CSCI 3104: Algorithms

Midterm Exam 1 (Fall 2014)

Student Name:

Student ID:

Email Address:

Honor Code Pledge: On my honor as a University of Colorado at Boulder student I have neither given nor received unauthorized assistance on this work.

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Student Signature

Instructions

1. This is a closed-book exam.
2. Keep one empty seat between you and your neighbor.
3. The total exam time is 45 minutes, from 11:05am to 11:50am.
4. Write down your name, student ID, email address, and sign the Honor Code Pledge.

Theorems

1. **Euclid's rule**: If x and y are positive integers with $x \geq y$, then $gcd(x, y) = gcd(x \ mod \ y, y)$.
2. **Fermat's little theorem**: If p is prime, then for every $1 \leq a < p$, $a^{p-1} \equiv 1 (mod \ p)$.
3. **Master theorem**: If $T(n) = aT(\lceil n/b \rceil) + O(n^d)$ for some constants $a > 0$, $b > 1$, and $d \geq 0$, then

$$T(n) = \begin{cases} O(n^d) & \text{if } d > \log_b a \\ O(n^d \log n) & \text{if } d = \log_b a \\ O(n^{\log_b a}) & \text{if } d < \log_b a. \end{cases}$$

1. In each of the following situations, determine whether $f = O(g)$, or $f = \Omega(g)$, or both (in which case $f = \Theta(g)$). Briefly explain why.

(a)	$f(n) = 8n\log(8n)$	$g(n) = 3n^3 + 2n^2 - 100$
(b)	$f(n) = \log(10n + 5)$	$g(n) = \log(n^7)$
(c)	$f(n) = 100n^7 + 10n^5 - n^3$	$g(n) = 2^{n+1} + 2n^2$
(d)	$f(n) = 10n^{1/2} + (\frac{7}{3})^n$	$g(n) = 8n^8 + 4\log n$

2. Provide a brief answer for each of the following questions.

   (a) In the RSA public-key scheme, what information can an eavesdropper obtain? How can the eavesdropper determine the original message that was sent?

   (b) Given an undirected graph G and a starting vertex S, can we find all vertices in the graph using the **explore(G, S)** algorithm discussed in class? Explain why.

   (c) If an algorithm solves a problem of size n by dividing it into nine subproblems of size $n/3$, recursively solving each subproblem, and then combining the solutions in $O(n^2)$ time, what is the time complexity of this algorithm?

   (d) Show the key steps of using the mergesort algorithm to sort the following array of values into ascending order: [139, 72, 89, 254, 35, 331, 158, 40]

3. Given the following directed graph,

   (a) Using the adjacency list representation, how many linked lists are needed to represent this graph? Draw the linked list for vertex 3.

   (b) Draw the DFS search forest, breaking ties by examining smaller-valued vertex first.

   (c) Draw the corresponding meta-graph. Is the meta-graph a directed acyclic graph (DAG)?

   (d) Show the result of topological sort (linearization) of the meta-graph.