Catastrophe Insurance and Solvency Regulation

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Abstract

This paper provides a welfare analysis of solvency regulation for catastrophe insurance. In our model, we characterize catastrophic risks by assuming that claims are correlated across policyholders. Because insurers cannot commit to a specific level of capital, solvency regulation can generate welfare gains. We show that the best solvency regulation restores the best insurance contract and features some insurer default in catastrophic states. Numerical simulations highlight that the optimal solvency regulation depends on risk lines and capital costs, and that a solvency regulation imposing constraints on insurers' expected shortfall is usually more valuable for policyholders than a solvency regulation imposing a maximal default rate.

Keywords: insurance, catastrophic risk, default risk, solvency regulation.

JEL classification: G22, G28, G52, H11, H84, Q54.

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1 Introduction

In modern economies, insurance companies have an important role to play in supplying risk coverage to risk-exposed agents to improve their welfare. Insurance contracts consist of premiums received by insurers ex ante in exchange for indemnities, transferred to insureds ex post, in case of damage. An important issue is that policyholders can be exposed to commitment issues or moral hazard from the insurer. Governments can deal with this issue by imposing capital requirements via solvency regulations. This raises the question of how stringent should solvency regulations be, which is a particularly sensitive matter for catastrophe insurance. Indeed, catastrophic risks (e.g., climate events, pandemics, or cyber-attacks) are characterized by significant loss dependence across risk-exposed agents. In this context, by contrast to non-catastrophic risks, insurance companies face highly uncertain liabilities and have to secure a high amount of costly capital to limit default on due indemnities. Thus, the trade-off between capital cost and insurer default determining the choice of solvency regulation is strongly affected by the presence of risk dependence across insureds. The present paper studies solvency regulation and insureds' welfare in the context of catastrophic risks, insurer default risks, and costly capital.

The main contributions of the paper are threefold. First, we develop an original catastrophe insurance model with explicit modeling of both the supply and demand sides of the insurance market. Second, we show that, in the presence of insurer commitment issues, the best solvency regulation restores the best insurance contract and features some insurer default risk. Third, we highlight that a solvency regulation imposing constraints on insurers' expected shortfall can be more valuable for insureds than a solvency regulation imposing a maximal default rate, in the sense that the former can be close to optimal for a larger set of risk lines than the latter.

The model features a continuum of agents exposed to a risk of loss. The fraction of agents incurring a loss is assumed to be a random variable, which constitutes the risk catastrophe dimension. Risk-exposed agents may purchase insurance contracts from insurance companies in competition. Because the fraction of affected insureds is uncertain, insurers face uncertain liabilities. We introduce standard measures of risks. The "Value at Risk" VaR_ε for an insurer is such that the probability to lose a higher amount than VaR_ε is ε. The "Expected Shortfall" ES_ε for an insurer is the expected loss, in monetary units, in the worst cases with cumulative probability ε. To limit default on due indemnities in catastrophic states (i.e., when a large fraction of policyholders are affected), insurers may purchase contingent capital supplied above the actuarially fair price (e.g., reinsurance or financial contracts). Moreover, insurers may divert some of their capital to pay dividends.

In a decentralized economy with commitment issues, the insurance market collapses because risk-exposed agents anticipate that insurers will not purchase contingent capital, will use premiums to pay dividends, and will always default on due indemnities. This calls for solvency regulation imposing some capital requirements. We show that the Pareto optimal insurance contract can be restored with a specific solvency regulation. Moreover, we show that this solvency regulation imposes a maximum default risk that is not null. If default is prohibited, insurers will have to secure costly contingent capital for catastrophic states and thus will supply insurance above the actuarially fair price, meaning that risk-exposed agents will purchase insurance with partial coverage. If some marginal default is allowed, insurance prices will fall, insurance purchases will increase, and the welfare gain generated for policyholders will dominate the welfare loss due to default in catastrophic states. Nevertheless, high default would damage policyholders' welfare by excessively decreasing indemnities in catastrophic states.

An important issue is that the capital requirement of the first best solvency regulation depends on the characteristics of the risk line and the capital market. In a numerical application, we derive the insurance market equilibrium under different stringencies and types of solvency regulations for different capital costs and risk lines (i.e., correlation across agents and probability-loss characteristics). We consider two different types of solvency regulation: a very common one imposing a maximal value for the default rate ε and another one imposing a maximal value for $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ to take into account the shape of the tail of insurers' loss distribution. Less stringent solvency regulations correspond to higher ε and higher $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$, respectively. In both cases, the inverted U-shape of the welfare gain for insureds with respect to solvency regulation stringency provides information about the optimal solvency regulation. The two types of solvency regulation feature some differences. On the one hand, the optimal ε is slightly less sensitive to a change of correlation across agents than the optimal $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$. Since a solvency constraint typically applies to all risk lines, indiscriminately of their correlation, probability, or loss parameters, this suggests that the ε policy is slightly more adapted than the $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ policy when risk lines feature heterogeneous correlation parameters. On the other hand, the optimal ε is substantially more sensitive to a change of probability-loss characteristics than the optimal $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$. This suggests that the latter policy is substantially better than the former when risk lines feature heterogeneous probability-loss characteristics.

The paper contributes to the literature on catastrophe insurance (i.e., when claims are correlated across policyholders). One branch of the literature focuses on insurance demand (e.g., Charpentier & Le Maux (2014); Raykov (2015)). These papers assume that insurers' default risks stem from their limited capital, without addressing their capital costs and choices. Another branch of the literature considers insurers' capital costs and choices but focuses on insurance supply (e.g., Ibragimov et al.

(2009); Zanjani (2002)). The latter branch highlights the quality-price trade-off in the supply of insurance. If insurers secure a low level of capital, they will supply insurance at a low price but they will often default (i.e., bad insurance quality). On the other hand, if they secure a high level of capital, they will hardly ever default (i.e., good insurance quality) but they will supply insurance at a high price. Our paper bridges the gap between the two branches of the literature. We explicitly model insurance demand in line with the former and we consider insurers' capital costs and choices in accordance with the latter. In this way, we show that the insurance quality-price trade-off translates into a quality-quantity trade-off for policyholders and that the optimal trade-off necessarily features some default risk. Moreover, our integrated approach complements the literature by analyzing how the optimal trade-off depends on risk lines and capital costs.

The paper also contributes to the literature on insurer default risk and solvency regulation (which is not limited to catastrophe insurance) (Klein 2013). In the papers dealing with insurer default risks, solvency regulation often has no purpose because it is (implicitly or explicitly) assumed that policyholders can observe default risks and that insurers can commit to a specific level of capital (Rees et al. 1999).[1] Biener et al. (2019) and Peter & Ying (2019) question the former assumption, while Boonen (2019) and Filipoviè et al. (2015) question the latter assumption. Similarly to Boonen (2019) and Filipoviè et al. (2015), we study the case where insurers cannot commit to a specific level of capital. However, in contrast to these authors, we focus on catastrophe insurance by modeling the risk correlation across risk-exposed agents.[2] Moreover, our work is the first, to our knowledge, to develop

[1]Relative to the lack of purpose of solvency regulation in some papers, see for instance Charpentier & Le Maux (2014); Cummins & Mahul (2003); Doherty & Schlesinger (1990); Raykov (2015) for papers focusing on insurance demand and Cagle & Harrington (1995); Cummins & Danzon (1997); Ibragimov et al. (2009); Zanjani (2002) for papers focusing on insurance supply.

[2]Filipoviè et al. (2015) highlight that when insurers cannot commit to a specific level of capital, the insurance market constitutes a principal-agent problem in which the principal is the policyholder

a model allowing a welfare comparison between different types of solvency regulation (i.e., one imposing a maximal default rate and another taking into consideration the tail of the loss distribution of insurers).

The remainder of the paper is structured as follows. Section 2 presents our setting. In Section 3, we derive the Pareto optimal insurance contract for a given level of insurer profit. In Section 4, we show that the insurance market collapses in the presence of insurer commitment issues. Section 5 demonstrates that the best solvency regulation restores the optimal insurance contract and that this solvency regulation allows some insurer default risk. In Section 6, our numerical analysis highlights how different types of solvency regulation behave relative to changes in risk lines and capital costs. Section 7 concludes.

2 Setting

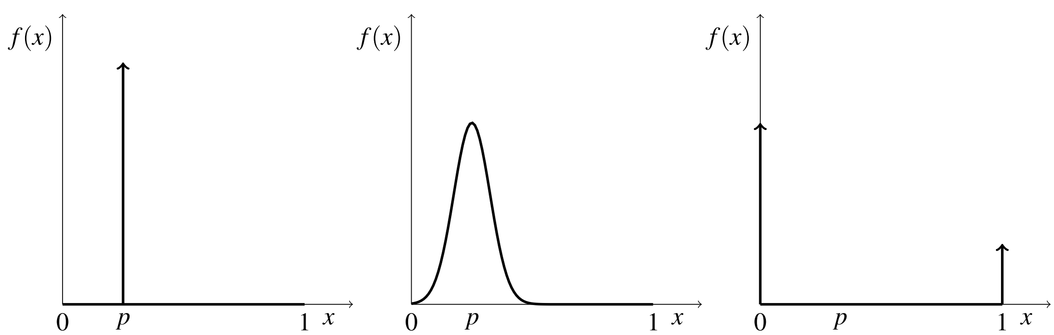
2.1 Risk-exposed agents

We consider a continuum of mass one, of ex ante identical agents exposed to a risk of loss. All agents have an initial wealth w and face a potential loss of size L with probability p. They are expected utility maximizers with a twice continuously differentiable Bernoulli utility function denoted $u(.)$. Moreover, they have non-satiated preferences (i.e., $u(.)$ strictly increasing) and are strictly risk-averse (i.e., $u(.)$ strictly concave). Without an insurance contract, the expected utility of a risk-exposed agent writes $(1-p)u(w)+pu(w-L)$.

The fraction of the population that incurs a loss is a random variable, denoted $\tilde{x}$ and distributed over the interval $[\underline{x},\overline{x}] \subseteq [0,1]$ with probability density function

and the insurer is the agent. Moreover, the problem has similarities with the risk-shifting problem widely studied in corporate finance between shareholders and debtholders (Green 1984; William H et al. 1976).

$f(.)$ and cumulative distribution function $F(.)$.[3] The states of nature where $\tilde{x}$ is high are referred to as catastrophic, and the larger the outcome of $\tilde{x}$, the more catastrophic the state. In this model, the probability that an agent incurs the loss L when $\tilde{x}$ takes the value x is simply x and the unconditional probability of loss is $p = \int_{\underline{x}}^{\overline{x}} x f(x) dx$. Moreover, assuming that conditional on x the individual losses of two agents are independent, the correlation coefficient between these losses is $\rho = \frac{1}{p(1-p)} \int_{\underline{x}}^{\overline{x}} (x-p)^2 f(x) dx$ (see proof in Appendix A). Figure 1 depicts three probability density functions $f(.)$ associated with more or less risk correlation between individual losses. Figure 1a corresponds to the absence of uncertainty for the affected fraction of the population, which is associated with the absence of correlation between individual losses ($\rho = 0$). Figure 1c corresponds to the case where either everyone or no one is affected by a loss, which is associated with a full correlation between individual losses ($\rho = 1$). Figure 1b corresponds to one of the numerous intermediate cases with a partial correlation ($0 < \rho < 1$).



(a) Case $\rho = 0$ (b) Case $0 < \rho < 1$ (c) Case $\rho = 1$

Figure 1: Different probability density functions $f(x)$ with $p = 0.25$.

[3] $\underline{x}$ and $\overline{x}$ are defined such that, for any $\delta > 0$, there exists $x \in [\underline{x}, \underline{x} + \delta]$ such that $f(x) > 0$, and there exists $x \in [\overline{x} - \delta, \overline{x}]$ such that $f(x) > 0$.

2.2 Insurers

We consider a finite number of identical expected profit maximizer insurers in competition that supply insurance contracts to the risk-exposed agents.[4] An insurance contract between an agent and an insurer comprises an indemnity I ($0 \leq I \leq L$), received by the agent when she is affected by a loss, in exchange for a premium P, paid to the insurer whether the agent is affected or not.[5] Given that risk-exposed agents are ex ante identical, we assume that they purchase the same insurance contract (I,P). Moreover, we assume that each insurer contracts with a continuum of agents to reach maximal diversification. Thus, an insurer receives the amount P per policyholder and pays out the indemnity I to a fraction x of policyholders when the fraction of the population that incurs a loss is x. Importantly, maximal diversification does not mean perfect diversification since the fraction of the population that incurs a loss is a priori uncertain (due to risk correlation). In this context of uncertainty for insurers, we can introduce standard measures of risks. The "Value at Risk" VaR_ε for an insurer can be defined such that the probability to lose a higher amount than VaR_ε is ε. In the present model, the VaR_ε per policyholder for an insurer thus corresponds to xI where x is such that $\int_x^1 f(x')dx' = \varepsilon$. The "Expected Shortfall" ES_ε for an insurer can be defined as the expected loss in monetary units in the worst cases with cumulative probability ε. In the present model, the ES_ε per policyholder for an insurer thus corresponds to $\frac{1}{\varepsilon}\int_x^1 Ix'f(x')dx'$ where x is such that $\int_x^1 f(x')dx' = \varepsilon$.

An insurer may default on due indemnities if its capital is not sufficient. The basic capital, which is composed of the premium P (per policyholder), may be in-

[4]Equivalently, insurers are represented by a single representative insurer and are assumed to make a generic reservation profit.

[5]When there is only one possible loss level for the agent, insurance above a deductible, co-insurance, and upper-limit insurance correspond to the same contract, which is the one considered here.

creased or decreased by the insurer. On the one hand, the insurer may divert one part $D \in [0,P]$ (per policyholder) from the purpose of solvency management in order to deliver an excess profit, hence increasing the probability of default. On the other hand, the insurer may secure an additional capital $R \geq 0$ (per policyholder) on top of the premium P. We assume that this additional capital is secured through reinsurance or financial contracts (which means that it is contingent capital).[6] Without contingent capital R, the capital $P-D$ is not sufficient to pay all claims if a high fraction of the population is affected by a loss and xI is larger than $P-D$. The insurer, therefore, faces a shortfall $S(x) = xI + D - P$ per policyholder, which can translate into partial default on due indemnities. The insurer may purchase contingent capital R to limit default on due indemnities in bad states of nature. We assume that it can purchase contingent capital by tranches of shortfall, as it is the case in practice (Froot 2001). The contingent capital by tranches is represented in Figure 2. The infinitesimal tranche associated with a given level of shortfall $S(x_i)$ provides a payment $S(x_i + dx_i) - S(x_i) = Idx_i$ when the fraction $\tilde{x}$ of policyholders affected by a loss is higher than x_i. This infinitesimal tranche, therefore, provides the insurer with a payment Idx_i triggered with probability $1 - F(x_i)$. The cost of this infinitesimal tranche is assumed to be $(1 + c(x_i))(1 - F(x_i))Idx_i$, in which $c(x_i) \geq 0$ is a loading factor. Since higher tranches are sold with higher loading factors in practice (Froot 2001), $c(x_i)$ is assumed to be strictly increasing with x_i. R corresponds to the level up to which the insurer purchases contingent capital for shortfalls. Thus, the insurer defaults partially when the fraction $\tilde{x}$ of policyholders affected by a loss is higher than $x_d = \frac{P-D+R}{I}$. In other terms, the probability of default or default rate is $\int_{\frac{P-D+R}{I}}^{1} f(x)dx$. The total expected value of the contingent capital is $\int_{\frac{P-D}{I}}^{\frac{P-D+R}{I}} (1 - F(x))Idx$ and its total cost is $\int_{\frac{P-D}{I}}^{\frac{P-D+R}{I}} (1 + c(x))(1 - F(x))Idx$.

[6] We leave aside here the question of the optimal mix between internal and external capital.

Finally, we assume that the available capital is shared between all policyholders in proportion to their due indemnities when the insurer defaults.

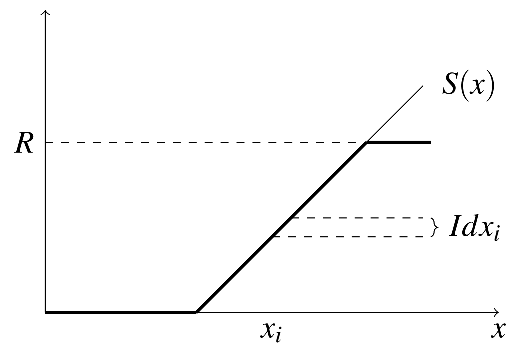


Figure 2: The thin line represents the shortfall $S(x)$, while the thick line represents the contingent capital for shortfalls in $[0,R]$. The infinitesimal tranche associated with a level of shortfall $S(x_i)$ provides a payment Idx_i with probability $1-F(x_i)$.

2.3 Payoffs

With insurance contract (I,P) and insurer default level x_d, the expected utility of a risk-exposed agent writes:

$$\begin{aligned} U(I,P,x_d) = &\int_0^{x_d} \Big((1-x)u(w-P) + xu(w-P-L+I)\Big) f(x)dx \\ &+ \int_{x_d}^1 \Big((1-x)u(w-P) + xu\Big(w-P-L+\frac{x_d}{x}I\Big)\Big) f(x)dx, \end{aligned} \tag{1}$$

in which the first term corresponds to the expected utility when x is low enough to avoid insurer default and the second term corresponds to the expected utility when x is not low enough to avoid insurer default.

With insurance contracts (I,P), diverted capital D and contingent capital R, the expected profit of an insurer (per unit of policyholders) writes:

$$
\begin{aligned}
\Pi(I,P,D,R) =& P - \int_0^{\frac{P-D+R}{I}} xIf(x)dx - \int_{\frac{P-D+R}{I}}^1 (P-D+R)f(x)dx \\
& - \int_{\frac{P-D}{I}}^{\frac{P-D+R}{I}} c(x)(1-F(x))Idx,
\end{aligned} \tag{2}
$$

in which the four terms correspond respectively to the insurance premiums raised, the insurance indemnities given when the insurer does not default, the insurance indemnities given when the insurer defaults, and the total net cost of contingent capital.

3 Pareto optimal allocation

By definition, an allocation (I,P,x_d,D,R) is Pareto optimal if it is not possible to find another allocation that increases the welfare of risk-exposed agents without decreasing the profit of insurers. Mathematically, (I,P,x_d,D,R) is Pareto optimal if there exists Π_0 such that:

$$
\begin{aligned}
\max_{I,P,x_d,D,R} \quad & U(I,P,x_d) \\
\text{s.t.} \quad & \Pi(I,P,D,R) \geq \Pi_0, \quad 0 \leq I \leq L, \quad 0 \leq D \leq P, \quad R \geq 0, \\
& \text{and} \quad \frac{P-D+R}{I} = x_d.
\end{aligned} \tag{3}
$$

Given that $U(I,P,x_d)$ does not depend directly on D and R, (3) can be rewritten:

$$\begin{aligned} \max_{I,P,x_d} \quad & U(I,P,x_d) \\ \text{s.t.} \quad & 0 \le I \le L, \quad \Pi(I,P,\tilde{D},\tilde{R}) \ge \Pi_0, \\ & \text{and} \quad (\tilde{D},\tilde{R}) = \underset{\substack{D,R \\ \text{s.t. } 0\le D\le P,\ R\ge 0, \text{ and } \frac{P-D+R}{I}=x_d}}{\arg\max} \Pi(I,P,D,R) \quad . \end{aligned} \tag{4}$$

The characterization (4) of Pareto optimal allocations will be useful to analyze whether a given regulation is a first-best policy or not. Moreover, since policyholders are identical, we will be able to Pareto rank regulations giving the same profit level for insurers. Indeed, if one regulation provides a higher utility to policyholders than another regulation, then the former Pareto dominates the latter. In the following section, we show that in a decentralized economy with a lack of insurer commitment, the insurance market collapses and the allocation is suboptimal, which calls for regulatory intervention.

4 Insurance market equilibrium

We characterize the insurance market equilibrium in the case where insurers cannot commit to a specific level of capital. In other words, risk-exposed agents choose their insurance contract first, insurers choose their level of capital second, and nature chooses x and who is affected third (Figure 3). A risk-exposed agent chooses her insurance contract (I,P) in order to maximize her expected utility, given that she anticipates the insurer default level x_d as a function of diverted capital D^* and contingent capital R^* chosen afterward by her insurer. Insurance contracts (I,P) supplied to risk-exposed agents give at least a generic reservation profit Π_0 to insurers. For given insurance contracts (I,P), an insurer chooses the diverted

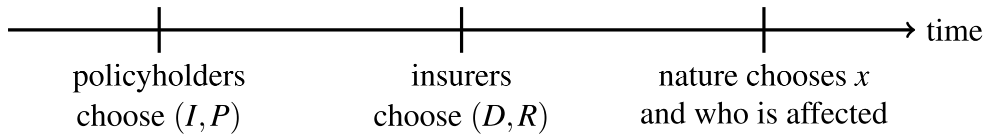


Figure 3: Sequential market structure.

capital D^* and the contingent capital R^* in order to maximize its expected profit. Mathematically, the equilibrium is thus determined by:

$$\begin{aligned} (I^*,P^*) = \arg\max_{I,P} \quad & U(I,P,x_d), \\ \text{s.t.} \quad & 0 \leq I \leq L, \quad x_d = \frac{P - D^* + R^*}{I}, \quad \Pi(I,P,D^*,R^*) \geq \Pi_0, \\ & \text{and} \quad (D^*,R^*) = \underset{\substack{D,R \\ \text{s.t. } 0 \leq D \leq P \text{ and } R \geq 0}}{\arg\max} \; \Pi(I,P,D,R). \end{aligned} \tag{5}$$

The optimization problem (5) features similarities with the one in Filipovìc et al. (2015). They also study the insurance market when insurers cannot commit to a specific level of capital, but they do not focus on the catastrophe insurance market (i.e., insurance markets in which claims are correlated across policyholders). Filipovìc et al. (2015) highlight that insurance problems with insurer commitment issues can be interpreted as risk-shifting principal-agent problems studied in Holmström (1979) where the principal is the policyholder and the agent is the insurer.

When insurers do not have the ability to commit to a minimal value for the default threshold x_d, the insurance market collapses. Indeed, insurers have incentives not to purchase any contingent capital ($R^* = 0$) and to divert the whole premiums into excess profit ($D^* = P$), which implies that insurers fully default in any state (since $x_d = 0$) and risk-exposed agents do not have any incentives to purchase any insurance contracts.[7] We detail the formal proof in appendix B. The collapse of

[7] By contrast, the insurance market in Filipovìc et al. (2015) does not completely collapse because the insurer cannot fully divert premiums to deliver excess profit.

the insurance market is in general inefficient. It is easy to show the inefficiency if one assumes $\underline{x} > 0$. In this case, it is possible to implement an allocation that keeps the profit of insurers null and is strictly better than no insurance contracts for risk-exposed agents. Indeed, with $R = 0$, $D = 0$, $x_d = \underline{x}$, $I = L$, $P = \underline{x}L$, insurers make zero profit and risk-exposed agents transfer wealth from good states to bad states while keeping the same expected wealth. This result calls for a regulatory intervention, such as a solvency regulation, which is the subject of the next section.

5 Solvency regulation

Solvency regulation can play an important role in avoiding insurance market collapse and increasing welfare through a better allocation of risks. In our setting, solvency regulation consists in choosing x_s such that default is not allowed for $x \leq x_s$. The policy maker chooses the solvency regulation x_s ex ante. We characterize the insurance market equilibrium under solvency regulation. Mathematically, the equilibrium is determined by an optimization problem similar to (5), except that the insurer default level x_s is imposed on insurers by solvency regulation. Therefore, it writes:

$$
\begin{aligned}
(I^*, P^*) = \arg\max_{I,P} \quad & U(I, P, x_d), \\
\text{s.t.} \quad & 0 \leq I \leq L, \quad x_d = \frac{P - D^* + R^*}{I}, \quad \Pi(I, P, D^*, R^*) \geq \Pi_0, \\
& \text{and} \quad (D^*, R^*) = \arg\max_{\substack{D,R \\ \text{s.t. } 0 \leq D \leq P,\ R \geq 0, \text{ and } \frac{P-D+R}{I} \geq x_s}} \Pi(I, P, D, R) \quad .
\end{aligned}
\tag{6}
$$

Similarly to Section 4, an insurer has incentives to choose R as low as possible and D as large as possible, which leads the solvency constraint to bind (i.e., $x_d = x_s$).

Proposition 1. *The solvency regulation x_s^* is a first best policy if and only if $x_s^* = \arg\max_{x_s} U(I^*, P^*, x_s)$.*

Proof. ($\Leftarrow$) If $x_s^* = \arg\max_{x_s} U(I^*, P^*, x_s)$, the allocation satisfies (4) which characterizes a Pareto optimal allocation. Thus, the chosen solvency regulation is a first best policy. ($\Rightarrow$) If $x_s^* \neq \arg\max_{x_s} U(I^*, P^*, x_s)$, it is possible by definition to find another solvency regulation that provides higher welfare to risk-exposed agents while keeping the profit constant for insurers. Thus, the chosen solvency regulation is not a first best policy. □

To go further in the analysis and characterize the first best solvency regulation (i.e., $x_s^* = \arg\max_{x_s} U(I^*, P^*, x_s)$), we should specify how the fraction of the population affected is distributed. In the following, we assume $\Pi_0 = 0$.

We first consider the simple case without uncertainty (i.e., $\underline{x} = p = \overline{x}$). This case is associated with the absence of correlation between individual losses (i.e., $\rho = 0$) and corresponds to the context without catastrophic risks. In this case, the first best solvency regulation is such that $x_s^* = \underline{x} = \overline{x}$, which means that it does not allow any default (i.e., $x_s^* = \overline{x}$). Moreover, at equilibrium under the first best solvency regulation, we have $D^* = 0$, $R^* = 0$, $I^* = L$, and $P^* = pI^*$. Since insurers are not exposed to any uncertainty, they do not have to purchase any contingent capital to avoid default ($R^* = 0$). Thus, insurance contracts are supplied without any default and without any loading ($P^* = pI^*$) and policyholders purchase full coverage ($I^* = L$). The solvency constraint (i.e., $D^* = P^* - x_s^* I^* + R^*$) with the null profit constraint implies that $D^* = 0$.

We now consider the more interesting cases with uncertainty (i.e., $\underline{x} < p < \overline{x}$). These cases are associated with the presence of a correlation between individual losses (i.e., $\rho > 0$) and correspond to the context with catastrophic risks.

Proposition 2. *Assume that the affected fraction of the population is uncertain (i.e., $\underline{x} < p < \bar{x}$) and $f(.)$ is continuous over $[\underline{x}, \bar{x}]$. The first best solvency regulation x_s^* is such that $\underline{x} < x_s^* < \bar{x}$.*

Proof. See Appendix C. □

Proposition 2 shows that in the cases with uncertainty (i.e., $\underline{x} < p < \bar{x}$) the first best solvency regulation allows some insurer default (i.e., $x_s^* < \bar{x}$). With insurer default risk, the diversifiable part of individual risks is not fully eliminated. The reason is that insurance contracts are defined on individual risks without a split between the diversifiable and undiversifiable parts. In this context of incomplete contracts, allowing a marginal amount of insurer default enables to better share, between insurers and policyholders, the part of individual risks which is undiversifiable and costly to manage. Indeed, it introduces a marginal difference in coverage rate between low catastrophic states and high catastrophic states, which is beneficial for policyholders since it comes with a lower insurance loading and a higher coverage rate purchase. In our model, the welfare value of allowing some insurer default thus comes from the correlation of claims across policyholders, the cost of contingent capital, and the insurance contract incompleteness. By contrast, the welfare value of allowing some insurer default in Filipoviè et al. (2015) comes from the assumption that the insurer can invest in some risky assets which are not directly accessible to the policyholder.

Under any solvency regulation, the allocation at equilibrium is such that $D^* = 0$, as detailed in the proof of Proposition 2. Moreover, under any solvency regulation satisfying $\underline{x} < x_s < \bar{x}$, the allocation at equilibrium is such that $R^* > 0$. At least a marginal amount of contingent capital is purchased whatever its loading. Otherwise, insurers would have to raise higher premiums from policyholders to satisfy the solvency constraint. This explains why Froot (2001) finds that some reinsurance is purchased even when the reinsurance premium is as large as seven times the expected indemnity. Additionally, since contingent capital is costly for insurers, insurance contracts are supplied above actuarially fair prices ($P^* > \int_0^{x_s} xI^* f(x)dx + \int_{x_s}^1 x_s I^* f(x)dx$). Finally, policyholders may purchase partial or full coverage ($I^* \leq L$). This is ambiguous since costly insurance incentivizes to lower coverage rate purchase while low coverage rate in catastrophic states due to insurer default incentivizes to increase coverage rate purchase.

The following section develops a numerical application to study more deeply the characteristics of the insurance contract in the regulated market and the characteristics of the optimal solvency regulation.

6 Numerical application

In the numerical application, we compute the insurance contract purchased in the regulated market and display the associated loading factor, coverage rate, and welfare gain for different solvency regulations x_s, contingent capital costs $c(.)$, and risk lines (i.e., loss correlation ρ, loss probability p and loss level L). For the choice of x_s, we consider two different types of solvency regulations: one imposing a maximal value for the default rate ε and one imposing a maximal value for $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ to take into consideration the shape of the tail of insurers' loss distribution.

6.1 Parametrization

We represent policyholders' preferences with a Constant Absolute Risk Aversion (CARA) utility function[8]

$$u(y) = -\frac{1}{\eta}\exp(-\eta y) \tag{7}$$

where η is the Arrow-Pratt index of absolute risk aversion. The wealth level of a policyholder is set at $w = \$50,000$, which corresponds roughly to the yearly income of a median household in developed countries.[9] In line with laboratory and field studies (Gollier 2004; Harrison & Rutström 2008), the Arrow-Pratt index of absolute risk aversion is set at 2.10^{-5} in order to obtain a level of relative risk aversion in the no-loss state of the world $-wu''(w)/u'(w) = 1$.

For the fraction $\tilde{x}$ of the population that incurs a loss, we assume a Beta distribution with a probability density function

$$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt}, \quad \forall x \in [0,1], \tag{8}$$

where α and β are two strictly positive parameters characterizing the shape of the distribution. The Beta distribution is defined over the interval $[0,1]$ and changes in the values of α and β allow us to span a wide range of distribution shapes (uniform, increasing convex, decreasing convex, single-peaked, or bi-modal). Given values of α and β determine the correlation coefficient $\rho = \frac{1}{p(1-p)}\int_0^1 (x-p)^2 f(x)dx$ and the loss probability $p = \int_0^1 x f(x)dx$. We consider a range of values for ρ and p, $\{1\%, 4\%, 8\%, 15\%, 25\%\}$ and $\{0.1\%, 0.2\%, 0.4\%, 0.8\%, 1.5\%\}$, respectively. The

[8]An alternative specification, such as a Constant Relative Risk Aversion (CRRA) function for example, would not qualitatively change our results.

[9]For the United States, the United Kingdom, and France, the household income distribution can be found on government websites (www.census.gov/, www.ons.gov.uk/ and www.insee.fr/, respectively).

baseline specification is $\rho = 8\%$ and $p = 0.4\%$. Concerning the loss L, we maintain constant the expected loss $pL = \$100$ in the different simulations to facilitate the comparison. In Figure 4, the graph on the left displays the probability density function in the baseline specification. The graph to the center shows the probability density functions when ρ is modified and the graph to the right the probability density functions when p is modified. In the center, an increase in the correlation leads to a more spread-out distribution, for which the probability of reaching high values of x (catastrophic states) is higher. On the right, an increase in the probability shifts the distribution toward higher realizations of the random variable $\tilde{x}$.

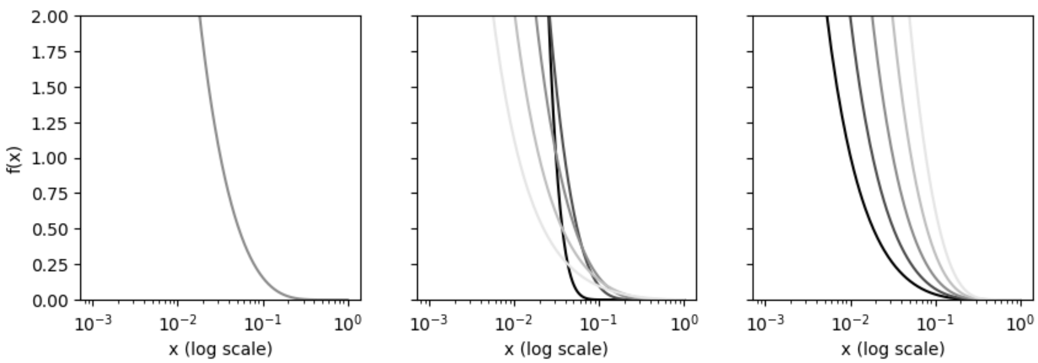


Figure 4: Probability density functions for the baseline specification $\rho = 8\%$ and $p = 0.4\%$ on the left graph, for $\rho \in \{1\%, 4\%, 8\%, 15\%, 25\%\}$ and $p = 0.4\%$ on the graph to the center, and for $p \in \{0.1\%, 0.2\%, 0.4\%, 0.8\%, 1.5\%\}$ and $\rho = 8\%$ on the graph to the right.

For the cost of contingent capital, we assume the loading $c(x) = kx$, with $k > 0$, to reflect the fact that higher tranches of reinsurance are sold at higher loadings (Froot 2001). We consider a range of values for k, $\{0.5, 1.5, 3, 6, 10\}$, with $k = 3$ as the baseline specification.

Finally, we consider different types and stringencies for the solvency regulation. Actual regulators may choose x_s in various ways. A common solvency regulation imposes to secure an amount of (contingent) capital at least as large as the Value at Risk VaR_ε for a given level ε. In other terms, it imposes to maintain the default rate below a given level ε. This corresponds in our model to choosing x_s such that $\int_{x_s}^1 f(x)dx = \varepsilon$. In the European Union, the first pillar of Solvency 2 imposes capital requirements corresponding to a 0.5% default rate on a 1-year period (Klein 2013). In the USA, the Risk-Based Capital framework developed by the National Association of Insurance Commissioners also determines a level of capital meant to guarantee a 0.5% annual default rate. Another type of solvency regulation could be more sensitive to the shape of the tail of insurers' loss distribution by imposing some constraint on the Expected Shortfall. For instance, it may impose to secure an amount of (contingent) capital to maintain the Expected Shortfall minus the Value at Risk multiplied by the default rate below a given level $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$. In our model, this type of regulation corresponds to choosing x_s such that $\int_{x_s}^1 I(x-x_s)f(x)dx = (ES_\varepsilon - VaR_\varepsilon)\varepsilon$. In contrast to the former type of regulation, the latter allows different default rates for different risk lines and market conditions. More specifically, it imposes a lower default rate and a higher (contingent) capital budgeting for distributions with larger tails above the VaR_ε. In terms of stringency, we consider solvency regulations imposing any given $\varepsilon \in [0\%, 2.5\%]$ and any given $(ES_\varepsilon - VaR_\varepsilon)\varepsilon \in [0\$, 30\$]$, respectively for each type of solvency regulation.

6.2 Analysis

Our goal is to highlight the impacts of different solvency regulations on insurance market equilibrium and policyholders' welfare. For that purpose, we provide information on insurance price, insurance coverage, and policyholders' welfare at equilibrium, as a function of solvency regulation. We present successively the results for the set of contingent capital costs considered (Figure 5), the set of risk correlations considered (Figure 6), and the set of risk probabilities considered (Figure 7). In each figure, the left column of graphs depicts characteristics at equilibrium as a function of solvency regulation imposing a maximal value for the default rate ε while the right column depicts the same characteristics as a function of solvency regulation imposing a maximal value for $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$. In each column, the first graph displays the loading factor of a full coverage insurance contract, defined as $(P_L - P_L^{fair})/P_L^{fair}$, in which P_L is the price of a contract with full coverage and a given default probability while P_L^{fair} is the expected indemnity of a contract with the same coverage and default conditions.[10] The second graph depicts the coverage rate I^*/L purchased by a risk-exposed agent, in which the indemnity I^* solves (15). The third graph displays the relative welfare gain $(u_I - u_0)/(u_M - u_0)$, in which u_I, u_M, and u_0 are the utility levels reached with the purchased indemnity I^*, with full coverage contract $(I = L, P = pL)$, and without any insurance contract, respectively. Therefore, $(u_I - u_0)/(u_M - u_0)$ characterizes the welfare gain obtained with the insurance contract purchased on a $[0,1]$ scale, whereby 0 corresponds to the absence of insurance contract and 1 corresponds to a full coverage insurance contract obtained when insurers do not pay loading cost for contingent capital. For given parameters of the model (i.e., k, ρ, p and L), regulations are Pareto ranked: if one regulation provides a higher welfare gain to policyholders than another regulation, then the former Pareto dominates the latter.

Figure 5 shows the impacts of solvency regulations with $k \in \{0.5, 1.5, 3, 6, 10\}$. In each graph, each curve is associated with a different k: the lighter the curve, the higher the k. In the first row of graphs, for a given k, the loading is a decreasing function of ε or $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ since an increase of these values reduces the contingent capital burden of insurers. Higher values of k, depicted by the lighter curves,

[10]Premiums P_L^{fair} and P_L are therefore such that $P_L^{fair} = \int_0^{x_s} Lxf(x)dx + \int_{x_s}^1 \frac{x_s L}{x} xf(x)dx$ and $\Pi(L, P, 0, x_s L - P) = 0$, respectively.

entail a higher loading, but this effect is attenuated when solvency regulation is less stringent. The second row of graphs shows the coverage rate as a function of solvency regulation. An increase in ε or $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ increases insurance purchases. Firstly, the price drop makes insurance more affordable. Secondly, the possibility of a partial default makes coverage even more desirable since, for a given solvency regulation, an increase in I offers a higher indemnity Ix_s/x in the event of a default. For the higher values of k, the gain in terms of coverage rate can be substantial (for instance, from 30% to 85% in our simulation for $k = 10$). The third row of graphs shows the relative welfare gain, achieved thanks to the purchased insurance contract, as a function of solvency regulation. The inverted U-shape is a direct illustration of Proposition 2. In 0, utility is increasing in ε or $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$. However, as the stringency of the solvency regulation decreases, the default risk born by policyholders becomes excessive, which leads to a decrease in welfare despite the price drop. An optimal solvency regulation can therefore be identified at the peak of the welfare curves. We can see that both the optimal ε and the optimal $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ increase with the contingent capital loading k. This means that a solvency regulation with a given ε or $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ cannot be optimal for different k. Since the curves obtained with the two different types of policies are very similar, there is no clear advantage of taking one type of policy or the other.

Figure 6 allows the risk correlation ρ to vary. The graphs show that a change in ε or $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ has a higher impact on the change in insurance price, insurance purchase, and policyholders' welfare when the correlation is higher (lighter curves). This is because capital requirements are more significantly affected by a change in solvency regulation for density functions with higher variance. Indeed, with a high variance, a stringent solvency regulation requires a high level of capital, which translates into a high loading and a low coverage rate. By contrast, a loose solvency regulation implies a low capital requirement, which translates into a low insurance loading and a high coverage rate. From a welfare point of view, there is, here again, an optimal trade-off for the default rate between the quantity of coverage (i.e., coverage rate) and the quality of coverage (i.e., default rate). In contrast to the comparative statics on k, we can see here that the curves differ between the two different types of solvency regulation. Indeed, as the tail fattens with higher levels of correlation, a policy imposing a given $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ features a decreasing default rate. Therefore, while the optimal ε does not change significantly with ρ, the optimal $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ increases with ρ. In a context where solvency regulations apply to all risk lines, irrespective of their precise characteristics, the fact that the optimal ε is slightly less sensitive to a change of correlation than the optimal $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ suggests that imposing a maximal ε is a slightly better choice than imposing a maximal $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$.

Figure 7 allows the individual probability p of loss to vary while maintaining the expected loss fixed. As a consequence, a decrease in p is accompanied by an increase in L. For risks with a lower probability and a higher loss (darker curves), the amount of indemnities due by insurers in catastrophic states is higher, or, in other words, insurers face greater volatility. Similar to the comparative statics on ρ, the curves differ between the two different types of solvency regulation. In contrast to the comparative statics on ρ, the optimal default rate changes substantially when p changes. For risks with a lower probability and a higher loss, insurance is much more valuable for policyholders. Therefore, as highlighted by the bottom left graph, the optimal default rate is substantially lower for low probability high loss risks than for high probability low loss risks. This means that a policy imposing a given maximal ε is necessarily far from being optimal for risks with different probability-loss characteristics. On the other hand, the bottom right graph shows that the optimal $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ is less sensitive to a change of the probability-loss characteristics than the optimal ε is. This is due to the fact that a policy imposing a given maximal level for $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ features a decreasing default rate when the volatility increases. Thereby, the latter type of policy can be much more valuable than a policy imposing a given maximal ε. For instance, in our simulations a solvency regulation imposing $(ES_\varepsilon - VaR_\varepsilon)\varepsilon = 15\$$ provides a higher welfare than a solvency regulation imposing $\varepsilon = 1.25\%$ for every risk line considered. This result suggests here that a policy imposing a given maximal $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ should be implemented rather than a policy imposing a given maximal ε.

7 Conclusion

This paper delivers policy recommendations for the solvency regulation of catastrophe insurance. Solvency regulation has a purpose because insurers cannot commit to a specific level of capital. We show that solvency regulation should be chosen wisely for catastrophe insurance since insurers are exposed to uncertain aggregate claims and have to purchase costly contingent capital to limit default in catastrophic states. For these reasons, the optimal solvency regulation features some insurer default risk. We show that a solvency regulation imposing constraints on insurers' expected shortfall can be more valuable for insureds than a solvency regulation imposing a maximal default rate, in the sense that the former can be close to optimal for a larger set of risk lines than the latter.

As usual, policy recommendations should be considered with care given the stylized modeling. One limit of the modeling is that agents are assumed to be identical with simple binomial monetary losses. Another limit is that they are assumed to have an accurate perception of insurer default risk, and no additional default costs, such as loss of trust in insurance, are considered. These elements should be taken into account when implementing a policy for catastrophe insurance. However, the present analysis of solvency regulation for catastrophe insurance takes into account the main pros and cons of default, namely insurance price and insurance quality. Moreover, it shows how solvency regulation should depend on a relevant range of risk features and highlights that imposing constraints on expected shortfall instead of default rate can increase the welfare of insureds.

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Conflict of interest

This contribution is free from any conflicts of interest, including all financial and non-financial interests and relationships.

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References

Biener, C., Landmann, A., & Santana, M. I. (2019). Contract nonperformance risk and uncertainty in insurance markets. *Journal of Public Economics*, 175, 65–83.

Boonen, T. J. (2019). Equilibrium recoveries in insurance markets with limited liability. *Journal of Mathematical Economics*, 85, 38–45.

Cagle, J. A. & Harrington, S. E. (1995). Insurance supply with capacity constraints and endogenous insolvency risk. *Journal of Risk and Uncertainty*, 11(3), 219–232.

Charpentier, A. & Le Maux, B. (2014). Natural catastrophe insurance: How should the government intervene? *Journal of Public Economics*, 115, 1–17.

Cummins, J. D. & Danzon, P. M. (1997). Price, financial quality, and capital flows in insurance markets. *Journal of Financial Intermediation*, 6, 3–38.

Cummins, J. D. & Mahul, O. (2003). Optimal insurance with divergent beliefs about insurer total default risk. *Journal of Risk and Uncertainty*, 27(2), 121–138.

Doherty, N. A. & Schlesinger, H. (1990). Rational insurance purchasing: Consideration of contract nonperformance. *Quarterly Journal of Economics*, 105(1), 243–253.

Filipoviè, D., Kremslehner, R., & Muermann, A. (2015). Optimal investment and premium policies under risk shifting and solvency regulation. *Journal of Risk and Insurance*, 82(2), 261–288.

Froot, K. A. (2001). The market for catastrophe risk: A clinical examination. *Journal of Financial Economics*, 60(2), 529–571.

Gollier, C. (2004). *The economics of risk and time*. Massachusetts Institute of Technology Press.

Green, R. C. (1984). Investment incentives, debt, and warrants. *Journal of Financial Economics*, 13(1), 115–136.

Harrison, G. W. & Rutström, E. E. (2008). Risk aversion in the laboratory. *Research in Experimental Economics*, 12(8), 41–196.

Holmström, B. (1979). Moral hazard and observability. *The Bell Journal of Economics*, 10(1), 74–91.

Ibragimov, R., Jaffee, D., & Walden, J. (2009). Nondiversification traps in catastrophe insurance markets. *Review of Financial Studies*, 22(3), 959–993.

Klein, R. W. (2013). Insurance market regulation: Catastrophe risk, competition, and systemic risk. In *Handbook of Insurance* (pp. 909–939).

Peter, R. & Ying, J. (2019). Do you trust your insurer? Ambiguity about contract nonperformance and optimal insurance demand. *Journal of Economic Behavior and Organization*.

Raykov, R. S. (2015). Catastrophe insurance equilibrium with correlated claims. *Theory and Decision*, 78(1), 89–115.

Rees, R., Gravelle, H., & Wambach, A. (1999). Regulation of insurance markets. *The Geneva Papers on Risk and Insurance Theory*, 24(1), 55–68.

William H, M., Michael, J., et al. (1976). Theory of the firm: managerial behavior, agency costs and ownership structure. *Journal of Financial Economics*.

Zanjani, G. (2002). Pricing and capital allocation in catastrophe insurance. *Journal of Financial Economics*, 65, 283–305.

Appendix

A Risk correlation coefficient

Let $\tilde{\ell}_i$ be the loss of agent i, equal to L with probability p and 0 with probability $1-p$. The variance of the loss of agent i and the co-variance between losses of agents i and j are respectively:

$$\text{var}(\tilde{\ell}_i) \equiv \mathbb{E}(\tilde{\ell}_i^2) - (\mathbb{E}\tilde{\ell}_i)^2 = L^2 p(1-p), \tag{9}$$

$$\text{cov}(\tilde{\ell}_i, \tilde{\ell}_j) \equiv \mathbb{E}(\tilde{\ell}_i \tilde{\ell}_j) - \mathbb{E}\tilde{\ell}_i \mathbb{E}\tilde{\ell}_j = L^2 \Big(\int_{\underline{x}}^{\overline{x}} x^2 f(x) dx - p^2 \Big). \tag{10}$$

Thus, the coefficient of risk correlation between two agents can be written as:

$$\rho \equiv \frac{\text{cov}(\tilde{\ell}_i, \tilde{\ell}_j)}{\text{var}(\tilde{\ell}_i)} = \frac{1}{p(1-p)} \int_{\underline{x}}^{\overline{x}} (x-p)^2 f(x) dx. \tag{11}$$

B Market collapse

We analyze problem (5) by solving the insurer profit maximization problem first. The derivatives of $\Pi(I,P,D,R)$ relative to D and R are respectively:

$$\begin{aligned} \frac{\partial \Pi}{\partial D}(I,P,D,R) = \int_{\frac{P-D+R}{I}}^{1} f(x)dx + c\Big(\frac{P-D+R}{I}\Big)\Big(1 - F\Big(\frac{P-D+R}{I}\Big)\Big) \\ - c\Big(\frac{P-D}{I}\Big)\Big(1 - F\Big(\frac{P-D}{I}\Big)\Big), \end{aligned} \tag{12}$$

$$\frac{\partial \Pi}{\partial R}(I,P,D,R) = -\int_{\frac{P-D+R}{I}}^{1} f(x)dx - c(R)\Big(1 - F\Big(\frac{P-D+R}{I}\Big)\Big). \tag{13}$$

Since the derivative of $\Pi(I,P,D,R)$ relative to R is negative, insurers choose $R^* = 0$. At $R^* = 0$, the derivative of $\Pi(I,P,D,R)$ relative to D is positive, which implies that insurers choose $D^* = P$. Thus, insurers have incentives not to purchase any contingent capital and to divert the whole premiums.

We continue the analysis of the problem (5) by solving the policyholder utility maximization problem. By choosing $R^* = 0$ and $D^* = P$, insurers fully default (i.e., $x_d = 0$). With $x_d = 0$, risk-exposed agents do not have any incentives to purchase insurance contracts since they would pay a premium without receiving any indemnities in any state of nature.

C Proof of Proposition 2

C.1 Simplification of problem (6)

In problem (6), we already know that the solvency constraint is binding (i.e., $D^* = P - x_s I + R^*$). The insurer maximization problem simplifies to:

$$\begin{aligned} R^* = \arg\max_{R} \quad & P - \int_0^{x_s} xIf(x)dx - \int_{x_s}^1 x_s I f(x)dx - \int_{x_s - \frac{R}{I}}^{x_s} c(x)(1-F(x))Idx, \\ \text{s.t.} \quad & 0 \leq P - x_s I + R \leq P \quad \text{and} \qquad R \geq 0. \end{aligned} \tag{14}$$

In the case where $R^* > 0$, the condition $\frac{d\Pi}{dR}(I, P, P - x_s I + R, R) \leq 0$ implies that $0 \leq P - x_s I + R$ is binding, which implies $R^* = x_s I - P$ and $D^* = 0$. Otherwise, $R^* = 0$ and $D^* = P - x_s I$. Thus, it is not optimal for insurers to purchase costly contingent capital ($R^* > 0$) and at the same time to divert capital ($D^* > 0$).

We continue the analysis of the problem (6) by solving the policyholder utility maximization problem. We can show that the combination $R^* = 0$ and $D^* = P - x_s I > 0$ is not possible. In this case, the insurer would make a strictly positive profit since $\Pi(I, P, P - x_s I, 0) = P - \int_0^{x_s} xIf(x)dx - \int_{x_s}^1 x_s I f(x)dx = \int_0^{x_s}(x_s - x)If(x)dx +$

$D^* > 0$. This is not possible because in this case, another insurer would provide the same coverage with the same default risk at a lower price by purchasing some contingent capital, which would be preferred by policyholders. Thus, we have the combination $R^* = x_s I - P$ and $D^* = 0$. Given that $D^* = 0$, we can state the following lemma:

Lemma 1. *At equilibrium, insurers do not divert capital ($D^* = 0$) and they may purchase contingent capital or not ($R^* = x_s I^* - P^* \geq 0$). Moreover, policyholders purchase insurance contracts such that:*

$$
\begin{aligned}
(I^*, P^*) = \arg\max_{I,P} \quad & U(I, P, x_s), \\
s.t. \quad & 0 \leq I \leq L \quad and \quad \Pi(I, P, 0, x_s I - P) = 0.
\end{aligned}
\tag{15}
$$

We show below that the first best solvency regulation x_s^* (i.e., which is such that $x_s^* = \arg\max_{x_s} U(I^*, P^*, x_s)$) satisfies $\underline{x} < x_s^* < \bar{x}$. We denote $W(x_s) = U(I^*, P^*, x_s)$ the indirect utility function derived from problem (15).

C.2 The first best solvency regulation satisfies $x_s^* > \underline{x}$

To show that the first best solvency regulation satisfies $x_s^* > \underline{x}$, we aim at showing that $\frac{dW}{dx_s}(x_s) > 0$ for any $x_s \leq \underline{x}$. When $x_s \leq \underline{x}$, insurers are not exposed to any uncertainty (i.e., they pay $x_s I$ per unit of policyholders in any state of nature), which implies that they do not purchase any contingent capital ($R^* = 0$). Thus, problem (15) gives $P = x_s I$ and $I^* = L$. Finally, we have for any $x_s \leq \underline{x}$:

$$
\frac{dW}{dx_s}(x_s) = -L(1-p)u'(w - x_s L) + L \int_{x_s}^{1} u'\Big(w - x_s L - L + \frac{x_s}{x} L\Big)(1-x) f(x) dx > 0.
\tag{16}
$$

Since $f(.)$ has finite values, $\frac{dW}{dx_s}(x_s)$ is continuous in $\underline{x}$ and the first best solvency regulation x_s^* is such that $x_s^* > \underline{x}$.

C.3 The first best solvency regulation satisfies $x_s^* < \overline{x}$

To show that the first best solvency regulation satisfies $x_s^* < \overline{x}$, we aim at showing that $\frac{dW}{dx_s}(x_s) < 0$ for any x_s close enough to $\overline{x}$. For $x_s > \underline{x}$, problem (15) writes:

$$\begin{aligned}(I^*, P^*) = \arg\max_{I,P} \quad & U(I, P, x_s), \\ \text{s.t.} \quad 0 \leq I \leq L \quad & \text{and} \quad \Pi(I, P, 0, x_s I - P) = 0,\end{aligned} \tag{17}$$

with:

$$\begin{aligned}U(I, P, x_s) =& (1-p)u(w-P) + \int_0^{x_s} u(w-P-L+I)xf(x)dx \\ & + \int_{x_s}^1 u\Big(w-P-L+\frac{x_s}{x}I\Big)xf(x)dx,\end{aligned} \tag{18}$$

$$\begin{aligned}\Pi(I, P, 0, x_s I - P) =& P - \int_0^{x_s} xIf(x)dx - \int_{x_s}^1 x_s If(x)dx \\ & - \int_{\frac{P}{I}}^{x_s} c(x)(1-F(x))Idx.\end{aligned} \tag{19}$$

The first order conditions of (17) are:

$$\frac{\partial U}{\partial I}(I, P, x_s) = -\lambda \frac{d\Pi}{dI}(I, P, 0, x_s I - P) + \mu - \nu, \tag{20}$$

$$\frac{\partial U}{\partial P}(I, P, x_s) = -\lambda \frac{d\Pi}{dP}(I, P, 0, x_s I - P), \tag{21}$$

in which λ is the Lagrangian multiplier associated with the null expected profit constraint, μ is the Lagrangian multiplier associated with $I \leq L$ and ν is the Lagrangian multiplier associated with $0 \leq I$. We have $I = L$ and $\mu \geq 0$, or $I < L$ and $\mu = 0$. We have $I=0$ and $\nu \geq 0$, or $I>0$ and $\nu = 0$. Moreover, we have:

$$\frac{\partial U}{\partial I}(I,P,x_s) = \int_0^{x_s} u'(w-P-L+I)xf(x)dx + \int_{x_s}^1 u'\Big(w-P-L+\frac{x_s}{x}I\Big)x_s f(x)dx, \tag{22}$$

$$\frac{\partial U}{\partial P}(I,P,x_s) = -(1-p)u'(w-P) - \int_0^{x_s} u'(w-P-L+I)xf(x)dx - \int_{x_s}^1 u'\Big(w-P-L+\frac{x_s}{x}I\Big)xf(x)dx, \tag{23}$$

$$\frac{d\Pi}{dI}(I,P,0,x_sI-P) = -\int_0^{x_s} xf(x)dx - \int_{x_s}^1 x_s f(x)dx - \int_{\frac{P}{I}}^{x_s} c(x)(1-F(x))dx - c\left(\frac{P}{I}\right)\left(1-F\left(\frac{P}{I}\right)\right)\frac{P}{I}, \tag{24}$$

$$\frac{d\Pi}{dP}(I,P,0,x_sI-P) = 1 + c\left(\frac{P}{I}\right)\left(1-F\left(\frac{P}{I}\right)\right). \tag{25}$$

Let us first show that $I<L$ for any x_s close enough to $\bar{x}$. For $x_s = \bar{x}$, (21) rewrites:

$$\lambda = \frac{(1-p)u'(w-P) + pu'(w-P-L+I)}{1+c(\frac{P}{I})(1-F(\frac{P}{I}))}. \tag{26}$$

Thus, for $x_s = \bar{x}$, (20) rewrites:

$$pu'(w-P-L+I) = \frac{(1-p)u'(w-P) + pu'(w-P-L+I)}{1+c(\frac{P}{I})(1-F(\frac{P}{I}))} \left(p + \int_{\frac{P}{I}}^{x_s} c(x)(1-F(x))dx + c\left(\frac{P}{I}\right)\left(1-F\left(\frac{P}{I}\right)\right)\frac{P}{I}\right) + \mu - \nu. \tag{27}$$

(27) tells that $I < L$. Otherwise, we would have $I = L$, $\mu \geq 0$ and $\nu = 0$, and (27) would give:

$$1 = \frac{1 + \int_{\frac{P}{I}}^{x_s} c(x)(1-F(x))\frac{1}{p}dx + c(\frac{P}{I})(1-F(\frac{P}{I}))\frac{P}{pI}}{1 + c(\frac{P}{I})(1-F(\frac{P}{I}))} + \frac{\mu}{pu'(w-P)}. \tag{28}$$

The latter equality is not possible since $\int_{\frac{P}{I}}^{x_s} c(x)(1-F(x))\frac{1}{p}dx + c(\frac{P}{I})(1-F(\frac{P}{I}))\frac{P}{pI}$ is strictly larger than $c(\frac{P}{I})(1-F(\frac{P}{I}))$. Thus, $I < L$ and $\mu = 0$ in $x_s = \bar{x}$. By continuity, $I < L$ and $\mu = 0$ for solvency regulation constraint above some $x_s < \bar{x}$.

Let us now show that $\frac{dW}{dx_s}(x_s) < 0$ for any x_s close enough to $\bar{x}$. If I is binding in 0 for $x_s = \bar{x}$, it is clear that it lowers $W(x_s)$ to increase the solvency regulation constraint above some $x_s < \bar{x}$ because it leads to the insurance market collapse. Thus, we focus on the case in which $0 < I < L$ for $x_s = \bar{x}$, which implies $\mu = 0$ and $\nu = 0$ for $x_s = \bar{x}$ and by continuity for any x_s close enough to $\bar{x}$. With $\mu = 0$ and $\nu = 0$, we have:

$$\frac{dW}{dx_s}(x_s) = \frac{\partial U}{\partial x_s}(I,P,x_s) + \lambda \frac{\partial \Pi}{\partial x_s}(I,P,0,x_sI-P), \tag{29}$$

with $\lambda = -\frac{\frac{\partial U}{\partial I}(I,P,x_s)}{\frac{d\Pi}{dI}(I,P,0,x_sI-P)}$ given (20). Moreover, we have:

$$\frac{\partial U}{\partial x_s}(I,P,x_s) = \int_{x_s}^{1} u'\left(w-P-L+\frac{x_s}{x}I\right)If(x)dx, \tag{30}$$

$$\frac{\partial \Pi}{\partial x_s}(I,P,0,x_sI-P) = -\int_{x_s}^{1} If(x)dx - c(x_s)(1-F(x_s))I. \tag{31}$$

We also have:

$$\lim_{x_s \to \bar{x}} \frac{1}{(1-F(x_s))I}\frac{\partial U}{\partial x_s}(I,P,x_s) = u'\left(w-P-L+I\right), \tag{32}$$

$$\lim_{x_s \to \bar{x}} \frac{1}{(1-F(x_s))I} \frac{\partial \Pi}{\partial x_s}(I, P, 0, x_s I - P) = -(1 + c(\bar{x})), \tag{33}$$

$$\lim_{x_s \to \bar{x}} \frac{\partial U}{\partial I}(I, P, x_s) = pu'(w - P - L + I), \tag{34}$$

$$\lim_{x_s \to \bar{x}} \frac{d\Pi}{dI}(I, P, 0, x_s I - P) = -p - \int_{\frac{P}{I}}^{\bar{x}} c(x)(1-F(x))dx - c\left(\frac{P}{I}\right)\left(1 - F\left(\frac{P}{I}\right)\right)\frac{P}{I}. \tag{35}$$

Thanks to an integration by parts, (35) rewrites:

$$\lim_{x_s \to \bar{x}} \frac{d\Pi}{dI}(I, P, 0, x_s I - P) = -p - \int_{\frac{P}{I}}^{\bar{x}} c(x) x f(x) dx + \int_{\frac{P}{I}}^{\bar{x}} c'(x) x (1-F(x)) dx. \tag{36}$$

Thus, we have:

$$\lim_{x_s \to \bar{x}} \frac{1}{(1-F(x_s))I} \frac{dW}{dx_s}(x_s) = u'\Big(w - P - L + I\Big) - \frac{pu'(w - P - L + I)(1 + c(\bar{x}))}{p + \int_{\frac{P}{I}}^{\bar{x}} c(x) x f(x) dx - \int_{\frac{P}{I}}^{\bar{x}} c'(x) x (1-F(x)) dx}. \tag{37}$$

Since $c(\bar{x})) > \int_{\frac{P}{I}}^{\bar{x}} c(x) \frac{x}{p} f(x) dx - \int_{\frac{P}{I}}^{\bar{x}} c'(x) \frac{x}{p} (1-F(x)) dx$, we have:

$$\lim_{x_s \to \bar{x}} \frac{1}{(1-F(x_s))I} \frac{dW}{dx_s}(x_s) < 0. \tag{38}$$

Thus: $\frac{dW}{dx_s}(x_s) \underset{x_s \to \bar{x}}{\sim} -A(1-F(x_s))I$ with $A > 0$. This concludes the proof.

D Output of the numerical application

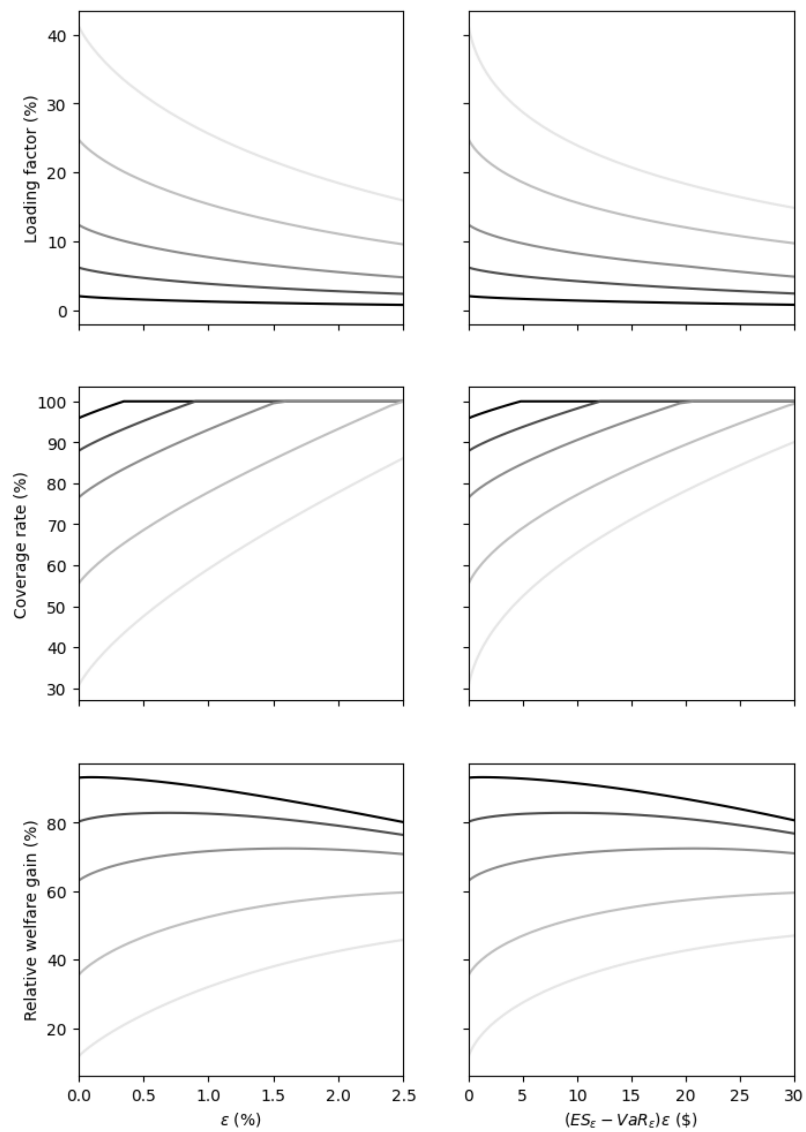

Figure 5: Comparative static analysis relative to k. From darker to lighter, the curves correspond to $k = [0.5, 1.5, 3, 6, 10]$. In each column, the first, second, and third figures display the loading factor of a full coverage insurance contract, the coverage rate purchased by a risk-exposed agent, and her relative welfare gain with this coverage, respectively, as a function of ε (column 1) or $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ (column 2).

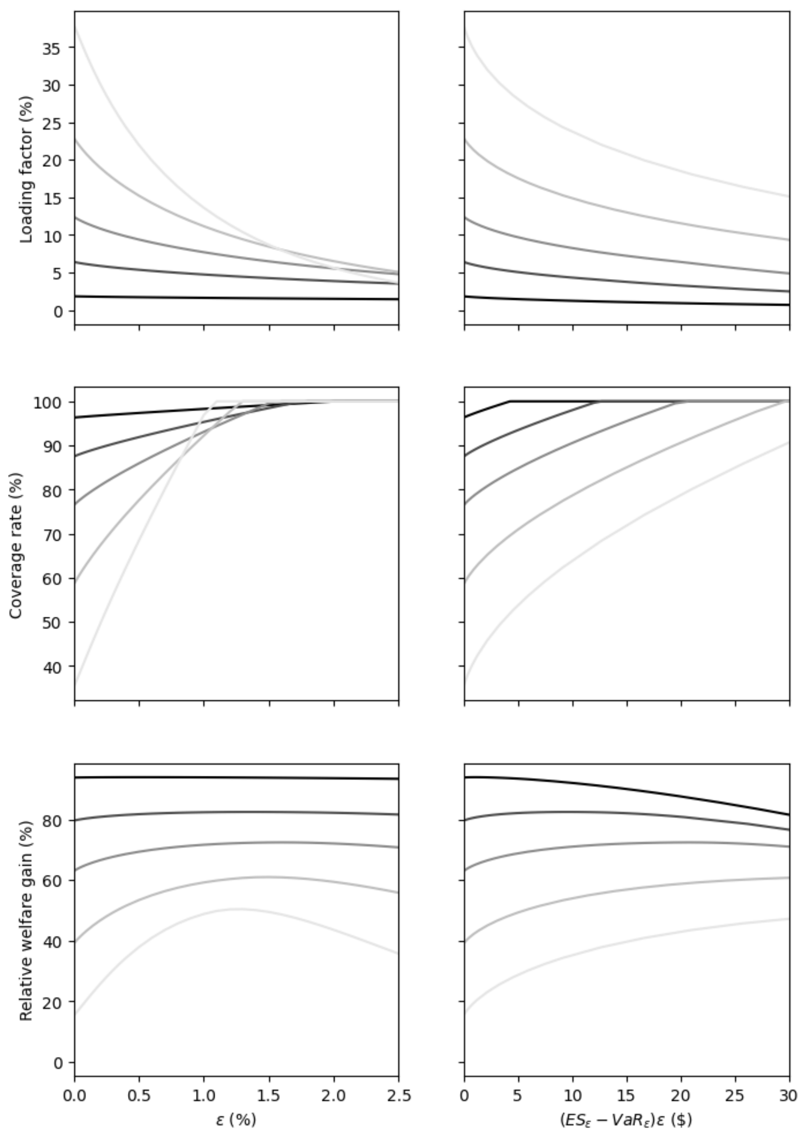

Figure 6: Comparative static analysis relative to ρ. From darker to lighter, the curves correspond to $\rho = [1\%, 4\%, 8\%, 15\%, 25\%]$. In each column, the first, second, and third figures display the loading factor of a full coverage insurance contract, the coverage rate purchased by a risk-exposed agent, and her relative welfare gain with this coverage, respectively, as a function of ε (column 1) or $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ (column 2).

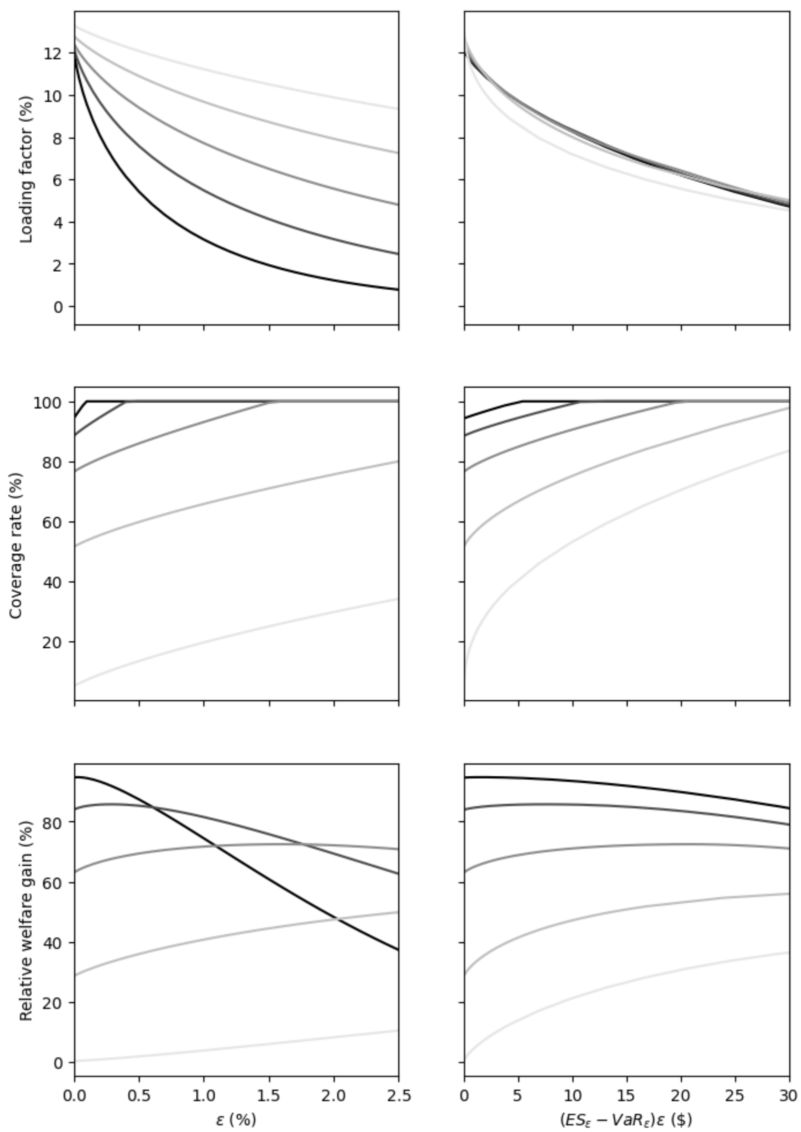

Figure 7: Comparative static analysis relative to p. From darker to lighter, the curves correspond to $p = [0.1\%, 0.2\%, 0.4\%, 0.8\%, 1.5\%]$, keeping the expected loss constant $pL = \$100$. In each column, the first, second, and third figures display the loading factor of a full coverage insurance contract, the coverage rate purchased by a risk-exposed agent, and her relative welfare gain with this coverage, respectively, as a function of ε (column 1) or $(ES_\varepsilon - VaR_\varepsilon)\varepsilon$ (column 2).