6. Raytracing

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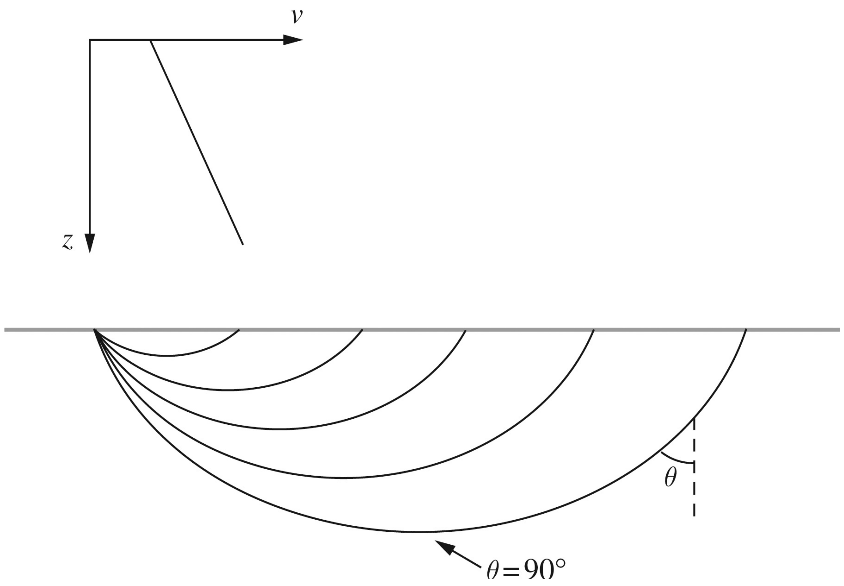
ERSE 210 Seismology

Eikonal Equation

$$|\nabla T|^2 = \frac{1}{\alpha^2}$$

Traveltime

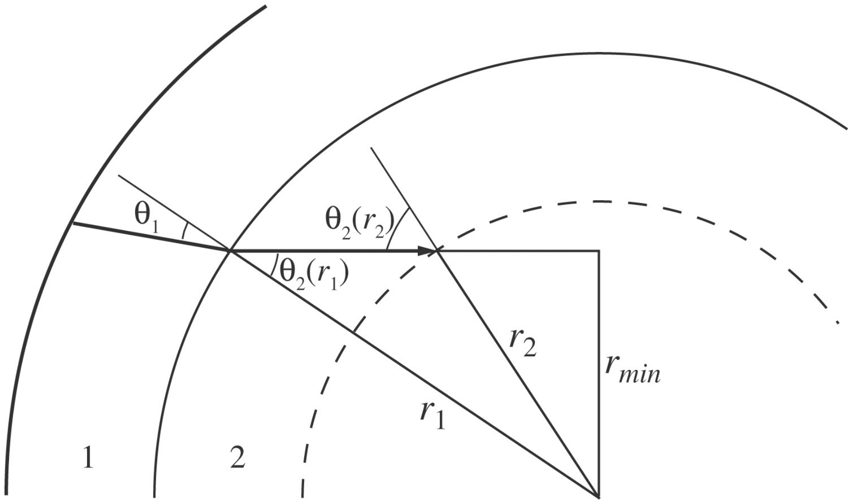
Raytracing in layered medium



$$X(p_x) = 2p_x \int_0^{z_p} \frac{dz}{\sqrt{u^2(p_x) - p_x^2}}$$

$$T(p_x) = 2 \int_0^{z_p} \frac{u^2(p_x)dz}{\sqrt{u^2(p_x) - p_x^2}}$$

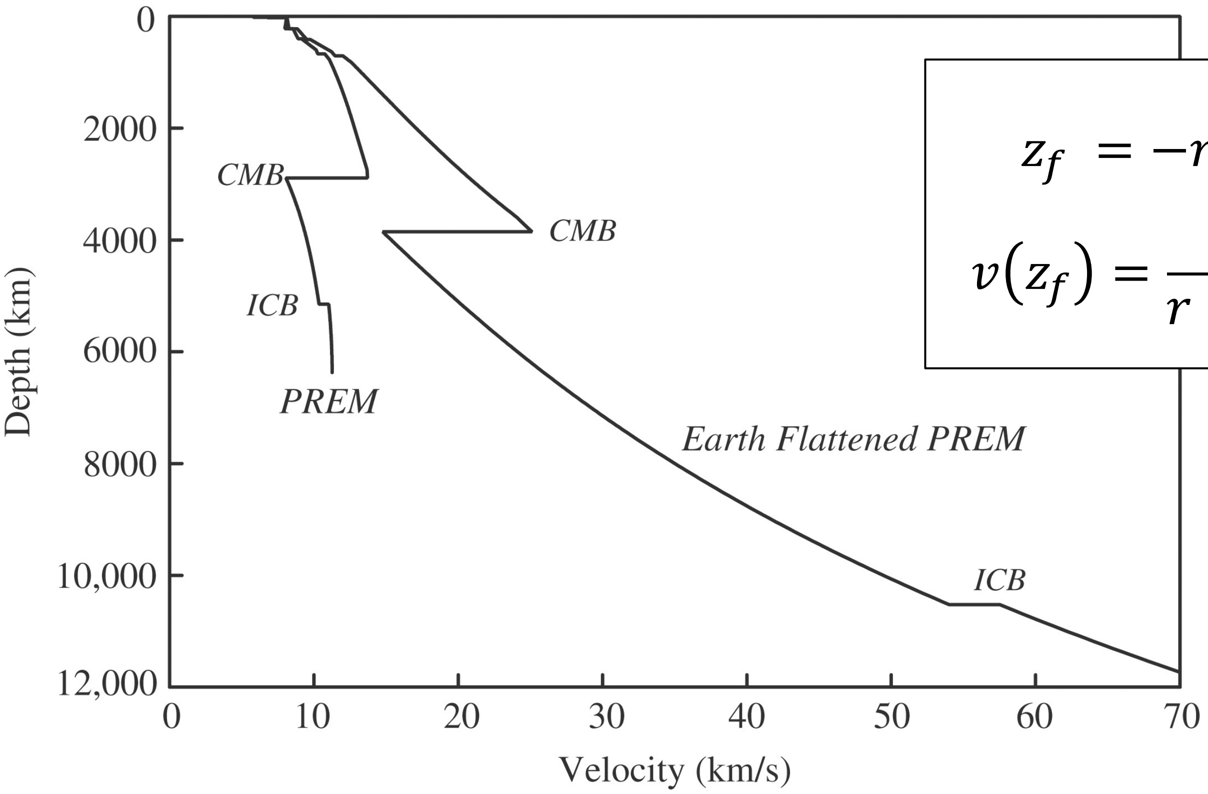
Raytracing in spherical medium



$$\Delta(p_{sph}) = 2p_{sph}\int_{r_{tp}}^{r_e} \frac{1}{\sqrt{(ur)^2 - p_{sph}^2}} \frac{dr}{r}$$

$$T(p_{sph}) = 2\int_{r_{tp}}^{r_e} \frac{(ur)^2}{\sqrt{(ur)^2 - p_{sph}^2}} \frac{dr}{r}$$

Earth-flattening transformation



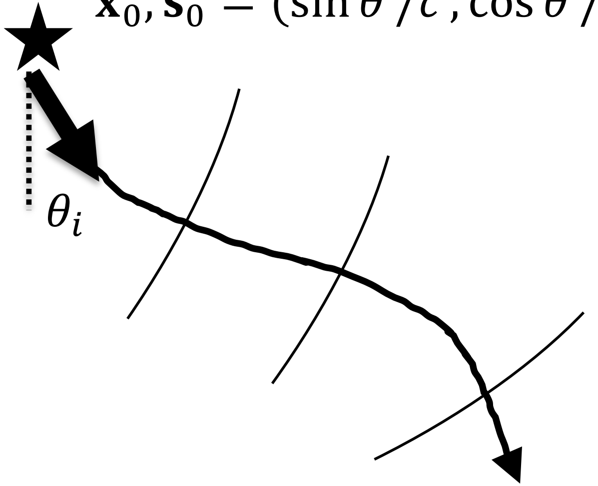
$$z_f = -r_e \ln\left(\frac{r_e - z_s}{r_e}\right)$$

$$v(z_f) = \frac{r_e}{r - z_s} v(r_e - z_s)$$

Raytracing in heterogenous medium

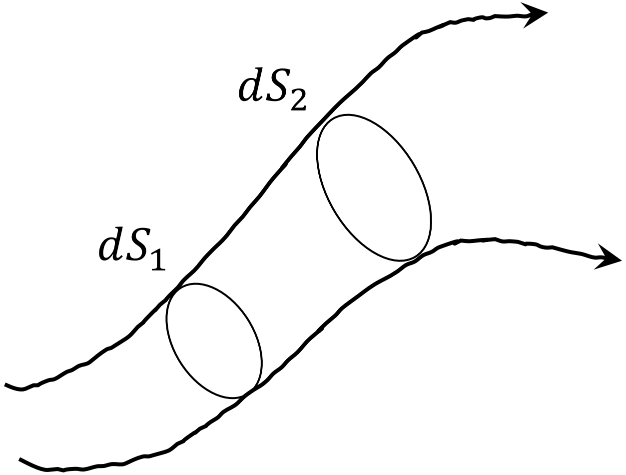
$$\begin{cases} \dfrac{d\mathbf{x}}{ds} = \dfrac{\mathbf{s}}{u} \\ \dfrac{d\mathbf{s}}{ds} = \nabla u \\ \mathbf{s}(s=0) = \mathbf{s}_0 \\ \mathbf{x}(s=0) = \mathbf{x}_0 \end{cases}$$

$\mathbf{x}_0, \mathbf{s}_0 = (\sin\theta\,/c\,, \cos\theta\,/c)$

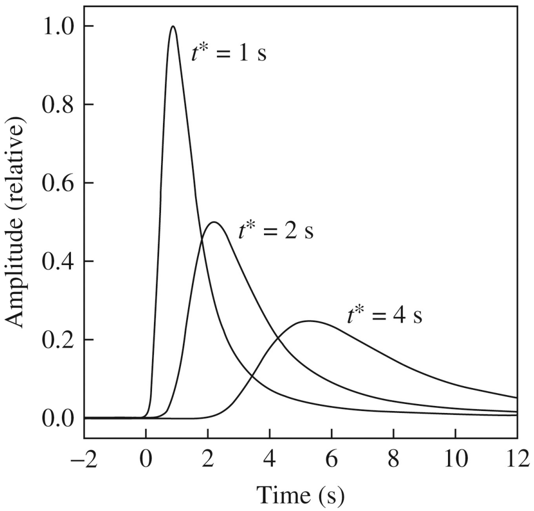


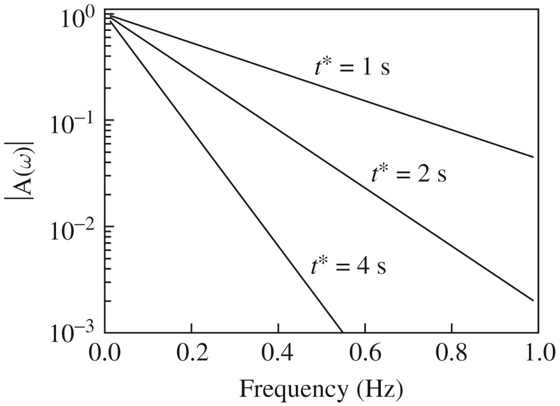
Ray amplitude (geometrical spreading)

$$\frac{A_1}{A_2} = \sqrt{\frac{dS_1}{dS_2}}$$



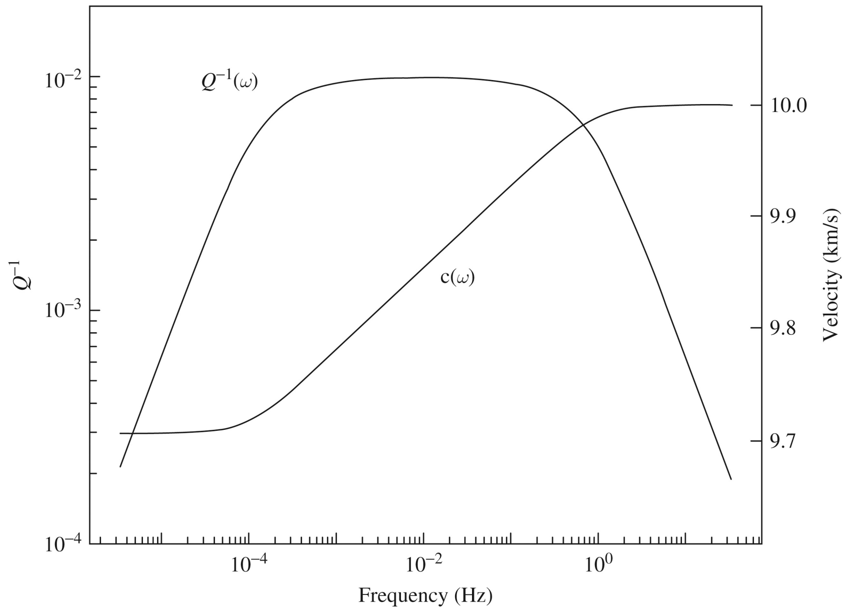
Ray amplitude (attenuation)





$$t^* = \int_{path} \frac{dt}{Q(\mathbf{r})}$$

Absorption model



$$Q^{-1}(\omega) = 2Q_m^{-1} D_Q(\omega)$$

$$c(\omega) = c_\infty \left(1 - \frac{D_c(\omega)}{Q_m} \right)$$