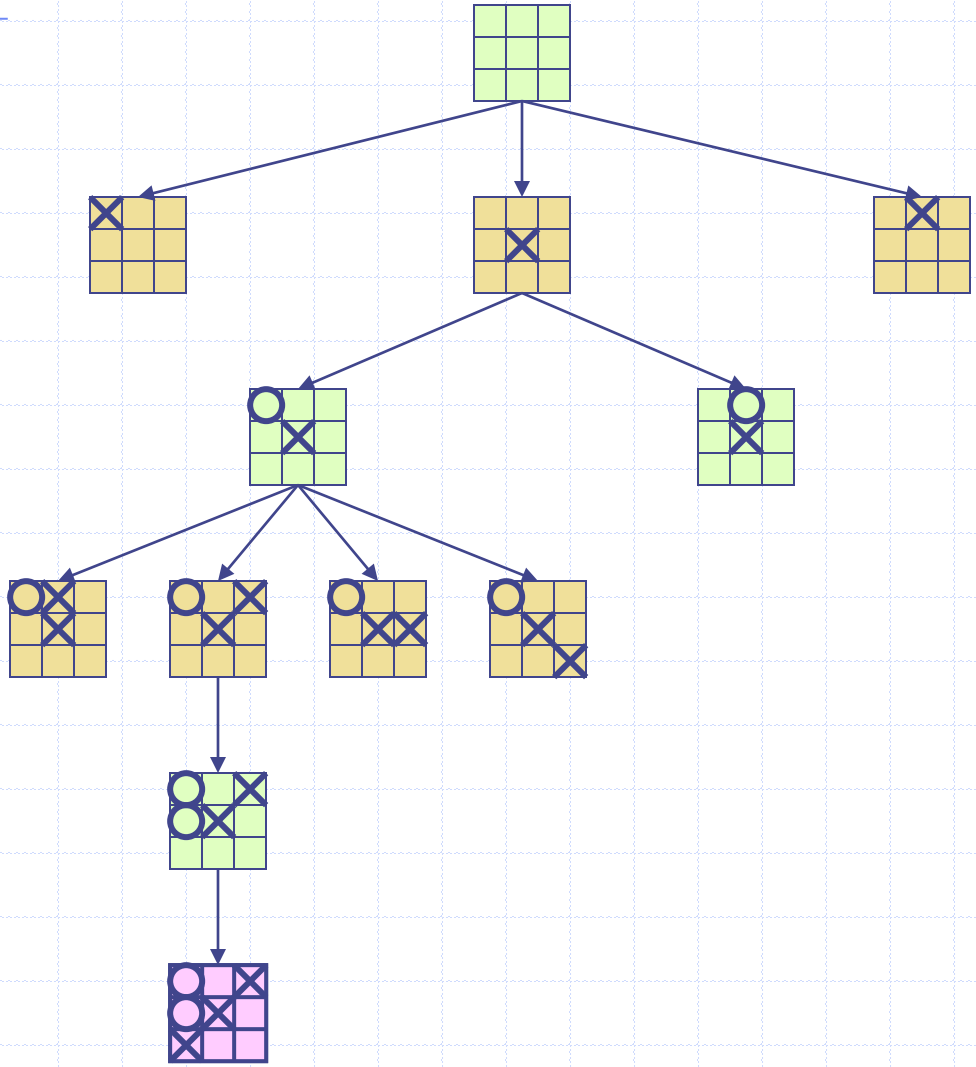
Adversarial Search and Game Playing

Russell and Norvig: Chapter 6

Perfect Two-Player Game

- Two players MAX and MIN take turn (with MAX playing first)
- State space
- Initial state
- Successor function
- Terminal test
- Score function, that tells whether a terminal state is a win (for MAX), a loss, or a draw
- Perfect knowledge of states, no uncertainty in successor function

Partial Tree for Tic-Tac-Toe



But in general the search tree is too big to make it possible to reach the terminal states!

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Examples:

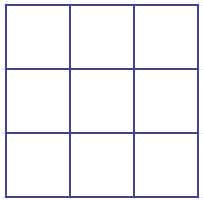
- Checkers: $\sim 10^{40}$ nodes
- Chess: $\sim 10^{120}$ nodes

Evaluation Function of a State

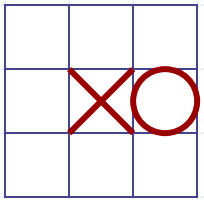
- e(s) = $+\infty$ if s is a win for MAX
- e(s) = $-\infty$ if s is a win for MIN
- e(s) = a measure of how "favorable" is s for MAX
  - > 0 if s is considered favorable to MAX
  - < 0 otherwise

Example: Tic-Tac-Toe

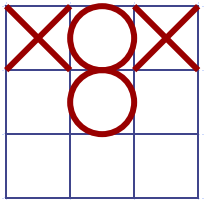
e(s) = number of rows, columns, and diagonals open for MAX
- number of rows, columns, and diagonals open for MIN



8-8 = 0



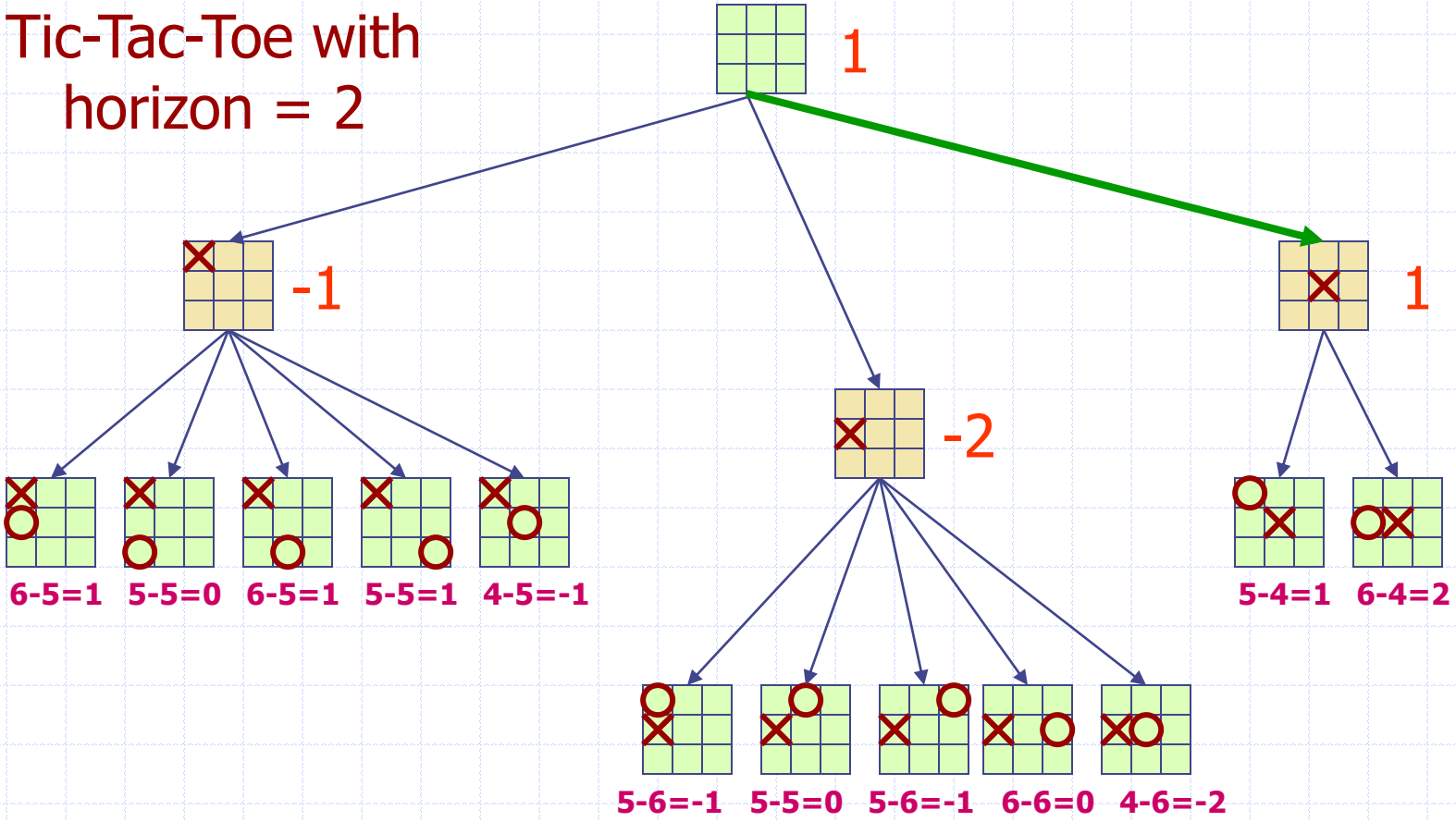
6-4 = 2



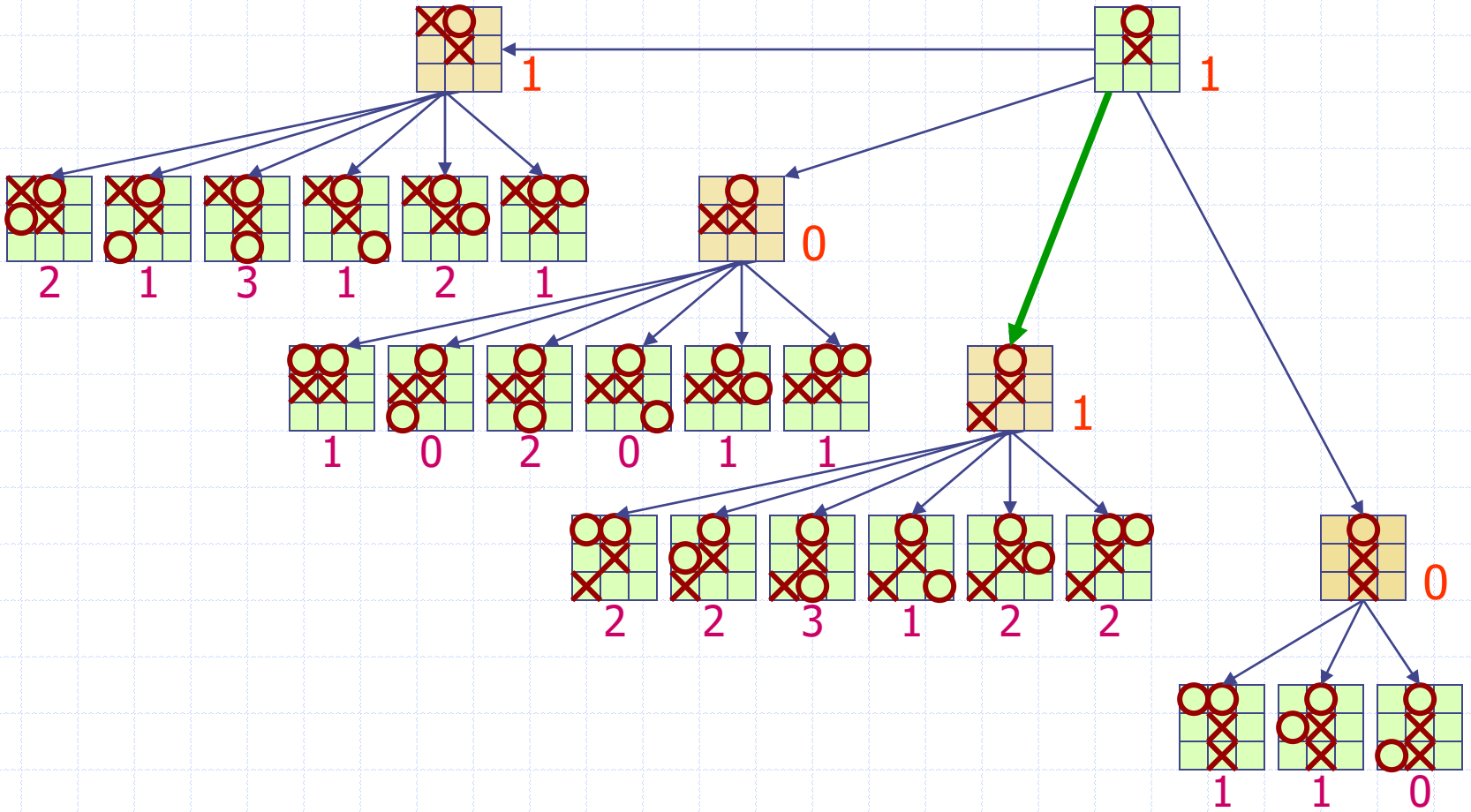
3-3 = 0

Example

Tic-Tac-Toe with horizon = 2



Example



Minimax procedure

1. Expand the game tree uniformly from the current state (where it is MAX's turn to play) to depth h
2. Compute the evaluation function at every leaf of the tree
3. Back-up the values from the leaves to the root of the tree as follows:
    1. A MAX node gets the maximum of the evaluation of its successors
    2. A MIN node gets the minimum of the evaluation of its successors
4. Select the move toward the MIN node that has the maximal backed-up value

Minimax procedure

1. Expand the game tree uniformly from the current state (where it is MAX's turn to play) to depth h
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   1. A MAX r
      succesc
   2. A MIN n
      succesc
4. Select the he maximal backed-up value

Horizon of the procedure

Needed to limit the size of the tree or to return a decision within allowed time

Game Playing (for MAX)

Repeat until *win*, *lose*, or *draw*

1. Select move using Minimax procedure
2. Execute move
3. Observe MIN's move

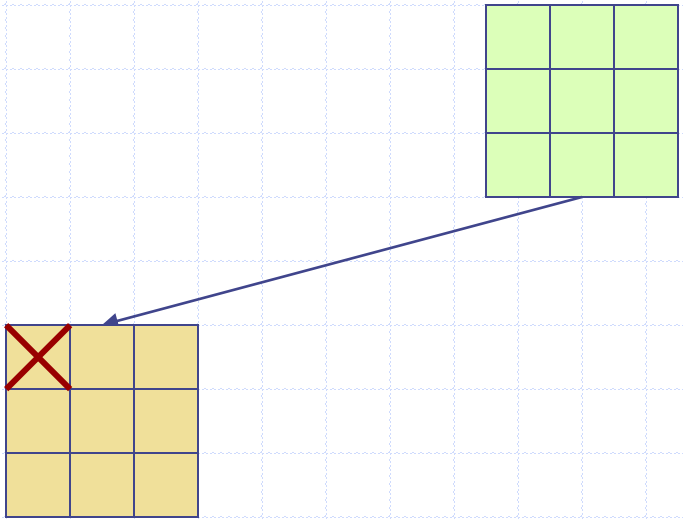
Issues

- Choice of the horizon
- Size of memory needed
- Number of nodes examined

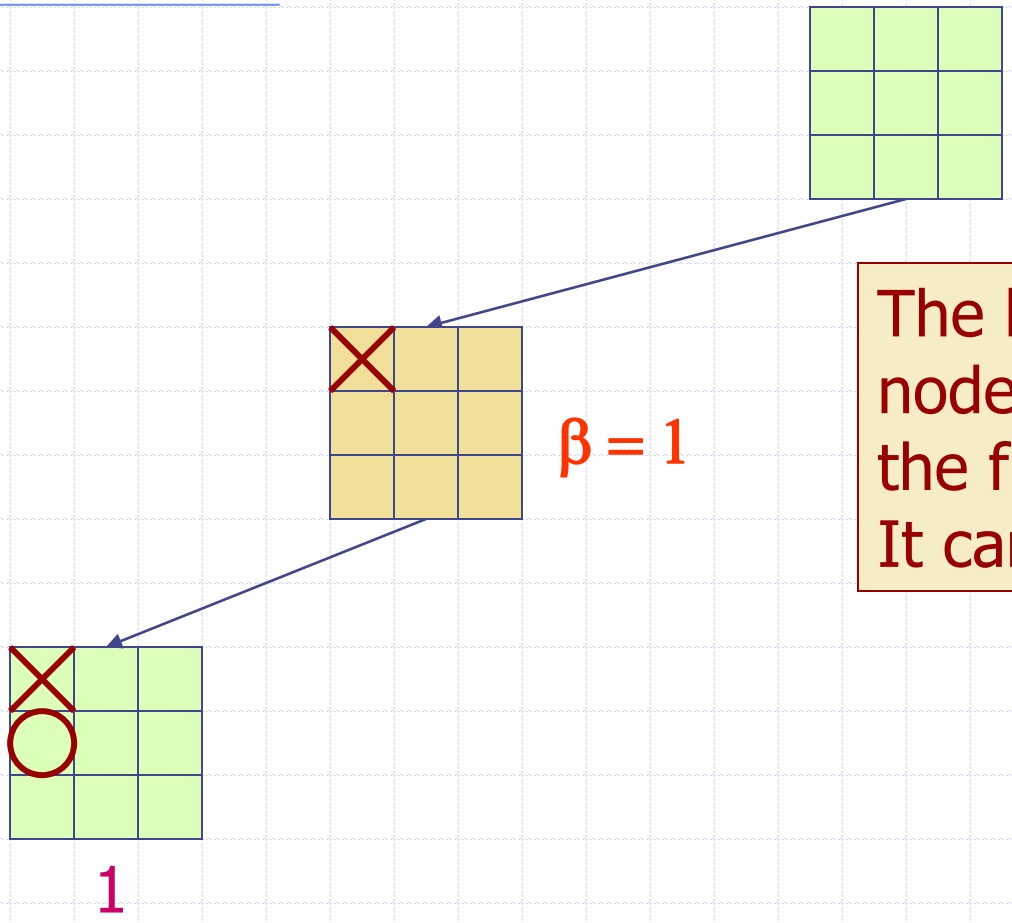
Alpha-Beta Procedure

- Generate the game tree to depth h in depth-first manner
- Back-up estimates (alpha and beta values) of the evaluation functions whenever possible
- Prune branches that cannot lead to changing the final decision

Example

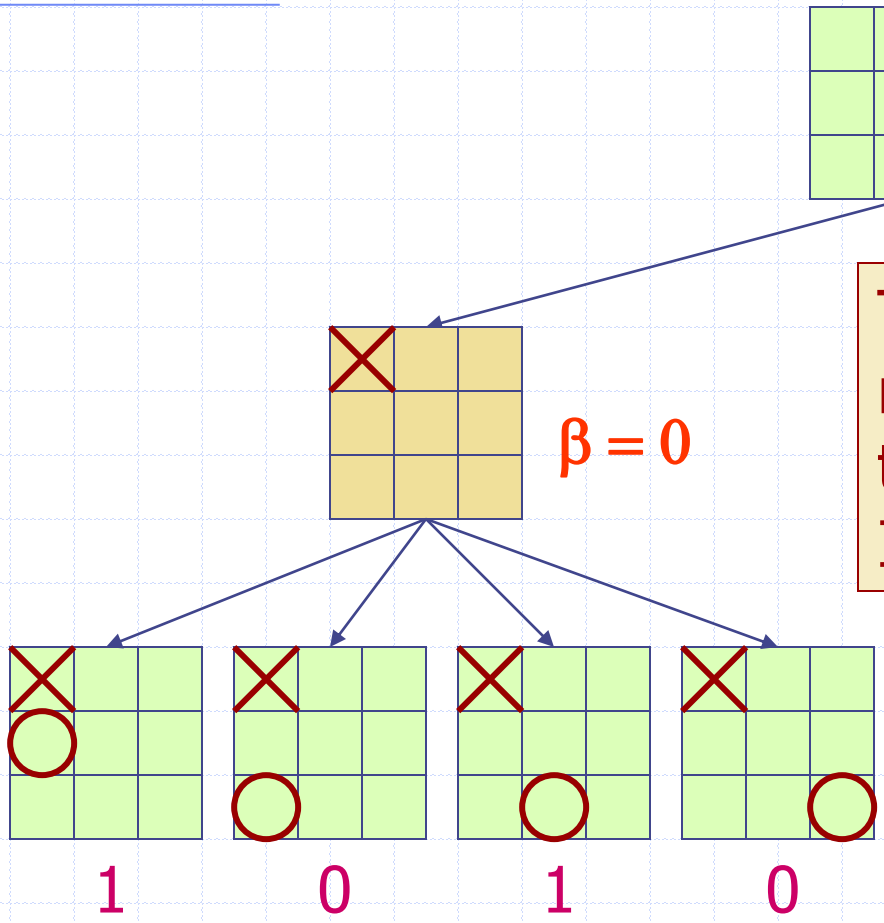


Example



The beta value of a MIN node is a higher bound on the final backed-up value. It can never increase

Example



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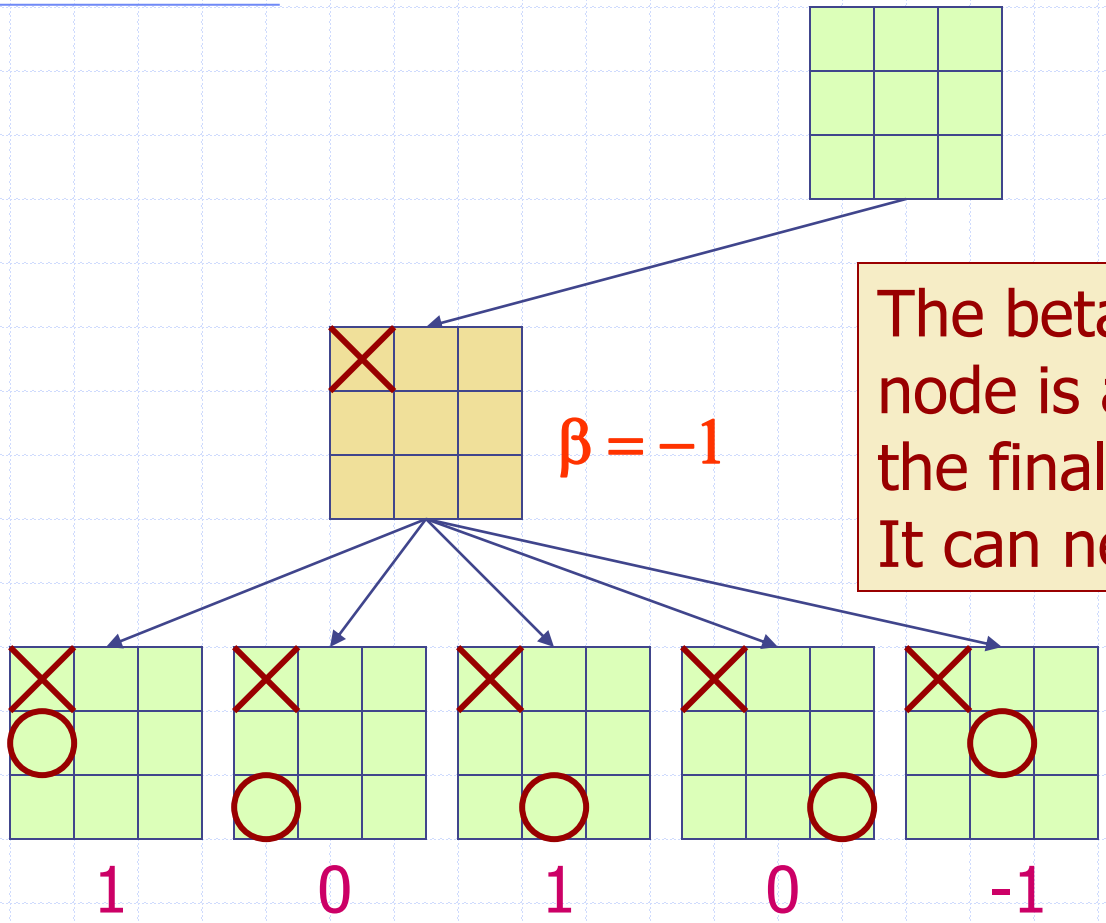

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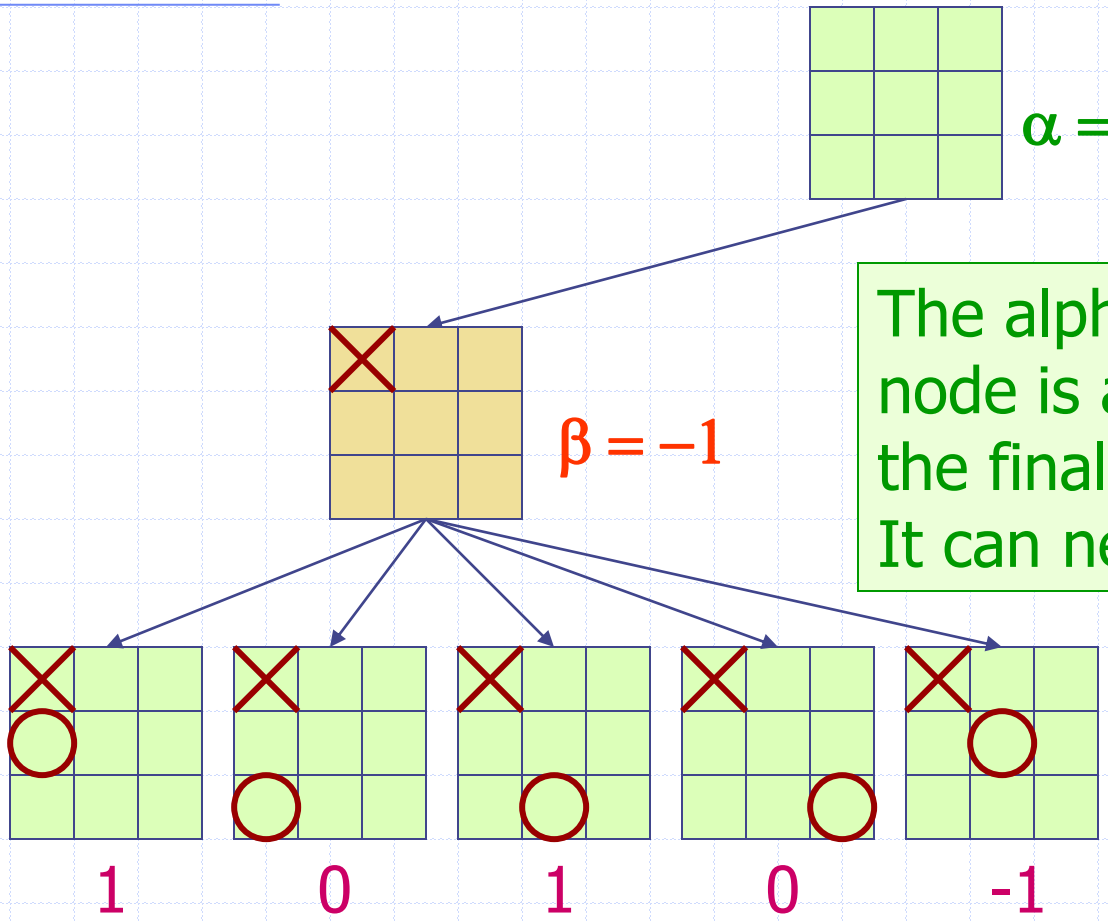
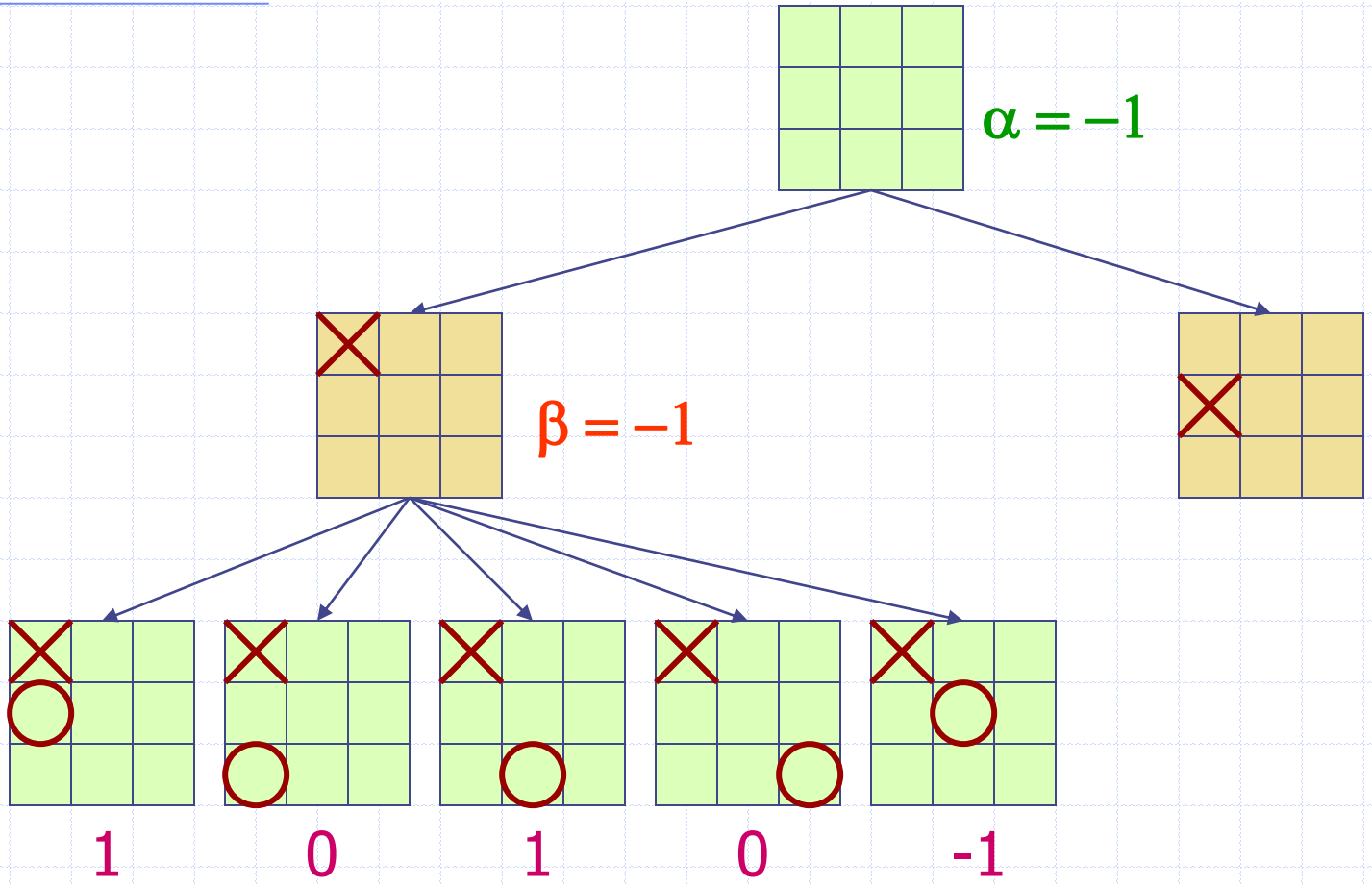
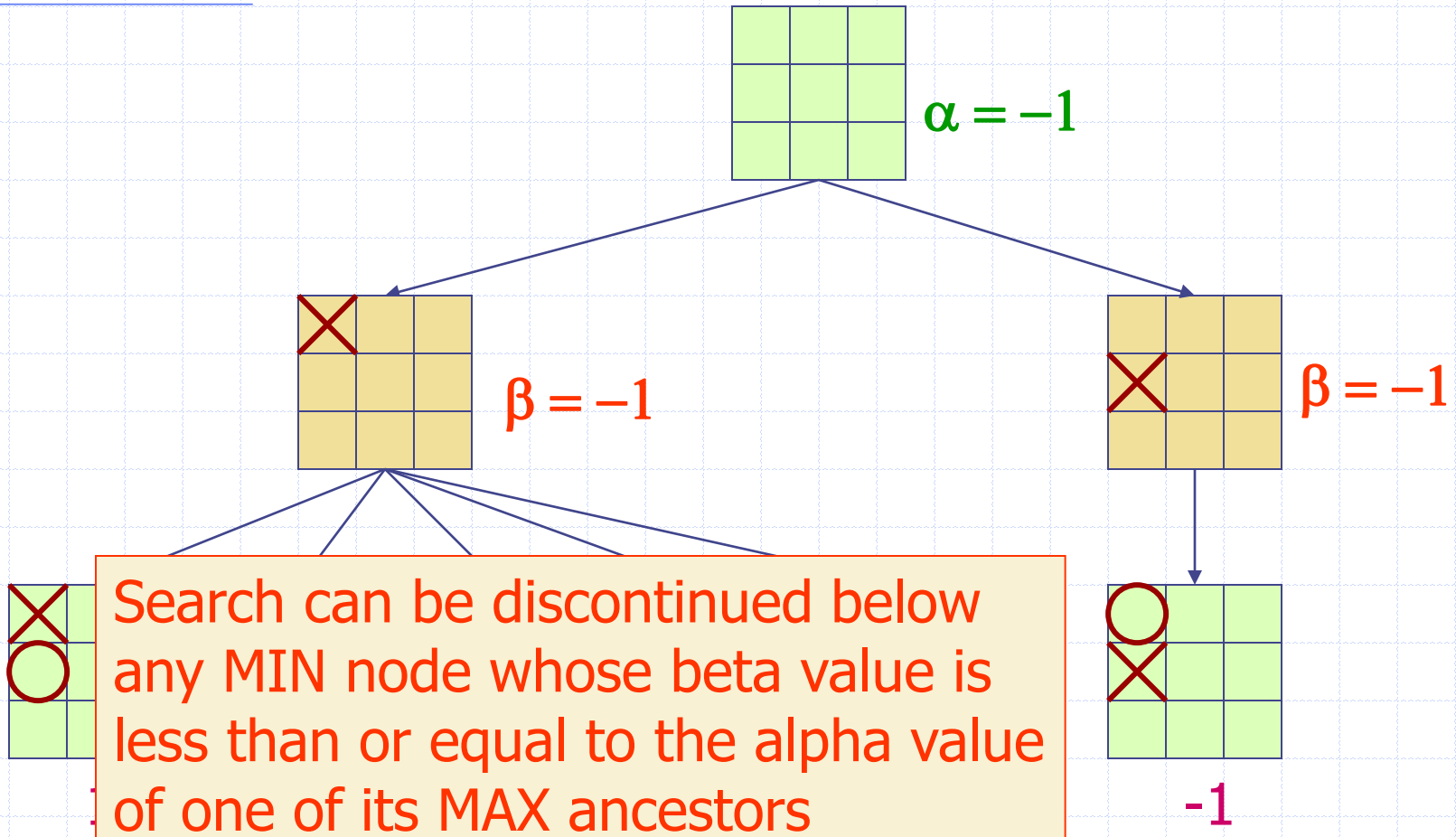


The alpha value of a MAX node is a lower bound on the final backed-up value. It can never decrease

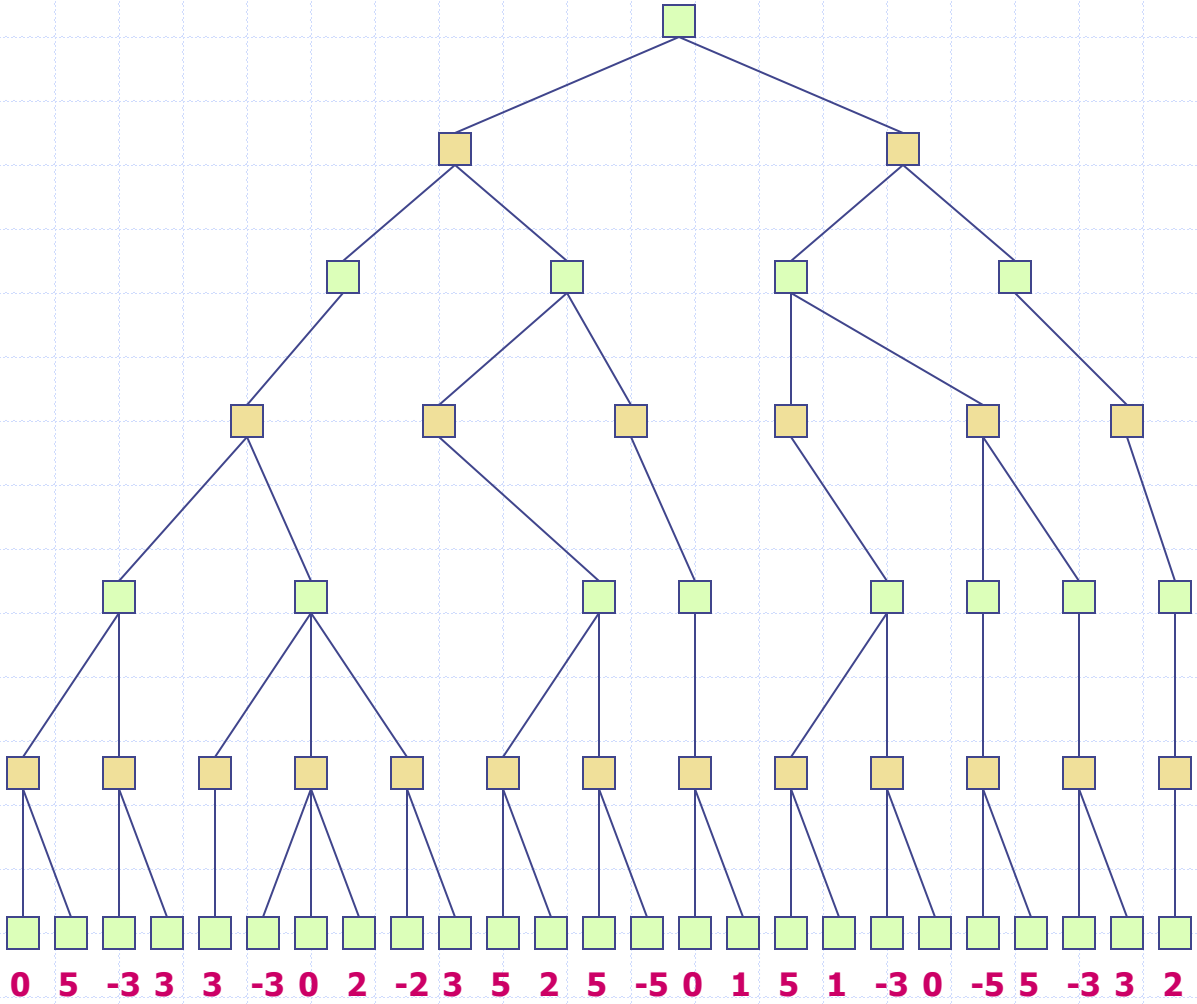
Example



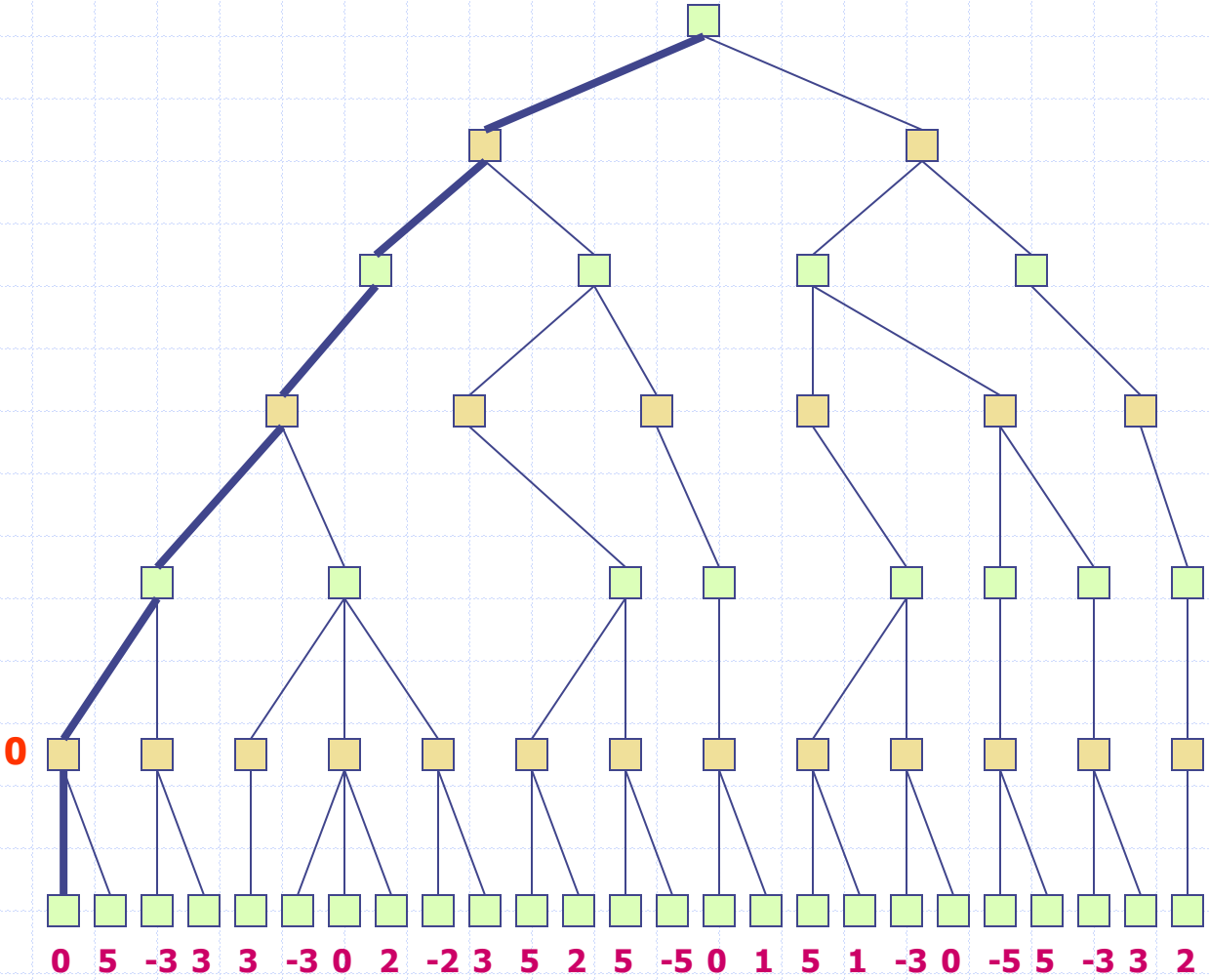
Example


α = –1
β = –1
β = –1
-1
Search can be discontinued below any MIN node whose beta value is less than or equal to the alpha value of one of its MAX ancestors

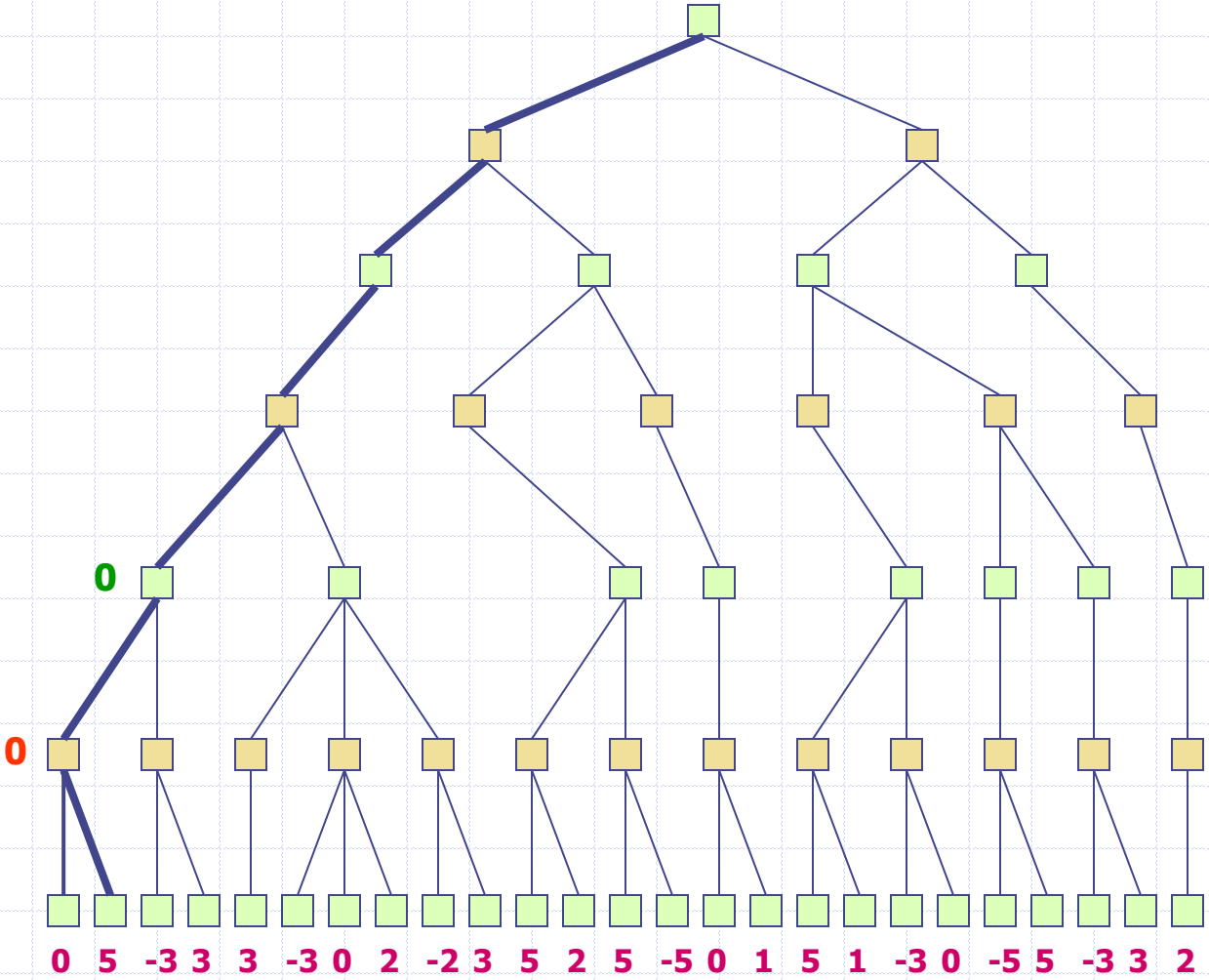
Alpha-Beta Example



Alpha-Beta Example



Alpha-Beta Example



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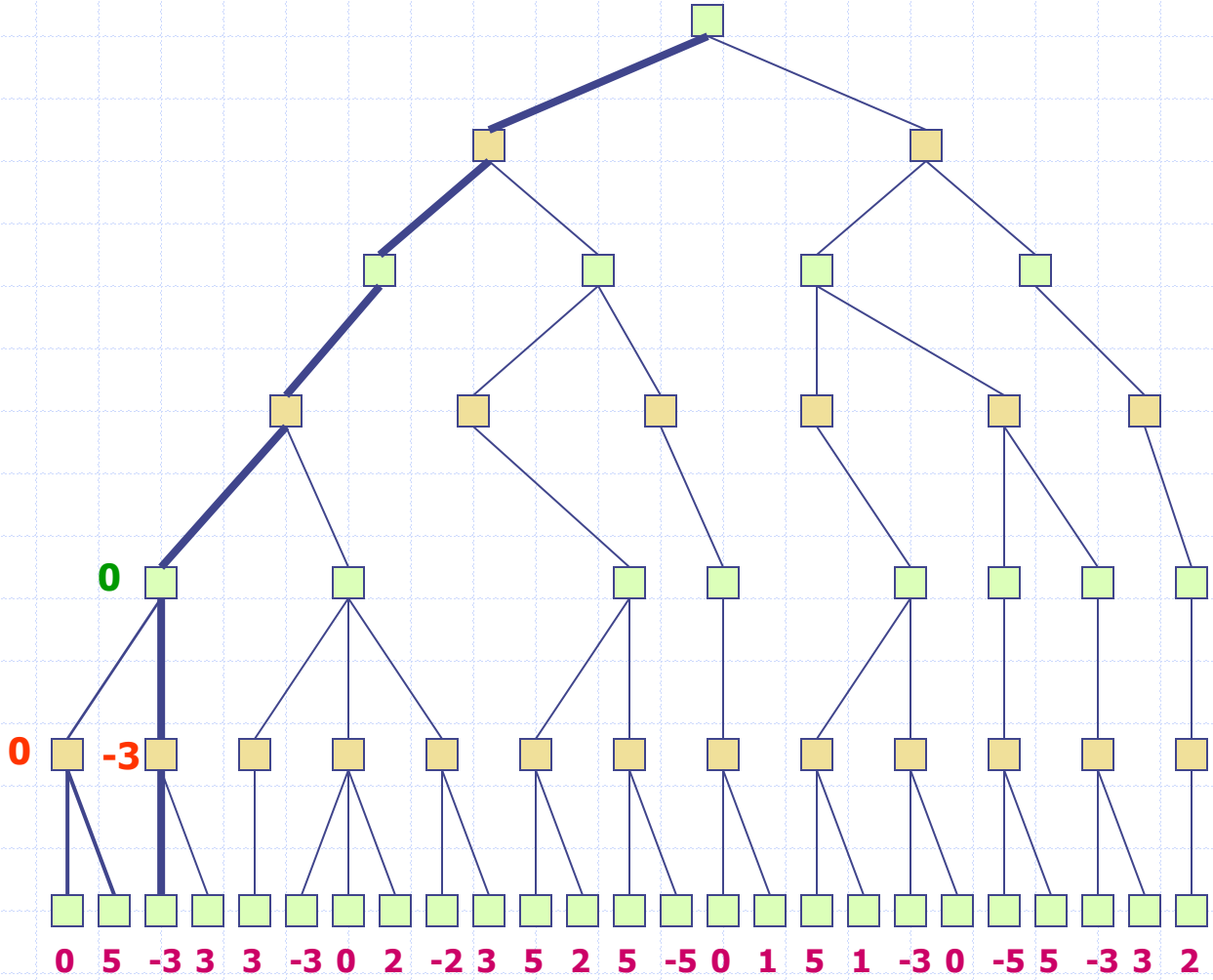

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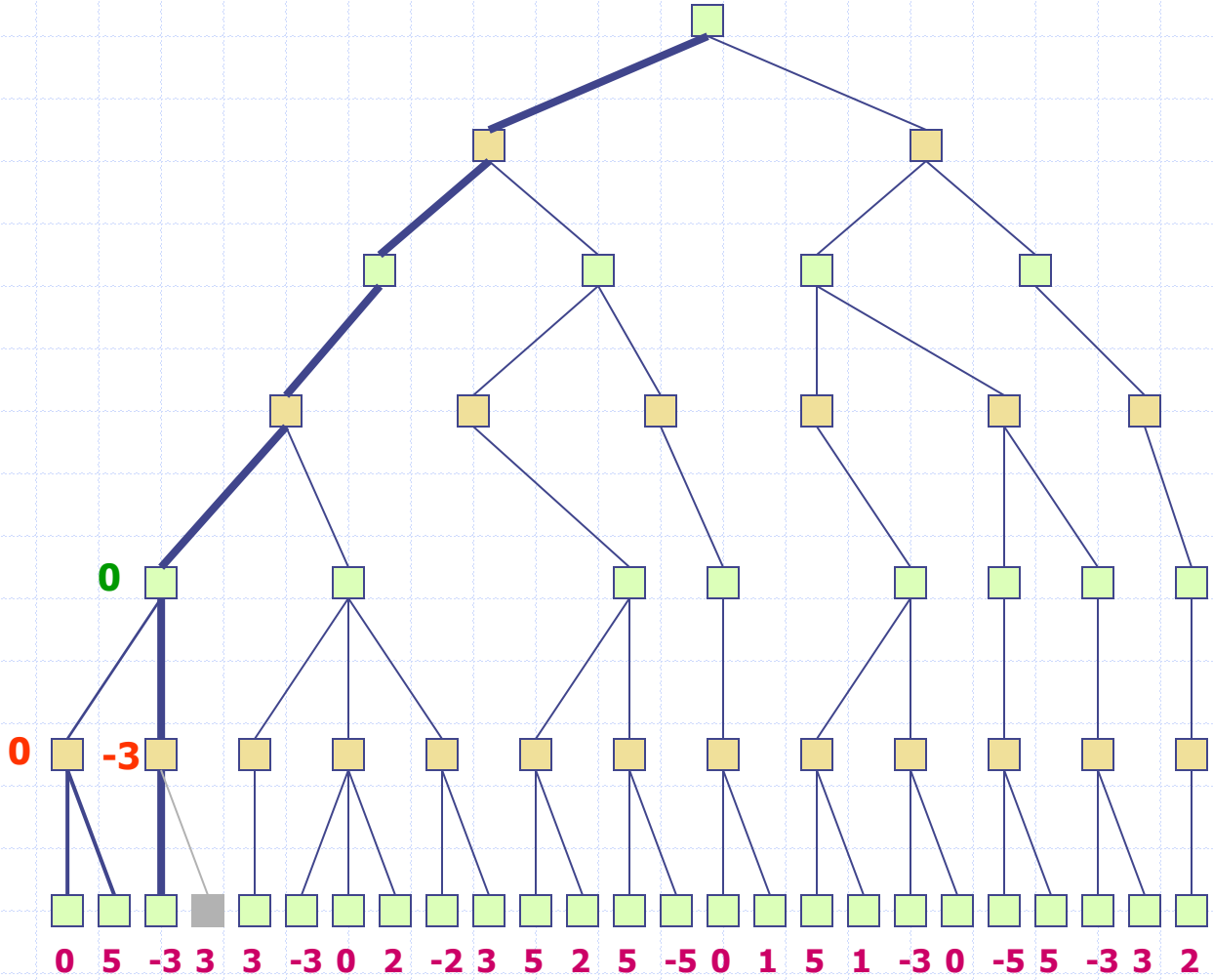

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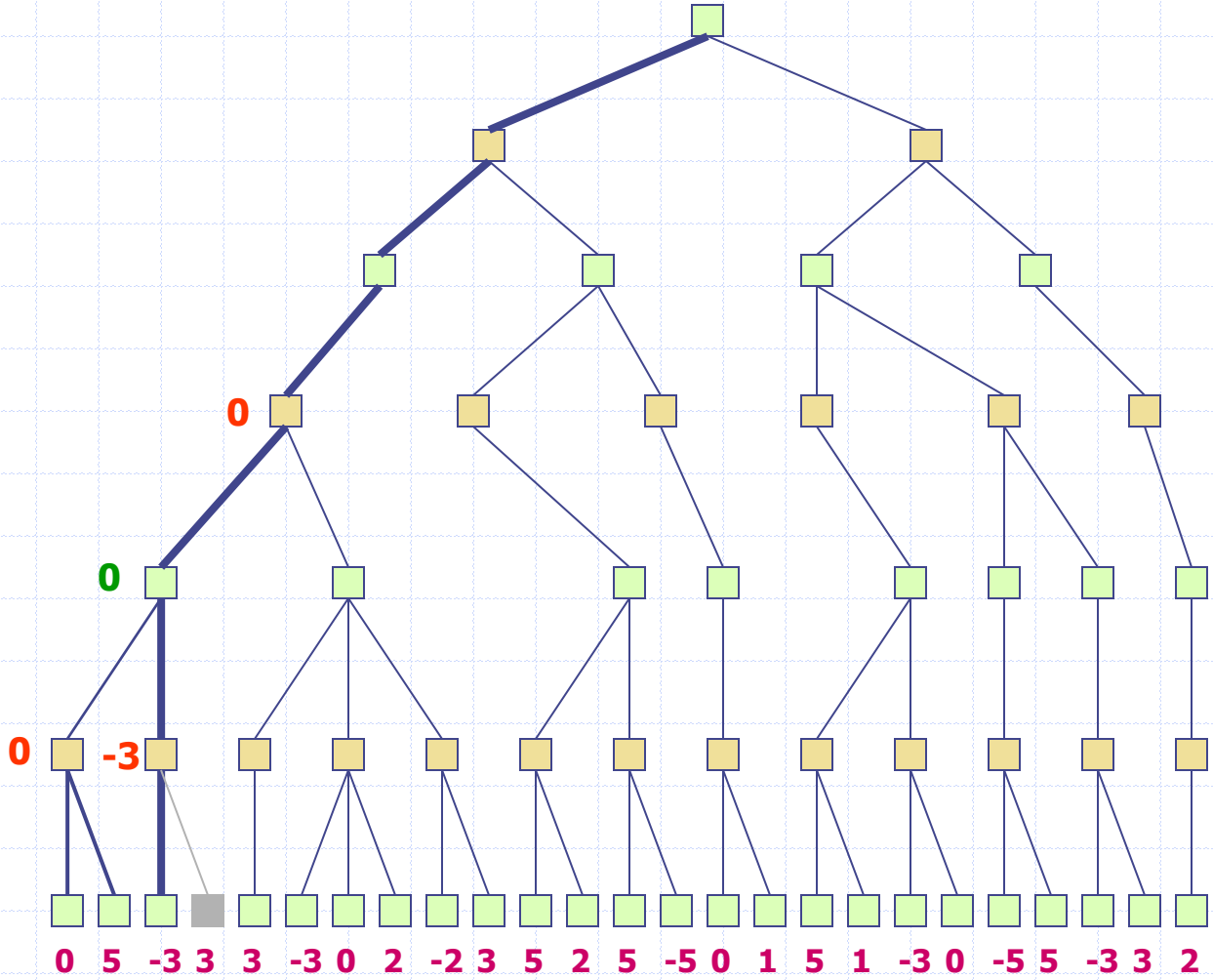

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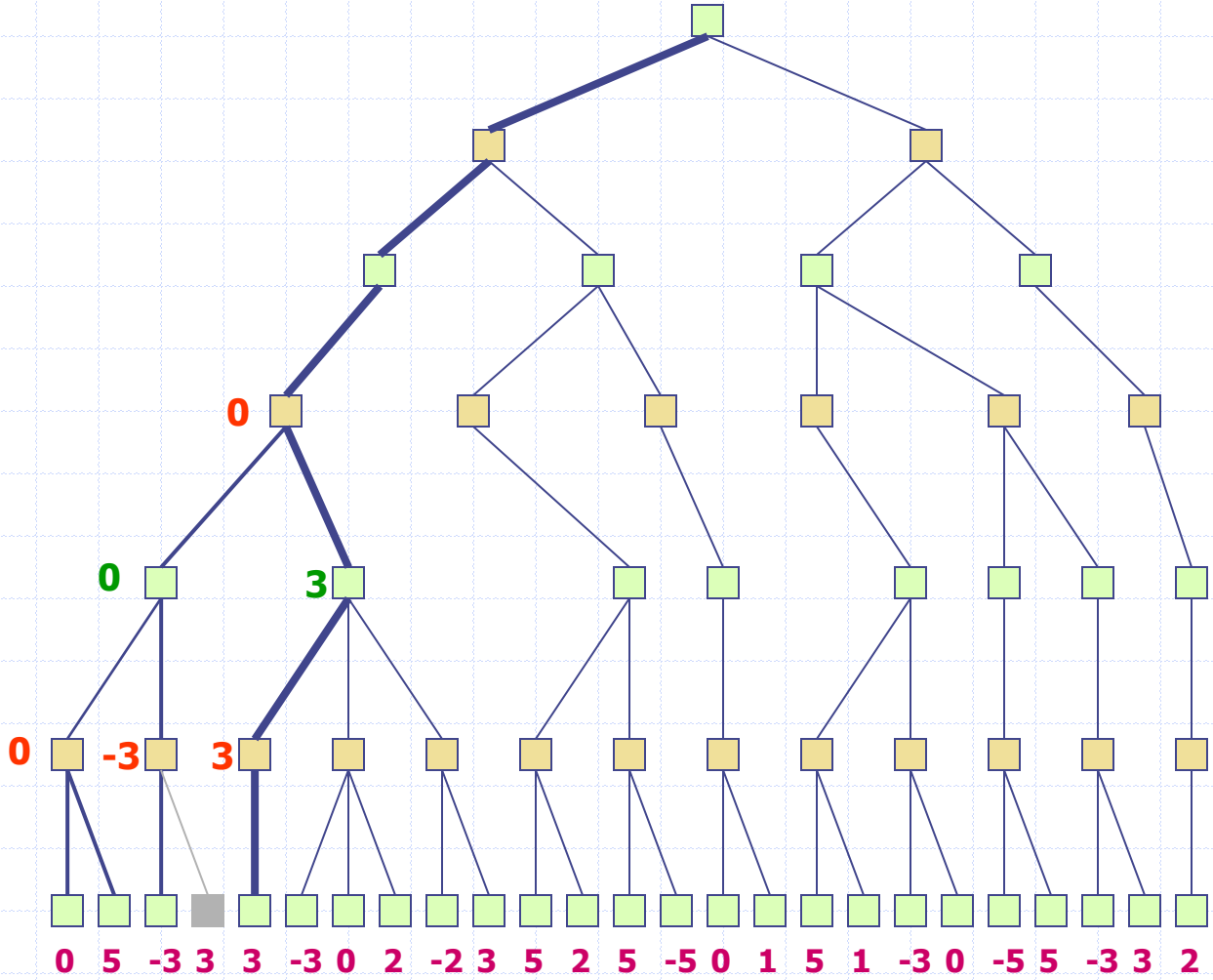

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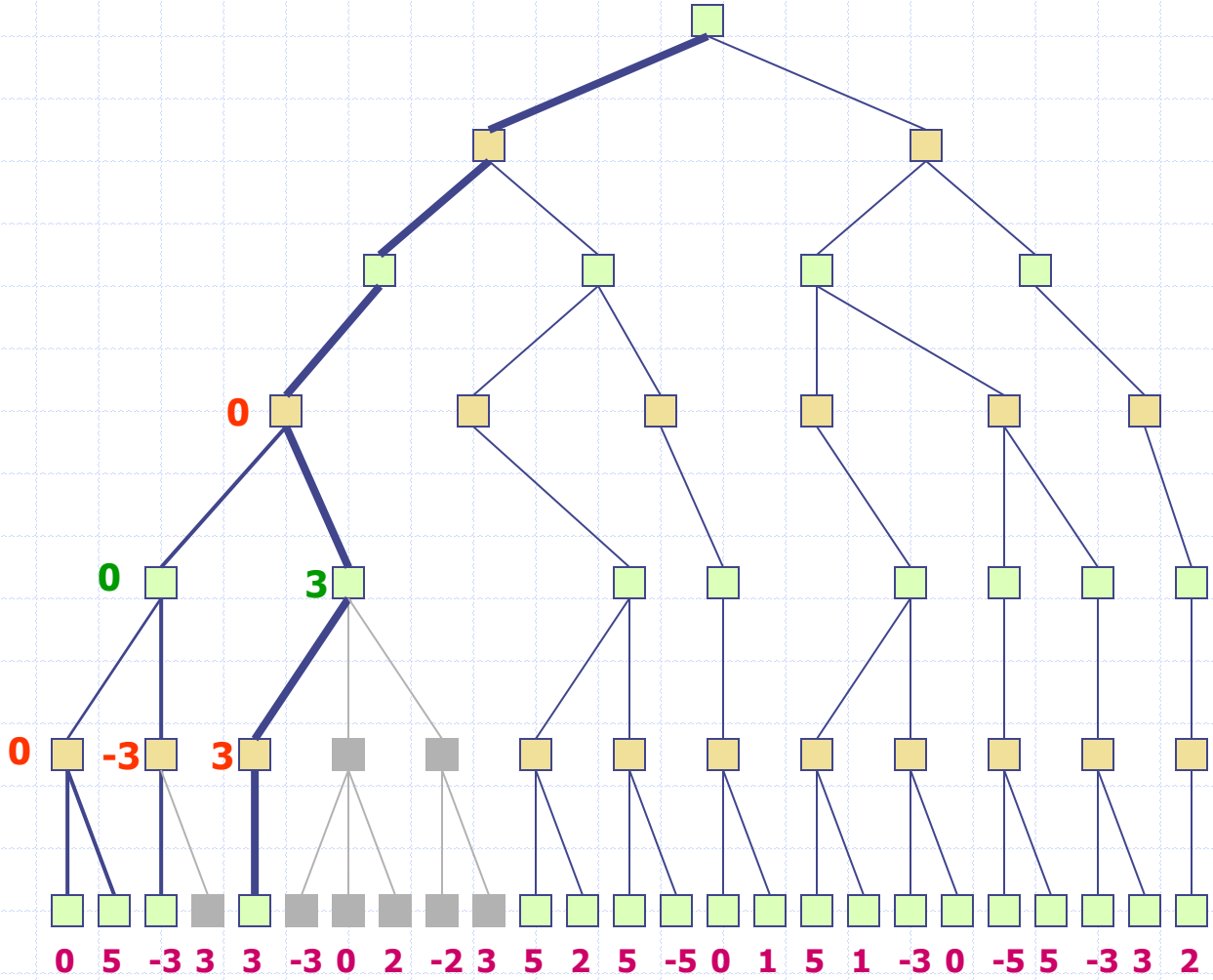

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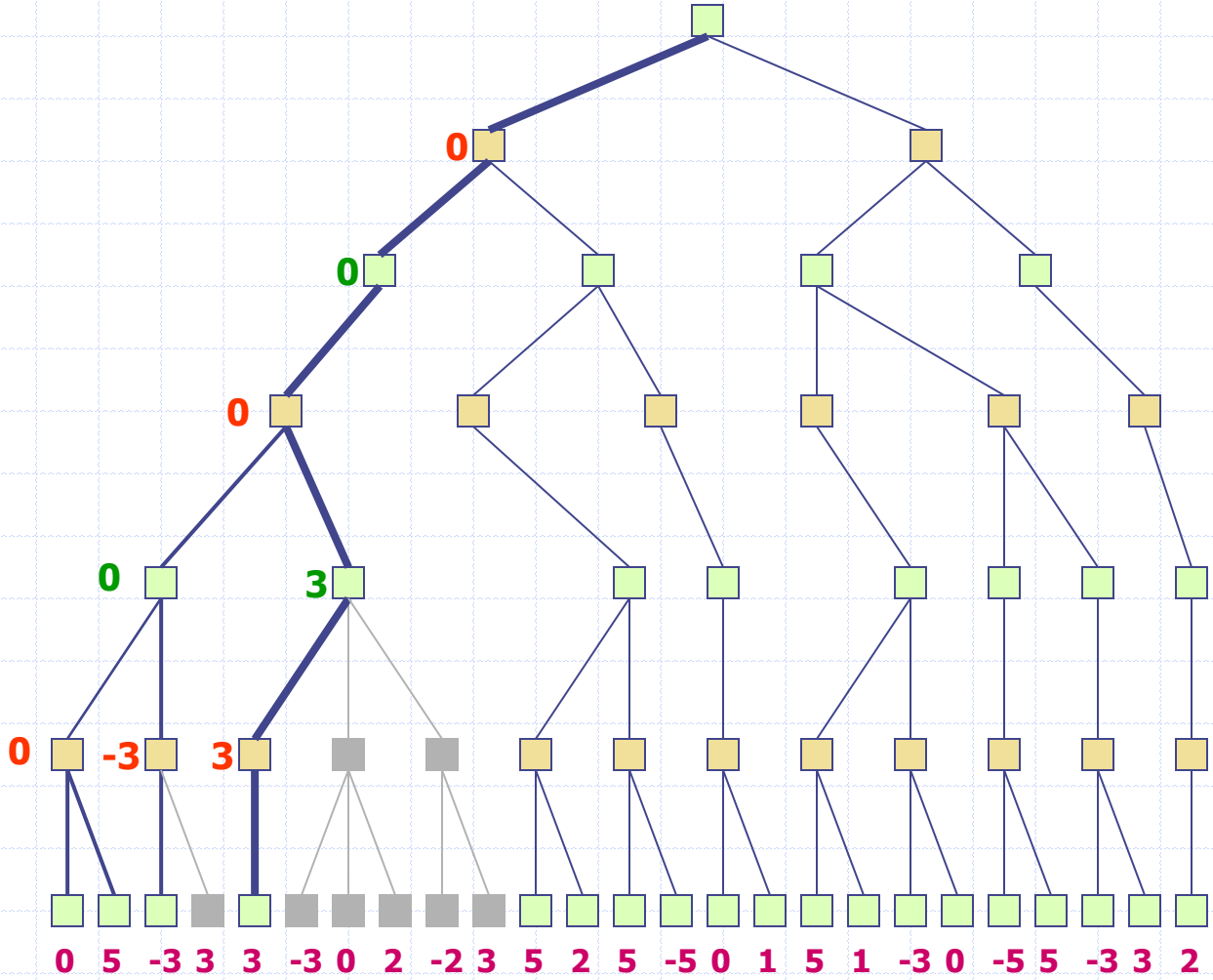

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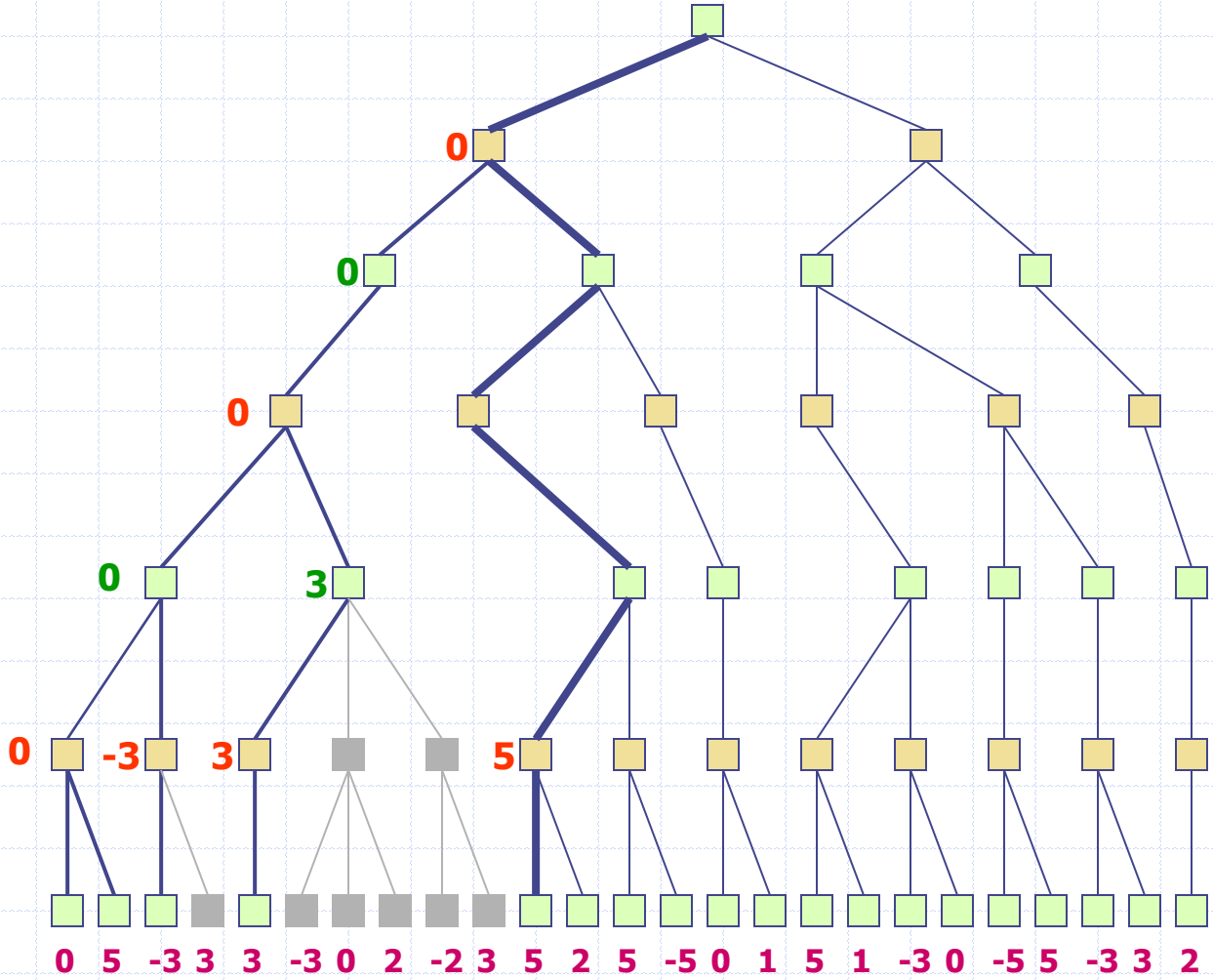

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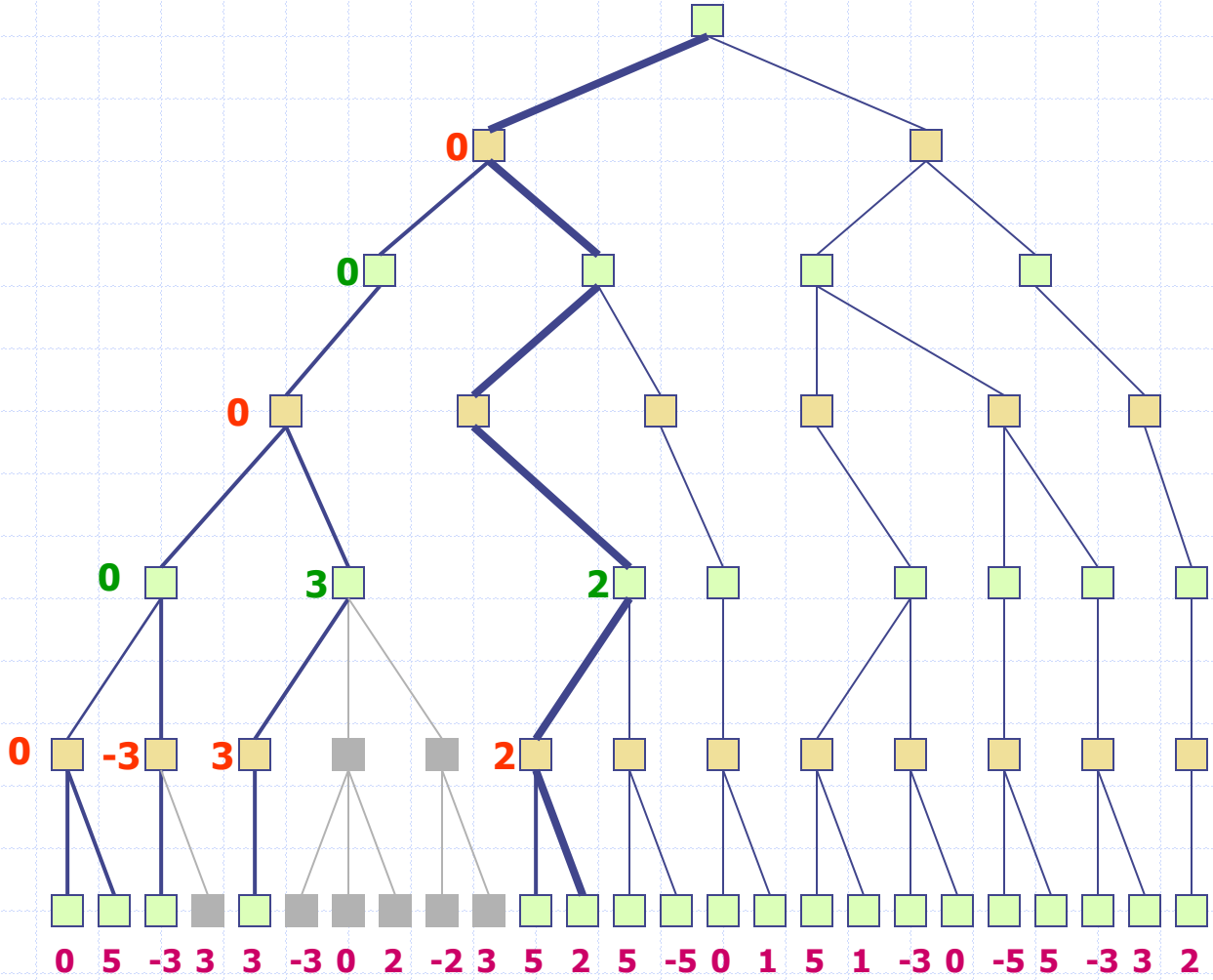

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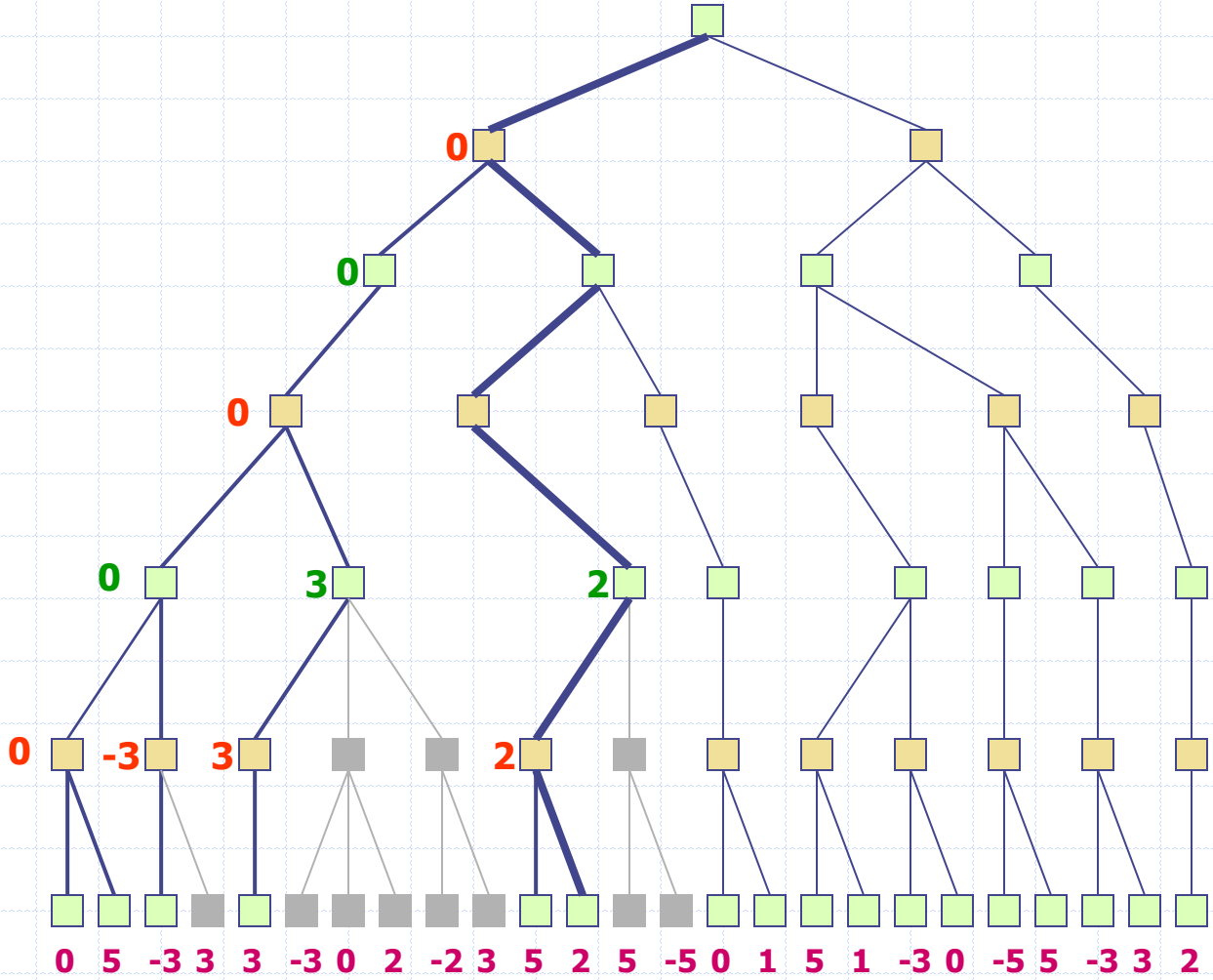

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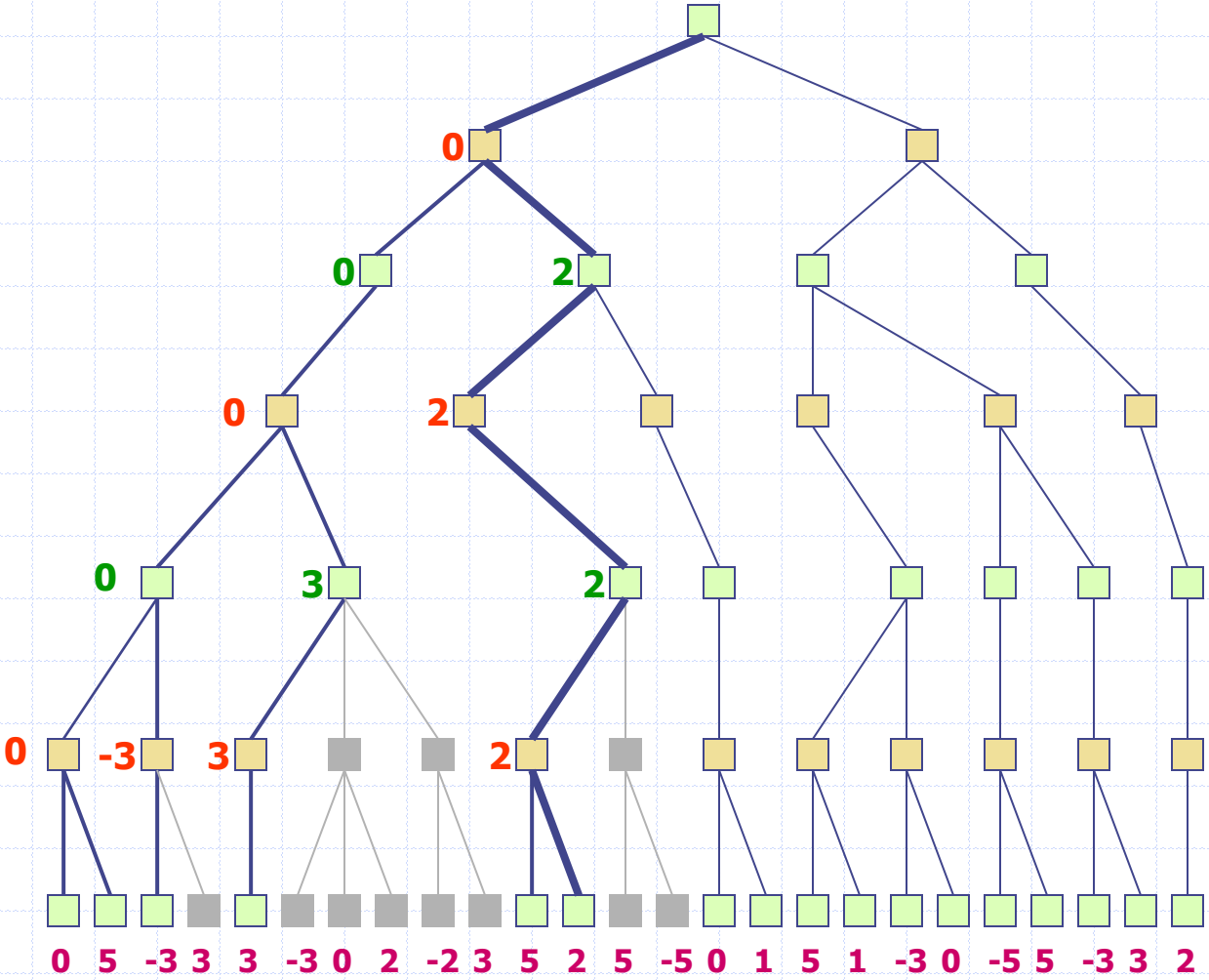

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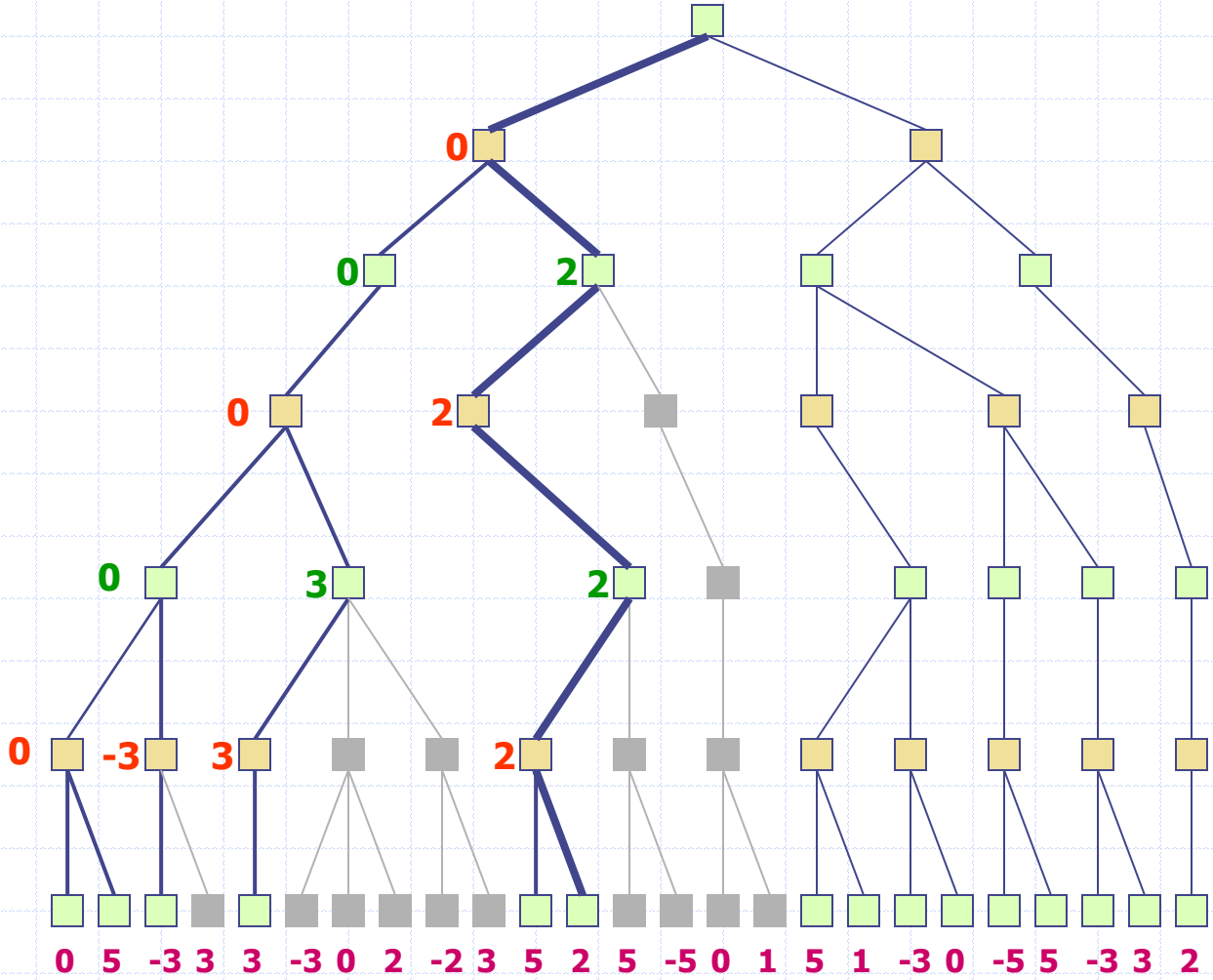

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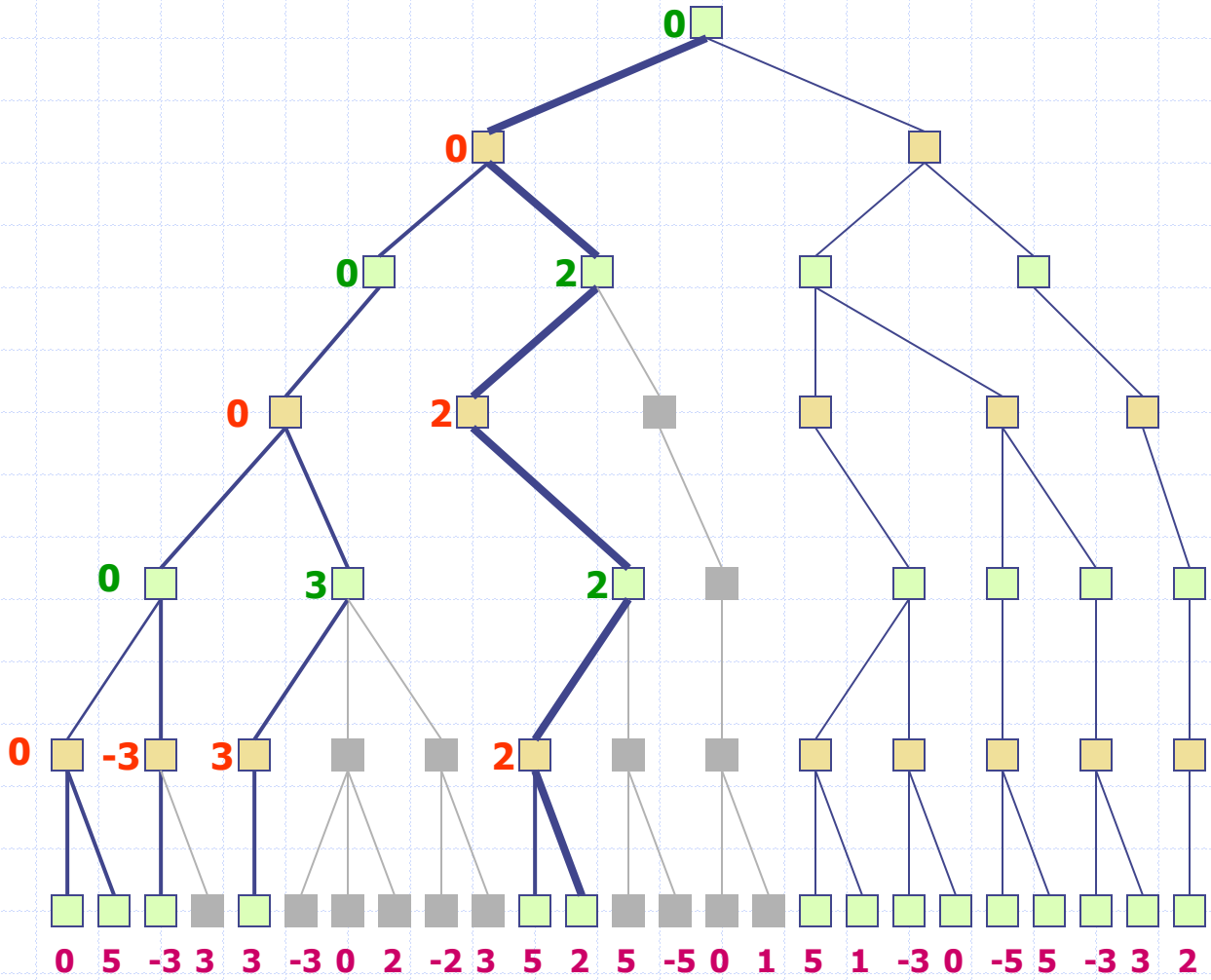

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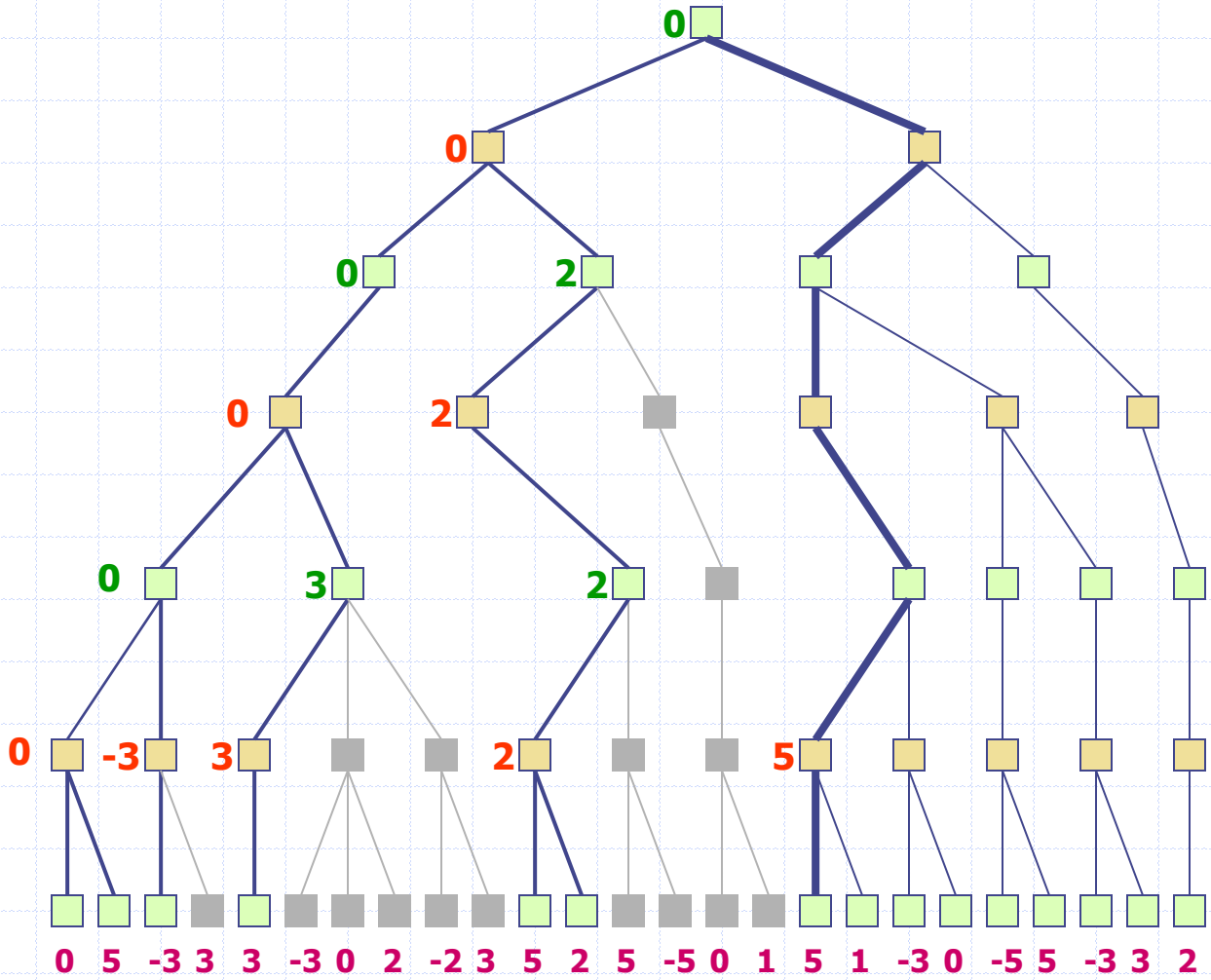

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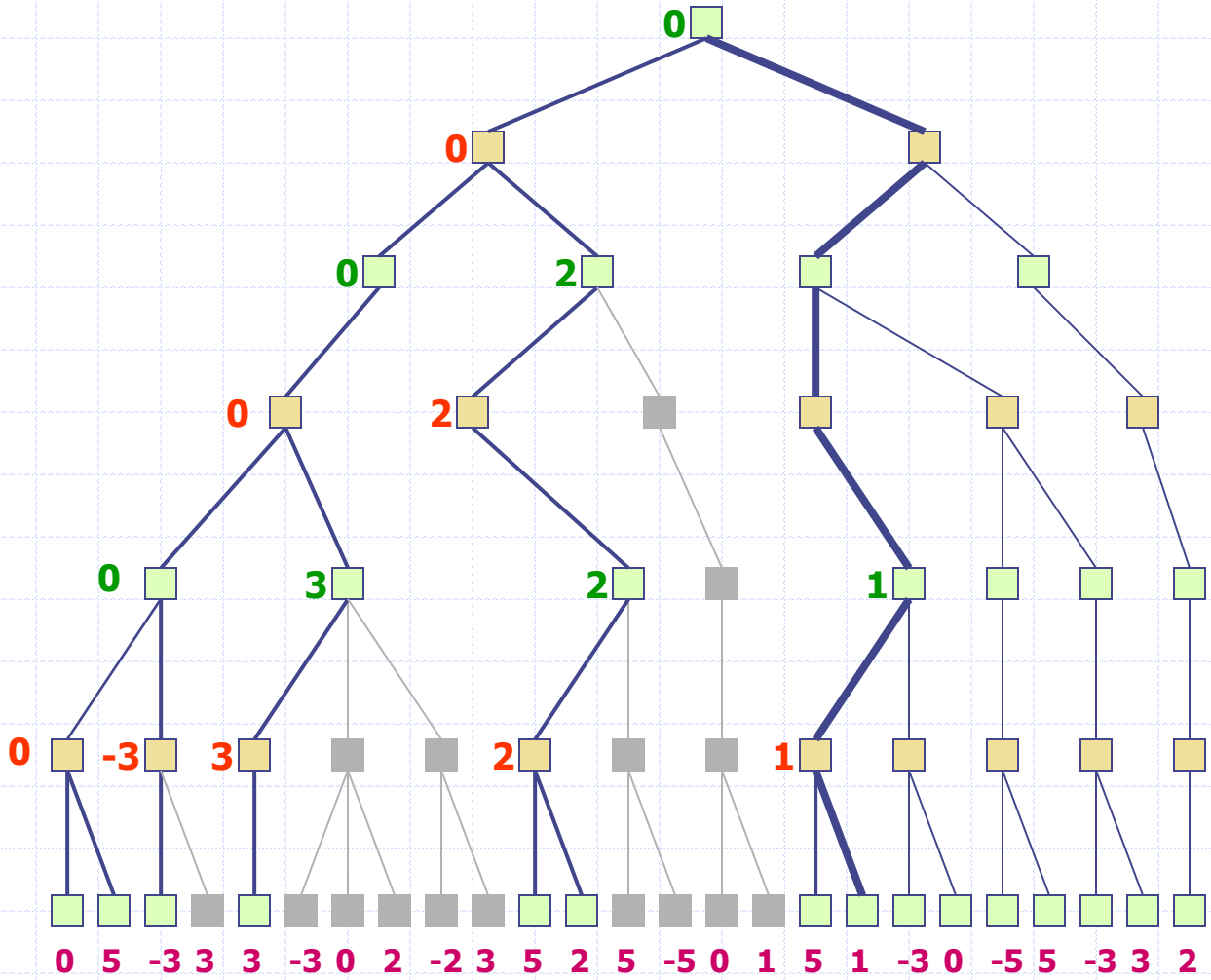

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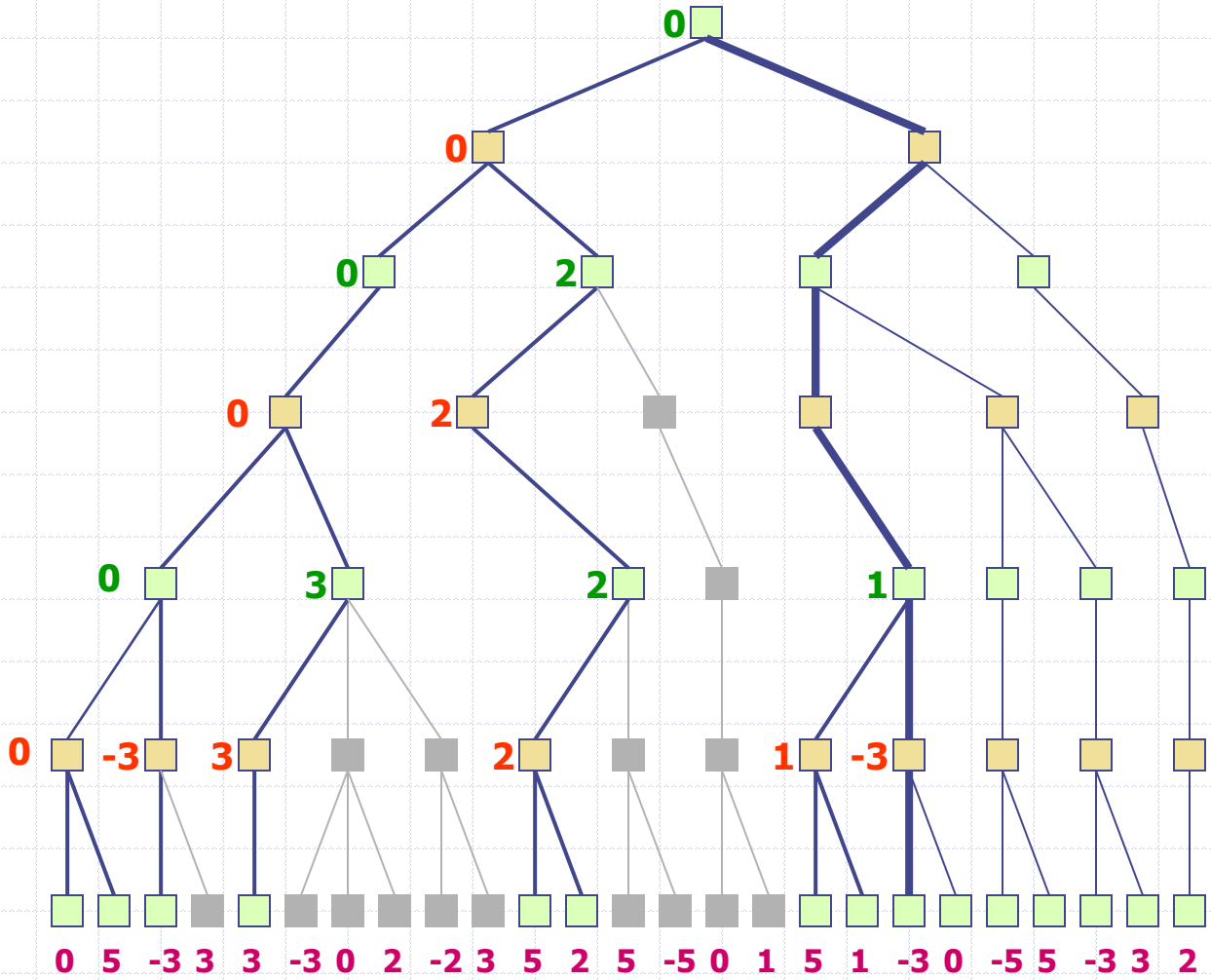

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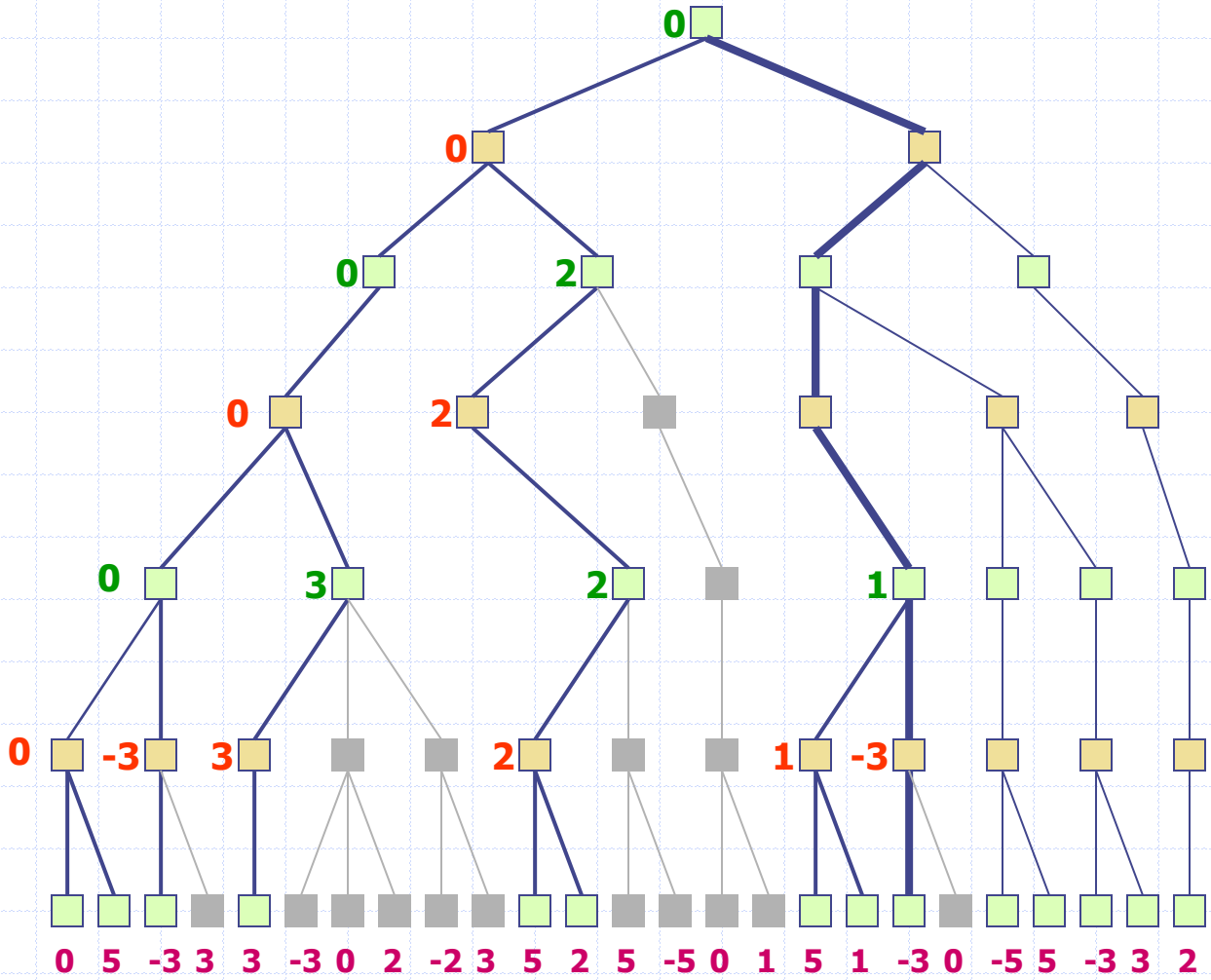

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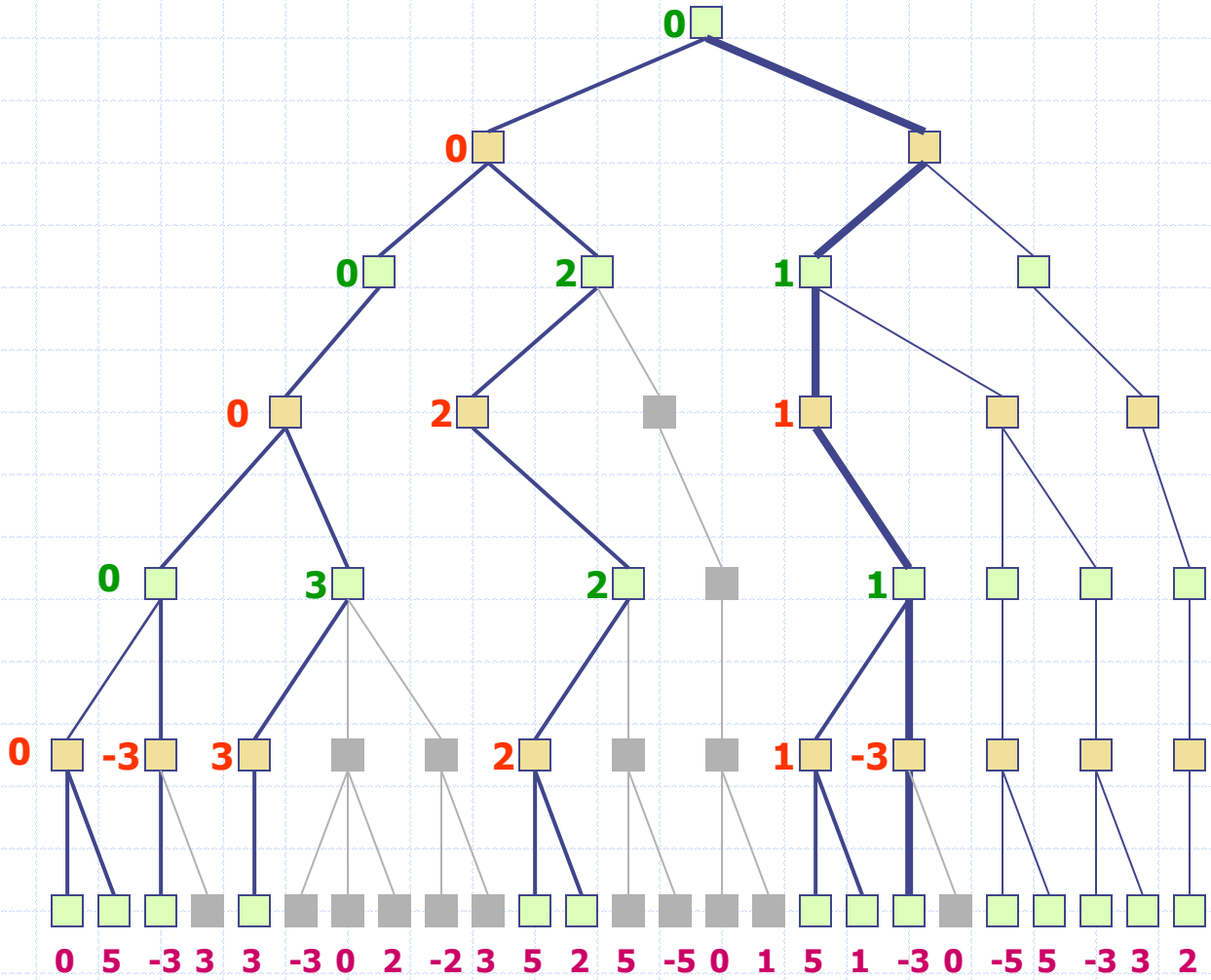

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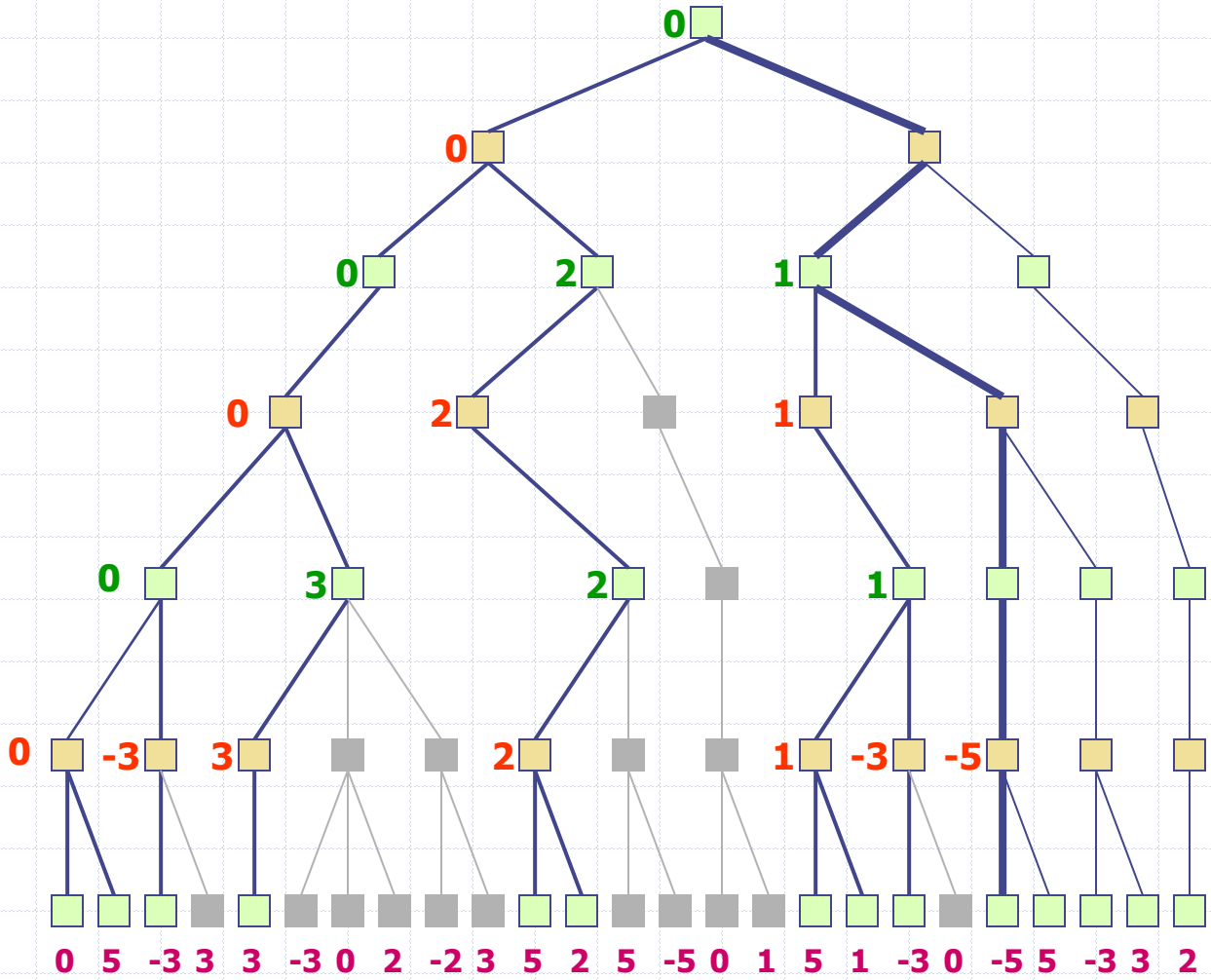

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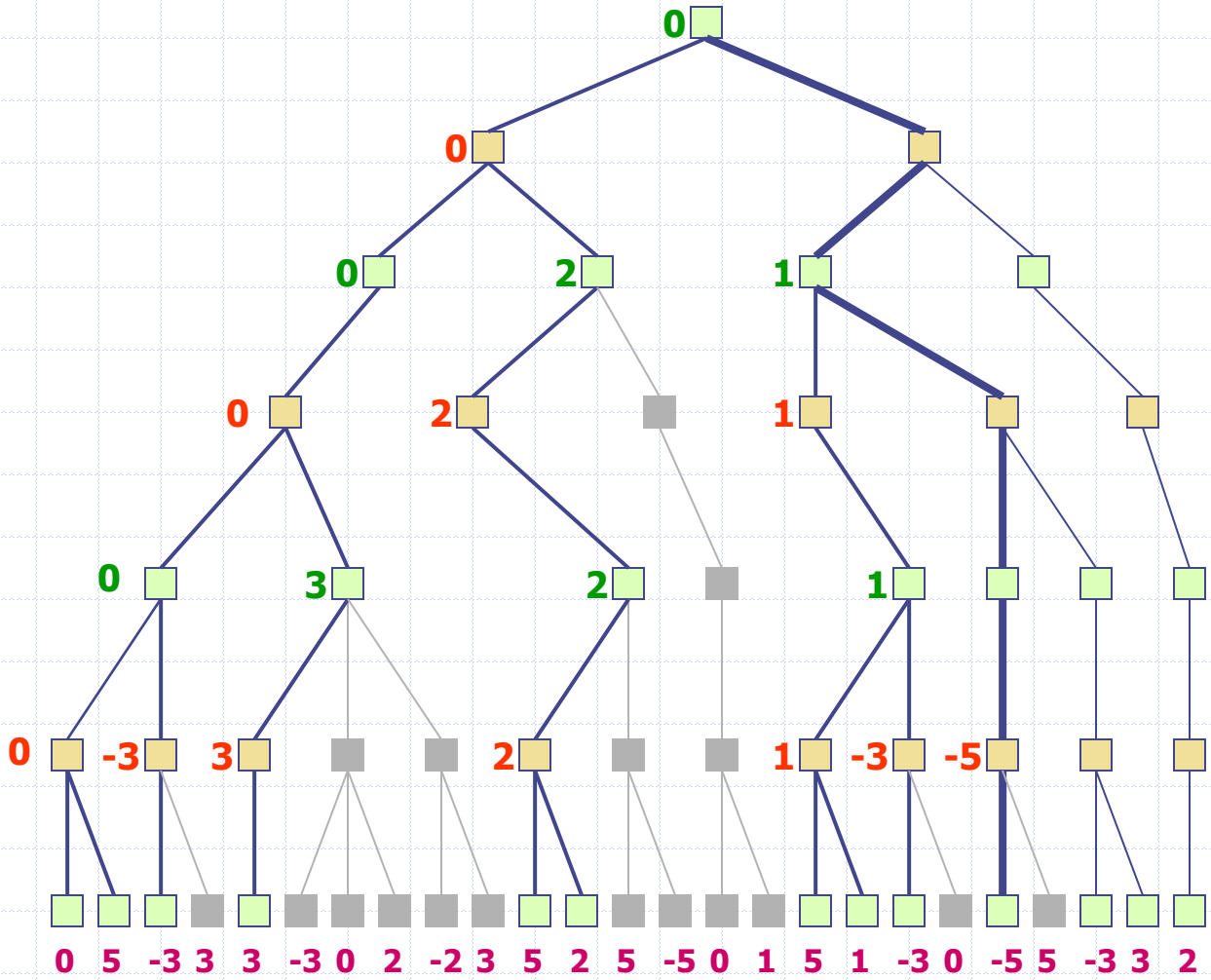

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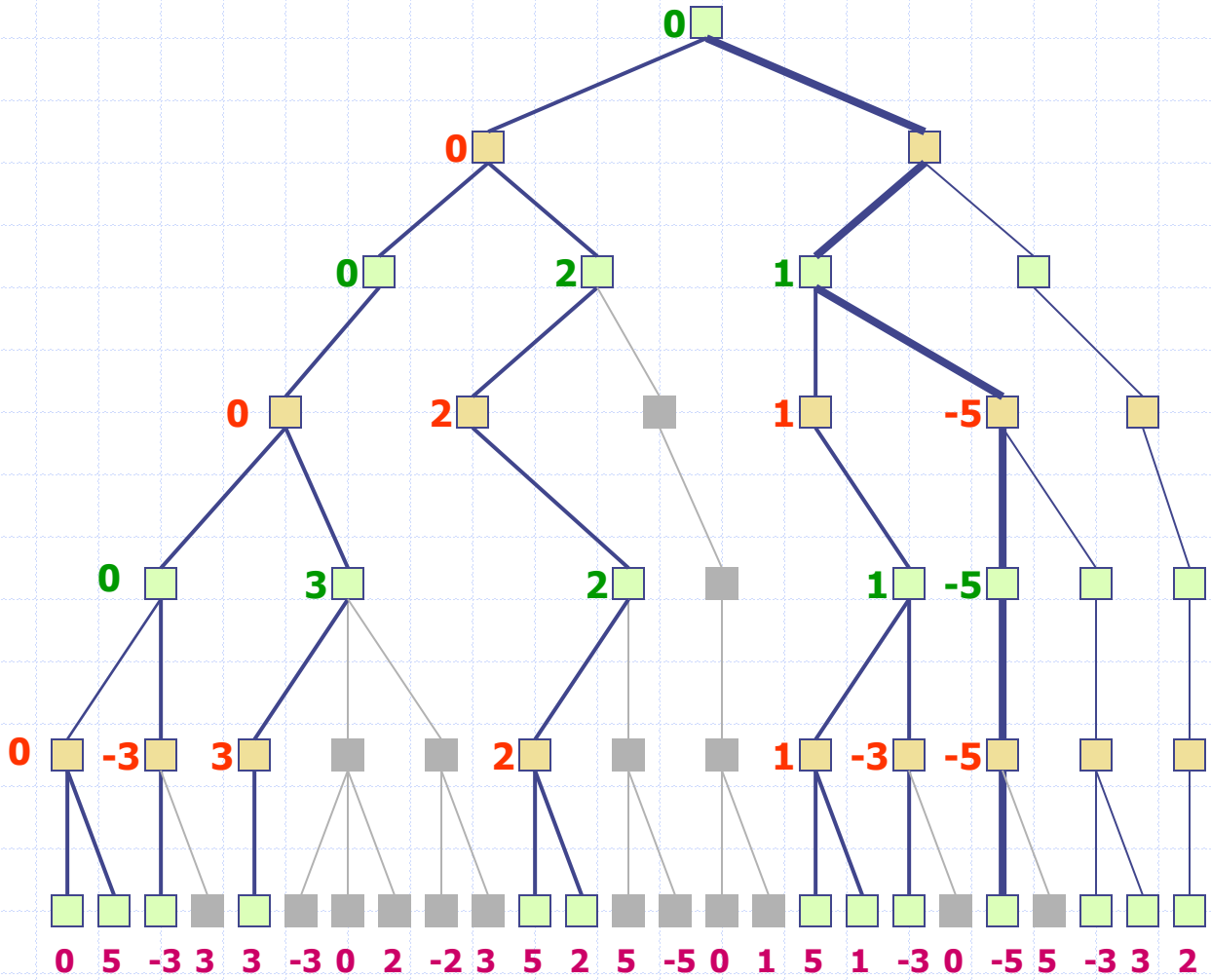

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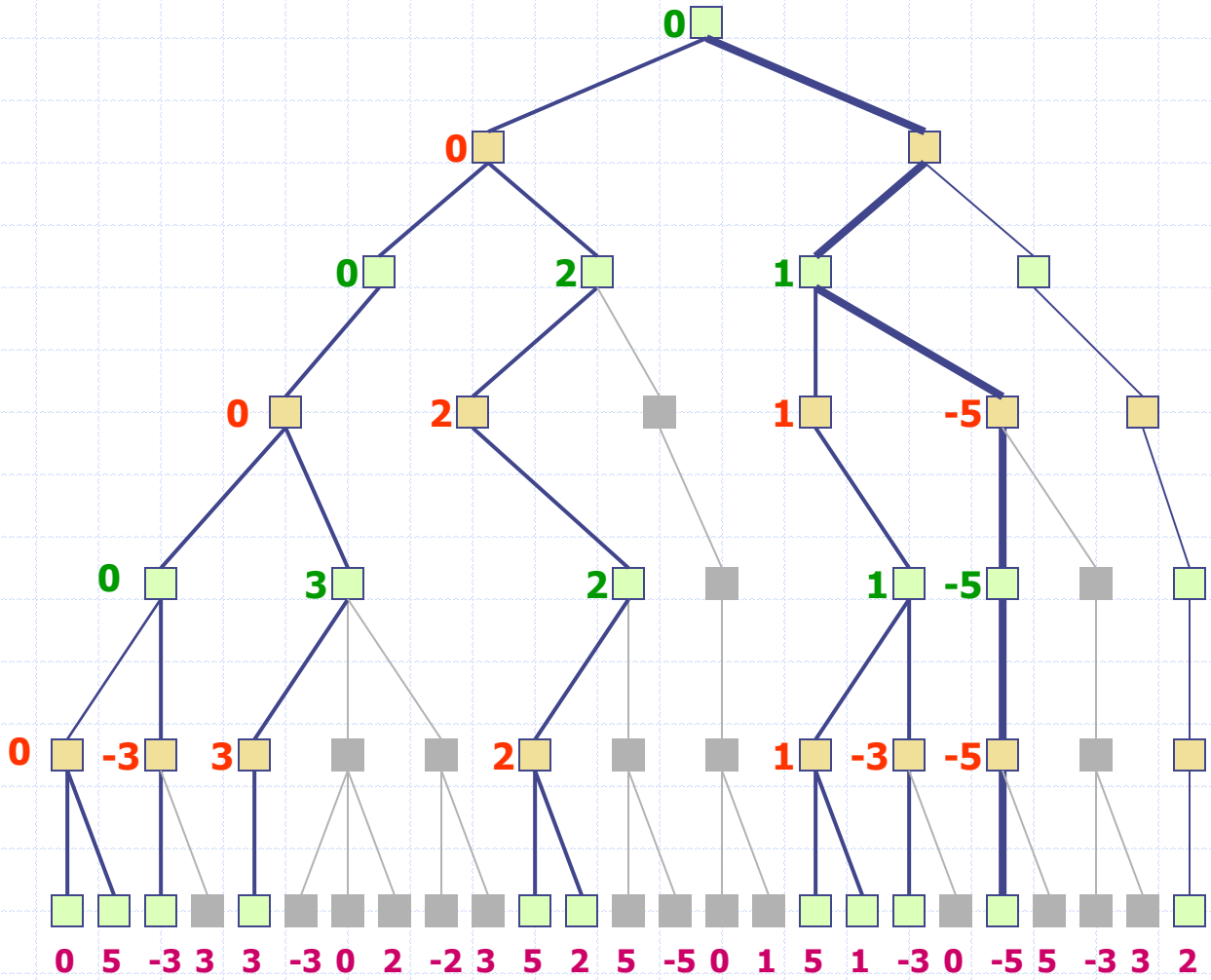

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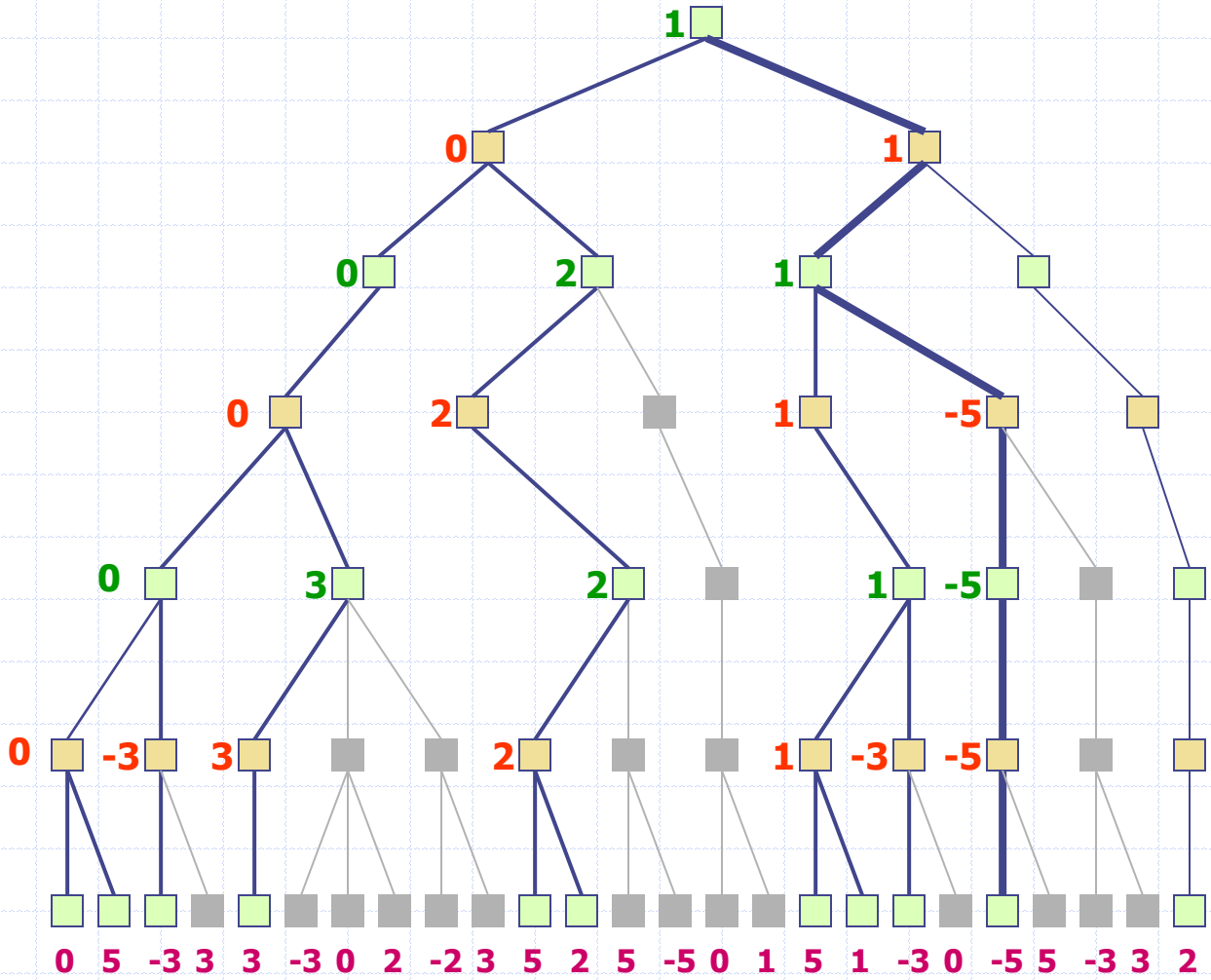

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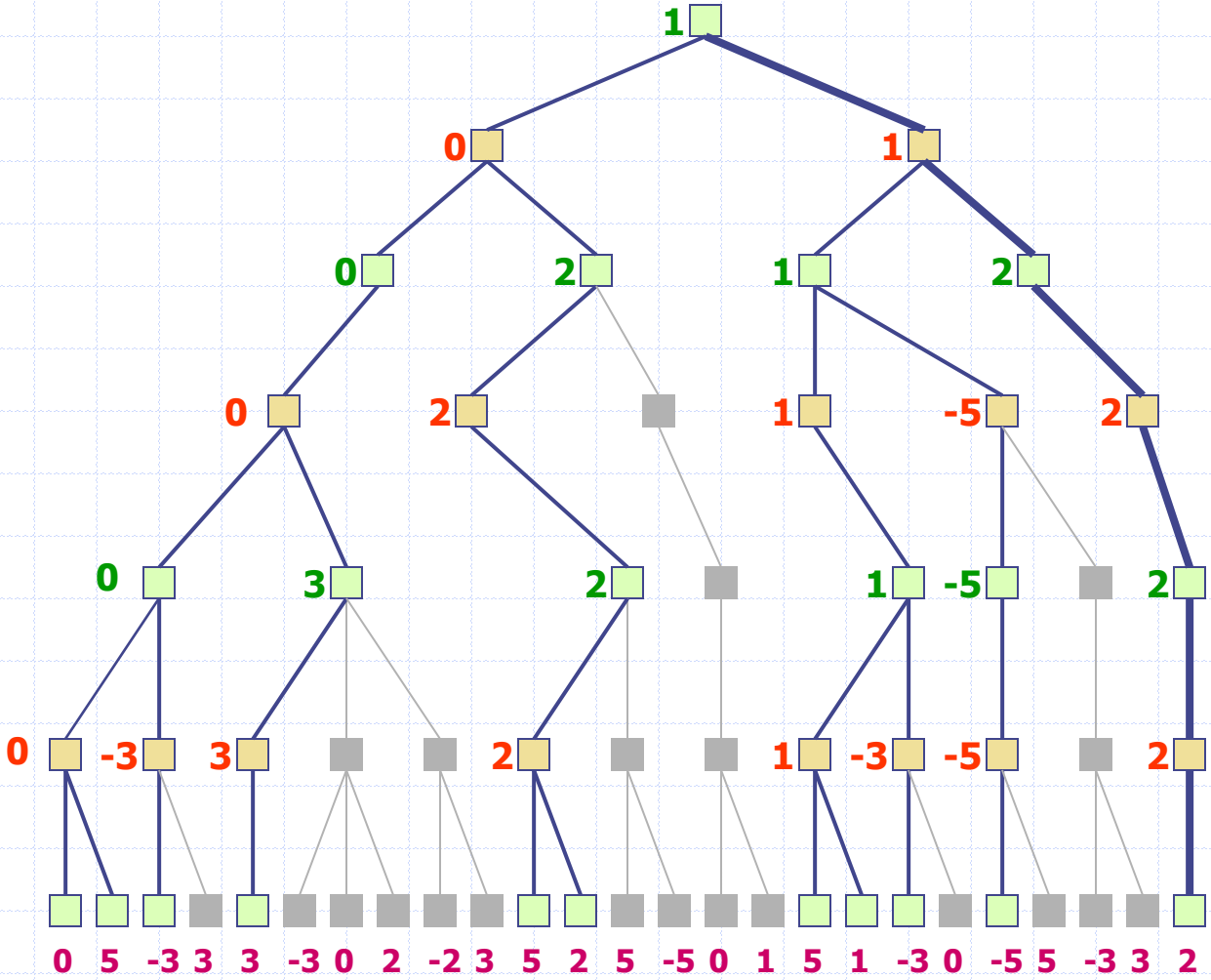

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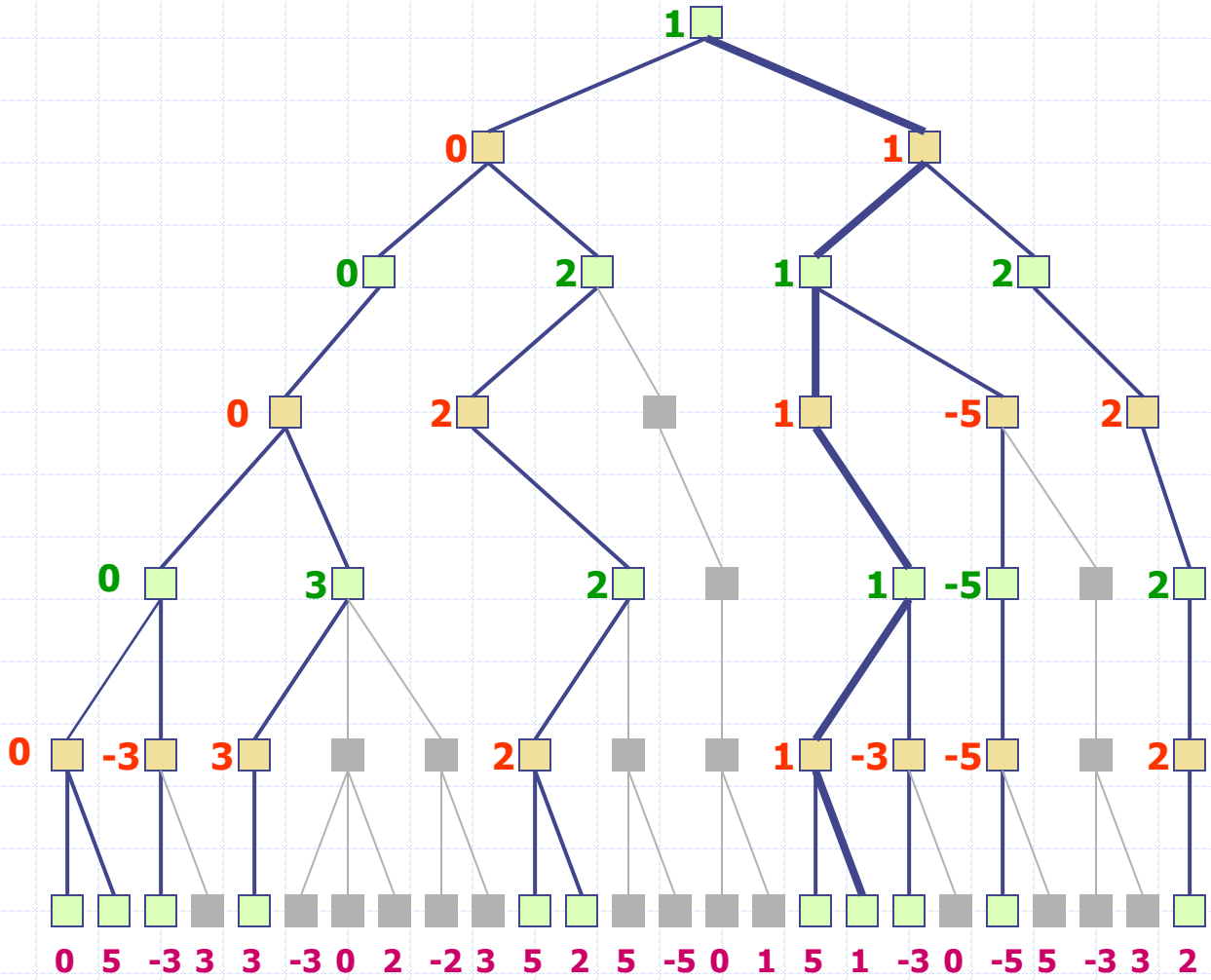
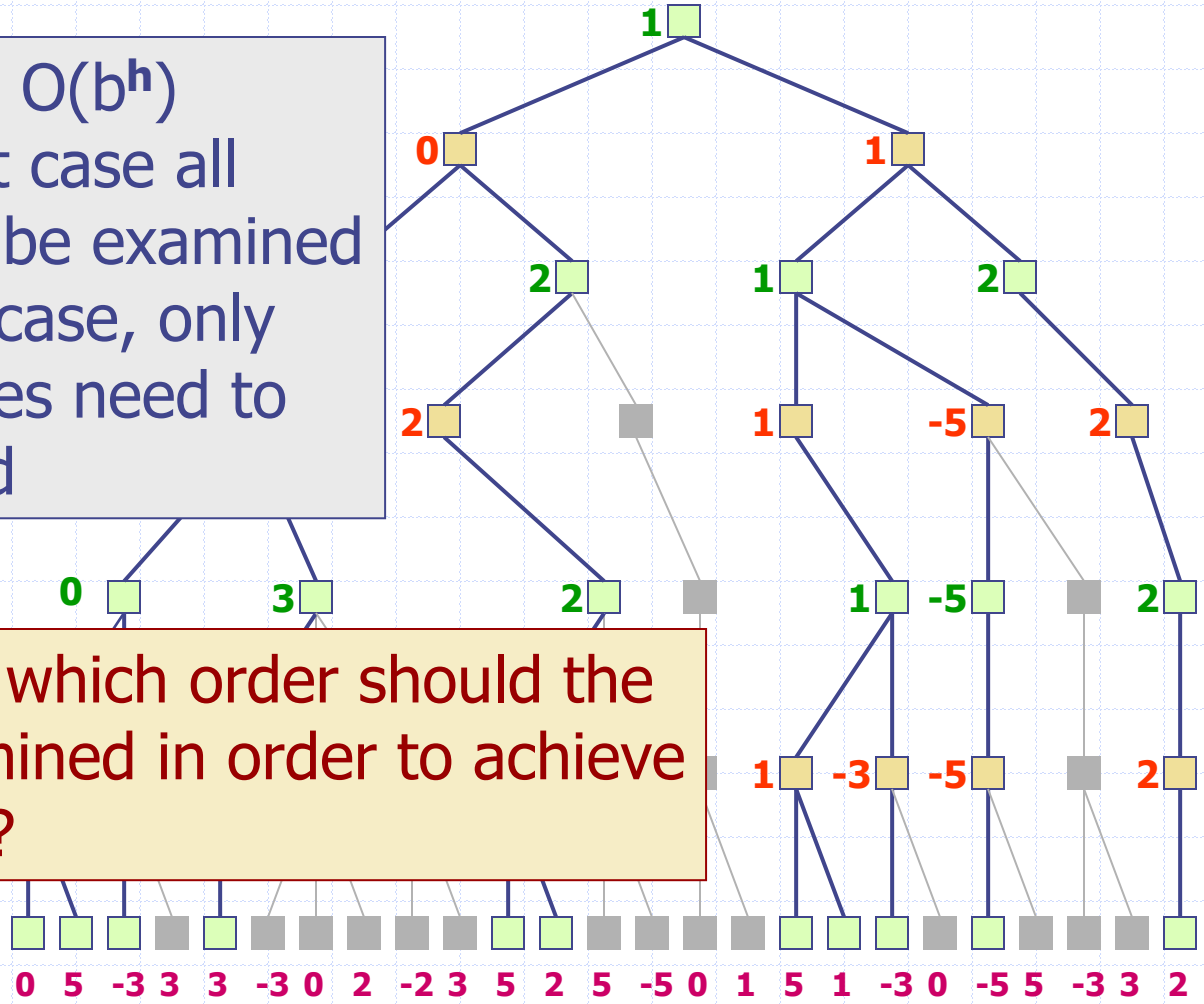


How Much Do We Gain?

Size of tree = $O(b^h)$

- In the worst case all nodes must be examined
- In the best case, only $O(b^{h/2})$ nodes need to be examined



Exercise: In which order should the node be examined in order to achieve the best gain?

Alpha-Beta Procedure

- The alpha of a MAX node is a lower bound on the backed-up value
- The beta of a MIN node is a higher bound on the backed-up value
- Update the alpha/beta of the parent of a node N when all search below N has been completed or discontinued

Alpha-Beta Procedure

- The alpha of a MAX node is a lower bound on the backed-up value
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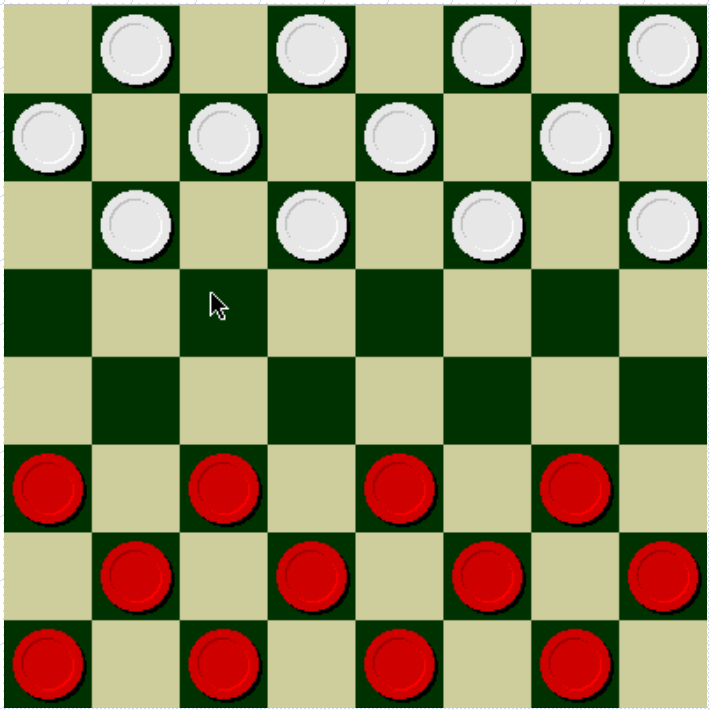
- Update the alpha/beta of the parent of a node N when all search below N has been completed or discontinued
- Discontinue the search below a MAX node N if its alpha is ≥ beta of a MIN ancestor of N
- Discontinue the search below a MIN node N if its beta is ≤ alpha of a MAX ancestor of N

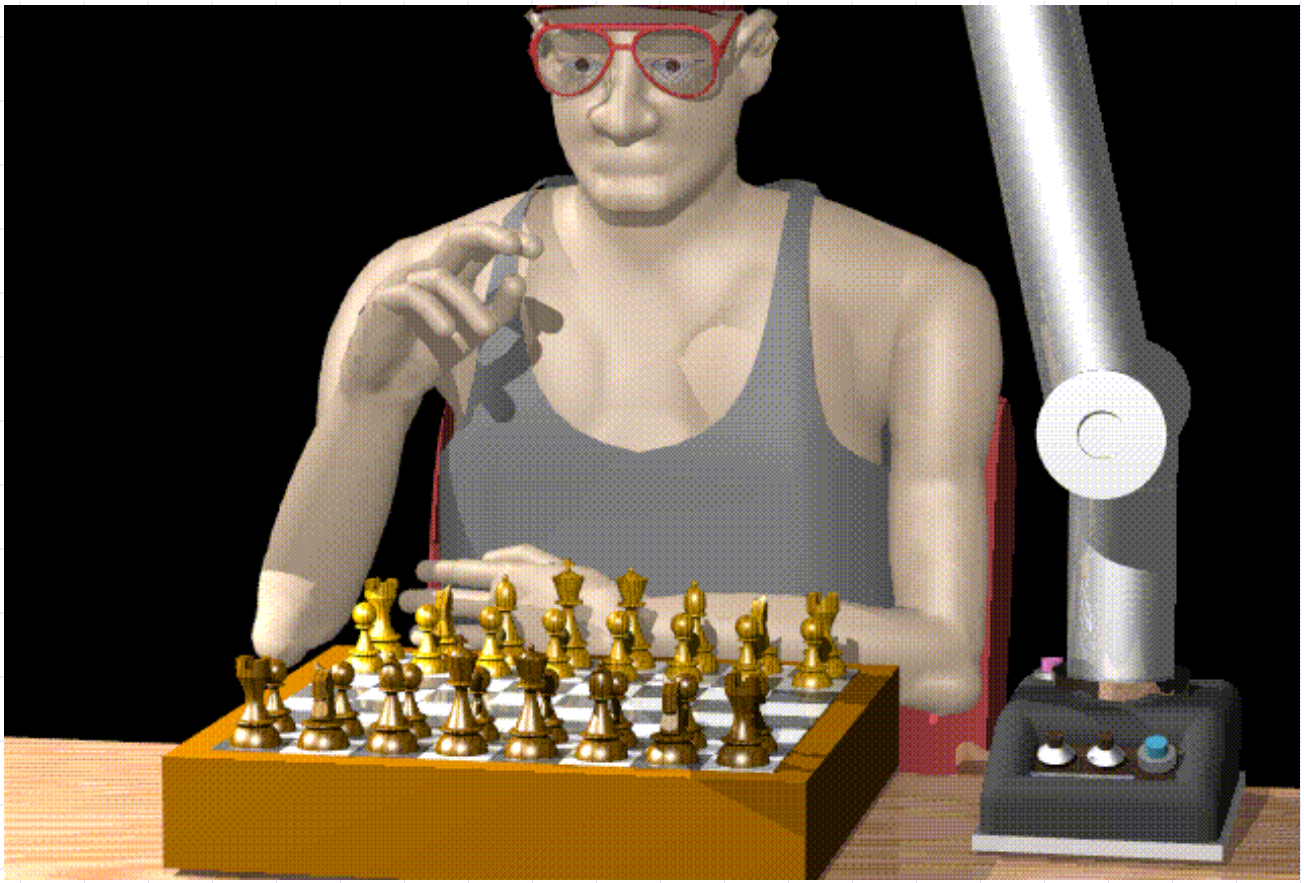
Alpha-Beta + ...

- Iterative deepening
- Singular extensions

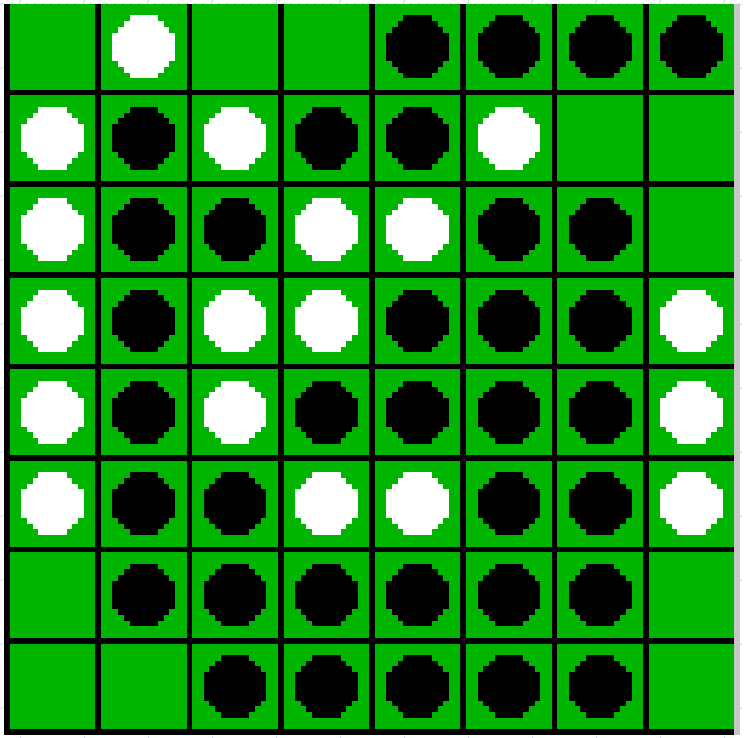
Checkers



Chess



Reversi/Othello



Summary

- Two-players game as a domain where action models are uncertain
- Optimal decision in the worst case
- Game tree
- Evaluation function / backed-up value
- Minimax procedure
- Alpha-beta procedure