2. Acoustic Wave Equation

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ERSE 210 Seismology

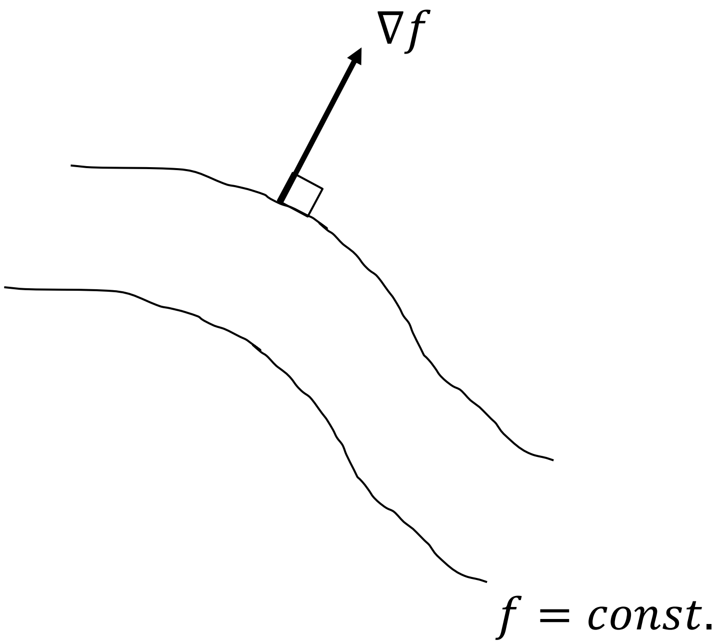
Wave equation(s)

- **Principle of Inertia** (aka **Equation of motion**)

- **Hooke's law** (aka **Deformation equation**)

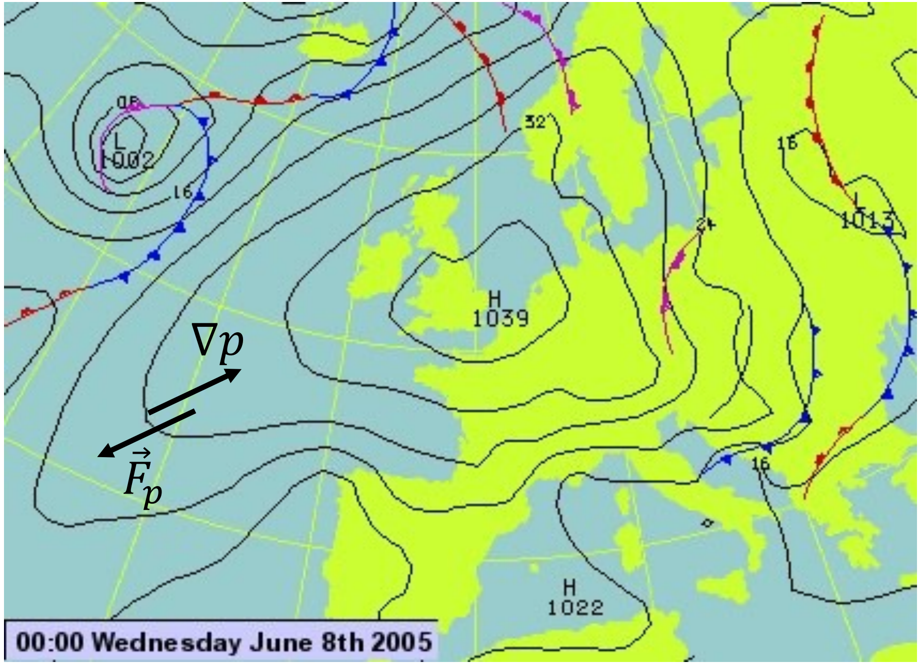
Gradient

$$\nabla f = \begin{bmatrix} \partial f / \partial x \\ \partial f / \partial y \\ \partial f / \partial z \end{bmatrix}$$



Pressure force

$$\vec{F}_p = \frac{\vec{F}}{\delta V} = -\nabla p$$



Force from high to low pressure

Total and partial derivatives

Lagrangian view

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + (\vec{v} \cdot \nabla f)$$

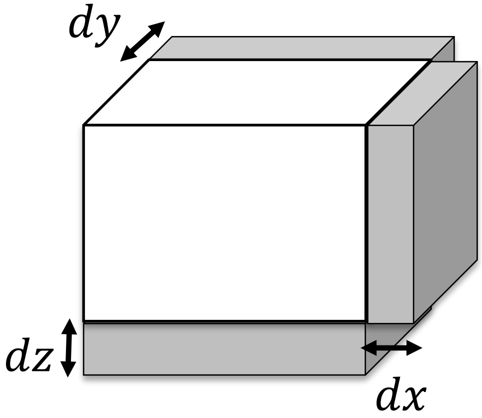
Eulerian view

Divergence

$$\nabla \cdot \vec{f} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

Flux

$$\frac{d\Phi_f}{dV} = \nabla \cdot \vec{f}$$



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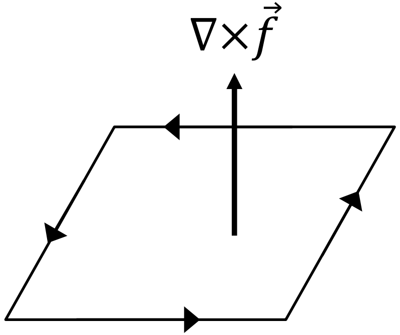
$$\nabla \times \vec{f} = \begin{vmatrix} \vec{i}_x & \vec{i}_y & \vec{i}_z \\ \partial x & \partial y & \partial z \\ f_x & f_y & f_z \end{vmatrix}$$



Acoustic wave equation constituents

- **Principle of Inertia** $$-\nabla p = \rho_0 \frac{\partial \vec{v}}{\partial t}$$

- **Hooke's law** $$-\nabla \cdot \vec{v} = \frac{1}{K} \frac{\partial p}{\partial t}$$

Acoustic wave equation

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

Propagation velocity

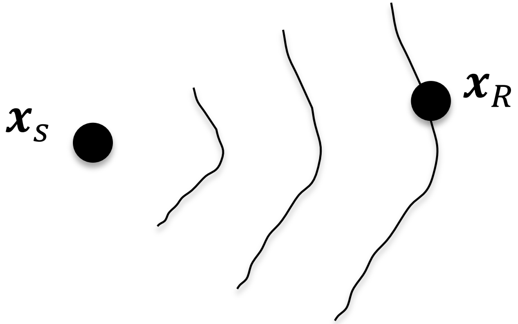
Helmoltz equation

$$\nabla^2 p - k^2 p = 0$$

Wavenumber: $k = \frac{\omega}{c}$

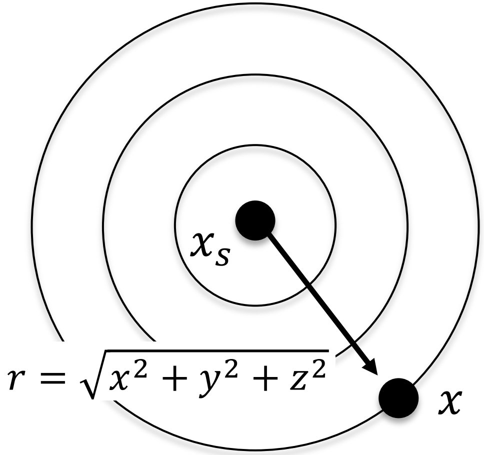
Green's function

$$\nabla^2 G - \frac{1}{c^2}\frac{\partial^2 G}{\partial t^2} = \delta(\boldsymbol{x} - \boldsymbol{x}')\delta(t - t')$$



$$p(t, \boldsymbol{x}_R) = s(t) * G(t, \boldsymbol{x}_R)$$

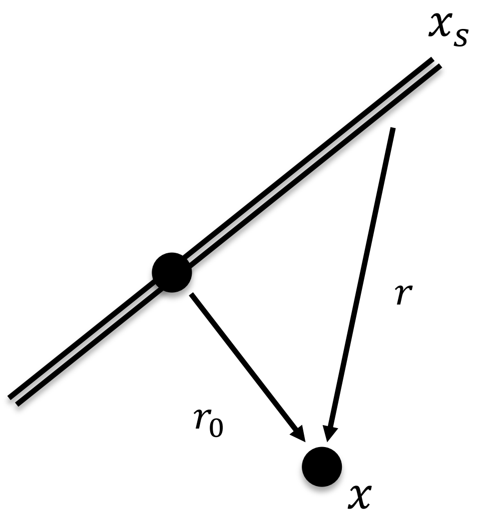
3D Spherical GF



$$G_{3D}(t, \boldsymbol{x}) = \frac{1}{4\pi r}\delta\left(t - \frac{r}{c}\right)$$

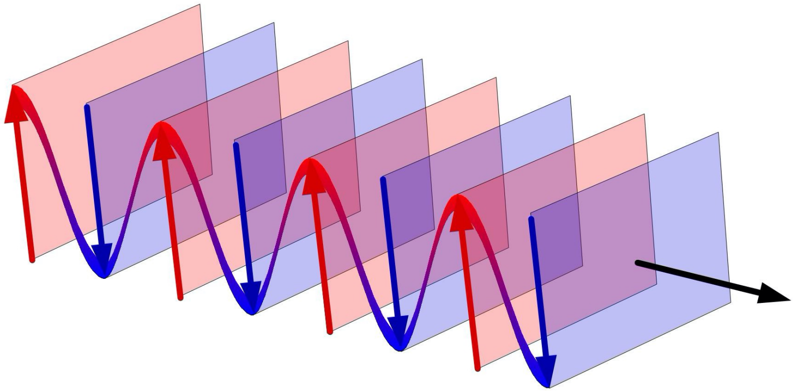
$$G_{3D}(f, \boldsymbol{x}) = \frac{1}{4\pi r} e^{j2\pi f\frac{r}{c}}$$

2D Cilindrical GF



$$G_{2D}(t,\boldsymbol{x}) = \frac{1}{2\pi}\frac{\mathrm{H}(t-\frac{r}{c})}{\sqrt{t^2-\frac{r^2}{c^2}}}$$

1D Plane GF



$$G_{1D}(t, \boldsymbol{x}) = \frac{2}{c}\delta\left(t - \frac{\boldsymbol{x} * \boldsymbol{i}_S}{c}\right)$$

$$G_{1D}(f, \boldsymbol{x}) = \frac{1}{4\pi} e^{j2\pi f \frac{\boldsymbol{x} * \boldsymbol{i}_S}{c}}$$