**OpAmp-Circuits**

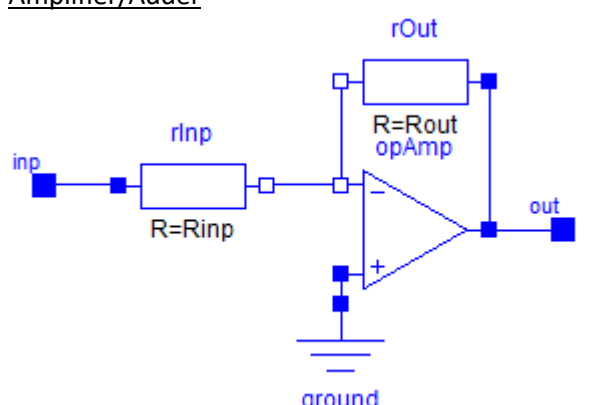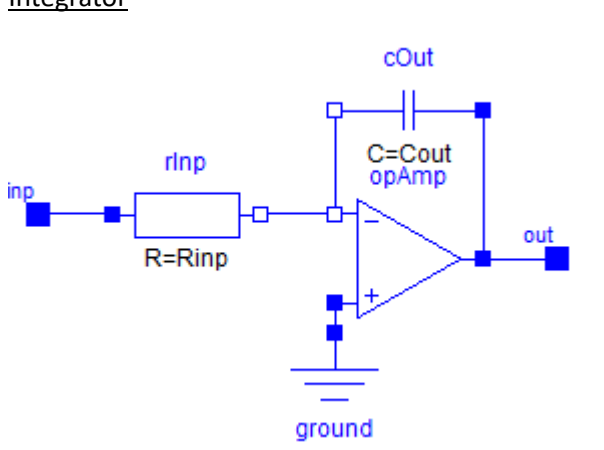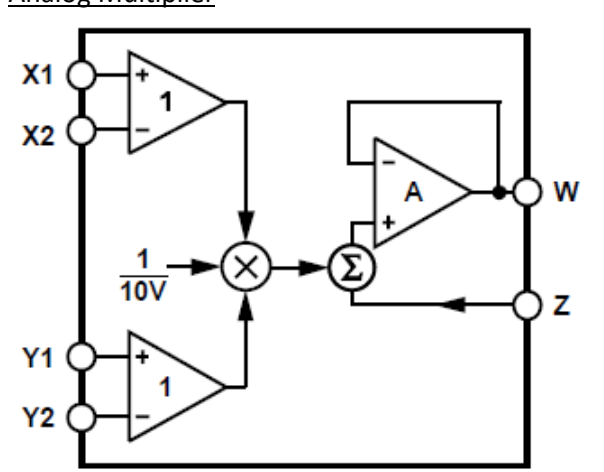| | |
|---|---|
| Amplifier/Adder  | $$\frac{inp.v}{R_{inp}} + \frac{out.v}{R_{out}} = 0$$ $$-out.v = k \cdot inp.v$$ $$k = \frac{R_{out}}{R_{inp}}$$ It is possible to add several inputs. |
| Integrator  | $$\frac{inp.v}{R_{inp}} + C_{out} \cdot \frac{d\ out.v}{dt} = 0$$ $$-out.v = v_0 + \frac{1}{\tau} \cdot \int_0^t inp.v \cdot dt$$ $$\tau = R_{inp} \cdot C_{out}$$ It is possible to integrate the sum of several inputs. |
| Analog Multiplier  | Functional Block Diagram of AD633 Division by $10\ V$ (scaling) inhibits overflow. Additional summing input Z is omitted. Negative inputs of X- and Y-amplifiers are connected to ground. <br><br> Possible implementations: <br> • Gilbert cell <br> • $y = e^{ln(x_1)+ln(x_2)}$ <br> • $y = \frac{(x_1+x_2)^2-(x_1-x_2)^2}{4}$ |

# 1_Lotka-Volterra

https://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equations

https://en.wikipedia.org/wiki/Competitive_Lotka%E2%80%93Volterra_equations

https://sprott.physics.wisc.edu/pubs/paper288.pdf

As a 2-dimensional predator-prey-model no chaos is reported:

$$\frac{dx}{dt} = r_x \cdot x - d_x \cdot x \cdot y$$
$$\frac{dy}{dt} = r_x \cdot x \cdot e_y \cdot y - d_y \cdot y$$

We might interpret $x$ as number of hares (prey) and $y$ as foxed (predator).

$r_x$ is the reproduction rate of hares, $d_x$ the deathrate of hares due to foxes.

$e_y$ is the efficiency in growing foxes from hares, $d_y$ the (natural) deathrate of foxes.

A n-dimensional Lotka-Volterra model is defined for $1 \leq i \leq n$ ($n$ designates the number of species):

$$\frac{dx_i}{dt} = r_i \cdot x_i \cdot \left( 1 - \sum_{j=1}^{n} a_{ij} \cdot x_j \right)$$

The vector $[r]$ describes the reproduction rates of the species whereas the quadratic matrix $[a]$ describes the competition between species.

## 2_Lorenz System

Developed 1963 by Edward Lorenz to model atmospheric convection.

$x$ is proportional to the rate of convection, $y$ to the horizontal temperature variation and $z$ to the vertical temperature variation. $\sigma$ depicts the Prandtl number, $\rho$ the Rayleigh number and $\beta$ the physical dimensions.

The original parameters were: $\sigma = 10, \rho = 28, \beta = \frac{8}{3}$

$$\frac{dx}{dt} = \sigma \cdot (y - x)$$

$$\frac{dy}{dt} = x \cdot (\rho - z) - y$$

$$\frac{dz}{dt} = x \cdot y - \beta \cdot z$$

$\beta = \frac{1}{3}$ leads to a periodic solution.

For an implementation as an electronic circuit, the equations have to be scaled to keep the variables within the desired range. This can be compared with calculating per-unit values by dividing by reference values:

$$x' = \frac{x}{k_x}$$

$$y' = \frac{y}{k_y}$$

$$z' = \frac{z}{k_z}$$

$$t' = \frac{t}{\tau}$$

We also have to take into account that the analog multiplier divides by $E_R$ to avoid overflow of the output. After that, none of the computing block should encounter an overflow.

This leads to the following set of equations:

$$\frac{1}{\tau} \cdot \frac{dx'}{dt'} = -\sigma \cdot x' + \sigma \cdot \frac{k_y}{k_x} \cdot y'$$

$$\frac{1}{\tau} \cdot \frac{dy'}{dt'} = \rho \cdot \frac{k_x}{k_y} \cdot x' - y' - \frac{k_x \cdot k_z \cdot V_S}{k_y} \cdot \frac{x' \cdot z'}{V_S}$$

$$\frac{1}{\tau} \cdot \frac{dz'}{dt'} = \frac{k_x \cdot k_y \cdot V_S}{k_z} \cdot \frac{x' \cdot y'}{V_S} - \beta \cdot z'$$

These equations can easily get implemented as blocks or as an electronic circuit.

# 3_Roessler System

A simple system of 3 ordinary nonlinear differential equations to study chaos without physical background.

$$\frac{dx}{dt} = -y - z$$

$$\frac{dy}{dt} = x + a \cdot y$$

$$\frac{dz}{dt} = b + (x - c) \cdot z$$

$a = 0.2$, $b = 0.2$ and $c = 1$ give periodic results. Changing $c = 5.7$ reveals chaotic results.

For an implementation as an electronic circuit, the equations have to be scaled to keep the variables within the desired range. This can be compared with calculating per-unit values by dividing by reference values:

$$x' = \frac{x}{k_x}$$

$$y' = \frac{y}{k_y}$$

$$z' = \frac{z}{k_z}$$

$$t' = \frac{t}{\tau}$$

We also have to take into account that the analog multiplier divides by $E_R$ to avoid overflow of the output. After that, none of the computing block should encounter an overflow.

This leads to the following set of equations:

$$\frac{1}{\tau} \cdot \frac{dx'}{dt} = -\frac{k_y}{k_x} \cdot y' - \frac{k_z}{k_x} \cdot z'$$

$$\frac{1}{\tau} \cdot \frac{dy'}{dt} = \frac{k_x}{k_y} \cdot x' + a \cdot y'$$

$$\frac{1}{\tau} \cdot \frac{dz'}{dt} = \frac{b}{k_z} + k_x \cdot V_S \cdot \frac{x' \cdot z'}{V_S} - c \cdot z'$$

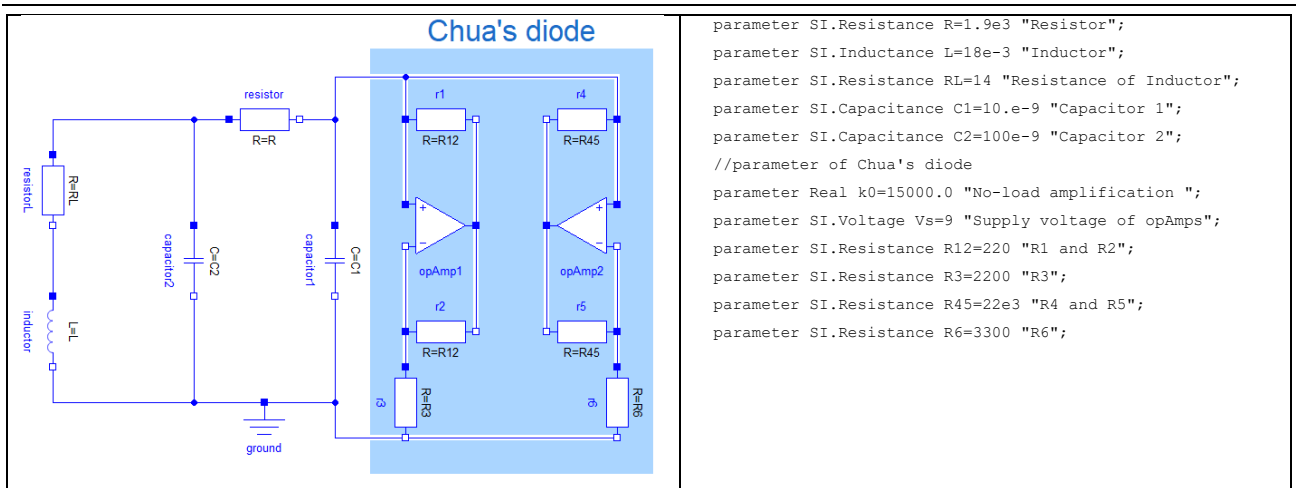These equations can easily get implemented as blocks or as an electronic circuit.
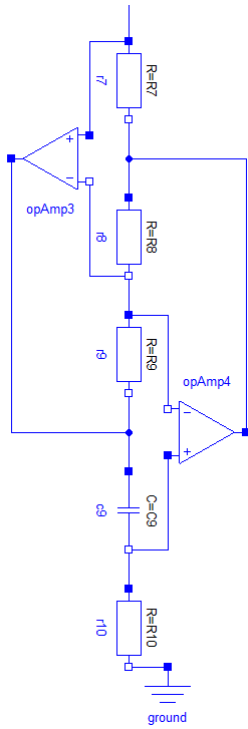
# 4_Chua's Circuit

$$L \cdot \frac{di_L}{dt} = v_2 - R_L \cdot i_L$$

$$C_2 \cdot \frac{dv_2}{dt} = -i_L - \frac{v_2 - v_1}{R}$$

$$C_1 \cdot \frac{dv_1}{dt} = -i_{NL} + \frac{v_2 - v_1}{R}$$

$$-i_{NL}(v_1) = \begin{cases} -\infty < v_1 < -V_e \rightarrow G_b \cdot (v_1 + V_e) - G_a \cdot V_e \\ -V_e < v_1 < +V_e \rightarrow G_a \cdot v_1 \\ +V_e < v_1 < +\infty \rightarrow G_b \cdot (v_1 - V_e) + G_a \cdot V_e \end{cases}$$

$$-\frac{i_{NL}}{v_1} = \begin{cases} -\infty < v_1 < -V_e \rightarrow G_b - (G_a - G_b) \cdot \frac{V_e}{v_1} \\ -V_e < v_1 < +V_e \rightarrow G_a \\ +V_e < v_1 < +\infty \rightarrow G_b + (G_a - G_b) \cdot \frac{V_e}{v_1} \end{cases}$$

$$-\frac{di_{NL}}{dv_1} = \begin{cases} -\infty < v_1 < -V_e \rightarrow G_b \\ -V_e < v_1 < +V_e \rightarrow G_a \\ +V_e < v_1 < +\infty \rightarrow G_b \end{cases}$$

---

$$\tau_L \cdot \dot{v}_{RL} = v_2 - v_{RL}$$

$$\tau_2 \cdot \dot{v}_2 = +v_1 - v_2 - \frac{R}{R_L} \cdot v_{RL}$$

$$\tau_1 \cdot \dot{v}_1 = -v_1 + v_2 + R \cdot g \cdot v_1$$

$$g(v_1) = \begin{cases} |v_1| > V_e \rightarrow G_b + (G_a - G_b) \cdot \frac{V_e}{|v_1|} \\ |v_1| < V_e \rightarrow G_a \end{cases}$$



```
parameter SI.Resistance R=1.9e3 "Resistor";
parameter SI.Inductance L=18e-3 "Inductor";
parameter SI.Resistance RL=14 "Resistance of Inductor";
parameter SI.Capacitance C1=10.e-9 "Capacitor 1";
parameter SI.Capacitance C2=100e-9 "Capacitor 2";
//parameter of Chua's diode
parameter Real k0=15000.0 "No-load amplification ";
parameter SI.Voltage Vs=9 "Supply voltage of opAmps";
parameter SI.Resistance R12=220 "R1 and R2";
parameter SI.Resistance R3=2200 "R3";
parameter SI.Resistance R45=22e3 "R4 and R5";
parameter SI.Resistance R6=3300 "R6";
```

# Chua's Circuit: Inductor Replacement

$$R_7 = 100 \; \Omega$$
$$R_8 = 1 \; k\Omega$$
$$R_9 = 1 \; k\Omega$$
$$R_{10} = 1{,}8 \; k\Omega$$
$$C_9 = 100 \; nF$$

$$R_7 \cdot i + R_8 \cdot (i + i_{OA4}) = 0$$
$$R_9 \cdot (i + i_{OA4}) + v_{c9} = 0$$
$$i + i_{OA4} + i_{OA3} = i_{C9}$$
$$i_{C9} = C_9 \cdot \frac{dv_{C9}}{dt}$$
$$v = R_{10} \cdot i_{C9}$$

$$(i + i_{OA4}) = -\frac{R_7}{R_8} \cdot i$$
$$v_{c9} = \frac{R_7 \cdot R_9}{R_8} \cdot i$$
$$i_{C9} = C_9 \cdot \frac{R_7 \cdot R_9}{R_8} \cdot \frac{di}{dt}$$
$$v = C_9 \cdot \frac{R_7 \cdot R_9 \cdot R_{10}}{R_8} \cdot \frac{di}{dt}$$

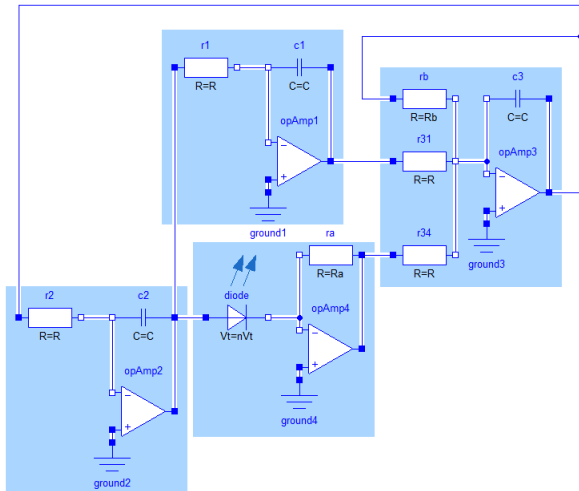$$L = C_9 \cdot \frac{R_7 \cdot R_9 \cdot R_{10}}{R_8} = 18 \; mH$$

# 5_Chaotic Diode Circuit

$$C \cdot \frac{dv_1}{dt} = -\frac{v_2}{R}$$

$$C \cdot \frac{dv_2}{dt} = -\frac{v_3}{R}$$

$$C \cdot \frac{dv_3}{dt} = -\frac{v_1}{R} - \frac{v_3}{R_b} - \frac{v_4}{R}$$

$$\frac{v_4}{R_a} = -I_{ds} \cdot \left(e^{\frac{v_2}{nV_t}} - 1\right)$$

---

$$\tau = R \cdot C$$

$$\tau \cdot \frac{dv_1}{dt} = -v_2$$

$$\tau \cdot \frac{dv_2}{dt} = -v_3$$

$$\tau \cdot \frac{dv_3}{dt} = -v_1 - \frac{R}{R_b} \cdot v_3 + R_a \cdot I_{ds} \cdot \left(e^{\frac{v_2}{nV_t}} - 1\right)$$

---

$$a = \frac{R_a \cdot I_{ds}}{nV_t}$$

$$b = \frac{R}{R_b}$$

$$\tau \cdot \dot{x}_1 = -x_2$$

$$\tau \cdot \dot{x}_2 = -x_3$$

$$\tau \cdot \dot{x}_3 = -x_1 + a \cdot (e^{x_2} - 1) - b \cdot x_3$$



```
parameter SI.Current Ids=1e-9 "Diode: saturation current";

parameter SI.Voltage nVt=2*26e-3
" n*voltage equ. of temperature";

//time constant and dimensionless parameters

parameter SI.Time Tau=0.1e-3 "Time constant";

parameter Real a=1/5200 "Parameter a = Ra*Ids/nVt";

parameter Real b=1.5 "Parameter b = R/Rb";

//configuration of the opAmps

parameter SI.Capacitance C=10e-9
"Capacitance of {c1, c2, c3}";

parameter SI.Resistance R=Tau/C "Resistance of {r1,r2,r3,r4}";

parameter SI.Resistance Ra=a*nVt/Ids "Resistance of ra";

parameter SI.Resistance Rb=R/b "Resistance of rb";
```

# 6_Chaotic Oscillator

$$i_L = C \cdot \frac{dv_C}{dt}$$

$$L \cdot \frac{di_L}{dt} = \left(k - 1 - \frac{R_L}{R}\right) \cdot R \cdot i_L - v_C - v_{C^*}$$

$$k = 1 + \frac{R_2}{R_1}$$

$$C^* \cdot \frac{dv_{C^*}}{dt} = I_0 + i_L - I_{DS} \cdot \left(e^{\frac{v_{C^*}}{nV_t}} - 1\right)$$

$$I_0 \approx \frac{V_b}{R_0}$$

---

$$\tau = \sqrt{L \cdot C}$$

$$Z = \sqrt{\frac{L}{C}}$$

$$a = \left(k - 1 - \frac{R_L}{R}\right) \cdot \frac{R}{Z}$$
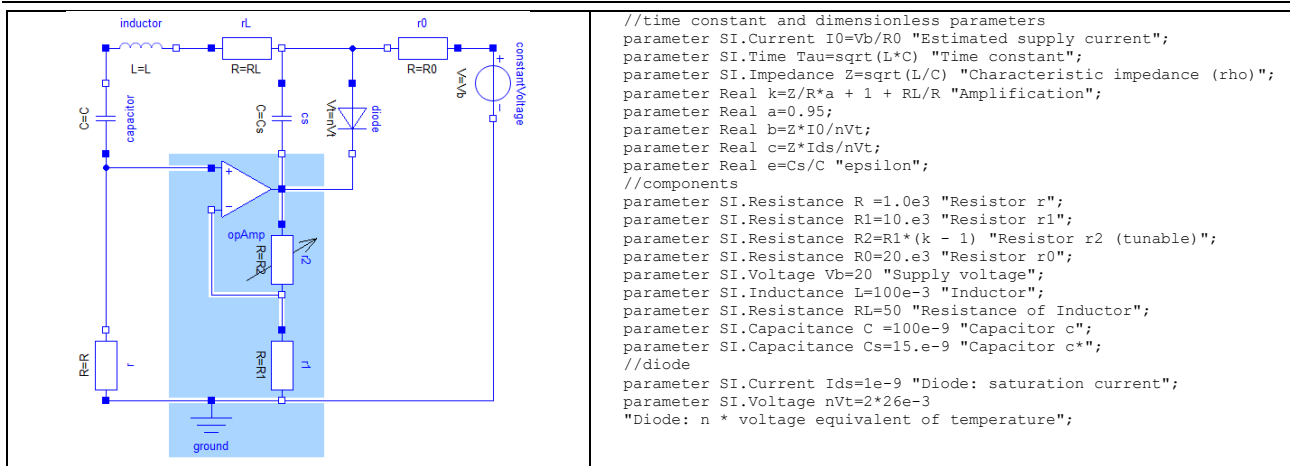
$$b = \frac{Z \cdot I_0}{nV_t}$$

$$c = \frac{Z \cdot I_{DS}}{nV_t}$$

$$e = \frac{C^*}{C}$$

---

$$\tau \cdot \frac{\dot{v}_C}{nV_t} = \frac{Z \cdot i_L}{nV_t}$$

$$\tau \cdot \frac{Z \cdot \dot{i}_L}{nV_t} = \left(k - 1 - \frac{R_L}{R}\right) \cdot \frac{R}{Z} \cdot \frac{Z \cdot i_L}{nV_t} - \frac{v_C}{nV_t} - \frac{v_{C^*}}{nV_t}$$

$$\tau \cdot e \cdot \frac{\dot{v}_{C^*}}{nV_t} = \frac{Z \cdot I_0}{nV_t} + \frac{Z \cdot i_L}{nV_t} - \frac{Z \cdot I_{DS}}{nV_t} \cdot \left(e^{\frac{v_{C^*}}{nV_t}} - 1\right)$$

---



```
//time constant and dimensionless parameters
parameter SI.Current I0=Vb/R0 "Estimated supply current";
parameter SI.Time Tau=sqrt(L*C) "Time constant";
parameter SI.Impedance Z=sqrt(L/C) "Characteristic impedance (rho)";
parameter Real k=Z/R*a + 1 + RL/R "Amplification";
parameter Real a=0.95;
parameter Real b=Z*I0/nVt;
parameter Real c=Z*Ids/nVt;
parameter Real e=Cs/C "epsilon";
//components
parameter SI.Resistance R =1.0e3 "Resistor r";
parameter SI.Resistance R1=10.e3 "Resistor r1";
parameter SI.Resistance R2=R1*(k - 1) "Resistor r2 (tunable)";
parameter SI.Resistance R0=20.e3 "Resistor r0";
parameter SI.Voltage Vb=20 "Supply voltage";
parameter SI.Inductance L=100e-3 "Inductor";
parameter SI.Resistance RL=50 "Resistance of Inductor";
parameter SI.Capacitance C =100e-9 "Capacitor c";
parameter SI.Capacitance Cs=15.e-9 "Capacitor c*";
//diode
parameter SI.Current Ids=1e-9 "Diode: saturation current";
parameter SI.Voltage nVt=2*26e-3
"Diode: n * voltage equivalent of temperature";
```

# 7_Colpitts Oscillator

https://link.springer.com/book/10.1007/978-3-319-05900-6 (1.3)
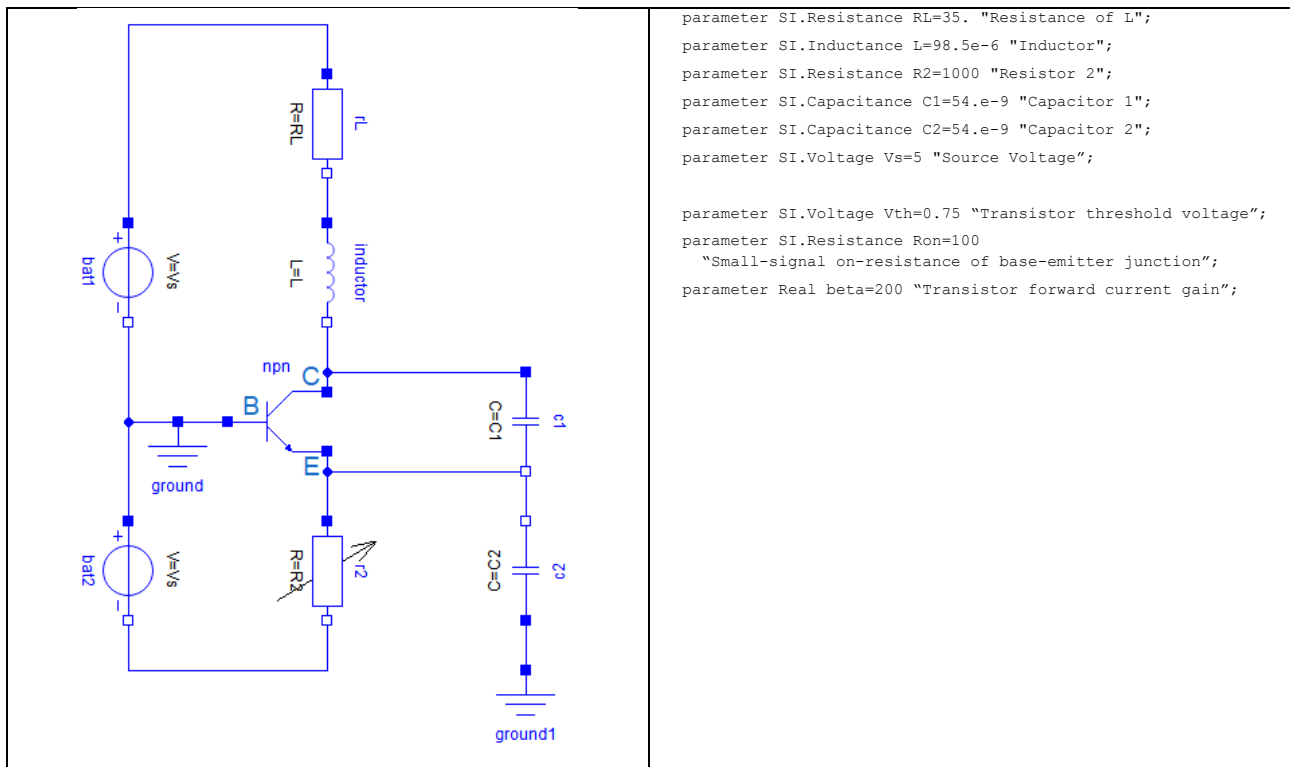
https://ieeexplore.ieee.org/document/331536

$$C_1 \cdot \frac{dv_1}{dt} = i_L - \beta \cdot i_B$$

$$C_2 \cdot \frac{dv_2}{dt} = -\frac{V_{s-} + v_2}{R_2} - i_L - i_B$$

$$L \cdot \frac{di_L}{dt} = V_{s+} - v_1 + v_2 - R_L \cdot i_L$$

---

$$i_B = \begin{cases} v_2 = v_{BE} \leq V_{th} \to & 0 \\ v_2 = v_{BE} > V_{th} \to & \dfrac{v_2 - V_{th}}{R_{on}} \end{cases}$$

---



```
parameter SI.Resistance RL=35. "Resistance of L";
parameter SI.Inductance L=98.5e-6 "Inductor";
parameter SI.Resistance R2=1000 "Resistor 2";
parameter SI.Capacitance C1=54.e-9 "Capacitor 1";
parameter SI.Capacitance C2=54.e-9 "Capacitor 2";
parameter SI.Voltage Vs=5 "Source Voltage";

parameter SI.Voltage Vth=0.75 "Transistor threshold voltage";
parameter SI.Resistance Ron=100
  "Small-signal on-resistance of base-emitter junction";
parameter Real beta=200 "Transistor forward current gain";
```
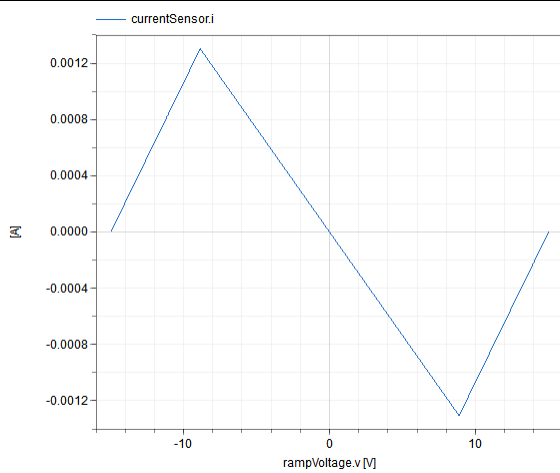
# 8_Shinriki Oscillator

$$i_z = \begin{cases} |v_z| < V_{bt} & 0 \\ |v_z| \geq V_{bt} & sign(v_z) \cdot [a \cdot (|v_z| - V_{bt}) + b \cdot (|v_z| - V_{bt})^3 + c \cdot (|v_z| - V_{bt})^5] \end{cases}$$

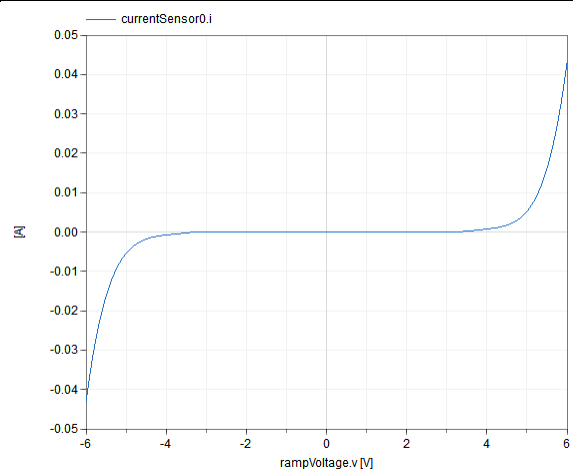$$C_1 \cdot \frac{dv_1}{dt} = -i_{NIC} - \frac{v_1}{R_1} - i_z$$

$$C_2 \cdot \frac{dv_2}{dt} = i_z - i_L$$

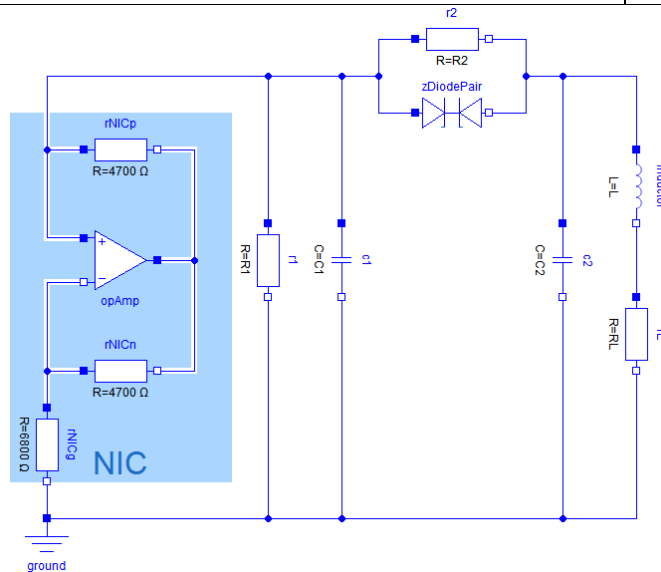$$v_2 = L \cdot \frac{di_L}{dt} + R_L \cdot i_L$$

$$g_{NIC} = \frac{di_{NIC}}{dv_{NIC}} = \begin{cases} |v_{NIC}| > V_{Lim} & G_+ \\ |v_{NIC}| \leq V_{Lim} & G_- \end{cases}$$



Characteristic of negative impedance converter



Characteristic of Zener diode pair



```
parameter SI.Inductance L=320e-3 "Inductor";
parameter SI.Resistance RL=100. "Resistor of L";
parameter SI.Resistance R1=60e3 "Resistor 1";
parameter SI.Resistance R2=20e3 "Resistor 2";
parameter SI.Capacitance C1=10.e-9 "Capacitor 1";
parameter SI.Capacitance C2=100e-9 "Capacitor 2";
```

$$a = 1{,}0862 \ \frac{mA}{V}$$

$$b = -0{,}1615 \ \frac{mA}{V^3}$$

$$c = 0{,}3021 \ \frac{mA}{V^5}$$

$$V_{Lim} = 8{,}870565 \ V$$

$$G_+ = 0{,}212766 \ mS$$
$$G_- = -0{,}147018 \ mS$$

# 9_Jerk Circuit

The name of the system stems from the third derivative of $x$, which – in a mechanical system – is the derivative of acceleration called jerk.

$$\dddot{x} + A \cdot \ddot{x} + f(\dot{x}) + x = 0$$

$$f(\dot{x}) = R \cdot I_S \cdot \left( e^{\frac{\dot{x}}{nV_t}} - 1 \right)$$
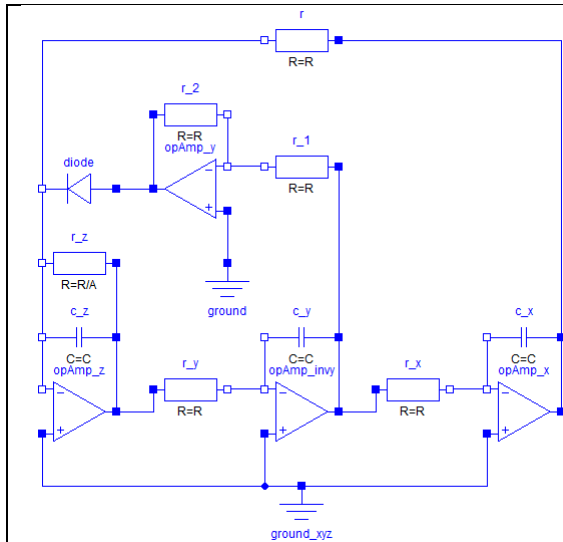
$$\dot{x} = y$$

$$\ddot{x} = \dot{y} = z$$

$$\dddot{x} = \ddot{y} = \dot{z}$$

$$\dot{z} = -A \cdot z - x - f(y)$$

The values stay pretty inside a practicable range for a normal voltage supply.

Using 4 operational amplifiers and an acceleration factor of 1000, the circuit can be implemented as follows. The parameter A influences only the feedback resistor at opAMp_z:



```
parameter Real A=0.3 "Parameter to be varied";
parameter SI.Resistance R=1e3 "Resistance";
parameter SI.Capacitance C=1e-6 "Capacitance";
parameter SI.Current Ids=1e-12 "Sat.current";
parameter SI.Voltage nVt=26e-3 " voltage equ.";
```

A=0.3 for periodic results

A=1.0 for chaotic results