Analysis of Linear Algebraic Operations of Convolutional Neural Networks

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Abstract

Introduction

This research project aims to explore the linear algebraic foundations of Convolutional Neural Networks (CNNs). CNNs are foundational to modern machine learning applications such as image and video processing, object detection, and medical imagining, to name a few.

Convolution

A convolution is a mathematical operation that is defined as

$$s(t) = (x * w)(t) \tag{1}$$

and

$$s(t) = \int x(a)w(t-a)da \tag{2}$$

Generally, the above function is defined for any two functions, $w(t-a)$ and $x(a)$ where the integral is defined. Goodfellow et al. [2016] Convolution has useful algebraic properties such as associativity, commutativity, and distributivity. We will prove commutativity as it will be useful for our purposes later on.

Theorem 1. *For two functions x and w, $x * w = w * x$*

Proof. By definition,

$$(x * w)(t) = \int_0^t x(a)w(t-a)da.$$

Let

$$v = t - a$$
$$a = t - v$$
$$dv = -da$$

Substituting back into the original equation,

$$
\begin{aligned}
&= \int_t^0 -x(t-v)w(a)da \\
&= \int_0^t w(a)x(t-v)da = (w * x)(t)
\end{aligned}
$$

□

Computers approximate real numbers as integers or floating point numbers. We would need an infinite amount of memory to represent the set of real numbers between 0 and 1 (since $\{\mathbb{R}\}$ is an infinite set).

References

Ian Goodfellow, Yoshua Bengio, and Aaron Courville. *Deep Learning*. MIT Press, 2016. `http://www.deeplearningbook.org`.