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1)

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]$$

a)

$$= \boxed{\sum_{k=0}^{4} x[n-k]}, \quad H(e^{jw}) = \sum_{k=-\infty}^{\infty} h[n].e^{-jwk}$$

$$h[n] = \sum_{k=0}^{4} \delta[n-k] \qquad = \sum_{k=0}^{4} 1.e^{-jwk}$$

$$\boxed{H(e^{jw}) = 1 + e^{-jw} + e^{-j2w} + e^{-j3w} + e^{-j4w}}, (-\pi \le w < \pi)$$

b)

$$H(e^{j(w+w_0)}) = H(e^{jw})$$

$$\sum_{k=0}^{4} (e^{-jwk}.e^{-jw_0k})$$

$$e^{-jw_0k} = 1$$

$$\boxed{\begin{array}{l} w_0 = 2\pi \\ \text{Period} = 2\pi \end{array}}$$

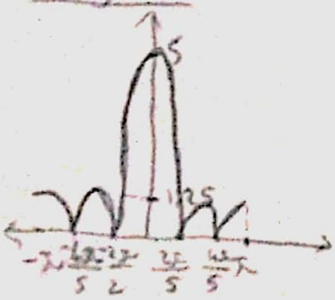
c)

$$H(e^{jw}) = \underbrace{\frac{\sin(wL/2)}{\sin(w/L)}}_{\text{Magnitude}} . \underbrace{e^{-jw(L-1)/2}}_{\text{Phase}},$$

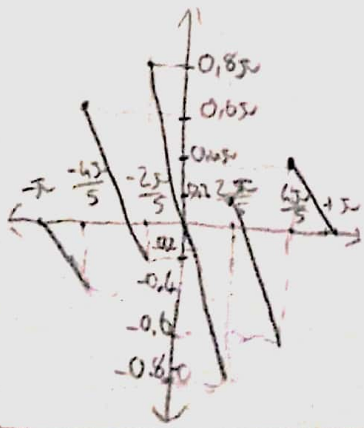
$\boxed{L=5}$ =>

$$\text{magnitude} = \left| \frac{\sin(5w/2)}{\sin(w/2)} \right|$$

Magnitude

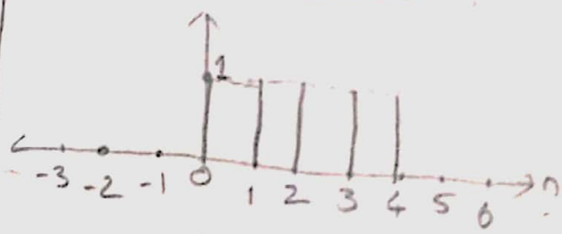


Phase



d)

impulse Response



e)

$$x[n] = x_1[n] + x_2[n] - x_3[n]$$

$$x[n] = 4 + 2\cos[0.5\pi(n-1)] - 3\cos[0.3\pi n]$$

$$y[n] = x[n] * h[n]$$

$$= x[n].H(e^{jw})$$

$$= x_1[n].H(e^{jw_1}) + x_2[n].H(e^{jw_2}) + x_3[n].H(e^{jw_3})$$

$x_1[n]$	$x_2[n]$	$x_3[n]$
$w_1 = 0$	$w_2 = \frac{\pi}{2}$	$w_3 = 0.3\pi$
$\lvert H \rvert = 5$	$\lvert H \rvert = 1$	$\lvert H \rvert = 1.56$
$\angle H = 0$	$\angle H = 0$	$\angle H = -0.6\pi$

$$x_1[n].H(e^{jw_1}) = 4.(5.e^{j0}) = 20$$

$$x_2[n].H(e^{jw_2}) = \left(e^{j(0.5\pi(n-1))} + e^{-j(0.5\pi(n-1))}\right).(1.e^{j0}) = 2\cos[0.5\pi(n-1)]$$

$$x_3[n].H(e^{jw_3}) = \frac{3}{2}\left(e^{j0.3\pi n} + e^{-j0.3\pi n}\right).(1.56\,e^{-j.0,6\pi})$$

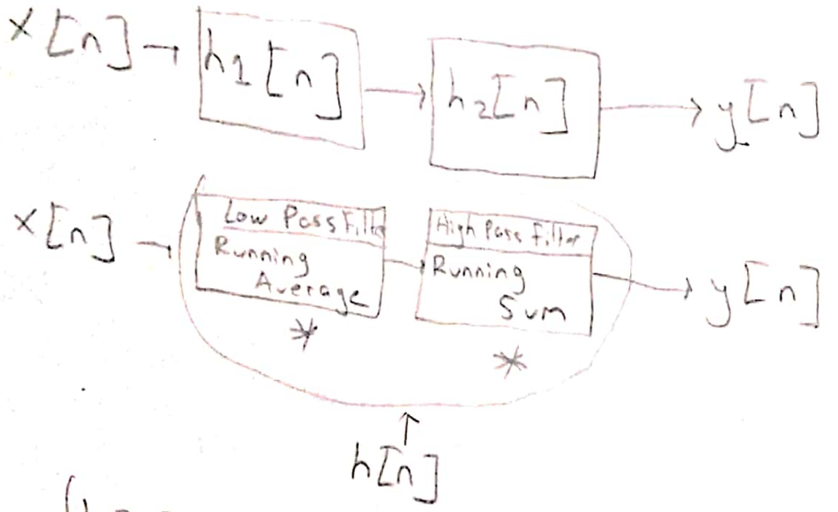
$$= 4.68.\cos[0.3\pi n - 0.6\pi]$$

$$\boxed{y[n] = x_1[n].H(w_1) + x_2[n].H(w_2) + x_3[n].(H w_3) = 20 + 2\cos[0.5\pi(n-1)] - 4.68.\cos[0.3\pi n - 0.6\pi]}$$

1.f) <u>Blurs</u>. Because of it is a Low-Pass filter. The image will be removes from high frequencies. Details will gone.

2)

i)



ii)

$$\left(h[n] = h_1[n] * h_2[n]\right) = H(e^{j\omega}) = H_1(e^{j\omega}) \cdot H_2(e^{j\omega})$$

(low)

$h_1[n]$ $\circledast$ $h_2[n]$ $\xrightarrow{DTFT}$ $H(e^{j\omega})$

Convolution (high)

iii) First Low filter will get rid of and noise, so if you use with signal or pictures it will be smooth. After that with high pass filter you can do "object detection", "edge detection" or "line detection" or "signal peaks" etc.

3) i)

$$x_1[n] = n\left(\frac{1}{3}\right)^n u[n] + e^{j(0,3\pi n + \frac{\pi}{4})}$$

DTFT Pairs

$$a^n u[n] \Leftrightarrow \frac{1}{1 - a e^{-j\hat{\omega}}}$$

$$= \left(\frac{1}{3}\right)^n u[n] \Leftrightarrow \frac{1}{1 - \frac{1}{3}e^{-j\hat{\omega}}}$$

$$n\left(\frac{1}{3}\right)^n u[n] \Leftrightarrow j\frac{d}{d\hat{\omega}}\left[\frac{1}{1-\frac{1}{3}e^{-j\hat{\omega}}}\right] = \frac{\frac{1}{3}e^{-j\hat{\omega}}}{\left(1-\frac{1}{3}e^{-j\hat{\omega}}\right)^2}$$

$$x[n]e^{j\hat{\omega}_0 n} \Leftrightarrow X(\hat{\omega} - \hat{\omega}_0)$$

$$e^{j0,3\pi n} \Leftrightarrow 2\pi\delta(\hat{\omega} - 0,3\pi)$$

DTFT

$$x_1[n] + x_2[n] \Rightarrow X_1(\hat{\omega}) + X_2(\hat{\omega})$$

$$x_1[n] = n\left(\frac{1}{3}\right)^n u[n]$$

$$= e^{j(0.3\pi n + \frac{\pi}{4})}$$

$$= e^{-j\frac{\pi}{4}} \cdot e^{-j(0,3\pi)n}$$

$$\boxed{\hat{\omega}_0 = 0,3\pi}$$

$$\boxed{X(\hat{\omega}) = \frac{\frac{1}{3}e^{-j\hat{\omega}}}{\left(1-\frac{1}{3}e^{-j\hat{\omega}}\right)^2} + e^{-j\frac{\pi}{4}} \cdot 2\pi\delta(\hat{\omega} - 0,3\pi)}$$

ii)

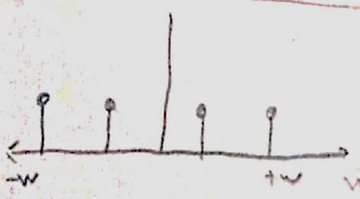
$$X(\hat{\omega}) = \sum_{n=-3}^{n=+3} h[n] e^{-j\hat{\omega}n}$$

$$\sum_{n=-3}^{+3} 3 - |n| e^{-j\omega n} = 0 + e^{2j\hat{\omega}} + 2e^{j\hat{\omega}} + 3 + 2e^{-j\hat{\omega}} + e^{-2j\hat{\omega}} + 0$$

$$= 3 + 2\left[e^{j\hat{\omega}} + e^{-j\hat{\omega}}\right] + \left[e^{2j\hat{\omega}} + e^{-2j\hat{\omega}}\right] \Rightarrow \boxed{X(\hat{\omega}) = 3 + 4\cos(\hat{\omega}) + 2\cos(2\hat{\omega})}$$

iii)

$$\frac{\sin(\hat{\omega} n)}{\pi n} \overset{DTFT}{\Leftrightarrow}$$



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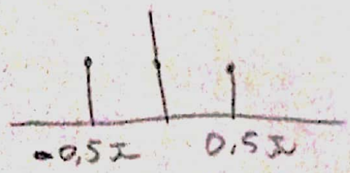
$$\frac{\sin(0,8\pi n)}{\pi n} \overset{DTFT}{\Leftrightarrow}$$

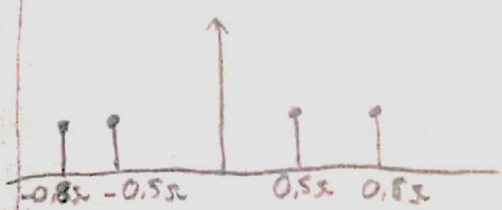
$$\frac{\sin(0,5\pi n)}{\pi n} \overset{DTFT}{\Leftrightarrow}$$



, .

$$X(\hat{\omega}) = X_1(\hat{\omega}) - X_2(\hat{\omega})$$

$X(\hat{\omega})$



4) a) $X(e^{jw}) = \dfrac{2+\frac{1}{4}e^{-jw}}{-\frac{1}{8}e^{-j2w}+\frac{1}{4}e^{-jw}+1} =$

$= \dfrac{2}{e^{-jw}+2} - \dfrac{4}{e^{-jw}-4}$

$= \left(\dfrac{1}{1-(-\frac{1}{2})\cdot e^{-jw}} = (X_1)\right) + \left(\dfrac{1}{1-\frac{1}{4}e^{-jw}} = (X_2)\right)$

$= X_1(e^{jw}) + X_2(e^{jw})$

$\boxed{\text{inverse DTFT}}$

$X(e^{jw}) \Rightarrow$

$x[n] = u[n]\cdot\left(-\frac{1}{2}\right)^n + u[n]\left(\frac{1}{4}\right)^n$

$x[n] = x_1[n] + x_2[n]$

b) $X(e^{jw}) = \begin{cases} 0{,}25\pi \le |w| < 0{,}75\pi \Rightarrow 1 \\ \text{other} \Rightarrow 0 \end{cases}$

$X(e^{jw}) = X_1(e^{jw}) - X_2(e^{jw})$

$X(e^{jw}) = \begin{cases} |w| < 0{,}75\pi \Rightarrow 1 \\ \text{other} \Rightarrow 0 \end{cases} - \begin{cases} |w| < 0{,}25\pi \Rightarrow 1 \\ \text{other} \Rightarrow 0 \end{cases}$

$x[n] = x_1[n] + x_2[n] = \dfrac{\sin(0{,}75\pi n)}{\pi n} - \dfrac{\sin(0{,}25\pi n)}{\pi n}$