Project Assignment

Automated design and control for multi-drone systems

1 Background

Recent advances of drone technologies have seen promising applications of using multi-drone systems in the detection, monitoring, and rescue for targets in large open areas. Interesting application examples involve the usage of drones to monitor wide open forests for early bushfire detection and the commissioning of drones and thermal imaging to locate koalas injured in bushfires (Fig. 1).

Figure 1: A drone hovers a distance from a koala, in a bushfire-stricken Victorian forest, Australia

However, the current application of drone technology in area coverage, search-and-rescue, and bushfire-fighting missions depends heavily on human operations and interactions. The development of autonomous drone techniques, in particular autonomous multi-drone systems that can adapt fast to different environments and working conditions, is urgently needed.

Multi-robot system control with complex tasks is challenging due to the high dimensions and the substantial amount of constraints. Taking the search-and-rescue task as an example, autonomous drones have to sequentially achieve a series of subtasks, including locating the survivors, navigating to the rescue spots, and transporting the survivors to a designated safe zone. In this type of complex task, drone swarms should autonomously change formations to cooperatively execute sensing and communication operations or to traverse narrow spaces. Such practical tasks involve complex navigational requirements imposed by the sequential subtasks in combination with formation control requirements on the multi-drone system. While the former navigational requirements have been successfully solved by first specifying them with Linear-time Temporal Logic (LTL) and using related tools, there is not much work that explicitly incorporates the formation requirements.

In this project, we will investigate certain key design problems and control principles of multi-robot systems (Fig. 2) for complex tasks represented by LTL specifications with automated formations.

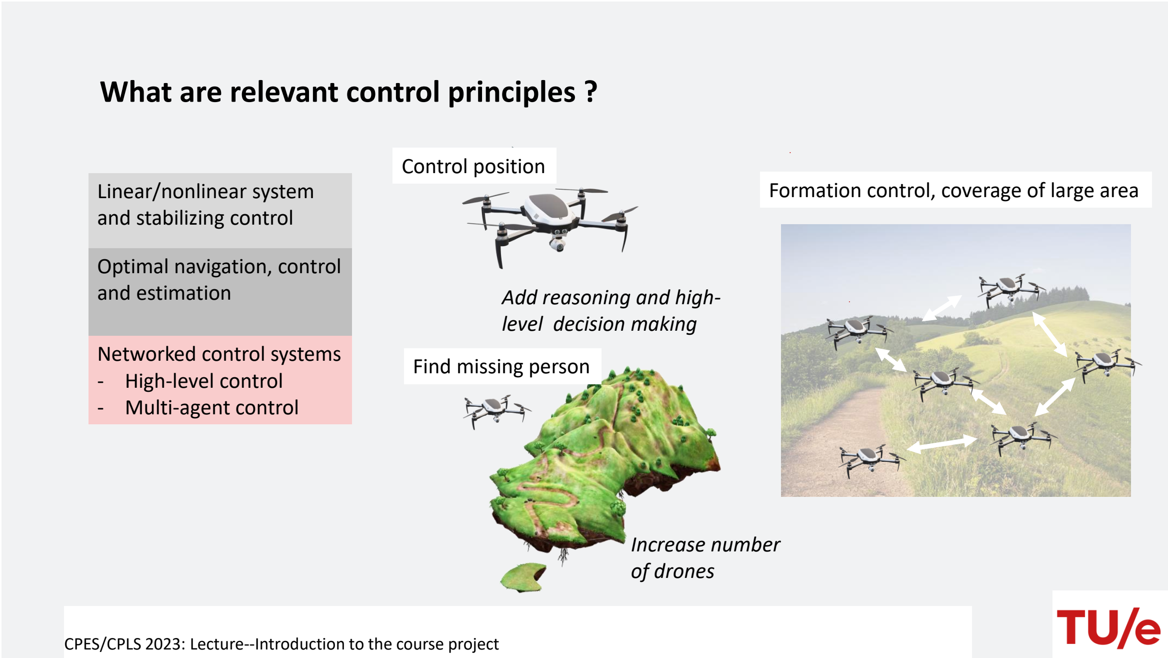

Figure 2: Control principles for multi-drone systems

2 Questions

2.1 Single-robot system

We consider the horizontal flight path of a quadrotor UAV. The continuous-time dynamics of a quadrotor drone is simplified as represented by the following double integrator model,

$$\begin{aligned}\dot{x} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix} \\ y &= \begin{bmatrix} p_x \\ p_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x \end{aligned} \tag{1}$$

where the system state $x = [p_x, p_y, v_x, v_y]^T$ consists of position vector $[p_x, p_y]$ and velocity vector $[v_x, v_y]$, and the control input is the acceleration vector $u = [a_x, a_y]^T$. The output y consists of the position of the UAV. Fig. 3 depicts the 2-dimensional environment in which the drone can operate.

1. **(Tracking control)** As shown in Fig. 3, five waypoints are already given, including

   $$s_1 = (10, 10), s_2 = (50, 10), s_3 = (80, 10), s_4 = (80, 30), s_5 = (80, 50).$$

   In the figure, some of the waypoints are connected with an arc $\leftrightarrow$. In this question, you have to develop a controller that can move the UAV from one waypoint to another connected waypoint.

   1.a Consider as a constant reference the position of the waypoint s_i with $i \in [1, \dots, 5]$. Design a continuous-time LQR controller that can track any of these waypoints. Show the derivations in the report and implement the code.

   1.b Show that the controlled UAV initialized at s_0 can follow the sequence of waypoints $s_1 \to s_2 \to s_3 \to s_4 \to s_5$. Give the simulation results.

2. **(Verification)** Consider the transition system built on predefined waypoints $[s_1, s_2, s_3, s_4, s_5]$, interconnected edges, area labels, and the initial position s_1 as shown in Fig. 3. Give the transition system and for each of the following specifications verify whether the transition system can be used to design a control strategy such that the controlled transition system satisfies them:

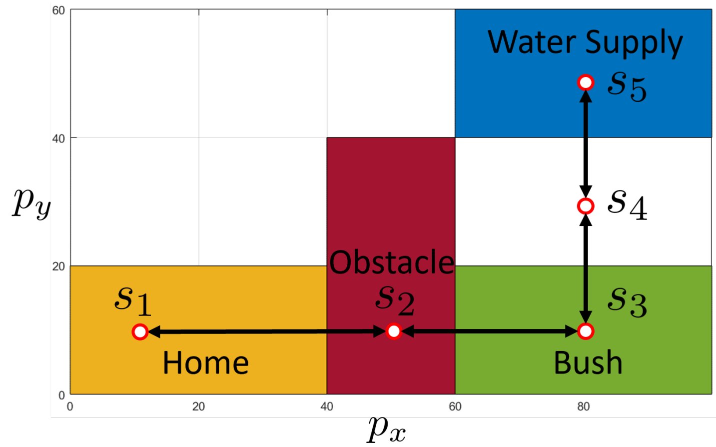

Figure 3: Planar environment for quadrotor UAVs with the size of $100m \times 60m$. The yellow, red, blue, and green areas in the environment are labeled as home, obstacle, water, and bush, respectively.

- $(\Diamond \text{bush}) \ \wedge \ (\Diamond\Box\neg \text{obstacle})$
- $(\Box\Diamond \text{water}) \ \wedge \ (\Box\neg \text{obstacle})$

If a feasible controller can be found to satisfy a specification, please provide the feasible prefix path and suffix cycle. If not, provide the reason based on the analysis of the product automata.

In this subquestion, you do not need to design the controller for the state space model (1).

3. **(High-level control)** Design a controller for the state-space model in (1) such that the following LTL specification

$$\psi = (\Box\Diamond \text{home}) \ \wedge \ (\Box\Diamond \text{water}) \ \wedge \ (\Box\Diamond \text{bush}) \ \wedge \ (\Box\neg \text{obstacle})$$

is satisfied.

3.a Extend the number of waypoints and define the new transition system. Requirements:
  - There should be waypoints in all the different labeled areas.
  - Every transition between two waypoints should represent a continuous trajectory of your state space model. Make sure that you do not have transitions in your transition system in which the corresponding continuous trajectory switches labels more than once.

3.b Design a controller for your finite transition system. If there does not exist a controller go back to **3.a** and add extra waypoints.

3.c Use the controller designed in Question 1 and the solution of **3.b** to design a controller for the state space model (1) that ensures the satisfaction of the LTL specification. Give a simulation of this controller.

2.2 Multi-robot system

In this section on the multi-agent systems, we still use the double-integrator equation to model the dynamics of quadrotor drones.

4. **(Formation shape control)** In the task of bushfire fighting, drone swarms are expected to transform into specific formations for different tasks. Considering two formations for a 4-drone swarm as shown in Figure 4, you are supposed to achieve the formation switch between them.

- Specify the incidence matrix and Laplacian matrix of each formation in Fig. 4, and give the spectrum of the Laplacian. Determine respectively whether the formation shapes in Fig. 4 are rigid or not.

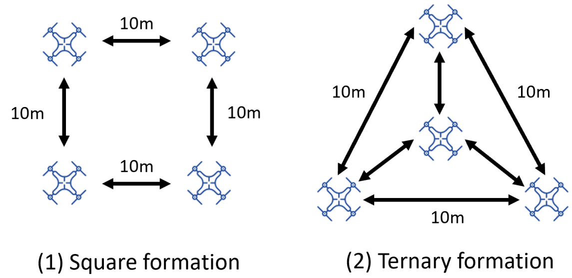

Figure 4: Formation Sketch

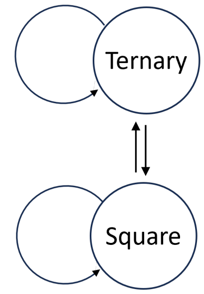

Figure 5: Formation Automata

- In terms of formation rigidity, design the displacement-based controller for the square formation, and the distance-based controller for the ternary formation, respectively. Prove the stability of the formation shape control system for randomly generated initial positions. Plot simulation results and show the convergence of the formation shape stabilization.
- Combine the two designed formation controllers and achieve the autonomous switch as depicted in the transition system shown in Fig. 5. Initialized with square formation, the swarm is required to keep one formation for 20 seconds and then switch to another formation repetitively.

5. **(Leader-follower flocking control)** Implement a leader-follower flocking system for the drone swarm by controlling the leader and followers based on the last question. Assume the drones are randomly initialized in the surrounding area.

   - Construct the flocking controller and explain the structure of your flocking control system (you can use either the displacement-based approach or distance-based approach to design the formation control part in the flocking controller; see the lecture slides or [1]);
   - Select a drone as the leader of the swarm, which is tasked with additional inputs for swarm motion. Consider the following two cases:
     - The leader is moving in a straight line (i.e., $[f_{x,l}, f_{y,l}]^T$ is a constant vector). You can choose a constant vector $[f_{x,l}, f_{y,l}]^T$ in your simulation.
     - The leader is moving in a circle. You will have to specify a vector function $[f_{x,l}, f_{y,l}]^T$ so that the leader moves with a circular motion.

   Simulate the flocking dynamics under the above two cases, and present your findings. (We expect to see 2D plots showing the trajectories of the leader and followers drones)

6. **(Formation control under LTL specifications)** In the process of search-and-rescue and fire fighting, drone swarms are expected to switch different formations in different areas to fulfill tasks. Consider the following LTL specifications about the navigation and formation of the drone swarm:

$$\psi = \Box\Diamond(\text{water} \wedge \text{square}) \;\wedge\; \Box\Diamond(\text{home} \wedge \text{square}) \;\wedge\; \Box\Diamond(\text{bush} \wedge \text{ternary}) \;\wedge\; (\Box\neg\text{obstacle})$$

   Suppose that a drone swarm is initialized at the home area with square formation. Design a combined transition system based on sampled waypoints and appropriate corresponding formations, and further develop a control strategy for the combined transition system satisfying ψ.

   In addition, drive the drone swarm to execute the synthesized strategy in combination with the flocking controller developed in the last question. To simplify the problem, we only consider obstacle avoidance for the leader drone. Provide simulation results to demonstrate the drone swarm can be driven to satisfy the LTL specification ψ.

References

[1] K.-K. Oh, M.-C. Park, and H.-S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, 2015.