

Given a matrix of weights:  $W = \begin{bmatrix} W_{0,0} & \cdots & W_{0,k} \\ \vdots & \ddots & \vdots \\ W_{m,0} & \cdots & W_{m,k} \end{bmatrix}$

An input vector:  $x = \begin{bmatrix} x_0 \\ \vdots \\ x_k \end{bmatrix}$  A bias vector:  $b = \begin{bmatrix} b_0 \\ \vdots \\ b_m \end{bmatrix}$  And a target vector of desired outputs:  $T = \begin{bmatrix} T_0 \\ \vdots \\ T_m \end{bmatrix}$

$$\text{Weighted Sum of Output Neuron } i = S_i = b_i + \sum_{j=0}^k W_{i,j} * x_j$$

$$\text{Weighted Sum of all Output Neurons (vector)} = S = W * x + b$$

$$\text{Sigmoid Output of Output Neuron } i = O_i = \frac{1}{1 + e^{-S_i}} = (1 + e^{-S_i})^{-1}$$

$$\text{Sigmoid Output of all Output Neurons (vector)} = O = \begin{bmatrix} O_0 \\ \vdots \\ O_m \end{bmatrix}$$

$$\text{Mean Squared Error of all Output Neurons} = MSE = \sum_{n=0}^m (O_n - T_n)^2$$

$$\frac{\partial MSE}{\partial O_i} = 2(O_i - T_i)$$

$$\frac{\partial O_i}{\partial S_i} = O_i(1 - O_i) = \frac{1}{1 + e^{-S_i}} * \left(1 - \frac{1}{1 + e^{-S_i}}\right)$$

$$\frac{\partial S_i}{\partial W_{i,j}} = x_j$$

$$\frac{\partial MSE}{\partial W_{i,j}} = x_j * 2(O_i - T_i) * O_i(1 - O_i)$$