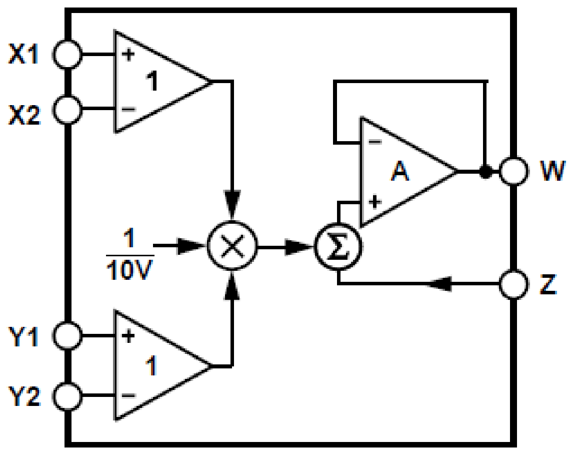
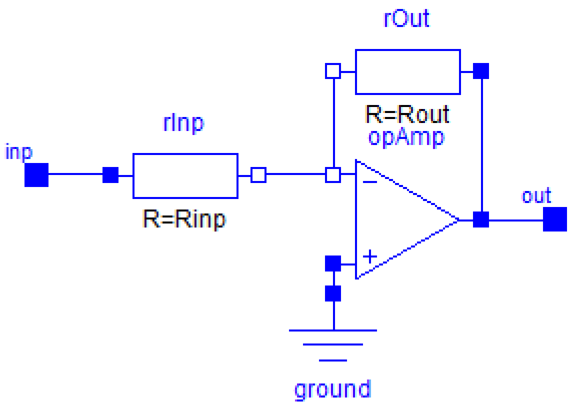
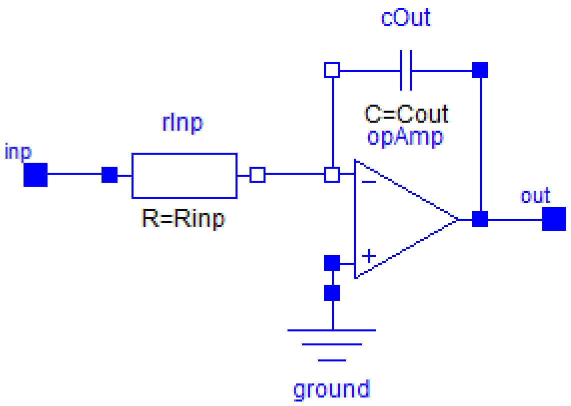
OpApm-Circuits

Analog Multiplier 	Functional Block Diagram of AD633 Division by $10\ V$ (scaling) inhibits overflow. Additional summing input Z is omitted. Negative inputs of X- and Y-amplifiers are connected to ground.
Amplifier/Adder 	$$\frac{inp.v}{R_{inp}} + \frac{out.v}{R_{out}} = 0$$ $$-out.v = k \cdot inp.v$$ $$k = \frac{R_{out}}{R_{inp}}$$ It is possible to add several inputs.
Integrator 	$$\frac{inp.v}{R_{inp}} + C_{out} \cdot \frac{d\ out.v}{dt} = 0$$ $$-out.v = v_0 + \frac{1}{\tau} \cdot \int_0^t inp.v \cdot dt$$ $$\tau = R_{inp} \cdot C_{out}$$ It is possible to integrate the sum of several inputs.

1 _Lorenz System

http://en.wikipedia.org/wiki/Lorenz_attractor

Developed 1963 by Edward Lorenz to model atmospheric convection.

x is proportional to the rate of convection, y to the horizontal temperature variation and z to the vertical temperature variation. σ depicts the Prandtl number, ρ the Rayleigh number and β the physical dimensions.

The original parameters were: $\sigma = 10, \rho = 28, \beta = \frac{8}{3}$

$$\frac{dx}{dt} = \sigma \cdot (y - x)$$
$$\frac{dy}{dt} = x \cdot (\rho - z) - y$$
$$\frac{dz}{dt} = x \cdot y - \beta \cdot z$$

$\beta = \frac{1}{3}$ leads to a periodic solution.

For an implementation as an electronic circuit, the equations have to be scaled to keep the variables within the desired range. This can be compared with calculating per-unit values by dividing by reference values:

$$x' = \frac{x}{k_x}$$
$$y' = \frac{y}{k_y}$$
$$z' = \frac{z}{k_z}$$
$$t' = \frac{t}{\tau}$$

We also have to take into account that the analog multiplier divides by E_R to avoid overflow of the output. After that, none of the computing block should encounter an overflow.

This leads to the following set of equations:

$$\frac{1}{\tau} \cdot \frac{dx'}{dt'} = -\sigma \cdot x' + \sigma \cdot \frac{k_y}{k_x} \cdot y'$$
$$\frac{1}{\tau} \cdot \frac{dy'}{dt'} = \rho \cdot \frac{k_x}{k_y} \cdot x' - y' - \frac{k_x \cdot k_z \cdot V_S}{k_y} \cdot \frac{x' \cdot z'}{V_S}$$
$$\frac{1}{\tau} \cdot \frac{dz'}{dt'} = \frac{k_x \cdot k_y \cdot V_S}{k_z} \cdot \frac{x' \cdot y'}{V_S} - \beta \cdot z'$$

These equations can easily get implemented as blocks or as an electronic circuit.

2 Chua's Circuit

https://link.springer.com/book/10.1007/978-3-319-05900-6 (1.1)

https://nonlinear.eecs.berkeley.edu/chaos/chaos.html#_Working_With_Chaos_Simulation

$$L \cdot \frac{di_L}{dt} = v_2 - R_L \cdot i_L$$

$$C_2 \cdot \frac{dv_2}{dt} = -i_L - \frac{v_2 - v_1}{R}$$

$$C_1 \cdot \frac{dv_1}{dt} = -i_{NL} + \frac{v_2 - v_1}{R}$$

$$-i_{NL}(v_1) = \begin{cases} -\infty < v_1 < -V_e \rightarrow G_b \cdot (v_1 + V_e) - G_a \cdot V_e \\ -V_e < v_1 < +V_e \rightarrow G_a \cdot v_1 \\ +V_e < v_1 < +\infty \rightarrow G_b \cdot (v_1 - V_e) + G_a \cdot V_e \end{cases}$$

$$-\frac{i_{NL}}{v_1} = \begin{cases} -\infty < v_1 < -V_e \rightarrow G_b - (G_a - G_b) \cdot \dfrac{V_e}{v_1} \\ -V_e < v_1 < +V_e \rightarrow G_a \\ +V_e < v_1 < +\infty \rightarrow G_b + (G_a - G_b) \cdot \dfrac{V_e}{v_1} \end{cases}$$

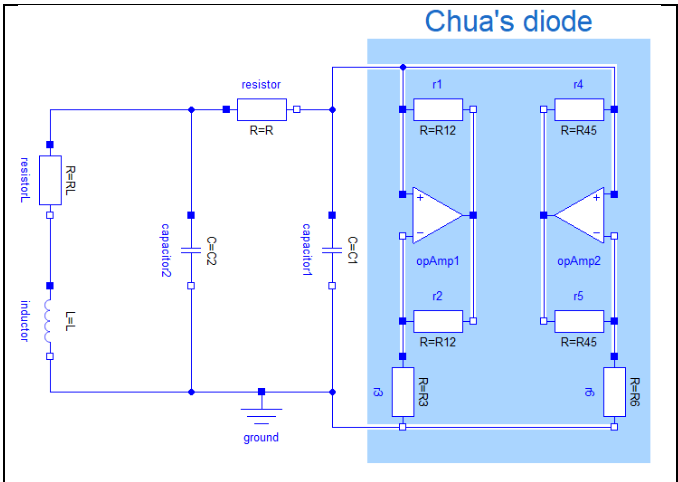
$$-\frac{di_{NL}}{dv_1} = \begin{cases} -\infty < v_1 < -V_e \rightarrow G_b \\ -V_e < v_1 < +V_e \rightarrow G_a \\ +V_e < v_1 < +\infty \rightarrow G_b \end{cases}$$

$$\tau_L \cdot \dot{v}_{RL} = v_2 - v_{RL}$$

$$\tau_2 \cdot \dot{v}_2 = +v_1 - v_2 - \frac{R}{R_L} \cdot v_{RL}$$

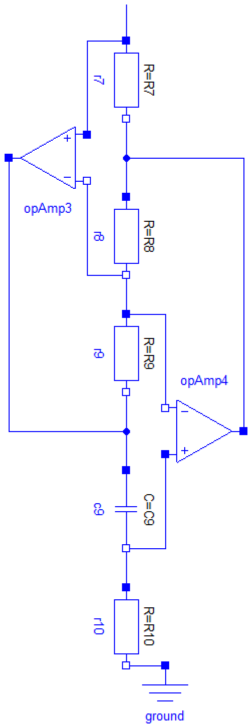
$$\tau_1 \cdot \dot{v}_1 = -v_1 + v_2 + R \cdot g \cdot v_1$$

$$g(v_1) = \begin{cases} |v_1| > V_e \rightarrow G_b + (G_a - G_b) \cdot \dfrac{V_e}{|v_1|} \\ |v_1| < V_e \rightarrow G_a \end{cases}$$




```
parameter SI.Resistance R=1.9e3 "Resistor";
parameter SI.Inductance L=18e-3 "Inductor";
parameter SI.Resistance RL=14 "Resistance of Inductor";
parameter SI.Capacitance C1=10.e-9 "Capacitor 1";
parameter SI.Capacitance C2=100e-9 "Capacitor 2";
//parameter of Chua's diode
parameter Real k0=15000.0 "No-load amplification ";
parameter SI.Voltage Vs=9 "Supply voltage of opAmps";
parameter SI.Resistance R12=220 "R1 and R2";
parameter SI.Resistance R3=2200 "R3";
parameter SI.Resistance R45=22e3 "R4 and R5";
parameter SI.Resistance R6=3300 "R6";
```

Chua's Circuit: Inductor Replacement



$$R_7 = 100\ \Omega$$
$$R_8 = 1\ k\Omega$$
$$R_9 = 1\ k\Omega$$
$$R_{10} = 1{,}8\ k\Omega$$
$$C_9 = 100\ nF$$

$$R_7 \cdot i + R_8 \cdot (i + i_{OA4}) = 0$$
$$R_9 \cdot (i + i_{OA4}) + v_{c9} = 0$$
$$i + i_{OA4} + i_{OA3} = i_{C9}$$
$$i_{C9} = C_9 \cdot \frac{dv_{C9}}{dt}$$
$$v = R_{10} \cdot i_{C9}$$

$$(i + i_{OA4}) = -\frac{R_7}{R_8} \cdot i$$
$$v_{c9} = \frac{R_7 \cdot R_9}{R_8} \cdot i$$
$$i_{C9} = C_9 \cdot \frac{R_7 \cdot R_9}{R_8} \cdot \frac{di}{dt}$$
$$v = C_9 \cdot \frac{R_7 \cdot R_9 \cdot R_{10}}{R_8} \cdot \frac{di}{dt}$$

$$\boxed{L = C_9 \cdot \frac{R_7 \cdot R_9 \cdot R_{10}}{R_8} = 18\ mH}$$

3_Chaotic Diode Circuit

https://www.researchgate.net/publication/309351711_A_simple_chaotic_circuit_with_a_light-emitting_diode

$$C \cdot \frac{dv_1}{dt} = -\frac{v_2}{R}$$

$$C \cdot \frac{dv_2}{dt} = -\frac{v_3}{R}$$

$$C \cdot \frac{dv_3}{dt} = -\frac{v_1}{R} - \frac{v_3}{R_b} - \frac{v_4}{R}$$

$$\frac{v_4}{R_a} = -I_{ds} \cdot \left(e^{\frac{v_2}{nV_t}} - 1\right)$$

$$\tau = R \cdot C$$

$$\tau \cdot \frac{dv_1}{dt} = -v_2$$

$$\tau \cdot \frac{dv_2}{dt} = -v_3$$

$$\tau \cdot \frac{dv_3}{dt} = -v_1 - \frac{R}{R_b} \cdot v_3 + R_a \cdot I_{ds} \cdot \left(e^{\frac{v_2}{nV_t}} - 1\right)$$

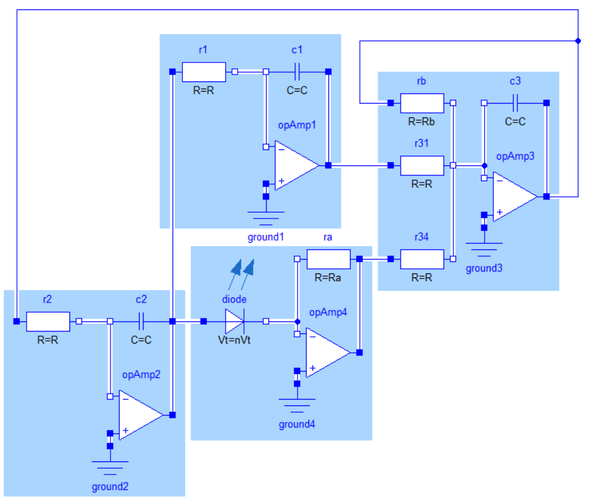
$$a = \frac{R_a \cdot I_{ds}}{nV_t}$$

$$b = \frac{R}{R_b}$$

$$\tau \cdot \dot{x}_1 = -x_2$$

$$\tau \cdot \dot{x}_2 = -x_3$$

$$\tau \cdot \dot{x}_3 = -x_1 + a \cdot (e^{x_2} - 1) - b \cdot x_3$$




```
parameter SI.Current Ids=1e-9 "Diode: saturation current";
parameter SI.Voltage nVt=2*26e-3
" n*voltage equ. of temperature";
//time constant and dimensionless parameters
parameter SI.Time Tau=0.1e-3 "Time constant";
parameter Real a=1/5200 "Parameter a = Ra*Ids/nVt";
parameter Real b=1.5 "Parameter b = R/Rb";
//configuration of the opAmps
parameter SI.Capacitance C=10e-9
"Capacitance of {c1, c2, c3}";
parameter SI.Resistance R=Tau/C "Resistance of {r1,r2,r3,r4}";
parameter SI.Resistance Ra=a*nVt/Ids "Resistance of ra";
parameter SI.Resistance Rb=R/b "Resistance of rb";
```

4_Chaotic Oscillator

https://www.researchgate.net/publication/230925506_A_simple_chaotic_oscillator_for_educational_purposes

https://www.researchgate.net/publication/259216097_NUMERICAL_TREATMENT_OF_EDUCATIONAL_CHAOS_OSCILLATOR

$$i_L = C \cdot \frac{dv_C}{dt}$$

$$L \cdot \frac{di_L}{dt} = \left(k - 1 - \frac{R_L}{R}\right) \cdot R \cdot i_L - v_C - v_{C^*}$$

$$k = 1 + \frac{R_2}{R_1}$$

$$C^* \cdot \frac{dv_{C^*}}{dt} = I_0 + i_L - I_{DS} \cdot \left(e^{\frac{v_{C^*}}{nV_t}} - 1\right)$$

$$I_0 \approx \frac{V_b}{R_0}$$

$$\tau = \sqrt{L \cdot C}$$

$$Z = \sqrt{\frac{L}{C}}$$

$$a = \left(k - 1 - \frac{R_L}{R}\right) \cdot \frac{R}{Z}$$

$$b = \frac{Z \cdot I_0}{nV_t}$$

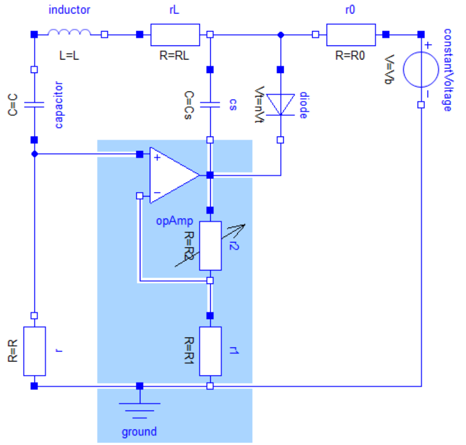
$$c = \frac{Z \cdot I_{DS}}{nV_t}$$

$$e = \frac{C^*}{C}$$

$$\tau \cdot \frac{\dot{v}_C}{nV_t} = \frac{Z \cdot i_L}{nV_t}$$

$$\tau \cdot \frac{Z \cdot \dot{i}_L}{nV_t} = \left(k - 1 - \frac{R_L}{R}\right) \cdot \frac{R}{Z} \cdot \frac{Z \cdot i_L}{nV_t} - \frac{v_C}{nV_t} - \frac{v_{C^*}}{nV_t}$$

$$\tau \cdot e \cdot \frac{\dot{v}_{C^*}}{nV_t} = \frac{Z \cdot I_0}{nV_t} + \frac{Z \cdot i_L}{nV_t} - \frac{Z \cdot I_{DS}}{nV_t} \cdot \left(e^{\frac{v_{C^*}}{nV_t}} - 1\right)$$




```
//time constant and dimensionless parameters
parameter SI.Current I0=Vb/R0 "Estimated supply current";
parameter SI.Time Tau=sqrt(L*C) "Time constant";
parameter SI.Impedance Z=sqrt(L/C) "Characteristic impedance (rho)";
parameter Real k=Z/R*a + 1 + RL/R "Amplification";
parameter Real a=0.95;
parameter Real b=Z*I0/nVt;
parameter Real c=Z*Ids/nVt;
parameter Real e=Cs/C "epsilon";
//components
parameter SI.Resistance R =1.0e3 "Resistor r";
parameter SI.Resistance R1=10.e3 "Resistor r1";
parameter SI.Resistance R2=R1*(k - 1) "Resistor r2 (tunable)";
parameter SI.Resistance R0=20.e3 "Resistor r0";
parameter SI.Voltage Vb=20 "Supply voltage";
parameter SI.Inductance L=100e-3 "Inductor";
parameter SI.Resistance RL=50 "Resistance of Inductor";
parameter SI.Capacitance C =100e-9 "Capacitor c";
parameter SI.Capacitance Cs=15.e-9 "Capacitor c*";
//diode
parameter SI.Current Ids=1e-9 "Diode: saturation current";
parameter SI.Voltage nVt=2*26e-3
"Diode: n * voltage equivalent of temperature";
```

5_Colpitts Oscillator

https://link.springer.com/book/10.1007/978-3-319-05900-6 (1.3)

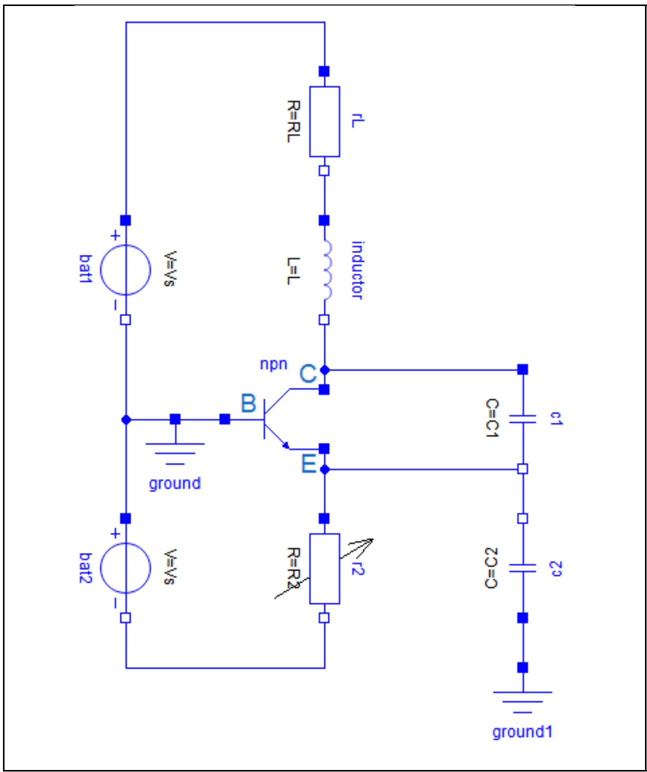
https://ieeexplore.ieee.org/document/331536

$$C_1 \cdot \frac{dv_1}{dt} = i_L - \beta \cdot i_B$$

$$C_2 \cdot \frac{dv_2}{dt} = -\frac{V_{s-} + v_2}{R_2} - i_L - i_B$$

$$L \cdot \frac{di_L}{dt} = V_{s+} - v_1 + v_2 - R_L \cdot i_L$$

$$i_B = \begin{cases} v_2 = v_{BE} \leq V_{th} \rightarrow & 0 \\ v_2 = v_{BE} > V_{th} \rightarrow & \dfrac{v_2 - V_{th}}{R_{on}} \end{cases}$$




```
parameter SI.Resistance RL=35. "Resistance of L";
parameter SI.Inductance L=98.5e-6 "Inductor";
parameter SI.Resistance R2=1000 "Resistor 2";
parameter SI.Capacitance C1=54.e-9 "Capacitor 1";
parameter SI.Capacitance C2=54.e-9 "Capacitor 2";
parameter SI.Voltage Vs=5 "Source Voltage";

parameter SI.Voltage Vth=0.75 "Transistor threshold voltage";
parameter SI.Resistance Ron=100
  "Small-signal on-resistance of base-emitter junction";
parameter Real beta=200 "Transistor forward current gain";
```

6_Shinriki Oscillator

https://pawn.physik.uni-wuerzburg.de/~slueck/PhyAmSa09/Home_files/Examensarbeit_Lueck.pdf

https://ieeexplore.ieee.org/abstract/document/1456241

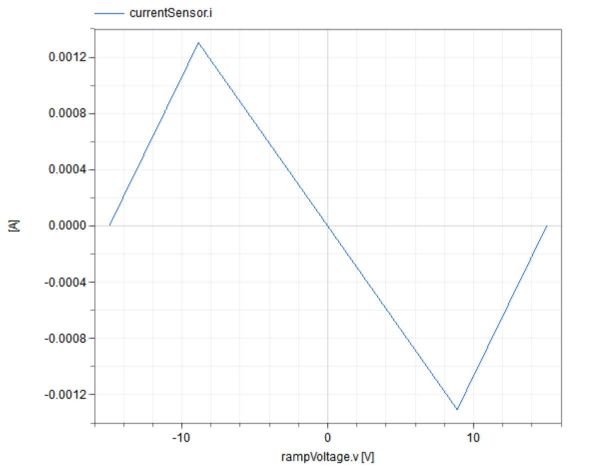
$$i_z = \begin{cases} |v_z| < V_{bt} & 0 \\ |v_z| \geq V_{bt} & sign(v_z) \cdot [a \cdot (|v_z| - V_{bt}) + b \cdot (|v_z| - V_{bt})^3 + c \cdot (|v_z| - V_{bt})^5] \end{cases}$$

$$C_1 \cdot \frac{dv_1}{dt} = -i_{NIC} - \frac{v_1}{R_1} - i_z$$

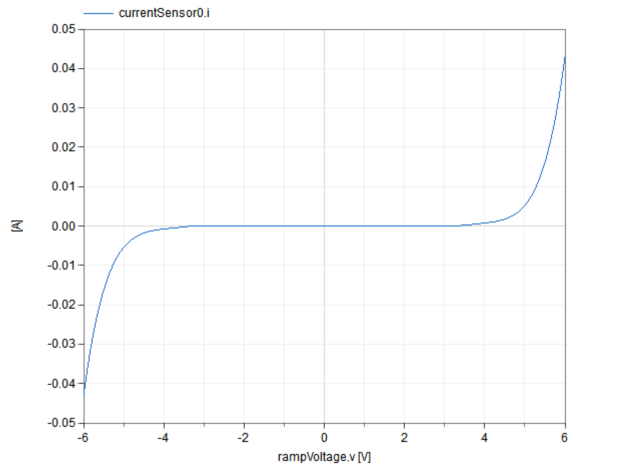
$$C_2 \cdot \frac{dv_2}{dt} = i_z - i_L$$

$$v_2 = L \cdot \frac{di_L}{dt} + R_L \cdot i_L$$

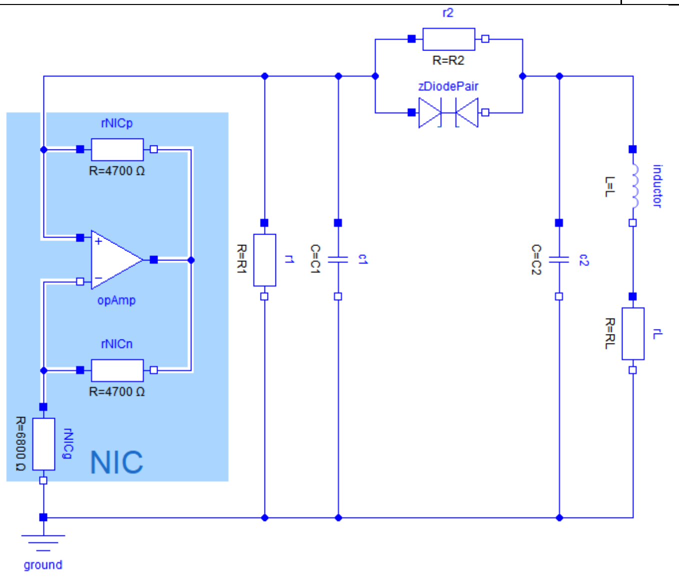
$$g_{NIC} = \frac{di_{NIC}}{dv_{NIC}} = \begin{cases} |v_{NIC}| > V_{Lim} & G_+ \\ |v_{NIC}| \leq V_{Lim} & G_- \end{cases}$$



Characteristic of negative impedance converter



Characteristic of Zener diode pair




```
parameter SI.Inductance L=320e-3 "Inductor";
parameter SI.Resistance RL=100. "Resistor of L";
parameter SI.Resistance R1=60e3 "Resistor 1";
parameter SI.Resistance R2=20e3 "Resistor 2";
parameter SI.Capacitance C1=10.e-9 "Capacitor 1";
parameter SI.Capacitance C2=100e-9 "Capacitor 2";
```

$$a = 1{,}0862 \ \frac{mA}{V}$$

$$b = -0{,}1615 \ \frac{mA}{V^3}$$

$$c = 0{,}3021 \ \frac{mA}{V^5}$$

$$V_{Lim} = 8{,}870565 \ V$$

$$G_+ = 0{,}212766 \ mS$$

$$G_- = -0{,}147018 \ mS$$