Formation and Evaporation of Nonsingular Black Holes

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Regular (nonsingular) space-times are given that describe the formation of a (locally defined) black hole from an initial vacuum region, its quiescence as a static region, and its subsequent evaporation to a vacuum region. The static region is Bardeen-like, supported by finite density and pressures, vanishing rapidly at large radius and behaving as a cosmological constant at small radius. The dynamic regions are Vaidya-like, with ingoing radiation of positive-energy flux during collapse and negative-energy flux during evaporation, the latter balanced by outgoing radiation of positive-energy flux and a surface pressure at a pair creation surface. The black hole consists of a compact space-time region of trapped surfaces, with inner and outer boundaries that join circularly as a single smooth trapping horizon.

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Introduction.—Black holes, predicted by Einstein gravity, appear to exist in the Universe. The singularities that were also predicted to form inside them [1], however, are generally regarded as indicating the breakdown of the theory, requiring modifications that presumably include quantum theory. A first step in this direction, quantum field theory on a stationary black-hole background, predicted Hawking radiation [2]. The ingoing radiation has negative-energy flux, which contradicts the assumptions of the singularity theorems and, in a semiclassical approximation, causes the black hole to shrink. In the usual picture [3], the black hole shrinks until the central singularity is reached. However, if the singularity does not exist, such a picture cannot be correct [4,5].

Regular (i.e., nonsingular) black holes have sometimes been considered, dating back at least to Bardeen [6]. One can find metrics that are spherically symmetric, are static, are asymptotically flat, and have regular centers, as well as for which the resulting Einstein tensor is physically reasonable, satisfying the weak energy condition and having components that are bounded and fall off appropriately at large distance. The simplest causal structure is similar to that of a Reissner-Nordström black hole, with the internal singularities replaced by regular centers (Fig. 1). Such space-times have been dismissed as unphysical, due to the presence of a Cauchy horizon, but if such a black hole evaporates, the Cauchy horizon is no more real than the event horizon, as examples show.

Imagine removing astrophysically irrelevant regions to the past and future of two consecutive advanced times (Fig. 1), then adjoining a past that describes gravitational collapse and a future that describes evaporation. The static region contains inner and outer horizons that no longer have global significance, but still have local significance as trapping horizons [7]. The key issue is how the trapping horizons develop, which has been predicted on general principles [5]. In this Letter, concrete models are given for the collapse and evaporation phases, using Vaidya-like regions [8] with ingoing or outgoing radiation.

Regular static black holes.—Consider static, spherically symmetric metrics of the form

$$ds^2 = r^2 dS^2 + dr^2/F(r) - F(r)dt^2, \tag{1}$$

where t is the static time, r is the area radius, and $dS^2 = d\theta^2 + d\phi^2 \sin^2\theta$. A surface has area $4\pi r^2$, is trapped if $F(r) < 0$, and is untrapped if $F(r) > 0$. Trapping horizons, in this case also Killing horizons, are located at the zeros $F(r) = 0$, and there is a standard procedure to match regions across such horizons [9]. For an asymptotically flat space-time with total mass m,

$$F(r) \sim 1 - 2m/r \quad \text{as } r \to \infty. \tag{2}$$

Similarly, flatness at the center requires

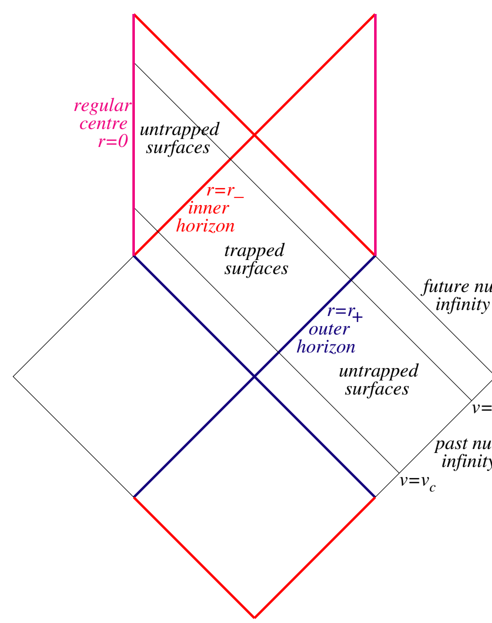

FIG. 1 (color online). Penrose diagram of a regular nonextreme black hole, to be identified vertically with isometric regions.

$$F(r) \sim 1 - r^2/l^2 \quad \text{as } r \to 0, \tag{3}$$

where l is a convenient encoding of the central energy density $3/8\pi l^2$, assumed positive. A sketch of $F(r)$ indicating where it might dip below zero (Fig. 2) shows that there will be a range of parameters for which there is no black hole, and that the simplest black-hole cases will generically have an inner and outer Killing horizon, the two cases separated by an extreme black hole with degenerate Killing horizon.

It can be shown that, for a metric g of the form (1) and (3), the Einstein tensor has the cosmological-constant form

$$G \sim -\Lambda g \quad \text{as } r \to 0, \qquad \Lambda = 3/l^2. \tag{4}$$

Thus, there is an effective cosmological constant at small distances, with Hubble length l. Such behavior has been proposed previously by Sakharov [10] as the equation of state of matter at high density and by Markov and others [11] based on an upper limit on density or curvature, to be ultimately justified by a quantum theory of gravity. Since l gives the approximate length scale below which such effects dominate, one might expect l to be the Planck length or of the same order, though larger length scales are not excluded.

A minimal model.—For definiteness, take a particularly simple metric satisfying the above conditions:

$$F(r) = 1 - \frac{2mr^2}{r^3 + 2l^2m}, \tag{5}$$

where (l, m) are constants that will be assumed positive. This is similar to the Bardeen black hole, reduces to the Schwarzschild solution for $l = 0$, and is flat for $m = 0$. Poisson and Israel [12] derived an equivalent form of $g^{rr} = F$ (without fixing g_{tt}) based on a simple relation between vacuum energy density and curvature.

Elementary analysis of the zeros of $F(r)$ reveals a critical mass $m_* = (3\sqrt{3}/4)l$ and radius $r_* = \sqrt{3}l$ such that, for $r > 0$, $F(r)$ has no zeros if $m < m_*$, one double zero at $r = r_*$ if $m = m_*$, and two simple zeros at $r = r_\pm$ if $m > m_*$ (Fig. 2). These cases therefore describe, respectively, a regular space-time with the same causal structure as flat space-time, a regular extreme black hole with degenerate Killing horizon, and a regular nonextreme black hole with

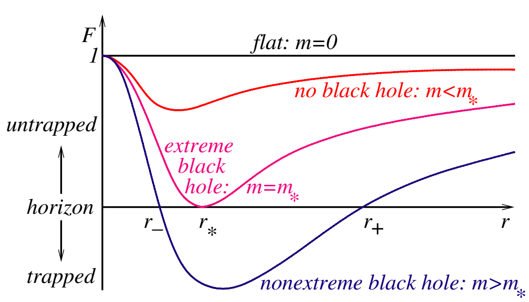

FIG. 2 (color online). The metric function $F = g^{rr}$, whose sign determines gravitational trapping, for fixed core radius l and different total masses m.

both outer and inner Killing horizons, located at $r_+ \approx 2m$ and $r_- \approx l$ for $m \gg m_*$ (Fig. 1). The horizon radii $r_\pm$ determine the mass (Fig. 3)

$$m(r_\pm) = \frac{\frac{1}{2}r_\pm^3}{r_\pm^2 - l^2}. \tag{6}$$

Note the existence of a mass gap: such black holes cannot form with mass $m < m_*$. Also, the inner horizon has radius $r_- > l$, which is very close to l for all but the smallest masses. In this sense, the black-hole core has a universal structure.

If the Einstein equation $G = 8\pi T$ is used to interpret components of the energy tensor T, these metrics are supported by density $-T_t^t$, radial pressure T_r^r, and transverse pressure $T_\theta^\theta = T_\phi^\phi$ given by

$$G_t^t = G_r^r = -\frac{12l^2m^2}{(r^3 + 2l^2m)^2}, \tag{7}$$

$$G_\theta^\theta = G_\phi^\phi = \frac{24(r^3 - l^2m)l^2m^2}{(r^3 + 2l^2m)^3}. \tag{8}$$

They fall off very rapidly, $O(r^{-6})$, at large distance. In terms of the energy E defined by

$$g^{rr} = 1 - 2E/r, \tag{9}$$

one finds the energy density $-T_t^t = (3l^2/2\pi)(E/r^3)^2$, proportional to the square of the curvature E/r^3. Poisson and Israel [12] assumed such proportionality as a property of vacuum energy density; then the component $dE/dr = -4\pi r^2 T_t^t$ of the Einstein equation implies g^{rr} equivalent to (5).

Adding radiation.—Next rewrite the static space-times in terms of advanced time

$$v = t + \int \frac{dr}{F(r)} \tag{10}$$

so that

$$ds^2 = r^2 dS^2 + 2dvdr - Fdv^2. \tag{11}$$

Now allow the mass to depend on advanced time, $m(v)$, defining $F(r, v)$ by the same expression (5). Then the density $-T_v^v$, radial pressure T_r^r, and transverse pressure T_θ^θ have the same form (7) and (8), but there is now an additional independent component, radially ingoing energy flux (or energy-momentum density) T_v^r given by

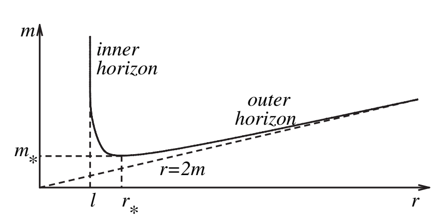

FIG. 3. Horizon mass-radius relation: a pair of horizons appears when mass m exceeds critical mass m_*.

$$G_v^r = \frac{2r^4 m'}{(r^3 + 2l^2 m)^2}, \tag{12}$$

where $m' = dm/dv$. This describes pure radiation, recovering the Vaidya solutions for $l = 0$ and at large radius. In the Vaidya solutions, the ingoing radiation creates a central singularity, but in these models, the center remains regular, with the same central energy density given by (3). It seems that the effective cosmological constant protects the core.

The ingoing energy flux is positive if m is increasing and negative if m is decreasing. A key point is that trapping horizons still occur where the invariant $g^{rr} = F(r, v)$ vanishes [7]. Then one can apply the previous analysis to locate the trapping horizons in (v, r) coordinates parametrized by m, given by $m(r_\pm)$ in (6) and a mass profile $m(v)$, qualitatively, by inspecting Figs. 3 and 4.

Ingoing radiation.—One can now model formation and evaporation of a static black-hole region. Introduce six consecutive advanced times $v_a < v_b < \cdots < v_f$ and consider smooth profiles of $m(v)$, meaning $m'(v)$ at least continuous, such that (Fig. 4)

$$\forall\ v \in (-\infty, v_a): m(v) = 0, \tag{13}$$

$$\forall\ v \in (v_a, v_c): m'(v) > 0, \tag{14}$$

$$\forall\ v \in (v_c, v_d): m(v) = m_0 > m_*, \tag{15}$$

$$\forall\ v \in (v_d, v_f): m'(v) < 0, \tag{16}$$

$$\forall\ v \in (v_f, \infty): m(v) = 0. \tag{17}$$

Then

$$\exists\ v_b \in (v_a, v_c): m(v_b) = m_*, \tag{18}$$

$$\exists\ v_e \in (v_d, v_f): m(v_e) = m_*. \tag{19}$$

These transition times mark the appearance and disappearance of a pair of trapping horizons: for $v < v_b$ and $v > v_e$, there is no trapping horizon, while for $v_b < v < v_e$, there are outer and inner trapping horizons, in the sense of the author's local classification [7]. These horizons join smoothly at the transitions and therefore unite as a single smooth trapping horizon enclosing a compact region of trapped surfaces (Fig. 5, for $r < r_0$).

Outgoing radiation.—Thus far, only the ingoing Hawking radiation has been modeled, since outgoing radiation does not enter the equation of motion of the trapping horizon; in terms of retarded time u, T_{vv} and T_{uv} enter, but T_{uu} does not [7]. Outgoing Hawking radiation will now be modeled by adapting an idea of Hiscock [13]: select a certain radius $r_0 > 2m_0$ outside the black hole, and adopt the above negative-energy radiation only inside that radius, balanced by outgoing positive-energy radiation outside that radius, with the same mass profile (Fig. 5). This is an idealized model of pair creation of ingoing particles with negative energy and outgoing particles with positive energy, locally conserving energy.

In more detail, consider an outgoing Vaidya-like region

$$ds^2 = r^2 dS^2 - 2du dr - F du^2 \tag{20}$$

with $F(r, u)$ as before (5), with m replaced by a mass function $n(u)$. Fix the zero point of the retarded time u so that $r = r_0$ corresponds to $u = v$. Now take the above model only for $v < v_d$ (13)–(15). For $v > v_d$, keep the profiles (16) and (17) for $r < r_0$, but for $r > r_0$, take an outgoing Vaidya-like region with

$$\forall\ u < v_d: n(u) = m_0, \tag{21}$$

$$\forall\ u > v_d: n(u) = m(u). \tag{22}$$

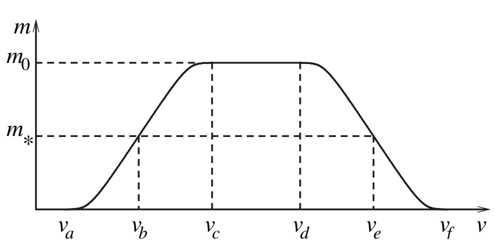

FIG. 4. A mass profile $m(v)$ in advanced time v.

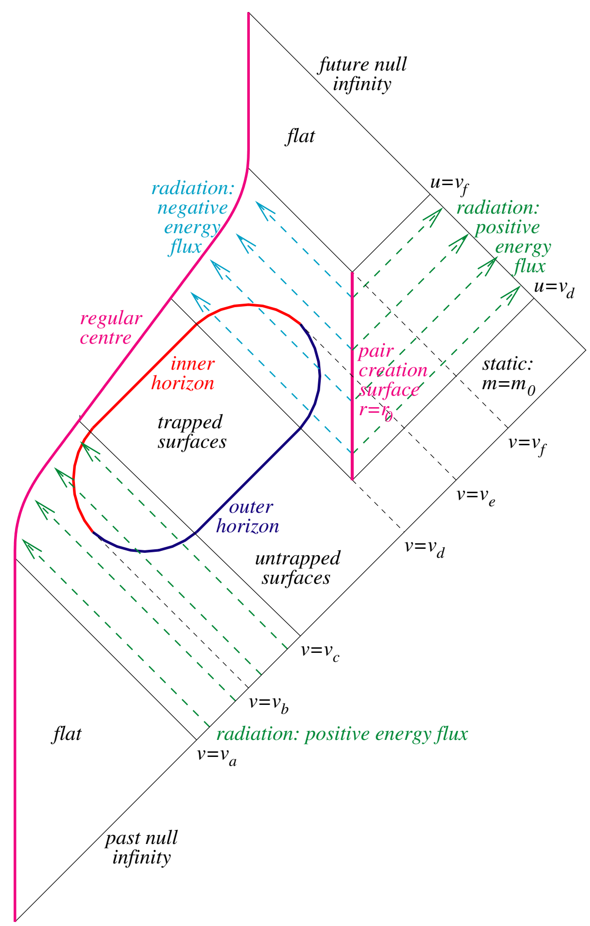

FIG. 5 (color online). Penrose diagram of formation and evaporation of a regular black hole in the given models.

Then there is a static region with total mass m_0 for $v > v_d$, $u < v_d$, and a flat region for $v > v_f$, $u > v_f$. Since the ingoing and outgoing radiation has no net energy but a net outward momentum, one might expect the pair creation surface $r = r_0$ to have a surface layer with no surface energy density but surface tension $\tau < 0$. This is confirmed using the Israel formalism [14], yielding

$$-\frac{16\pi(g^{rr})^{3/2}}{r}\tau = [G^{rr}] = -\frac{4r^4 m'}{(r^3 + 2l^2 m)^2} \qquad (23)$$

at $r = r_0$, $v_d < v < v_f$.

The whole picture is given in Fig. 5. Action begins at $v = v_a$, and a black hole begins to form at $v = v_b$, has collapsed completely at $v = v_c$ to a static state with mass m_0, begins to deflate at $v = v_d$, and eventually evaporates at $v = v_e$, leaving flat space finally after $v = v_f$, $u = v_f$. There is no singularity and no event horizon.

Remarks.—A trapping horizon with both inner and outer sections typically develops in numerical simulations of binary black-hole coalescence, in analytical examples of gravitational collapse such as Oppenheimer-Snyder collapse and according to general arguments [15]. A key point here is that the inner horizon never reaches the center, where a singularity would form [5]. This is compatible with the classical singularity theorems [1], which make assumptions that are already not satisfied by a Bardeen black hole, such as the strong energy condition. The negative-energy nature of ingoing Hawking radiation shows that such theorems do not apply to a black hole that might someday begin to evaporate.

In contrast to the usual picture [3], the end point $v = v_e$ of evaporation, defined locally by the disappearance of trapped surfaces, occurs when the outer and inner sections of the trapping horizon reunite. The subsequent time scale until the effective cessation of particle production at $v = v_f$ can be expected to be of the same order as l. Another logical possibility is that the inner and outer horizons approach each other asymptotically, forming the horizon of an extreme black hole with $m = m_*$, but such a delicately balanced situation would require justification.

The possibility of a circular trapping horizon has, in fact, been conjectured before [16]. Since there is no event horizon, long accepted as the defining property of a black hole, it seems necessary to stress that the static region looks just like a black hole over time scales that can be arbitrarily long. Thus, it should be regarded as a black hole by any practical definition, as in the local, dynamical paradigm for black holes in terms of trapping horizons [7]. The nonexistence of an event horizon for a black hole that eventually evaporates seems to have been recently accepted by its most influential proponent [17].

Most discussions of black-hole evaporation mention a certain I-word, as a paradox, problem, or puzzle. The above space-times, regular as well as with the causal structure of flat space-time, show that this word need not be mentioned. To paraphrase an old gravitational adage: what goes in, must come out.

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