CSCI 3104: Algorithms Final Exam (Fall 2014)

Student Name:

Student ID:

Email Address:

Honor Code Pledge: On my honor as a University of Colorado at Boulder student I have neither given nor received unauthorized assistance on this work.

_____

Student Signature

Instructions

1. This is a closed-book exam.
2. Keep at least one empty seat between you and your neighbor.
3. Write down your name, student ID, email address, and sign the Honor Code Pledge.
4. The exam time is from 7:30pm to 10:00pm.

Theorems

1. **Euclid's rule**: If x and y are positive integers with $x \geq y$, then $gcd(x, y) = gcd(x \ mod \ y, y)$.
2. **Master theorem**: If $T(n) = aT(\lceil n/b \rceil) + O(n^d)$ for some constants $a > 0$, $b > 1$, and $d \geq 0$, then
$$T(n) = \begin{cases} O(n^d) & \text{if } d > \log_b a \\ O(n^d \log n) & \text{if } d = \log_b a \\ O(n^{\log_b a}) & \text{if } d < \log_b a. \end{cases}$$
3. **Duality theorem**: If a linear program has a bounded optimum, then so does its dual, and the two optimum values coincide.

$$\begin{array}{lcc} \text{Primal LP} & \max c^T x & \qquad \text{Dual LP:} \quad \min y^T b \\ & Ax \leq b & \qquad\qquad\quad y^T A \geq c^T \\ & x \geq 0 & \qquad\qquad\quad y \geq 0 \end{array}$$

1. (20 Points) Determine if the following statements are TRUE or FALSE. Briefly explain why.

   (a) If $f(n) = 10n^{10} + 5n^5 + 10n \log n$ and $g(n) = (2n^2 + 3^n + \log n)^2$, then $f = O(g)$.

   (b) In the RSA encryption scheme, an eavesdropper cannot have access to the encryption key (N, e).

   (c) Approximation algorithms do not always find the optimal solution.

   (d) If an NP-complete problem can be solved in polynomial time, then P equals NP.

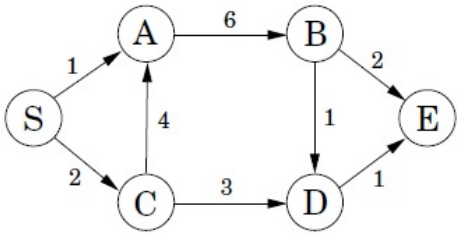
2. (20 Points) Provide a brief answer for each of the following questions.

   (a) Compute gcd(532, 190) using the Euclid algorithm.

   (b) If an algorithm solves a problem of size n by dividing it into eight subproblems of size n/3, recursively solving each subproblem, and then combining the solutions in $O(n^3)$ time, what is the time complexity of this algorithm?

   (c) Do adjacency matrix and adjacency list always require the same amount of space to represent the same graph? Briefly explain why.

   (d) What is the linearization result of the following DAG?



3. (15 Points) Show the key steps of the following computation problems.

   (a) Generate the Huffman coding for the 7 symbols with the following frequencies $\{A : 12, B : 17, C : 5, D : 10, E : 23, F : 20, G : 13\}$.

   (b) Draw the binary heap after the *make_queue*($[12, 24, 7, 18, 35, 10]$) operation. How does the binary heap change after the *delete_min*() operation?

   (c) Given the sequence $[5, 8, 3, 9, 6, 10]$, draw the DAG of increasing subsequences and find the longest increasing subsequence.

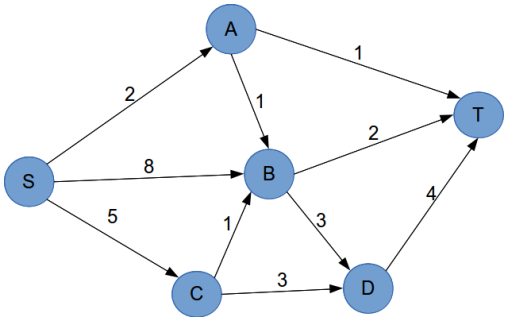
4. (20 Points) Given the following graph, the numbers represent edge lengths in (a) and (b), and edge capacity in (c) and (d):

   (a) Find the minimum spanning tree using Prim's algorithm and start vertex S , breaking ties alphabetically and ignore edge direction. Show the order of the edges added.

   (b) Using start vertex S and Dijkstra's algorithm, show for the first three iterations, the vertex removed from the priority queue and the changes of distance values.

   (c) Starting with zero flow in the network, if the augmenting path $S \to B \to D \to T$ is used, draw the residue network after the flow increase along that path.

   (d) Find the minimum $S - T$ cut of the network. Find a solution that achieves maximum flow in the network.



5. (15 Points) Lisa has an online jewelry shop where she sells earrings and necklaces. She sells earrings for $30 and necklaces for $40. It takes 30 minutes to make a pair of earrings and 1 hour to make a necklace, and, since Lisa is a math tutor, she only has 10 hours a week to make jewelry. In addition, she only has enough materials to make 15 total jewelry items per week. She makes a profit of $15 on each pair of earrings and $20 on each necklace. How many pairs of earrings and necklaces should Lisa make each week in order to maximize her profit, assuming she sells all her jewelry?

   (a) Formulate this problem as a linear programming problem.

   (b) Draw the region of feasible solutions, and find the optimum using the simplex method.

   (c) Based on the Duality theorem, write the dual LP of this problem.

6. (10 Points) Given an unlimited supply of coins of denominations $x_1, x_2, \ldots, x_n$, we wish to make change for a value v using at most k coins; that is, we wish to find a set of $\leq k$ coins whose total value is v. This might not be possible: for instance, if the denominations are 5 and 10 and $k = 6$, then we can make change for 55 but not for 65. Give an efficient dynamic-programming algorithm for the following problem.

   Input: $x_1, x_2, \ldots, x_n$; k; v

   Question: Is it possible to make change for v using at most k coins, of denominations $x_1, x_2, \ldots, x_n$?

   State the subproblems and provide the formula for calculating a larger subproblem using smaller subproblems. What is the time complexity of your algorithm?