

Two-Player Games

For example, let us consider **Tic-Tac-Toe** (although it would still be possible to search the complete game tree for this game).

What would be a suitable evaluation function for this game?

We could use the **number of lines** that are still open for the computer (**X**) minus the ones that are still open for its opponent (**O**).

Two-Player Games

| | | |
|--|--|--|
| | | |
| | | |
| | | |

$$e(p) = 8 - 8 = 0$$

| | | |
|---|---|--|
| X | | |
| O | X | |
| | | |

$$e(p) = 6 - 2 = 4$$

| | | |
|---|---|---|
| O | O | X |
| X | O | |
| X | | |

$$e(p) = 2 - 2 = 0$$

shows the weakness of this $e(p)$

How about these?

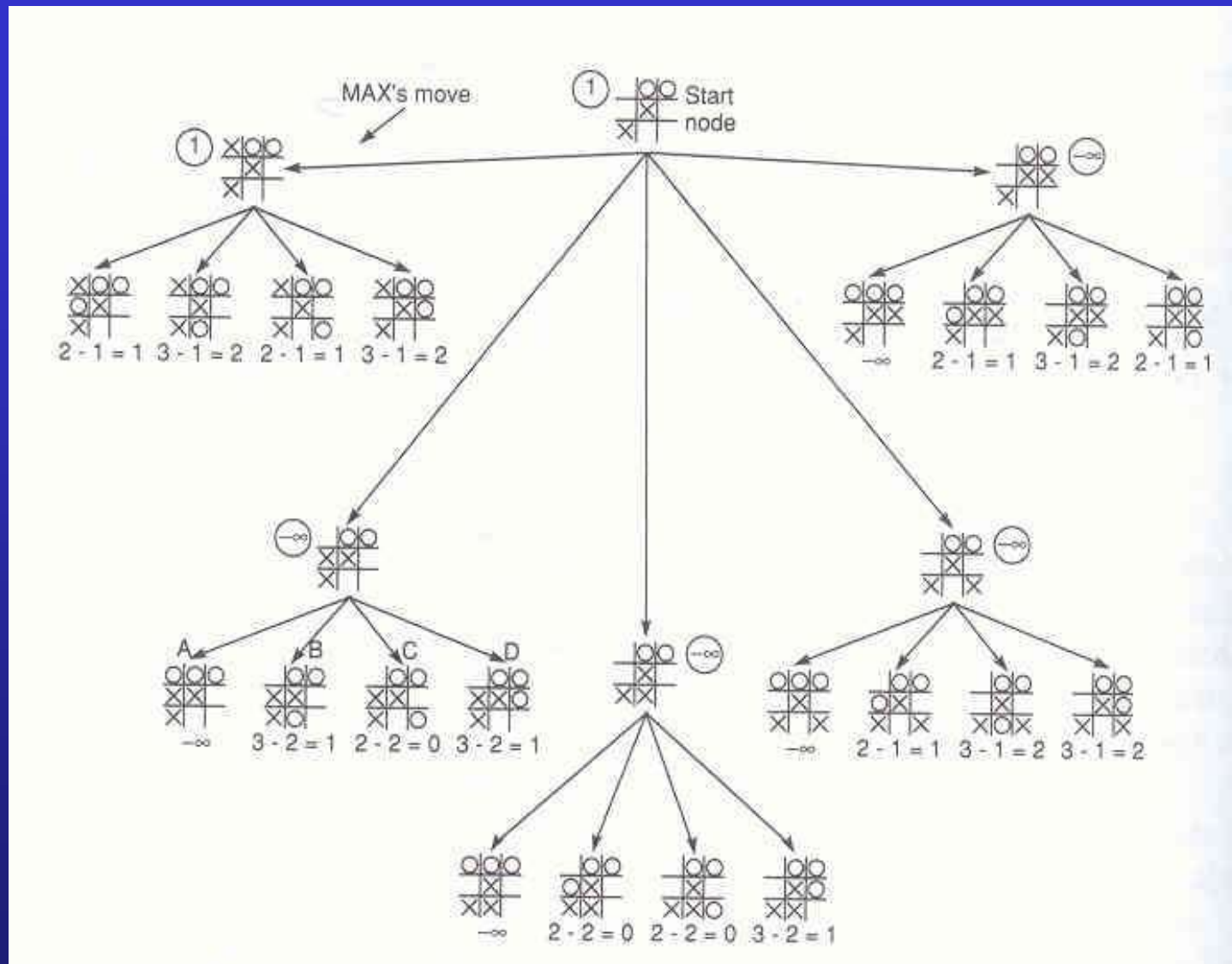
| | | |
|---|---|---|
| O | O | X |
| | X | |
| X | | |

$$e(p) = \infty$$

| | | |
|---|---|---|
| X | X | |
| O | O | O |
| | X | |

$$e(p) = -\infty$$

Two-Player Games



The Alpha-Beta Procedure

Now let us specify how to prune the Minimax tree in the case of a static evaluation function.

- Use two variables alpha (associated with MAX nodes) and beta (associated with MIN nodes).
- These variables contain the best (highest or lowest, resp.) $e(p)$ value at a node p that has been found so far.
- Notice that alpha can never decrease, and beta can never increase.

The Alpha-Beta Procedure

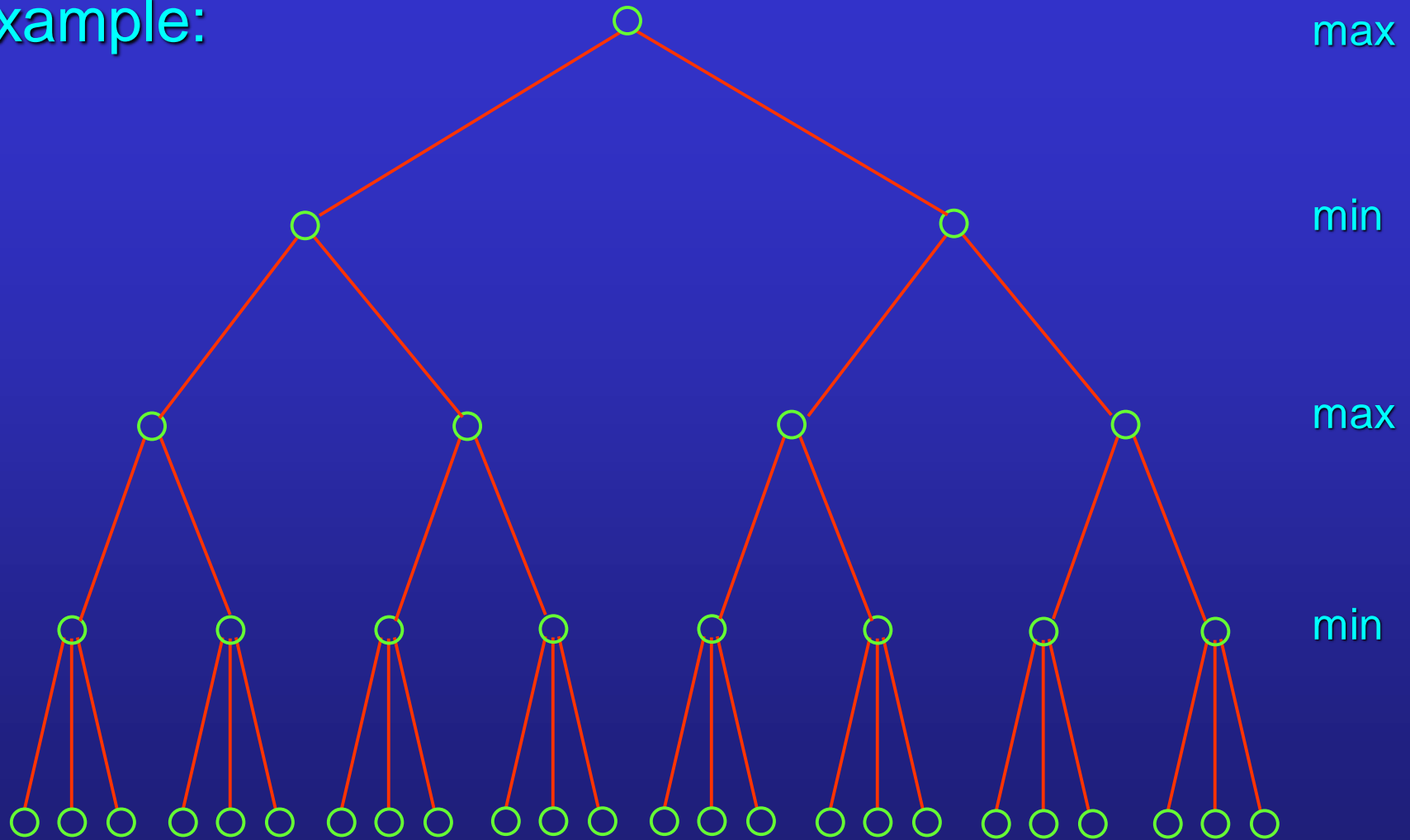
There are two rules for terminating search:

- Search can be stopped below any MIN node having a beta value less than or equal to the alpha value of any of its MAX ancestors.
- Search can be stopped below any MAX node having an alpha value greater than or equal to the beta value of any of its MIN ancestors.

Alpha-beta pruning thus expresses a relation between nodes at level n and level $n+2$ under which entire subtrees rooted at level $n+1$ can be eliminated from consideration.

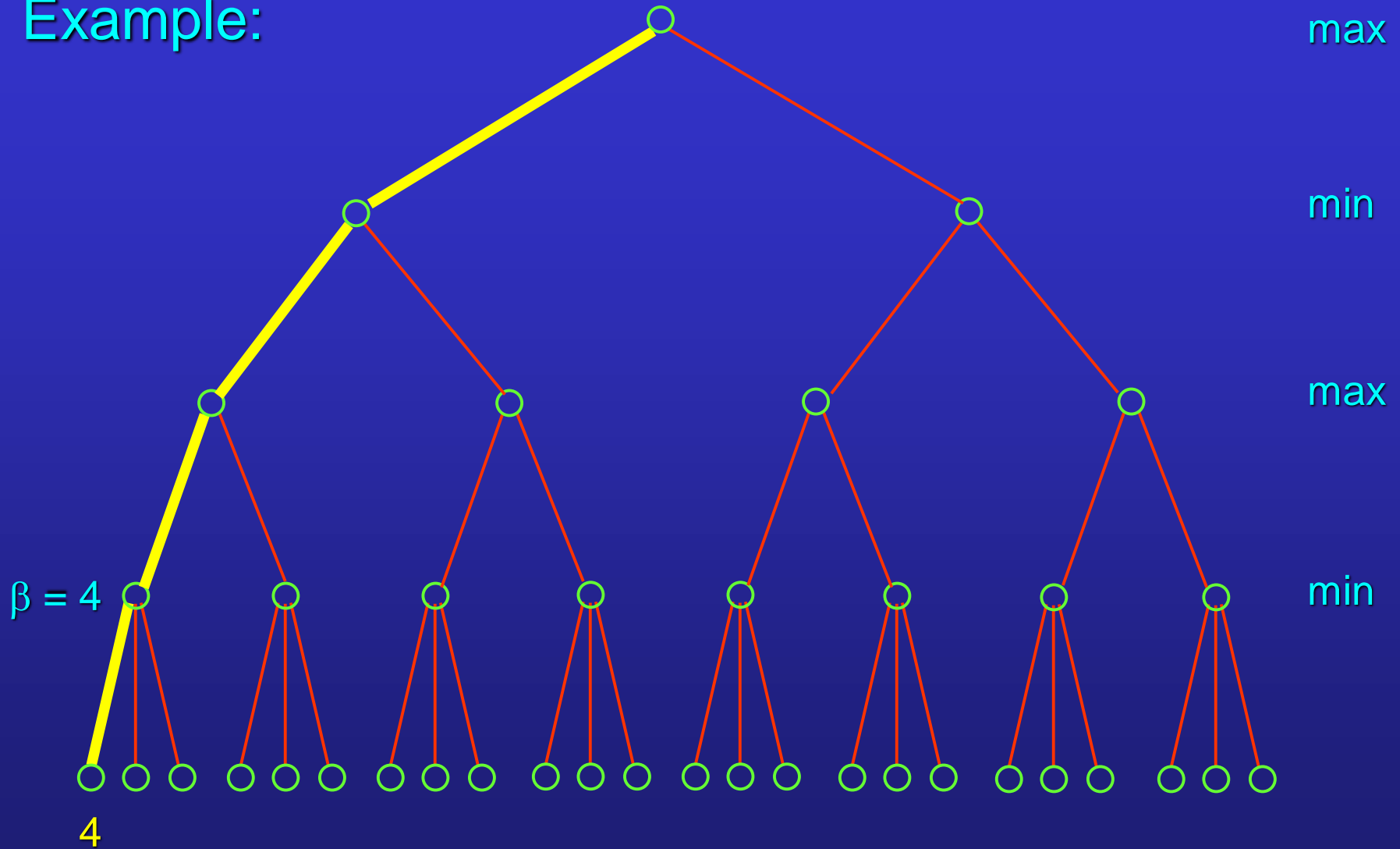
The Alpha-Beta Procedure

Example:



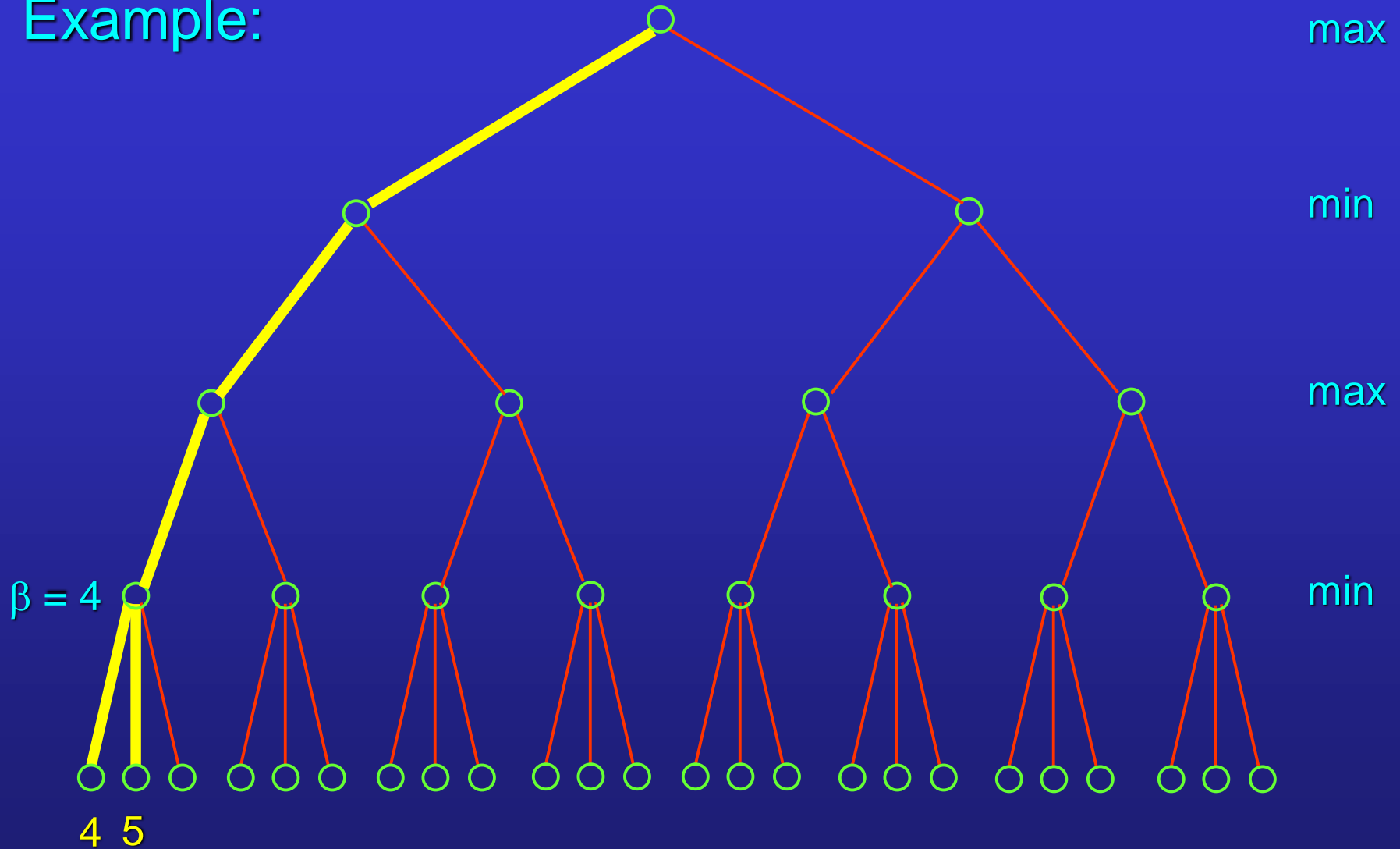
The Alpha-Beta Procedure

Example:



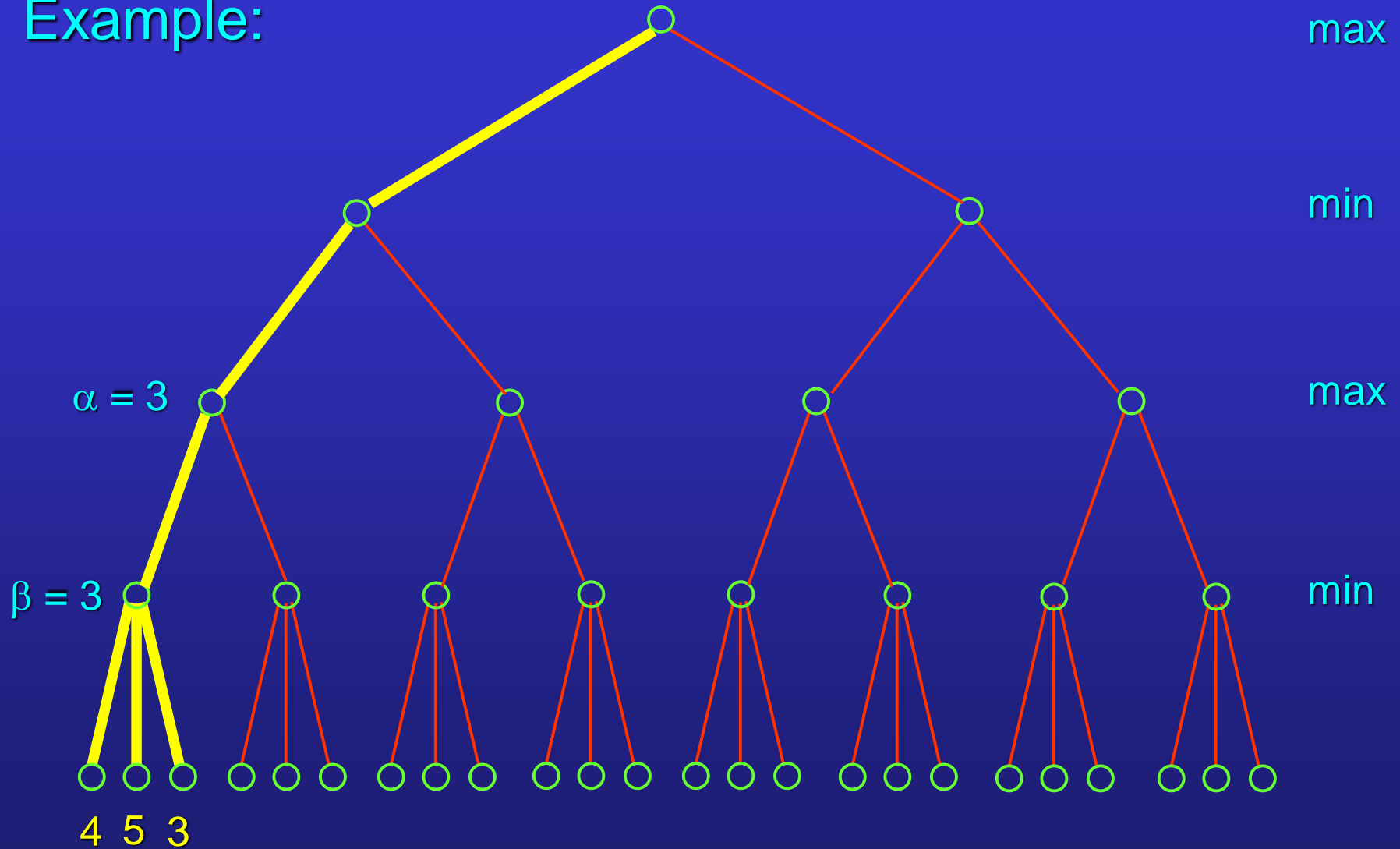
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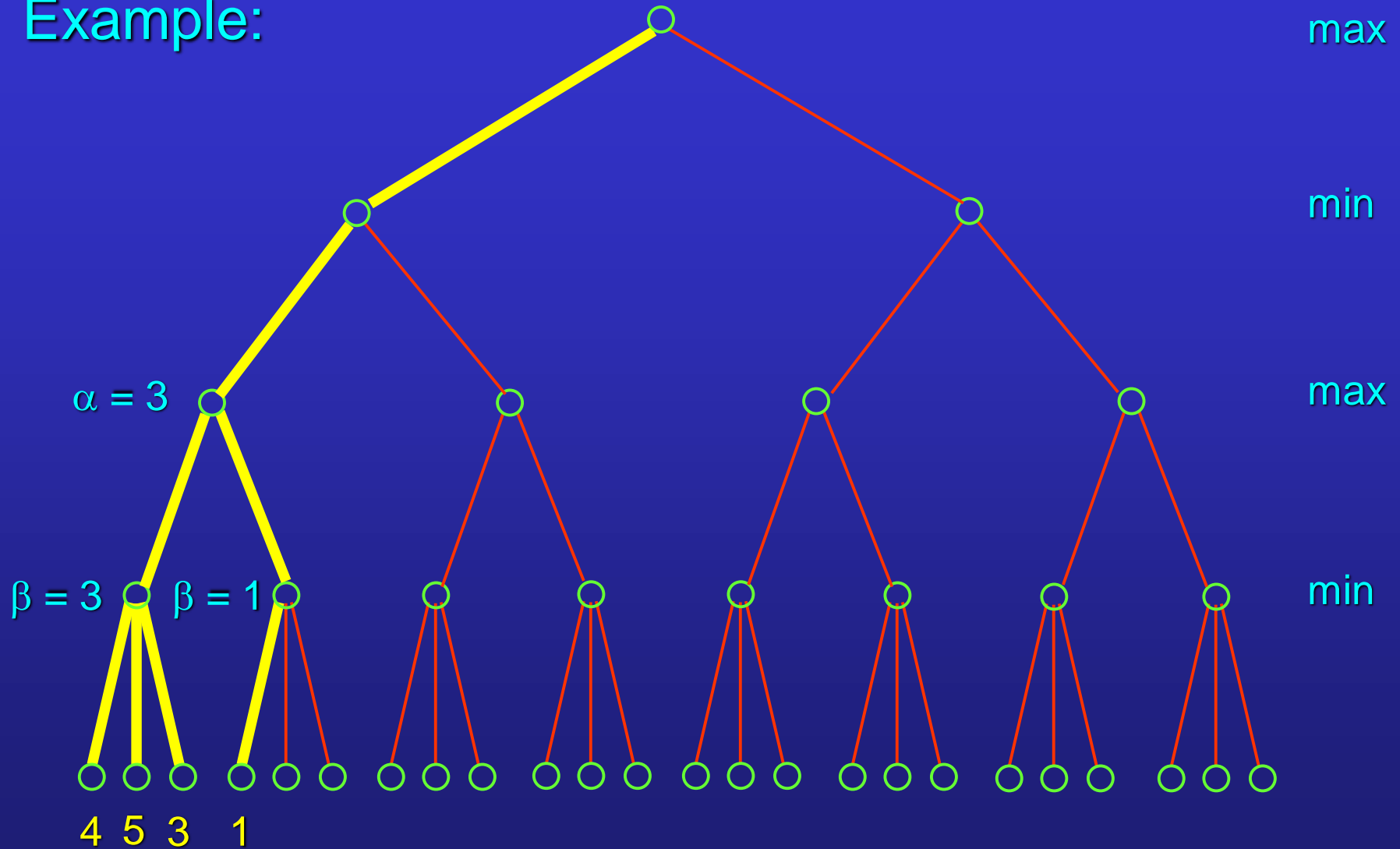
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Example:



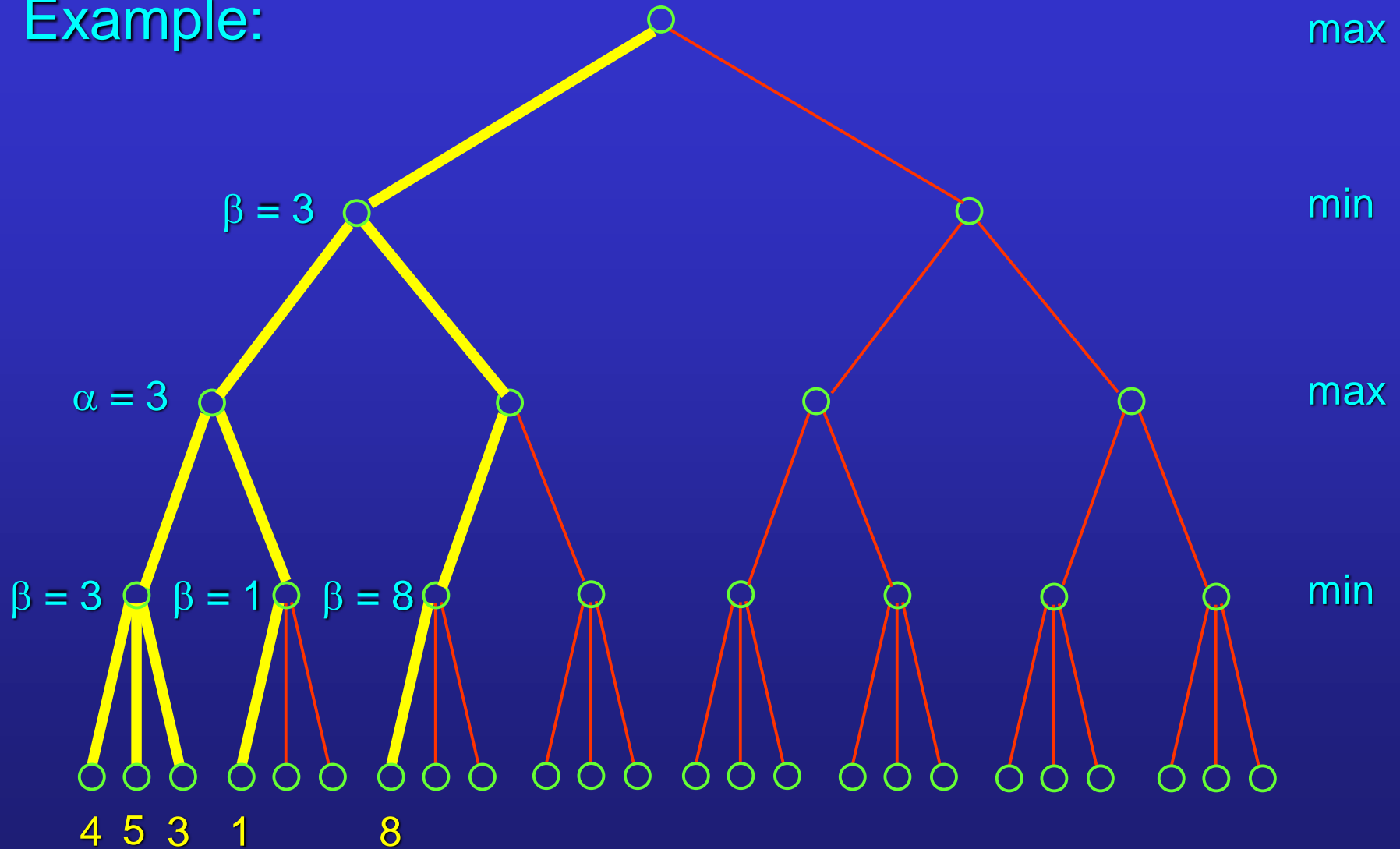
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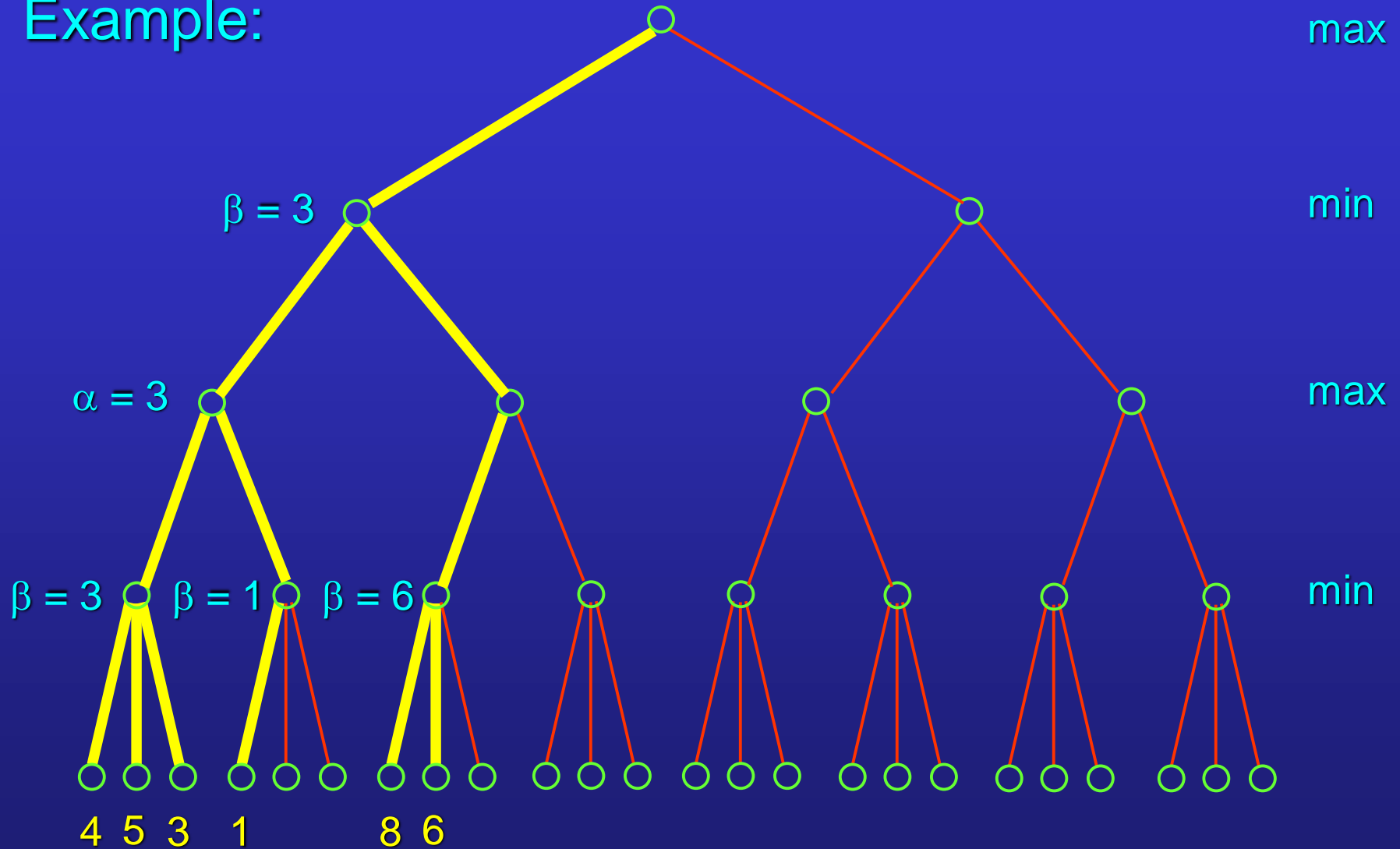
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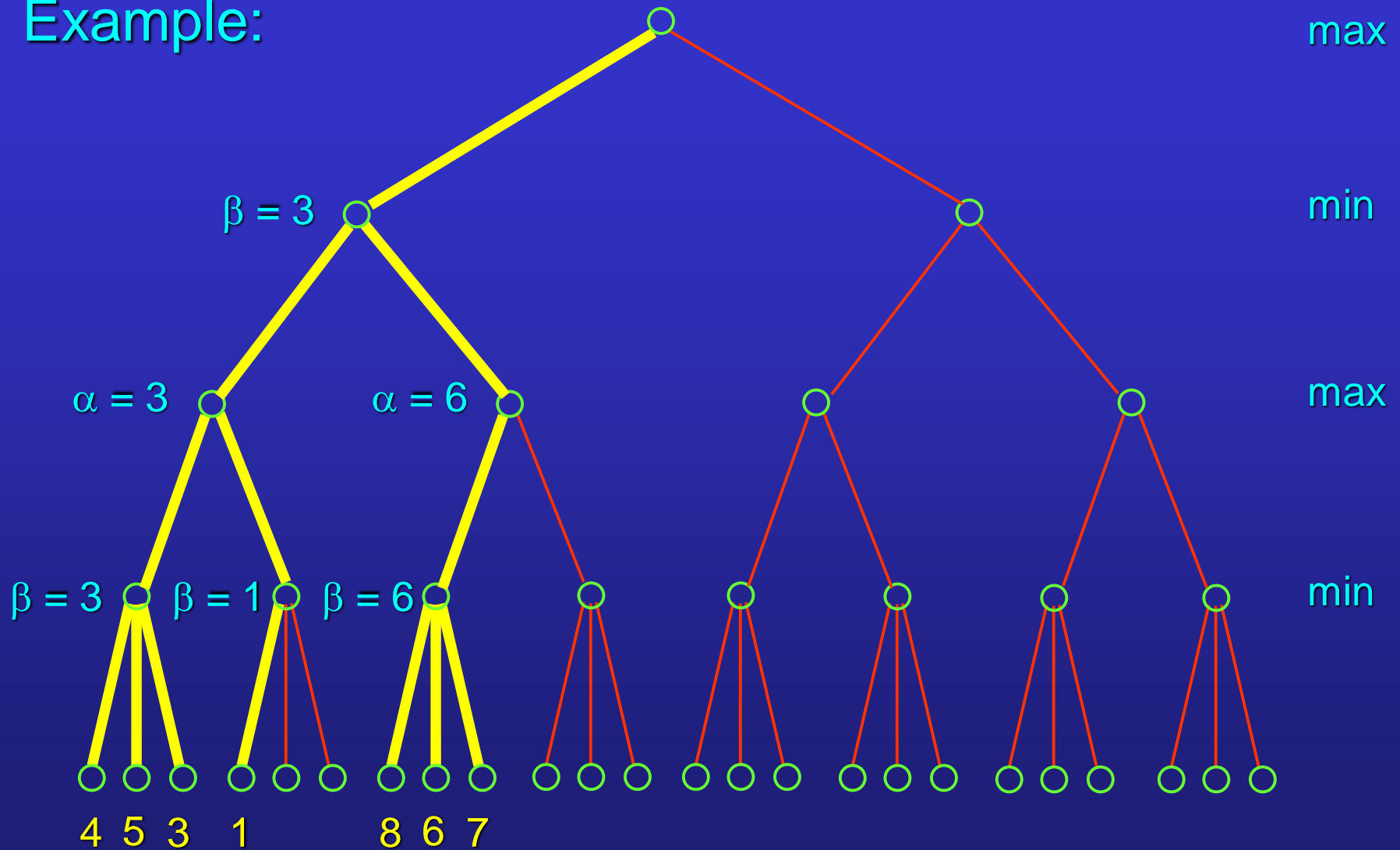
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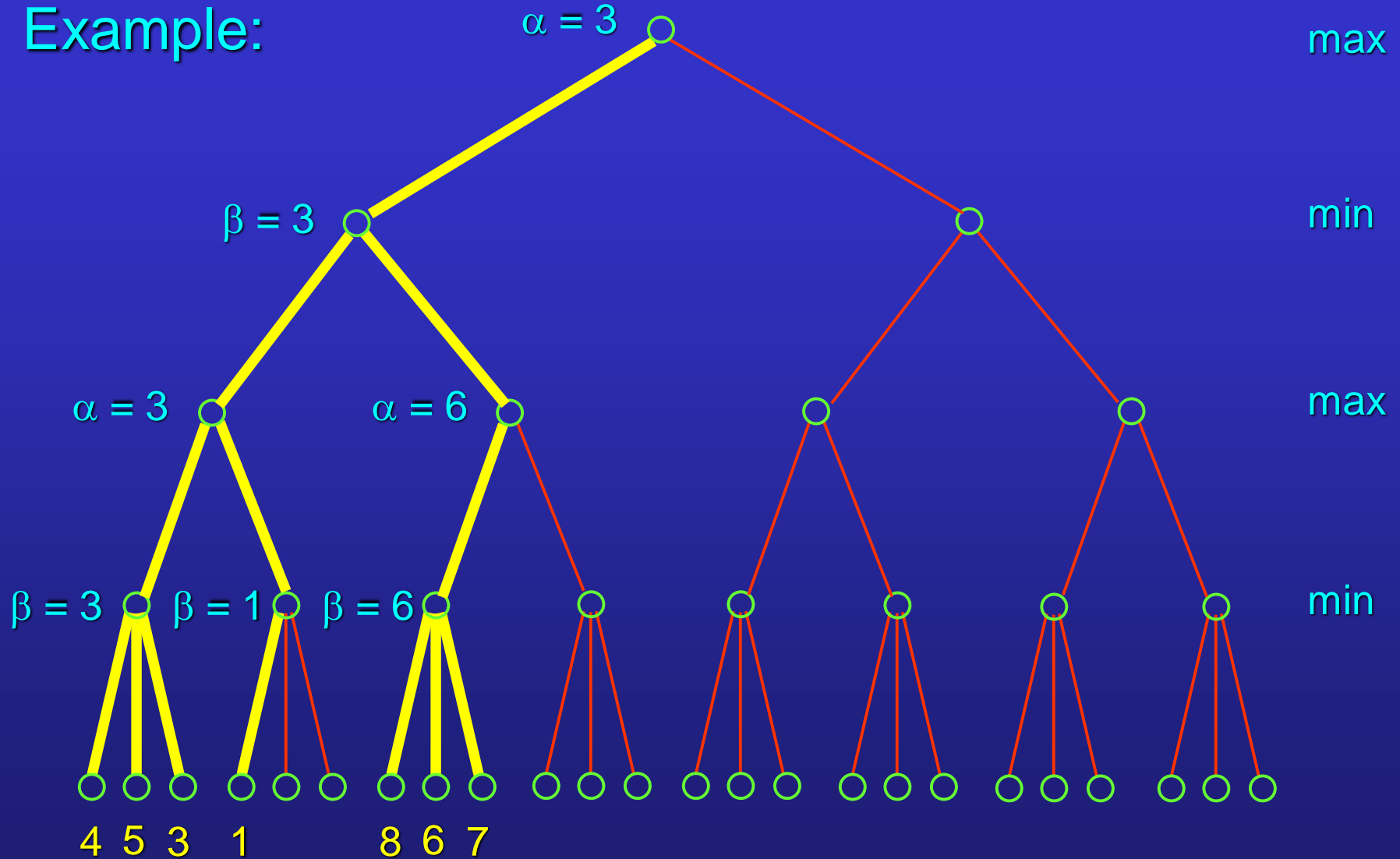
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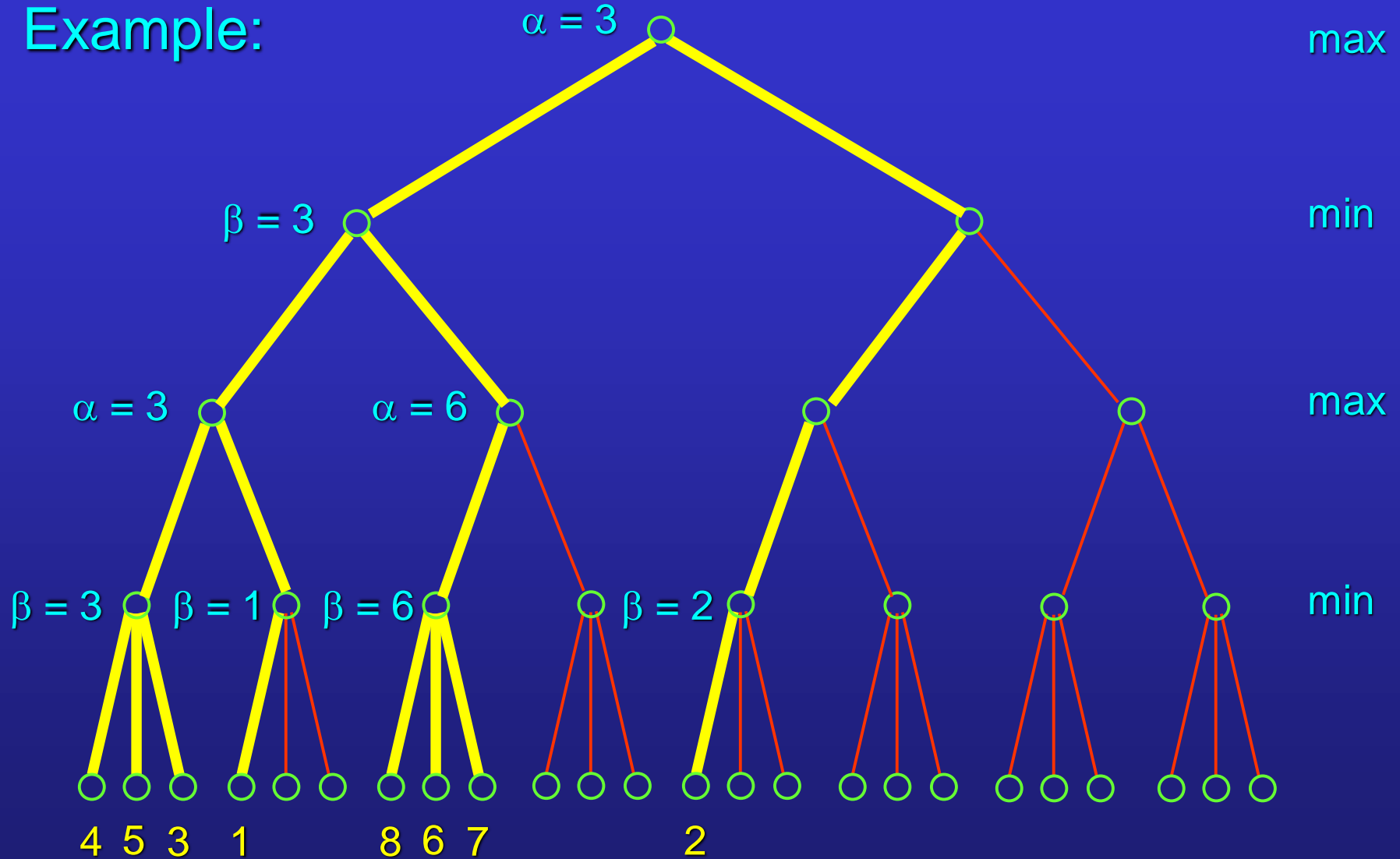
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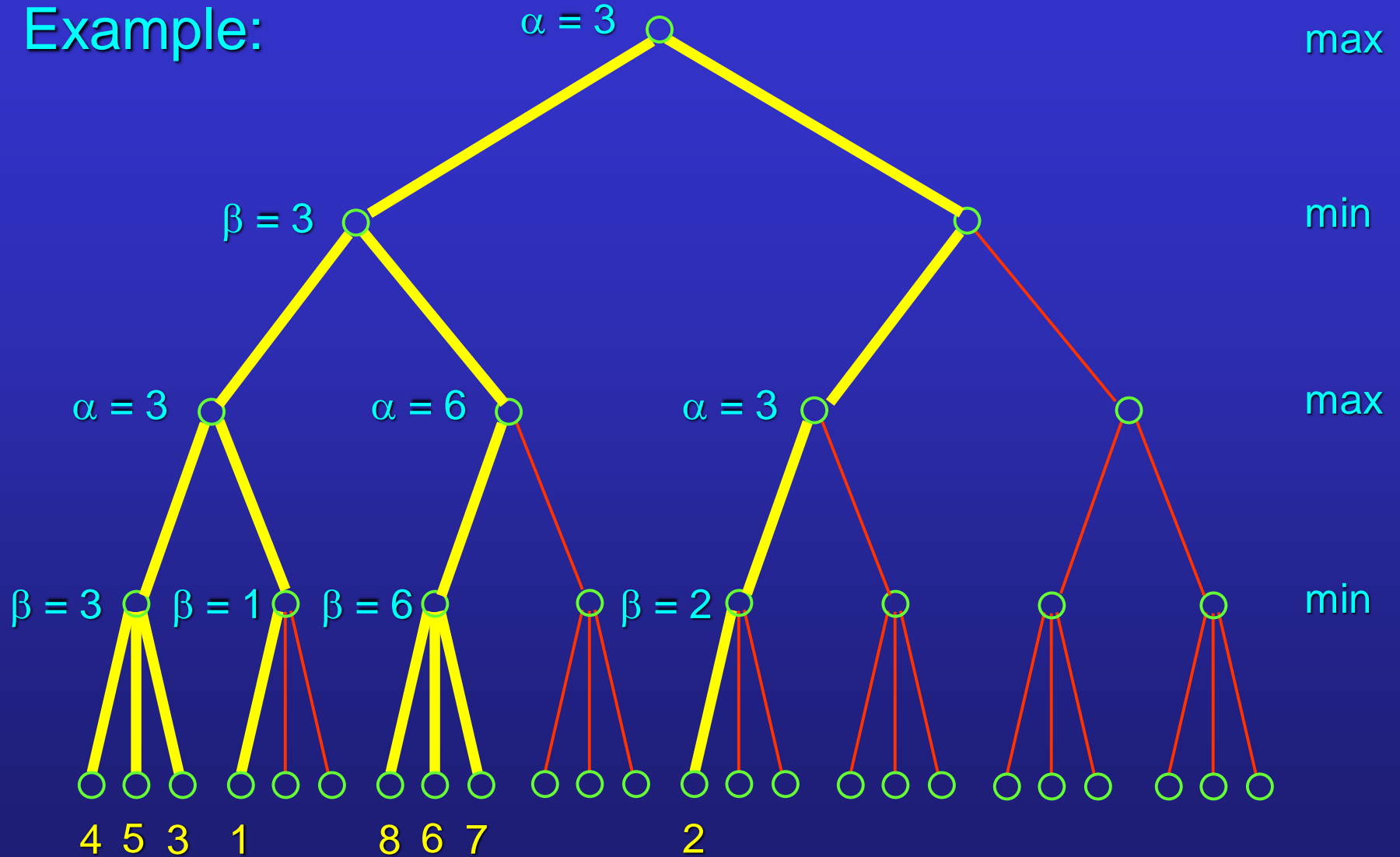
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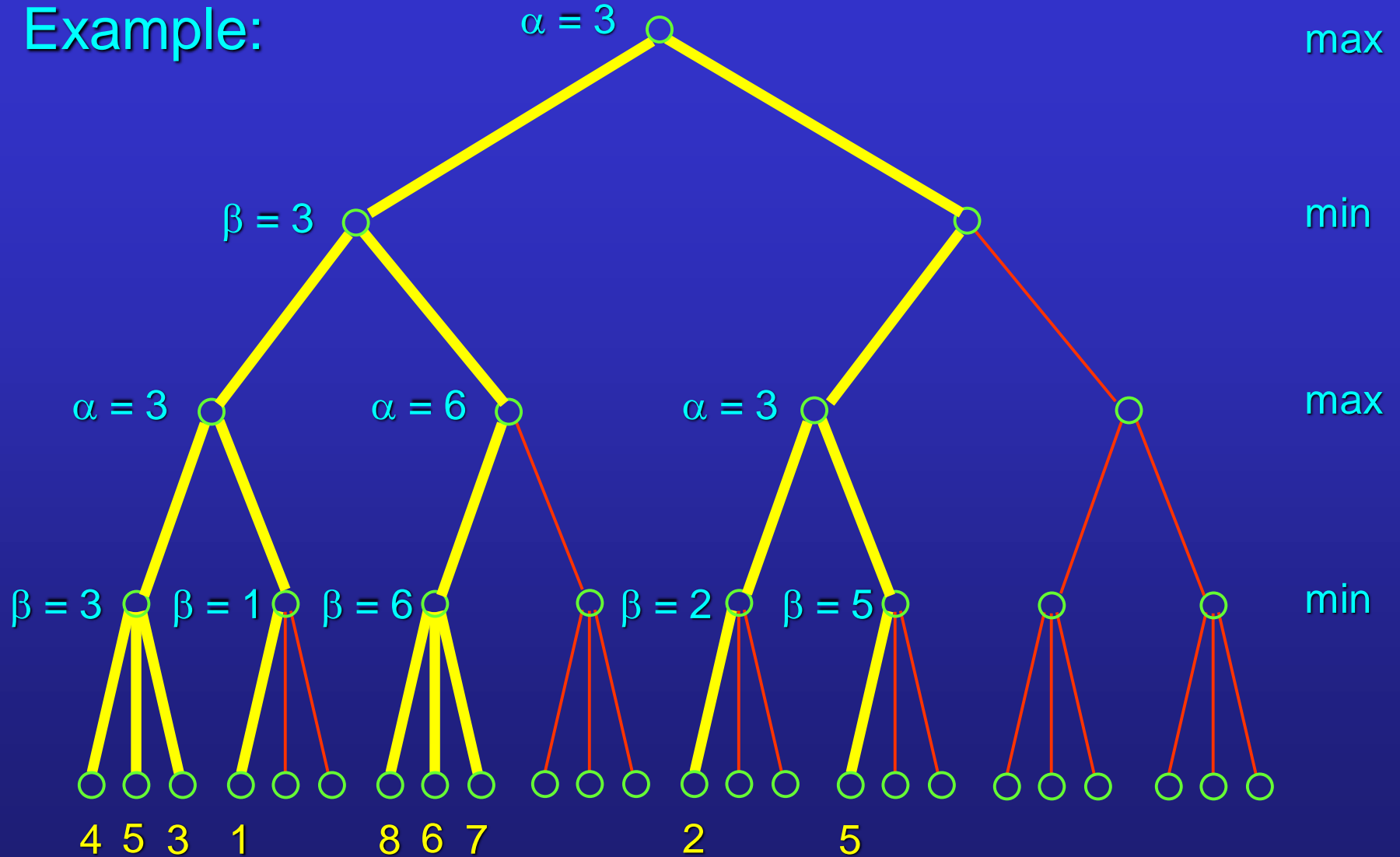
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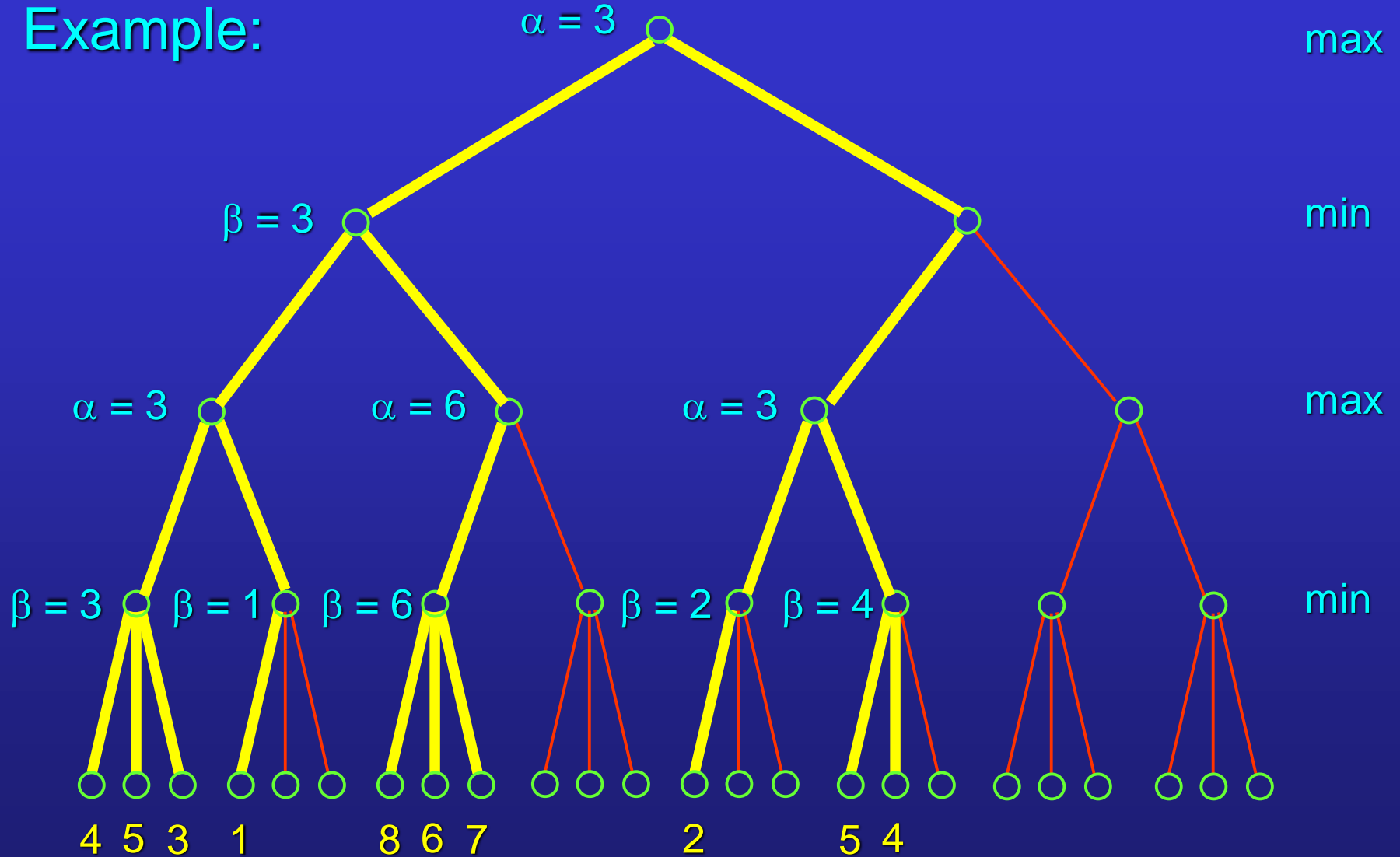
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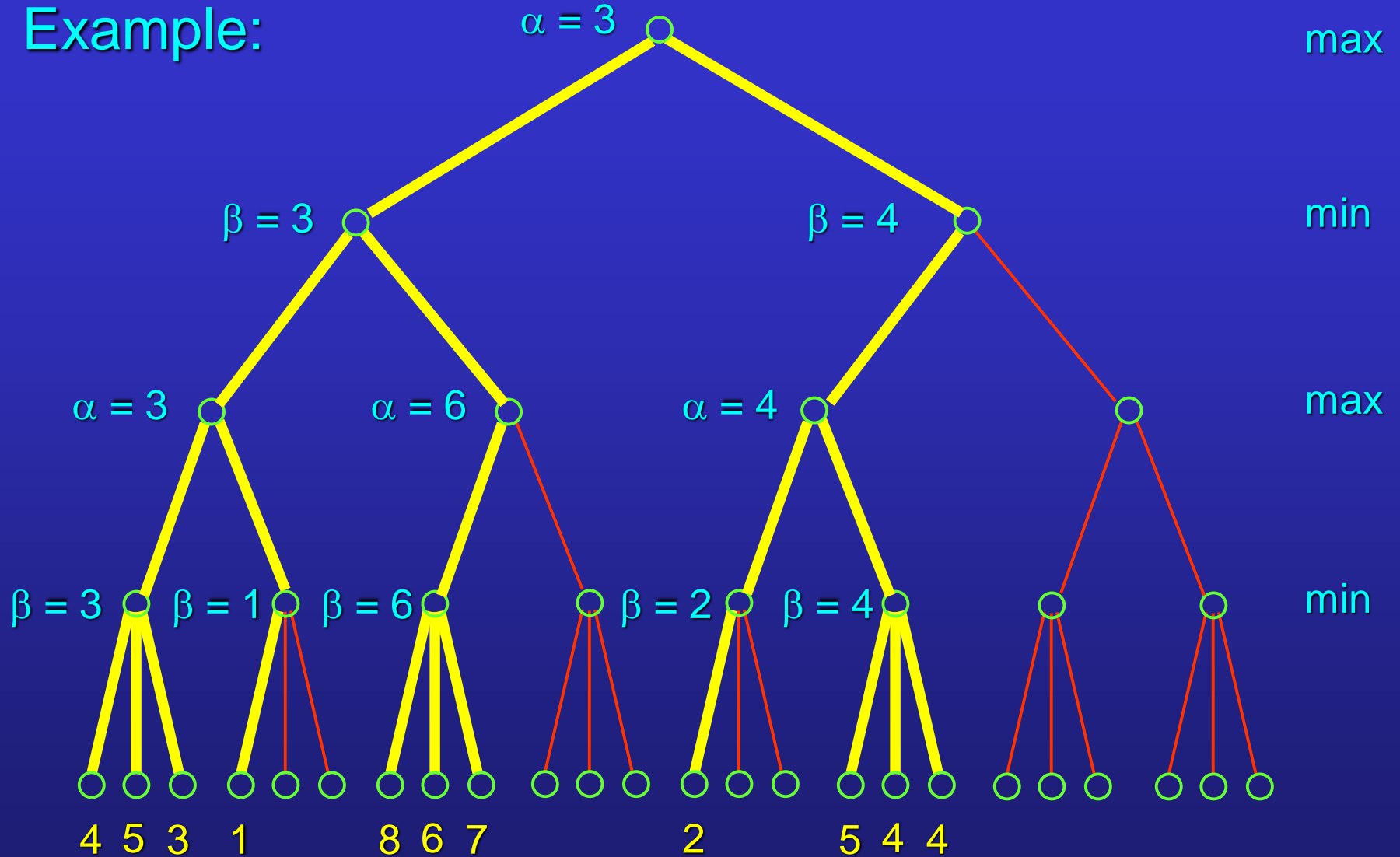
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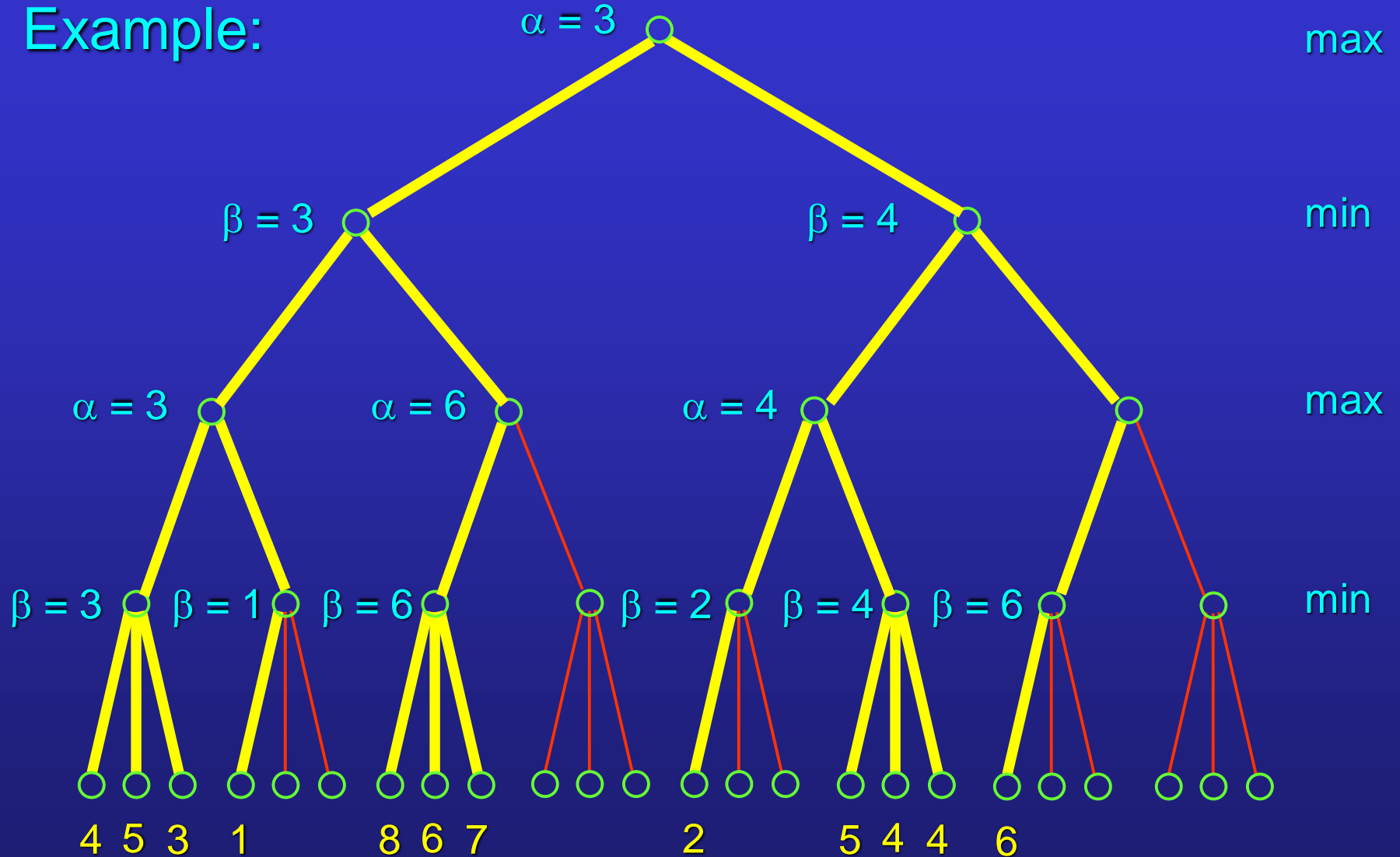
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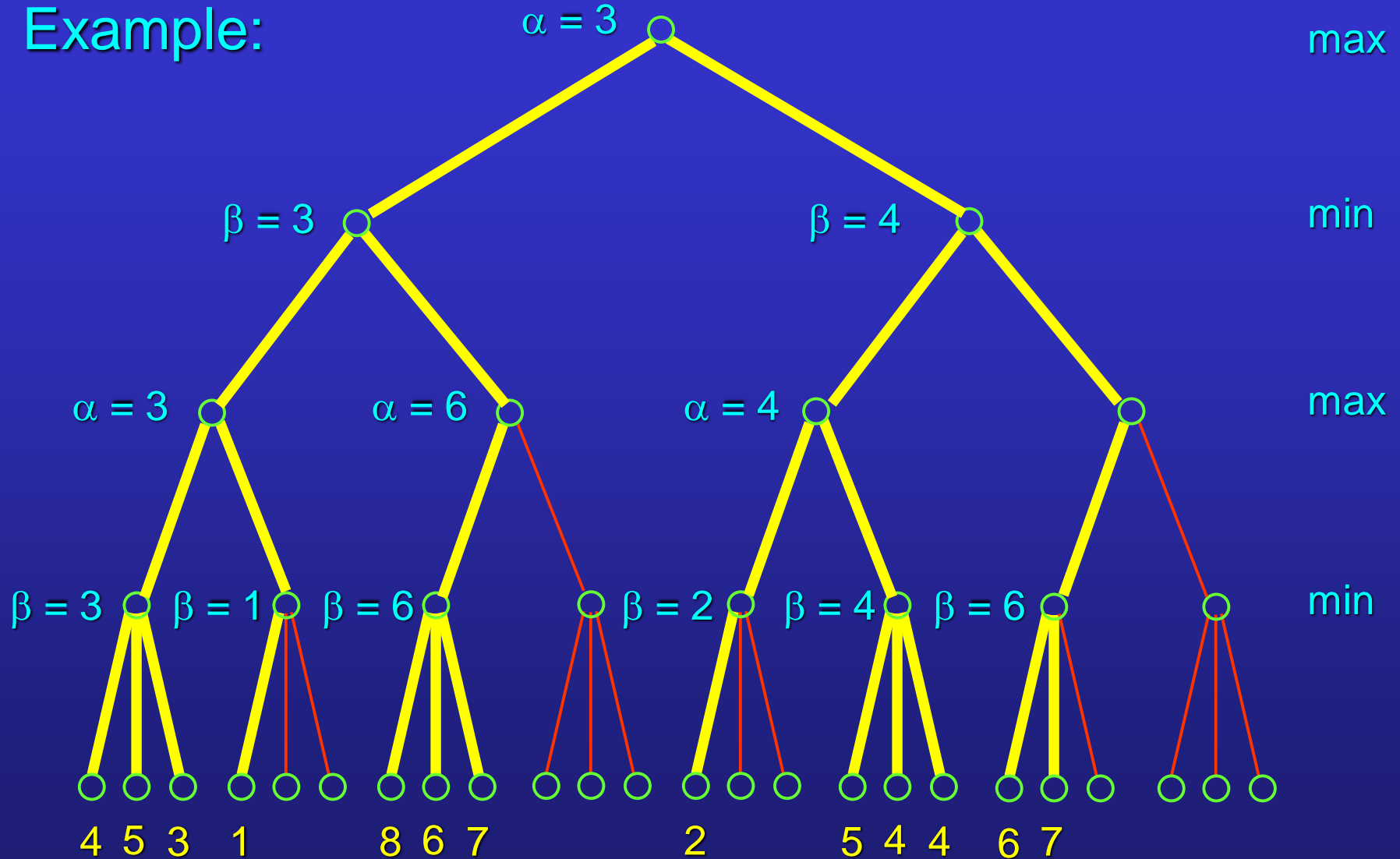
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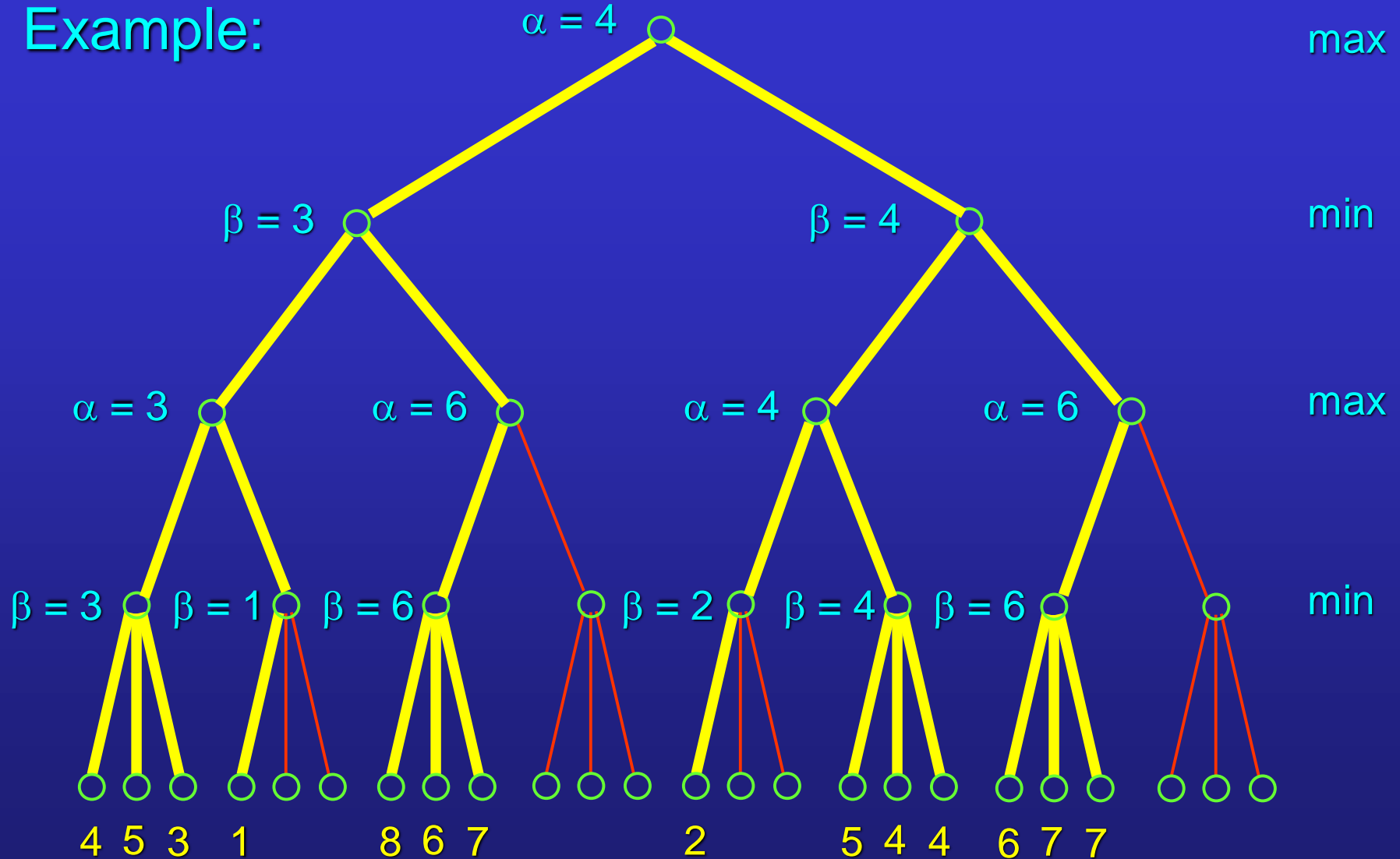
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