

Regular electrically charged vacuum structures with de Sitter center in Nonlinear Electrodynamics coupled to General Relativity

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We address the question of existence of regular spherically symmetric electrically charged solutions in Nonlinear Electrodynamics coupled to General Relativity. Stress-energy tensor of the electromagnetic field has the algebraic structure $T_0^0 = T_1^1$. In this case the Weak Energy Condition leads to the de Sitter asymptotic at approaching a regular center. In de Sitter center of an electrically charged NED structure, electric field, geometry and stress-energy tensor are regular without Maxwell limit which is replaced by de Sitter limit: energy density of a field is maximal and gives an effective cut-off on self-energy density, produced by NED coupled to gravity and related to cosmological constant Λ . Regular electric solutions satisfying WEC, suffer from one cusp in the Lagrangian $\mathcal{L}(F)$, which creates the problem in an effective geometry whose geodesics are world lines of NED photons. We investigate propagation of photons and show that their world lines never terminate which suggests absence of singularities in the effective geometry. To illustrate these results we present the particular example of the new exact analytic spherically symmetric electric solution with the de Sitter center.

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Introduction - Seventy years ago Born and Infeld proposed a Nonlinear Electrodynamics starting from *the principle of finiteness: a satisfactory theory should avoid letting physical quantities become infinite* [1]. In the standard picture based on the conception of a point charge, infinities come from the fact that its self-energy is infinite. Born and Infeld obtained a finite total energy for the field around a point charge by introducing an upper limit on the electric field strength E_{max} connected with an electron finite radius by $r_0 = \sqrt{e/E_{max}}$ which gives the boundary of applicability of their theory [1].

Today NED theories appear as low energy effective limits in certain models of string/M-theory (for review [2,3]).

In NED coupled to General Relativity there exists several regular solutions describing electrically charged black holes [4–8], magnetic black holes and monopoles [9], and regular black holes of hybrid type which are electrically charged but contain a magnetically charged core [10]. There exist also a number of theorems of non-existence ([9] and references therein) prohibiting existence of regular NED structures with an electric charge. They state that in NED with any $\mathcal{L}(F)$ such that $\mathcal{L} \sim F$ at small F (the Maxwell weak field limit) static spherically symmetric configurations with a non-zero electric charge cannot have a regular center [9].

Another doubt about regular electrically charged structures concerns the existence of singularities in the effective geometry in which NED photons propagate [11].

The aim of this paper is to show that Nonlinear Electrodynamics coupled to General Relativity and satisfying the Weak Energy Condition guarantees existence of electrically charged regular structures and provides a cutoff on self-energy which diverges for a point charge. One should only discard requirement of Maxwell weak field

limit at the center, on which non-existence theorems are based, because a field must not be weak to be regular.

We prove the existence of electrically charged structures with the regular center, in which geometry, field, and stress-energy tensor are regular without Maxwell limit as $r \rightarrow 0$. We study propagation of photons in the effective geometry and present the particular example of the new regular electrically charged solution.

The presented results apply directly to the cases when relevant NED scale is much lower than the Planck scale.

In NED coupled to GR each electric solution has its magnetic counterpart [9], whose existence is not forbidden, and we concentrate here on the electric structure with its existential problems.

Energy Conditions - The Weak Energy Condition (WEC), $T_{\mu\nu}\xi^\mu\xi^\nu \geq 0$ for any timelike vector ξ^μ , which is satisfied if and only if [12]

$$\rho \geq 0; \quad \rho + p_k \geq 0, \quad k = 1, 2, 3 \quad (1a)$$

guarantees that the energy density as measured by any local observer is non-negative.

The Dominant Energy Condition (DEC), $T^{00} \geq |T^{ik}|$ for each $i, k = 1, 2, 3$, which holds if and only if [12]

$$\rho \geq 0; \quad \rho + p_k \geq 0; \quad \rho - p_k \geq 0; \quad k = 1, 2, 3 \quad (1b)$$

includes WEC and requires each principal pressure $p_k = -T_k^k$ never exceed the energy density which guarantees that speed of sound cannot exceed the speed of light.

The Strong Energy Condition (SEC) requires [12]

$$\rho + \sum p_k \geq 0 \quad (1c)$$

and defines the sign of the gravitational acceleration.

Symmetry of a source term - Spherically symmetric electromagnetic field with an arbitrary gauge-invariant Lagrangian $\mathcal{L}(F)$, $F = F_{\mu\nu}F^{\mu\nu}$, has stress-energy tensor with the algebraic structure

$$T_t^t = T_r^r \quad (2)$$

It is invariant under rotations in the (r, t) plane, which enables to identify it as a vacuum defined by the symmetry of its stress-energy tensor [13,14]. An observer moving through a medium with stress-energy tensor of structure (2), cannot measure his velocity with respect to it which is typical for motion in a vacuum [15,16].

For the class of regular spherically symmetric geometries with the symmetry of a source term given by (2), the Weak Energy Condition leads inevitably to de Sitter asymptotic at approaching a regular center [17].

The basic fact of any geometry with de Sitter center generated by a source term of type (2), which does not depend on whether we identify it as a vacuum or not, is that the ADM mass of an object is related to both de Sitter vacuum trapped inside and smooth breaking of space-time symmetry from the de Sitter group in the origin to the Poincaré group at infinity [17].

For the spherically symmetric stress-energy tensor with the algebraic structure (2) the equation of state relating density $\rho = T_t^t$ with the radial pressure $p_r = -T_r^r$ and tangential pressure $p_\perp = -T_\theta^\theta = -T_\phi^\phi$, reads [13,17]

$$p_r = -\rho; \quad p_\perp = -\rho - \frac{r}{2}\rho' \quad (3)$$

where prime denotes differentiation with respect to r .

Basic equations - In nonlinear electrodynamics minimally coupled to gravity, the action is given by (in geometrical units $G = c = 1$)

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} (R - \mathcal{L}(F)); \quad F = F_{\mu\nu}F^{\mu\nu} \quad (4)$$

Here R is the scalar curvature, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field. The gauge-invariant electromagnetic Lagrangian $\mathcal{L}(F)$ is an arbitrary function of F which should have the Maxwell limit, $\mathcal{L} \rightarrow F$, $\mathcal{L}_F \rightarrow 1$ in the weak field regime.

The action (2) gives the dynamic field equations

$$(\mathcal{L}_F F^{\mu\nu})_{;\mu} = 0; \quad {}^*F^{\mu\nu}_{;\mu} = 0 \quad (5)$$

where $\mathcal{L}_F = d\mathcal{L}/dF$. In the spherically symmetric case the only essential components of $F_{\mu\nu}$ are a radial electric field $F_{01} = -F_{10} = E(r)$ and a radial magnetic field $F_{23} = -F_{32}$.

The Einstein equations take the form [9]

$$G_\nu^\mu = -T_\nu^\mu = 2\mathcal{L}_F F_{\nu\alpha} F^{\mu\alpha} - \frac{1}{2}\delta_\nu^\mu \mathcal{L} \quad (6)$$

Definition of $T_{\mu\nu}$ here differs from standard definition (see, e.g., [16]) by 8π , so that $T_{0(here)}^0 = 8\pi\rho$, etc.

The density and pressures for electrically charged structures are given by

$$\rho = -p_r = \frac{1}{2}\mathcal{L} - F\mathcal{L}_F; \quad p_\perp = -\frac{1}{2}\mathcal{L} \quad (7)$$

and scalar curvature is

$$R = 2(\mathcal{L} - F\mathcal{L}_F) = 2(\rho - p_\perp) \quad (8)$$

Symmetry of a source term (2) leads to the metric

$$ds^2 = g(r)dt^2 - \frac{dr^2}{g(r)} - r^2 d\Omega^2 \quad (9)$$

where $d\Omega^2$ is the line element on a unit sphere. The metric function and mass function are given by

$$g(r) = 1 - \frac{2\mathcal{M}(r)}{r}; \quad \mathcal{M}(r) = \frac{1}{2} \int_0^r \rho(x)x^2 dx \quad (10)$$

Dynamical equations (5) yield

$$r^2 \mathcal{L}_F F^{01} = q, \quad (11)$$

where q is constant of integration identified as an electric charge by asymptotic behavior in the weak field limit.

As follows from (10),

$$F = 2F_{01}F^{01} = -\frac{2q^2}{\mathcal{L}_F^2 r^4} \quad (12)$$

Theorems of non-existence require the Maxwell behavior at the regular center, $\mathcal{L} \rightarrow 0$, $\mathcal{L}_F \rightarrow 1$ as $F \rightarrow 0$. The proof is that regularity of stress-energy tensor requires $|F\mathcal{L}_F| < \infty$ as $r \rightarrow 0$, while $F\mathcal{L}_F^2 \rightarrow -\infty$ by virtue of (12), it follows that $\mathcal{L}_F \rightarrow \infty$ and $F \rightarrow 0$, which is strongly non-Maxwell behavior [9].

This sentence reads that a regular electrically charged structure does not compatible with the Maxwell weak field limit $\mathcal{L} \rightarrow 0$, $\mathcal{L}_F \rightarrow 1$ as $F \rightarrow 0$, in the center.

However, if the density does not vanish as $r \rightarrow 0$, then \mathcal{L} must not vanish there, although F vanishes in all cases of the regular center. Moreover, for solutions satisfying the Weak Energy Condition, density takes maximum there, since the WEC requires, by (1a) and (3), $\rho' \leq 0$. Then ρ is maximal at the center, and one cannot expect validity of the weak field limit in the region of maximal energy density of the field.

Let us fix the basic properties of electrically charged NED configurations obligatory for any Lagrangian $\mathcal{L}(F)$:

i) First is the fundamental observation of Bronnikov's theorem [9] - that F must vanish as $r \rightarrow 0$ to guarantee regularity, and the electric field strength is zero in the center of any regular electrically charged NED structure.

ii) Second is another observation from [9] - since F vanishes at both zero and infinity where it should follow

the Maxwell weak field limit, F must have at least one minimum in between where an electric field strength has a maximum. This leads to branching of $\mathcal{L}(F)$ as a function of F . This inevitable feature of electrically charged solutions creates problems in an effective geometry whose geodesics are world lines of NED photons [11].

iii) Third is the existence of surface of zero gravity at which Strong Energy Condition is violated. For all electric NED configurations this reads $2p_\perp = -\mathcal{L} \geq 0$, and SEC is violated at the surface $\mathcal{L} = 0$.

NED structures satisfying WEC - The Weak Energy Condition requires density be non-zero and maximal in the origin, since with $\rho \geq 0$ and $\rho' \leq 0$, a density cannot decrease beyond zero being obliged to be non-negative. Combined with the first property this raises the question - whose energy density is maximal in the center of structures where electric field tension vanishes?

The basic feature of all solutions of class (2) is de Sitter behavior at approaching the regular center [17]. Indeed, regularity of $\rho(r)$ requires $r\rho'/2 \rightarrow 0$ as $r \rightarrow 0$ (which is easily to check by taking $r\rho' = \text{const}$ and calculating ρ). With $|\rho'| < \infty$ the equation of state, by (3), tends to $p_r = p_\perp = -\rho$ as $r \rightarrow 0$ which gives de Sitter asymptotic

$$g(r) = 1 - \frac{\Lambda}{3}r^2 \quad (13)$$

with cosmological constant $\Lambda = 8\pi\rho(0)$. For electric NED structure Lagrangian $\mathcal{L}(F) \rightarrow 2\rho(0)$ as $r \rightarrow 0$, by (7), so that Lagrangian is positive and takes its maximal value at the center which testifies that the limiting density as $r \rightarrow 0$ is of electromagnetic origin.

Here we can answer the question *whose density is maximal as $r \rightarrow 0$ where electric field vanishes*. The T_0^0 component of electromagnetic stress-energy tensor does not vanish (neither diverges) as $r \rightarrow 0$ and provides an effective cutoff on self-interaction by relating it, through Einstein equations, with cosmological constant Λ corresponding to energy density of a vacuum, in this case the electromagnetic vacuum (2).

The WEC requirement $\rho + p_\perp \geq 0$ leads to $-F\mathcal{L}_F \geq 0$. It gives $\mathcal{L}_F \geq 0$. It gives also a constraint on a Lagrangian $\mathcal{L} \geq 2F\mathcal{L}_F$ as its obligatory low boundary.

The DEC requirement $\rho - p_\perp \geq 0$, is satisfied when $\mathcal{L} \geq F\mathcal{L}_F$. With $F\mathcal{L}_F \leq 0$ by WEC, this constraint is satisfied in the whole region surrounding the center including a certain region outside the surface of violation of the strong energy condition at which $\mathcal{L} = 0$.

Electrically charged solutions are typically found in the alternative form of NED obtained by the Legendre transformation: one introduces the tensor $P_{\mu\nu} = \mathcal{L}_F F_{\mu\nu}$ with its invariant $P = P_{\mu\nu}P^{\mu\nu}$ and consider Hamiltonian-like function $\mathcal{H}(P) = 2F\mathcal{L}_F - \mathcal{L}$ as a function of P ; the theory is then reformulated in terms of P and specified by

$\mathcal{H}(P)$ [19]. P frame is related with F frame by [19]

$$\mathcal{L} = 2P\mathcal{H}_P - \mathcal{H}; \quad \mathcal{L}_F\mathcal{H}_P = 1; \quad F = P\mathcal{H}_P^2 \quad (14)$$

Here $\mathcal{H}_P = d\mathcal{H}/dP$. The electric invariant is

$$P = -2P_{01}P^{01} = -\frac{2q^2}{r^4} \quad (15)$$

The metric in P frame is calculated from (10) with

$$\rho(r) = -\frac{1}{2}\mathcal{H} \quad (16)$$

FP duality coincides with conventional electric-magnetic duality only in the Maxwell limit where $\mathcal{L} = F = P = \mathcal{H}$ [9]. Interpretation of the results obtained in P framework depends essentially on transformation to F framework where Lagrangian dynamics is specified. The two frames are equivalent only when the function $F(P)$ is monotonic [9].

The function $F(P)$ which vanishes at both center and infinity has at least one minimum in which

$$\mathcal{L}_{FF} = \frac{1}{2} \left(\frac{\mathcal{H}_P}{F_P} - \mathcal{L}_F \right)$$

tends to infinities of opposite signs and $\mathcal{L}(F)$ suffers additional branching. Additional branching is related to extrema of the function $\mathcal{H}(P)$ [9].

While the first kind of branching is inevitable, the second is avoided by WEC, since $\mathcal{L}_F \geq 0$ results in $\mathcal{H}_P \geq 0$. When $\mathcal{H}(P)$ is monotonic function, the function $\mathcal{L}(F)$ has only two branches related to one minimum of F [9]. This loses problems with restoring F-frame Lagrangian dynamics from P-frame results. With one cusp interpretation is transparent and inevitable cusp becomes the source of information about most interesting behavior of electrically charged NED structures which displays in propagation of photons in an effective geometry. Typical behavior of Lagrangian \mathcal{L} as a function of F is depicted in Fig.1.

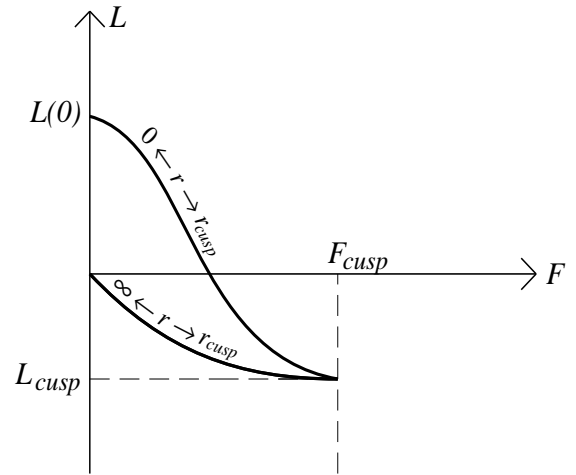


FIG. 1. Typical behavior of a Lagrangian $\mathcal{L}(F)$.

At the cusp surface $r = r_{cusp}$ the invariant F has minimum as a function of P and a function of r (since $P(r)$ is monotonic). The Lagrangian r -derivative there $\mathcal{L}' = \mathcal{L}_F F' = 0$ and \mathcal{L} takes its minimal value. The Lagrangian $\mathcal{L}(F)$ which is monotonic function of F ($\mathcal{L}_F \geq 0$), first decreases smoothly along the first branch from its maximal value $\mathcal{L}(0)$ to \mathcal{L}_{cusp} as F decreases from $F = -0$ at $r = 0$ to $F_{min} = F_{cusp}$; then the Lagrangian increases along the second branch from its minimal value $\mathcal{L}_{cusp} < 0$ to its Maxwell limit $\mathcal{L} \rightarrow F \rightarrow -0$ as F increases from F_{cusp} to $F \rightarrow -0$ as $r \rightarrow \infty$.

At the cusp the electric field $E^2(r) = -F(r)/2$ achieves its maximum.

Tangential pressure is maximal at the cusp surface, where $p'_\perp = -\mathcal{L}'/2 = 0$. In one-cusp configurations tangential pressure has one extremum, this is actually dictated by WEC which defines also the number of horizons. The function $g(r)$ has only one minimum and geometry described by the metric (9) can have not more than two horizons [17].

With $\mathcal{H}_P \geq 0$ the electric susceptibility $\epsilon = 1/\mathcal{H}_P$ is everywhere positive. When $P \rightarrow -\infty$ at the center $\mathcal{H}_P \rightarrow +0$ (since $\mathcal{L}_F \rightarrow \infty$ there), this leads to $\epsilon \rightarrow +\infty$, electric susceptibility is divergent, so that electrically charged NED configurations demonstrate ideal conducting behavior at approaching the regular center where the electric field tension vanishes.

Summarizing we conclude that *regular electrically charged NED structures satisfying Weak Energy Condition, have de Sitter center, not more than two horizons and precisely one cusp of $\mathcal{L}(F)$ where the electric field strength achieves its only maximum.*

New exact electric NED solution - Let us choose the function $\mathcal{H}(P)$ in the form

$$\mathcal{H}(P) = \frac{P}{(1 + \alpha\sqrt{-P})^2} \quad (17)$$

where α is characteristic parameter of the NED theory. Then we get

$$\mathcal{H}_P = \frac{1}{(1 + \alpha\sqrt{-P})^3} \quad (18)$$

$$F = \frac{P}{(1 + \alpha\sqrt{-P})^6}; \quad F_P = \frac{(1 - 2\alpha\sqrt{-P})}{(1 + \alpha\sqrt{-P})^7} \quad (19)$$

With P defined by (15) this gives

$$\mathcal{H} = -\frac{2q^2}{(r^2 + r_0^2)^2}; \quad \mathcal{H}_P = \frac{r^6}{(r^2 + r_0^2)^3} \quad (20)$$

The parameter $r_0^2 = \alpha\sqrt{2q^2}$ is fixed by integrating (10) with the density (16) which connects r_0 with the

total mass $m = \mathcal{M}(r \rightarrow \infty)$. This gives

$$r_0 = \frac{\pi q^2}{8m} \quad (21)$$

as classical electromagnetic radius modified by numerical coefficient of chosen particular NED model (17).

The only minimum of $F(P)$ is at $2\alpha\sqrt{-P} = 1$ and the cusp surface is given by

$$r_{cusp} = \sqrt{2}r_0 \quad (22)$$

The density and pressure are (up to 8π mentioned above)

$$\rho(r) = \frac{q^2}{(r^2 + r_0^2)^2}; \quad p_\perp = \frac{q^2(r^2 - r_0^2)}{(r^2 + r_0^2)^3} \quad (23)$$

Function $\rho(r)$ is monotonically decreasing, function $p_\perp(r)$ achieves maximum at the cusp surface.

The electric field is given by

$$F = -\frac{2q^2r^8}{(r^2 + r_0^2)^6}; \quad E^2 = \frac{q^2r^8}{(r^2 + r_0^2)^6} \quad (24)$$

It achieves its maximum at the cusp surface

$$E_{max} = \frac{4}{27} \frac{q}{r_0^2} \quad (24a)$$

and Maxwell limit $E \rightarrow 0$ as $r \rightarrow \infty$.

Lagrangian and its derivative are

$$\mathcal{L} = \frac{2q^2(r_0^2 - r^2)}{(r^2 + r_0^2)^3}; \quad \mathcal{L}_F = \frac{(r^2 + r_0^2)^3}{r^6} \quad (25)$$

The scalar curvature for this Lagrangian is given by

$$R = \frac{4q^2r_0^2}{(r^2 + r_0^2)^3} \quad (26)$$

It is positive everywhere, and the Dominant Energy Condition is satisfied (although we did not impose it) which is the good feature, since e.g., propagation of NED photons in an effective geometry resembles propagation inside a dielectric medium [11], and DEC makes it free of effects produced by speed of sound exceeding speed of light.

Integrating (10) with the density profile (23) we get the metric

$$g(r) = 1 - \frac{4m}{\pi r} \left(\arctg \frac{r}{r_0} - \frac{rr_0}{r^2 + r_0^2} \right) \quad (27)$$

For $r \gg r_0$ it reduces to

$$g(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2} - \frac{2}{3} \frac{q^2 r_0^2}{r^4} \quad (28)$$

and has Reissner-Nordström limit as $r \rightarrow \infty$.

At small values of r , $r \ll r_0$ we get de Sitter asymptotic (13) with the cosmological constant

$$\Lambda = \frac{q^2}{r_0^4} \quad (29)$$

which gives proper expression for a cutoff on self-energy density by the finite value of electromagnetic density $T_0^0(r \rightarrow 0)$ related to the cosmological constant $\Lambda = 8\pi\rho(0)$ which appears at the regular center.

The mass, of electromagnetic origin, is related to this cutoff by $m = \pi^2\rho(0)r_0^3$, where r_0 is the classical electromagnetic radius.

Characteristic parameter which decides if a solution describes a regular electrically charged black hole either self-gravitating particle-like structure with de Sitter vacuum inside, is given by

$$\beta = \frac{8}{\pi^2} \left(\frac{2m}{q} \right)^2 = \frac{2}{\pi} \frac{r_g}{r_0} \quad (30)$$

where $r_g = 2m$ is the characteristic Schwarzschild radius.

For $\beta > \beta_{crit} = 2.816$ solution describes a black hole.

For $r_g \gg r_0$, two horizons are

$$r_- \simeq r_S \left(1 + 1.4 \frac{r_0}{r_g} \right); \quad r_+ \simeq r_g \left(1 + 1.3 \frac{r_0}{r_g} \right) \quad (31a)$$

Internal horizon in this limit is close to de Sitter horizon $r_S = \sqrt{3/\Lambda}$, and an event horizon to the Schwarzschild horizon r_g .

For $\beta = \beta_{crit}$ there is a double horizon

$$r_{\pm} = 1.825 r_0 \quad (31b)$$

The global structure of space-time with horizons is precisely the same as for de Sitter-Schwarzschild geometry [18] (pictures for two and one horizon cases are presented in [20], Fig.3), and differs from Reissner-Nordström case only in that the timelike surface $r = 0$ is regular.

In terms of $q/2m$ black hole exists for $q/2m \leq 0.536$, and for $q/2m > 0.536$ we have electrically charged self-gravitating particle-like NED structure.

NED electrically charged structure from the point of view of photons - In nonlinear electrodynamics photons do not follow null geodesics of background geometry, but propagate along null geodesics of an effective geometry [21]. In the spherically symmetric case it is described by the metric [11]

$$ds_{eff}^2 = \frac{g(r)}{\Phi(r)} dt^2 - \frac{dr^2}{g(r)\Phi(r)} - \frac{r^2}{\mathcal{L}_F(r)} d\Omega^2 \quad (32)$$

where

$$\Phi(r) = \frac{\mathcal{H}_P}{F_P} = \mathcal{L}_F + 2\mathcal{L}_{FF} \quad (33)$$

For any NED background geometry satisfying WEC, $\mathcal{H}_P \geq 0$ everywhere, and F has one minimum. The function Φ is negative for $r < r_{cusp}$ where F decreases, and positive for $r > r_{cusp}$ where F increases, so that $\Phi \rightarrow +\infty$

when $r \rightarrow r_{cusp} + 0^*$. At infinity, where $\mathcal{H} = P = F = \mathcal{L}$, we get $\Phi \rightarrow 1$ and $\mathcal{L}_F \rightarrow 1$. So, for a distant observer an effective metric is close to his background metric.

One cusp in background geometry creates one problem in the effective geometry. The most essential consequence of a cusp is the redshift as measured by a distant observer which is given by [11]

$$1 + z = \frac{\Phi}{\sqrt{g}} \quad (34)$$

It diverges at the BH horizon where $g(r)$ vanishes, and at the cusp surface $r = r_{cusp}$ where Φ diverges.

For a distant observer photons disappear beyond the surface $r = r_{cusp}$ in the same way as they disappear beyond the event horizon of a black hole.

To investigate what is going with photons after crossing the cusp surface, we should change coordinates r, t (which exchange their roles after crossing r_{cusp}), to coordinates R, τ given by

$$R = t + \int \frac{dr}{g(r)\sqrt{1 - g(r)/\Phi(r)}} \quad (35a)$$

$$\tau = t + \int \left(\frac{1}{g} - \frac{1}{\Phi} \right) \frac{dr}{\sqrt{1 - g/\Phi}} \quad (35b)$$

In the case of background geometry without horizons the metric function $g(r)$ is everywhere positive. We concentrate now on this case to show how photons feel a cusp itself, when there is no other peculiarities in space-time which they penetrate. (Their behavior on horizons related to $g(r)$ is similar to the case without a cusp.)

In coordinates (R, τ) the metric (32) transforms to

$$ds_{eff}^2 = d\tau^2 - \left(1 - \frac{g}{\Phi} \right) dR^2 - \frac{r^2(R, \tau)}{\mathcal{L}_F(r(R, \tau))} d\Omega^2 \quad (36)$$

The function g_{RR} never change its sign. The function Φ achieves its minimal positive value $\Phi \rightarrow 1$ as $r \rightarrow \infty$ (where (32) coincides with the background metric).

In coordinates (R, τ) surfaces $r = const$ are straight lines $\tau - R = const$ (more precise, some $f(r = const)$, see, e.g., [16]). Equation of light cones are given by

$$\frac{d\tau}{dR} = \pm \sqrt{1 - \frac{g}{\Phi}} \quad (37)$$

At infinity $g \rightarrow 1, \Phi \rightarrow 1$ and $d\tau/dR \rightarrow 0$, the cones are entirely open (compare with the Schwarzschild case where $d\tau/dR = \pm \sqrt{r_g/r}$). Light cones in an effective geometry (36) are shown in Fig.2.

*For the above exact solution $\Phi(r) = \frac{(r^2 + r_0^2)^4}{r^6(r^2 - 2r_0^2)}$.

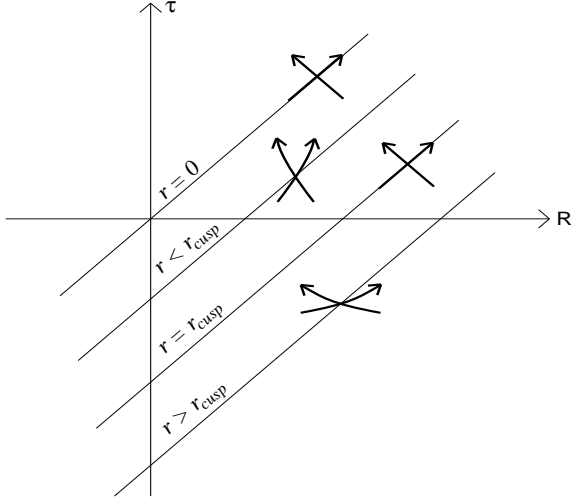


FIG. 2. Light cones (37) in an effective geometry (36).

For $r > r_{cusp}$, $\frac{d\tau}{dR} < 1$, outgoing photons move towards a distant observer, ingoing photons towards a center.

For $r = r_{cusp}$, $\frac{d\tau}{dR} = \pm 1$, ingoing photons moves toward a center, outgoing photons are kept on the surface $r = r_{cusp}$ (as it should be for a horizon which is the null surface, in Schwarzschild case $\frac{d\tau}{dR} = \pm \sqrt{\frac{r_g}{r}} = \pm 1$).

For $r < r_{cusp}$ cones are directed inward - all photons are ingoing. The cone is maximally narrow at the surface where $1 - \frac{q}{\Phi}$ is maximal, this is near the surface of zero gravity (for considered solution, $r \simeq r_0$). Beyond this surface cones start to widen again, still being inward-oriented (actually this is T_- region, a black hole in the effective geometry).

At approaching the center, $\mathcal{H}_P = \mathcal{L}_F^{-1} \rightarrow 0$ (since $\mathcal{L}_F \rightarrow \infty$ as $r \rightarrow 0$ for any electrically charged solution), while $F_P < \infty$. Therefore $\frac{d\tau}{dR} \rightarrow \pm 1$ as $r \rightarrow 0$. Some of photons are kept on the surface $r = 0$! Let us compare with Schwarzschild case where $\frac{d\tau}{dR} = \pm \sqrt{\frac{r_g}{r}} \rightarrow \infty$ as $r \rightarrow 0$, cones are entirely closed and perpendicular to the surface $r = 0$ which is the space-like singularity of the Schwarzschild geometry. In space-time created by NED electrically charged structure for photons, one of ruling of a light cone is on the surface $r = 0$, which appears to be the *null surface* of the effective geometry.

The further analysis is straightforward - one changes signs in (35b) to map T_+ region (a white hole in the effective geometry), and introduces Kruskal coordinates to follow photons everywhere and construct global structure of space-time as seen by NED photons.

The main conclusion is that photons world lines never terminate which suggests the absence of singularities in the background geometry satisfying WEC.

NED and cosmological constant - In Quantum Field Theory relativistic field is considered as collection

of harmonic oscillators of all possible frequencies, and vacuum as a superposition of ground states of all fields with the energy $E_{vac} = \sum \frac{1}{2} \hbar \omega$, where $\frac{1}{2} \hbar \omega$ is a zero-point energy of each particular field mode. Zero-point vacuum contribution to a stress-energy tensor has the form $\langle T_{\mu\nu} \rangle = \langle \rho_{vac} \rangle g_{\mu\nu}$ and behaves like a cosmological term. In QFT an upper cutoff on a vacuum energy density is estimated by a scale at which our confidence in formalism of QFT is broken, $E_{Pl} \sim 10^{19}$ GeV. With $\rho_{vac} \sim \rho_{Pl} \sim 10^{93} g cm^{-3}$, a QFT zero-point contribution to gravity through Einstein equations, is incompatible with observational data which constraint the vacuum contribution by 70 % of total density in the Universe, $\rho_{total} \sim 10^{-30} g cm^{-3}$.

This creates the *problem of cosmological constant* [22] which traces back directly to divergent self-energy of a point charge. In QED the infinite electromagnetic mass connected with self-energy of a point charge, is renormalized to a finite observable value of m_e by introducing an equally infinite negative mass of non- electromagnetic (and actually still unknown) origin [23].

As it was noted already in [1], theories based on geometrical assumptions about the "shape" of charge, etc, break down on similar reason: they are compelled to introduce *cohesive forces* of non-electromagnetic origin.

From above analysis it follows that NED coupled to GR makes it possible to get a regular extended charged structure without involving cohesive forces of non-electromagnetic origin. Finite self-energy is of purely electromagnetic origin since it comes to Einstein equations from electromagnetic Lagrangian, but then gravity, through Einstein equations, transforms *cohesive forces of electromagnetic origin* (negative pressure), into repulsive gravitational forces acting beyond the surface at which strong energy condition is violated. Beyond this surface NED structure displays rather de Sitter than Maxwell behavior, with vacuum energy density Λ coming from GR equations as an effective cutoff on a self-energy.

Summarizing we conclude that *NED coupled to GR and satisfying WEC, provides at the classical level a cut-off on self-energy related to cosmological constant Λ .*

At the classical level a NED structure with de Sitter center can be considered as an elementary excitation of an electromagnetic vacuum (2). A simplest quantum version can be obtained in frame of a minisuperspace model. A spherical object with de Sitter vacuum trapped inside can be described by a minisuperspace model with a single degree of freedom. Quantization leads to the Wheeler-De Witt equation which reduces to the Schrödinger equation with the potential $V(r) = -\mathcal{M}(r)/r$ [17]. Near the minimum of the potential, Schrödinger equation in turn reduces to the equation for the harmonic oscillator whose basic frequency gives the energy of a zero-point vacuum mode $E_0 \sim \hbar \sqrt{\Lambda}$ which never exceeds the absolute value of the negative binding energy V_{min} . One can expect that

a cutoff on a self-energy existing in the classical model would survive in its quantum version.

Conclusion - In Nonlinear Electrodynamics coupled to General Relativity and satisfying the Weak Energy Condition, regular electrically charged structures exist and have de Sitter regular center. In de Sitter center of a NED structure, electric field, geometry and stress-energy tensor are regular without Maxwell limit which is replaced by de Sitter limit: while the electric field tension vanishes, energy density of a field takes its maximal value (vacuum density in the center) which gives an effective cut-off on self-energy density, produced by NED coupled to gravity and related to cosmological constant Λ . Mass of objects which is of purely electromagnetic origin, is related to both de Sitter vacuum trapped inside and smooth breaking of space-time symmetry from the de Sitter group in the origin to the Poincaré group at infinity. All this concerns also structures possessing magnetic charge or both, since in NED coupled to GR each electric solution has its magnetic counterpart.

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