Analysis of Linear Algebraic Operations of Convolutional Neural Networks

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Abstract

Introduction

This research project aims to explore the linear algebraic foundations of Convolutional Neural Networks (CNNs). CNNs are foundational to modern machine learning applications such as image and video processing, object detection, and medical imagining, to name a few.

Background

A convolution is a mathematical operation that is defined as

$$s(t) = (x * w)(t) \tag{1}$$

and

$$s(t) = \int x(a)w(t-a)da \tag{2}$$

Generally, the above function is defined for any two functions, $w(t-a)$ and $x(a)$ where the integral is defined. Goodfellow et al. [2016] Convolution has useful algebraic properties such as associativity, commutativity, and distributivity. We will prove commutativity as it will be useful for our purposes later on.

Theorem 1. *For two functions x and w, $x * w = w * x$*

Proof. By definition,

$$(x * w)(t) = \int_0^t x(a)w(t-a)da.$$

Let

$$\begin{aligned} v &= t - a \\ a &= t - v \\ dv &= -da \end{aligned}$$

Substituting back into the original equation,

$$= \int_t^0 -x(t-v)w(a)da$$
$$= \int_0^t w(a)x(t-v)da = (w * x)(t)$$

□

Research Questions

This study seeks to answer the following questions:

Methodology and Outcomes

This project will include a unique blend of both theoretical and practical approaches for examining these optimization algorithms.

Timeline

The timeline for this undergraduate research project is expected to go as follows:

1. **September - October:** Gather all sources. Research linear algebraic foundations of CNNs.

References

Ian Goodfellow, Yoshua Bengio, and Aaron Courville. *Deep Learning*. MIT Press, 2016. `http://www.deeplearningbook.org`.