University of Colorado
Boulder

CSCI 3104 Algorithms

Fall 2015
Lecture 17 (Oct 5)

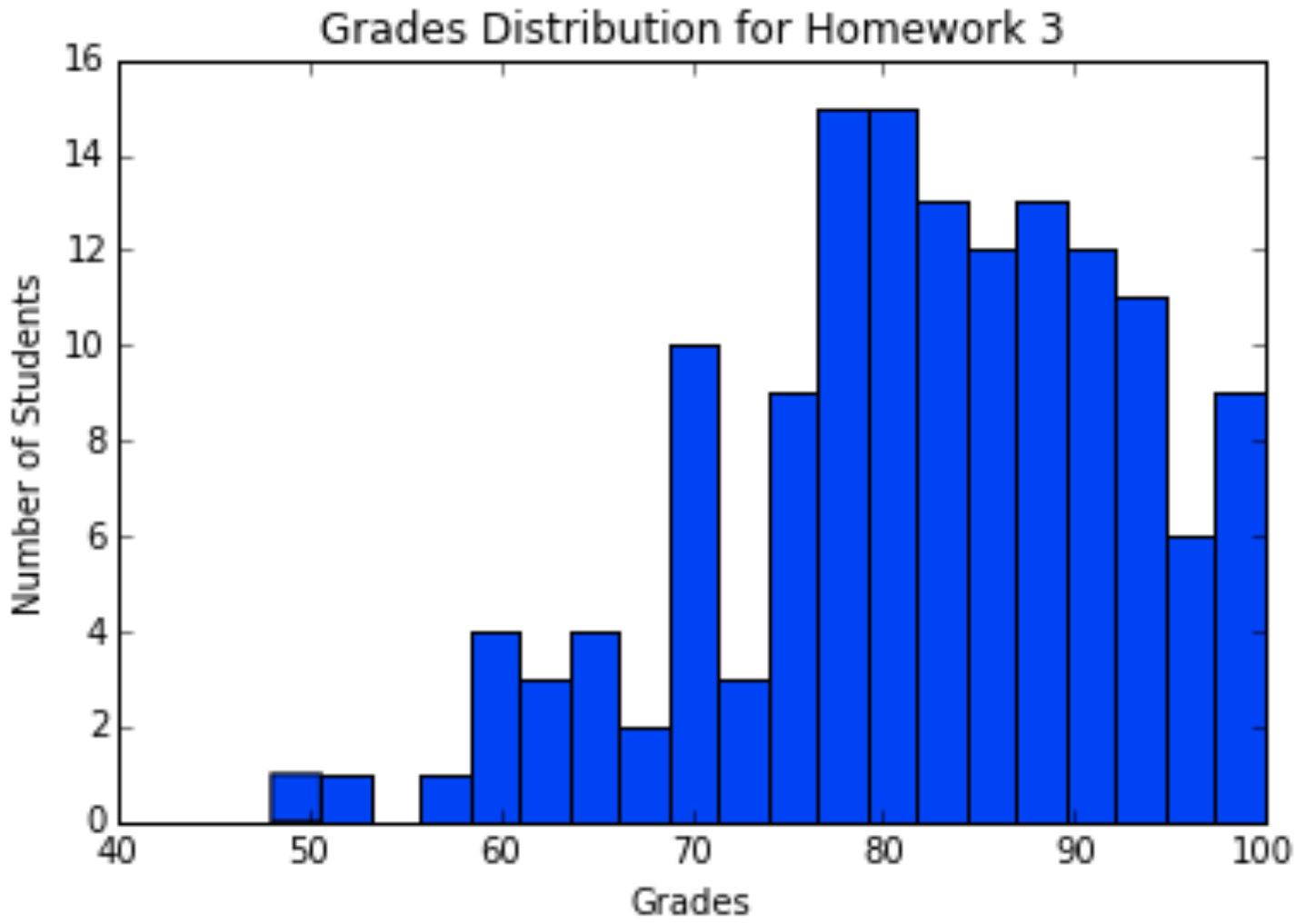
Announcements

- Homework 1 & 2 & 3, midterm exam 1
  - pick up graded work
  - check grades at moodle

- Homework 4
  - posted at moodle
  - due at 11am, Wed Oct 7

Midterm Exam 1

- Q1: 4 x 5 points
- Q2: 4 x 10 points
- Q3: 4 x 10 points
- Median = 82
- Mean = 81.7
- Stdev = 10.7
- Max = 100



Problem 1

- In each of the following situations, indicate whether f = O(g) or f = Ω(g), or both (in which case f = Θ(g)). Briefly explain why.
  - definitions of O(), Ω(), Θ()
  - exponential, polynomial, logarithm
  - lower order terms, leading coefficient

Problem 1 (cnt'd)

- In each of the following situations, indicate whether f = O(g) or f = Ω(g), or both (in which case f = Θ(g)). Briefly explain why.
  - (a) $f(n) = 6n^6+9n^3-300n$, $g(n) = 3n^6+6n^2+900n$
  - (b) $f(n) = 7\log(n^7)+6n^{10}$,
    - $g(n) = 5^{n/3}+7n \log (10n+5)$
  - (c) $f(n) = 3n^3 + (n+2)!$, $g(n) = (n+5)^3 + 100n^5$
  - (d) $f(n) = 7(n+2) \log n + (7/2)^n$, $g(n) = n^{10}$

Problem 2

- Provide a brief answer for each of the following questions.
  - (a) Show the key steps of finding the greatest common divisor of 105 and 595 using Euclid's algorithm.
    - gcd (595, 105) = gcd (105, 70)
    - = gcd (70, 35) = gcd (35, 0) = 35

Problem 2 (cnt'd)

- Provide a brief answer for each of the following questions.
  - (b) In cryptography, what is the key difference between private-key schemes (e.g., one-time pad) and public-key schemes (e.g., RSA)?
    - private-key schemes: need prearrangement of private key by both parties (Alice, Bob)
    - public-key schemes: no prearrangement needed, only recipient (Bob) has private key

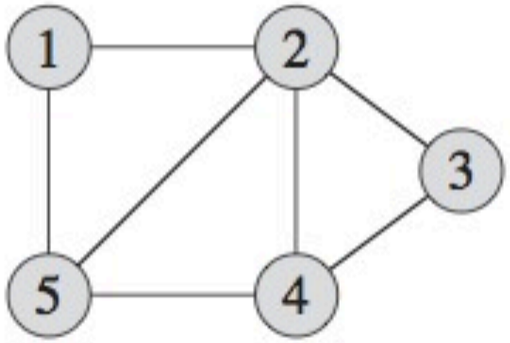
Problem 2 (cnt'd)

- Provide a brief answer for each of the following questions.
  - (c) If an algorithm solves a problem of size n by dividing it into 8 subproblems of size n/2, recursively solving each subproblem, and then combining the solutions in $O(n^2)$ time, what is the time complexity of this algorithm?
    - $T(n) = 8T(n/2) + O(n^2)$
    - $a = 8, b = 2, d = 2 \Longrightarrow O(n^3)$

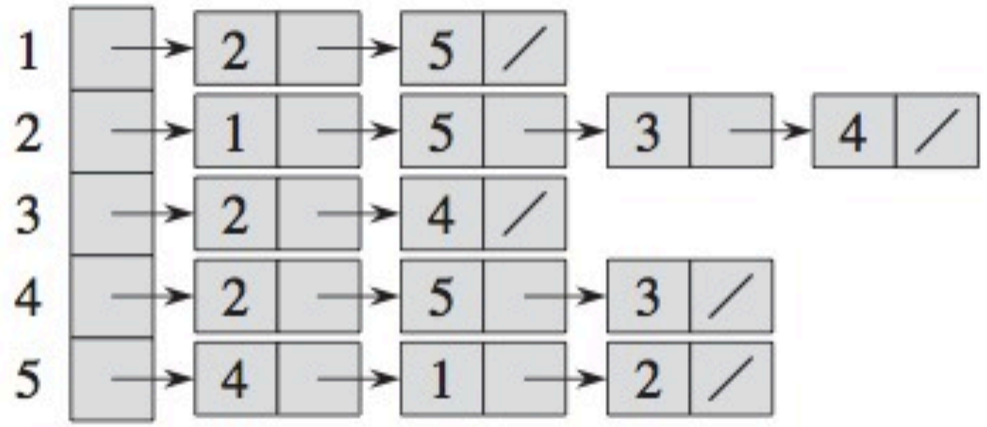
Problem 2 (cnt'd)

- Provide a brief answer for each of the following questions.
  - (d) Given a graph G = (V, E), what are the time complexities of constructing its reverse graph using adjacency matrix and adjacency list, respectively?
    - adjacency matrix: O(|V| x |V|)
    - adjacency list: O(|V| + |E|)

Graph Representation Examples



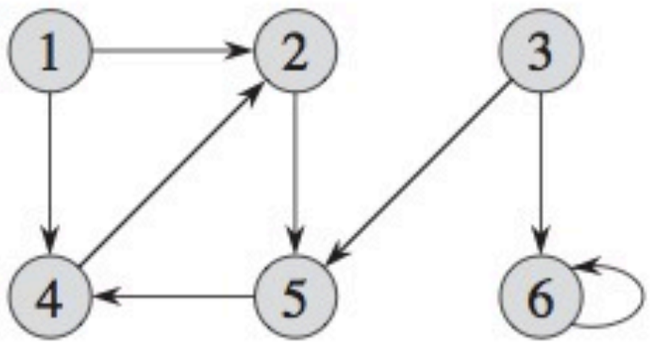
(a)



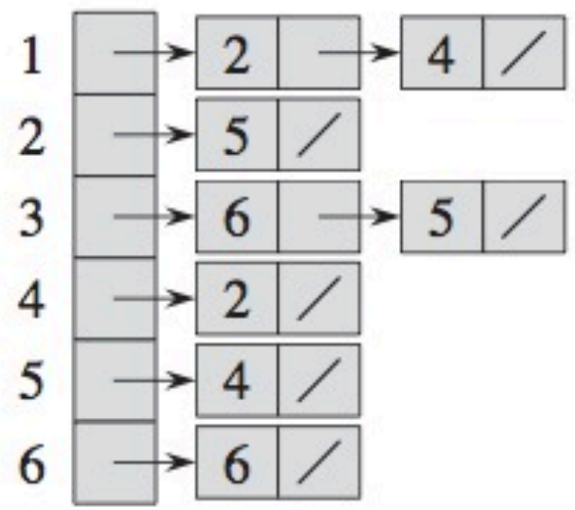
(b)

	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

(c)



(a)



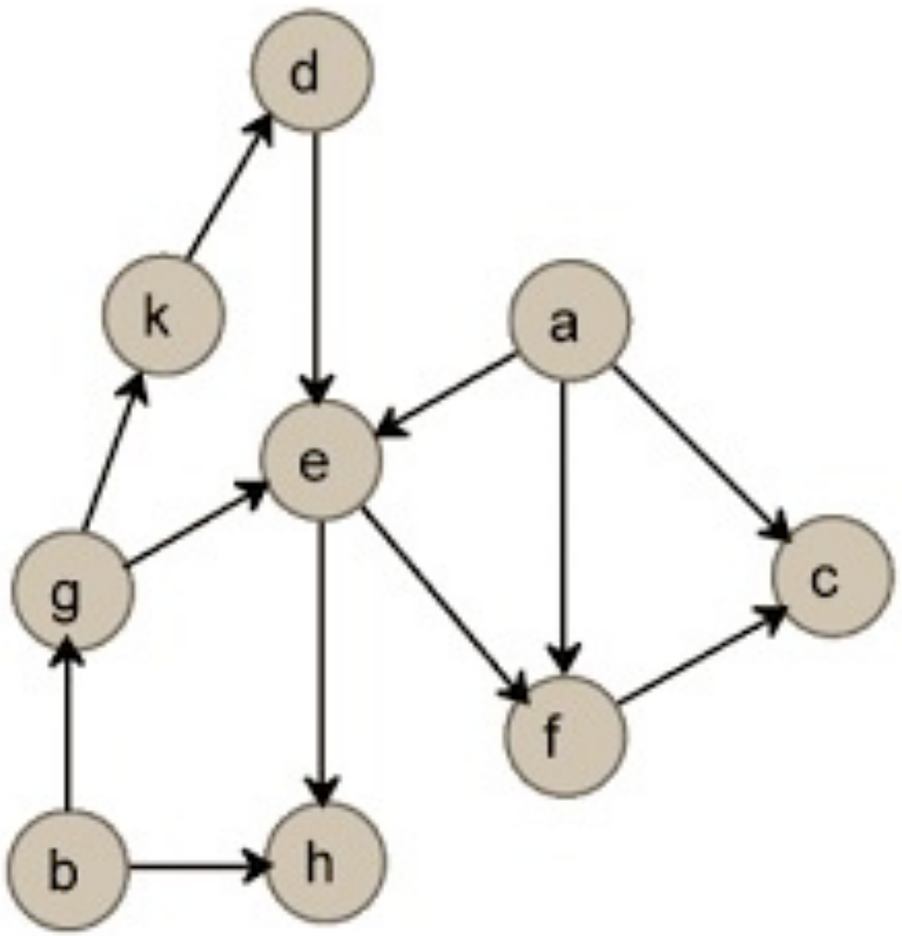
(b)

	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

(c)

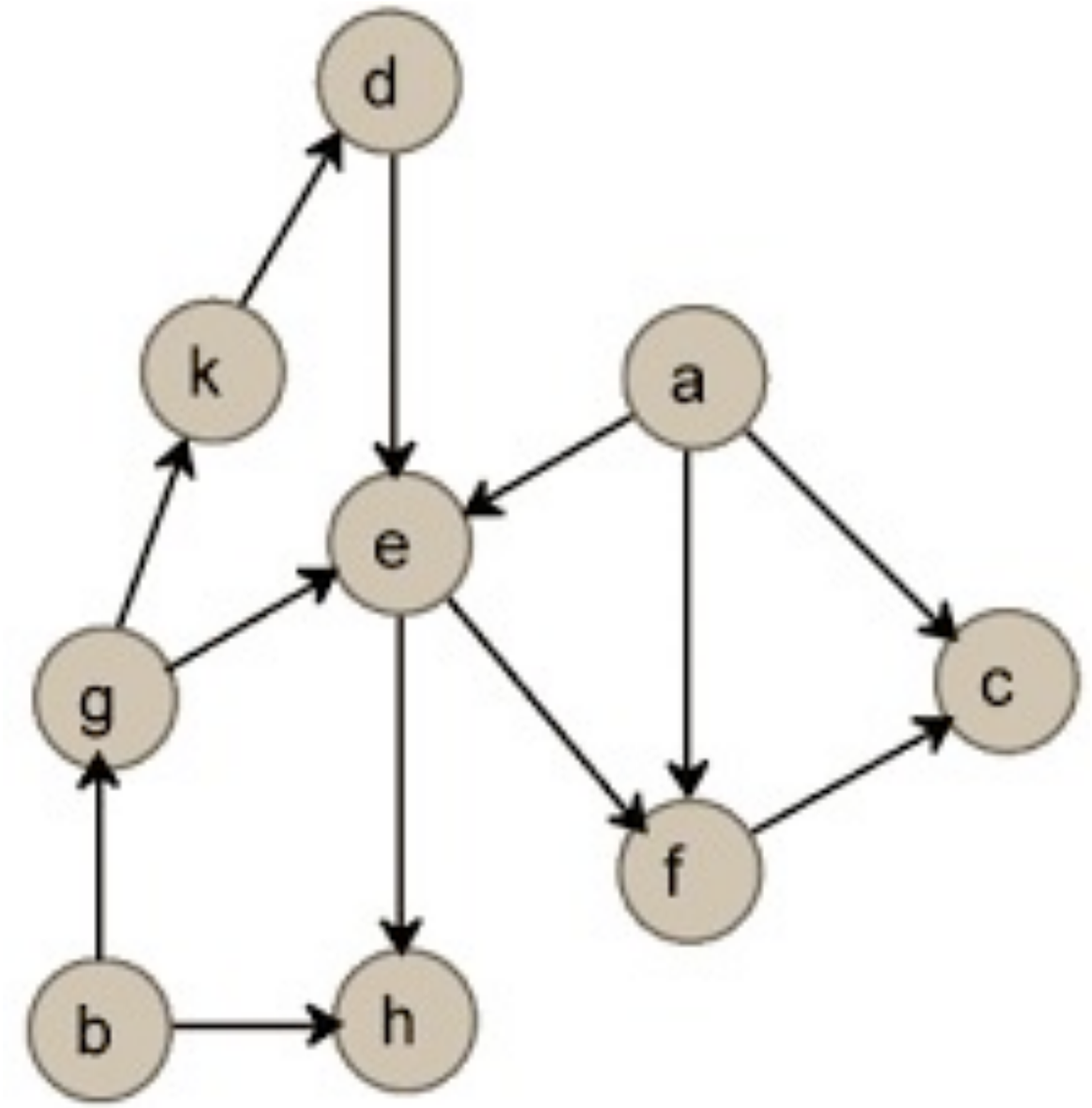
Problem 3: Graph Algorithms

- Given the following directed graph,
- (a) What is the size of the adjacency matrix? Draw the adjacency list for vertex e.
  - |V| = 9
  - adjacency matrix: 9 x 9
  - e: f => h



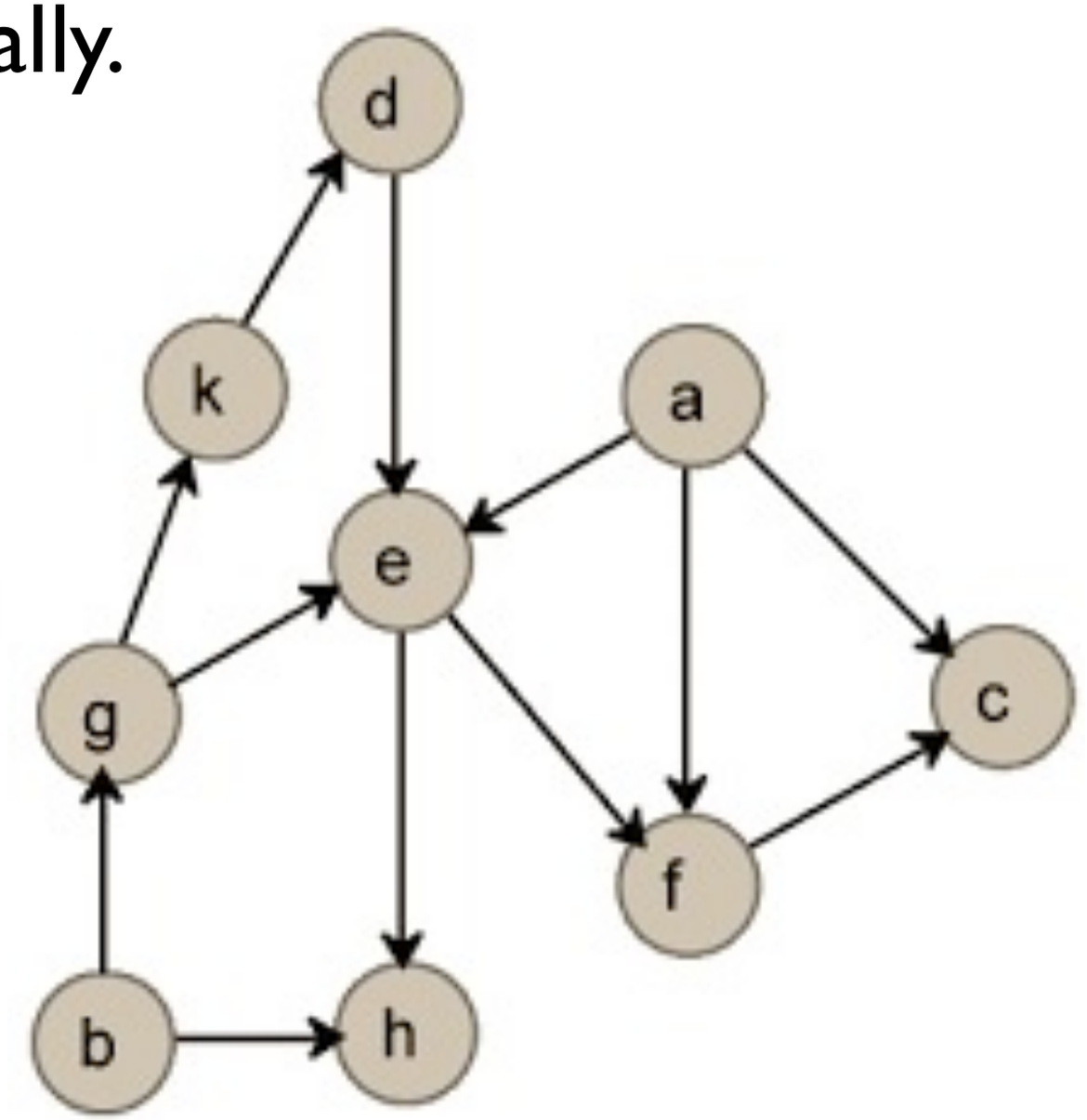
Problem 3: Graph Algorithms

- (b) Draw the DFS forest, breaking ties alphabetically.
  - draw the forest
  - a => c, a => e
  - e => f, e => h
  - b => g
  - g => k
  - k => d



Problem 3: Graph Algorithms

- (c) Draw the BFS tree with starting vertex g, breaking ties alphabetically.
  - draw the tree
  - g
  - g => e, g=> k
  - e => f, e => h, k => d
  - f => c



Problem 3: Graph Algorithms

- (d) Show the result of topological sort (linearization) of the graph.
  - linear ordering
  - {b, g, k, d, e, f, c}
  - a before e
  - h after e