Linear Control Systems Project:
Balancing and Transferring of a Ball Segway

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Deadline: Saturday–5 Bahman

Figure 1: A ball segway balancing on the floor [1]

In this project, you will learn how to balance and control a "ball segway", a people-carrier robot that could turn 360^o and drive itself freely with a spherical wheel. The ball segway is similar to a spherical inverted pendulum with a car-like structure mounted on top that moves on a ball. The ball segway in this project is a people-carrying robot that is sufficiently large enough for a human to ride.

Step 1: Derive Dynamic Model of the Ball Segway

As described in [1], motion of a ball segway can be described by three individual sets of dynamic equations in the $x-z$ plane (Equation 7), $y-z$ plane (Equation 8), and $x-y$ plane (Equation 13).

(a) Let the state vector $x_r = \begin{bmatrix} y_k \ \theta_x \ \dot{y_k} \ \dot{\theta}_x \end{bmatrix}^T$ represent the states of the dynamic equations in the $y-z$ plane. Use the ball segway parameters in Table 1 of [1], and convert the dynamic equations of the $x-z$ plane into a standard nonlinear state-space form with state vector x_r. Hint: The resultant state space is given in Equation 9.

(b) Let the state vector $x_p = \begin{bmatrix} x_k \ \theta_y \ \dot{x_k} \ \dot{\theta}_y \end{bmatrix}^T$ represent the states of the dynamic equations in the $x-z$ plane. Use the ball segway parameters in Table 1 of [1], and convert the dynamic equations of the $y-z$ plane into a standard nonlinear state-space form with state vector x_p. Hint: The resultant state space is given in Equation 10.

(c) Let the state vector $x_t = \begin{bmatrix} \theta_z \ \dot{\theta}_z \end{bmatrix}^T$ represent the states of the dynamic equations in the $x-y$ plane. Use the ball segway parameters in Table 1 of [1], and convert the dynamic equations of the $x-y$ plane into a standard nonlinear state-space form with state vector x_t.

Step 2: Build the Nonlinear Ball Segway Model in Simulink

(a) Build a Simulink model of the first subsystem in step 1 (part a) with one input and three measurable state variables y_k, θ_x, $\dot{y_k}$, and $\dot{\theta}_x$.

(b) Build a Simulink model of the second subsystem in step 1 (part b) with one input and three measurable state variables x_k, θ_y, $\dot{x_k}$, and $\dot{\theta}_y$.

(c) Build a Simulink model of the third subsystem in step 1 (part c) with one input and two state variables θ_z and $\dot{\theta}_z$.

Step 3: Implement the Torque Conversion

Set $\alpha = 65.5^o$ (as given in Table 1) and use Equation (20) to form the relationship between the actual motor torques ($\tau_1,\ \tau_2,\ \tau_3$), and the torques along the $x,\ y$, and z axes.

step 4: Design a Rotating Controller

A proportional-derivative (PD) controller is utilized to control the rotation of the segway around the vertical axis. The controller feeds back the yaw angle θ_z and yaw angular velocity $\dot{\theta}_z$ as given in part (c) of step 1 and step 2. The controller is in the form of,

$$\tau_z = K_{dz}\dot{\theta}_z + K_{pz}(\theta_z - \theta_{zd}).$$

(a) Design a PD rotating controller by appropriate selection of controller parameters K_{dz} and K_{pz}.

(b) Use the Simulink model of the third subsystem developed in step 2 to investigate the rotating motion of the ball segway. Explain the importance of this controller in the motion of ball segway, i.e., what happens if this controller does not work appropriately?

Step 5: Design Balancing Controllers

Consider the double-loop controller shown in Figure 1, consisting of two LQR controllers of the form,

$$v_y = -K_r \begin{bmatrix} \dot{\theta}_x \\ \theta_x \\ \dot{y_k} \\ y_k - y_{kd} \\ x_{5r} \end{bmatrix}, \qquad v_x = -K_p \begin{bmatrix} \dot{\theta}_y \\ \theta_y \\ \dot{x_k} \\ x_k - x_{kd} \\ x_{5p} \end{bmatrix},$$

two proportional-integral (PI) controllers of the form,

$$\begin{aligned} u_{PIy} &= K_{pp}(v_x - \dot{x_k}) + K_{ip}(x_{5p} - x_k), \\ u_{PIx} &= K_{pr}(v_y - \dot{y_k}) + K_{ir}(x_{5r} - y_k), \end{aligned}$$

and two feedforward compensation terms u_{yf}, and u_{xf}.

(a) Use [1] to explain the role of each controller in this mechanism.

(b) Construct the control mechanism of Figure 2 in Simulink. Use the Simulink models of steps 2 and 3 for the ball segway and torque conversion blocks. Assume that the LQR controller is optimally given as,

$$\begin{aligned} K_r &= \begin{bmatrix} -6.98 & -4.45 & 20.88 & 6.09 & 5.11 \end{bmatrix}, \\ K_p &= \begin{bmatrix} -3.27 & 4.01 & 29.07 & 8.54 & 5.72 \end{bmatrix}. \end{aligned}$$

Follow the instructions in [1] to design PI controller parameters and the feedforward compensation terms. Feel free to design your own PI controllers with appropriate selection of ζ and ω_n.

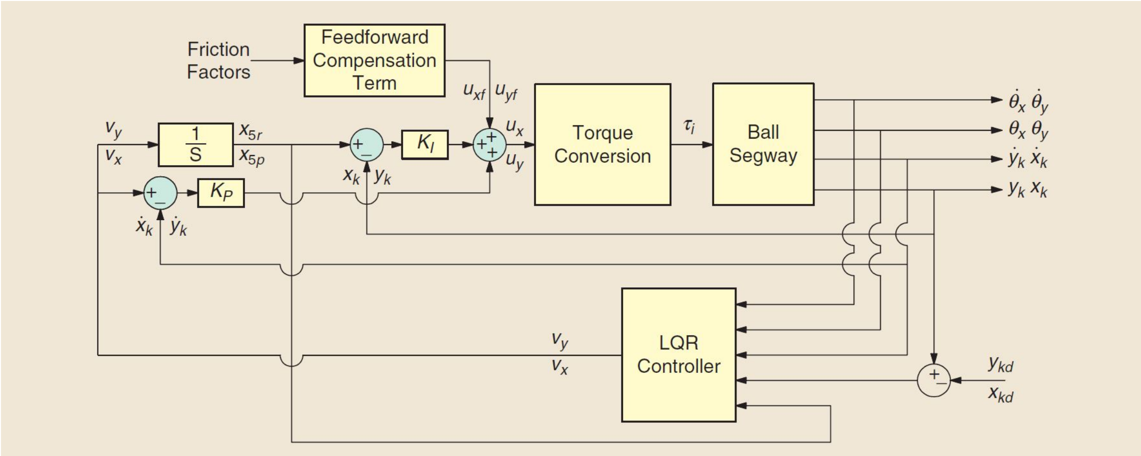

Figure 2: The double-loop control system for designing the balancing controllers. [1]

Step 6: Balancing with an initial nonzero tilt angle

Assume that the ball segway is initially motionless with a roll angle of 2^o in the x-axis and a pitch angle of 2^o in the y-axis from the upright position. Plot the roll angle θ_x, the pitch angle θ_y, and the control signals, $u_x = u_{xf} + u_{PIx}$ and $u_y = u_{yf} + u_{PIy}$. Interpret the results. You may compare your results with Figure 8 in [1].

Step 7: Balancing with External Disturbances

(a) Apply an impulse disturbance force with an amplitude of 50 N to the ball segway in the yz plane at 15s. Plot the roll angle θ_x, the pitch angle θ_y, and the control signals. You may compare your results with Figure 9 in [1].

(b) Repeat the kicking disturbance in part (a) to the ball segway in the xz plane, and demonstrate the results as well.

(c) Show the trajectory of the ball on the floor by plotting y_k versus x_k.

Step 8: Transferring to the Desired Position

Assume that the ball segway should transfer from the initial position $(x_k, y_k) = (0, 0)$ to the desired position $(x_k, y_k) = (-0.13m, 0.37m)$.

(a) Implement the transferring scenario by applying appropriate step signals for x_{kd} and y_{kd}. Plot the x-position x_k, the y-position y_k, the roll angle θ_x, the pitch angle θ_y, and the control signals. You may compare your results with Figure 10 in [1].

(b) Repeat the transferring scenario in part (a) by applying appropriate ramp signals for x_{kd} and y_{kd}, and demonstrate the results as well.

(c) Compare the effectiveness of transferring to a desired position by step and ramp reference signals.

References

[1] Pham, Dinh Ba, et al. *Balancing and transferring control of a ball segway using a double-loop approach [applications of control]*. *IEEE Control Systems Magazine* 38.2 (2018): 15-37.