Quantifying the Consistency of Uncertainty Propagation Algorithms in Cislunar Space

MOTIVATION

- Consistently propagate statistical moments in nonlinear systems
- Earth-Moon CR3BP
- Statistical moments: mean and covariance

$$\boldsymbol{\mu} = E[\boldsymbol{x}] = \int \boldsymbol{x} p(\boldsymbol{x})\, d\boldsymbol{x}$$

$$\Sigma = E\left[(\boldsymbol{x}-\boldsymbol{\mu})(\boldsymbol{x}-\boldsymbol{\mu})^T\right] = \frac{1}{N}\sum_i^N (\boldsymbol{x}_i-\boldsymbol{\mu})(\boldsymbol{x}_i-\boldsymbol{\mu})^T$$

ALGORITHMS

Monte Carlo (MC)

- Benchmark algorithm since its creation in the 1940s
- Given an initial distribution, MC randomly samples points
- Independently propagates points
- Increasing the number of points by 100 typically decreases error by 0.1

Unscented Transform (UT)

- Samples points on primary axes to create a 2nd-order accurate estimate
- Points are weighted equally
- Independently propagate points through time

Conjugate Unscented Transform (CUT)

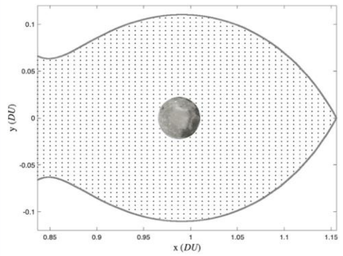
- Samples points on conjugate axes to create a 4th-, 6th-, or 8th-order accurate estimate
- Used CUT 8 (8th-order) in this study
- Points assigned weights to satisfy the constraint equations

$$1 = \sum_i^N w_i \qquad \boldsymbol{\mu} = \sum_i^N w_i \boldsymbol{x}_i$$

$$\Sigma = \sum_i^N w_i (\boldsymbol{x}_i-\boldsymbol{\mu})(\boldsymbol{x}_i-\boldsymbol{\mu})^T$$

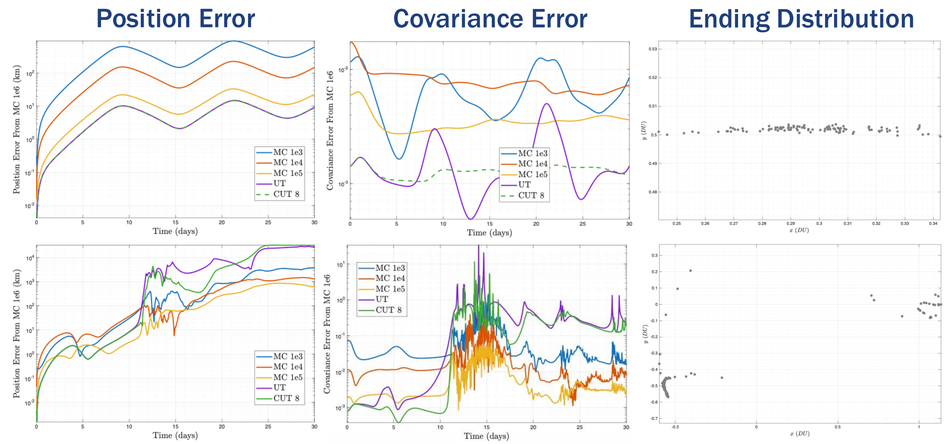
	MC	UT	CUT 8
4 dimensions	100,000	8	161
6 dimensions	100,000	12	745

L_1 to L_2 Grid Points



- 1810 grid points between L_1 and L_2
- Jacobi energy of L_2
- Perilune initial conditions
- Propagated for 30 days
- 5 km position and 1 m/s velocity deviation
- UT, CUT 8,100,000 MC and 1,000,000 MC
- 1,810,000,000 trajectories

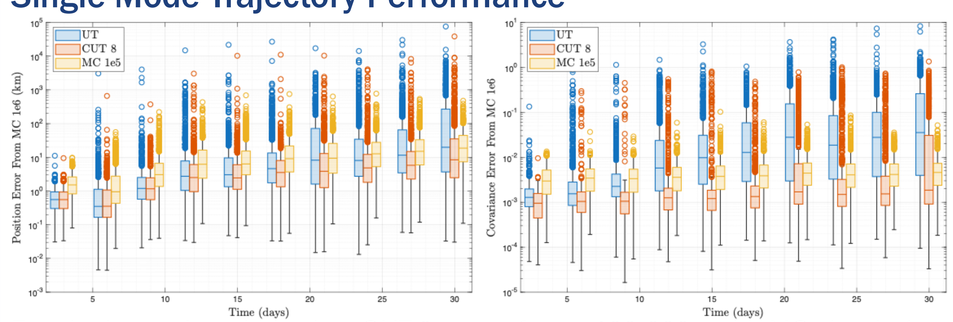
Individual Grid Points: 10 and 500



Multi-Modal Problem

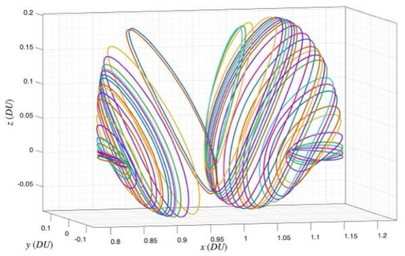
When the distribution becomes multi-modal, statistical moments break down. Trajectories became multi-modal after propagating through the Moon or escaping the L_2 gateway. UT and CUT 8 perform badly for these trajectories.

Single-Mode Trajectory Performance



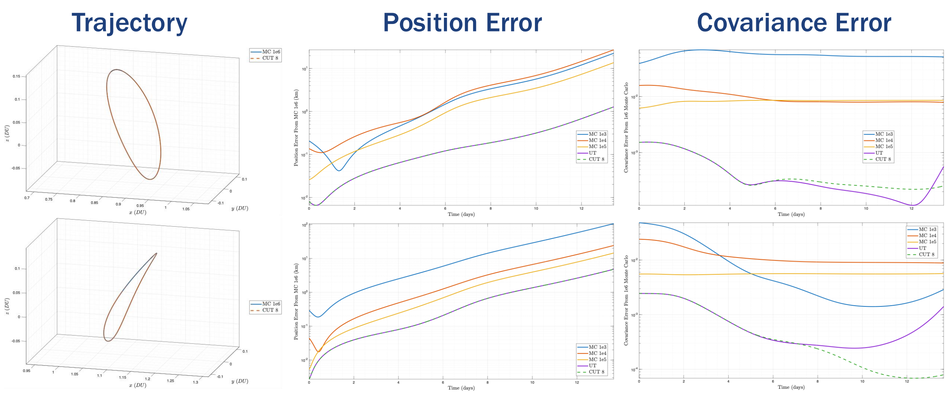
For single-mode trajectories, CUT 8 outperforms 100,000-point MC simulation for most trajectories in estimation of both mean and covariance. UT and CUT 8 are prone to outliers.

L_1 and L_2 Halo Orbits

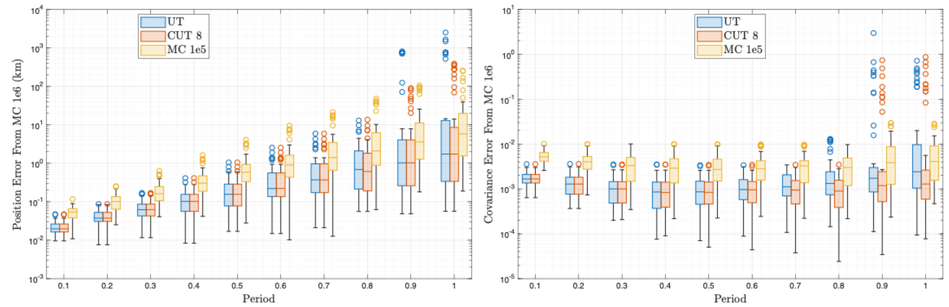


- 23 L_1 and 27 L_2 Halo orbits
- Propagated for one time periods
- 5 km position and 0.1 m/s velocity deviation
- UT, CUT 8,100,000 and 1,000,000 MC

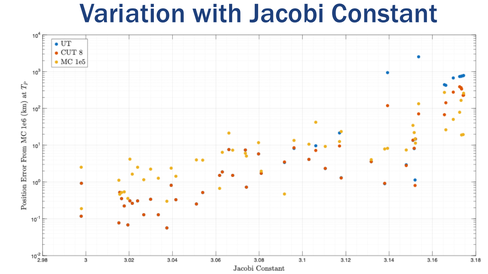
Individual Halo Orbits:



Overall Trajectory Performance



Variation with Jacobi Constant



Over 1 period, both UT and CUT 8 provide better estimates than 100,000-point MC. The difference between UT and CUT 8 is noticed in the covariance. The position error at one time period increased with Jacobi constant.

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