Two Countries Heterogenous Agent New Keynesian Model (2CHANK), Numerical solutions*

Alexandre Gaillard
Toulouse School of Economics

Tannous Kass-Hanna
Toulouse School of Economics

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Abstract

We compare result of a DSGE representative household model with a Heterogenous Agent New Keynesian (HANK) model in the spirit of Kaplan et al. (2016), applied to an monetary union with two countries. Using HFCS household data for Germany and the rest of european countries, we calibrate our model to match wealth, income and consumption distribution. We show that distributional impact are important. In particular, fiscal boost implemented by Germany and Hungary have non-negligible impact on inequalities in other countries. Moreover, this changes in inequalities impact overal interest rate creating a crowd-out effect in the domestic country. Our result suggests that representative model in international markets miss important features and channels through which a country-specific policy affects the rest of the world.

1 Steady State economy

There are two countries indexed with subscript $s = H$ for the rest of the European countries and $s = F$ for the domestic country. The two countries have different population weights given by $\mho_D$ and $(1 - \mho_D)$ respectively for the domestic and the foreign economy.

1.1 Household

To make things easy, we assume a simple household problem with two poisson idiosyncratic income shocks, such that $z_s \in \{z_{s,1}, z_{s,2}\}$. For simplicity, we assume that agent can save only

*Corresponding author: alexandre.gaillard@tse-fr.eu. This draft is based on Achdou et al. (2017). All the mathematics behind this come from their work.

in one type of asset a_i. He works l_t hours. We adopt a continuous-time approach in line with Achdou et al. (2017).

The problem of an individual with wealth a_i, in country s and with income process $z_{s,j}$ can be summarized by:

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_{i,j,s,t}, l_{i,j,s,t}) dt$$

We assume that labor is not transferable between countries. The budget constraint of the individual is given by:

$$\dot{a}_{i,j,s} = w_s z_{s,j} l_{i,j,s} + r_s a_i - c_{i,j,s} \tag{1}$$

$$c_{i,j,s} = \left[\omega^{\frac{1}{\zeta}} (c^s_{H,t})^{\frac{\zeta-1}{\zeta}} + (1-\omega)^{\frac{1}{\zeta}} (c^s_{F,t})^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}} \tag{2}$$

where w_s and r_s are determined by equilibrium conditions[1]. In addition, households consume a bundle $c_{i,j,s}$ of goods which are divided between foreign goods $c_{F,i,j,s}$ and home goods $c_{H,i,j,s}$, such that

$$c_{i,j,s} = \left[\omega^{\frac{1}{\zeta}} c_{H,i,j,s}^{\frac{\zeta-1}{\zeta}} + (1-\omega)^{\frac{1}{\zeta}} c_{F,i,j,s}^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}} \tag{3}$$

where ω translate the preferences over home goods relative to foreign goods. Solutions (see appendix **??**) of the household problem implies that

$$c_{H,t} = \omega \left(\frac{P_{H,t}}{P_t} \right)^{-\zeta} c_t \tag{4}$$

$$c_{F,t} = (1-\omega) \left(\frac{P_{F,t}}{P_t} \right)^{-\zeta} c_t \tag{5}$$

$$P_t = \left[\omega^{\frac{1}{\zeta}} P_{H,t}^{\frac{\zeta-1}{\zeta}} + (1-\omega)^{\frac{1}{\zeta}} P_{F,t}^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}} \tag{6}$$

To solve the model, we refer to the method developped by Achdou et al. (2017). That is, our discretized continuous time formulation for numerical solution of this simple household problem is given by

$$\begin{aligned}
(continuous) \qquad & \rho v(a_i, z_j, s) = u(c_{i,j,s}, l_{i,j,s}) + \partial_a v(a_i, z_j, s) \dot{a_{i,j,s}} + \lambda_j (v(a_i, z_{j'}, s) - v(a_i, z_j, s)) \\
& \qquad + \partial_t v(a_i, z_j, s) \\
(discrete) \qquad & \rho v^{n+1}_{i,j,s} + \frac{v^{n+1}_{i,j,s} - v^n_{i,j,s}}{\Delta} = u^n_{i,j,s} + \frac{v^{n+1}_{i+1,j,s} - v^{n+1}_{i,j,s}}{\Delta_a} \dot{a}^+_{i,j,s} + \frac{v^{n+1}_{i,j,s} - v^{n+1}_{i-1,j,s}}{\Delta_a} \dot{a}^-_{i,j,s} \\
& \qquad + \lambda_j (v^{n+1}_{i,j',s} - v^{n+1}_{i,j,s})
\end{aligned}$$

[1]Note that because we do not distinguish between type of bonds, r_s has to be consistent with european bond market. Alternatively, we can assume that households can not invest in the other country.

where I replace $v(a_i, z_j, s)$ by $v_{i,j,s}$ and index n means iteration n. $\dot{a}^-_{i,j,s}$ and $\dot{a}^+_{i,j,s}$ can be replaced by:

$$\dot{a}^-_{i,j,s} = min\{0, w_s z_{j,s} l_{i,j,s} + r_s a_i - c^B_{i,j,s}\}$$
$$\dot{a}^+_{i,j,s} = max\{0, w_s z_{j,s} l_{i,j,s} + r_s a_i - c^F_{i,j,s}\}$$

This allows us to rewrite:

$$(discrete) \qquad \rho v^{n+1}_{i,j,s} + \frac{v^{n+1}_{i,j,s} - v^n_{i,j,s}}{\Delta} = u^n_{i,j,s} + v^{n+1}_{i,j,s} y_{i,j,s} + v^{n+1}_{i-1,j,s}\zeta_{i,j,s} + v^{n+1}_{i+1,j,s} x_{i,j,s} + \lambda_j (v^{n+1}_{i,j',s} - v^{n+1}_{i,j,s})$$
$$(matrix\ form) \qquad \rho v^{n+1} + (v^{n+1} - v^n)\frac{1}{\Delta} = u^n + A^n v^{n+1} \qquad A^n = B^n + \Lambda$$

where we have

$$y_{i,j,s} = \dot{a}^-_{i,j,s} - \dot{a}^+_{i,j,s} \qquad x_{i,j,s} = \dot{a}^+_{i,j,s} \qquad \zeta_{i,j,s} = \dot{a}^-_{i,j,s}$$

Therefore, matrices B^n and C are given by:

$$B^n_s = \begin{bmatrix} y_{1,1,s} & x_{1,1,s} & 0 & \cdots & & & & \\ \zeta_{2,1,s} & y_{2,1,s} & x_{2,1,s} & 0 & & & & \\ 0 & \ddots & \ddots & \ddots & & & & \\ \vdots & 0 & \zeta_{I,1,s} & y_{I,1,s} & & & & \\ & & & & \ddots & & & \\ & & & & y_{1,J,s} & x_{1,J,s} & 0 & \dots \\ & & & & \zeta_{2,J,s} & y_{2,J,s} & x_{2,J,s} & 0 \\ & & & & 0 & \ddots & \ddots & \ddots \\ & & & & \vdots & \ddots & \zeta_{i,j,s} & y_{i,j,s} \end{bmatrix}$$

$$\Lambda_s = \begin{bmatrix} 0 & \dots & 0 & \lambda_{1,s} & 0 & \dots & \dots & \\ 0 & \dots & \dots & 0 & \lambda_{1,s} & 0 & \dots & \\ & & & & \ddots & & & \\ \lambda_{2,s} & 0 & \dots & \dots & & & \dots & 0 \\ 0 & \lambda_{2,s} & 0 & \dots & & & \dots & 0 \end{bmatrix}$$

1.2 Firms

Based on https://www3.nd.edu/~esims1/new_keynesian_model.pdf and http://www.princeton.edu/~moll/ECO521_2016/Lecture2_ECO521.pdf.

Final good producers In each country, there is a representative final goods producer which aggregates a continuum of intermediate inputs indexed by $k \in [0,1]$, such that:

$$Y_s = \Big(\int_0^1 y_{k,s}^{\frac{\epsilon-1}{\epsilon}} dk\Big)^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 0$ is the elasticity of substitution across goods. Cost minimization implies that demand for intermediate good j is

$$y_{k,s}(p_{k,s}) = \Big(\frac{p_{k,s}}{P_s}\Big)^{-\epsilon} Y_t \qquad \text{where} \qquad P_s = \Big(\int_0^1 p_{k,s}^{1-\epsilon} dk\Big)^{\frac{1}{1-\epsilon}}$$

Intermediate goods producers Each intermediate good is produced by a monopolistically competitive producer which use only labor $n_{k,s}$ as input, such that:

$$y_{k,s} = Z_s n_{k,s}$$

where Z_s is a country-specific aggregate TFP shock. From cost-minization problem, we have

$$w_s = \frac{\epsilon - 1}{\epsilon} Z_s n_s \tag{7}$$

Aggregation

$$Y_s = \int_0^1 y_{k,s} = \int_0^1 Z_s n_{k,s} = Z_s L_s^d \tag{8}$$

where, according to market clearing condition, we must have:

$$L_s^d = L_s^s = \bar{z}_s w_s^{\kappa_s} \tag{9}$$

Accounting implies that Y_s is equal to total demand. Moreover, we assume that government spending and home profit are consumed in the home country.

$$Y_s = C_s^H + G_s^H + \Pi_s^H + C_{s'}^F$$

Such that the profit is equal to:

$$\Pi_s = p_s^H Y_s^H - p_s^H w_s n_s \tag{10}$$

1.3 Monetary policy

We assume that the monetary policy adopt a simple taylor rule, such that:

$$i_t = \bar{r}_{ss} + \phi \pi_t$$

Inflation in our economy behave according to the following law of motion, using a Rotemberg [1982] and a quadratic price adjustement cost, we have for price setting:

$$\rho\pi = \frac{\epsilon - 1}{\theta}\Big(\frac{\epsilon}{\epsilon - 1}\frac{w_t}{Z_t} - 1\Big) + \dot{\pi}_t$$
$$w_t = \frac{\theta}{\epsilon}(\rho\pi_t - \dot{\pi}_t) + 1$$

1.4 Equilibrium

At the equilibrium, bond demand should be equal to supply.

$$B_d = B_s \tag{11}$$

We assume that $B_d = 0.1$. For B_s, we have

$$B_s = \int_0^1 ag(a) \tag{12}$$

Equilibrium implies that $B_d = B_s$, such that interest rate r adjust. Condition (7) holds by Walras law.

2 Algorithm

- Start by fixing $P_F = 1.0$, guess P_H and compute P_t. Guess also r_s in the two country.
- Given prices, solve for the HJB equation in the two countries.
- Given saving and consumption decision, compute the associated aggregate demand for the two countries.
- Check that bond markets are clear, if not, then update r_s. Check that aggregate demand is equal to aggregate production in the two countries, if not, update $P_{H,t}$ accordingly.
- If $B_s^d > B_s^S$ then increases r_s. If $Y_H < D_H$, then lower P_H.

References

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