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Generowanie sumy kontrolnej pliku:
cat plik | tr -d "[:space:]" | md5sum

#### 1. Makefile

```
b5b7c66cc0f4193b9ec2c3f33ca878f3\\ CXXFLAGS = -Wall - Wextra - Wshadow - Wunused - std=c++17 - 03 - fsanitize=address\\ \hookrightarrow - fsanitize=undefined
```

#### 2. Pragma

```
// wpisujemy na samym poczatku kodu
#pragma comment(linker, "/stack:200000000")
#pragma GCC optimize("03")
#pragma GCC optimize("0fast")
#pragma GCC optimize("unroll-loops")
#pragma GCC target("sse, sse2, sse3, sse4, avx, avx2")
```

#pragma GCC target("popcnt,abm,mmx,avx,tune=native")

#### 3. Vimrc

```
syn on
set nu cin sw=4 ts=4
ino jk <ESC>
no <C-e> :w <bar> :!make %:r && ./%:r <cr>
no <F4> :w <bar> :!make %:r && ./%:r <cr>
```

f2c727b8f588b09620aac2f114d1770f

dd7c731ec381c16234b3b75e11daf308

#### 4. Policy-Based Data Structures

```
cout << M.find_by_order(0) -> second << " " << M.order_of_key(3);</pre>
                                        5. RMQ
   f1c82285d5563e4325f8b48d558ade25
// uzycie - RMQ < int, max > R(n, t);
template < typename TP, const TP& (*F)(const TP&, const TP&) >
struct RMO {
       int n; vector < TP > t;
        vector < vector < TP > > res;
        RMQ(vector < TP > T) : n(sz(T)), t(T) {
                res.push_back(t);
                for(int p = 2; p <= n; p <<= 1) {</pre>
                        vector < TP > tmp(n - p + 1);
                        for(int j = 0; j <= n - p; j++)
                                tmp[j] = F(res.back()[j], res.back()[j + p / 2]);
                        res.push_back(tmp);
                }
        TP query(int a, int b) {
                int d = b - a + 1, lg = 31 - __builtin_clz(d);
                return F(res[lg][a], res[lg][b - (1 << lg) + 1]);</pre>
};
                                      6. KD Tree
   4176f5e9557c3f86af88fb8e33f32399
const int d = 2; //dimension
struct point {
 int p[d];
  bool operator !=(const point &a) const {
   bool ok = 1;
    for(int i = 0; i < d; i++) ok &= (p[i] == a.p[i]);</pre>
    return !ok:
};
struct kd node {
 int axis, value:
 point p;
 kd_node *left, *right;
};
struct cmp_points {
 int axis;
 cmp_points() {}
```

```
cmp_points(int x): axis(x) {}
  bool operator () (const point &a, const point &b) const {
   return a.p[axis] < b.p[axis];</pre>
};
typedef kd node* node ptr;
int tests, n;
point arr[N], pts[N];
node_ptr root;
long long ans;
long long squared_distance(point a, point b) {
  long long ans = 0;
  for(int i = 0; i < d; i++) ans += (a.p[i] - b.p[i]) * 111 * (a.p[i] - b.p[i]);
 return ans:
void build_tree(node_ptr &node, int from, int to, int axis) {
  if(from > to) {
   node = NULL;
    return;
  node = new kd_node();
  if(from == to) {
   node->p = arr[from];
   node->left = NULL;
    node->right = NULL;
    return;
  int mid = (from + to) / 2;
  nth_element(arr + from, arr + mid, arr + to + 1, cmp_points(axis));
  node->value = arr[mid].p[axis];
  node->axis = axis;
 build tree (node->left, from, mid, (axis + 1) % d);
 build_tree(node->right, mid + 1, to, (axis + 1) % d);
void nearest_neighbor(node_ptr node, point q, long long &ans) {
  if(node == NULL) return;
  if (node->left == NULL && node->right == NULL) {
    if(q != node->p) ans = min(ans, squared_distance(node->p, q)); ///Beware!!! take
    return:
```

```
if(g.p[node->axis] <= node->value) {
   nearest_neighbor(node->left, q, ans);
   if(q.p[node->axis] + sqrt(ans) >= node->value) nearest_neighbor(node->right, q,
   → ans);
  else {
   nearest_neighbor(node->right, q, ans);
   if(q.p[node->axis] - sqrt(ans) <= node->value) nearest_neighbor(node->left, q, ans);
                                   7. LI CHAO TREE
   2c59c8cbf4632218545950f1afa7ef8d
    minimum value of linear functions
    1 a b - add linear function f(x) = a * x + b
    2 \times -  find minimum value of f(x), 0 <= x <= 1e6
namespace discreteLinear {
    typedef double T;
    const T INF = 1e15:
    struct line {
       T a, b;
       line(T A = 0, T B = INF) : a(A), b(B) {}
       T val(T x) { return a * x + b; }
    const int MAX = 1 \ll 20;
    line tree[2 * MAX];
    void update(line f, int node = 1, int l = 0, int r = MAX - 1) {
       if(tree[node].val(1) > f.val(1)) swap(tree[node], f);
       if(tree[node].val(r) < f.val(r)) return;</pre>
        int m = (1 + r) / 2;
        if(tree[node].val(m) < f.val(m))</pre>
            update(f, 2 * node + 1, m + 1, r);
            swap(tree[node], f), update(f, 2 * node, 1, m);
    T query(int x, int node = 1, int l = 0, int r = MAX - 1) {
        if(l == r) return tree[node].val(x);
        int m = (1 + r) / 2;
        if(x <= m) return min(tree[node].val(x), query(x, 2 * node, 1, m));</pre>
        return min(tree[node].val(x), query(x, 2 * node + 1, m + 1, r));
```

# 8. LCT Jarek

```
1d8a7c903746f7e7f981f5718cfa5468
struct node {
   typedef int T;
   static const T def = 0;
   node *left = 0, *right = 0, *up = 0, *path_parent = 0;
   T val, agg = def;
   bool rev = false;
   node(T v = def) : val(v) { update(); }
   static int f(int a, int b) {
       return max(a, b);
   void update() {
       agg = f(left ? left->agg : def, f(val, right ? right->agg : def));
   bool left_son() {
       return up->left == this;
   void swap_ups(node *n) {
       n->up = up;
       n->path_parent = path_parent;
       up = n;
       path_parent = nullptr;
       n->update();
       if(n->up) {
           if(n->up->left == this) n->up->left = n;
           else n->up->right = n;
           n->up->update();
   void rot left() {
       node *n = left;
       left = n->right;
       if(left) left->up = this;
       n->right = this;
       update();
       swap_ups(n);
   void rot right() {
       node *n = right;
       right = n->left;
       if(right) right->up = this;
       n->left = this;
       update();
       swap_ups(n);
   void rot up() {
       if(left_son()) up->rot_left();
       else up->rot_right();
```

```
void update_rev() {
    if(up) up->update_rev();
    if(rev) {
        rev = false:
        swap(left, right);
        if(left) left->rev = !left->rev;
       if(right) right->rev = !right->rev;
node* go_up() {
   update_rev();
    while(up) {
       if(up->up)
            if(up->left_son() == left_son()) {
                up->rot_up();
                rot_up();
            else {
                rot_up();
                rot_up();
        else rot_up();
    return this;
node* access() {
   node *last = nullptr;
   node *n = this;
    while(n) {
        n->go_up();
        if(n->left)
            swap(n->left->up, n->left->path_parent);
        n->left = last;
        if(last) swap(last->up, last->path_parent);
        n->update();
        last = n;
        n = n->path_parent;
    go_up();
    return last;
void link(node *n) {
   access();
    rev = !rev;
   path parent = n;
void cut() {
   access();
   if (right) {
        right -> up = 0;
        right->path_parent = 0;
        right = 0;
```

```
node* root() {
        access();
        node *n = this;
        while (n->right)
           n = n->right;
        n->go_up();
        return n;
};
node* lca(node *a, node *b) {
    a->access();
    return b->access();
node::T on_path(node *a, node *b) {
    node *c = lca(a, b);
    a->go_up();
    c->qo_up();
    if(a == c) return c->left->agg;
    if (b == c) return a->agg;
    return node::f(a->agg, c->left->agg);
node* insert(node *n, T k) {
    if(n == 0) return new node(k);
    n = n->splay(k);
    node *r = new node(k);
    if(n->val < k) {
        r->right = n->right; n->right = 0;
        if(r->right) r->right->up = r;
        r->left = n; n->up = r;
        n->update(); r->update();
        r->left = n->left; n->left = 0;
        if(r\rightarrow left) r\rightarrow left\rightarrow up = r;
        r->right = n; n->up = r;
        n->update(); r->update();
    return r;
node* erase(node *n, T k) {
    if(n == 0) return 0;
    n = n -> splay(k);
    if (n->val != k) return n;
    if(n->left==0) {
        if(n->right == 0) return 0;
        n->right->up = 0;
        return n->right;
    n->left->up = 0;
    node *r = n \rightarrow left \rightarrow splay(k);
    r->right = n->right;
```

```
if(r->right) r->right->up = r;
   r->update(); delete n;
   return r:
                                   9 LCT BOHUN
   3de26330f43cb16a3249de56e7802dbf
struct SplayTree {
 struct Node {
   int ch[2] = \{0, 0\}, p = 0;
   long long self = 0, path = 0;
                                        // Path aggregates
   long long sub = 0, vir = 0;
                                        // Subtree aggregates
   bool flip = 0;
                                        // Lazy tags
 vector<Node> T;
 SplayTree(int n) : T(n + 1) {}
 void push(int x) {
   if (!x || !T[x].flip) return;
   int 1 = T[x].ch[0], r = T[x].ch[1];
   T[1].flip ^= 1, T[r].flip ^= 1;
   swap(T[x].ch[0], T[x].ch[1]);
   T[x].flip = 0;
 void pull(int x) {
   int 1 = T[x].ch[0], r = T[x].ch[1]; push(1); push(r);
   T[x].path = T[1].path + T[x].self + T[r].path;
   T[x].sub = T[x].vir + T[1].sub + T[r].sub + T[x].self;
 void set(int x, int d, int y) {
   T[x].ch[d] = y; T[y].p = x; pull(x);
 void splay(int x) {
   auto dir = [&](int x) {
     int p = T[x].p; if (!p) return -1;
     return T[p].ch[0] == x ? 0 : T[p].ch[1] == x ? 1 : -1;
   auto rotate = [&](int x) {
     int y = T[x].p, z = T[y].p, dx = dir(x), dy = dir(y);
     set (y, dx, T[x].ch[!dx]);
     set (x, !dx, y);
     if (\simdy) set(z, dy, x);
     T[x].p = z;
```

```
};
    for (push(x); \sim dir(x);) {
     int y = T[x].p, z = T[y].p;
     push(z); push(y); push(x);
     int dx = dir(x), dy = dir(y);
     if (\simdy) rotate(dx != dy ? x : y);
     rotate(x);
};
struct LinkCut : SplayTree {
 LinkCut(int n) : SplayTree(n) {}
 int access(int x) {
   int u = x, v = 0;
   for (; u; v = u, u = T[u].p) {
     splay(u);
     int& ov = T[u].ch[1];
     T[u].vir += T[ov].sub;
     T[u].vir -= T[v].sub;
     ov = v; pull(u);
    return splay(x), v;
  void reroot(int x) {
   access(x); T[x].flip ^= 1; push(x);
  // Podczep u jako dziecko v
  void Link(int u, int v) {
   reroot(u); access(v);
   T[v].vir += T[u].sub;
   T[u].p = v; pull(v);
  // Usuń krawędz (u, v)
  void Cut(int u, int v) {
   reroot(u); access(v);
   T[v].ch[0] = T[u].p = 0; pull(v);
  // Rooted tree LCA. Returns 0 if u and v arent connected.
  int LCA(int u, int v) {
   if (u == v) return u;
   access(u); int ret = access(v);
   return T[u].p ? ret : 0;
  // Query subtree of u where v is outside the subtree.
  long long Subtree(int u, int v) {
```

```
reroot(v); access(u); return T[u].vir + T[u].self;
  // Query path [u..v]
 long long Path(int u, int v) {
   reroot (u); access (v); return T[v].path;
 // Update vertex u with value v
 void Update(int u, long long v) {
   access(u); T[u].self = v; pull(u);
};
                                    10. Treap Adam
   5eb594d6af64d9ef8446db75b51d7500
struct treap // treap z zadania odwracane sortowanie
        struct node
                node* 1:
                node* r;
                int val;
                int min:
                int rev;
                int size;
                int rank:
                node (int val = 0) : val(val), min(val), rev(false), l(NULL), r(NULL),
                \hookrightarrow size(1), rank(rand()) {}
        };
        node pool[1000005];
        int head = 0;
        node* mynew (int val) {
                pool[head] = node(val);
                return pool + head ++;
        node* root = NULL;
        int size(node* u) { return u == NULL ? 0 : u->size;
        int min(node* u) { return u == NULL ? le9 : u->min; }
        void pull(node* u) {
                u \rightarrow size = 1 + size(u \rightarrow 1) + size(u \rightarrow r);
                u->min = std::min(u->val, std::min(min(u->l), min(u->r)));
```

```
void push(node* u){
        if (u->rev) {
                swap(u->1, u->r);
                if (u->1 != NULL) u->1->rev ^= 1;
                if (u->r != NULL) u->r->rev ^= 1;
                u->rev = false;
        }
pair <node*, node*> split(node* u, int k){
        if (u == NULL) return {NULL, NULL};
        push(u);
        if (k <= size(u->1)) {
                auto p = split(u->1, k);
                u\rightarrow 1 = p.s;
                pull(u);
                return {p.f, u};
        } else {
                auto p = split(u->r, k - size(u->1) - 1);
                u \rightarrow r = p.f;
                pull(u);
                return {u, p.s};
        }
node* merge(node* a, node* b) {
        if (a == NULL) return b;
        if (b == NULL) return a;
        push(a); push(b);
        if (a->rank > b->rank) {
                a \rightarrow r = merge(a \rightarrow r, b);
                pull(a);
                return a;
        } else {
                b->1 = merge(a, b->1);
                pull(b);
                return b;
void push_back(int val) {
        root = merge(root, mynew(val));
void pop_front(){
        root = split(root, 1).s;
int find min(){
        int res = 0;
        node* u = root;
```

```
push(u);
                while (u->val != u->min) {
                       push(u);
                       if (\min(u->1) < \min(u->r))
                               u = u -> 1;
                       else {
                               res += size(u->1) + 1;
                               u = u -> r;
               push(u);
               res += size(u->1);
               return res;
       void reverse_pref(int k){
               auto p = split(root, k);
               p.f->rev ^= 1;
               root = merge(p.f, p.s);
};
                                    11. Wavelet
   bldf6bfeb814126836a6397c6686a611
//array values can be negative too, use appropriate minimum and maximum value
struct wavelet_tree {
 int lo, hi;
 wavelet_tree *1, *r;
 int *b, *c, bsz, csz; // c holds the prefix sum of elements
 wavelet_tree() {
   10 = 1;
   hi = 0;
   bsz = 0;
   csz = 0, l = NULL;
   r = NULL;
 void init(int *from, int *to, int x, int y) {
   lo = x, hi = y;
   if(from >= to) return;
   int mid = (lo + hi) >> 1;
   auto f = [mid](int x) {
     return x <= mid;</pre>
   b = (int*)malloc((to - from + 2) * sizeof(int));
   bsz = 0;
   b[bsz++] = 0;
   c = (int*)malloc((to - from + 2) * sizeof(int));
```

```
csz = 0;
    c[csz++] = 0;
    for(auto it = from; it != to; it++) {
     b[bsz] = (b[bsz - 1] + f(*it));
     c[csz] = (c[csz - 1] + (*it));
     bsz++;
     csz++;
   if(hi == lo) return;
    auto pivot = stable_partition(from, to, f);
   1 = new wavelet_tree();
   1->init(from, pivot, lo, mid);
   r = new wavelet_tree();
   r->init(pivot, to, mid + 1, hi);
  //kth smallest element in [1, r]
  //for array [1,2,1,3,5] 2nd smallest is 1 and 3rd smallest is 2
  int kth(int 1, int r, int k) {
   if(1 > r) return 0;
   if(lo == hi) return lo;
   int inLeft = b[r] - b[1 - 1], 1b = b[1 - 1], rb = b[r];
   if(k <= inLeft) return this->l->kth(lb + 1, rb, k);
   return this->r->kth(l - lb, r - rb, k - inLeft);
  //count of numbers in [1, r] Less than or equal to k
  int LTE(int 1, int r, int k) {
   if(1 > r || k < 10) return 0;
   if(hi <= k) return r - 1 + 1;
   int 1b = b[1 - 1], rb = b[r];
   return this->1->LTE(1b + 1, rb, k) + this->r->LTE(1 - 1b, r - rb, k);
  //count of numbers in [1, r] equal to k
  int count(int 1, int r, int k) {
   if(1 > r || k < lo || k > hi) return 0;
   if(lo == hi) return r - l + 1;
   int 1b = b[1 - 1], rb = b[r];
   int mid = (lo + hi) >> 1;
   if(k <= mid) return this->l->count(lb + 1, rb, k);
   return this->r->count(1 - 1b, r - rb, k);
  //sum of numbers in [1 ,r] less than or equal to k
  int sum(int 1, int r, int k) {
   if(1 > r or k < 1o) return 0;
   if(hi <= k) return c[r] - c[l - 1];</pre>
   int 1b = b[1 - 1], rb = b[r];
    return this->1->sum(lb + 1, rb, k) + this->r->sum(l - lb, r - rb, k);
  ~wavelet tree() {
   delete 1;
   delete r:
};
```

## 12. Subset Sum DP

```
9852cf0f880d1f38b9a53e7e6bcf0328
vector <int> solve(int W, vector <int> coins)
    int n = coins.size();
   if (n == 0) {
       return 0;
    vector <int> dp[2];
    for (int t = 0; t < 2; ++t) {
       dp[t].resize(W + W + 1);
   int all sum = 0;
    for (auto v: coins) {
       all_sum += v;
    int goal = all sum / 2;
    int start_idx = 0;
   int cur_sum = 0;
    while (cur_sum + coins[start_idx] <= goal) {</pre>
       cur_sum += coins[start_idx++];
    start_idx--;
    for (int i = 0; i < 2; ++i) {</pre>
       for (int j = 0; j \le W + W; ++j) {
            dp[i][j] = -1;
    auto update = [&](const int id) {
        for (int i = W + W; i >= 0; --i) {
            for (int j = max(0, dp[id ^ 1][i]); j < dp[id][i]; ++j) {</pre>
                if (i - coins[j] >= 0) {
                    dp[id][i - coins[j]] = max(dp[id][i - coins[j]], j);
    dp[start idx \& 1][cur sum + W - goal] = start idx + 1;
    update(start_idx & 1);
    for (int i = start_idx + 1; i < n; ++i) {</pre>
```

```
const int id = i & 1;
    dp[id].assign(W + W + 1, -1);

for (int j = 0; j + coins[i] <= W + W; ++j) {
        dp[id][j] = max(dp[id][j], dp[id ^ 1][j]);
        dp[id][j + coins[i]] = max(dp[id][j + coins[i]], dp[id ^ 1][j]);
    }

    update(id);
}

const int id = (n - 1) & 1;
return dp[id];

13. OTOCZKA WYPUKŁA

47743f9c1902e90fbb2b5810c8e8ec05

#define f first
#define s second</pre>
```

```
#define f first
#define s second
#define x first
#define y second
#define all(x) x.begin(), x.end()
#define ss(x) ((int)((x).size()))
#define rep(i,a,b) for(int i=(a);i \le (b);i++)
template <class p, class q> pair<p,q> operator-(pair<p,q> a, pair<p,q> b) { return
\hookrightarrow mp(a.f - b.f, a.s - b.s); }
template <class p, class g> pair<p,q> operator+(pair<p,q> a, pair<p,q> b) { return
\hookrightarrow mp(a.f + b.f, a.s + b.s); }
template <class p, class q> void umin(p &a, const q &b) { if (a > b) a = b; }
template <class p, class q> void umax(p &a, const q &b) { if (a < b) a = b; }
using 11 = long long;
using cll = const ll;
using point = pair <11,11>;
using cpoint = const point;
11 IS(cpoint& a, cpoint& b) { return a.x * b.x + a.y * b.y; }
11 IW(cpoint& a, cpoint& b) { return a.x * b.v - a.v * b.x; }
11 IW(cpoint& a, cpoint& b, cpoint& c) { return IW(a - c, b - c); }
point rot90(point a) { return point(-a.y, a.x); }
vector <point> convex hull(vector <point> points, bool strict) {
        sort(all(points));
        vector <point> hull;
        rep(phase, 0, 1) {
                int start = ss(hull);
                for (cpoint& p : points) {
                        while (hull.size() >= start + 2) {
                                 cll iw = IW(p, hull.back(), hull[ss(hull) - 2]);
                                 if (iw < 0 || iw == 0 && strict == false) break;</pre>
```

```
hull.pop_back();
                       hull.pb(p);
               hull.pop_back();
               reverse (all (points));
        if (ss(hull) == 2 && hull[0] == hull[1])
               hull.pop_back();
        return hull;
struct cht
       vector <point> hull;
       vector <point> vecs;
       vector <int> ids;
       void insert(point p, int id) // utrzymuje dolną otoczkę; p.x powinny rosnąć
                while (!vecs.empty() && IS(vecs.back(), p - hull.back()) <= 0)</pre>
                       hull.pop_back();
                       vecs.pop_back();
                       ids.pop_back();
               if (!hull.empty())
                       vecs.pb(rot90(p - hull.back()));
               hull.pb(p);
               ids.pb(id);
        pair <11, int > query(11 x) // zwraca minimum w punkcie x / minimalny iloczyn

⇒ skalarny + do kogo należy

                point p(x, 1);
               auto it = lower_bound(all(vecs), p, [] (point a, point b) {
                       return IW(a, b) > 0;
               int i = it - vecs.begin();
                return { IS(p, hull[i]), ids[i] };
};
                                14. Punkty i proste
   cd161c446c970b07550e805e8b129c1a
typedef long double T:
```

```
typedef long double T;
int sgn(T x) {
   const T eps = 1e-9;
```

```
if(abs(x) <= eps) return 0;</pre>
    return x > 0 ? 1 : -1;
struct Vec {
    T x, y;
};
Vec operator+(const Vec &a, const Vec &b) {
    return { a.x + b.x, a.y + b.y };
Vec operator-(const Vec &a, const Vec &b) {
    return { a.x - b.x, a.y - b.y };
Vec operator* (const Vec &a, const T &b) {
    return { a.x * b, a.y * b };
Vec operator/(const Vec &a, const T &b) {
    return { a.x / b, a.y / b };
T dot(const Vec &a, const Vec &b) {
    return a.x * b.x + a.y * b.y;
T prod(const Vec &a, const Vec &b) {
    return a.x * b.y - a.y * b.x;
T len(const Vec &v) {
    return sqrt(dot(v, v));
T len2 (const Vec &v) {
    return dot(v, v);
T alpha (const Vec &v)
    return atan2(v.y, v.x);
struct ParamLine {
    Vec p, v;
    ParamLine() = default;
    ParamLine (Vec a, Vec b, bool norm=false) {
        p = a; v = b - a;
        if(norm) v = v / len(v);
Vec project (const ParamLine &1, const Vec &u) {
    return 1.p + 1.v * dot((u - 1.p), 1.v) / len2(1.v);
T dist(const ParamLine &1, const Vec &u) {
    return abs(prod(1.v, u - 1.p) / len(1.v));
Vec intersection (ParamLine a, ParamLine b) {
        T \text{ norm} = prod(a.v, b.v);
        T t = prod((b.p - a.p), b.v) / norm;
        return a.p + a.v * t;
```

# 15. Kółka

```
71ef43e8cdc940998eee383d3e1ed76e
struct Circle {
    Vec o:
    Tr:
pair<Vec, Vec> intersect (const Circle &a, const Circle &b) {
   Vec v = b.o - a.o; T d = len(v);
   T x = (d * d + a.r * a.r - b.r * b.r) / (2 * d);
   T y = sqrt(a.r * a.r - x * x);
   v = v / d; Vec u { -v.y, v.x };
   return { a.o + v * x + u * y, a.o + v * x - u * y };
pair<Vec, Vec> intersect(const Circle &c, const ParamLine &l) {
   Vec pr = project(1, c.o);
   T t = sqrt(c.r * c.r - len2(pr - c.o));
   Vec v = l.v / len(l.v);
   return { pr + v * t, pr - v * t };
pair<Vec, Vec> tangent (const Circle &c, const Vec &v) {
   Vec d = v - c.o;
   T r = sqrt(len2(d) - c.r * c.r);
    return intersect(c, Circle { v, r } );
void tangents(Vec c, double r1, double r2, vector<ParamLine> & ans) {
    double r = r2 - r1;
    double z = c.x * c.x + c.y * c.y;
    double d = z - r * r;
   if(sgn(d) == -1) return;
   d = sqrt (abs (d));
   Vec v { c.x * d - c.y * r, c.x * r + c.y * d };
   Vec p = Vec \{ -v.v, v.x \} * r1 / z;
   ans.emplace_back(p, p + v);
vector<ParamLine> tangents (Circle a, Circle b) {
   vector<ParamLine> ans;
    for (int i=-1; i<=1; i+=2)</pre>
        for (int j=-1; j<=1; j+=2)
            tangents (b.o-a.o, a.r*i, b.r*j, ans);
    for (size_t i=0; i<ans.size(); ++i)</pre>
       ans[i].p = ans[i].p + a.o;
    return ans;
```

#### 16. Przeciecie półpłaszczyzn

b51a3286beabb7a52727669ccd1b534c

```
#define ALL(X) X.begin(), X.end()
#define FORW(I, A, B) for (int(I) = (A); (I) < (B); (I) ++)
#define SIZE(X) int(X.size())
#define PB push back
inline int type(const Vec &p) {
    return sgn(p.y) == 1 or (sgn(p.y) == 0 \&\& sgn(p.x) == 1);
bool compare_angle(const Vec &a, const Vec &b) {
    int at = type(a), bt = type(b);
    if (at != bt)
        return at < bt;</pre>
    int p = sqn(prod(a, b));
    if(p) return p > 0;
    return sgn(len2(a) - len2(b)) == -1;
bool operator<(const ParamLine &a, const ParamLine &l) {</pre>
    if (sgn(a.v.x - 1.v.x) \text{ or } sgn(a.v.y - 1.v.y))
        return compare_angle(
            a.v, 1.v);
    return sqn(prod(a.v, l.p - a.p)) < 0;</pre>
int sqn_dist(const ParamLine &1, const Vec &u) {
    return sgn(prod(l.v, u - l.p));
vector<Vec> halfcoat(vector<ParamLine> h) {
    const static int MAXN = 40020;
    static const T Z = 1e6;
    static Vec p[MAXN];
    static Vec box[4] = {
        \{ Z, -Z \},
        { Z, Z },
        \{ -Z, Z \},
        \{ -Z, -Z \} \};
    FORW(i, 0, 4)
    h.PB(ParamLine(box[i], box[(i + 1) % 4], true));
    int n = SIZE(h), z = 0;
    sort (ALL(h));
    FORW(i, 0, n)
    if (i == 0 \text{ or } (sqn(h[i].v.x - h[i - 1].v.x) \text{ or } sqn(h[i].v.y - h[i - 1].v.y)))
        h[z++] = h[i];
    n = z;
    int m = 0, del = 0;
    FORW(i, 1, n)
        while (m > del and sqn dist(h[i], p[m - 1]) <= 0)
        while (m > del and sqn_dist(h[i], p[del]) <= 0)</pre>
        if (del == m \text{ and } sqn(prod(h[m].v, h[i].v)) <= 0)
            return {};
        Vec q = intersection(h[i], h[m]);
        if (sqn_dist(h[del], q) >= 0)
```

```
p[m++] = q, h[m] = h[i];
    rotate(p, p + del, p + m);
    rotate(h.begin(), h.begin() + del, h.end());
    m -= del;
   if (m == 0)
       return {};
   Vec q = intersection(h[0], h[m]);
    p[m++] = q;
    return vector<Vec>(p, p + m);
                                  17. Trapezoidacja
   56abea681ae28d4d928da0a8b0bc23cb
bool operator<(const Vec &a, const Vec &b) {</pre>
    return make_pair(a.x, a.y) < make_pair(b.x, b.y);</pre>
struct Trapezoid {
   Vec bl, br, tl, tr;
T get_y(const ParamLine &1, T x) {
    return 1.p.y + 1.v.y * (x - 1.p.x) / 1.v.x;
struct CmpY {
   T \star x;
    T y (const ParamLine &1) const {
        return get_y(l, *x);
    bool operator() (const ParamLine &a, const ParamLine &b) const {
        return v(a) < v(b);</pre>
vector<Trapezoid> trapezoidation(const vector<Vec> &poly) {
    const T leps = 1e-9;
    int n = polv.size();
    auto previ = [&](int k) { return (k + n - 1) % n; };
    auto nexti = [\&] (int k) { return (k + 1) % n; };
    vector<int> pts(n);
    for(int i = 0; i < n; i++) pts[i] = i;</pre>
    sort(pts.begin(), pts.end(), [&](int a, int b) {
        return polv[a] < polv[b];</pre>
    vector<bool> added(n);
    vector<Trapezoid> ans;
   T x; CmpY compare { &x };
    multimap<ParamLine, bool, CmpY> st(compare);
    for(int i: pts) {
        if(added[i]) continue;
        added[i] = true;
```

};

```
x = poly[i].x;
int i1 = i, i2 = i;
int pi = previ(i1), ni = nexti(i2);
while (sgn(poly[i1].x - poly[pi].x) == 0) {
   added[pi] = true;
   i1 = pi; pi = previ(pi);
while(sgn(poly[i2].x - poly[ni].x) == 0) {
   added[ni] = true;
   i2 = ni; ni = nexti(ni);
if(!added[pi] && !added[ni]) {
   if((i1 == i2 && prod(poly[pi] - poly[i], poly[ni] - poly[i]) < 0)</pre>
    | | sgn(poly[i1].y - poly[i2].y) > 0) {
       swap(pi, ni); swap(i1, i2);
    auto it1 = st.insert({ ParamLine(poly[i1], poly[pi]), true });
    auto it2 = st.insert({ ParamLine(poly[i2], poly[ni]), false });
   if(it1 != st.begin() && prev(it1)->second == true) {
       swap(it1->second, it2->second);
        auto &a = prev(it1) ->first, &b = next(it2) ->first;
       Vec na { x, compare.y(a) }, nb { x, compare.y(b) };
       ans.push_back({ a.p, na, b.p, nb });
       a.p = na; b.p = nb;
} else if(added[pi] && added[ni]) {
   if((i1 == i2 && prod(poly[pi] - poly[i], poly[ni] - poly[i]) > 0)
    | | sqn(poly[i1].y - poly[i2].y) > 0) 
       swap(pi, ni); swap(i1, i2);
    auto it1 = st.lower_bound(ParamLine(poly[pi], poly[i1] - Vec { 0, leps }));
   auto it2 = prev(st.upper_bound(ParamLine(poly[ni], poly[i2] + Vec { 0, leps
   if(it1->second == true) {
        ans.push_back({ it1->first.p, poly[i1], it2->first.p, poly[i2] });
    } else {
       auto &a = prev(it1) -> first, &b = next(it2) -> first;
       Vec na { x, compare.y(a) }, nb { x, compare.y(b) };
       ans.push_back({ a.p, na, it1->first.p, poly[i1] });
       ans.push_back({ it2->first.p, poly[i2], b.p, nb });
       a.p = na; b.p = nb;
    st.erase(it1); st.erase(it2);
} else {
    if(!added[pi]) {
       swap(pi, ni); swap(i1, i2);
   auto it = st.lower_bound(ParamLine(poly[pi], poly[i1] - Vec { 0, leps }));
   assert(it != st.end());
   if(it->second) {
        auto &a = next(it)->first;
```

```
Vec na { x, compare.y(a) };
                                     ans.push_back({ it->first.p, poly[i1], a.p, na });
                                     a.p = na;
                            } else {
                                     auto &a = prev(it)->first;
                                     Vec na { x, compare.y(a) };
                                     ans.push_back({ a.p, na, it->first.p, poly[i1] });
                                     a.p = na;
                            it->first.p = poly[i2];
                            it->first.v = poly[ni] - poly[i2];
         ans.erase(remove_if(ans.begin(), ans.end(), [](Trapezoid t) {
                  return sqn(t.bl.x - t.br.x) == 0;
         }), ans.end());
         return ans:
                                                                                        18. Voronoi
        b7616e903d792310c7c1605db6d8f1f5
#define Oi(e) ((e)->oi)
#define Dt(e) ((e)->dt)
#define On(e) ((e)->on)
#define Op(e) ((e)->op)
#define Dn(e) ((e)->dn)
#define Dp(e) ((e)->dp)
#define Other(e, p) ((e) -> oi == p ? (e) -> dt : (e) -> oi)
#define Next(e, p) ((e) -> oi == p ? (e) -> on : (e) -> dn)
#define Prev(e, p) ((e)->oi == p ? (e)->op : (e)->dp)
#define V(p1, p2, u, v) (u = p2->x - p1->x, v = p2->y - p1->y)
#define C2(u1, v1, u2, v2) (u1 * v2 - v1 * u2)
#define C3(p1, p2, p3) ((p2->x - p1->x) * (p3->y - p1->y) - (p2->y - p1->y) * (p3->x - p1->y) + (p3->x - p1->y) * (p3-
\hookrightarrow p1->x))
#define Dot(u1, v1, u2, v2) (u1 * u2 + v1 * v2)
#define dis(a,b) (sqrt( (a->x - b->x) * (a->x - b->x) + (a->y - b->y) * (a->y - b->y) ))
const int maxn = 110024:
const int aix = 4;
const double eps = 1e-7;
int n, M, k;
struct gEdge {
                  int u, v; double w;
                  bool operator <(const gEdge &el) const { return w < el.w - eps; }</pre>
} E[aix * maxn], MST[maxn];
struct point {
                  double x, y; int index; edge *in;
                  bool operator <(const point &p1) const { return x < p1.x - eps || (abs(x - p1.x)
                   \hookrightarrow <= eps && v < p1.v - eps); }
```

```
struct edge { point *oi, *dt; edge *on, *op, *dn, *dp; };
point p[maxn], *O[maxn];
edge mem[aix * maxn], *elist[aix * maxn];
int nfree;
void Alloc_memory() { nfree = aix * n; edge *e = mem; for (int i = 0; i < nfree; i++)</pre>
⇔ elist[i] = e++; }
void Splice(edge *a, edge *b, point *v) {
        edge *next;
        if (Oi(a) == v) next = On(a), On(a) = b; else next = Dn(a), Dn(a) = b;
        if (Oi(next) == v) Op(next) = b; else Dp(next) = b;
        if (Oi(b) == v) On(b) = next, Op(b) = a; else Dn(b) = next, Dp(b) = a;
edge *Make_edge(point *u, point *v) {
        edge *e = elist[--nfree];
        e->on = e->op = e->dn = e->dp = e; e->oi = u; e->dt = v;
        if (!u->in) u->in = e;
        if (!v->in) v->in = e;
        return e;
edge *Join(edge *a, point *u, edge *b, point *v, int side) {
        edge *e = Make edge(u, v);
        if (side == 1) {
                if (Oi(a) == u) Splice(Op(a), e, u);
                 else Splice(Dp(a), e, u);
                 Splice(b, e, v);
        } else {
                 Splice(a, e, u);
                if (Oi(b) == v) Splice(Op(b), e, v);
                 else Splice (Dp(b), e, v);
        } return e;
void Remove(edge *e) {
        point *u = Oi(e), *v = Dt(e);
        if (u->in == e) u->in = e->on;
        if (v->in == e) v->in = e->dn;
        if (Oi(e->on) == u) e->on->op = e->op; else e->on->dp = e->op;
        if (0i(e\rightarrow p) == u) e\rightarrow p\rightarrow n = e\rightarrow n; else e\rightarrow p\rightarrow dn = e\rightarrow n;
        if (Oi(e->dn) == v) e->dn->op = e->dp; else e->dn->dp = e->dp;
        if (Oi(e->dp) == v) e->dp->on = e->dn; else e->dp->dn = e->dn;
        elist[nfree++] = e;
void Low_tangent(edge *e_1, point *o_1, edge *e_r, point *o_r, edge **l_low, point **OL,

→ edge **r low, point **OR) {
        for (point *d 1 = Other(e 1, o 1), *d r = Other(e r, o r); ; )
                if (C3(o 1, o r, d 1) < -eps)
                                                     e l = Prev(e l, d l), o l = d l, d l
                 \hookrightarrow = Other(e_1, o_1);
                else if (C3(o_1, o_r, d_r) < -eps) e_r = Next(e_r, d_r), o_r = d_r, d_r
                \hookrightarrow = Other(e r, o r);
                 else break;
        *OL = o_1, *OR = o_r; *l_low = e_1, *r_low = e_r;
```

```
void Merge(edge *lr, point *s, edge *rl, point *u, edge **tangent) {
        double 11, 12, 13, 14, r1, r2, r3, r4, cot L, cot R, u1, v1, u2, v2, n1, cot n,
        \hookrightarrow P1, cot P;
        point *O, *D, *OR, *OL; edge *B, *L, *R;
        Low tangent (lr, s, rl, u, &L, &OL, &R, &OR);
        for (*tangent = B = Join(L, OL, R, OR, O), O = OL, D = OR; ;)
                edge *El = Next(B, O), *Er = Prev(B, D), *next, *prev;
                point *l = Other(El, O), *r = Other(Er, D);
                V(1, 0, 11, 12); V(1, D, 13, 14); V(r, 0, r1, r2); V(r, D, r3, r4);
                double c1 = C2(11, 12, 13, 14), cr = C2(r1, r2, r3, r4);
               bool BL = cl > eps, BR = cr > eps;
                if (!BL && !BR) break;
                if (BL) {
                        double dl = Dot(11, 12, 13, 14);
                        for (cot_L = dl / cl; ; Remove(El), El = next, cot_L = cot_n) {
                                next = Next(El, O); V(Other(next, O), O, ul, vl);
                                \hookrightarrow V(Other(next, O), D, u2, v2);
                                n1 = C2(u1, v1, u2, v2); if (!(n1 > eps)) break;
                                cot n = Dot(u1, v1, u2, v2) / n1;
                                if (cot n > cot L) break;
                } if (BR) {
                        double dr = Dot(r1, r2, r3, r4);
                        for (cot_R = dr / cr; ; Remove(Er), Er = prev, cot_R = cot_P) {
                                prev = Prev(Er, D); V(Other(prev, D), O, u1, v1);
                                \hookrightarrow V(Other(prev, D), D, u2, v2);
                                P1 = C2(u1, v1, u2, v2); if (!(P1 > eps)) break;
                                cot P = Dot(u1, v1, u2, v2) / P1;
                                if (cot P > cot R) break;
                        }
                1 = Other(El, O); r = Other(Er, D);
                if (!BL || (BL && BR && cot_R < cot_L)) B = Join(B, O, Er, r, 0), D = r;</pre>
                else B = Join(El, 1, B, D, 0), O = 1;
       }
void Divide(int s, int t, edge **L, edge **R) {
        edge *a, *b, *c, *ll, *lr, *rl, *rr, *tangent;
        int n = t - s + 1;
        if (n == 2) *L = *R = Make_edge(Q[s], Q[t]);
        else if (n == 3) {
                a = Make\_edge(Q[s], Q[s + 1]), b = Make\_edge(Q[s + 1], Q[t]);
                Splice(a, b, Q[s + 1]);
                double v = C3(Q[s], Q[s + 1], Q[t]);
                if (v > eps)
                                  c = Join(a, Q[s], b, Q[t], 0), *L = a, *R = b;
                else if (v < -eps) c = Join(a, Q[s], b, Q[t], 1), *L = c, *R = c;
                else *L = a, *R = b;
       } else if (n > 3) {
                int split = (s + t) / 2;
                Divide(s, split, &ll, &lr); Divide(split + 1, t, &rl, &rr);
                Merge(lr, Q[split], rl, Q[split + 1], &tangent);
                if (Oi(tangent) == Q[s]) 11 = tangent;
```

14

```
if (Dt(tangent) == Q[t]) rr = tangent;
                \starL = 11; \starR = rr;
void Make_Graph() {
        edge *start, *e; point *u, *v;
        for (int i = 0; i < n; i++) {</pre>
                start = e = (u = &p[i]) -> in;
                do\{ v = Other(e, u);
                        if (u < v) E[M++].u = (u - p, v - p, dis(u, v)); // M < aix *
                        → maxn
                } while ((e = Next(e, u)) != start);
int b[maxn];
int Find(int x) { while (x != b[x]) { b[x] = b[b[x]]; x = b[x]; } return x; }
void Kruskal() {
        memset(b, 0, sizeof(b)); sort(E, E + M);
        for (int i = 0; i < n; i++) b[i] = i;</pre>
        for (int i = 0, kk = 0; i < M && kk < n - 1; i++) {
                int m1 = Find(E[i].u), m2 = Find(E[i].v);
                if (m1 != m2) b[m1] = m2, MST[kk++] = E[i];
void solve() {
        scanf("%d", &n);
        for (int i = 0; i < n; i++) scanf("%lf%lf", &p[i].x, &p[i].y), p[i].index = i,</pre>
        \hookrightarrow p[i].in = NULL;
        Alloc_memory(); sort(p, p + n);
        for (int i = 0; i < n; i++) Q[i] = p + i;</pre>
        edge *L, *R; Divide(0, n - 1, &L, &R);
        M = 0; Make_Graph(); Kruskal();
                                        19. Dinic
    ee279b38c7ec82ffad2853c69d09fc8a
#define sz(x) (int)(x).size()
// indeksowany od 1
// uzupełniamy [n, s, t] i dodajemy krawedzie
typedef int T;
const int N = 2005;
const T INF = 1e9;
struct edge {
    int a, b;
    T cap, flow;
int n, s, t, d[N], ptr[N], q[N];
vector < edge > e;
```

vector < int > q[N];

```
void clear() {
        e.clear();
        for (int i = 1; i <= n; ++i)</pre>
                g[i].clear();
/* Edges can be added only using add_edge to use this function! */
void clear flow() {
        for(int i = 0; i < (int)e.size(); i += 2)</pre>
                e[i].flow = 0, e[i + 1].flow = e[i + 1].cap;
int add_edge(int a, int b, T cap) {
    edge e1 = { a, b, cap, 0 };
    edge e2 = { b, a, cap, cap };
    g[a].push_back(sz(e));
    e.push back(e1);
    g[b].push_back(sz(e));
    e.push_back(e2);
    return sz(e) - 2;
bool bfs() {
    int qh = 0, qt = 0;
    q[qt++] = s;
    memset(d + 1, -1, n * sizeof d[0]);
    d[s] = 0;
    while (gh < gt && d[t] == -1) {
        int v = q[qh++];
        for(int i = 0; i < sz(g[v]); ++i) {</pre>
            int id = q[v][i], to = e[id].b;
            if(d[to] == -1 && e[id].flow < e[id].cap) {</pre>
                q[qt++] = to;
                d[to] = d[v] + 1;
    return d[t] != -1;
T dfs(int v, T flow) {
    if(flow <= 0) return 0;</pre>
    if(v == t) return flow;
    T res = 0;
    for(; ptr[v] < sz(q[v]); ++ptr[v]) {</pre>
        int id = g[v][ptr[v]], to = e[id].b;
        if(d[to] != d[v] + 1) continue;
        T pushed = dfs(to, min(flow, e[id].cap - e[id].flow));
        e[id].flow += pushed;
        e[id^1].flow -= pushed;
        res += pushed;
        flow -= pushed;
        if(flow == 0) break;
    return res;
```

15

```
T dinic(int _n, int _s, int _t) {
   n = _n; s = _s; t = _t;
   T flow = 0;
   for (;;) {
       if(!bfs()) break;
       memset(ptr, 0, (n + 1) * sizeof ptr[0]);
       flow += dfs(s, INF);
    return flow;
                                20. Gomory-Hu Tree
   5516aac48c56368bc1a189c391d931e2
struct edge {
       int u, v;
       long long w;
int n;
vector <int> p, w, c;
vector <edge> tree;
void dfs(int u) {
       c[u] = 1;
        for(const int &id: Dinic::g[u]) {
                int v = Dinic::e[id].b;
                if(!c[v] and Dinic::e[id].flow < Dinic::e[id].cap)</pre>
                        dfs(v);
/* Clears and runs */
vector <edge> run(int _n, const vector <edge> &ed) {
       n = _n;
        tree.clear();
       p.resize(n + 1), w.resize(n + 1), c.resize(n + 1);
        for(const auto &e: ed) {
                Dinic::add_edge(e.u, e.v, e.w);
                Dinic::add_edge(e.v, e.u, e.w);
       p[1] = 0, fill(p.begin() + 2, p.end(), 1);
        for(int i = 2; i <= n; ++i) {</pre>
                w[i] = Dinic::dinic(n, i, p[i]);
                fill(c.begin(), c.end(), 0);
                dfs(i);
                for(int j = i + 1; j <= n; ++j)
                        if(c[j] && p[j] == p[i])
                                p[j] = i;
                if(p[p[i]] && c[p[p[i]]]) {
                        int pi = p[i];
                        swap(w[i], w[pi]);
```

```
p[i] = p[pi];
                        p[pi] = i;
                Dinic::clear flow();
        tree.clear();
        for (int i = 1; i <= n; ++i) {</pre>
               if(p[i])
                        tree.push_back(edge{i, p[i], w[i]});
        return tree;
                               21. Min-cost max-flow
   9f227bfb2cfcd5fb30c148c86ab2dd1b
typedef int flow t;
typedef int cost_t;
struct edge {
   int u, v;
    flow_t flow, capa;
   cost t cost;
const int N = 10000;
const cost_t cinf = 1e9;
const flow_t finf = 1e9;
vector<int> g[N];
cost_t d[N];
cost_t p[N];
int pre[N];
vector<edge> e;
inline bool remin(cost_t &a, cost_t b) {
    return a > b ? a = b, true : false;
void init(int n) { //wierzcholki numerowane od 0 do n - 1
    for(int i = 0; i < n; i++)</pre>
       g[i].clear();
   e.clear();
void add edge(int u, int v, flow t capa, cost t cost) {
   g[u].push_back(e.size());
    e.push_back({ u, v, 0, capa, cost });
    g[v].push_back(e.size());
    e.push_back({ v, u, 0, 0, -cost });
pair<flow_t, cost_t> flow(int s, int t) {
    fill(p, p + n, 0);
    bool improved = true;
```

```
while(improved) {
   improved = false;
    for (auto &ed: e)
        if(ed.flow < ed.capa && remin(p[ed.v], p[ed.u] + ed.cost))</pre>
            improved = true;
flow_t fans = 0;
cost_t cans = 0;
while(true) {
   fill(d, d + n, cinf);
   priority_queue<pair<cost_t, int>> q;
    d[s] = 0; q.push({0, s});
    while(!q.empty()) {
        auto u = q.top().second, c = -q.top().first;
        q.pop();
        if(c != d[u]) continue;
        for(int ed: q[u]) {
            if(e[ed].flow == e[ed].capa) continue;
            auto v = e[ed].v;
            if(remin(d[v], c + p[u] - p[v] + e[ed].cost)) {
                pre[v] = ed;
                q.push({ -d[v], v });
    if(d[t] == cinf) break;
    vector<int> path;
   int v = t, ed = pre[v];
    flow_t flow = finf;
    while (v != s) {
       path.push_back(ed);
        flow = min(flow, e[ed].capa - e[ed].flow);
        v = e[ed].u;
        ed = pre[v];
    for(auto ed: path) {
        e[ed].flow += flow;
        e[ed^1].flow -= flow;
    fans += flow;
    cans += flow * (d[t] + p[t] - p[s]);
    for(int i = 0; i < n; i++)</pre>
        p[i] += d[i];
return { fans, cans };
```

#### 22. Hungarian

bf99fa2858cd62d5c725aa617899112d

```
typedef int T;
const int N = 507;
const T INF = 1e9 + 7;
int n, max_match;
T cost[N][N], lx[N], ly[N], slack[N], slackx[N];
int xy[N], yx[N], prev[N];
bool S[N], U[N];
void update_labels() {
   T delta = INF:
    FOR(y, n) if(!U[y]) delta = min(delta, slack[y]);
    FOR(x, n) if(S[x]) lx[x] -= delta;
    FOR(y, n) if(U[y]) ly[y] += delta;
    FOR(y, n) if(!U[y]) slack[y] -= delta;
void add_to_tree(int x, int f) {
    S[x] = true; prev[x] = f;
    FOR(y, n) \ \textbf{if}(lx[x] + ly[y] - cost[x][y] < slack[y]) \ \{
        slack[y] = lx[x] + ly[y] - cost[x][y];
        slackx[y] = x;
void augment() {
    if (max_match == n) return;
    int root, q[N], wr = 0, rd = 0;
    memset(S, false, sizeof(S));
    memset(U, false, sizeof(U));
    memset(prev, -1, sizeof(prev));
    FOR(x, n) if(xy[x] == -1) {
        q[wr++] = root = x;
        prev[x] = -2;
        S[x] = true;
        break;
    FOR(y, n) {
        slack[y] = lx[root] + ly[y] - cost[root][y];
        slackx[y] = root;
    int x, y;
    while (true) {
        while(rd < wr) {</pre>
            x = q[rd++];
            for (y = 0; y < n; y++) {
                if(cost[x][y] == lx[x] + ly[y] && !U[y]) {
                    if (yx[y] == -1) break;
                    U[v] = true;
                    q[wr++] = yx[y];
                    add_to_tree(yx[y], x);
            if(y < n) break;</pre>
        if (v < n) break;</pre>
```

```
update_labels();
        wr = rd = 0;
        for (y = 0; y < n; y++) {
            if(!U[y] && slack[y] == 0) {
                if(yx[y] == -1) {
                    x = slackx[y];
                    break;
                } else {
                    U[y] = true;
                    if(!S[yx[y]]) {
                       q[wr++] = yx[y];
                       add_to_tree(yx[y], slackx[y]);
               }
        if(y < n) break;</pre>
    if(y < n) {
        max_match++;
        for (int cx = x, cy = y, ty; cx != -2; cx = prev[cx], cy = ty) {
           ty = xy[cx];
           yx[cy] = cx;
            xy[cx] = cy;
       augment();
T maxCostMatching()
    T res = 0; max_match = 0;
    FOR(i, n) xy[i] = yx[i] = -1, lx[i] = ly[i] = 0;
    FOR(x, n) FOR(y, n) lx[x] = max(lx[x], cost[x][y]);
    augment();
    FOR(x, n) res += cost[x][xy[x]];
    return res;
                                   23. Dominatory
   b0236cf0296506f82cff4f349582e533
#define NODES 100007
int parent[NODES+1], ancestor[NODES+1], vertex[NODES+1];
int label[NODES+1], semi[NODES+1];
int dom[NODES+1]; // drzewo dominacji, dom[v] - ojciec v w drzewie, dom[root] = 0
vi succ[NODES+1], pred[NODES+1], bucket[NODES+1];
int czas, N, root;
void czysc (int _N, int _root) {
```

N = \_N;
root = \_root;

```
czas = 0;
    FOR(i,N+1) semi[i] = 0;
    FOR(i,N+1) pred[i].clear();
    FOR(i,N+1) succ[i].clear();
    FOR(i,N+1) bucket[i].clear();
void dfs (int v) {
    semi[v] = ++czas;
    label[v] = vertex[czas] = v;
    ancestor[v] = 0;
    for(auto w: succ[v]){
       if (semi[w] == 0) { parent[w] = v; dfs(w); }
       pred[w].push_back(v);
void compress (int v) {
    if (ancestor[ancestor[v]] == 0) return;
    compress(ancestor[v]);
    if (semi[label[ancestor[v]]] < semi[label[v]]) label[v] = label[ancestor[v]];</pre>
    ancestor[v] = ancestor[ancestor[v]];
int eval(int v) {
   if (ancestor[v] == 0) return v;
    compress(v);
    return label[v];
void dominacja () {
    dfs(root);
    for(int i = czas; i >= 2; i--) {
        int w = vertex[i];
        for(auto v: pred[w]) {
            int u = eval(v);
            if (semi[u] < semi[w]) semi[w] = semi[u];</pre>
        bucket[vertex[semi[w]]].push_back(w);
        ancestor[w] = parent[w];
        for(auto v: bucket[parent[w]]) {
            int u = eval(v);
            dom[v] = (semi[u] < semi[v] ? u : parent[w]);</pre>
       bucket[parent[w]].clear();
    for(int i = 2; i <= czas; i++) {</pre>
        int w = vertex[i];
        if (dom[w] != vertex[semi[w]]) dom[w] = dom[dom[w]];
    dom[root] = 0;
int main() { int n = 10; czysc(n,1); /*dod krawedzie do succ*/ dominacja(); }
```

#### 24. SKIEROWANE MST

```
ebb02b3ab6583bb66d9998e92f93c675
// directed mst z wierzcholka 0 w grafie 0..n-1
// preconditions: 1. nie ma krawedzi wchodzacych do wierzcholka 0
                2. wszystkie wierzcholki sa osiagalne z wierzcholka 0
// przed uzyciem ustawic n, m, MAXN, MAXM i wypelnic
//edge[0..m-1] (wystarczy pola: u, v, key)
const int MAXN = 100007, MAXM = 100007;
struct edge { // krawedz/element kolejki zlaczalnej
    int u, v; // IN: poczatek i koniec krawedzi
    int key; // IN: waga krawedzi (zmienia sie!)
   edge *left, *right; // poczatkowo: 0, 0
                       // poczatkowo: 1, 0
   int len, add;
};
struct node1 { // element zbioru
   node1 *parent; int size, scc;
struct node2 { // j.w.
   node2 *parent; // poczatkowa wartosc: this
                  // poczatkowa wartosc: 1
   int size;
// Operacje na zbiorach rozlacznych
template < class T > T *set_find(T *p) { // znajduje reprezentanta
   if (p->parent != p) p->parent = set_find(p->parent);
    return p->parent;
template<class T> T *set_union(T *p1, T *p2) { // laczy zbiory
   if (p1->size < p2->size) swap(p1, p2);
   p2->parent = p1;
   p1->size += p2->size;
   return p1;
// Operacje na kolejkach zlaczalnych
void tree push(edge *p) {
   p->key += p->add;
   if (p->left) p->left->add += p->add;
   if (p->right) p->right->add += p->add;
   p->add = 0;
edge *tree_union(edge *p1, edge *p2) { // laczy kolejki
   if (!p1) return p2; if (!p2) return p1;
    if (p2->key+p2->add < p1->key+p1->add) swap(p1, p2);
   tree push (p1);
   p1->right = tree union(p1->right, p2);
    if (!p1->left || p1->left->len < p1->right->len) swap(p1->left, p1->right);
   p1->len = p1->right ? p1->right->len+1 : 1;
   return p1;
edge *tree_extract(edge *p) { // usuwa z kolejki element najmniejszy
    tree push(p); return tree union(p->left, p->right);
```

```
void tree add(edge *p, int x) { // dodaje x do wszystkich wartosci w kolejce
   if (p) p->add += x;
                 // IN: liczba wierzcholkow, liczba krawedzi
int n, m;
edge edges[MAXM]; // IN: tablica wszystkich krawedzi
node1 scc set[MAXN];
node2 wcc set[MAXN];
int upper[2*MAXN], lower[2*MAXN];
edge *adj[2*MAXN];
edge *res[2*MAXN]; // OUT: krawedz do rodzica w drzewie (korzen ma NULL)
int compute_branching() { // zwraca wage drzewa
   FOR(i,n) {
       scc_set[i].parent = scc_set+i;
       scc set[i].size = 1;
       scc_set[i].scc = i;
        wcc_set[i].parent = wcc_set+i;
       wcc_set[i].size = 1;
       upper[i] = lower[i] = -1;
       adj[i] = res[i] = 0;
   FOR(j,m) {
       edges[j].left = edges[j].right = 0;
       edges[i].len = 1;
       edges[j].add = 0;
        adj[edges[j].v] = tree_union(adj[edges[j].v], edges+j);
   int scc_c=n, value=0;
   FOR(i,n) {
       int c = set_find(scc_set+i)->scc;
        while (adj[c] && !res[c]) {
           edge *e = adi[c];
            adj[c] = tree_extract(adj[c]);
            nodel *s1 = set_find(scc_set+e->v), *s2 = set_find(scc_set+e->u);
           if (s1==s2) continue;
            res[c] = e;
            value += e->kev;
            tree_add(adj[c], -e->key);
            node2 *w1 = set_find(wcc_set+e->v), *w2 = set_find(wcc_set+e->u);
            if (w1!=w2) { set union(w1, w2); continue; }
            upper[c] = scc_c;
            do {
               e = res[s2->scc];
               upper[s2->scc] = scc_c;
               adj[c] = tree union(adj[c], adj[s2->scc]);
               s1 = set union(s1, s2);
               s2 = set_find(scc_set+e->u);
            } while (s1!=s2);
            s1->scc = scc\_c;
            upper[scc_c] = lower[scc_c] = -1;
            adj[scc_c] = adj[c];
            res[scc cl = 0;
```

```
c = scc_c++;
    REPD(c, scc_c - 1, n) {
       if (lower[c]==-1)
          for (int i=res[c]->v; i!=c; i=upper[i]) lower[upper[i]] = i;
       res[lower[c]] = res[c];
    return value:
                        25. Matching dowolny (random)
    97192338fddf7d1f2771187687264d4c
const int N = 555;
mt19937 rnd(time(0));
struct Matching {
                        // [1, n]
       vector<int> e[N];
       int mate[N], vis[N];
        void add(int a, int b) {
               if (a == b) return;
               e[a].push_back(b);
               e[b].push_back(a);
        bool dfs(int a) {
               shuffle(e[a].begin(), e[a].end(), rnd);
               vis[a] = 1;
               for (auto b : e[a]) {
                       int c = mate[b];
                       if (vis[c]) continue;
                       mate[a] = b; mate[b] = a; mate[c] = 0;
                       if (!c || dfs(c)) return 1;
                       mate[a] = 0; mate[b] = c; mate[c] = b;
                return 0;
        vector<pair<int, int>> matching(int n) {
               vector<pair<int, int>> res;
                rep(_, 1, 20) {
                       memset(mate, 0, sizeof mate);
                       rep(i, 1, n) {
                               if (!mate[i]) {
                                       memset(vis, 0, sizeof vis);
                                       dfs(i);
                       vector<pair<int, int>> cur;
                       rep(i, 1, n) if (mate[i] > i) cur.push_back({i, mate[i]});
                       if (cur.size() > res.size()) res = cur;
```

```
return res;
} ;
                              26. Matching dowolny
   fa9b662370e56f0b1a48ac4fe9352938
#define N 307
                      // IN: liczba wierzcholkow
int n;
                      // IN: macierz sasiedztwa (mozna zmienic na liste)
bool edge[N][N];
                      // OUT: wierzcholek skojarzony (-1 oznacza brak)
int mate[N];
int label[N], base[N], prev1[N], prev2[N];
bool mark[N];
bool prepare (int v) {
    while(1) {
       mark[v] = !mark[v];
       if (mate[v] == -1) return mark[v];
       v = base[prev2[mate[v]]];
int shrink (int v, int b1, int b2, queue<int> &Q) {
    while (mark[v]) {
       prev1[v] = b1; prev2[v] = b2;
       mark[mate[v]] = 1;
       Q.push(mate[v]);
       v = base[prev2[mate[v]]];
    return v;
bool make blos (int i, int j, int bi, int bj, queue<int> &O) {
   if (label[i]!=1 || i==j) return 0;
    if (prepare(i), prepare(j)) return 1;
    int b = (shrink(i, bi, bj, Q), shrink(j, bj, bi, Q));
    FOR(v,n) if (mark[base[v]]) base[v] = b;
    return 0;
void rematch(int i, int i) {
    int nxt = mate[i];
    mate[i] = j;
   if (nxt==-1) return;
   mate[nxt] = -1;
   rematch(prev2[nxt], prev1[nxt]);
   rematch(prev1[nxt], prev2[nxt]);
bool augment() {
    queue<int> 0;
    FOR(i,n) {
       label[i] = mate[i] == -1;
       if (mate[i]==-1) Q.push(i);
```

};

```
mark[i] = 0;
       base[i] = i;
   while (!Q.empty()) {
       int cur = Q.front(); Q.pop();
       FOR(i,n) /*tu zmienic*/ if (edge[cur][i] && i!=mate[cur]) {
                                        // (nie wiem co "zmienic", dziala jak jest)
           if (!label[i]) {
               label[i] = -1;
               label[mate[i]] = 1;
               Q.push(mate[i]);
               prev1[i] = i; prev2[i] = cur;
            } else if (make_blos(base[i], base[cur], i, cur, Q)) {
               rematch(i, cur); rematch(cur, i);
               return 1:
    return 0;
int compute_gcm() { // zwraca licznosc maksymalnego skojarzenia
   fill_n(mate, n, -1);
   int res = 0;
    while (augment()) ++res;
    return res;
                   27. Matching dowolny ważony (random)
   bf3d0e46ae9fb4933a125fa1c8ccdcc9
struct RandomizedMatching {
       long long G[N][N], dis[N];
       int match[N];
       int mat[N], stk[N], id[N], vis[N];
       int n, top;
       const long long inf = 1e18;
       RandomizedMatching() {}
       RandomizedMatching(int _n) {
               n = n; top = 0;
               memset (match, 0, sizeof match);
               for (int i = 1; i <= n + 1; i++) {
                       for (int j = 1; j \le n + 1; j++) {
                               G[i][i] = 0;
        void add_edge(int u, int v, long long w) {
               G[u][v] = max(G[u][v], w);
               G[v][u] = max(G[v][u], w);
```

bool spfa(int u) {

```
stk[top ++] = u;
        if (vis[u]) return true;
        vis[u] = true;
        for (int i = 1; i <= n; ++ i) {</pre>
                if (i != u && i != mat[u] && !vis[i]) {
                         int v = mat[i];
                         if (dis[v] < dis[u] + G[u][i] - G[i][v]) {
                                 dis[v] = dis[u] + G[u][i] - G[i][v];
                                 if (spfa(v)) return true;
        top --; vis[u] = false;
        return false;
long long maximum_matching() {
        for (int i = 1; i <= n; ++ i) id[i] = i;
        for (int i = 1; i <= n; i += 2) mat[i] = i + 1, mat[i + 1] = i;</pre>
        for (int times = 0, flag; times < 3; ) { //increase the iteration value</pre>

    → for higher probability

                memset(dis, 0, sizeof(dis));
                memset(vis, 0, sizeof(vis));
                top = 0; flag = 0;
                for (int i = 1; i <= n; ++ i) {</pre>
                         if (spfa(id[i])) {
                                 flaq = 1:
                                 int t = mat[stk[top - 1]], j = top - 2;
                                 while (stk[j] != stk[top - 1]) {
                                         mat[t] = stk[j];
                                         swap(t, mat[stk[j]]);
                                          -- j;
                                 mat[t] = stk[j]; mat[stk[j]] = t;
                                 break:
                if (!flag) times ++;
                if (!flag) random_shuffle(id + 1, id + n + 1);
        long long ans = 0;
        for (int i = 1; i <= n; ++ i) {</pre>
                if (mat[i] <= n && i < mat[i]) {</pre>
                         if (G[i][mat[i]] != 0) ans += G[i][mat[i]], match[i] =

    mat[i], match[mat[i]] = i;

        return ans;
```

## 28. Turbo Matching

```
565529bc2bd2546c12d5ac514d100004
struct Matching {
    int n; vector < int > *G, match, vis;
   bool dfs(int v) {
       vis[v] = 1;
        for(auto u: G[v]) if(!match[u] || (!vis[match[u]] && dfs(match[u]))) 
                match[v] = u; match[u] = v; return true;
        return false;
   Matching(int N, vector < int > *g) : n(N), G(g) {
       match.resize(n + 1, 0), vis.resize(n + 1, 0); bool ok = 1;
        while (ok) {
            ok = 0;
            for(int i = 1; i <= n; i++) if(!match[i] && dfs(i)) ok = 1;</pre>
            for(int i = 1; i <= n; i++) vis[i] = 0;</pre>
};
                                       29. 2 SAT
```

```
b4b2a54c9e18151894b58de59501f2ca
struct twosat {
   int n:
   vector <vector <int> > G, R;
   vector <int> order, comp, ans;
   vector <bool> vis;
   twosat() {}
   twosat(int _n) : n(_n) {
       G.resize(n + n);
       R.resize(n + n);
       comp.resize(n + n);
       vis.resize(n + n);
       ans.resize(n);
       order.reserve(n + n);
   void add edge(int u, int v) {
       G[u].push back(v);
       R[v].push_back(u);
   /* 0-indexed, subtract if necessary */
   void add clause(int u, bool fu, int v, bool fv) {
       add_edge(u << 1 | !fu, v << 1 | fv);
       add_edge(v << 1 | !fv, u << 1 | fu);
```

```
void dfs(int u) {
       vis[u] = true;
       for (const auto &v: G[u])
           if (!vis[v]) dfs(v);
       order.push_back(u);
    void scc(int u, int id) {
       vis[u] = true, comp[u] = id;
       for (const auto &v: R[u])
           if (!vis[v]) scc(v, id);
   bool run() {
       for (int i = 0; i < n + n; ++i)</pre>
           if (!vis[i]) dfs(i);
        fill(vis.begin(), vis.end(), false);
        reverse(order.begin(), order.end());
        int cnt = 0;
        for (const auto &v: order)
           if (!vis[v]) scc(v, ++cnt);
        for (int i = 0; i < n; ++i) {</pre>
           if (comp[i << 1] == comp[i << 1 | 1]) return false;</pre>
           ans[i] = comp[i << 1] < comp[i << 1 | 1];
        return true;
};
                        30. Ogólna transformata Adama
   7d9acfc445ea90039f75c68e876dde44
const int mod = 998244353;
const int gen = 3;
const int inv2 = (mod + 1) / 2;
struct tran // przodek
       virtual int size () {}
       virtual void apply (int* A, bool inverse, cint& jump = 1) {}
struct xor_tran : tran // binarny XOR (Walsh-Hadamard)
```

```
int size () { return 2; }
        void apply (int* A, bool inverse, cint& jump = 1)
                int& na = A[0 * jump];
                int \& nb = A[1 * jump];
                cint pa = na;
                cint pb = nb;
                na = pa + pb;
                nb = pa - pb;
                if (na >= mod)
                       na -= mod;
                if (nb < 0)
                       nb += mod;
                if (inverse)
                        na = 111 * na * inv2 % mod;
                        nb = 111 * nb * inv2 % mod;
};
/* uwaga na modulo
struct and_tran : tran // binarny AND
        int size () { return 2; }
        void apply (int* A, bool inverse, cint& jump = 1)
                int \& na = A[0 * jump];
                int \& nb = A[1 * jump];
                cint pa = na;
                cint pb = nb;
                if (inverse == false)
                        na = pb;
                        nb = pa + pb;
                if (inverse == true)
                        na = -pa + pb;
                        nb = pa;
1:
struct or_tran : tran // binarny OR (Mobius)
        int size () { return 2; }
        void apply (int * A, bool inverse, cint& jump = 1)
                int \& na = A[0 * jump];
```

```
int \& nb = A[1 * jump];
                cint pa = na;
                cint pb = nb;
                if (inverse == false)
                        na = pa + pb;
                        nb = pa;
                if (inverse == true)
                        na = pb;
                        nb = pa - pb;
};
*/
struct sum_tran : tran // suma (Fourier)
       int mysize;
       int size () { return mysize; }
       sum_tran (int s) : mysize(s) {}
        void apply (int* A, bool inverse, cint& jump = 1)
                int n, i, j, k, w, a, b, root;
                n = mysize;
                  dla jumpa > 1 odkomentować, zmienić argument A na B
                assert(jump == 1);
                  vector <int> A(n);
                  for (i=0; i<n; i++)
                          A[i] = B[i * jump];
                for (i=1, j=0; i<n; i++) {</pre>
                        for (k=n/2; j>=k; k/=2)
                                j ^= k;
                        j ^= k;
                        if (i < j)
                                swap(A[i], A[j]);
                for (k=1; k < n; k *= 2) {
                        root = pw(qen, (mod-1)/(k+k));
                        if (inverse == true)
                                root = pw(root, mod-2);
                        for (i=0; i<n; i+=k+k) {</pre>
                                w = 1;
                                for (j=i; j<i+k; j++) {</pre>
                                        a = A[j];
                                         b = 111 * A[j+k] * w % mod;
```

```
A[j] = a + b;
                                        A[j+k] = a - b;
                                        if (A[j] >= mod)
                                                A[j] -= mod;
                                        if (A[j+k] < 0)
                                                A[j+k] += mod;
                                        w = 111 * w * root % mod;
                if (inverse == true) {
                        w = pw(n, mod-2);
                        for (i=0; i<n; i++)
                               A[i] = 111 * A[i] * w % mod;
                  for (i=0; i<n; i++)
                          B[i * jump] = A[i];
};
struct russian : tran // niezależne podkonwolucje (kolejność od najmłodszej do
→ najstarszej)
        int mysize;
        vector <tran*> T;
        int size () { return mysize; }
        russian (vector <tran*> vec = {})
                T = \text{vec};
                mvsize = 1;
                for (tran* t : T)
                        mysize \star = t -> size();
        void apply (int* A, bool inverse, cint& jump = 1)
                int i, j, s = 1;
                for (tran* t : T) {
                        for (i = 0; i < this->size(); i += s * t->size())
                                for (j = 0; j < s; j++)
                                        t->apply(A + (i + j) * jump, inverse, s * jump);
                        s *= t->size();
};
vector <int> convolute (tran& t, vector <int> A, vector <int> B) // konwolucja w
→ ogólności
```

```
int n = t.size();
        assert(A.size() <= n); A.resize(n, 0);</pre>
        assert(B.size() <= n); B.resize(n, 0);</pre>
        t.apply(&A[0], false);
       t.apply(&B[0], false);
        for (int i=0; i<n; i++)</pre>
               A[i] = 111 * A[i] * B[i] % mod;
       t.apply(&A[0], true);
        return A;
                               31. POLYNOMIAL DIVISION
   ea39ca4b66d63e4fcd3faa4f25f03fd6
vector <int> polydiv(vector <int> a, vector <int> b)
       int n, m, i, k, s;
       n = a.size();
        m = b.size();
       if (m > n) return {0};
        reverse(a.begin(), a.end());
        reverse(b.begin(), b.end());
        s = moddiv(1, b[0], MOD);
        for (int& x : b)
                x = modmul(x, s, MOD);
        k = n - m + 1;
        vector \langle int \rangle r, v, g = \{1\};
        for (int w = 1; (1 << (w - 1)) < k; w++) {
                r = polymul(q, q);
                r.resize(1 << w);
                v = b;
                v.resize(1 << w);
                r = polymul(r, v);
                r.resize(1 << w);
                q.resize(1 << w);
                polyadd(g, g);
                polysub(g, r);
       g = polymul(g, a);
        q.resize(k);
        reverse(g.begin(), g.end());
        for (int& x : g)
                x = modmul(x, s, MOD);
```

```
32. Karatsuba
    ee27a5660b46a795436be1a725683b98
#pragma GCC optimize("Ofast,unroll-loops")
#pragma GCC target("avx,avx2,fma")
namespace {
    template<int n, typename T>
    void mult(const T *__restrict a, const T *__restrict b, T *__restrict res) {
        if (n <= 64) { // if length is small then naive multiplication if faster</pre>
            for (int i = 0; i < n; i++) {</pre>
                for (int j = 0; j < n; j++) {</pre>
                    res[i + j] += a[i] \star b[j];
        } else {
            const int mid = n / 2;
            alignas (64) T btmp[n], E[n] = \{\};
            auto atmp = btmp + mid;
            for (int i = 0; i < mid; i++) {</pre>
                atmp[i] = a[i] + a[i + mid]; // atmp(x) - sum of two halfs a(x)
                btmp[i] = b[i] + b[i + mid]; // btmp(x) - sum of two halfs b(x)
            mult<mid>(atmp, btmp, E); // Calculate E(x) = (alow(x) + ahigh(x)) *
            \hookrightarrow (blow(x) + bhigh(x))
            mult < mid > (a + 0, b + 0, res); // Calculate rlow(x) = alow(x) * blow(x)
            mult < mid > (a + mid, b + mid, res + n); // Calculate rhigh(x) = ahigh(x) *
            for (int i = 0; i < mid; i++) { // Then, calculate rmid(x) = E(x) - rlow(x)
            \hookrightarrow - rhigh(x) and write in memory
                const auto tmp = res[i + mid];
                res[i + mid] += E[i] - res[i] - res[i + 2 * mid];
                res[i + 2 * mid] += E[i + mid] - tmp - res[i + 3 * mid];
                                        33. NTT
    06b16ee6c824db06b0859775d87d3992
typedef unsigned long long ull;
typedef unsigned T;
//ma byc unsigned int albo 11 albo ull
//jak sie przekrecimy przez inta to zly wynik!
const T P = 998244353;
const T ROOT = 570984967:
```

return q;

```
const int MN=21;
T omega[1<<MN];
T pw (T x, T n) {
        T res = 1:
        while(n) {
                         if (n\&1) res = (ull) res*x%P;
                         x = (ull) x * x % P;
                        n >> = 1;
        return res;
void fft(vector<T> &a, int n, bool inverse=false) {
        int N = 1 << n;
        a.insert(a.end(), N-SZ(a), 0);
        T root = pw(ROOT, (1 << 23) / N*(inverse?(N-1):1));
        omega[0] = 1;
        rep(i, 1, N) omega[i] = (ull) omega[i-1] *root%P;
        rep(i, 0, n) {
                         rep(j, 0, 1 << i) {
                                         rep(k, 0, 1 << (n-i-1)) {
                                                          int s = (j << (n-i)) + k;
                                                          int t = s + (1 << (n-i-1));
                                                          T w = omega[k << i];
                                                          T \text{ temp} = a[s] + a[t];
                                                           if (temp >= P) temp -= P;
                                                          T t2 = a[s] - a[t] + P;
                                                          a[t] = (ull) w * t2 % P;
                                                          a[s] = temp;
        rep(i, 0, N) {
                         int x=i, y=0;
                         rep(j, 0, n) y=(y<<1)+(x&1), x>>=1;
                         if (i<y) swap(a[i],a[y]);</pre>
        if (inverse) {
                         T inv = pw(N, P-2);
                         rep(i, 0, N) a[i] = (ull)a[i]*inv%P;
vector<T> conv(vector<T> A, vector<T> B) {
        int n = 31- builtin clz(2*(SZ(A)+SZ(B))-1);
        fft(A, n);
        fft(B, n);
        rep(i, 0, (1<<n)) A[i] = (ull) A[i] * B[i] % P;
        fft(A, n, true);
        return A;
vector<T> square(vector<T> A) {
        int n = 32-_builtin_clz(2*SZ(A)-1);
```

```
fft(A, n);
       rep(i, 0, (1<<n)) A[i] = (ull) A[i] * A[i] % P;
       fft(A, n, true);
       return A:
                                    34. Josephus
   421dd32d75d61954519d94f3fc96fc4a
ll josephus(ll n, ll k, ll m) {
 m = n - m;
  if (k <= 1) return n - m;</pre>
  11 i = m;
  while (i < n) {
   11 r = (i - m + k - 2) / (k - 1);
   if ((i + r) > n) r = n - i;
   else if (!r) r = 1;
   i += r;
   m = (m + (r * k)) % i;
  } return m + 1;
                       35. Knights Moves in Infinity Grid
   964cf59287e3516922e3704eeba9201e
#define 11 long long int
// Minimum number of knight moves from (x,v) to
// (0,0) in non-negative infinite chessboard
ll knight move(ll x, ll v) {
  11 cnt = \max(\{(x + 1) / 2, (y + 1) / 2, (x + y + 2) / 3\});
  while((cnt % 2) != (x + y) % 2) cnt++;
  if(x == 1 && !v) return 3;
  if(y == 1 && !x) return 3;
  if(x == y && x == 2) return 4;
  return cnt;
                              36. Mnożenie nimberów
   26a891adbcc465fbff392ab63dbe4c0f
typedef unsigned long long T;
const int POWERS = 5;
const T vals[] = {1, 4, 16, 256, 65536, 4294967296ull};
/* Działa w log^2, jeśli trzeba szybciej to można skorzystać z rodzielności względem

→ ksora (preprocessing) */
T multiply(T a, T b) {
       if(!a or !b)
               return 0:
```

```
T pa = 0, pb = 0;
        for(int i = POWERS; i >= 0; --i) {
                if(!pa and a >= vals[i])
                                                pa = vals[i];
                if(!pb and b >= vals[i])
                                                pb = vals[i];
        if(pa != pb) {
                if(pa < pb)</pre>
                                   swap(pa, pb), swap(a, b);
                return pa * multiply(a / pa, b) ^ multiply(a % pa, b);
        if(pa == 1) {
                if(a == 1 \text{ or } b == 1)
                                            return a * b;
                if(a != b)
                                  return 1:
                return a ^ 1;
        T a1 = a / pa, a2 = a % pa;
        T b1 = b / pb, b2 = b % pb;
        T p1 = multiply(a1, b1);
        T p2 = multiply(a2, b2);
        T p3 = multiply(a1 ^ a2, b1 ^ b2);
       T p4 = multiply(p1, pa >> 1);
        T p5 = p2 ^ p3;
        return p2 ^ p4 ^ p5 * pa;
                                        37. FFT
   4d6c99e53e9c2eab4ea8fffea35c5218
typedef double T;
const T PI = acos(-1.0);
struct C {
   T re, im;
    C () {}
   C (T r) : re(r), im(0) {}
    C (T r, T i) : re(r), im(i) {}
    C operator * (const C &c) const {
        return C(re * c.re - im * c.im, im * c.re + re * c.im);
    C operator + (const C &c) const {
        return C(re + c.re, im + c.im);
    C operator - (const C &c) const {
        return C(re - c.re, im - c.im);
    void operator += (const C &c) {
        re += c.re, im += c.im;
    C conj() const {
        return C(re, -im);
};
```

```
typedef vector < C > VC;
typedef vector < LL > VLL;
inline void FFT(C *a, int n, int dir) {
    for(int i = 0, j = 0; i < n; i++) {
        if(i > j) swap(a[i], a[j]);
        for (int k = n >> 1; (j ^= k) < k; k >>= 1);
    for(int p = 2; p <= n; p <<= 1) {</pre>
        C wn(cos(2.0 * dir * PI / p), sin(2.0 * dir * PI / p));
        for (int k = 0; k < n; k += p) {
            C w = 1;
            for (int j = 0; j < (p >> 1); j++) {
                C xx = a[k + j];
                C yy = w * a[k + j + (p >> 1)];
                a[k + j] = xx + yy;
                a[k + j + (p >> 1)] = xx - yy;
                w = w * wn;
        }
void multiply(VLL &a, VLL &b, VLL &res) {
   int n = max(a.size(), b.size()), p = 2;
    while((p >> 1) < n) p <<= 1;</pre>
    C *fa = new C[p + 4];
    for(int i = 0; i < p; i++) fa[i] = 0;</pre>
    for(int i = 0; i < sz(a); i++) fa[i] += C(a[i], 0);</pre>
    for(int i = 0; i < sz(b); i++) fa[i] += C(0, b[i]);</pre>
    FFT(fa, p, 1);
    for(int i = 0; i <= p / 2; i++) {</pre>
       C bp = fa[i] + fa[p - i == p ? 0 : p - i].conj();
        C_q = fa[p - i == p ? 0 : p - i] - fa[i].conj();
        C q(_q.im, _q.re);
        fa[i] = (bp * q) * C(0.25);
        if(i > 0) fa[p - i] = fa[i].conj();
    FFT(fa, p, -1);
    res.resize(sz(a) + sz(b) - 1);
    for(int i = 0; i < sz(res); i++) {
        res[i] = round(fa[i].re / p);
    delete [] fa;
```

# 38. FFT Radecki

#### e7aa770b0e04b5fb8e2ee1dacac4a4cf

/\* Prec. error max\_ans/1e15 (2.5e18) for (long) doubles, so int rounding works for doubles with answers 0.5e15, e.g. for sizes  $2^2$ 0 and RANDOM ints in [0,45k], assuming DBL\_MANT\_DIG=53 and LDBL\_MANT\_DIG=64. Consider normalizing and brute.\*/

```
#define REP(i,n) for(int i = 0; i < int(n); ++i)
typedef double ld; // 'long double' is 2.2 times slower
struct C { ld re, im;
        C operator * (const C & he) const {
                return C{re * he.re - im * he.im,
                                re * he.im + im * he.re};
        void operator += (const C & he) { re += he.re; im += he.im; }
void dft(vector<C> & a, bool rev) {
        const int n = a.size();
        for (int i = 1, k = 0; i < n; ++i) {
                for(int bit = n / 2; (k ^= bit) < bit; bit /= 2);;;</pre>
                if(i < k) swap(a[i], a[k]);</pre>
        for (int len = 1, who = 0; len < n; len \star= 2, ++who) {
                static vector<C> t[30];
                vector<C> & om = t[who];
                if(om.empty()) {
                        om.resize(len);
                        const ld ang = 2 * acosl(0) / len;
                        REP(i, len) om[i] = i%2 || !who ?
                                        C(\cos(i*ang), \sin(i*ang)) : t[who-1][i/2];
                for (int i = 0; i < n; i += 2 * len)
                        REP(k, len) {
                                 const C x = a[i+k], y = a[i+k+len]
                                                 * C{om[k].re, om[k].im * (rev ? -1 :
                                                 \hookrightarrow 1)};
                                 a[i+k] += v;
                                a[i+k+len] = C\{x.re - y.re, x.im - y.im\};
       if(rev) REP(i, n) a[i].re /= n;
template<typename T>vector<T> multiply(const vector<T> & a, const vector<T> & b,
                bool split = true, bool normalize = false) {
        if(a.empty() || b.empty()) return {};
        T big = 0; if (normalize) { // [0,B] into [-B/2, B/2]
                assert(a.size() == b.size()); // equal size!!!
                for(T x : a) big = max(big, x);
                for (T x : b) big = max(big, x);
                big /= 2;
       int n = a.size() + b.size();
        vector<T> ans (n-1);
        /* if(min(a.size(),b.size()) < 190) { // BRUTE FORCE
                REP(i, a.size()) REP(j, b.size()) ans[i+j] += a[i]*b[j];
                return ans; } */
        while (n&(n-1)) ++n;
        auto foo = [&](const vector<C> & w, int i, int k) {
```

27

```
int j = i ? n - i : 0, r = k ? -1 : 1;
       return C{w[i].re + w[j].re * r, w[i].im
                       -w[j].im * r * (k ? C{0, -0.5} : C{0.5, 0});
if(!split) { // standard fast version
       vector<C> in(n), done(n);
       REP(i, a.size()) in[i].re = a[i] - big;
       REP(i, b.size()) in[i].im = b[i] - big;
       dft(in, false);
       REP(i, n) done[i] = foo(in, i, 0) * foo(in, i, 1);
       dft(done, true);
       REP(i, ans.size()) ans[i] = is_integral<T>::value ?
                       llround(done[i].re) : done[i].re;
//REP(i,ans.size())err=max(err,abs(done[i].re-ans[i]));
else { // Split big INTEGERS into pairs a1*M+a2,
       const T M = 1<<15; // where M = sqrt(max_absvalue).</pre>
       vector<C> t[2]; // This version is 2.2-2.5 times slower.
       REP(x, 2) {
               t[x].resize(n);
               auto & in = x ? b : a; // below use (in[i]-big) if normalized
               REP(i, in.size()) t[x][i]=C\{ld(in[i]\%M), ld(in[i]/M)\};
               dft(t[x], false);
       T \text{ mul} = 1;
       for(int s = 0; s < 3; ++s, mul = (mul*M)%mod) {</pre>
               vector<C> prod(n);
               REP(x, 2) REP(y, 2) if(x + y == s) REP(i, n)
                       prod[i] += foo(t[0], i, x) * foo(t[1], i, y);
               dft(prod, true); // remember: llround(prod[i].re)%MOD*mul !!!
               REP(i, ans.size()) ans[i]=
               if(normalize) {
       T so far = 0;
       REP(i, ans.size()) {
               if(i < (int) a.size()) so_far += a[i] + b[i];</pre>
               else so_far -= a[i-a.size()] + b[i-a.size()];
               ans[i] += big * so_far - big * big * min(i + 1, (int) ans.size()

→ - i);
return ans;
```

# 39. Faktoryzacja Rho-Pollarda

2221309c189efb5908813dc33d7351b7

```
typedef long long int LL;
const int maxv = 50;
const int maxc = 5007;
const int maxp = 1e6 + 7:
int cnt;
int ptot;
int d[maxp];
int pr[maxp];
LL ans[maxc];
inline LL mod(LL a, LL n) {
       if(a >= n)
                     a -= n;
       return a;
inline LL add(LL a, LL b, LL n) {
        a += b; mod(a, n);
        return a;
inline LL mul(LL x, LL y, LL p) {
        LL ret = x * y - (LL)((long double)x * y / p + 0.5) * p;
        return ret < 0 ? ret + p : ret;</pre>
LL fast(LL x, LL k, LL p) {
       LL ret = 1\%p;
        for (; k > 0; k >>= 1, x = mul(x, x, p))
                (k & 1) && (ret = mul(ret, x, p));
        return ret;
bool rabin(LL n) {
       if(n == 2) return 1;
        if(n < 2 | | !(n & 1))
                return 0;
        LL s = 0, r = n - 1;
        for(; !(r & 1); r >>= 1, ++s);
        for(int i = 0; pr[i] < n && pr[i] < maxv; ++i) {</pre>
                LL cur = fast(pr[i], r, n), nxt;
                for (int j = 0; j < s; ++j) {
                        nxt = mul(cur, cur, n);
                        if(nxt == 1 && cur != 1 && cur != n - 1) return false;
                        cur = nxt;
                if(cur != 1) return false;
        return true;
LL gcd(LL a, LL b) {
        LL tmp;
        while (b) {
                tmp = b;
                b = a%b;
                a = tmp:
        return a:
```

```
LL factor(LL n) {
        static LL seq[maxp];
        while (true) {
                LL x = rand()%n, y = x, c = rand()%n;
                LL *px = seq, *py = seq, tim = 0, prd = 1;
                         *py++ = y = add(mul(y, y, n), c, n);
                         *py++ = y = add(mul(y, y, n), c, n);
                         if((x = *px++) == y) break;
                         LL tmp = prd;
                         prd = mul(prd, abs(y - x), n);
                         if(!prd) return gcd(tmp, n);
                         if((++tim) == maxv){
                                 if((prd = gcd(prd, n)) > 1 && prd < n) return prd;</pre>
                                 tim = 0;
                if(tim && (prd = gcd(prd, n)) > 1 && prd < n) return prd;</pre>
void decompose(LL n) {
        if(n < maxp) {
                while (n > 1)
                        ans [cnt++] = d[n], n /= d[n];
        else if(rabin(n))
                ans[cnt++] = n;
        else{
                LL fact = factor(n);
                decompose(fact), decompose(n / fact);
void init(){
        d[1] = 1;
        for(int i = 2; i * i < maxp; ++i)</pre>
                if(!d[i])
                         for(int j = i * i; j < maxp; j += i)</pre>
                                 d[j] = i;
        for(int i = 2; i < maxp; ++i)</pre>
                if(!d[i]){
                        d[i] = i;
                        pr[ptot++] = i;
void clear(){
        cnt = 0;
```

#### 40. Drzewo Sterna-Brocota

```
c508131f936fb1d35fde4f83ec43c823
typedef double D;
typedef long long int LL;
#define st first
#define nd second
#define PII pair <int, int>
const int INF = 1e9;
PII operator*(int t, PII a) {
       return {a.st * t, a.nd * t};
PII operator+(PII a, PII b) {
        return {a.st + b.st, a.nd + b.nd};
bool operator<=(PII a, PII b) {</pre>
        return 1LL * a.st * b.nd <= 1LL * b.st * a.nd;</pre>
PII getLeft (PII &Left, PII &Right, PII &a, int &d) {
        int p = 1, k = (a.nd - Left.nd) / Right.nd;
        if(k < 1)
                          assert(false);
        while(p < k){
                int m = (p + k + 1) / 2;
                PII maybe = m * Right + Left;
                if(maybe <= a)</pre>
                        p = m;
                else
                        k = m - 1;
        return p * Right + Left;
PII getRight (PII &Left, PII &Right, PII &a, int &d) {
       int p = 1, k = (d - Right.nd - 1) / Left.nd;
                         assert(false);
        if(k < 1)
        while (p < k) {
                int m = (p + k + 1) / 2;
                PII maybe = m * Left + Right;
                if (maybe <= a)</pre>
                        k = m - 1;
                else
                        p = m;
        return p * Left + Right;
                                       //Zwraca najmniejszy ulamek wiekszy niz a, o
PII getFraction(PII a, int d) {
\hookrightarrow mianowniku < d, d >= 2, a < 1
       PII Left = \{0, 1\}, Right = \{1, 1\};
        while (true) {
                PII Mid = Left + Right;
                if(Mid <= a)</pre>
                         Left = getLeft(Left, Right, a, d);
```

```
else if(Mid.nd >= d)
                       break;
                else
                       Right = getRight(Left, Right, a, d);
       assert(Right.nd < d);
       return Right;
                            41. Pierwiastek Dyskretny
   3181eabf11c32bfb2d74eed98ebf27b5
int fast(int a, int b, int mod){
        int ret = 1;
        while (b) {
               if(b & 1)
                       ret = (1LL * ret * a) %mod;
               b >>= 1;
               a = (1LL * a * a) %mod;
       return ret:
int get(int p){
        int t = 2;
        while (fast (t, (p - 1) / 2, p) == 1)
               ++t;
        return t;
int dsr(int v, int p) {
                             //Jeśli -1 to nie ma rozwiązania, wpp. rozwiązanie to r i

→ -r, p jest pierwsze

        if (v == 0)
               return 0:
        if(p == 2)
                return 1;
        if (fast (v, (p-1) / 2, p) == p-1)
               return -1;
        int q = p - 1, s = 0;
        while(!(q & 1))
               q /= 2, ++s;
        if(s == 1)
               return fast (v, (p + 1) / 4, p);
        int z = get(p);
        int c = fast(z, q, p);
        int r = fast(v, (q + 1) / 2, p);
        int t = fast(v, q, p);
        int m = s;
        while(t != 1){
               int tt = t;
               int i = 0;
                while(tt != 1){
```

```
tt = (1LL * tt * tt)%p;
                        ++i;
                int b = fast(c, fast(2, m - i - 1, p - 1), p);
                int b2 = (1LL * b * b) %p;
                r = (1LL * r * b) %p;
               t = (1LL * t * b2) %p;
               c = b2;
               m = i;
       return r;
                               42. Berlekamp-Massey
   0d0ab87e89132a8fab9e9a9b81638992
#define FOR(i, n) for(int i = 0; i < (n); i++)
#define sz(x) (int)(x).size()
const int N = 5005;
const int M = 1e9 + 7;
LL fast(LL a, LL n) {
   LL x = 1; a %= M;
    while(n) {
       if (n \& 1) x = x * a % M;
       a = a * a % M; n >>= 1;
   return x;
vector < LL > BM(vector < LL > x) {
   vector < LL > ls, cur;
    LL lf = 0, ld = 0;
   FOR(i, sz(x)) {
       LL t = 0;
       FOR(j, sz(cur)) t = (t + (LL)x[i - j - 1] * cur[j]) % M;
       if((t - x[i]) % M == 0) continue;
       if(cur.emptv()) {
            cur.resize(i + 1);
            lf = i; ld = (t - x[i]) % M;
            continue;
       LL k = -(x[i] - t) * fast(ld, M - 2) % M;
       vector \langle LL \rangle c(i - lf - 1); c.push back(k);
       for(auto y: ls) c.push_back(-y * k % M);
       if(sz(c) < sz(cur)) c.resize(sz(cur));</pre>
       FOR(j, sz(cur)) c[j] = (c[j] + cur[j]) % M;
       if(i - lf + sz(ls) >= sz(cur))
            ls = cur, lf = i, ld = (t - x[i]) % M;
       cur = c:
    for(auto &y: cur) y = (y % M + M) % M;
```

```
return cur;
int m;
LL a[N], h[N], t_[N], s[N], t[N];
void mull(LL *p, LL *q) {
    FOR(i, 2 * m) t_[i] = 0;
    FOR(i, m) if(p[i]) FOR(j, m)
       t_{i} = (t_{i} + j) + p[i] * q[j]) % M;
    for(int i = 2 * m - 1; i >= m; i--) if(t_[i])
        for(int j = m - 1; ~j; --j)
            t_{i} = (t_{i} - j - 1) = (t_{i} - j - 1) + t_{i} * h[j] % M;
    FOR(i, m) p[i] = t_[i];
LL calc(LL k) {
    for(int i = m; ~i; i--) s[i] = t[i] = 0;
    s[0] = 1; (m != 1 ? t[1] = 1 : t[0] = h[0]);
    while(k) {
       if(k & 1) mull(s, t);
        mull(t, t); k >>= 1;
    LL su = 0;
    FOR(i, m) su = (su + s[i] * a[i]) % M;
    return (su % M + M) % M;
LL nth_element(vector < LL > x, LL n) {
    if(n < (int)sz(x)) return x[n];</pre>
    vector < LL > v = BM(x); m = v.size(); if(!m) return 0;
    FOR(i, m) h[i] = v[i], a[i] = x[i];
    return calc(n);
```

# 43. CHIŃSKIE TWIERDZENIE O RESZTACH

5101903203c6e2f4fd3b35236a2dfc78

```
typedef long long T;
pair<T, T> gcd_ext(T a, T b)
{
    if(b == 0) return { 1, 0 };
    auto p = gcd_ext(b, a % b);
    return { p.second, p.first - a / b * p.second };
}
pair<T, T> CRT(vector<pair<T, T>> con)
{
    T k = 0, m = 1;
    for(auto c: con)
    {
        T k1 = k, m1 = m, k2 = c.first, m2 = c.second;
        auto q = gcd_ext(m1, m2);
        T gcd = m1 * q.first + m2 * q.second;
        m = m1 / gcd * m2;
```

```
if(k1 % gcd != k2 % gcd)
            return { -1, -1 };
        k = (k1 / gcd) * m2 * q.second + (k2 / gcd) * m1 * q.first + k1 % gcd;
       k %= m;
       if (k < 0) k += m;
    return { k, m };
                        44. Liczba punktów w trójkacie
   ed44d297144492d6a9fb968a8a3fbfbe
LL go(LL n, LL p, LL g) {
   if(n * p < q) return n + 1;
    if(p < q) {
       LL rect = (n + 1) * (n * p / q + 1);
       LL diag = n / q + 1;
       return diag + rect - go(n * p / q, q, p);
    LL k = p / q;
    return k * n * (n + 1) / 2 + qo(n, p - k * q, q);
LL in_triangle(LL n, LL p, LL q) {
    LL d = \underline{gcd(p, q)};
    return go(n, p / d, g / d);
                       45. Sumy ciagów arytmetycznych
   4e1e30dd460407d3ca69674351ea4e1f
ll sumsq(ll n) {
 return n / 2 * ((n - 1) | 1);
// \sum_{i=0}^{n-1} (a + d * i) / m}, O(\log m)
11 floor sum(ll a, ll d, ll m, ll n) {
 ll res = d / m * sumsq(n) + a / m * n;
 d %= m; a %= m;
 if (!d) return res;
 11 to = (n * d + a) / m;
 return res + (n-1) * to - floor sum(m-1-a, m, d, to);
// \sum_{i=0}^{n-1} \{(a+d*i) % m\}
11 mod sum(11 a, 11 d, 11 m, 11 n) {
 a = ((a \% m) + m) \% m;
 d = ((d % m) + m) % m;
  return n * a + d * sumsq(n) - m * floor_sum(a, d, m, n);
```

typedef vector <T> VT;

#### 46. Suma prefiksowa funkcji multiplikatywnej

```
ca2c8162f93b86c71e8efba7f759ce9a
          Prefix sum of (not necessarily?) multiplicative functions:
               p_f: the prefix sum of f(x) (1 <= x <= th).
               p_g: the prefix sum of q(x) (0 <= x <= N).
               p_c: the prefix sum of (f * g)(x) (0 \le x \le N).
               th : the thereshold, generally should be n ^{\circ} (2 / 3).
struct prefix_mul
        typedef ll (*func) (ll);
        func p_f, p_q, p_c;
       ll n, th, inv;
       unordered_map <11,11> mem;
       prefix_mul(func p_f, func p_g, func p_c) : p_f(p_f), p_g(p_g), p_c(p_c) {}
       ll calc(ll x)
               if (x <= th) return p_f(x);</pre>
               auto d = mem.find(x);
               if (d != mem.end()) return d->second;
               11 \text{ ans} = 0;
                for (11 i = 2, 1a; i \le x; i = 1a + 1) {
                        la = x / (x / i);
                        ans = ans + (p_g(la) - p_g(i - 1)) * calc(x / i);
               ans = (p_c(x) - ans) / inv;
                return mem[x] = ans;
       ll solve(ll n, ll th)
                if (n <= 0) return 0;
               prefix mul::n = n;
               prefix_mul::th = th;
               inv = p q(1);
               return calc(n);
};
```

#### 47. Xor- And- Convolution

```
aa28e1d6bd7e255bafcc3ae14d55a779
typedef long long T;
int n, invn;
T mod;
```

```
void convolve(T *tab, vector<vector<int>> M) {
    for (int i = 1; i < n; i *= 2)
        for (int j = 0; j < n; j += 2 * i)
            for(int k = 0; k < i; k++)
                T \& a = tab[j + k], \& b = tab[j + i + k];
                T = (M[0][0] * a + M[0][1] * b) * mod, nb = (M[1][0] * a + M[1][1] *

→ b) % mod;

                a = na; b = nb;
void mult(T *a, T *b, T *c) {
    for (int i = 0; i < n; i++)</pre>
       c[i] = a[i] * b[i] % mod;
//tych funkcji uzywac
//N MUSI byc potega dwojki
void init(int N, T Mod) {
   mod = Mod;
    n = N;
    invn = inv(n); //Trzeba sobie napisać funkcję inv, która liczy odwrotność modulo
//UWAGA modyfikuje tablice a i b, robic kopie jesli potrzebne pozniej
void xor conv(T *a, T *b, T *res) {
    convolve(a, \{ \{ 1, 1 \}, \{ 1, -1 \} \});
    if(a != b) convolve(b, { { 1, 1 }, { 1, -1 } });
    mult(a, b, res);
    convolve(res, \{ \{ 1, 1 \}, \{ 1, -1 \} \});
    for(int i = 0; i < n; i++)</pre>
        res[i] = res[i] * invn % mod;
void and_conv(T *a, T *b, T *res) {
    convolve(a, { { 0, 1 }, { 1, 1 } });
   if(a != b) convolve(b, { { 0, 1 }, { 1, 1 } });
   mult(a, b, res);
   convolve(res, \{ \{ -1, 1 \}, \{ 1, 0 \} \});
void or_conv(T *a, T *b, T *res) {
    convolve(a, { { 1, 0 }, { 1, 1 } });
   if(a != b) convolve(b, { { 1, 0 }, { 1, 1 } });
   mult(a, b, res);
   convolve(res, { { 1, -1 }, { 0, 1 } });
                                      48. SIMPLEX
   2c86cc90476d4687f17c0061f9956edf
const double EPS = 1e-7;
typedef long double T;
```

```
typedef vector < int > vi;
#define FOR(i,n) for (int i = 0; i < (n); i++)
vector < VT > A; VT b, c, res; vi kt, N; int m;
inline void pivot(int k, int l, int e) {
        int x = kt[1]; T p = A[1][e];
        FOR(i, k) A[1][i] /= p; b[1] /= p; N[e] = 0;
        FOR(i, m) if(i != 1) b[i] -= A[i][e] * b[1], A[i][x] = A[i][e] * -A[1][x];
        FOR(j, k) if(N[j]) {
               c[j] -= c[e] * A[1][j];
                FOR(i, m) if(i != 1) A[i][j] -= A[i][e] * A[1][j];
        kt[1] = e; N[x] = 1; c[x] = c[e] * -A[1][x];
VT doit(int k) {
        VT res; T best;
        while(1) {
                int e = -1, 1 = -1; FOR(i, k) if(N[i] && c[i] > EPS) {e = i; break;}
                if(e == -1) break;
                FOR(i, m) if(A[i][e] > EPS && (1 == -1 || best > b[i] / A[i][e]))
                        best = b[ l = i ] / A[i][e];
                if (1 == -1)
                        return VT();
                pivot(k, l, e);
        res.resize(k, 0); FOR(i, m) res[kt[i]] = b[i];
        return res;
VT simplex(const vector < VT > &AA, const VT &bb, const VT &cc) {
        int n = AA[0].size(), k;
        m = AA.size(); k = n + m + 1; kt.resize(m); b = bb; c = cc; c.resize(n + m);
        A = AA; FOR(i, m) \{ A[i].resize(k); A[i][n + i] = 1; A[i][k - 1] = -1; kt[i] = n
        N = vi(k, 1); FOR(i, m) N[kt[i]] = 0;
        int pos = min_element(b.begin(), b.end()) - b.begin();
        if(b[pos] < -EPS) {</pre>
                c = VT(k, 0); c[k - 1] = -1; pivot(k, pos, k - 1); res = doit(k);
                if(res[k - 1] > EPS) return VT();
                FOR(i, m) if (kt[i] == k - 1) {
                        FOR(j, k-1) if(N[j] && (A[i][j] < -EPS || EPS < A[i][j])) {
                                pivot(k, i, j); break;
                c = cc; c.resize(k, 0); FOR(i, m) FOR(j, k) if(N[j]) c[j] -= c[kt[i]] *

→ A[i][i];

        res = doit(k - 1); if(!res.empty()) res.resize(n);
        return res;
```

#### 49. Subset Convolution

5ca2aa1714889771ee5c7e75a28c9a1d

```
typedef long long T;
const int B = 17, N = 1 << B;</pre>
const T mod = 1000000007;
T tA[B+1][N];
T tB[B+1][N];
T conv[B+1][N];
int cnt[N];
void sum_over_subset(T *a, T *b, int k = -1)
    for(int i = 0; i < N; i++)</pre>
        b[i] = (k == -1 | | k == cnt[i]) * a[i];
    for(int i = 0; i < B; i++)</pre>
        for(int j = 0; j < N; j++)</pre>
            if(j & (1 << i))
                b[j] += b[j ^ (1 << i)];
    for (int i = 0; i < N; i++)</pre>
        b[i] %= mod;
void mobius_transform(T *a, T b[B+1][N])
    for (int i = 0; i <= B; i++)</pre>
        sum_over_subset(a, b[i], i);
void subset_conv(T *a, T *b, T *res)
    for(int i = 0; i < N; i++)</pre>
        cnt[i] = __builtin_popcount(i);
    mobius_transform(a, tA);
    if(a != b)
        mobius_transform(b, tB);
    else copy(tA[0], tA[0] + (B+1)*N, tB[0]);
    for (int i = 0; i <= B; i++)</pre>
        for (int j = 0; j < N; j++)
            conv[i][i] = 0;
    for(int i = 0; i <= B; i++)</pre>
        for (int i = 0; i + i \le B; i++)
             for (int k = 0; k < N; k++)
                 conv[i+j][k] += tA[i][k] * tB[j][k];
                 if(conv[i+j][k] >= (1LL << 62))
                     conv[i+i][k] %= mod;
    for(int i = 0; i <= B; i++)</pre>
        for (int j = 0; j < N; j++)
            conv[i][j] %= mod;
            if(cnt[j] & 1)
                 conv[i][j] *= -1;
```

```
for(int i = 0; i <= B; i++)</pre>
        sum_over_subset(conv[i], conv[i]);
    for(int i = 0; i < N; i++)</pre>
        res[i] = conv[cnt[i]][i];
       if(cnt[i] & 1)
            res[i] \star = -1;
                                     50 FAST RAND
   db8d516d7e531668d3c8c57d1703abe6
int getInt(int a = INT_MIN, int b = INT_MAX){
        static mt19937
        → rng(chrono::steady_clock::now().time_since_epoch().count());
        return uniform_int_distribution <int> (a, b) (rng);
}
long long getLL(long long a = LLONG_MIN, long long b = LLONG_MAX) {
        static mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());
        return uniform_int_distribution <long long int> (a, b) (rng);
double getReal(double a = 0.0, double b = 1.0) {
        static mt19937
        → rng(chrono::steady_clock::now().time_since_epoch().count());
        return uniform_real_distribution <double> (a, b) (rng);
                                    51. Yarın Sieve
   dbd7f9d81b80e583a4cc973cf5149222
// This is the famous "Yarin sieve", for use when memory is tight.
#define MAXSIEVE 100000000 // All prime numbers up to this
#define MAXSIEVEHALF (MAXSIEVE/2)
#define MAXSORT 5000 // sgrt (MAXSIEVE) /2
char a[MAXSIEVE/16+2];
\#define\ isprime(n)\ (a[(n)>>1)&(1<<(((n)>>1)&7)))\ //\ Works\ when\ n\ is\ odd
int main()
  int i, j;
  memset(a, 255, sizeof(a));
  a[0]=0xFE;
  for(i=1;i<MAXSORT;i++)</pre>
  if (a[i>>3]&(1<<(i&7)))</pre>
  for(j=i+i+i+1; j<MAXSIEVEHALF; j+=i+i+1)</pre>
```

```
a[j>>3] &=\sim (1<<(j&7));
 //isprime(n) - mowi czy n pierwsze
                              52. Green Hackenbush
   cc6ba075e6892c806d17df34ace68424
struct GreenHack {
   /* Set ground should be called before adding edges! */
   int n;
   vector <int> id;
   vector <vector <int> > G;
   int T;
   vector <int> low, pre;
   GreenHack(int _n) : n(_n) {
       id.resize(n + 1);
       iota(id.begin(), id.end(), 0);
       G.resize(n + 1);
       low.resize(n + 1);
       pre.resize(n + 1);
   void set_ground(int u) {
       id[u] = 0;
    void add edge(int u, int v) {
       G[id[u]].push_back(id[v]);
       if (id[u] != id[v]) {
           G[id[v]].push_back(id[u]);
   int dfs(int u, int p) {
       int ans = 0;
       pre[u] = low[u] = ++T;
       for (const int &v: G[u]) {
           if (v == p) {
               p = -1;
                continue;
           if (pre[v] == 0) {
               int res = dfs(v, u);
               low[u] = min(low[u], low[v]);
```

```
if (low[v] > pre[u]) {
                   ans ^= res + 1;
                } else {
                   ans ^= res ^ 1;
            } else if (pre[v] < pre[u]) {</pre>
               low[u] = min(low[u], pre[v]);
            } else {
               ans ^= 1;
       return ans;
   int run() {
       return dfs(0, -1);
};
                            53. Red Blue Hackenbush
   aa85012a35331797c1b1f69c95f26b77
struct Surreal {
   int value = 0, offset = 0;
   set <int> powers;
    void clear() {
       value = offset = 0;
       powers.clear();
    int size() {
        return powers.size();
   int sign() {
       const int tmp = 2 * value + !powers.empty();
        return tmp < 0 ? -1 : (tmp > 0);
   int add power(int power) {
       while (power) {
            if (!powers.count(power - offset)) {
```

powers.insert(power - offset);

powers.erase(power - offset);

break;

--power;

```
return !power;
    void operator += (const Surreal &v) {
        value += v.value;
        for (const int &power: v.powers) {
            value += add_power(power + v.offset);
    void divide(int power) {
        offset += power;
        int to_add = 0;
        for (int i = 0; i < power; ++i) {</pre>
            if (value & 1) {
               to_add += add_power(power - i);
            value >>= 1;
        value += to_add;
    void get_next(int t) {
        int power = max(0, -t * value);
       value += t * (power + 1);
        if (value == -1 || (value == 1 && powers.empty())) {
            power++;
            value += t;
        divide (power);
};
struct RedBlueHack {
    /* Weights on edges should be -1 or 1 */
    vector <int> id;
    vector <Surreal> ans;
    vector <vector <pair <int, int> > > G;
    RedBlueHack(int _n) : n(_n) {
        id.resize(n + 1);
        iota(id.begin(), id.end(), 0);
        ans.resize(n + 1);
```

```
G.resize(n + 1);
    void add_edge(int u, int v, int t) {
       G[u].push_back({v, t});
       G[v].push_back({u, t});
    void dfs(int u, int p)
        ans[u].clear();
        for (auto &[v, w]: G[u])
            if (v == p) {
                continue;
            dfs(v, u);
            ans[id[v]].get_next(w);
            if (ans[id[u]].size() < ans[id[v]].size()) {</pre>
                swap(id[u], id[v]);
            ans[id[u]] += ans[id[v]];
    }
    int run() {
       dfs(1, 1);
        return ans[id[1]].sign();
};
```

192bcedab9c2be0ca62e73bc72c47976

11 phi = power(p, cnt) - power(p, cnt - 1);

#### 54. DISTINCT K-TH POWERS

```
11 u = phi / \underline{gcd(k, phi)};
 return u + f(p, cnt - k, k);
// returns the number of distinct values of (a^k % n) over all integers a
ll vo(ll k, ll n) {
 11 \text{ ans} = 1;
 for (11 i = 2; i * i <= n; i++) {</pre>
   if (n % i == 0) {
     int cnt = 0;
     while (n % i == 0) {
       cnt++;
       n /= i;
     ans \star = f(i, cnt, k);
 if (n > 1) ans *= f(n, 1, k);
 return ans;
                               55. Max Convolution
   7c138e61281f52dde1cb2939c0dbd2ba
// a[i + 1] - a[i] >= a[i] - a[i - 1] -> convex
// b[i + 1] - b[i] >= b[i] - b[i - 1] -> convex
// compute ans(i + j) = max(a(i) + b(j))
vector<int> multiply(vector<int> a, vector<int> b) {
 int n = a.size() - 1, m = b.size() - 1;
 vector<int> ans(n + m + 1);
 int sum = a[0] + b[0]; ans[0] = sum;
 int 1 = 0, r = 0;
 while (1 < n && r < m) {
   if (a[l+1] - a[l] > b[r+1] - b[r]) {
     sum += a[1 + 1] - a[1];
     1++;
   } else {
     sum += b[r + 1] - b[r];
     r++;
   ans[l + r] = sum;
 while (1 < n) sum += a[1 + 1] - a[1], 1++, ans[1 + r] = sum;
 while (r < m) sum += b[r + 1] - b[r], r++, ans[1 + r] = sum;
 return ans;
                                 56. Liczba partycji
```

b9fc1294ad577760b8e822e37ab9fa3b

```
vector<mint> solve(int n) {
  vector < mint > ans(n + 1);
  vector<pair<int, int>> qp; // (siqn, generalized pentagonal numbers)
  qp.emplace_back(0, 0);
  for (int i = 1; gp.back().second <= n; i++) {</pre>
   gp.emplace_back(i % 2 ? 1 : -1, i * (3 * i - 1) / 2);
   gp.emplace_back(i % 2 ? 1 : -1, i * (3 * i + 1) / 2);
  ans[0] = 1;
  for (int i = 1; i <= n; i++) {</pre>
   for (auto it : qp) {
     if (i >= it.second) ans[i] += ans[i - it.second] * it.first;
      else break;
  return ans;
                                  57. Aho-Corasick
   396bcb66ecd505afb85059b4568b6ef7
struct aho {
        static const int A = 26;
        static const int off = 'a';
        int n;
       vector <int> par, fail;
        vector <char> last;
        vector <vector <int> > nxt;
                                           //mozna mape przy duzym alfabecie
        aho() {
                n = 0;
                par = \{ 0 \};
                nxt = { vector < int > (A, -1) };
               last = \{0\};
                fail = { };
        int create(int parent, char letter) {
                nxt.push_back(vector <int> (A, -1));
                par.push back(parent);
                last.push_back(letter);
                return v;
        void add(string &s) {
                int cur = 0;
                for(auto c: s) {
                        if(nxt[cur][c - off] == -1)
```

```
nxt[cur][c - off] = create(cur, c);
                       cur = nxt[cur][c - off];
       void make() {
               queue <int> Q;
               Q.push(0);
                fail.resize(n + 1);
               while(!Q.empty()) {
                       int u = Q.front();
                       Q.pop();
                        fail[u] = u ? nxt[fail[par[u]]][last[u]] : 0;
                        for (int i = 0; i < A; ++i)</pre>
                               if(nxt[u][i] == -1)
                                       nxt[u][i] = u ? nxt[fail[u]][i] : 0;
                               else
                                       Q.push(nxt[u][i]);
};
                           58. Drzewo Palindromiczne
   44dcb230d61198956a6f35d6950e6316
struct node {
   int len;
   unordered_map<char, node*> ch;
   node *suf;
   char *pos = 0;
   node(int l = 0) : len(l), suf(this) { }
struct eertree {
   node *r0, *r1;
   vector<node*> nodes;
   eertree() {
       r0 = new node(0);
       r1 = new node(-1);
       r0->suf = r1;
   vector<node*> update(char *c) {
       vector<node*> ans;
       node *cur = r1;
       while(*c) {
            char x = *c;
            while (x != c[-1 - cur->len])
               cur = cur->suf;
           if(!cur->ch.count(x)) {
```

#### 59. ALL SUBSTRING LCS

c5282ef0a4b5607d9f10834d57249cee

```
const int N = 2002;
int f[N][N], q[N][N];
void solve(string s, string t)
    int n = s.size(), m = t.size();
    s = "#" + s;
    t = "#" + t;
    for (int i = 1; i <= m; ++i) f[0][i] = i;</pre>
    for (int i = 1; i <= n; ++i) {</pre>
        for (int j = 1; j \le m; ++j) {
            if (s[i] == t[j]) {
                f[i][j] = q[i][j-1];
                q[i][j] = f[i - 1][j];
            else {
                f[i][j] = max(f[i-1][j], g[i][j-1]);
                g[i][j] = min(g[i][j-1], f[i-1][j]);
    for (int i = 1; i <= m; ++i) {</pre>
        for (int j = i, ans = 0; j <= m; ++j) {</pre>
            if (i > f[n][j]) ++ans;
            //ans is lcs of s and t[i, j]
```

```
60. Cyclic LCS
   95d7ef1685641b9253ddd64e1bbe322c
const int N = 2010;
/*Cyclic Longest Common Subsequence
maximum of lcs(any cyclic shift of s, any cyclic shift of t)
O(nm) */
int dp[N * 2][N], from[N * 2][N];
int solve(string s, string t) {
   int n = s.size(), m = t.size();
    auto eq = [&](int a, int b) {
        return s[(a - 1) % n] == t[(b - 1) % m];
    dp[0][0] = from[0][0] = 0;
    for (int i = 0; i <= n * 2; ++i) {
       for (int j = 0; j \le m; ++j) {
            dp[i][j] = 0;
            if (j && dp[i][j - 1] > dp[i][j]) {
                dp[i][j] = dp[i][j - 1];
                from[i][j] = 0;
            if (i && j && eq(i, j) && dp[i - 1][j - 1] + 1 > dp[i][j]) {
                dp[i][j] = dp[i - 1][j - 1] + 1;
                from[i][j] = 1;
            if (i && dp[i - 1][j] > dp[i][j]) {
                dp[i][j] = dp[i - 1][j];
                from[i][i] = 2;
    int ret = 0;
    for (int i = 0; i < n; ++i) {
       ret = max(ret, dp[i + n][m]);
        // re-root
        int x = i + 1, y = 0;
        while (y \le m \&\& from[x][y] == 0) ++y;
        for (; v <= m && x <= n * 2; ++x) {
            from[x][y] = 0, --dp[x][m];
            if (x == n * 2) break;
            for (; v <= m; ++v) {</pre>
                if (from[x + 1][y] == 2) break;
                if (y + 1 \le m \&\& from[x + 1][y + 1] == 1) {
                    break:
```

```
return ret;
                                       61. KMP
   d89d819c3e88f1fbed457befb8d21f89
vector<int> KMP(char *s) {
   int n = strlen(s + 1);
   vector<int> T(n + 1, 0);
    for (int i = 2; i <= n; i++) {</pre>
       int t = T[i - 1];
       while (t > 0 \&\& s[i] != s[t + 1]) {
           t = T[t];
       if (s[i] == s[t + 1])
           t++;
       T[i] = t;
    return T;
                             62. MINIMAL CYCLIC SHIFT
   2b037c0badead7f860b57ca5d7b01414
string min_cyclic_string(string s) {
   s += s;
   int n = s.size();
   int i = 0, ans = 0;
   while (i < n / 2) {
       ans = i;
       int j = i + 1, k = i;
       while (j < n \&\& s[k] <= s[j]) {
           if (s[k] < s[j])
               k = i;
           else
               k++;
           j++;
        while (i <= k)</pre>
           i += j - k;
    return s.substr(ans, n / 2);
                                    63. Manacher
```

63. ib9d984af25d554d8831d1e2746d0db3b

```
//wynik dla palindromu parzystego o środku między pozycjami i, i + 1 znajduje się 2 * i
\hookrightarrow + 1
//wynik dla palindromu nieparzystego o środku na pozycji i znajduje się w 2 * i
struct manacher{
      int n;
       string s;
       vector <int> rad;
       void init(int N, string &in) {
              n = N;
              s.resize(n + n);
              rad.resize(n + n);
              for(int i = 0; i < n + n; ++i)</pre>
                     s[i] = '#';
                                              //znak niewystepujacy w tekscie
              for(int i = 0; i < n; ++i)</pre>
                     s[i + i] = in[i];
       void go() {
              int m = 2 * n - 1;
              int x = 0;
              for(int i = 1; i < m; ++i){</pre>
                     int &r = rad[i] = 0;
                     if(i <= x + rad[x])
                             r = min(rad(x + x - i), x + rad(x) - i);
                      while(i - r - 1 >= 0 && i + r + 1 < m && s[i - r - 1] == s[i + r
                     if(i + r >= x + rad[x])
                            x = i;
              for(int i = 0; i < m; ++i){</pre>
                     if(i - rad[i] == 0 || i + rad[i] == m - 1)
                             ++rad[i];
                     rad[i] /= 2;
};
                            64. Manacher Radecki
   bec3c3aa22e11af07ca65aba34f8f78e
// @s[0..n-1] - napis długości @n.
// @r[0..2n-21 - tablica promieni palindromów.
//s:abaabaacaabbbbaacac
void Manacher(const char* s, int n) {
 for (int i = 0, m = 0, k = 0, p = 0; i < 2 * n - 1; m = i + + - 1) {
   while (p < k \text{ and } i / 2 + r[m] != k)
     r[i++] = min(r[m--], (k + 1 - p++) / 2);
```

**while** (k + 1 < n and p > 0 and s[k + 1] == s[p - 1])

```
k++, p--;
r[i] = (k + 1 - p++) / 2;
```

#### 65. Tablica sufiksowa

```
struct SA {
        char s[N];
```

4e725887a0b7d6b22f995808e00ed07c

```
int n, sa[N], x[N], y[N], lcp[N], cnt[N];
bool dif(int a, int b, int k) {
        return y[a] != y[b] || (a + k <= n ? y[a + k] : -1) != (b + k <= n ? y[b
        \hookrightarrow + k] : -1);
void build() {
       n = strlen(s + 1);
        int A = 26;
       rep(i, 1, n) cnt[x[i] = s[i] - 'a' + 1]++;
        rep(i, 1, A) cnt[i] += cnt[i - 1];
        per(i, 1, n) sa[cnt[x[i]]--] = i;
        for (int k = 1; k < n; k *= 2) {
                int p = 0:
                rep(i, n - k + 1, n) y[++p] = i;
                rep(i, 1, n) if (sa[i] > k) y[++p] = sa[i] - k;
                rep(i, 1, A) cnt[i] = 0;
                rep(i, 1, n) cnt[x[i]]++;
                rep(i, 1, A) cnt[i] += cnt[i - 1];
                per(i, 1, n) sa[cnt[x[y[i]]]--] = y[i];
                swap(x, y);
                A = x[sa[1]] = 1;
                rep(i, 2, n) x[sa[i]] = (A += dif(sa[i - 1], sa[i], k));
                if (n == A) break;
        }
        int j = 0;
        rep(i, 1, n) {
                if (x[i] == n) {
                        lcp[x[i]] = 0;
                        continue;
                int nxt = sa[x[i] + 1];
                while (\max(i, nxt) + j \le n \&\& s[i + j] == s[nxt + j]) j++;
                lcp[x[i]] = j;
                j = max(j - 1, 0);
```

```
};
                           66. Tablica sufiksowa (KMR)
   5d82fcf670689a48eedc3c4ee85342d5
int n, logn;
vector<int> suf;
vector<vector<int>> kmr_tab;
void init(string input) {
    n = input.size();
    logn = 0;
    kmr_tab.clear();
    vector<int> val(input.begin(), input.end());
    vector<long long> pairs(n);
    suf.resize(n);
    for(int i = 0; i < n; i++)</pre>
        suf[i] = i;
    for (int t = 1; logn++, t *= 2) {
        //zakomentowac jesli niepotrzebne, dodaje n log n do pamieci
        kmr_tab.push_back(val);
        if(t >= n) break;
        for (int i = 0; i < n; i++) {</pre>
            pairs[i] = (i + t < n ? val[i+t] : 0) | ((long long)val[i] << 30LL);</pre>
        sort(suf.begin(), suf.end(),
            [&] (int a, int b) { return pairs[a] < pairs[b]; });</pre>
        int k = 0;
        for(int i = 0; i < n; i++) {</pre>
            if(i == 0 || pairs[suf[i]] != pairs[suf[i-1]])
                k++;
            val[suf[i]] = k;
int common_prefix(int a, int b) {
    int v = 0;
    for(int i = logn; i >= 0; i--) {
        if(a == n || b == n) break;
        if(kmr tab[i][a] == kmr tab[i][b]) {
            int k = 1 << i;
            a += k;
            b += k;
            v += k;
    return v;
```

vector<vector<int>> q;

#### 67. Tablica sufiksowa

```
5fa11091fb1d8efef4a3cb97123672ae
inline bool leq(int a1, int a2, int b1, int b2) {
        return(a1 < b1 || a1 == b1 && a2 <= b2);
inline bool leq(int a1, int a2, int a3, int b1, int b2, int b3) {
        return(a1 < b1 || a1 == b1 && leq(a2,a3, b2,b3));
static void radixPass(int* a, int* b, int* r, int n, int K) {
        int* c = new int[K + 1];
        for (int i = 0; i <= K; i++) c[i] = 0;</pre>
        for (int i = 0; i < n; i++) c[r[a[i]]]++;</pre>
        for (int i = 0, sum = 0; i <= K; i++) {
                int t = c[i]; c[i] = sum; sum += t;
        for (int i = 0; i < n; i++)</pre>
                b[c[r[a[i]]]++] = a[i];
        delete [] c:
// require s[n]=s[n+1]=s[n+2]=0, n>=2
void suffixArray(int* s, int* SA, int n, int K) {
                                                         //ma problemy dla n = 1
        int n0=(n+2)/3, n1=(n+1)/3, n2=n/3, n02=n0+n2;
        int* s12 = new int[n02 + 3]; s12[n02] = s12[n02+1] = s12[n02+2] = 0;
        int* SA12 = new int[n02 + 3]; SA12[n02]=SA12[n02+1]=SA12[n02+2]=0;
        int* s0 = new int[n0];
        int* SA0 = new int[n0];
        for (int i=0, j=0; i < n+(n0-n1); i++) if (i%3 != 0) s12[j++] = i;
        radixPass(s12, SA12, s+2, n02, K);
        radixPass(SA12, s12, s+1, n02, K);
        radixPass(s12 , SA12, s , n02, K);
        int name = 0, c0 = -1, c1 = -1, c2 = -1;
        for (int i = 0; i < n02; i++) {</pre>
                if (s[SA12[i]] != c0 || s[SA12[i]+1] != c1 || s[SA12[i]+2] != c2){
                        name++; c0 = s[SA12[i]]; c1 = s[SA12[i]+1]; c2 = s[SA12[i]+2];
                if (SA12[i] % 3 == 1)
                        s12[SA12[i]/3] = name;
                        s12[SA12[i]/3 + n0] = name;
        if (name < n02) {
                suffixArray(s12, SA12, n02, name);
                for (int i = 0; i < n02; i++) s12[SA12[i]] = i + 1;
        else
                for (int i = 0; i < n02; i++) SA12[s12[i] - 1] = i;</pre>
        for (int i=0, j=0; i < n02; i++) if (SA12[i] < n0) s0[j++] = 3*SA12[i];</pre>
        radixPass(s0, SAO, s, n0, K);
        for (int p=0, t=n0-n1, k=0; k < n; k++) {
                #define GetI() (SA12[t] < n0 ? SA12[t] *3+1: (SA12[t] - n0) * 3 + 2)
```

```
int i = GetI();
                int j = SA0[p];
                if(SA12[t] < n0 ? leq(s[i], s12[SA12[t] + n0], s[j], s12[j/3]) :</pre>
                \hookrightarrow leg(s[i],s[i+1],s12[SA12[t]-n0+1], s[j],s[j+1],s12[j/3+n0])){
                        SA[k] = i; t++;
                        if (t == n02)
                                for (k++; p < n0; p++, k++) SA[k] = SA0[p];</pre>
                else
                        SA[k] = j; p++;
                        if (p == n0)
                                            for (k++; t < n02; t++, k++) SA[k] = GetI();
        delete [] s12; delete [] SA12; delete [] SA0; delete [] s0;
void lcp(int *s, int *SA, int *ans, int n) {
       int* tmp = new int[n];
       int last = 0;
        for (int i = 0; i < n; ++i)</pre>
                tmp[SA[i]] = i;
        for (int i = 0; i < n; ++i) {
                if(last)
                                --last:
                if(tmp[i] == 0)
                                       continue;
                int who = SA[tmp[i] - 1];
                while(s[i + last] == s[who + last])
                        ++last;
                ans[tmp[who]] = last;
        delete [] tmp;
                               68 AUTOMAT SUFIKSOWY
   9d23628122d8f32efb9e4496ad25441d
// len -> largest string length of the corresponding endpos-equivalent class
// link -> longest suffix that is another endpos-equivalent class.
// firstpos -> 1 indexed end position of the first occurrence of the largest string of

→ that node

// minlen(v) -> smallest string of node v = len(link(v)) + 1
// terminal nodes -> store the suffixes
struct SuffixAutomaton {
    struct node {
        int len, link, firstpos;
        map<char, int> nxt;
    int sz. last:
    vector<node> t;
    vector<int> terminal:
    vector<long long> dp;
```

```
SuffixAutomaton() {}
SuffixAutomaton(int n) {
   t.resize(2 * n); terminal.resize(2 * n, 0);
   dp.resize(2 * n, -1); sz = 1; last = 0;
   g.resize(2 * n);
   t[0].len = 0; t[0].link = -1; t[0].firstpos = 0;
void extend(char c) {
   int p = last;
   if (t[p].nxt.count(c)) {
       int q = t[p].nxt[c];
       if (t[q].len == t[p].len + 1) {
           last = q;
            return;
       int clone = sz++;
       t[clone] = t[q];
       t[clone].len = t[p].len + 1;
       t[q].link = clone;
       last = clone;
       while (p != -1 \&\& t[p].nxt[c] == q) {
           t[p].nxt[c] = clone;
           p = t[p].link;
       return;
   int cur = sz++;
   t[cur].len = t[last].len + 1;
   t[cur].firstpos = t[cur].len;
   p = last;
    while (p != -1 \&\& !t[p].nxt.count(c)) {
       t[p].nxt[c] = cur;
       p = t[p].link;
   if (p == -1) t[cur].link = 0;
   else {
       int q = t[p].nxt[c];
       if (t[p].len + 1 == t[q].len) t[cur].link = q;
           int clone = sz++;
           t[clone] = t[q];
           t[clone].len = t[p].len + 1;
           while (p != -1 \&\& t[p].nxt[c] == q) {
               t[p].nxt[c] = clone;
                p = t[p].link;
           t[q].link = t[cur].link = clone;
   last = cur:
void build tree() {
```

```
for (int i = 1; i < sz; i++) q[t[i].link].push_back(i);</pre>
    void build(string &s) {
        for (auto x: s) {
            extend(x);
            terminal[last] = 1;
       build tree();
   long long cnt(int i) { //number of times i-th node occurs in the string
        if (dp[i] != -1) return dp[i];
       long long ret = terminal[i];
        for (auto &x: g[i]) ret += cnt(x);
        return dp[i] = ret;
};
                                  69. TABLICE PREF
   8319c71e02aa9ee6d697aa03dc9db4f1
//numeracja od zera
struct pref{
       int n;
        string s;
        vector <int> rad;
        void init(int N, string &in) {
               n = N;
                s = in;
                rad.resize(n);
        void go() {
                int x = 0; rad[0] = -1;
                for(int i = 1; i < n; ++i){</pre>
                        int &r = rad[i] = -1;
                        if(i <= x + rad[x])
                                r = min(rad[i - x], x + rad[x] - i);
                        while (i + r + 1 < m \&\& s[r + 1] == s[i + r + 1])
                        if(i + r >= x + rad[x])
                                x = i;
};
```

#### 70. Wzory

Sumy:  $\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ ,  $\sum_{k=0}^{n} k^3 = \frac{n^2(n+1)^2}{4}$ ,  $\sum_{i=n}^{m} {i \choose i} = {m+1 \choose n+1}$ ,  $\sum_{i=0}^{k} {n \choose i} {n \choose k-i} = {n+m \choose k}$ .

Całki:  $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b|$ ,  $\int \tan x dx = -\ln|\cos x|$ ,  $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \arctan \frac{x}{a}$ ,  $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln|\frac{x-a}{x+a}|$ ,  $\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin \frac{x}{a}$ ,  $\int \frac{1}{\sqrt{x^2+q}} dx = \ln|x+\sqrt{x^2+q}|$ ,  $\int a^x dx = \frac{a^x}{\ln a}$ .

Liczby Catalana:  $C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}$ ,  $C_n = \frac{1}{n+1} {2n \choose n} = {2n \choose n} - {2n \choose n+1}$ ,  $C_{n+1} = C_n \frac{4n+2}{n+2}$ , 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 74290, ...

Suma dzielników:  $\sigma(n) = \sigma(p_1^{\alpha_1}, \dots, p_k^{\alpha_k}) = \prod_{i=1}^k \frac{p_i^{\alpha_i+1}-1}{p_i-1}$ .

Funkcja Eulera:  $\phi(p^k)=p^k-p^{k-1}, \phi(ab)=\phi(a)\phi(b)$  dla  $a\perp b, \sum_{d\mid n}\phi(d)=n.$ 

Funckja Mobiusa: 1 dla liczb bezkwadratowych z parzystą liczbą czynników, -1 – nieparzystą, 0 dla liczb nie bezkwadratowych. Inaczej  $\mu(n)$  to suma pierwotnych pierwiastków z jedności stopnia n,  $\sum_{d|n} \mu(d) = 0$  dla n > 1.

Związek między  $\phi$  a  $\mu$ :  $\phi(n) = \sum_{d|n} \mu(d) \frac{n}{d}$ .

**Problem znaczków pocztowych:** Niech a, b względnie pierwsze. Jest dokładnie  $\frac{1}{2}(a-1)(b-1)$  liczb, których nie da się zapisać w postaci  $ax + by(x, y \le 0)$ . Największa z nich to (a-1)(b-1) - 1.

**Lemat Burnside'a:** Liczba orbit grupy G na zbiorze X:  $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X_g|$ , gdzie  $X_g = \{x \in X : g(x) = x\}$ . ("Średnia liczba punktów stałych")

Metoda Simpsona:  $\int_a^{b=a+2h} f(x)dx = \frac{b-a}{6}(f(a)+4f(a+h)+f(b))+O(h^5f^{(4)}(\xi))$ . Liczby Stirlinga pierwszego rodzaju: Opisują liczbę sposobów na rozmieszczenie n liczb w k cyklach,  $\begin{bmatrix} n \\ k \end{bmatrix} = (n-1) \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}$ .

Liczby Stirlinga drugiego rodzaju: Opisują liczbę sposobów podziału zbioru n elementowego na k niepustych podzbiorów,  $\binom{n}{k} = k\binom{n-1}{k} + \binom{n-1}{k-1}$ ,  $\binom{n}{k} = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$ .

Liczby Bella: Liczba podziałów zbioru n elementowego,  $\mathcal{B}_{n+1} = \sum_{k=0}^{n} {n \choose k} \mathcal{B}_k$ .

**Nieuporządkowania:** Permutacje bez elementu stałego,  $!n = \lfloor \frac{n!}{e} + \frac{1}{2} \rfloor$ .

Liczby harmoniczne:  $H_n = \sum_{k=1}^n \frac{1}{k}, \frac{1}{2n+1} < H_n - \ln n - \gamma < \frac{1}{2n}, \gamma = 0.57721\,56649\,01532\,86060\,65120\dots$ 

Trygonometria:  $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$ ,  $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$ , tw.  $\sin(\alpha)\sin(\beta) = \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} = 2R$ , tw. cosinusów:  $c^2 = a^2 + b^2 - 2ab\cos(\gamma)$ ,  $R = \frac{abc}{4S}$ ,  $r = \frac{2S}{a+b+c}$ ,  $S = \sqrt{s(s-a)(s-b)(s-c)}$ ,

Wzór Picka:  $P = W + \frac{B}{2} - 1$  gdzie P – pole, W — wewnętrzne, B – brzegowe.

gdzie S – pole trójkąta,  $s=\frac{a+b+c}{2}, r, R$  – promień okręgu wpisanego/opisanego.

Reguła Warnsdorffa obchodzenia skoczkiem szachownicy: W każdym kroku idź na pole, z którego można zrobić najmniejszą liczbę ruchów do nieodwiedzonych pól.

Optymalizacja Knutha:  $dp[i][j] = \min_{i < k < j} \{dp[i][k] + dp[k][j]\} + C[i][j]$ , potrzebujemy  $opt[i][j-1] \le opt[i][j] \le opt[i+1][j]$ , gdzie opt[i][j] daje najmniejsze optymalne k dla dp[i][j], wystarcza też  $C[a][c] + C[b][d] \le C[a][d] + C[b][c]$  i  $C[b][c] \le C[a][d]$ , dla wszytskich  $a \le b \le c \le d$ .

Pokrycie wierzchołkowe i zbiór niezależny: Niech M,C,I – maksymalne skojarzenie, minimalne pokrycie wierzchołkowe i maksymalny zbiór niezależny, wtedy  $|M| \leq |C| = N - |I|$ , równość zachodzi dla grafów dwudzielnych. Dodatkowo  $C^c = I$  (zawsze). Znajdowanie C,I (dla grafu dwudzielnego (A,B)): łączymy źródło z wierzchołkami z A, wierzchołki z B z ujściem (przepustowość taka jak wagi wierzchołków lub 1 dla nieważonego), krawędzie między A i B mają przepustowość  $\infty$ . Znajdujemy minimalne cięcie (S,T). Wtedy  $C = (A \cap T) \cup (B \cap S)$  i  $I = (A \cap S) \cup (B \cap T)$ .

Macierz sąsiedztwa a liczba drzew spinających: Niech macierz  $T = [t_{ij}]$ , gdzie  $t_{ij}$  to liczba krawędzi z wierzchołka i do j dla  $i \neq j$ ,  $t_{ii} = -\text{deg}(i)$ . Liczba drzew spinających jest równa wyznacznikowi macierzy T po usunięciu k-tego wiersza i k-tej kolumny (k dowolne). Uwaga: dla niespójnych odpalać osobno dla każdej spójnej składowej.

**Macierz a perfect matching:** Tutte matrix:  $A_{ij} = x_{ij}$  if  $(i, j) \in E$  and  $i < j, -x_{ij}$  if i > j, 0 if there is no edge.

**Tw.** Erdősa-Gallai: Ciąg  $d_1, d_2, ..., d_n$   $(n-1 \ge d_1 \ge \cdots \ge d_n \ge 0)$  jest ciągiem stopni wierzchołków pewnego nieskierowanego grafu prostego  $\iff 2|\sum d_i$  i  $(\forall k \in \{1, ..., n-1\})\sum_{i=1}^k d_i \le k(k-1) + \sum_{i=k+1}^n \min(k, d_i).$ 

**Liczby Bernoulliego:**  $B_0 = 1$ ;  $\sum_{k=0}^{m} {m+1 \choose k} B_k = 0$ ;  $1, \frac{-1}{2}, \frac{1}{6}, 0, \frac{-1}{30}, ...$ ;  $\sum_{v=1}^{n} v^k = \frac{1}{k+1} \sum_{j=0}^{k} {k+1 \choose j} B_j n^{k+1-j}$ 

**Dni tygodnia:** 01.01.1600 – sobota, 01.01.1900 – poniedziałek, 13.06.2042 – piątek, 01.04.2008 – wtorekm 31.12.1999 - piątek, 01.01.3000 – środa, 04.04.2019 – czwartek (dzień finałów).