$$\mathscr{H} = \begin{pmatrix} L_1 & L_2 & L_3 & L_4 & \dots & L_p \\ L_2 & L_3 & L_4 & L_5 & \dots & L_{p+1} \\ L_3 & L_4 & L_5 & L_6 & \dots & L_{p+2} \\ L_4 & L_5 & L_6 & L_7 & \dots & L_{p+3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ L_q & L_{q+1} & L_{q+2} & L_{q+3} & \dots & L_k \end{pmatrix}$$

$$\mathscr{H} = \sum_{i=1}^{n} \sigma_{i} \begin{pmatrix} u_{i1}v_{i1} & \cdots & u_{i1}v_{i(\alpha\eta)} \\ \vdots & \ddots & \vdots \\ u_{i(\beta\mu)}v_{i1} & \cdots & u_{i(\beta\mu)}v_{i(\alpha\eta)} \end{pmatrix}$$

$$\mathcal{H} \approx \mathcal{H}_r = \sum_{i=1}^r \sigma_i \begin{pmatrix} u_{i1}v_{i1} & \cdots & u_{i1}v_{i(\alpha\eta)} \\ \vdots & \ddots & \vdots \\ u_{i(\beta\mu)}v_{i1} & \cdots & u_{i(\beta\mu)}v_{i(\alpha\eta)} \end{pmatrix}$$

$$\mathcal{H} \approx \mathcal{H}_r = \sum_{i=1}^r \sigma_i \left\{ \begin{array}{ccc} & \ddots & \vdots \\ u_{i(\beta\mu)} v_{i1} & \cdots & u_{i(\beta\mu)} v_{i(\alpha\eta)} \end{array} \right\}$$

$$\left\{ \begin{array}{ccc} L_{1_{(1)}}(r) & L_{2_{(1)}}(r) & L_{3_{(1)}}(r) & L_{4_{(1)}}(r) \\ L_{2_{(2)}}(r) & L_{3_{(2)}}(r) & L_{4_{(2)}}(r) & L_{5_{(2)}}(r) \end{array} \right\}$$

$$L_{1(3\mu)} = \begin{pmatrix} L_{1(1)}(r) & L_{2(1)}(r) & L_{3(1)}(r) & L_{4(1)}(r) & \dots & L_{\alpha(1)}(r) \\ L_{2(2)}(r) & L_{3(2)}(r) & L_{4(2)}(r) & L_{5(2)}(r) & \dots & L_{\alpha+1(2)}(r) \\ L_{3(3)}(r) & L_{4(3)}(r) & L_{5(3)}(r) & L_{6(3)}(r) & \dots & L_{\alpha+2(3)}(r) \\ L_{4(4)}(r) & L_{5(4)}(r) & L_{6(4)}(r) & L_{7(4)}(r) & \dots & L_{\alpha+3(4)}(r) \end{pmatrix}$$

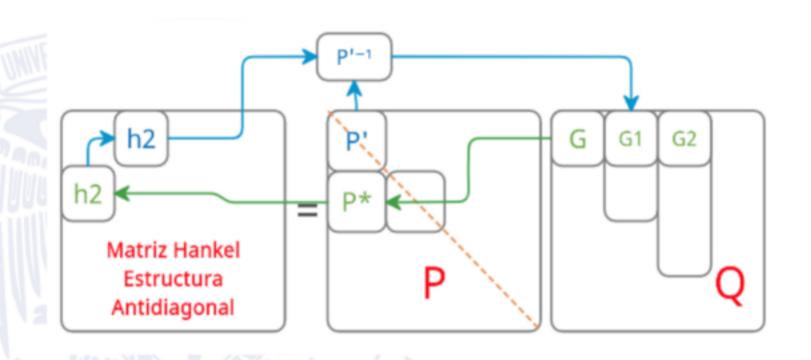
$$\mathcal{E}_{r} = \begin{bmatrix} L_{3_{(3)}}(r) & L_{4_{(3)}}(r) & L_{5_{(3)}}(r) & L_{6_{(3)}}(r) & \dots & L_{\alpha+2_{(3)}}(r) \\ L_{4_{(4)}}(r) & L_{5_{(4)}}(r) & L_{6_{(4)}}(r) & L_{7_{(4)}}(r) & \dots & L_{\alpha+3_{(4)}}(r) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ L_{\beta_{(\beta)}}(r) & L_{\beta+1_{(\beta)}}(r) & L_{\beta+2_{(\beta)}}(r) & L_{\beta+3_{(\beta)}}(r) & \dots & L_{k_{(\beta)}}(r) \end{bmatrix}$$

 $L_{\alpha_{(1)}}(r)$ 

$$P(r)_{:,1} = \frac{1}{Q(r)_{i,i}} (\mathscr{H}_r)_{1:,1} \quad P(r)_{i:,i} = \frac{(\mathscr{H}_r)_{i:,i} - \sum_{k=1}^{i-1} P(r)_{i:k} Q(r)_{k,i}}{Q(r)_{i,i}}$$

$$Q(r)_{1,:} = (\mathcal{H}_r)_{1,:}$$
  $Q(r)_{i,i:} = (\mathcal{H}_r)_{i,i:} - \sum_{k=1}^{i-2} P(r)_{i,k} Q_{k,i:}$ 

$$\mathscr{H}_r = P(r)Q(r) = egin{pmatrix} 1 & 0 & \cdots & 0 \\ p_{2,1} & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ p_{\beta\mu,1} & \cdots & p_{\beta\mu,\beta\mu-1} & 1 \end{pmatrix} egin{pmatrix} q_{11} & q_{12} & \cdots & q_{1,\alpha\eta} \\ 0 & q_{22} & \cdots & q_{2,\alpha\eta} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & q_{\beta\mu,\alpha\eta} \end{pmatrix}$$



$$F = (P')^{-1}P^*$$

$$G=Q_{0:\mu,:}$$

$$H = P'$$

$$\mathcal{H}_{r} \xrightarrow{\qquad \qquad \qquad } \mathcal{H}_{r} \xrightarrow{\qquad \qquad \qquad } \mathcal{H}_{r} \xrightarrow{\qquad \qquad }$$