CSCI 3104: Algorithms
Homework 7

Due at **11:00am on Wednesday, October 28, 2015**. Submit your solutions electroni-
cally at moodle (name file as **LastName_FirstName_HW7.pdf**) or submit in paper before
class. Also, submit your python source code electronically at moodle (name your file as **Last-
Name_FirstName_HW7.py**). Make sure to include your name and student ID. Digital
submission should also include the Honor Code Pledge (`http://honorcode.colorado.edu/`
`about-honor-code`), and paper submission should include your signature indicating adher-
ence to the Honor Code Pledge.

1. Yuckdonalds is considering opening a series of restaurants along Quaint Valley Highway
   (QVH). The n possible locations are along a straight line, and the distances of these
   locations from the start of QVH are, in miles and in increasing order, $m_1, m_2, \ldots, m_n$.
   The constraints are as follows:

   - At each location, Yuckdonalds may open at most one restaurant. The expected
     profit from opening a restaurant at location i is p_i , where $p_i > 0$ and $i =
     1, 2, \ldots, n$.
   - Any two restaurants should be at least k miles apart, where k is a positive integer.

   Give an efficient algorithm to compute the maximum expected total profit subject to
   the given constraints. What is the running time of your algorithm?

2. A subsequence is **palindromic** if it is the same whether read left to right or right
   to left. For instance, the sequence $A, C, G, T, G, T, C, A, A, A, A, T, C, G$ has many
   palindromic subsequences, including A, C, G, C, A and A, A, A, A (on the other hand,
   the subsequence A, C, T is not palindromic). Devise an algorithm that takes a sequence
   $x[1...n]$ and returns the (length of the) longest palindromic subsequence. Its running
   time should be $O(n^2)$.

3. **Cutting cloth**. You are given a rectangular piece of cloth with dimensions $X \times Y$,
   where X is width and Y is height (both are positive integers), and a list of n products
   that can be made using the cloth. For each product $i \in [1, n]$, you know that a rectangle
   of cloth of dimensions $a_i \times b_i$ is needed and that the final selling price of the product is
   c_i. Assume the a_i, b_i, and c_i are all positive integers. You have a machine that can cut
   any rectangular piece of cloth into two pieces either horizontally or vertically. Write a
   python program that determines the best return on the $X \times Y$ piece of cloth, that is, a
   strategy for cutting the cloth so that the products made from the resulting pieces give
   the maximum sum of selling prices. You are free to make as many copies of a given
   product as you wish, or none if desired.

   Name your code as LastName_FirstName_HW7.py. Your code reads two filenames as
   the input arguments, the first is input file and the second is output file. A sample

input file is provided at moodle via a separate link. The first line contains the X and Y values, second line is the n value, and each of the following n lines contains the a_i, b_i, and c_i values for the i-th product $i \in [1, n]$. For the output file, the first line contains

$$P, T$$

where P is the maximal profit and T is the total number of cuts, and the following lines are the individual cuts (one cut per line) in the format:

$$t, x, y, z, g$$

where $t = 1, 2, \ldots$ is the t-th cut, x and y are the width and height of the current cloth to cut, z is either 0 (horizontal cut) or 1 (vertical cut), and g is the point of cut. For example, "10 5 1 2" means to cut a 10×5 cloth vertically at 2, resulting in two pieces of cloth 2×5 and 8×5.