Quasi-normal modes of near-extremal black holes in dRGT massive gravity using Physics-Informed Neural Networks (PINNs)

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Abstract

In this study, we demonstrate the use of physics-informed neural networks (PINNs) for computing the quasinormal modes (QNMs) of black holes in de Rham-Gabadadze-Tolley (dRGT) massive gravity. These modes describe the oscillation frequencies of perturbed black holes and are important in understanding the behavior of these objects. We show that by carefully selecting the hyperparameters of the PINN, including the network architecture and the training data, it is possible to achieve good agreement between the computed QNMs and the approximate analytical formula in the near-extremal limit for the smallest mode number. Our results demonstrate the effectiveness of PINNs for solving inverse problems in the context of QNMs and highlight the potential of these algorithms for providing valuable insights into the behavior of black holes.

1 Introduction

Quasinormal modes (QNMs) describe the oscillation modes of a perturbed black hole and are important in understanding the behavior of these objects. They are complex numbers, with the real part corresponding to the oscillation frequency and the imaginary part corresponding to the damping rate. Determining QNMs is a challenging problem, as it requires solving a differential equation with suitable boundary conditions in a spacetime that may be highly complex.

Physics-informed neural networks (PINNs) are a type of machine learning algorithm that have gained significant attention in recent years for their ability to solve complex physics problems. These algorithms are particularly well-suited for inverse problems, which involve determining the unknown parameters of a system from observed data. One of the key advantages of PINNs is their ability to incorporate physical constraints and laws directly into the neural network architecture. This allows them to learn the solution to a problem by taking into account the underlying physics, rather than simply fitting to a set of data points.

In recent years, PINNs have been applied to the problem of computing QNMs of black holes. In the near-extremal limit, the effective potential of the radial wave equation for QNMs can be expressed in terms of the Poschl-Teller potential, for which the exact solution is known. This allows the QNMs problem to be viewed as an inverse problem that can be solved using PINNs. By training a PINN to approximate the QNMs using the known solution of the Poschl-Teller potential [2], it is possible to obtain accurate approximations of the QNMs for a wide range of black hole spacetimes.

One of the challenges in using PINNs to compute QNMs is the choice of hyperparameters, which can significantly affect the performance of the algorithm. Careful selection of these parameters, including the network architecture, the type of activation functions, and the training data, is essential for achieving good results.

Overall, PINNs represent a promising approach for computing QNMs of black holes and have the potential to provide valuable insights into the behavior of these objects. As the field of machine learning continues to advance, it is likely that PINNs will play an increasingly important role in solving challenging problems in astrophysics and other areas of physics.

2 Computing QNMs with Physics-Informed Neural Networks

Deep learning is a machine learning technique that produces a model that predicts a result based on input data. It looks for patterns in the input data and uses them to reduce the difference between the forecast and the actual output, as well as generalize its predictions to new inputs. There are many different types of deep learning algorithms, but one of the most popular is neural networks, which look for correlations between input and output.

A neural network consists of several layers, including an input layer, hidden layers, and an output layer. The input layer collects the data, the hidden layers adjust weights and parameters to identify patterns in the data, and the output layer provides predictions based on these patterns.

Physics-informed neural networks (PINNs) are a specific type of deep learning algorithm that take into account the physics of the problem, which is often represented by nonlinear partial differential equations. In 2017, Raissi et al. were the first to propose the use of PINNs for solving such equations.

We can use a library called DeepXDE, which is designed for PINN applications, to compute quasinormal frequencies (QNFs). DeepXDE makes it easy to specify the problem using built-in modules for the computational domain, the PDE equations, boundary and initial conditions, constraints, training data, neural network architecture, and training hyperparameters. The process for using DeepXDE to solve differential equations is as follows:

Algorithm 1 The process for building PINN models in DeepXDE

1: The physical equations that drive the problem are all set and passed to `deepxde.data...(...)` as parameters.
2: Construct a neural network using the `deepxde.nn...(...)` module.
3: The PINN model is a combination of the neural network and physical constraints defined as parameters to `deepxde.Model(...)`.
4: The `Model.Compile(...)` function is used to define training parameters such as the specific choice for the optimiser.
5: The PINN model can then be run using `Model.train(...)` for a defined number of training epochs.
6: Call `Model.predict(...)` to predict the solution.

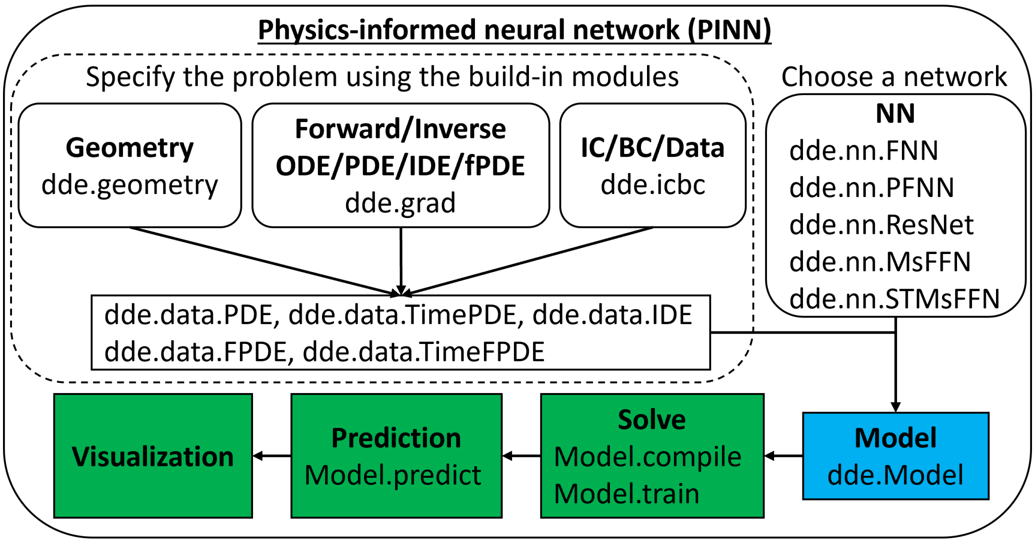

Figure 1: The PINNs algorithm in DeepXDE [1]

3 Quasi-normal modes of near-extremal black holes in dRGT massive gravity

3.1 Theoretical Framework

3.1.1 physics-informed neural networks

The theoretical framework for physics-informed neural networks (PINNs) involves the use of neural networks to solve partial differential equations (PDEs) that represent physical systems. These PDEs can be expressed in the form:

$$\mathcal{N}[u] = f \quad u \qquad = u(\mathbf{x}, t) \ \mathbf{x} \in \Omega \subset \mathbb{R}^d \ t \qquad \in [0, T]$$

Here, u is the unknown solution to the PDE, $\mathcal{N}$ is the differential operator representing the physical system, f is a known forcing function, $\mathbf{x}$ represents the spatial coordinates, and t is time. The domain Ω and time interval $[0, T]$ depend on the specific problem being considered.

To solve this PDE using a PINN, we introduce a neural network $u_{\mathbf{w}}(\mathbf{x}, t)$ parameterized by weights $\mathbf{w}$ that approximates the solution u. The neural network is trained to minimize the loss function:[6]

$$\begin{aligned}\mathcal{L}(\mathbf{w}) = \frac{1}{N_{pde}} \sum_{i=1}^{N_{pde}} |\mathcal{N}[u_{\mathbf{w}}(\mathbf{x}i, t_i)] - f_i|^2 \\ + \frac{\lambda}{Nic} \sum_{i=1}^{N_{ic}} |u_{\mathbf{w}}(\mathbf{x}i, t_i) - uex(\mathbf{x}i, t_i)|^2 + \frac{\lambda}{Nbc} \sum_{i=1}^{N_{bc}} |u_{\mathbf{w}}(\mathbf{x}i, t_i) - uex(\mathbf{x}i, t_i)|^2 \\ + \frac{\lambda}{Nt} \sum_{i=1}^{N_t} |u_{\mathbf{w}}(\mathbf{x}i, t_i) - uex(\mathbf{x}i, t_i)|^2 + \frac{\lambda}{Nx} \sum_{i=1}^{N_x} |u_{\mathbf{w}}(\mathbf{x}i, t_i) - uex(\mathbf{x}_i, t_i)|^2\end{aligned}$$

Here, N_{pde}, N_{ic}, N_{bc}, N_t, and N_x are the number of points at which the PDE, initial conditions, boundary conditions, and spatiotemporal points are enforced,

3.1.2 Quasi-normal modes of near-extremal black holes in dRGT massive gravity

To get to the differential equations of black hole perturbations, we start with the space-times of the dRGT black hole which is given in general as [4]:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1}$$

with $f(r)$ is a metric function which is given as [4] :

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2 + \frac{Q^2}{r^2} + \gamma r + \zeta, \tag{2}$$

The black hole mass and charge are labelled by M and Q respectively. Λ acts as a cosmological constant, while the set $\{\gamma, \zeta\}$ are two parameters from massive gravity, which are calculated in function of the graviton mass, and other parameters of the theory.

This metric has four possible roots as shown in Figure 2. Each real positive root corresponds to a black hole horizon. Those roots are written as r_-, r_0, r_h and r_c. r_- has a negative value, which is essentially unphysical, r_0 is the Cauchy horizon, also called the inner horizon, which covers the singularity, r_h is the event horizon and the r_c is the cosmological horizon.

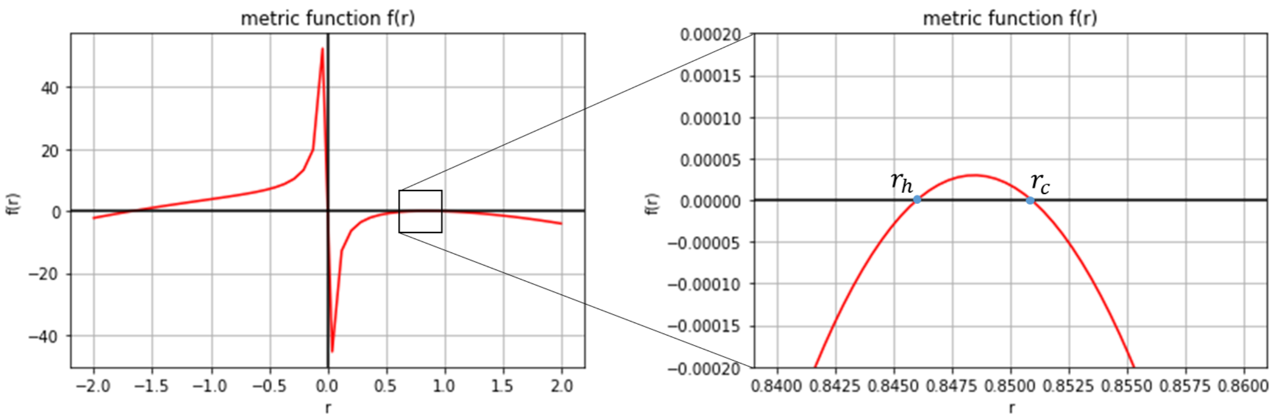

Figure 2: The metric function $f(r)$ profil for $M = 1$, $\Lambda = 5.0001$, $\gamma = 0.05$, $\zeta = 2.51465$, $Q = 0$. Event horizon $r_h = 0.8460$, cosmological horizon $r_c = 0.8509$

In near-extremal Black hole, where $r_h \sim r_c$, the metric (2) may be expressed in the tortoise coordinate r_* as [3] :

$$f(r_*) \sim \frac{r_h k_h}{(1 + \zeta + \gamma r_h - 2Q^2/r_h^2)\cosh^2(k_h r_*)}, \tag{3}$$

The wave equation of neutral scalar field of the near-extremal black holes,is given by the Klein–Gordon equation :

$$\Box\Phi = 0 \tag{4}$$

The scalar field Φ can be expressed as :

$$\Phi = \sum_{l}^{\infty}\sum_{m=0}^{l} e^{-i\omega t}\frac{\psi(r)}{r}Y_{lm}(\theta, \phi),$$

With $Y_{lm}(\theta, \phi)$ the spherical harmonics. The radial wave equation of $\psi(r)$ then takes the form :

$$\frac{d^2\psi}{dr_*^2} + [\omega^2 - V(r)]\psi = 0, \tag{5}$$

where :

$$V(r) = f(r)(m_s^2 + \frac{l(l+1)}{r^2} + \frac{f'(r)}{r}) \tag{6}$$

m_s stands for the mass of the scalar field, l is the angular azimuthal quantum number.

From the expression of the metric $f(r)$ (3) in the near extrimal limit, the Klein-Gordon radial equation (5) becomes :

$$\frac{d^2\psi}{dr_*^2} + [\omega^2 - \frac{V_0}{cosh^2(\kappa_h r_*)}]\psi = 0 \tag{7}$$

in which $V_0 = \dfrac{\kappa_h^2}{1+\zeta+\gamma r_h - 2Q^2/r^2}(m_s^2 r^2 + l(l+1))$.

The potential is the well-known Pöschl-Teller potential [5]. Applying the boundary condition for quasi-normal modes, the following quasi-normal frequencies can be obtained [2],

$$\omega_n = \sqrt{V_0 - \frac{\kappa_h^2}{4}} - i\kappa_h(n + \frac{1}{2}) \tag{8}$$

And the associated quasi-normal modes are given by [2] :

$$\psi = (D(D-1))^{-i\omega/2k_h} . {}_2F_1(1/2 + B - \frac{i\omega}{k_h}, 1/2 - B - \frac{i\omega}{k_h}; 1 - \frac{i\omega}{k_h}; D) \tag{9}$$

where : $D = \dfrac{1}{1+e^{-2k_h r_*}}$ and $B = \sqrt{\dfrac{1}{4} - \dfrac{V_0}{k_h^2}}$

3.2 Results

Given the infinite problem domain, the new coordinate $y = tanh(k_h r_*)$ was used to create a finite domain $(-1, 1)$ that is easier to implement in the code. Therefore, as a function of y, the perturbation equations (7) for the near extremal dRGT black holes are:

$$\kappa_h^2 \left(1-y^2\right)^2 \cdot \frac{d^2\Psi(y)}{dy^2} - 2\kappa_h^2 y \left(1-y^2\right) \cdot \frac{d\Psi(y)}{dy} + \left[\omega^2 - V_0\left(1-y^2\right)\right]\Psi(y) = 0 \tag{10}$$

To make implementation within the DeepXDE package easier, the equations 10 were split into real and imaginary parts.

The following hyperparameters were used to generate the results: two fully connected neural networks (PFNN) of depth 4 (i.e., 3 hidden layers) with the structure [1, [36, 36], [20, 20], [20, 20], 2]; the swish activation function; the L-BFGS-B and Adam optimizers with a learning rate of 0.0001 and loss weights of [0.01, 0.01, 0.001, 0.001, 100, 100, 100, 100]; 150,000 training epochs; training data consisting of 100 domain points; and a dataset of 100 uniformly distributed actual values of the quasinormal modes ($\Psi(y)$) in the domain [-0,7,0,7].

The PINN approximations results of the QNFs for massless scalar perturbations of dRGT neutral black holes, for $l = 1$ and $n = 0$ are listed in table 1 ana table 2, respectively.

$l = 1$			
n	PINN	Formula (8)	relative error
0	0.000031 - 0.000003	0.0 - 0.0j	...
1	0.0 - 0.012175j	0.0 - 0.012175j	0.0002072%
2	0.0 - 0.024352j	0.0 - 0.024352j	0.001769 %
3	0.0 - 0.036527j	0.0 - 0.036527j	0.004005 %
4	0.0 - 0.048703j	0.0 - 0.048703j	0.00032755%
5	0.0 - 0.060879j	0.0 - 0.060879j	0.0001240%
6	0.0 - 0.073055j	0.0 - 0.073055j	0.0004933%
7	0.0 - 0.085233j	0.0 - 0.085231j	0.002220%
8	0.0 - 0.094307j	0.0 - 0.097407j	3.1830%

Table 1: The PINN approximations results of the QNFs for massless scalar perturbations of dRGT neutral black holes, for $l = 1$ and different value of n, $M = 1$, $\Lambda = 5.0001$, $\gamma = 0.05$, $\zeta = 2.51465$, $Q = 0$, $m_s = 0$, $r_h = 0.8460$, $r_c = 0.8509$, $k_h = 0.012175$

$n = 0$			
l	PINN	Formula (8)	relative error
0	0.000031 - 0.000003j	0.0 - 0.0j	...
1	0.000031 - 0.000003j	0.0 - 0.0j	...
2	0.006804 - 0.006088j	0.006804 - 0.006088j	0.0004471 %
3	0.014593 - 0.006088j	0.014599 - 0.006088j	0.006659 %
4	0.021519 - 0.006088j	0.021519 - 0.006088j	0.0003780%
5	0.028222 - 0.006088j	0.028223 - 0.006088j	0.001197%
6	0.034832 - 0.006088j	0.034833 - 0.006088j	0.0004149%
7	0.041394 - 0.006088j	0.041394 - 0.006088j	0.0009156 %
. .			
30	0.190327 - 0.006088j	0.190327 - 0.006088j	0.0001256 %
. .			
50	0.319495 - 0.006082j	0.319498 - 0.006088j	0.0018230 %

Table 2: The PINN approximations results of the QNFs for massless scalar perturbations of dRGT neutral black holes, for $n = 0$ and different value of l, $M = 1$, $\Lambda = 5.0001$, $\gamma = 0.05$, $\zeta = 2.51465$, $Q = 0$, $m_s = 0$, $r_h = 0.8460$, $r_c = 0.8509$

4 Conclusion

In conclusion, the quasinormal modes (QNMs) of black holes in de Rham-Gabadadze-Tolley (dRGT) massive gravity can be accurately computed using physics-informed neural networks (PINNs). These algorithms are able to take into account the underlying physics of the problem, represented by nonlinear partial differential equations, and can effectively learn the solution to the QNMs problem by minimizing a loss function that encodes the physical constraints. By carefully selecting the hyperparameters of the PINN, including the network architecture and the training data, it is possible to achieve good agreement between the computed QNMs and the approximate analytical formula in the near-extremal limit. The use of PINNs for computing QNMs offers a promising approach for understanding the behavior of black holes and has the potential to provide valuable insights into these fascinating objects.

Important links

DeepXDE : `https://github.com/lululxvi/deepxde`

References

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