University of Colorado
Boulder

CSCI 3104 Algorithms

Review of Fall 2014
Midterm Exam 1

Problem 1

- In each of the following situations, indicate whether f = O(g) or f = Ω(g), or both (in which case f = Θ(g)). Briefly explain why.
  - definitions of O(), Ω(), Θ()
  - exponential, polynomial, logarithm
  - lower order terms, leading coefficient

Problem 1 (cnt'd)

- In each of the following situations, indicate whether f = O(g) or f = Ω(g), or both (in which case f = Θ(g)). Briefly explain why.
  - (a) f(n) = $8n \log (8n)$, g(n) = $3n^3 + 2n^2 - 100$
  - (b) f(n) = $\log (10n+5)$, g(n) = $\log (n^7)$
  - (c) f(n) = $100n^7 + 10n^5 - n^3$, g(n) = $2^{n+1} + 2n^2$
  - (d) f(n) = $10n^{1/2} + (7/3)^n$, g(n) = $8n^8 + 4 \log n$

Problem 2

- Provide a brief answer for each of the following questions.
  - (a) In the RSA public-key scheme, what information can an eavesdropper obtain? How can the eavesdropper determine the original message that was sent?
    - public key (N, e) and encoded message
    - either guess the original message or try to factor N into p and q

Problem 2 (cnt'd)

- Provide a brief answer for each of the following questions.
  - (b) Given an undirected graph G and a starting vertex S, can we find all vertices in the graph using the explore(G, S) algorithm discussed in class? Explain why.
    - No. explore(G, S) follows edges to neighbors, cannot find all vertices in graphs with disconnected components.

Problem 2 (cnt'd)

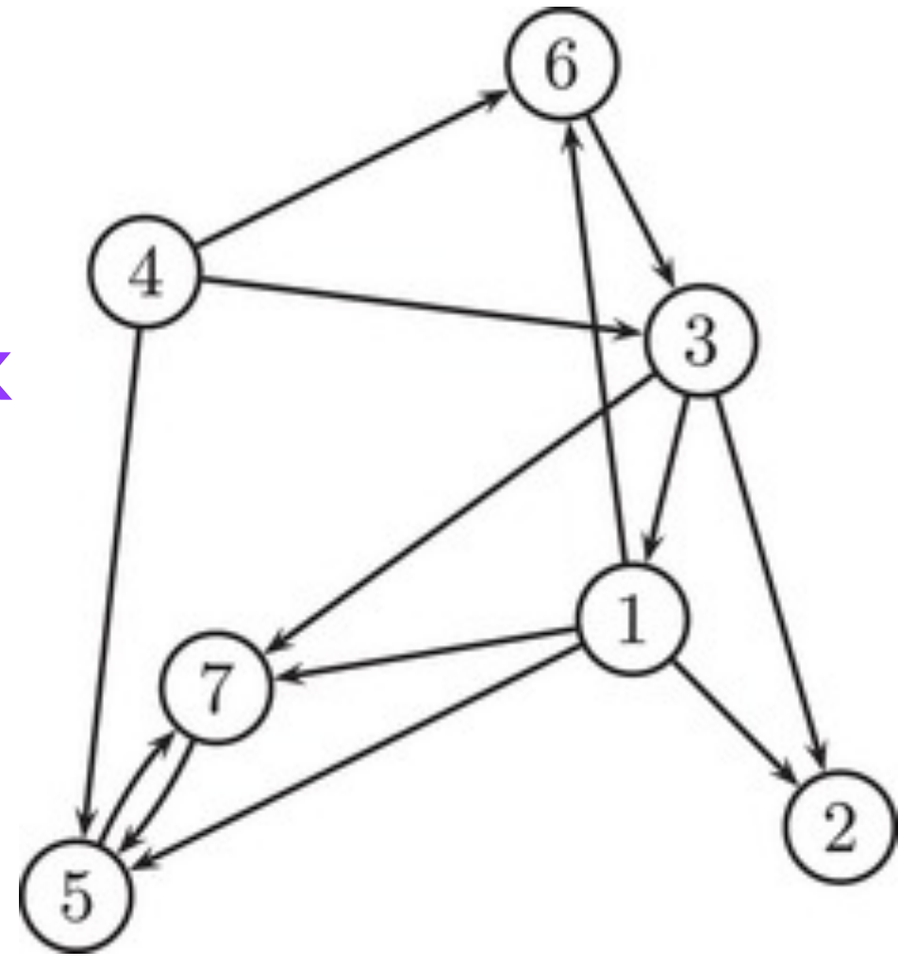
- Provide a brief answer for each of the following questions.
  - (c) If an algorithm solves a problem of size n by dividing it into nine subproblems of size n/3, recursively solving each subproblem, and then combining the solutions in $O(n^2)$ time, what is the time complexity of this algorith?
- $T(n) = 9T(n/3) + O(n^2) \implies O(n^2 \log n)$

Problem 2 (cnt'd)

- Provide a brief answer for each of the following questions.
  - (d) Show the key steps of using the mergesort algorithm to sort the following array of values into ascending order: [139, 72, 89, 254, 35, 331, 158, 40]
  - divide-and-conquer

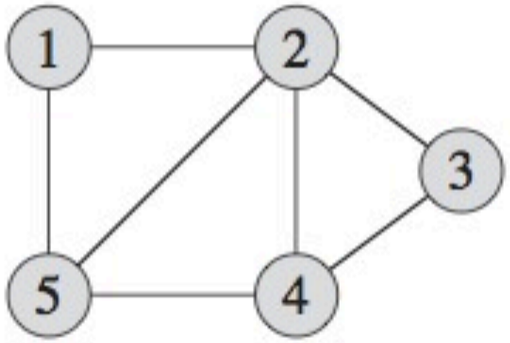
Problem 3: Graph Algorithms

- (a) Using the adjacency list representation, how many linked lists are needed to represent this graph? Draw the linked list for vertex 3.

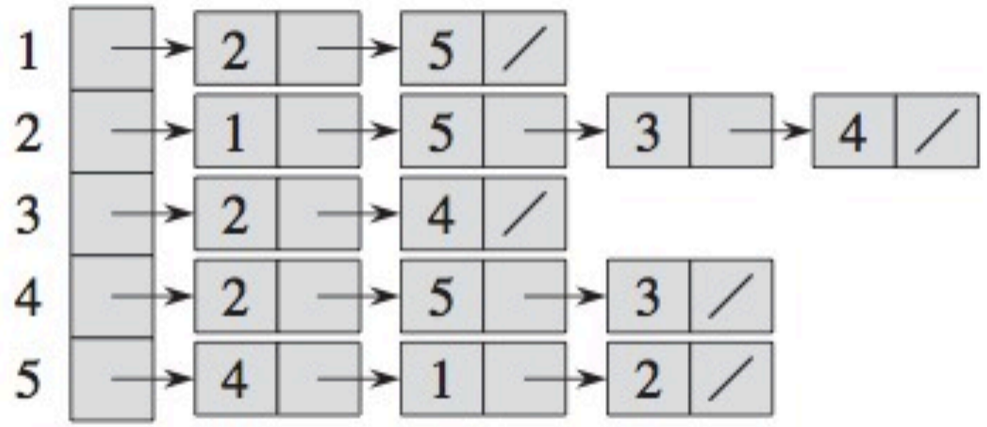


  - |V| = 7
  - 7 linked lists, one per vertex
  - V3: 1 => 2 => 7

Graph Representation Examples



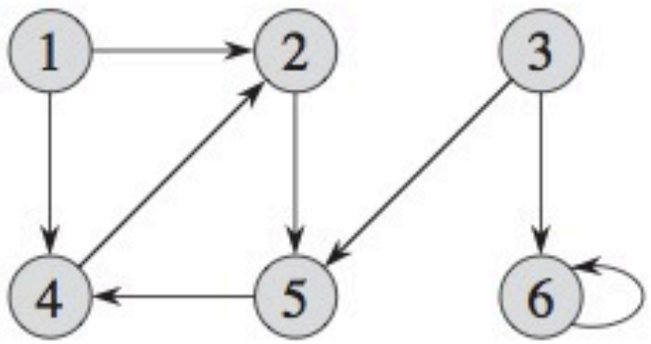
(a)



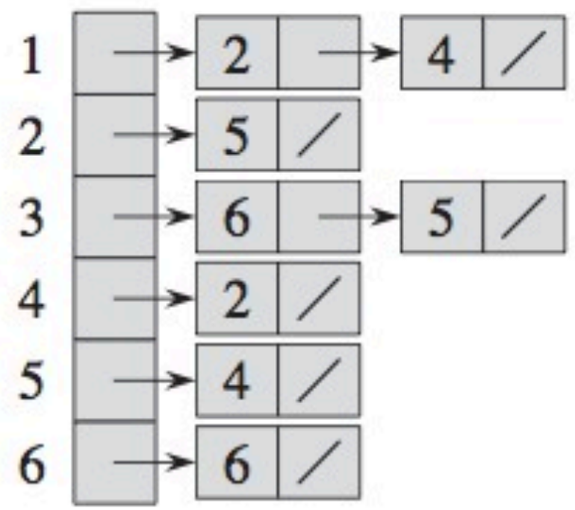
(b)

	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

(c)



(a)



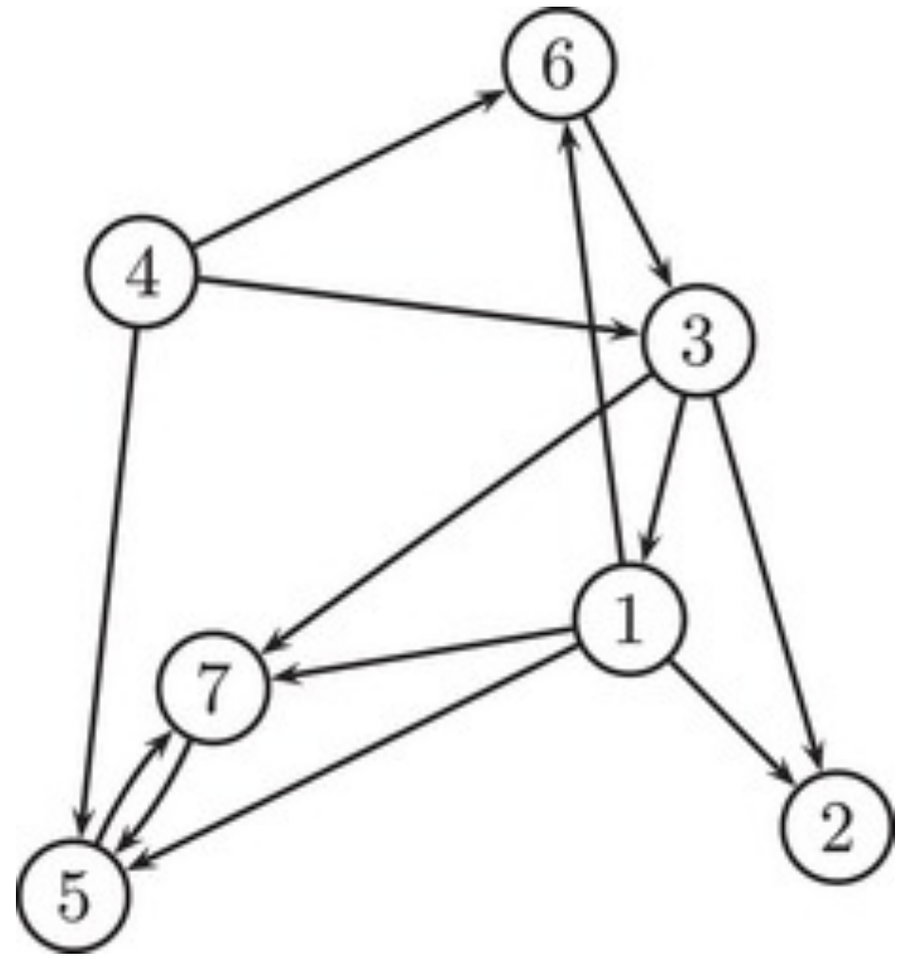
(b)

	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

(c)

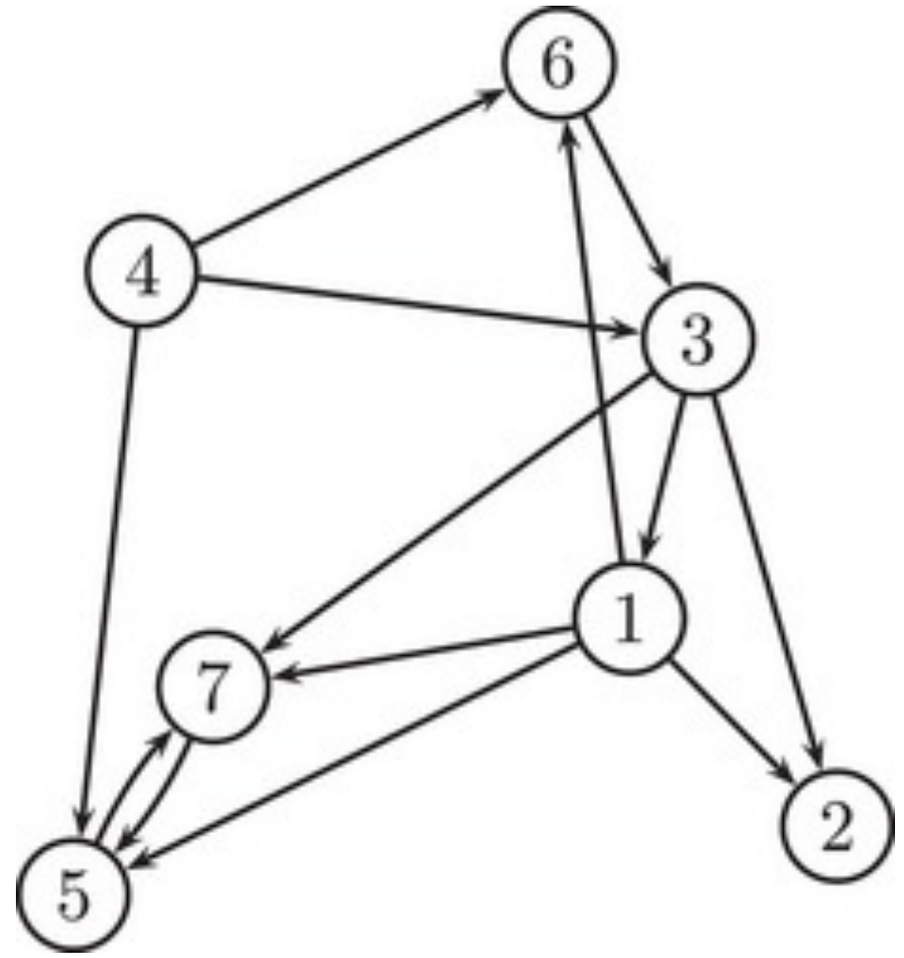
Problem 3: Graph Algorithms

- (b) Draw the DFS search forest, breaking ties by examining smaller-valued vertex first.



Problem 3: Graph Algorithms

- (c) Draw the corresponding meta-graph. Is the meta-graph a directed acyclic graph (DAG)?
  - SCCs => meta-nodes
  - {1, 3, 6}, {2}, {4}, {5, 7}
  - meta-graph
  - DAG



Problem 3: Graph Algorithms

- (d) Show the result of topological sort (linearization) of the meta-graph.
  - linear ordering
  - {4}
  - {1, 3, 6}
  - {2}
  - {5, 7}