944 to 1964, where more than 65 million children were born in the United States. In this respect, we can obtained the baby boomers effects throughout our simplified life-cycle model by increasing the mass of individuals in our economy by 2.5% each year during 20 years. In this respect, we display in figure 13 the corresponding active population growth rate since the 1960s. Then, this mass slowly propagates over our life-cycle structure according to the aging probabilities. The

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[38]Of course, our experiment assume that the structural parameters of the model remained unchanged between 1970 and 2007. Still, we think this experiment has the advantage to isolate the specific effect of the demographic change along the transition.
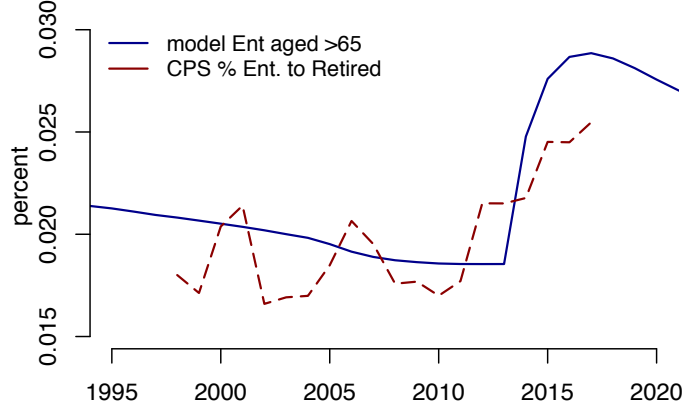
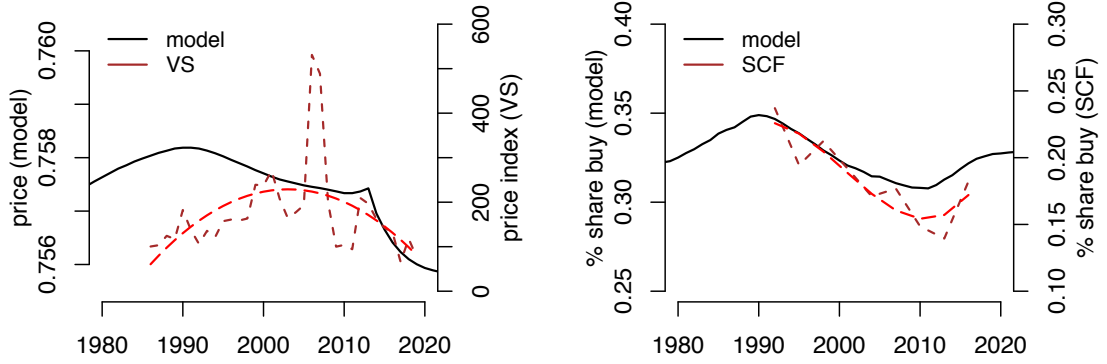**Figure 9.** Dynamic of retirement in the model and the data.

Figure 10. Dynamic of the Business Sale Market in the data and the model.

economy recover its stationary property when all the surplus of mass coming from the entry of baby boomers in the model die. We thus assume that some individuals do not reborn to recover a unit mass.[39]

[IN PROGRESS, COMING SOON]

To formalize our calibration strategy:

$$\widehat{\Phi}_{t_0} = \underset{\Phi_{t_0}}{\text{argmin}} \sum_{k=1}^{M} \left| m_{k,t_r} - \tilde{m}_{k,t_r}(\Phi_{t_0}, \{\mathbf{X}_t\}_{t_0}^{T}) \right|$$

where for instance: $\mathbf{X}_t = \{\phi_{d,t}, A_t, \tau_t\}$ and $\Phi_{t_0} = \{\tau_{up}, \tau_{down}, u_R, u_E\}$

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[39] We could also reduce slowly the probability of reborn, generating cycles in the demographic structure, but this is beyond the scope of this paper.
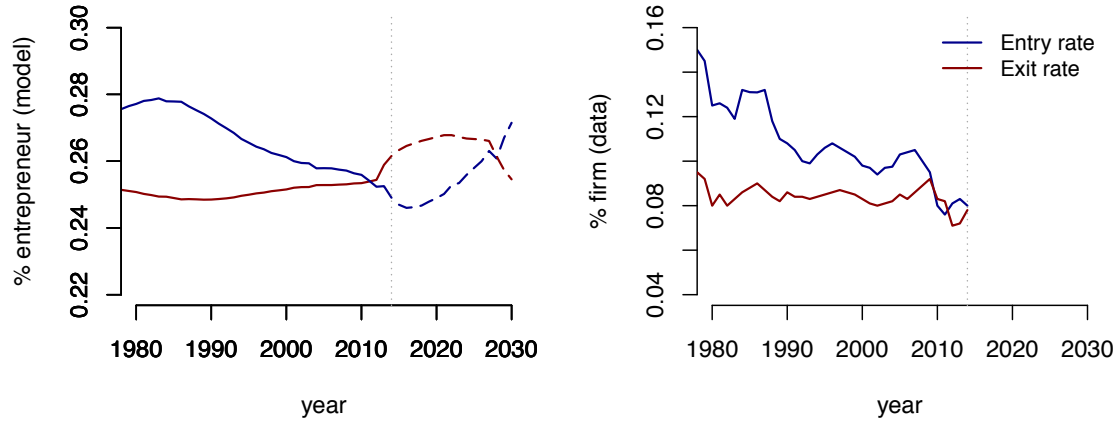
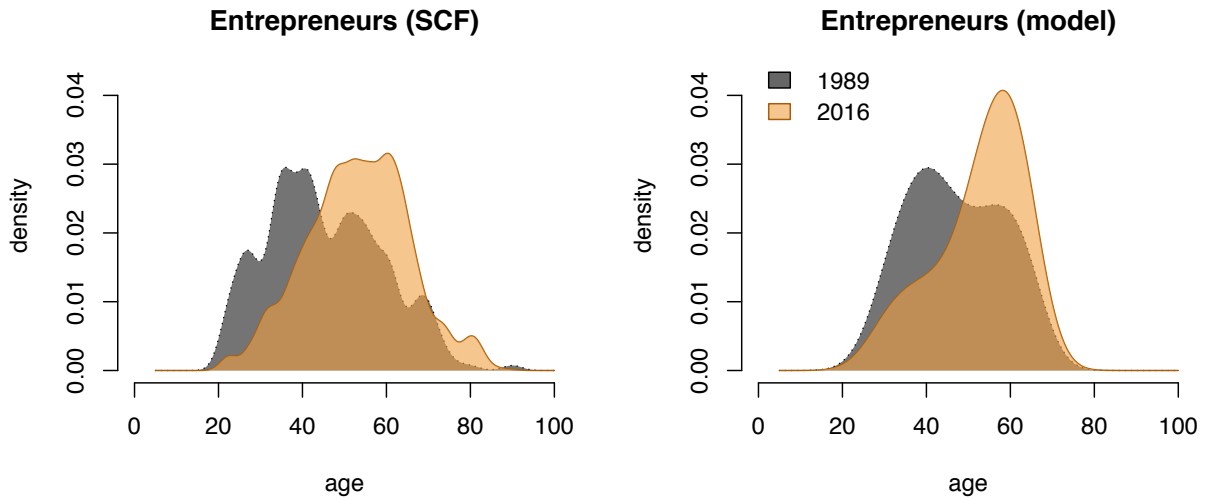**Figure 11.** Entry and exit rate in the US, data and model.

Figure 12. Distribution of entrepreneur at different periods.

7 Further empirical evidence

7.1 Financial constraint

Using the same SSBF data, we find that among those who needed a credit during the preceding three years, 27.5% of the entrepreneurs who started a new business did not apply because they thought the application would be turned down. This number goes down to 21.5% for buyers. Regarding recent loan applications[40], 73.5% were always approved and 19.3% were always denied for founders, against respectively 83.6% and 11.6% for purchasers. If the magnitude of denied loan applications are different between acquisition types, the reasons for it are also quite different. For founded businesses, the most recurrent reasons for denied loan applications are that the firm was not in business long enough (28.7%), that credit history is not favorable (22%) and that the firm has poor balance sheet or financial situation (19.8%). For purchased busi-

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[40]Unfortunately, we can not control for loan size. Those numbers have therefore to be taken with caution.
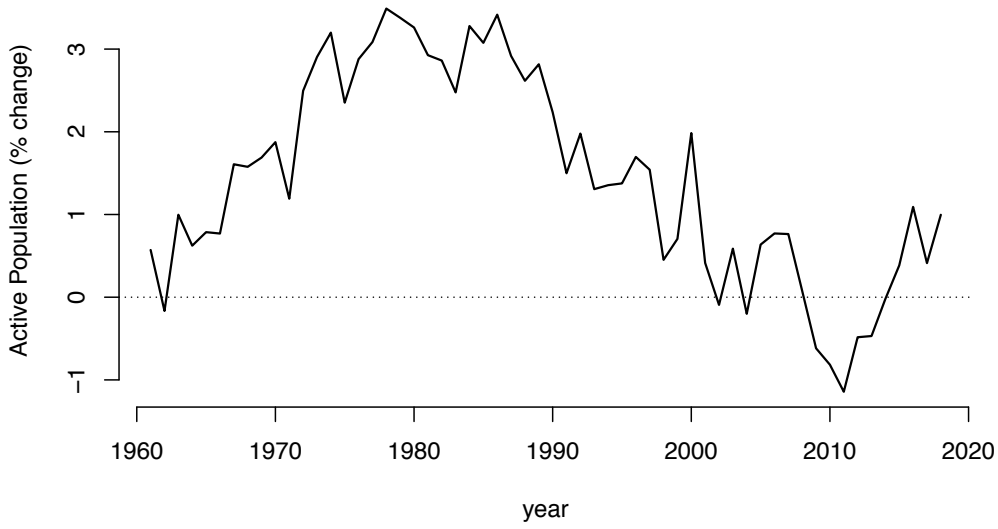
Figure 13. Active population aged between 24 and 55 (annual percent change).

nesses, credit history is the main reason (25.7%) followed with no particular reason (25%) and insufficient collateral or no guarantees available (19%). This means that purchased businesses are not only less risky in terms of income stream (higher profit), but they might take advantage of favorable credit conditions because the history of firm's performance is long enough for a bank to evaluate the quality of a business project. Consistent with this idea, we empirically test whether the type of business acquisition significantly impact the interest rate charged on the business, controlling for business owner's characteristics (net worth, age, education) and business characteristics (profit, capital invested, employment size). We run:

$$int.rate_{it} = \gamma_{it} + \beta \mathbf{X_{it}} + \epsilon_{it} \tag{29}$$

with γ_{it} a dummy variable indicating whether the business is purchased or founded, $\mathbf{X_{it}}$ a vector of controls and ϵ_{it} an error term. Table 7 gather the results for two business groups: ESB and the whole sample. We find that purchasing a business is significantly associated with a lower interest rate charged on debt, lowered by 1.6% for ESB, and 0.6% for the whole sample. The difference between the two samples can be accounted by the fact that started business self-select over time, and that more business history is available to creditors as time goes by.[41]

The above illustrate that the decision to purchase an existing business as opposed to founding a new one is an important dimension for entry into entrepreneurship. This translates to many facets such as the initial wealth level of the prospective entrepreneur but also to future credit conditions for the business; generating a wedge between the type of acquisition. Throughout this paper, we therefore argue that failure to transfer businesses could lead to

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[41]Those results are robust to a number of alternative specifications: ...

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Table. 7. Interest rate charged on debt and type of acquisition

	Interest rate charged on debt (%)	
	All firms	Young firms ($<$5y)
	(1)	(2)
Purchased	-0.58^{***}	-1.59^{***}
	(0.13)	(0.28)
Controls	Yes	Yes
N	8,919	1,811
R^2	0.12	0.31

Notes: p$<$0.1; **p$<$0.05; ***p$<$0.01. In parenthesis: std. deviation.

economic deadweight losses due to the loss of tangible assets (machine, properties) but also intangible assets (customers, history of sales and favorable credit conditions).

7.2 Lower investment close to retirement

8 Conclusion

In this paper, we develop an incomplete markets heterogeneous agents general equilibrium model with stylized life-cycle dynamics and occupational choices. Importantly, our model lets incumbent entrepreneurs sell their businesses and prospective workers buy or found businesses, subject to an endogenous business price and adjustments costs. We also build a new dataset using data from a major online business selling platform and show that selling a business takes much time and that the vast majority of businesses are not sold. The model accounts for the main empirical features about business selling and buying frictions and age and life-cycle dynamics found in the empirical data. We show that illiquid business assets, frictions on the business sale market and the life-cycle components of entrepreneurship are key to reproducing our empirical findings. We show that our model is tractable and can be used to simulate the demographic changes appearing on the labor and entrepreneurial markets due to the aging entrepreneurial population.

Appendix

A Algorithm

A.1 Pertubated solution

Taste shocks Following **?**, we introduce taste shocks to tackle the issue of kinks in the value function. Denoting the discrete worker's choice $d \in D \equiv \{d_b, d_f, d_c\}$ of either purchasing a business, founding a business or continuing as a worker, we define the perturbed value function of a worker as:

$$W_\sigma(a, \mathbf{x}_w, \epsilon_w) = \max_{d \in D} \left\{ \underbrace{\mathcal{B}(a, \mathbf{x}_w) - u_b}_{w_\sigma(d_b, a, \mathbf{x}_w)} + \sigma_w \epsilon_w(d_b), \underbrace{\widetilde{W}(a, j, 0)}_{w_\sigma(d_f, a, \mathbf{x}_w)} + \sigma_w \epsilon_w(d_f), \underbrace{W^c(a, \mathbf{x}_w)}_{w_\sigma(d_c, a, \mathbf{x}_w)} + \sigma_w \epsilon_w(d_c) \right\} \tag{30}$$

where ϵ_w is a *vector* of IID extreme value type I taste shock across the discrete choices and σ_w is a scale parameter of the Type I Extreme value distribution, proportional to the variance of the taste shock. We furthermore assume that taste shocks are contemporaneously independent across different discrete choices but are also intertemporally independent. Under the perturbed problem, the expected value of $W_\sigma(a, \mathbf{x}_w, \epsilon_w)$, before the realization of the taste shock, can be derived in a closed form as

$$\mathbb{E}\left[W_\sigma(a, \mathbf{x}_w)\right] = \sigma_w \log \left(\sum_{d \in D} \exp \left\{ \frac{w_\sigma(d, a, \mathbf{x}_w)}{\sigma_w} \right\} \right) \tag{31}$$

Similarly for an entrepreneur, we define the set of discrete choice $g \in \mathcal{G} \equiv \{g_s, g_l, g_c\}$ over selling or liquidating the business or pursuing the entrepreneurial activity. We write the perturbed problem of an entrepreneur as:

$$E_\sigma(a, k, \mathbf{x}_e, \epsilon_e) = \max_{g \in \mathcal{G}} \left\{ \underbrace{\mathcal{S}(a, k, \mathbf{x}_e) - u_s}_{e_\sigma(g_s, a, k, \mathbf{x}_e)} + \sigma_e \epsilon_e(g_s), \underbrace{E^e(a, k, \mathbf{x}_e, 0)}_{e_\sigma(g_l, a, k, \mathbf{x}_e)} + \sigma_e \epsilon_e(g_l), \underbrace{E^c(a, k, \mathbf{x}_e}_{e_\sigma(d_c, a, k, \mathbf{x}_e)} + \sigma_e \epsilon_e(g_c) \right\} \tag{32}$$

And as for a worker, we can derive the closed form for the expected value as:

$$\mathbb{E}\left[E_\sigma(a, k, \mathbf{x}_e, \epsilon_e)\right] = \sigma_e \log \left(\sum_{g \in \mathcal{G}} \exp \left\{ \frac{e_\sigma(g, a, k, \mathbf{x}_e)}{\sigma_e} \right\} \right) \tag{33}$$

Under this specification, taste shocks play two roles. First, it facilitates the numerical computation by smoothing the next period expected value over discrete choice and limit the propagation of the kinks over the grids of assets. Second, it allows for unobservable heterogeneity in the choice of becoming a worker (resp. an entrepreneur) that fit recent finding in the literature, as already pointed out. The larger are σ_w and σ_e and the larger are the variances of the taste shocks and the smoother are the value functions.

B Data

B.1 Public Micro Sample - Survey of Business Owners (SBO) (2007)

Provided by the U.S. Census Bureau, we use the SBO to recover information concerning both sellers' and buyers' side. The dataset contains information on whether owners established, purchased or acquired their businesses. The dataset also contain demographic variables such as age, education and past self-employment experience of business owners. On top of that, information concerning the amount of startup capital and how this capital was financed is provided. SBO 2007 includes variables for the four most important owners of a business. Following **?**, we define an entrepreneur as a business owner that actually manage day-to-day the business or participate to the production process and whose the business constitute the primary source of personal income. Those individuals choose option 1 or 2 to question (1) and *Yes* in question (2)[42]:

1. In 2007, which of the following best represents Owner X's function(s) in this business? (multiple choices allowed) – Providing services and/or producing goods – Managing day-to-day operations – Financial control with the authority to sign loans, leases, and contracts – none of the above.

2. In 2007, did the business provide Owner X's primary source of personal income? – Yes – No.

Own robustness check on the definition of an entrepreneur provided similar results than those shown in the paper. Taking all business owners, without controlling for actively managing the business yield a respective fraction of business founded and purchased of about X% and Y%. Other results are also comparable.

In addition to this, the SBO 2007 public micro sample allows to match business owners' characteristics with businesses' characteristics. This allows us to recover information on the reasons to cease businesses along the life-cycle. We select early-stage businesses those who were established, purchased or acquired in the past two years (in 2006 and 2007).

B.2 Annual Survey of Entrepreneurs (ASE)

Provided by the U.S. Census Bureau, we use the ASE, which contains 290,000 employer firms in the private, non-agricultural U.S. economy, in order to characterize recent small business dynamics concerning the evolution of reasons to cease a business and on the share of entrepreneurs that purchase or found a business. The dataset is not available as a micro public

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[42]We could also control for a minimum time spent in actually working for the business. This information is also contains in the survey questionary.

sample and we use the online API to recover informations[43].

B.3 Panel Study of Income Dynamics (PSID)

We use the PSID in order to get information on entrepreneurs' selling their businesses. Since 1989, the PSID contains variables that record whether or not an individual recently sold a business (available only every 5 years from 1989 to 1999 and every two years afterward). We use the panel by only considering the head of the family. In order to guarantee some consistency in our definition of entrepreneurship. We define an entrepreneur as someone owning a business and declaring being self-employed only (answering option 3 in question (1)).

1. On your main job, are you (HEAD) self-employed, are you employed by someone else, or what? – Someone else only – Both someone else and self – Self-employed only

2. Did you (or anyone else in the family there) own a business at any time in DATE or have a financial interest in any business enterprise? – Yes – No

Unfortunately, we could not control for actively managing a business/working in the business using the PSID[44]. We provide additional robustness on our results regarding our definition in the online appendix and we show that our main results are not very sensitive between business owners and self-employed business owners. Due to lack of business sold observations per year, we aggregate the number of business sold from 1989 to 2013 that we normalize using 2018 base prices. This provides an average profile of sellers in the US during the past 30 years.

B.4 Survey of Consumer Finance (SCF)

We use three SCF waves (1989, 2004, 2016) in order to draw the evolution of the entrepreneurs' characteristics over almost 30 years. As firstly used by **?**, the SCF contains additional question as compared to the PSID that allows to better quantify the number of entrepreneurs in the economy. Indeed, as pointed by **?**, total wealth reported in the SCF is close to the one reported for the US, making the comparison of moments close to aggregate data. We define an entrepreneur as a self-employed business owners owning at least a business of more than 5000$. In order to be consistent with other datasets, we do not control for business management role. We use the SCF to calibrate the model. Among other moments, we compute the median net worth between workers and entrepreneurs, as well as different moments between generations of entrepreneurs.

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[43]For more information: https://www.census.gov/programs-surveys/ase.html.

[44]Although, results from the SBO 2007 showed relatively few differences between these two definitions regarding the age profile of entrepreneurs and their propensity to sell businesses.

B.5   Current Population Survey (CPS)

When used as a panel, the CPS from the U.S. Bureau of Labor Statistics provide exhaustive information on the transition rate between occupations as highlighted in **?**. In this paper, we use the CPS to compute the flow of entrepreneurs that retire each year, as well as to compute with an important number of observations, the share of entrepreneurs with more than 60 years old. CPS does not contain clear definition of entrepreneurship. We therefore construct a variable *holdhubus* that translates whether or not an entrepreneur was previously business owner. Using this variables, we define an entrepreneur as a self-employed person that already declared owning a business[45].

B.6   Survey of Small Business Finance (SSBF)

Conducted by the U.S. Federal Reserve, the 2003's SSBF wave provides relevant information on how business were acquired (either purchased or founded) as well as information on business owners (age, net-worth etc.). The SSBF also provide a number of variable concerning small businesses' credit conditions. To ensure comparability with other dataset, we define an entrepreneur as a business owners who actually manages day-to-day his business. Finally, due to lack of observations, we define an early-stage business as a business who was acquired during the last five years.

B.7   Online platform BizBuySell (BBS)

We use data from a private company *BizBuySell.com* in order to get information related to the US business sale markets. BBS is currently one of the most active platform for selling/purchasing a business in US. We proceed a number of text mining in order to classify different variables. We first classify in four categories the reason to sell between *retirement*, *move to another location*, *family purposes* and *other reason*. We aggregate businesses into 20 main business types[46]. We observe the number of employees that are actually working in the business for sale, with additional information on the whether employees are full-time or part-time. We assign a weight 0.5 to part-time employees and 1 to full-time employees.

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[45]Some questions are not answered several times in the CPS, depending on the month of the survey.

[46]Those types include: agriculture, automotive and boat, beauty and personal care, building and construction, communication and media, education and children, entertainment and recreation, financial services, food and restaurants, health care and fitness, manufacturing, non-classifiable establishments, online and technology, pet services, real estate, retail, service businesses, transportation and storage, travel, wholesale and distribution.

C Supplement facts

C.1 Demographic of entrepreneurs

Figure 14 displays the density of workers selecting into entrepreneurship by age. As argued in the main text, we observe an increase in the mean age at which workers select into entrepreneurship, contributing to the overall aging of the entrepreneur's population. Figure 15 displays the evolution of the entrepreneur's density by age using the CPS.
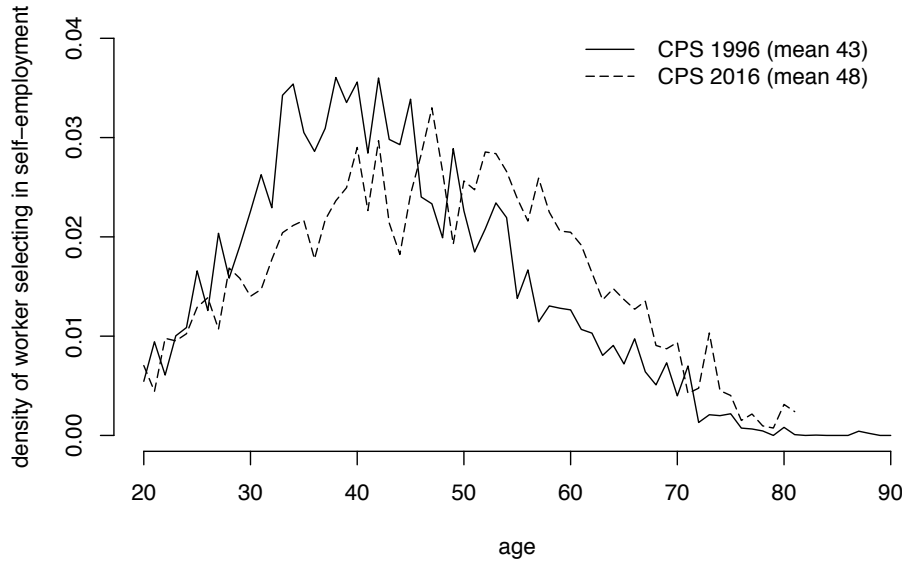

Figure 14. Workers selecting into entrepreneurship by age. *Source:* CPS 1996 and 2016 from author's own calculation.

Table 8 also provide this evidence by comparing early stage firms (with less than two years of operation) with the entrepreneur's population.

Age	25 and less	25 to 34	35 to 44	45 to 54	55 to 64	65 and more
All firms	0.5	5.5	16.5	27.5	30.5	19.6
Firms < 2 years	1.4	16.6	29.6	27.6	17.8	7.0

Table. 8. Fraction of entrepreneurs for different age brackets. *Source:* ASE (2016).

In table 9 we report the increase of the business value and total wealth held by the older entrepreneurs.

The reasons to cease a business because entrepreneurs sold or retire tends to increase a lots. We plot in figure 16 the projection of the 2016 distribution of reasons to cease given the increasing fraction of entrepreneurs who sold and retire (ASE - 2016). The underlying assumption is that we keep constant, for a given reason to cease, the share of individual in a given age
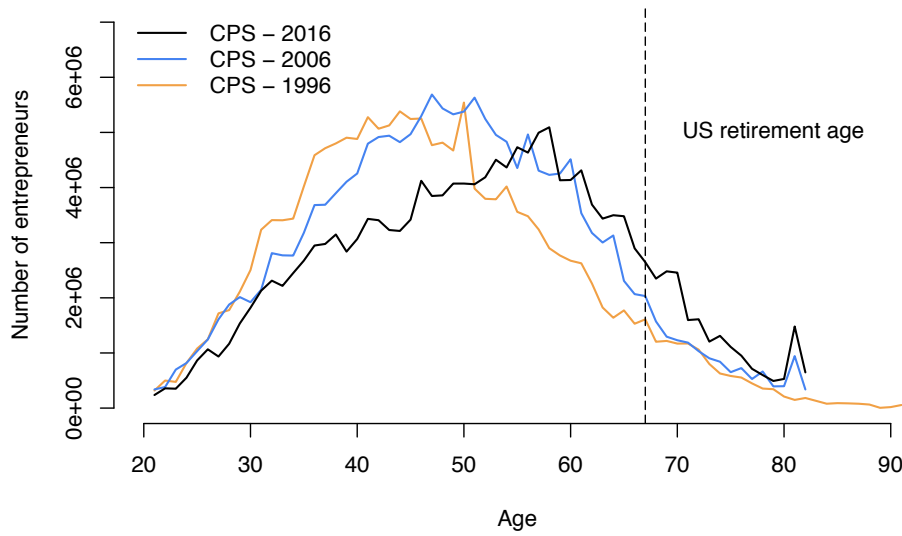
Figure 15. Entrepreneur's density by age. *Source:* CPS 1996-2016 from author's own calculation.

% of total entrepreneur's population	1989	2004	2016
55 and over	26.1	33.7	47.1
wealth	49.3	49.4	65.2
business value	48.6	45.1	63.9
60 and over	17.3	22.2	31.8
wealth	36.7	33.7	46.8
business value	36.6	29.9	45.4

Table. 9. Share of business capital and wealth relative to the entrepreneur's population. *Source:* SCF (1989,2004,2016).

bracket (for instance, in 2016 and in 2007, among those who sold their business, 27% were aged between 55 and 64 years).

In figure 17 we display the entrepreneur's profit distribution by acquisition type.

In figure 18, we use the SBO - 2007 to draw the distribution of bracketed startup capital by type of acquisition.

Concerning the financing source, 63% (resp. 83%) of purchasers (resp. founders) declare using personal assets and 65% (resp. 49%) declare using diverse loans. In Table 10, using the SSBF, we find that entrepreneurs who just purchased a business are substantially richer (around 1.5 times on average) than founders and make substantially more profits (around 5 times).[47]

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[47]Regarding the distribution of profits, we find large disparities among purchasers and founders as shown in Figure 17 in the Appendix, with a substantial fraction with losses. This is in line with many papers arguing that a

(a) 2007   **(b)** 2016 (projection)
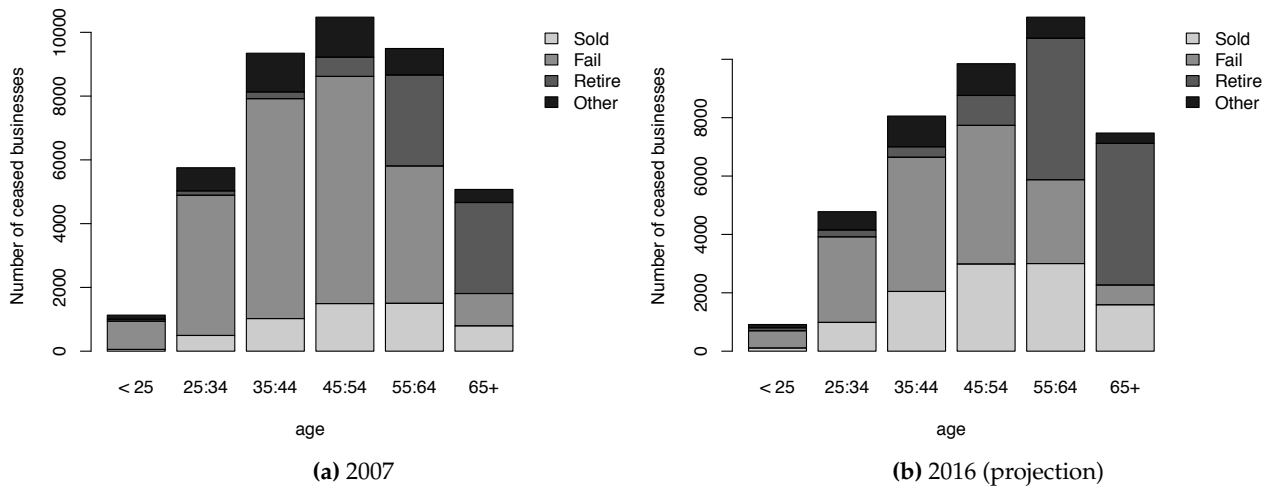
Figure 16. Age profile of ceasing entrepreneurs. SBO and projection using ASE.
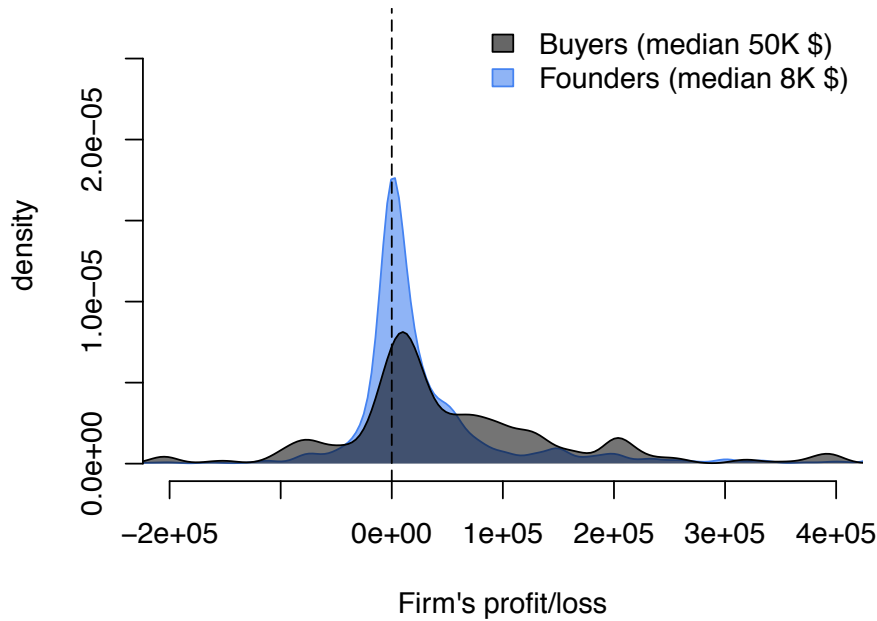
Figure 17. Profit/loss by acquisition type. *Source:* SSBF 2003, taking into account early stage businesses (less than 5 years in operation).

C.2 Small business sale market

In order to estimate the market price of businesses, we conduct a hedonic regression with respect to cash-flow, gross-revenue, state, number of years in operation, number of employees. We estimate:

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substantial number of entrepreneurs experience losses but persist in their entrepreneurial situation (see for instance **?**).
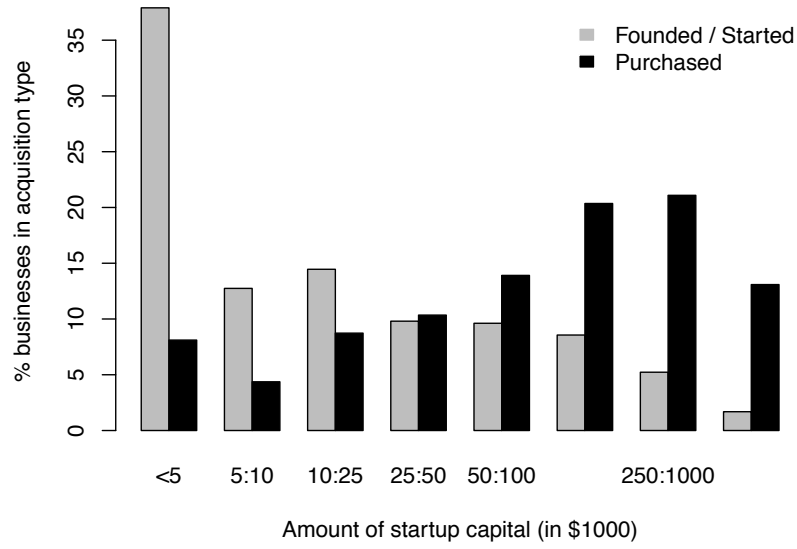
Figure 18. Distribution of startup capital invested by acquisition type. *Source:* SBO 2007 from author's own calculation.

Table. 10. Firm's and owner's characteristics depending on acquisition type

	Profit (K$)		Firm's equity (K$)		Owner's networth (K$)	
	mean	median	mean	median	mean	median
Founded	104	8	140	10	635	260
Purchased	505	50	555	37	1196	390
Ratio	4.8	6.1	3.7	3.9	1.5	1.9

Author's computation using the SSBF (2003), estimates correspond to early stage businesses (<5 years).

$$p_{j,s,q}^{sell} = \alpha_s + \alpha_q + cash.flow_j + gross.rev_j + inventory_j + employees_j + \epsilon_{j,s,q} \tag{34}$$

Finally, we also found that experienced worker tend to switch to entrepreneurship with lower probability, consistent with the model

	SBO			BBS		
	Founded	Purchased	Sold	For sale	Sold
Professional, Scientific, Technical Services	20	7	9	3	13
Construction	14	6	4	2	3
Finance, Insurance, Real Estate	11	7	9	2	3
Wholesale and Retail Trade	11	27	26	23	19
Health Care and Social Assistance	10	7	8	9	14
Administrative and Support	7	5	6	8	10
Transportation and Warehousing	5	6	6	1	<1
Manufacturing	3	6	6	4	2
Accomodation and Food Services	2	12	10	27	17
Other	17	17	16	21	19

Table. 11. Main category of businesses. *Sources:* SBO (2007) and *BizBuySell* (BBS, 2017).

	share_sold_reported
Time on market (in days) / 365	-0.008^{**}
	(0.003)
Constant	0.018^{***}
	(0.002)
N	46
R^2	0.108

Table. 12. Share of sold businesses using ASE (2016) and time on market (divided by 365 days) in BBS, by US state. *Notes:* $p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$. In parenthesis: std. deviation.
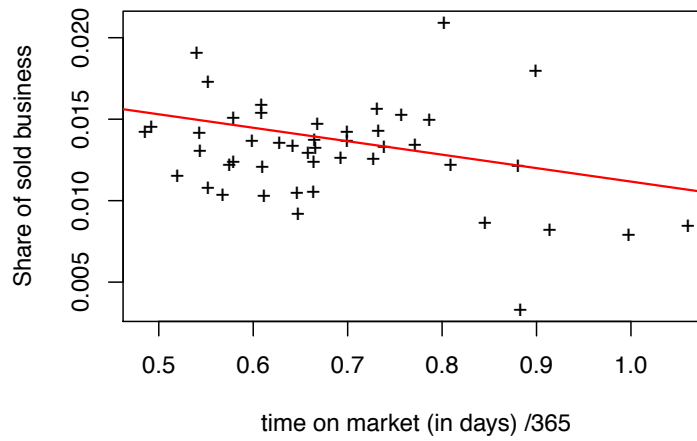
Figure 19. Share of sold businesses and share of walk away as exit strategy using ASE (2016) and probability to sell a business (in a year) in BBS, by US state.

	Number (th.)	%
Walk away from the business	494	11.2
Liquidate business assets & repay bus. debt	422	9.6
Sell the business to employees or manager	281	6.4
Sell ownership to external	1159	26.2
Sell ownership to another owner/family	806	18.2
No exit strategy	1085	24.6
Other exit strategy	171	3.9
Total report	4418	100

Table. 13. Exit strategy. *Source:* ASE (2016).