Introduction to Plack Stars

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Introduction

Einstein Field Equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \tag{1}$$

Minkowski metric

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2 \tag{2}$$

Black hole solutions

Classification of Black holes

	Non-rotating ($J = 0$)	Rotating ($J \neq 0$)
Uncharged ($Q = 0$)	Schwarzschild	Kerr
Charged ($Q \neq 0$)	Reissner-Nordström	Kerr-Newman

Schwarzschild Black hole

Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{3}$$

Kretschmann invariant

$$\kappa = R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} = \frac{48m^2}{r^6} \tag{4}$$

Regularized Schwarzschild metric

According to [Hayward, 2006], we cand find metrics that are:

- Spherically symmetric.
- Static.
- Asymptotically flat (minkowski).
- **Have regular center.**

The resulting stress-energy tensor is physically reasonable, satisfies the weak energy condition and has components that are bounded and fall off appropriately at large distance.

Energy conditions

Null energy condition
For every null vector x: $T_{\mu\nu}x^{\mu}x^{\nu} \geq 0.$

Weak energy condition
For every timelike vector x: $T_{\mu\nu}x^{\mu}x^{\nu} \geq 0.$

Dominant energy condition
For every timelike vector x: $T_{\mu\nu}x^{\mu}x^{\nu} \geq 0, \qquad T_{\mu\nu}T^{\nu}_{\lambda}x^{\mu}x^{\lambda} \leq 0.$

Strong energy condition
For every timelike vector x: $T_{\mu\nu}x^{\mu}x^{\nu} \geq \frac{1}{2}T^{\lambda}_{\lambda}x^{\sigma}x_{\sigma}.$

Regularized Schwarchild metric

Consider a static, spherically symmetric metric of the form:

$$ds^2 = -F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 d\Omega^2 \tag{5}$$

We demand

$$F(r) \sim 1 - \frac{2m}{r} \quad as \quad r \to \infty \tag{6}$$

$$F(r) \sim 1 - \frac{r^2}{l^2} \quad as \quad r \to 0 \tag{7}$$

Hayward metric

The so called Hayward metric [Hayward, 2006] satisfies all the required properties and is given by:

$$F(r) = 1 - \frac{2mr^2}{r^3 + 2ml^2} \tag{8}$$

where l encodes the central energy density $\frac{3}{8\pi l^2}$. Therefore

$$ds^2 = -\left(1 - \frac{2mr^2}{r^3 + 2ml^2}\right) dt^2 + \frac{1}{\left(1 - \frac{2mr^2}{r^3+2ml^2}\right)} dr^2 + r^2 d\Omega^2 \tag{9}$$

Hayward metric

Analyzing the zeros of $F(r)$, we get a critical mass $m_* = \frac{3\sqrt{3}}{4}l$ and a radius $r_* = \sqrt{3}l$.

- No zeros if $m < m_*$. ($\leftrightsquigarrow$ Regular space time with the same causal structure as a flat space-time).
- One double zero at $r = r_*$ if $m = m_*$. ($\leftrightsquigarrow$ Regular extreme black hole with degenerate Killing horizon).
- Two simple zeros at $r = r_\pm$ if $m > m_*$. ($\leftrightsquigarrow$ Regular nonextreme black hole with both outer and inner Killing horizons located at $r_+ \approx 2m$ and $r_- \approx l$ for $m \gg m_*$).

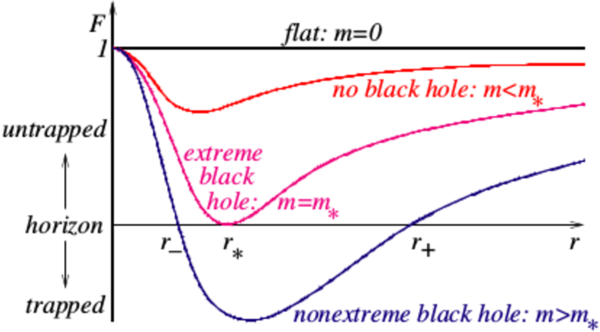

Figure : Behaviour[1] of $g_{tt} = F(r)$ for different values of the parameter m.

[1] Image taken from [Hayward, 2006].

Hayward metric

If we use field equations 1, we note that this metric is supported by density $-T_t^t$, radial pressure T_r^r, and transverse pressure $T_\theta^\theta = T_\phi^\phi$ given by:

$$G_t^t = G_r^r = -\frac{12 l^2 m^2}{(r^3 + 2 l^2 m)^2} \tag{10}$$

$$G_\theta^\theta = G_\phi^\phi = \frac{24 \left(r^3 - l^2 m\right) l^2 m^2}{(r^3 + 2 l^2 m)^3} \tag{11}$$

They fall off very rapidly $\mathcal{O}(r^{-6})$.

Quantum Field Theory

Spacetime metric describing 'non-singular' black holes are commonly studied in the literature [De Lorenzo, 2015, Rovelli, 2014] as effective modification to the Schwarzschild solution that mimic quantum gravity effects removing the central singuarity.

To begin with, two insights from quantum cosmology [Ashtekar, 2007]:

- The onset of quantum gravitational effects is when energy density reaches the Plank scale ($\sim 5.155 \cdot 10^{96}\ \frac{kg}{m^3}$).
- The dominant quantum effect at high density is a strong pressure, sufficient to counterbalance weight and reverse gravitational collapse.

Plack scale

Planck scale is given by

Quantity	SI equivalent
Planck time	$t_p = 5.39121 \cdot 10^{-44} s$
Planck mass	$m_p = 2.17645 \cdot 10^{-8} kg$
Planck length	$l_p = 1.616252 \cdot 10^{-35} m$

and the Plack density is the quotient

$$\rho_p = \frac{m_p}{l_p^3} \approx 5.155 \cdot 10^{96} \, \frac{kg}{m^3} \tag{12}$$

Quantum Field theory

For a black hole, the previous arguments imply that matter's collapse can be stopped before the central singularity is formed, yielding the formation of a central core, called a **Planck star** [Rovelli, 2014].

Nevertheless, several metrics describing non-singular black holes possess two unphysical characteristics:

- A clock in the regular center is not delayed with respect to a clock at infinity [Rovelli, 2014].
- They do not reproduce the corrections to Newton potential derived from an effective treatment of quantum gravitational theory [Bjerrum-Bohr,2003].

Newton potential

The quantum corrections to the Newton potential can be obtained using effective field theory [Bjerrum-Bohr,2003], and reintroducing the Planck length, they are given by:

$$\Phi(r) = -\frac{m}{r}\left(1 + \beta\frac{l_p^2}{r^2}\right) + \mathcal{O}(r^4) \tag{13}$$

Since

$$\Phi(r) = -\frac{1}{2}(1 + g_{tt}) \tag{14}$$

$$g_{tt} = -F(r) = -1 + \frac{2m}{r} - \frac{4l^2m^2}{r^4} + \mathcal{O}(r^{-5}) \tag{15}$$

We require additional adjustments to the Hayward metric.

Modified Hayward metric

The most general spherically symmetric, static metric that includes the previously mentioned corrections is [De Lorenzo, 2015]:

$$ds^2 = -G(r)F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 d\Omega^2 \tag{16}$$

The physical requirements imposed on $G(r)$ are:

- Preserve the Schwarzschild behaviour at large r.
- **Include the quantum corrections of the Newton potential.**
- Allow a final time dilatation between $r = 0$ and $r \to \infty$.
- Near the center, the metric is still de Sitter.

In particular, we can take

$$G(r) = 1 - \frac{\beta m \alpha}{\alpha r^3 + \beta m} \tag{17}$$

Conclusions

- Spacetime singularities are unavoidable in gravitational collapse, if the classical theory of general relativity is valid at all scales and the stress-energy tensor of matter satisfies the classical energy conditions.
- General relativity cannot be valid at all scales because of quantum mechanics.
- There is a certain expectation that near the center of a physical black hole quantum effects dominate, and prevent the formation of the singularity.
- Planck stars are one possible way to include effective QFT correction in general relativity.
- Hayward metric by itself does not cover all the desirable quantum correction into the Schwarzchild metric, therefore the proposal of a modified Hayward metric.

References

- Hayward, S.A.: Formation and Evaporation of regular black holes. Phys. Rev. Lett. **96**, 31103 (2006).
- De Lorenzo, T., Pacilio, C., Rovelli, C., Speziale, S.: On the effective metric of a Planck star. Gen. Relativ. Gravit. **47**, 41 (2015).
- Mazur, P. O., Mottola, E.: Gravitational condensate stars: An alternative to black holes, arXiv:gr-qc/0109035
- Ashtekar, A., Pawlowski, T., Singh, P., Vandersloot, K.: Loop quantum cosmology of k=1 FRW models. Phys. Rev. D **75**,24035 (2007).
- Rovelli, C., Vidotto, F.: Planck Stars. Int. J. Mod. Phys. D. **23**, 1142026, (2014).
- Bjerrum-Bohr, N.E.J., Donoghue, J.F., Holstein, B.R.: Quantum gravitational corrections to the non-relativistic scattering potential of two masses. Phys. Rev. D **67**, 084033 (2003).

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