1.a) $\alpha_t = \alpha > 0$ independent of round t

$(\frac{1}{2} - \epsilon_t) \geq \gamma > 0$, find optimal α

if $f(x) = \sum \alpha_t h_t(x)$

Distribution at $t+1$:

$$D_{T+1}(i) = \frac{e^{-y_i f(x_i)}}{m \prod Z_t}$$

$$\alpha_m = \frac{1}{2} \ln\left(\frac{1-\epsilon_m}{\epsilon_m}\right)$$ (given in book & wikipedia)

$$R(h) \leq \frac{1}{m} \sum_i e^{-y_i f(x_i)}$$

$$R(h) = \sum D_{T+1}(i) \prod Z_t = \prod(e^{-\alpha}(1-\epsilon_t) + e^{\alpha}\epsilon_t) = \prod(e^{-\alpha} - \epsilon_t e^{-\alpha} + e^{\alpha}\epsilon_t)$$

$$= \prod(\epsilon_t(e^{\alpha} - e^{-\alpha}) + e^{-\alpha})$$

& since $\gamma \geq (\frac{1}{2} - \epsilon_t) \geq \gamma > 0$ then:

$\frac{1}{2} - \gamma \geq \epsilon_t$

$$R(h) \leq \left((\tfrac{1}{2} - \gamma)(e^{\alpha} - e^{-\alpha}) + e^{-\alpha}\right)^T$$

$\left((\frac{1}{2}-\gamma)e^{\alpha} - (\frac{1}{2}-\gamma)e^{-\alpha} + \cancel{e^{-\alpha}}\right)' = 0$, then:

$(\frac{1}{2} - \gamma) e^{\alpha} = (\frac{1}{2} + \gamma) e^{-\alpha}$ take the log then:

$$e^{\alpha} = \frac{\frac{1}{2}+\gamma}{\frac{1}{2}-\gamma} e^{-\alpha}$$

$$2\alpha = \ln\left(\frac{\frac{1}{2}+\gamma}{\frac{1}{2}-\gamma}\right) \Rightarrow \alpha = \frac{1}{2} \ln\left(\frac{\frac{1}{2}+\gamma}{\frac{1}{2}-\gamma}\right)$$

b) going off a):

$p(\text{wrong}) = \epsilon_t e^{\alpha}$

$p(\text{correct}) = (1-\epsilon_t) e^{-\alpha}$ & since $\epsilon_t = \frac{1}{2} - \gamma$ & $\alpha = \frac{1}{2} \ln\left(\frac{\frac{1}{2}+\gamma}{\frac{1}{2}-\gamma}\right)$

Using mathematica:

$p(\text{wrong}) - p(\text{correct}) = 0$ so $p(\text{wrong}) = p(\text{right})$ for this α

c) $R(h) \leq \left((\frac{1}{2} - \gamma) e^{\alpha} + (\frac{1}{2} + \gamma) e^{-\alpha}\right)^T$ & substituting in $\alpha = \frac{1}{2} \ln\left(\frac{0.5+\gamma}{0.5-\gamma}\right)$

in mathematicas

$$\left(\sqrt{\frac{\frac{1}{2}+\gamma}{\frac{1}{2}-\gamma}}\,(\tfrac{1}{2}-\gamma) + \frac{\gamma+\frac{1}{2}}{\sqrt{\frac{\frac{1}{2}+\gamma}{\frac{1}{2}-\gamma}}}\right)^T$$

upper bound: $\exp(-2\gamma^2 T)$ ← from alternate forms in mathematica when set to initial γ bounds

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Machine Learning: Boosting

Ahmed Alomdara

☆ 2. $t: h_t$, $t+1: h_{t+1}$

Proof by Contradiction attempt:

$\alpha_t: \frac{1}{2} \ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right)$

$Z_t: 2\sqrt{\epsilon_t(1-\epsilon_t)}$

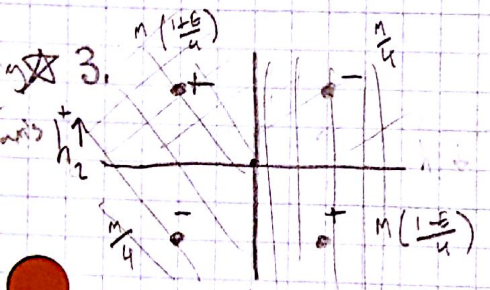
so assuming $\epsilon_t = \frac{1}{4}$:

$\alpha_t: \frac{1}{2} \ln\left(\frac{1-\frac{1}{4}}{\frac{1}{4}}\right) = 0.594$ | so for $t+1$ our distribution is:

$Z_t: 2\sqrt{\frac{1}{4}(1-\frac{1}{4})} = 0.866$ | $D_{t+1}(i) = \frac{D_t(i)\, e^{-\alpha_t y_i h_t(x_i)}}{Z_t} = \frac{\frac{1}{n} \cdot e^{0.594}}{0.866} = \frac{2.09}{n}$

$\epsilon_{t+1} = \frac{n}{4} \cdot \frac{2.09}{n} = \frac{2.09}{4} = 0.5225$ { Since $0 \le \epsilon < \frac{1}{2}$, then we can say h_{t+1} is different by contradiction.

☆ 3.



h_1 – everything left of y-axis

everything right of y axis – h_3

$\frac{1}{2} \ln\left(\frac{1-\sigma}{\sigma}\right)$, $\epsilon: \frac{1}{2}$

if $m=4$: $(-1, 1)$: $\frac{m}{4} = \frac{4}{4} = 1$

$\frac{m(1-\epsilon)}{4} = \frac{4(0.5)}{4} = \frac{1}{2}$

$\frac{m(1+\epsilon)}{4} = \frac{4(1.5)}{4} = \frac{3}{2}$

$t=1$

$\epsilon_1 = \frac{1}{\cancel{m}}\left(\frac{\cancel{m}}{4} + \frac{m}{4}(1-\epsilon)\right) = \frac{1}{4} + \frac{1}{4}(1-\frac{1}{2}) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$ D_1

$Z_1 = 2\sqrt{\epsilon_1(1-\epsilon)} = 2\sqrt{\frac{3}{8}(1-\frac{3}{8})} = 2\sqrt{\frac{15}{64}} = 0.968$

$\alpha_1 = \frac{1}{2} \ln\left(\frac{1-\epsilon}{\epsilon}\right) = \frac{1}{2} \ln\left(\frac{1-\frac{3}{8}}{\frac{3}{8}}\right) = 0.255$

wrong points   right points

$D_2 = \frac{D_1 e^{-\alpha_1 y_i h_1(x_i)}}{Z_1} = \frac{1.33}{m}$, $D_2 = \frac{0.8}{m}$

when it gets bigger than $\frac{1}{2}$ then we stop:

$\epsilon_2 = \frac{1.33}{m}\left(\frac{m}{4}(1-\epsilon)\right) + \frac{0.8}{m}\left(\frac{m}{4}\right) = 0.535$

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