Hwk 2 - CSE 250A   Andrew Ghafari   A59020215

P-1) a- $P(E=1 \mid A=1) = \dfrac{P(E, A)}{P(A)} = \dfrac{P(A=1 \mid E=1) \times P(E=1)}{P(A=1)}$

for numerator:

$P(E=1) = 0.002$

$P(A=1 \mid E=1) = \sum_b P(A=1, B=b \mid E=1)$ marginalisation.

$= \sum_b P(A=1 \mid E=1, B=b) . P(B=b)$

$= P(A=1 \mid E=1, B=0) . P(B=0) + P(A=1 \mid E=1, B=1) . P(B=1)$

$= 0.29 \times (1-0.001) + 0.95 \times 0.001$

$= 0.29066$

Den: using marginalization.

$P(A=1) = \sum_e \sum_b P(A=1, E=e, B=b)$ bcz E, B marginally independent.

$= \sum_e \sum_b P(A=1 \mid E=e, B=b) . P(E=e) . P(B=b)$

$= 0.001 \times (1-0.002).(1-0.001) + 0.94 \times 0.001 \times (1-0.002)$
$+ 0.29 \times 0.002 \times (1-0.001) + 0.95 \times 0.002 \times 0.001$

$= 0.0025$

Putting it all together: $P(E=1 \mid A=1) = \dfrac{0.29066 \times 0.002}{0.0025}$

$= \underline{0.2325}$

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b) $P(E=1 \mid A=1, B=0) = \dfrac{P(A=1, B=0, E=1)}{P(A=1, B=0)}$

m. indep.

$= \dfrac{P(A=1 \mid B=0, E=1) \cdot \cancel{P(B=0)} \cdot P(E=1)}{\cancel{P(B=0)}\ P(A=1 \mid B=0)}$

$= \dfrac{0.29 \times 0.002}{1.578 \times 10^{-3}} = \underline{\underline{0.3675}}$

$\cdot\ P(A=1 \mid B=0) = \sum_e P(A=1, E=e \mid B=0)$

$= \sum_e P(A=1 \mid E=e, B=0) \cdot P(E=e)$

$= 0.001 \times (1-0.002) + 0.29 \times 0.002$

$= 1.578 \times 10^{-3}$

c) $P(A=1 \mid M=1) = \dfrac{P(A=1, M=1)}{P(M=1)} = \dfrac{P(A=1) \times P(M=1 \mid A=1)}{P(M=1)}$

$= \dfrac{0.7 \times 0.0025}{0.0117} = \boxed{0.149}$

marginalize

$!\ P(M=1) = \sum_a P(M=1 \mid A=a)\, P(A=a)$

$= 0.7 \times 0.0025 + 0.01 \times (1-0.0025) = 0.0117$

d) $P(A=1 \mid M=1, J=0) = \dfrac{P(M=1, J=0 \mid A=1) \cdot P(A=1)}{P(M=1, J=0)}$

num.

$P(A=1) = 0.0025$ (calculated)   they are cond. indep. given A.

$P(M=1, J=0 \mid A=1) = P(M=1 \mid A=1) \cdot P(J=0 \mid A=1)$

$= 0.7 \times (1-0.9) = 0.07$

denom.

$\cancel{P(M=1, J=0) = \sum_a P(M=1, J=0, A=a)}$

$= \cancel{\sum P(A=a \mid M=1, J=0) \cdot P(M=1) \cdot P(J=0)}$

$= \cancel{\sum_a P(A=a \mid M=1) \cdot P(A=a \mid J=0) \cdot P(M=1) \cdot P(J=0)}$

$=$

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den: $P(M=1, J=0) = \sum_a P(A=a, M=1, J=0)$

$= \sum_a P(A=a) \cdot P(M=1 \mid A=a) \cdot P(J=0 \mid A=a)$

$= 0.0025 \times 0.7 \times (1-0.9) + (1-0.0025) \times 0.01 \times (1-0.05)$

$= 9.65 \times 10^{-3}$

<u>All in All:</u> $P(A=1 \mid M=1, J=0) = \dfrac{0.07 \times 0.0025}{9.65 \times 10^{-3}} = \boxed{0.018}$

c) $P(A=1 \mid M=0) = \dfrac{P(M=0, A=1)}{P(M=0)} = \dfrac{P(A=1) \cdot P(M=0 \mid A=1)}{P(M=0)}$.

$P(M=0) = \sum P(M=0, A=a) = \sum P(M=0 \mid A=a) \cdot P(A=a)$

$= (1-0.7) \times 0.0025 + (1-0.01) \times (1-0.0025)$

$= 0.988275$

$\Rightarrow = \dfrac{0.0025 \times (1-0.7)}{0.988} = 7.59 \times 10^{-4} = \boxed{0.000759}$

f) $P(A=1 \mid M=0, B=1) = \dfrac{P(M=0 \mid A=1, B=1) \cdot P(A=1 \mid B=1) \cdot \cancel{P(B=1)}}{\cancel{P(B=1)} \cdot P(M=0 \mid B=1)}$

→ num. $P(M=0 \mid A=1, B=1) = P(M=0 \mid A=1)$ d-sp. $= 0.3$.

$P(A=1 \mid B=1) = \sum_e P(A=1, E=e \mid B=1) = \sum_e P(A=1 \mid B=1, E=e) \cdot P(E=e)$

$= 0.002 \times 0.95 + 0.998 \times 0.94 = 0.94$.

→ den: $P(M=0 \mid B=1) = \sum_a P(M=0, A=a \mid B=1)$

$= \sum_a P(M=0 \mid A=a) \cdot P(A=a \mid B=1)$

$= 0.94 \times 0.3 + 0.06 \times 0.99 = 0.3414$

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All mall: $P(A=1 \mid M=0, B=1) = \frac{0.94 \times 0.3}{0.3414} \boxed{\approx 0.826}$

P 2.2

$P(D=0) = P(D=1) = 1/2$.

for $D=0$. $P(S_1 \mid D=0) = 1$.

$k \geq 2$ $P(S_k = 1 \mid D=0) = \frac{f(k-1)}{f(k)}$

w/ $f(k) = 2^k + (-1)^k$

for $D=1$ $P(S_k = 1 \mid D=1) = 1/2$.

a) $r_k = \frac{P(D=0 \mid S_1=1 \ldots S_k=1)}{P(D=1 \mid S_1=1 \ldots S_k=1)}$

$= \frac{[P(S_1=1 \ldots S_k=1 \mid D=0) \cdot \cancel{P(D=0)}^{1/2}] / \cancel{P(S_1=1 \ldots S_k=1)}}{(P(S_1=1 \ldots S_k=1 \mid D=1) \cdot \cancel{P(D=1)}^{1/2}) / \cancel{P(S_1=1, S_2=1 \ldots)}}$

S_i are indep $\mid D$. so. num: $\prod_{i=1}^{k} P(S_i \mid D=0)$

$= 1 \times \frac{f(1)}{\cancel{f(2)}} \cdot \frac{\cancel{f(2)}}{\cancel{f(3)}} \ldots \frac{\cancel{f(k-1)}}{f k}$.

$= \frac{f(1)}{f(k)} = \frac{2-1}{2^k + (-1)^k} = \frac{1}{2^k + (-1)^k}$

den $= \prod_{i=1}^{k} P(S_i \mid D=1) = \left(\frac{1}{2}\right)^k$

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$$r_k = \frac{\frac{1}{2^k + (-1)^k}}{\frac{1^k}{2^k}} = \frac{2^k}{2^k + (-1)^k}$$

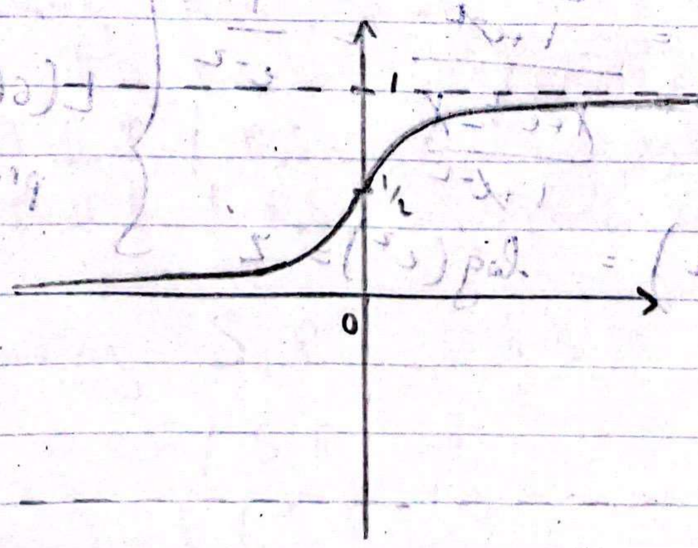
$(-1)^k$ is $\begin{cases} 1 & \text{when } k = 2n \\ -1 & \text{when } k = 2n+1 \end{cases}$

So on even days, $r_k < 1 \longrightarrow D = 1$
odd ", $r_k > 1 \longrightarrow D = 0$.

b) As more symptoms are observed, 2^k will be really close to $2^k - 1$ & $2^k + 1$.
(ex: $\frac{2}{3}$ & $\frac{999}{1000}$) → so ratio will converge to 1.

and hence diagnosis will become more & more harder (less + less certain) as k increases.

P.2.3



Sigmoid function

$\sigma(x) = (1 + e^{-x})^{-1}$

$\sigma(x) = \frac{1}{1 + e^{-x}}$.

a) $\sigma(z) \cdot \sigma(-z) = \dfrac{1}{1+e^{-z}} \cdot \dfrac{1}{1+e^{z}} = \dfrac{1}{1+e^{z}+e^{-z}+\cancel{e^{z} \cdot e^{-z}}}$

$$= \frac{1}{e^{z}+e^{-z}+2}$$

$$\sigma'(z) = \frac{d\,\sigma(z)}{dz} = \frac{-(-e^{-x})}{(1+e^{-x})^2} = \frac{e^{-z}}{1+e^{-2z}+2e^{-z}}$$

$$= \frac{\cancel{e^{-z}}}{\cancel{e^{-z}}(e^{z}+e^{-z}+2)} = \frac{1}{\underline{\underline{e^{z}+e^{-z}+2}}} \longleftarrow = \frac{e^{-z}}{e^{z} \cdot e^{-z}+e^{-2z}+2e^{-z}}$$

<u>proved</u>.

b) $\sigma(-z)+\sigma(z) = \dfrac{1}{1+e^{-z}} + \dfrac{1}{1+e^{z}}$

$$= \frac{1+e^{z}+1+e^{-z}}{(1+e^{-z})(1+e^{z})} = \frac{2+e^{z}+e^{-z}}{2+e^{z}+e^{-z}}$$

$= \underline{1}$. <u>Proved!</u>

c) $\dfrac{\sigma(z)}{1-\sigma(z)} = \dfrac{\frac{1}{1+e^{-z}}}{\frac{\cancel{1}+e^{-z}-\cancel{1}}{1+e^{-z}}} = \dfrac{1}{e^{-z}}$

$\log\left(\dfrac{1}{e^{-z}}\right) = \log(e^{z}) = z$

$L(\sigma(z)) = z$

proved!

d) $p_i = P(Y=1 \mid X_i = 1, X_j = 0 \text{ for all } i \neq j)$

$$L(p_i) = \log\left(\frac{p_i}{1-p_i}\right)$$

$$p_i = P(Y=1 / X_i = 1; X_j = 0)$$

$$= \sigma(w_i x_i + w_j x_j)$$

$$= \frac{1}{1+e^{-(w_i x_i + w_j x_j)}}$$

$$\log\left(\frac{\frac{1}{\cancel{1+e^{-(w_i x_i + w_j x_j)}}}}{\frac{\cancel{1}+e^{-(w_i x_i + w_j x_j)}-\cancel{1}}{\cancel{1+e^{-(w_i x_i + w_j x_j)}}}}\right)$$

$$\log\left(e^{(w_i x_i + w_j x_j)}\right)$$

$$= w_i x_i + w_j x_j$$

$x_i = 1 \quad x_j = 0$

$= w_i \times 1 + 0$

$= w_i$ (LHS) $\rightarrow$ Proved.

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Problem 2.4: Puddle (P), Sprinkler (S), Fall (F), Month (M), Rain (R)

+ Knowing Puddle: E{puddle}:

① $R \perp\!\!\!\perp F \mid P$. di

② $S \perp\!\!\!\perp F \mid P$ di

③ $M \perp\!\!\!\perp F \mid P$. di

* Knowing Month: E{Month}.

④ $R \perp\!\!\!\perp S \mid M$. di

* Knowing Rain, Sprinkler E{R, S}.

⑤ $M \perp\!\!\!\perp P \mid R,S$ di (both paths)

⑥ $M \perp\!\!\!\perp F \mid R,S$ di (both paths)

- Knowing S, P E{S, P}

⑦ $R \perp\!\!\!\perp F \mid S, P$ di

⑧ $M \perp\!\!\!\perp F \mid S, P$. di

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Knowing R, P   E {R, P}

9) M ⫫ F | E. di

10) S ⫫ F | E. di

E: {R, S, P}. X.Y = {M.F} 11. di

E: {M, S, P}: X.Y = {R, F} 12 di

13 E: {M, R, P}: X.Y: {S, F}. di

14 E {R, S, F}: X.Y = {M, P}. di

15 E {M, P}, X.Y = {R.F} di

16 E {M, P}: X.Y = {S.F} di

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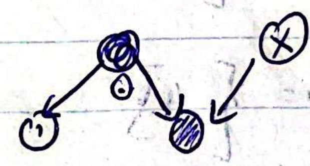
Problem 2.8:

a) < b) < c) < d) < e) = f) > g) <

(I will attach my scratch notes when solving these for reference if needed).

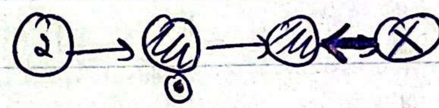
Problem 2.5

Case I: We have d-sep:

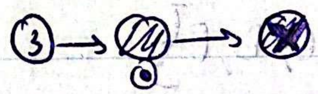


⊙ where d-sep condition applies.

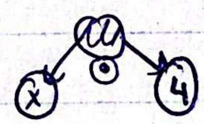
Case II: D-sep:



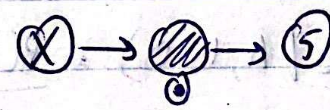
Case III: D-sep:



Case IV: D-sep:



Case V: D-sep:



Problem 2.6

1) F 2) T 3) T 4) F 5) T 6) T
7) F 8) T 9) T 10) F

Problem 2.7

1) $P(A|D) = P(A|D)$. $S = \{D\}$ Nothing else applies as per piazza

2) $S = \{B, D, C, E, F\}$.

5) $S = \{F\}$

3) $S = \{D, E, F\}$

6) $S = \{D, F\}$.

4) $S = \{A\}$

7) $S = \{A, B, C, D\}$

8) $S = \{\phi\}$

10) $S = \{A, B, C, D, E\}$

9) $S = \{D\}$

Problem 2.9

a) $P(C|A,B,D) = \dfrac{P(A,B,C,D)}{P(A,B,D)}$

$$= \frac{\cancel{P(A)} . \cancel{P(B|A)} . P(C|A,B) - P(D|A,B,C)}{\cancel{P(A)} - \cancel{P(B|A)} . P(D|A,B)}$$

① ② ③

=> ①: Given evidence = $\{A\}$ $C \perp\!\!\!\perp B | A$

-> $P(C|A,B) = P(C|A)$.

=> ② D, A are $\perp\!\!\!\perp$ given C -> $D \perp\!\!\!\perp A | C$

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$P(D \mid A, B, C) = P(D \mid B, C)$.

3) $P(D \mid A, B) = \sum_c P(D, C=c \mid A, B)$

marginalization $= \sum_c P(D \mid C=c, A, B) \times P(C=c \mid A, B)$.

$= \sum_c P(D \mid C=c, B) \times P(C=c \mid A)$

<u>Putting it together:</u>

(a) $P(C \mid A, B, D) = \dfrac{P(C \mid A) \cdot P(D \mid B, C)}{\sum_c P(D \mid B, C=c) \cdot P(C=c \mid A)}$

(b) ~~$P(E \mid A, B, D) = \dfrac{P(E, A, B, D)}{P(A, B, D)}$~~

$P(E \mid A, B, D) = \sum_c P(E, C=c \mid A, B, D) \times \cancel{P(C=c \mid A, B, D)}$.

$= \sum_c \underbrace{P(E \mid C=c, A, B, D)}_{1} \times \underbrace{P(C=c \mid A, B, D)}_{(a)}$

1 : Given $E = \{C\}$ $E \perp\!\!\!\perp A$ d; $E \perp\!\!\!\perp B$ d;; $E \perp\!\!\!\perp D$ d;; $\rightarrow P(E \mid C=c, A, B, D) = P(E \mid C=c)$

$= \sum_c P(E \mid C=c) \cdot \underbrace{P(C=c \mid A, B, D)}_{(a)}$.

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c) $P(G|A,B,D) = \sum_{e} P(G, E=e \mid A,B,D)$

$$= \sum_{e} \underbrace{P(G \mid E=e, A,B,D)}_{①} \times \underbrace{P(E=e \mid A,B,D)}_{(b)}$$

Given E., $G \perp\!\!\!\perp \{A,B,D\}$ d_i @ E $\rightarrow E \rightarrow G$

$\Rightarrow ① = P(G \mid E=e)$.

c) $\Rightarrow \sum_{e} P(G \mid E=e) \cdot \underbrace{P(E=e \mid A,B,D)}_{(b)}$

d) $P(F \mid A,B,D,G) = \dfrac{P(G,D,B,A,F)}{P(A,B,D,G)}$

$$= \frac{P(G \mid A,B,D,F)^{①} \times P(A,B,D,F)^{②}}{P(A,B,D,G)^{③}}$$

$$\frac{②}{③} = \frac{\cancel{P(F)}.\ \cancel{P(A,B,D)} \times P(F \mid A,B,D)}{\underbrace{P(G \mid A,B,D)}_{c} .\ \cancel{P(A,B,D)}}$$

now d) $= \dfrac{P(G \mid A,B,D,F)^{(1)} \times P(F \mid A,B,D)^{(2)}}{\underbrace{P(G \mid A,B,D)}_{(c)}}$

(1) becomes after marginalization

$$\sum_{e} P(G|E,A,B,D,F) \times \underbrace{P(E|A,B,D,F)}_{\Downarrow\ P(E|A,B,D)\ (b)}$$

$E \perp\!\!\!\perp F$ given A,B,D (case iii)

$G \perp\!\!\!\perp A,B,D$ given E,F. (case i @ E).

$$= \sum_{e} P(G/E.F) \times P(E/A,B,D)$$

(2). $P(F|A,B,D)$ is $P(F)$.

$A,B,D \perp\!\!\!\perp F$ given $E = \{\phi\}$ case iii @ G.

→ all together:

d) $$P(F|A,B,D,G) = \frac{P(F) \cdot \sum_{e} P(G/E,F) \times \overbrace{P(E|A,B,D)}^{(b)}}{\underbrace{P(G|A,B,D)}_{(c)}}$$

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