

Starting out, I compared the basic “improved” heuristic (player moves - opponent moves) and center heuristic (squares of distances to center of x and y coordinates) with a heuristic I found online to be a solution to the solo knight tour problem, which is to pick the square with the fewest options. If the least moves heuristic could apply to essentially two-player knight’s-tour, it could do well by the fact that it would prune branches with a higher branching factor. Here are the winrate results of running these three through tournament.py 3 times, along with the default AB_Improved:

AB Custom is (player moves - opponent moves), AB_Custom_2 is (distances to center squared), AB_Custom_3 is (least moves as long as there is one)

| AB_improved | AB_Custom | AB_custom_2 | AB_custom_3 |
|-------------|-----------|-------------|-------------|
| 50% | 44% | 54.3% | 47.1% |
| 44.3% | 40% | 64.3% | 40% |
| 47.1% | 42.9% | 45.7% | 44.7% |

The least moves heuristic didn’t perform too well. In hindsight, the fewest moves heuristic does well for single player knight’s tour because it inadvertently avoids situations where you create an unreachable space. Creating unreachable spaces is fine in this isolation game.

For my second run, I decided to try a closest to center approach that didn’t square the distances to the center, instead just taking the absolute value, and an approach that combined all three heuristics. The combined approach gave a score expressed as $((\text{distance_from_center})/2 + \text{player_move_count} - \text{opponent_move_count})$.

AB Custom is $((\text{distances to center squared})/2 + \text{player_move_count} - \text{opponent_move_count})$, AB_Custom_2 is (distances to center squared), AB_Custom_3 (absolute value of distances to center)

| AB_improved | AB_Custom | AB_custom_2 | AB_custom_3 |
|-------------|-----------|-------------|-------------|
| 47.1% | 44.3% | 48.6% | 45.7% |
| 48.6% | 51.4% | 50.0% | 37.1% |
| 47.1% | 54.3% | 41.4% | 47.1% |

Here the combined heuristic seemed to do best. After some thinking, I realized that the variance between opponent move count could only ever be one for scores in the same depth. So I decided to switch to an implementation where I did $(8 - \text{opponent moves} / 2)^2$, which essentially squared a value from 0 to 3.5, giving greater returns from limiting an opponent from 2 to 1 than from 8 to 7, which usually doesn’t lead to any sort of end game scenario. Playing through some games of isolation myself, I found myself often looking at specific critical squares that split the board and would cut my opponent off from many squares, and figuring out who could get to the square first, me or my opponent. This often lead to a win, so I tried to come up with a heuristic that would be able to discover moves that grant access to more of the board for one player. I came up with a method that creates coordinate sets from each player, and then alternates between players, picking out all the available moves picks out moves from

the list of empty spaces that could be reached from spaces that the players have traversed. It essentially plays out the game as if each player could make all moves simultaneously each move. This is expensive, but not as expensive as calculating every possible move to the endgame. Essentially, this heuristic has the effect of determining which player has more board control based on the fact that if they can move to a space before the opponent, they can potentially block off the opponent's moves. This heuristic is really good at finding critical moves that will block the opponent from a portion of the board, and rewards being closer to the center than the opponent.

AB Custom is $(\text{distances to center squared})/2 + \text{player move count} -$

$((8 - \text{opponent_move_count})/2)^2,$

AB_Custom_2 is $\text{player_move_count} - \text{distance_from_center}/2 - \text{opponent_move_count},$

AB_Custom_3 is the board control algorithm

AB_improved AB_Custom AB_custom_2 AB_custom_3

42.9% 42.9% 41.4% 60.0%

47.1% 51.4% 45.7% 51.4%

41.4% 42.9% 35.7% 50.0%

From these final runs of tournament.py I conclude that the final set of 3 heuristics are the the best 3 that I have tried. I would recommend the board control algorithm (AB_custom_3) because it consistently outperforms or matches the win rates of all the other heuristics. Board control also has consistently beaten or tied every other heuristic for each head to head across all these trials.

Here for additional insight I provide a 4th trial with more detail, switched around so that board control is AB_Custom and the other two heuristics from above are shifted to the right, meaning AB_Custom_2 is

$(\text{distances to center squared})/2 + \text{player move count} - ((8 - \text{opponent_move_count})/2)^2)$

and AB_Custom_3 is

$(\text{player_move_count} - \text{distance_from_center}/2 - \text{opponent_move_count}):$

| Match # | Opponent | AB_Improved | | AB_Custom | | AB_Custom_2 | | AB_Custom_3 | |
|-----------|-------------|-------------|------|-----------|------|-------------|------|-------------|------|
| | | Won | Lost | Won | Lost | Won | Lost | Won | Lost |
| 1 | Random | 6 | 4 | 8 | 2 | 8 | 2 | 8 | 2 |
| 2 | MM_Open | 2 | 8 | 2 | 8 | 2 | 8 | 3 | 7 |
| 3 | MM_Center | 4 | 6 | 8 | 2 | 8 | 2 | 4 | 6 |
| 4 | MM_Improved | 4 | 6 | 5 | 5 | 4 | 6 | 4 | 6 |
| 5 | AB_Open | 5 | 5 | 6 | 4 | 6 | 4 | 3 | 7 |
| 6 | AB_Center | 4 | 6 | 6 | 4 | 5 | 5 | 4 | 6 |
| 7 | AB_Improved | 5 | 5 | 7 | 3 | 3 | 7 | 6 | 4 |
| Win Rate: | | 42.9% | | 60.0% | | 51.4% | | 45.7% | |

What I find curious is that the board control seems to not be as dominant against the MM_Open and Random heuristics. It then makes that up by performing remarkably well against the "better" opponent heuristics that use alpha-beta, like the 7/3 performance against AB here. Since we assume that our opponents are going to be using more advanced heuristics, I would say that this odd poor performance against MM_Open and random, is less significant than the better performance against "better" heuristics. However, I am curious as to see why the board control heuristic does poorly vs random, my guess is that it overlooks a move that it considers suboptimal that can actually counter its plan in the late stages of the game. Perhaps switching to a faster heuristic that can completely map out the endgame toward the end could improve the board control heuristic further. I have also toyed with the idea of using transposition tables to speed up the heuristic.