EINDHOVEN UNIVERSITY OF TECHNOLOGY

Control Principles for Engineered Systems

Problem set IV: Networked Control System

1 Algebraic graph theory: Incidence matrix, Laplacian, connectivity and rigidity

Consider the following (unweighted) undirected graph in Fig. 1.

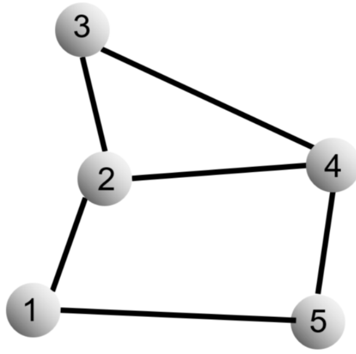

Figure 1: An undirected graph

1. Give the incidence matrix and the Laplacian matrix for the graph in Fig. 1.
2. Give the spectrum (all eigenvalues) of the Laplacian matrix (via e.g., Matlab).
3. Show that the graph is not rigid. Can you add one edge to the graph to make it rigid?
4. (Open research question): connectivity management for mobile networks is an active research topic in multi-agent systems. A naive idea to maintain connectivity for mobile networks is to preserve all existing edges in a connected graph when node dynamics are moving (i.e., when one edge exists, it should be preserved for all time). There are many other options for connectivity management in a dynamic environment.
   In the lecture we have discussed the algebraic connectivity (the second smallest eigenvalue of the graph Laplacian) as an index to quantify the connectivity of a static/dynamic graph. A possible solution is to maintain the positivity of this eigenvalue for a mobile graph. Read the survey paper [1] and summarize some key possible solutions on connectivity control of mobile robot networks.

2 Robotic coordination: continuous-time cyclic pursuit on the plane

(Modified from Exercise 1.6 of [2])

Consider four mobile robots on a plane with positions $p_i \in \mathbb{R}^2, i \in \{1, 2, 3, 4\}$, and moving according to $\dot{p}_i = u_i$, where $u_i \in \mathbb{R}^2$ are the velocity commands. The task of the robots is rendezvous at a common point (while using only onboard sensors). A simple strategy to achieve rendezvous is cyclic pursuit: each robot i picks another robot, say $i + 1$, and pursues it. (Here we follow the convention $4 + 1 \to 1$.) In other words, we set $u_i = p_{i+1} - p_i$ and obtain the closed-loop system (see also corresponding simulation below in Fig. 2):

$$
\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \\ \dot{p}_4 \end{bmatrix} = \left(\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix} \otimes I_2 \right) \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} \tag{1}
$$

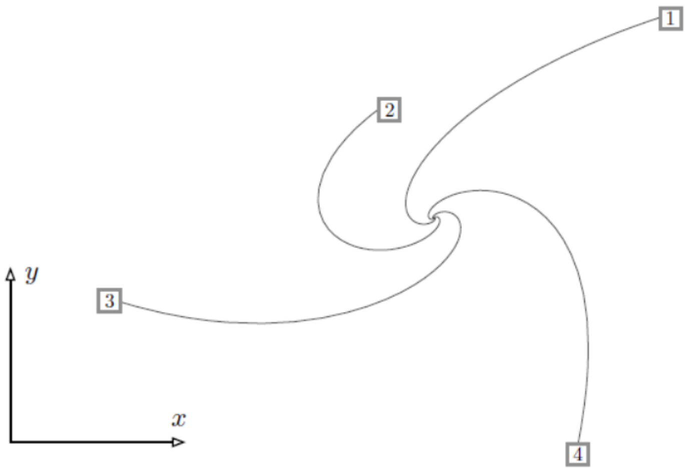

Figure 2: Robot rendezvous via cyclic pursuit.

1. Find the eigenvalues of the underlying Laplacian matrix.

2. Show that the average robot position $average(p(t)) = \sum_{i=1}^{4} p_i(t)/4$ remains constant for all $t \geq 0$;

3. Prove that the robots asymptotically rendezvous at the initial average robot position mass, that is,

$$
\lim_{t \to \infty} p_i(t) = average(p(0)), \quad i \in \{1, 2, 3, 4\} \tag{2}
$$

4. Show that if the robots are initially arranged in a square formation, then they remain in a square formation (the scaling of the shape is changing).

5. Simulate the four-robot dynamics of the system (1) under different initial conditions to verify the conclusions (2-4). Plot the trajectories of the four robots. You will expect to have similar trajectories as shown in Fig. 2.

6. (Open question) Can you speed up the convergence rate of the rendezvous by smartly adding one edge in the underlying directed graph? Justify your answer.

3 Formation control with obstacle avoidance

(Modified from Exercise 6.5 of [3])

If a team of robots is to drive in formation while avoiding obstacles as well as progressing toward a goal location, one can, for example, let the individual agent dynamics be given by

$$\dot{x}_i = F_{form} + F_{goal} + F_{obst}, \tag{3}$$

where F_{form} is used to maintain formations. However, F_{goal} is used to steer the robot towards a goal and F_{obst} is used to have it avoid obstacles.

1. Find reasonable F_{goal} and F_{obst} (if you are unable to design F_{obst}, search in Google Scholar via keywords such as "collision avoidance" "obstacle avoidance" and you will find lots of research papers that will give you some hints).

2. Simulate your proposed solution (you can use either displacement-based or distance-based approach for F_{form}.). The final result should look something like the plots in Fig. 3.

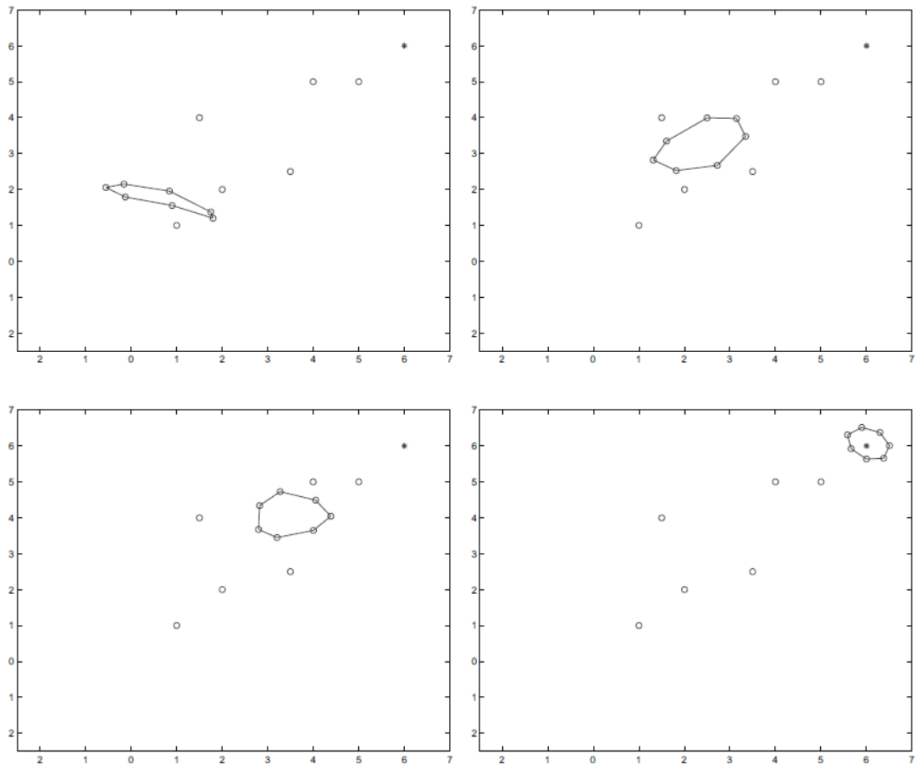

Figure 3: Formation movement while avoiding obstacles.

References

[1] M. M. Zavlanos, M. B. Egerstedt, and G. J. Pappas, "Graph-theoretic connectivity control of mobile robot networks," *Proceedings of the IEEE*, vol. 99, no. 9, pp. 1525–1540, 2011.

[2] F. Bullo, *Lectures on network systems.* Kindle Direct Publishing, http://motion.me.ucsb.edu/book-lns/, 2022.

[3] M. Mesbahi and M. Egerstedt, *Graph theoretic methods in multiagent networks.* Princeton University Press, 2010, vol. 33.