A study of the Reissner-Nordström black hole

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1 Introduction

General Relativity predicts that evolution of certain stars with spherical symmetry and very high masses can not end up in equilibrium. This happens because when the nuclear fuel of such a star is exhausted, gravitational effects will be stronger that the internal pressure of the star involved, leading to a collapse of all the mass of the star into a singularity placed in the center of the symmetry. Such bodies are known as black holes and are of much interest in current physics branches such as Cosmology. Our aim here is to make a survey about this topic.

The problem that arises is then to find the geometry that space-time will assume in the neighbourhood of a black hole. There exist several solutions to the previous question, depending on the characteristics of the body that one is dealing with. The fact that there are singularities in space-time raised since Schwarzschild presented his solution to the Einstein equations in vacuum.

An important result that allows us to classify black holes is known as *The no-hair Theorem*. This theorem asserts that all solutions for the Einstein-Maxwell equations in general relativity can be completely characterized by three external parameters: mass (m), charge (q) and angular momentum (j). The idea under this proposition is that the information encoded in the rest of observables of the black hole would be behind the event horizon and would be unreachable to an external observer, making black holes indistinguishable beyond the three parameters previously mentioned.

With that in mind, the following table would help us to characterize all types of black holes

	Non-rotating ($j = 0$)	Rotating ($j \neq 0$)
Uncharged ($q = 0$)	Schwarzschild	Kerr
Charged ($q \neq 0$)	Reissner-Nordström	Kerr-Newmann

The Schwarzschild black hole corresponds to a body of mass M without charge neither angular momentum that collapses; it is the simplest solution to the problem that concerns us here. One can also consider a Schwarzschild black hole and add a charge q to obtain the Reissner-Nordström solution (RN), which we are going to study in the present work.

Then, the main goal of our project is to study the space-time due to a non-rotating body of mass m and charge q that collapses into a singularity. To do so, it is necessary to find the solutions to the coupled Einstein-Maxwell field equations. This duty was first developed by Hans Reissner and Gunnar Nordström in 1916, for whom the solution receives its name.

It is worth to mention that it is difficult to find bodies that generate the RN solution in nature, that is because the charge in the interior of the black hole will tend to neutralize in a short time, becoming a Schwarzschild black hole. In spite of that, the RN solution is very useful to tackle the more complicated problems of black holes with mass m and angular momentum j (the Kerr solution) or the general case of a black hole that depends on the three possible parameters (the Kerr-Newmann solution), which actually appear in nature.

Finally, another reason for studying the RN solution are the physical results that can be interpreted from it, which are of much academical interest for us.

2 Preliminaries

2.1 Birkhoff's Theorem

In this section, we present an important result about uniqueness of the solutions that will allow us to characterize the geometry of space-time around the different kinds of black holes.

Theorem (Birkhoff,1923): *Let the geometry of a given region of space-time be such that is spherically symmetric and is a solution of Einstein's field equations in vacuum. Then this geometry is necessarily a piece of the Schwarzschild geometry.*

Proof: (We follow [1]). If we consider a spherical region of space-time, then we can introduce the coordinates (For a rigorous derivation of this line element see [1] Box 23.3)

$$ds^2 = -e^{2\alpha}dt^2 + e^{2\beta}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Where $\alpha = \alpha(r,t)$ and $\beta = \beta(r,t)$; applying Einstein field equations to this metric we obtain

$$G_{tt} = \frac{1-e^{-2\beta}}{r^2} + 2\frac{\beta_{,r}}{r}e^{-2\beta} = 0$$
$$G_{tr} = G_{rt} = 2\frac{\beta_{,r}}{r}e^{-\alpha+\beta} = 0$$
$$G_{rr} = 2\frac{\alpha_{,r}}{r}e^{-2\beta} + \frac{(e^{-2\beta}-1)}{r^2} = 0$$

$$G_{\theta\theta} = G_{\phi\phi} = (\alpha_{,rr} + \alpha_{,r}^2 - \alpha, r\beta, r + \frac{\alpha_{,r}}{r} - \frac{\beta_{,r}}{r})e^{-2\beta} - (\beta_{,tt} + \beta_{,t}^2 - \beta_{,t}\alpha_{,t})e^{-2\phi} = 0$$

From the previous equations we obtain expressions for α and β

$$\alpha = \frac{1}{2}\ln\left|1-\frac{2M}{r}\right| + f(t)$$
$$\beta = -\frac{1}{2}\ln\left|1-\frac{2M}{r}\right|$$

Here $f(t)$ is an arbitrary function depending on time, if we introduce these expressions in the line element it becomes

$$ds^2 = -e^{2f(t)}\left(1-\frac{2M}{r}\right)dt^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Finally if we redefine the time component as $t' = \int e^{f(t)}dt$, the metric becomes the well known Schwarzschild metric. $\square$

This theorem establishes the uniqueness of the Schwarzschild solution and tells us that any spherically symmetric and vacuum region of the universe obeys Schwarzschild geometry. Naturally, there exists an extension of Birkhoff theorem to the RN geometry, which is of interest for us in this project.

Theorem: *Let the geometry of a given region of space-time be spherically symmetric and a solution of the coupled Einstein-Maxwell field equations. Then the geometry in question is a necessarily a piece of the RN geometry.*

The previous theorem gives us uniqueness of the RN solution, so that we can start working on the explicit solutions for the behavior of space-time in the neighbourhood of a charged black hole. The proof is similar to that of the previous theorem but it requires some more calculations, for a detailed proof cf [1].

2.2 Singularities

As when working with black holes one generally encounters singularities, in this section we give a definition of singularity based on Hawking and Ellis [2].

A space-time is said to be complete if there is an endpoint for every smooth curve of finite length, measured by a generalized affine parameter. In other words, every geodesic can be maximally extended. If a space-time is incomplete, then it has a singularity, which implies unboundedness of the curvature.

Theorem 2 of chapter 8 in [2] gives conditions for existence of singularities. The hypothesis are three, roughly speaking: i) a condition over the energy/matter content of the fields coupled to gravity, ii) a condition over the global and causal structure of space-time, and iii) the validity of some geometric properties related to the behaviour of geodesics.

3 Development

As we mentioned before, RN metric is a static and spherically symmetric (SS) solution of the coupled Maxwell-Einstein equations. By Birkhoff's Theorem, we know that this solution is unique. In this section we will solve the equations to obtain the RN metric, and will analyse some important properties of this space-time. We follow [3].

Since we are considering spherical symmetry, then the metric can be expressed as follows (see chapter 7 of [4] or [5])

$$ds^2 = -e^{2\alpha(r)}dt^2 + e^{2\beta(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{1}$$

with $\alpha(r)$ and $\beta(r)$ functions that we must determine. The Einstein's field equations are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2}$$

where $R_{\mu\nu} = R^{\rho}_{\mu\rho\nu}$ is the Ricci curvature tensor obtained from the Riemann curvature tensor

$$R^{\rho}_{\gamma\alpha\beta} \equiv \partial_\alpha \Gamma^{\rho}_{\beta\gamma} + \Gamma^{\rho}_{\alpha\lambda}\Gamma^{\lambda}_{\beta\gamma} - \partial_\beta \Gamma^{\rho}_{\alpha\gamma} - \Gamma^{\rho}_{\beta\lambda}\Gamma^{\lambda}_{\alpha\gamma} \tag{3}$$

Coefficientes of the Levi-Civita connection (Christoffel Symbols) are given by

$$\Gamma^{\gamma}_{\mu\nu} \equiv \frac{1}{2} g^{\gamma\lambda} \left(g_{\lambda\nu,\mu} + g_{\lambda\mu,\nu} - g_{\mu\nu,\lambda} \right) \tag{4}$$

From Maxwell's electromagnetism we have the energy-momentum tensor $T_{\mu\nu}$, given by

$$T_{\mu\nu} = F_{\mu\rho} F^{\rho}_{\nu} - \frac{1}{4} g_{\mu\nu} F_{\sigma\rho} F^{\sigma\rho} \tag{5}$$

with $F_{\mu\nu}$ the electromagnetic field tensor. Given (5) it is easy to see that $T_{\mu\nu}$ is traceless because

$$T = g^{\mu\nu} T_{\mu\nu} = F_{\mu\rho} F^{\mu\rho} - F_{\sigma\rho} F^{\sigma\rho} = 0 \tag{6}$$

With (6) we can show that $R_{\mu\nu}$ is also traceless. Recalling $R = g^{\mu\nu} R_{\mu\nu}$, we obtain from (2)

$$R - 2R = 8\pi G T = 0$$

Therefore

$$R = 0 \tag{7}$$

Since $T = R = 0$, we can rewrite (2) as follows

$$R_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{8}$$

In addition, $F_{\mu\nu}$ must satisfy Maxwell's equations in source-free regions

$$\nabla_\mu F^{\mu\rho} = 0 \tag{9}$$

$$\nabla_{[\mu} F_{\nu\rho]} = 0 \tag{10}$$

where, for any two contravariant tensor $A^{\nu\sigma}$ we have

$$\nabla_\mu A^{\nu\sigma} = \partial_\mu A^{\nu\sigma} + \Gamma^{\sigma}_{\mu\lambda} A^{\lambda\nu} + \Gamma^{\nu}_{\mu\lambda} A^{\sigma\lambda}$$

The assumption that the field is due to a non rotating, static charged black hole, which we take to be situated at the origin of coordinates, means that the line element and the Maxwell tensor will become singular there. Moreover, the charged black hole will give rise to an electrostatic field $E(r)$ (no magnetic field) which is purely radial. This arguments lead us to the conclusion that the Maxwell tensor must take the form

$$F_{\mu\nu} = E(r) \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{11}$$

Plugging (1) and (11) into Maxwell's eqs. we see that (10) is satisfied automatically and (9) takes the form

$$\frac{d}{dr}\left(e^{-(\alpha(r)+\beta(r))} r^2 E(r)\right) = 0 \tag{12}$$

Therefore

$$E(r) = \frac{e^{(\alpha(r)+\beta(r))}}{r^2} q \tag{13}$$

With q a constat of integration. Our assumption that the solution is asymptotically flat (Minkowski) requires

$$\alpha(r), \beta(r) \to 0 \ as \ r \to \infty \tag{14}$$

so $E(r) \backsim \frac{q}{r^2}$ asymptotically. This latter result is exactly the same as the classical electric field due to a point charge q situated at the origin. We therefore interpret q as the charge of the black hole. Now, using (8) with $\mu\nu = tt$ and $\mu\nu = rr$ we find

$$\frac{d}{dr}\alpha(r) + \frac{d}{dr}\beta(r) = 0 \tag{15}$$

And taking into account (15) we conclude $\beta = -\alpha$. The equation $\mu\nu = \theta\theta$ is the one independent equation remaining and leads to

$$\frac{d}{dr}(re^{2\alpha(r)}) = 1 - \frac{q^2}{r^2} \tag{16}$$

From which we obtain

$$e^{2\alpha(r)} = 1 - \frac{2m}{r} + \frac{q^2}{r^2} \tag{17}$$

with m an constant of integration. Finally, we have the RN metric given by

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right) dt^2 + \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{18}$$

Where m is interpreted as the mass of the BH by comparing, to first order, the previous equation to Newton's metric

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Note that the RN solution has the following limits

- When $q \to 0$ we approach Schwarzschild metric, that is

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{19}$$

- When $r \to \infty$ we approach Minkowski space-time.

Now we will verify the value of the scalar curvature. Reading from (18), we have

$$g_{tt} = -\left(1 - \tfrac{2m}{r} + \tfrac{q^2}{r^2}\right) \quad g_{rr} = \left(1 - \tfrac{2m}{r} + \tfrac{q^2}{r^2}\right)^{-1}$$

$$g_{\theta\theta} = r^2 \qquad\qquad g_{\phi\phi} = r^2 \sin^2\theta$$

So the nonvanishing Christoffel symbols are

$$\Gamma^t_{rt} = \Gamma^t_{tr} = \tfrac{\left(mr - q^2\right)}{r(r^2 - 2mr + q^2)} \quad \Gamma^r_{tt} = \tfrac{\left(r^2 - 2mr + q^2\right)\left(mr - q^2\right)}{r^5} \qquad \Gamma^\phi_{r\phi} = \Gamma^\phi_{\phi r} = \tfrac{1}{r}$$

$$\Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \tfrac{1}{r} \qquad \Gamma^r_{\theta\theta} = -\tfrac{\left(r^2 - 2mr + q^2\right)}{r^2} \qquad \Gamma^r_{\phi\phi} = -\tfrac{\left(\sin^2\theta\right)\left(r^2 - 2mr + q^2\right)}{r}$$

$$\Gamma^r_{rr} = -\tfrac{\left(mr - q^2\right)}{r(r^2 - 2mr + q^2)} \qquad \Gamma^\theta_{\phi\phi} = -\left(\sin\theta\right)\left(\cos\theta\right) \qquad \Gamma^\phi_{\theta\phi} = \Gamma^\phi_{\phi\theta} = \cot\theta$$

Now we calculate some components of Riemann curvature tensor (see Appendix) to obtain the relevant components for the Ricci curvature tensor:

$$R_{tt} = \frac{-3(r^2-2mr+q^2)(mr-q^2)}{r^6} + \frac{2(r-m)(mr-q^2)}{r^5} + \frac{m(r^2-2mr+q^2)}{r^5} - \frac{2(mr-q^2)^2}{r^6}$$

$$R_{rr} = \frac{-2(mr-q^2)^2}{r^2(r^2-2mr+q^2)^2} - \frac{mr-q^2}{(r^2-2mr+q^2)} + \frac{2(r-m)(mr-q^2)}{r(r^2-2mr+q^2)^2} - \frac{m}{r(r^2-2mr+q^2)}$$

$$R_{\theta\theta} = \frac{q^2}{r^2}$$

$$R_{\phi\phi} = \frac{q^2 \sin^2\theta}{r^2}$$

Finally, the scalar curvature is given by

$$R = g^{tt}R_{tt} + g^{rr}R_{rr} + g^{\theta\theta}R_{\theta\theta} + g^{\phi\phi}R_{\phi\phi} = 0$$

As we have previously shown in (7). By an analogous procedure we can calculate the Kretchmann scalar [6]

$$K = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} = \frac{48m^2r^2 - 96mq^2 + 56q^2}{r^8} \tag{20}$$

From this curvature invariant we conclude that there is a physical singularity at $r = 0$, the same result one gets for the Schwarzschild metric. Now we analyse the structure of the event horizons, these are located at the radius where $-g_{tt}$ vanish, that is

$$1 - \frac{2m}{r} + \frac{q^2}{r^2} = 0 \tag{21}$$

this will occur at

$$r_{\pm} = m \pm \sqrt{m^2 - q^2} \tag{22}$$

Thus, we see that there are 3 possible results, depending on the discriminant of the latter equation. Let us analyse these cases separately

- **Case one:** $\left(m^2 - q^2\right) < 0$
  In this case $r_{\pm}$ are complex numbers, which implies that there are no event horizons and the metric is regular everywhere in the coordinates (t, r, θ, ϕ) except for $r = 0$. Thus, we get a naked singularity, which violates the Penrose cosmic censorship conjecture [7]. Note that this solution is not likely to be physical, since this condition states that the total energy of the black hole is less that the contribution to the energy just from the electromagnetic fields; therefore, the mass of the charged particle would be negative.

- **Case two:** $\left(m^2 - q^2\right) = 0$
  This case is known as an extremal black hole, since there is a single degenerate event horizon at $r_\pm = r = m$. In this solution the mass is balanced by the charge; nevertheless, this situation is unstable since adding a little bit of mass or charge would take it to one of the other 2 cases.

- **Case three:** $\left(m^2 - q^2\right) > 0$
  This situation in the most likely to be physical, and would apply in a real gravitational collapse, since the energy of the electromagnetic field is less than the total energy. There are two event horizons located at $r_\pm = m \pm \sqrt{m^2 - q^2}$, which represent coordinate singularities and can be removed by a change of coordinates. More precisely, r_+ represents an event horizon totally analogous to $r = 2M$ in the Schwarzschild metric, whereas r_- corresponds to a Cauchy horizon. The latter is characterized by the differentiation on the geometry of the space-time around it, since the coordinate r is timelike when $r_- < r < r_+$ and spacelike when $0 < r < r_-$. This horizon represents a limit to the domain of validity of the Cauchy problem (hence its name), for after crossing the border functions are no longer real and analytical (Cauchy-Kowalevski theorem, see [8]).

We now analyse qualitatively the radial null geodesics of the RN metric for case three, satisfying the following conditions

$$\dot{\theta} = \dot{\phi} = ds^2 = 0 \tag{23}$$

This leads to the equation

$$dt = \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1} dr \tag{24}$$

From which we take the solution for $r_+ < r$, and define a new time coordinate $\bar{t}$

$$\bar{t} = t + \frac{r_+^2}{r_+ - r_-} ln(r - r_+) - \frac{r_-^2}{r_+ - r_-} ln(r - r_-) \tag{25}$$

The coordinate system defined above is a type of advanced Eddington-Finkelstein coordinates. From (22) and defining

$$f = 1 - g_{tt} = \frac{2m}{r} - \frac{q^2}{r^2} \tag{26}$$

the line element then takes the form

$$ds^2 = -(1-f)d\bar{t}^2 + 2fd\bar{t}dr + (1+f)dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{27}$$

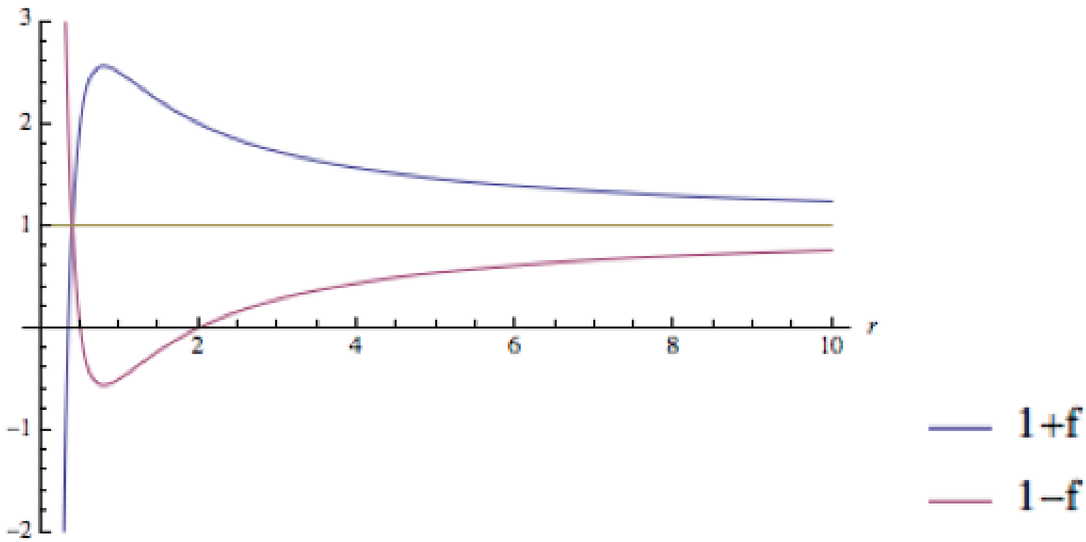

Figure 1: Graphs of the functions $1+f$ and $1-f$.

The latter leads to the ingoing family of null geodesics $\bar{t}+r = constant$ and the outgoing ones which satisfy the differential equation $\frac{d\bar{t}}{dr} = \frac{1+f}{1-f}$. In order to build a space-time diagram out of this, there is no need to solve the equations to extract useful information. For example, since f vanishes at infinity, the slope there would satisfy the Minkowski condition and therefore we would get an asymptotically flat solution as expected. As we come from infinity, $1+f$ increases and $1-f$ decreases (see figure 1), and so the slope increases until at $r = r_+$ take the value of infinity. In region II the slope increases from $-\infty$ at $r = r_+$ to some maximum negative value at $r = \frac{q^2}{m}$, and then decreases again to $-\infty$ as r tends to r_-. In region III, the slope decreases from ∞ until it reaches -1 at the origin (see figure 2).

It is clear from figure 2 that no signal can scape from region II to region I, that is why we call $r = r_+$ the event horizon. As we can see, in region II the light cones are slanted towards the center, so particles always move in direction to the origin, until they either reach asymptotically $r = r_-$ or cross that limit, arriving at region III. What is surprising

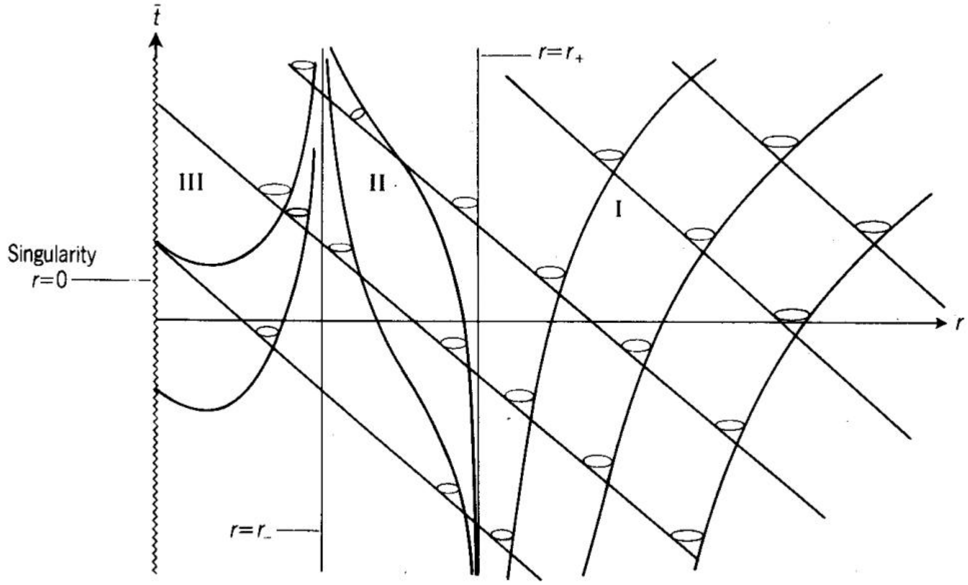

Figure 2: Reissner-Nordström geodesics ($q^2 < m^2$) in the advanced Eddington-Finkelstein type coordinates. From [3]

is that in region III particles do not need to fall into the singularity, as one would expect, since the light cones are no longer inclined towards the singularity, making it possible to move around "freely" inside the inner radius.

We now consider the maximal extension for this space-time. Starting from (26) and (27) we introduce the new variables

$$\nu = \bar{t} + r \tag{28}$$

$$\omega = 2t - v \tag{29}$$

The line element will transform into

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) d\nu d\omega + r^2 d\Omega^2 \tag{30}$$

It is worthwhile, trying to follow the Kruskal procedure for the Schwarzschild metric, to perform yet another transformation to the coordinates given by

$$\nu' = e^{\left(\frac{r_+ - r_-}{2r_+^2}\right)\nu} \tag{31}$$

$$\omega' = -e^{\left(\frac{r_- - r_+}{2r_+^2}\right)\omega} \tag{32}$$

Replacing this new coordinates in the metric (30) will yield the appropriate line element which presents maximal analytical extension to the RN solution, namely

$$ds^2 = -\left[\frac{4r_+^4(r - r_-)^{1+(\frac{r_-}{r_+})^2}}{r^2(r_+ - r_-)^2}\right] e^{\left(\frac{r_- - r_+}{r_+^2}\right)r} d\nu' d\omega' + r^2 d\Omega^2 \tag{33}$$

Where r is given implicitly by

$$\nu'\omega' = \left((r - r_+)(r - r_-)^{-\frac{r_-^2}{r_+^2}}\right) e^{\left(\frac{r_- - r_+}{r_+^2}\right)r}$$

This equation is valid only in the regime $q^2 < m^2$. From this we are ready to construct the most general Carter- Penrose Diagram for which the maximal extension, here performed, is physically relevant. Nevertheless it is a cumbersome calculation not included in the present work.

Therefore we construct the easier Carter-Penrose diagram for the case $q^2 > m^2$. In this regime, the RN metric 18 is already maximally extended for the coordinates here chosen; so the manifold is totally covered by the chart here used, except for $r = 0$ where we have a singularity, so the compactification of the space can be performed. With this in mind the diagram shown in figure 3 is achieved.

From the previous discussion and as we see from the diagram in figure 3 there exists only one asymptotically flat region in this case. Note that the singularity is not censored by an event horizon, so it is clearly visible to an external observer. Therefore all geodesics end in this singularity, as the lines $r = constant$ show.

4 Conclusions

After the development of this work we conclude that Birkhoff's theorem is a useful tool to study of the solutions of Einstein's field equations. The present work also allowed us

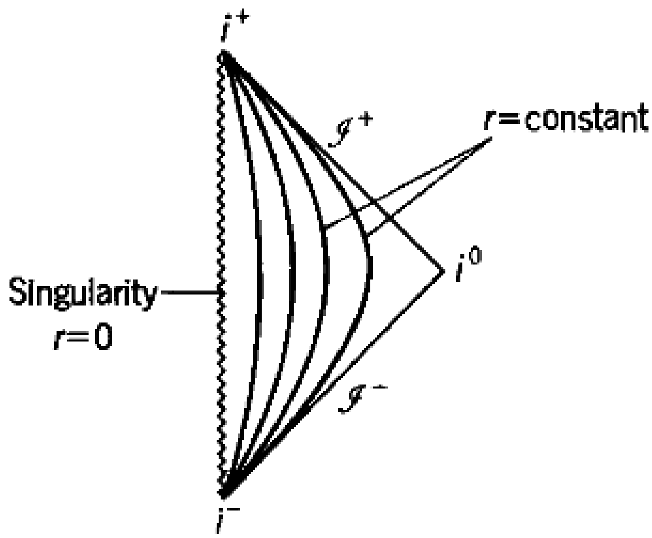

Figure 3: Carter-Penrose Diagram for RN metric in the case $q^2 < m^2$. From [3]

to study some relevant theorems in general relativity such as the no-hair theorem and the singularity theorems which we would like to study in detail in the future.

It is important to mention that we applied the knowledge acquired in the EET course to study a concrete problem in general relativity, in particular we could perform the explicit calculations of the Christoffel symbols, the Riemann and Ricci tensor components and the scalar curvature in two different ways. In addition, applying some useful coordinate changes, we studied the causal structure of the RN space-time and the Carter-Penrose diagram for the simplest case.

Finally, it would be of interest for us in the future to extend this work to the study of regular black holes (black holes without singularity) by considering non-linear electrodynamics. The previous, is an active research topic in Physics [9].

Appendix : Components of the Riemann tensor

$$R^t_{ttt} = 0$$

$$R^r_{trt} = -\frac{-5(r^2-2mr+q^2)(mr-q^2)}{r^6} + \frac{2(r-m)(mr-q^2)}{r^5} + \frac{m(r^2-2mr+q^2)}{r^5} - \frac{2(mr-q^2)^2}{r^6}$$

$$R^\theta_{t\theta t} = \frac{(r^2-2mr+q^2)(mr-q^2)}{r^6}$$

$$R^\phi_{t\phi t} = \frac{(r^2-2mr+q^2)(mr-q^2)}{r^6}$$

$$R^t_{rtr} = \frac{-2(mr-q^2)^2}{r^2(r^2-2mr+q^2)^2} + \frac{(mr-q^2)}{r^2(r^2-2mr+q^2)} + \frac{2(r-m)(mr-q^2)}{r(r^2-2mr+q^2)^2} - \frac{m}{r(r^2-2mr+q^2)}$$

$$R^r_{rrr} = 0$$

$$R^\theta_{r\theta r} = -\frac{mr-q^2}{r^2(r^2-2mr+q^2)}$$

$$R^\phi_{r\phi r} = -\frac{mr-q^2}{r^2(r^2-2mr+q^2)}$$

$$R^t_{\theta t\theta} = -\frac{(mr-q^2)}{r^2}$$

$$R^r_{\theta r\theta} = -\frac{(mr-q^2)}{r^2}$$

$$R^\theta_{\theta\theta\theta} = 0$$

$$R^\phi_{\theta\phi\theta} = \frac{(2mr-q^2)}{r^2}$$

$$R^t_{\phi t\phi} = -\frac{\sin^2\theta(mr-q^2)}{r^2}$$

$$R^r_{\phi r\phi} = -\frac{\sin^2\theta(mr-q^2)}{r^2}$$

$$R^\theta_{\phi\theta\phi} = \frac{\sin^2\theta(2mr-q^2)}{r^2}$$

$$R^\phi_{\phi\phi\phi} = 0$$

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