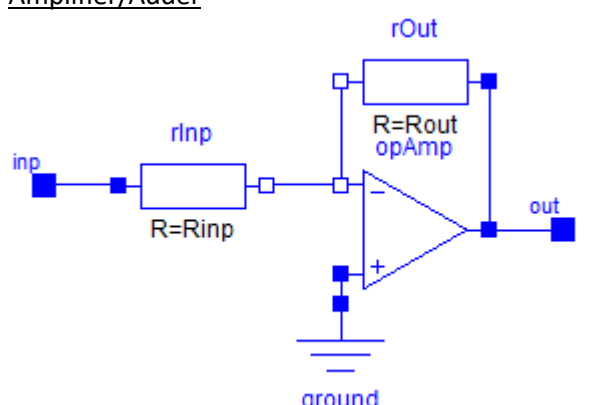
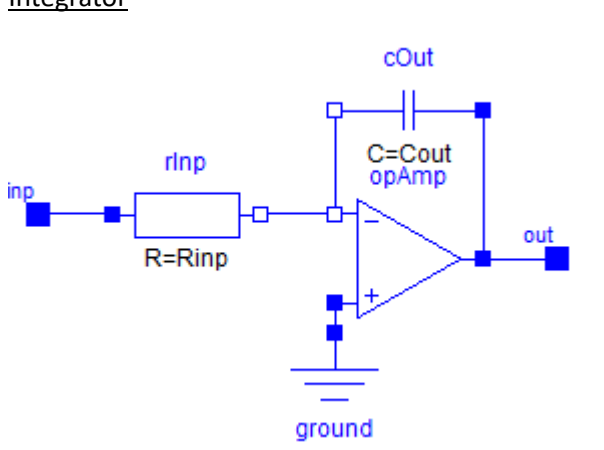
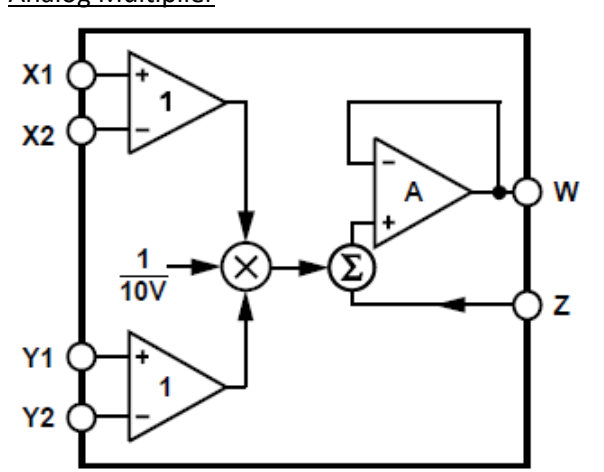


## OpAmp-Circuits

<p><u>Amplifier/Adder</u></p> 	$\frac{inp.v}{R_{inp}} + \frac{out.v}{R_{out}} = 0$ $-out.v = k \cdot inp.v$ $k = \frac{R_{out}}{R_{inp}}$ <p>It is possible to add several inputs.</p>
<p><u>Integrator</u></p> 	$\frac{inp.v}{R_{inp}} + C_{out} \cdot \frac{d out.v}{dt} = 0$ $-out.v = v_0 + \frac{1}{\tau} \cdot \int_0^t inp.v \cdot dt$ $\tau = R_{inp} \cdot C_{out}$ <p>It is possible to integrate the sum of several inputs.</p>
<p><u>Analog Multiplier</u></p> 	<p>Functional Block Diagram of AD633</p> <p>Division by 10 V (scaling) inhibits overflow.</p> <p>Additional summing input Z is omitted.</p> <p>Negative inputs of X- and Y-amplifiers are connected to ground.</p> <p>Possible implementations:</p> <ul style="list-style-type: none"> <li>• Gilbert cell</li> <li>• <math>y = e^{\ln(x_1) + \ln(x_2)}</math></li> <li>• <math>y = \frac{(x_1 + x_2)^2 - (x_1 - x_2)^2}{4}</math></li> </ul>

## 1 Lotka-Volterra

[https://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra\\_equations](https://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equations)

[https://en.wikipedia.org/wiki/Competitive\\_Lotka%E2%80%93Volterra\\_equations](https://en.wikipedia.org/wiki/Competitive_Lotka%E2%80%93Volterra_equations)

<https://sprott.physics.wisc.edu/pubs/paper288.pdf>

As a 2-dimensional predator-prey-model no chaos is reported:

$$\begin{aligned}\frac{dx}{dt} &= r_x \cdot x - d_x \cdot x \cdot y \\ \frac{dy}{dt} &= r_x \cdot x \cdot e_y \cdot y - d_y \cdot y\end{aligned}$$

We might interpret  $x$  as number of hares (prey) and  $y$  as foxes (predator).

$r_x$  is the reproduction rate of hares,  $d_x$  the deathrate of hares due to foxes.

$e_y$  is the efficiency in growing foxes from hares,  $d_y$  the (natural) deathrate of foxes.

A n-dimensional Lotka-Volterra model is defined for  $1 \leq i \leq n$  ( $n$  designates the number of species):

$$\frac{dx_i}{dt} = r_i \cdot x_i \cdot \left( 1 - \sum_{j=1}^n a_{ij} \cdot x_j \right)$$

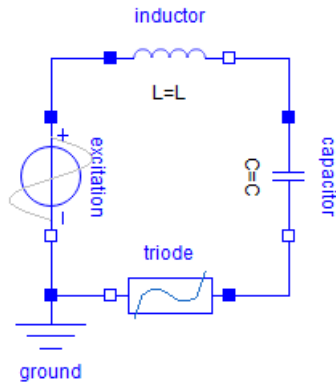
The vector  $[r]$  describes the reproduction rates of the species whereas the quadratic matrix  $[a]$  describes the competition between species.

## 2 van der Pol Circuit

[https://en.wikipedia.org/wiki/Van\\_der\\_Pol\\_oscillator](https://en.wikipedia.org/wiki/Van_der_Pol_oscillator)

B. van der Pol and J. van der Mark, „Frequency Demultiplication”, Nature 120 (1927), p. 363-364, ISSN 0028-0836, DOI <https://doi.org/10.1038/120363a0>

Balthasar van der Pol reported 1927 strange phenomena about oscillations in a series resonance circuit containing a vacuum electron triode. Due to the nonlinear characteristic of the triode the autonomous circuit is able to maintain periodic oscillations, and with harmonic excitation it is able to produce chaos.



$$i = C \cdot \frac{dv_C}{dt}$$

$$L \cdot \frac{di}{dt} + \tilde{R}(i) \cdot i + v_C = \begin{cases} 0 \\ \hat{V} \cdot \cos(\omega \cdot t - \pi) \end{cases}$$

$$\tilde{R}(i) = -R_0 \cdot \left[ 1 - \frac{1}{3} \cdot \left( \frac{i}{I_0} \right)^2 \right]$$

$$L \cdot \frac{d^2 i}{dt^2} - R_0 \cdot \left[ 1 - \left( \frac{i}{I_0} \right)^2 \right] \cdot \frac{di}{dt} + \frac{1}{C} \cdot i = \begin{cases} 0 \\ \hat{V} \cdot \omega \cdot \sin(\omega \cdot t) \end{cases}$$

$$\omega_0 = \frac{1}{\sqrt{L \cdot C}}$$

$$\mu = R_0 \cdot \sqrt{\frac{C}{L}}$$

$$\tau = \omega_0 \cdot t$$

$$x = \frac{i}{I_0}$$

$$A = \frac{\omega \cdot C \cdot \hat{V}}{I_0}$$

$$w = \frac{\omega}{\omega_0}$$

$$\frac{d^2 x}{d\tau^2} - \mu \cdot [1 - x^2] \cdot \frac{dx}{d\tau} + x = \begin{cases} 0 \\ A \cdot \sin(w \cdot \tau) \end{cases}$$

$$\frac{dx}{d\tau} = y$$

$$\frac{dy}{d\tau} = \mu \cdot [1 - x^2] \cdot y - x + \begin{cases} 0 \\ A \cdot \sin(w \cdot \tau) \end{cases}$$

For an implementation as an electronic circuit, the equations have to be scaled to keep the variables within the desired range. This can be compared with calculating per-unit values by dividing by reference values:

$$\begin{aligned}x' &= \frac{x}{k_x} \\y' &= \frac{y}{k_y} \\\tau' &= \frac{\tau}{k_t}\end{aligned}$$

We also have to take into account that the analog multiplier divides by  $V_S$  to avoid overflow of the output. After that, none of the computing block should encounter an overflow.

This leads to the following set of equations:

$$\begin{aligned}\frac{1}{k_t} \cdot \frac{dx'}{d\tau'} &= \frac{k_y}{k_x} \cdot y \\\frac{1}{k_t} \frac{dy'}{d\tau'} &= \mu \cdot \left[ \frac{1}{k_x^2 \cdot V_S} - \frac{x'^2}{V_S} \right] k_x^2 \cdot V_S^2 \cdot \frac{y'}{V_S} - \frac{k_x}{k_y} \cdot x + \frac{1}{k_y} \cdot A \cdot \sin(w \cdot k_t \cdot \tau')\end{aligned}$$

These equations can easily get implemented as blocks or as an electronic circuit.

Calculating back from per-unit-parameters\_

$$\mu = 0.2, w = 1.15, A = [0..1]$$

and some assumptions:

$$C = 1 \text{ mF}, \omega_0 = 1000 \frac{\text{rad}}{\text{s}}, I_0 = 1 \text{ A}$$

we obtain physical parameters:

$$\begin{aligned}L &= \frac{1}{\omega_0^2 \cdot C} = 1 \text{ mH} \\R_0 &= \mu \cdot \sqrt{\frac{L}{C}} = \frac{\mu}{\omega_0 \cdot C} = 0.2 \Omega \\\hat{V} &= A \cdot \frac{I_0}{w \cdot \omega_0 \cdot C} = \left[ 0 \dots \frac{5}{6} \right] \text{ V}\end{aligned}$$

Investigating correlation between physical and scaled values:

$$\begin{aligned}x &= \frac{i}{I_0} \\y &= \frac{dx}{d\tau} = \frac{1}{\omega_0 \cdot I_0} \cdot \frac{di}{dt} \\i &= C \cdot \frac{dv_C}{dt} \rightarrow v_C = v_{C0} + \frac{1}{C} \cdot \int_{t0}^t i \cdot dt' \\z &= v_C \cdot \frac{\omega_0 \cdot C}{I_0}\end{aligned}$$

Investigating the nonlinear resistance of the triode:

$$v_R = -R_0 \cdot I_0 \cdot \left[ \left( \frac{i}{I_0} \right) - \frac{1}{3} \cdot \left( \frac{i}{I_0} \right)^3 \right] = -R_0 \cdot i \cdot \left[ 1 - \frac{1}{3} \cdot \left( \frac{i}{I_0} \right)^2 \right]$$

$$\frac{v_R}{i} = -R_0 \cdot \left[ 1 - \frac{1}{3} \cdot \left( \frac{i}{I_0} \right)^2 \right]$$

$$\frac{dv_R}{di} = -R_0 \cdot \left[ 1 - \left( \frac{i}{I_0} \right)^2 \right]$$

$$x = \frac{i}{I_0}$$

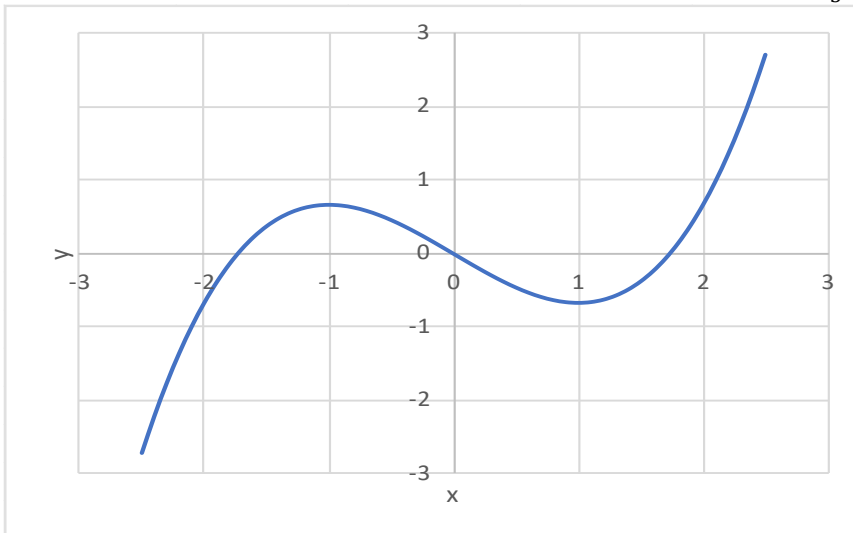
$$y = \frac{v_R}{R_0 \cdot I_0}$$

$$y = - \left( x - \frac{x^3}{3} \right) = -x \cdot \left( 1 - \frac{x^2}{3} \right)$$

$$\frac{y}{x} = - \left( 1 - \frac{x^2}{3} \right)$$

$$\frac{dy}{dx} = -(1 - x^2)$$

Characteristic of the triode  $y(x)$  with extrema at  $x = \pm 1$  with  $y = \pm \frac{2}{3}$ :



If current  $x$  is prescribed, voltage  $y$  can be unambiguously determined.

If  $y$  is prescribed, in the range  $-2 \leq x \leq +2$  i.e.  $-\frac{2}{3} \leq y \leq +\frac{2}{3}$   $x$  has 2 or 3 possible solutions.

For this application, this restriction has no influence.

### 3 Lorenz System

[http://en.wikipedia.org/wiki/Lorenz\\_attractor](http://en.wikipedia.org/wiki/Lorenz_attractor)

Developed 1963 by Edward Lorenz to model atmospheric convection.

$x$  is proportional to the rate of convection,  $y$  to the horizontal temperature variation and  $z$  to the vertical temperature variation.  $\sigma$  depicts the Prandtl number,  $\rho$  the Rayleigh number and  $\beta$  the physical dimensions.

The original parameters were:  $\sigma = 10, \rho = 28, \beta = \frac{8}{3}$

$$\begin{aligned}\frac{dx}{dt} &= \sigma \cdot (y - x) \\ \frac{dy}{dt} &= x \cdot (\rho - z) - y \\ \frac{dz}{dt} &= x \cdot y - \beta \cdot z\end{aligned}$$

$\beta = \frac{1}{3}$  leads to a periodic solution.

For an implementation as an electronic circuit, the equations have to be scaled to keep the variables within the desired range. This can be compared with calculating per-unit values by dividing by reference values:

$$\begin{aligned}x' &= \frac{x}{k_x} \\ y' &= \frac{y}{k_y} \\ z' &= \frac{z}{k_z} \\ t' &= \frac{t}{\tau}\end{aligned}$$

We also have to take into account that the analog multiplier divides by  $V_S$  to avoid overflow of the output.

After that, none of the computing block should encounter an overflow.

This leads to the following set of equations:

$$\begin{aligned}\frac{1}{\tau} \cdot \frac{dx'}{dt'} &= -\sigma \cdot x' + \sigma \cdot \frac{k_y}{k_x} \cdot y' \\ \frac{1}{\tau} \cdot \frac{dy'}{dt'} &= \rho \cdot \frac{k_x}{k_y} \cdot x' - y' - \frac{k_x \cdot k_z \cdot V_S}{k_y} \cdot \frac{x' \cdot z'}{V_S} \\ \frac{1}{\tau} \cdot \frac{dz'}{dt'} &= \frac{k_x \cdot k_y \cdot V_S}{k_z} \cdot \frac{x' \cdot y'}{V_S} - \beta \cdot z'\end{aligned}$$

These equations can easily get implemented as blocks or as an electronic circuit.

#### 4 Roessler System

[https://en.wikipedia.org/wiki/R%C3%B6ssler\\_attractor](https://en.wikipedia.org/wiki/R%C3%B6ssler_attractor)

A simple system of 3 ordinary nonlinear differential equations to study chaos without physical background.

$$\begin{aligned}\frac{dx}{dt} &= -y - z \\ \frac{dy}{dt} &= x + a \cdot y \\ \frac{dz}{dt} &= b + (x - c) \cdot z\end{aligned}$$

$a = 0.2$ ,  $b = 0.2$  and  $c = 1$  give periodic results. Changing  $c = 5.7$  reveals chaotic results.

For an implementation as an electronic circuit, the equations have to be scaled to keep the variables within the desired range. This can be compared with calculating per-unit values by dividing by reference values:

$$\begin{aligned}x' &= \frac{x}{k_x} \\ y' &= \frac{y}{k_y} \\ z' &= \frac{z}{k_z} \\ t' &= \frac{t}{\tau}\end{aligned}$$

We also have to take into account that the analog multiplier divides by  $V_S$  to avoid overflow of the output. After that, none of the computing block should encounter an overflow.

This leads to the following set of equations:

$$\begin{aligned}\frac{1}{\tau} \cdot \frac{dx'}{dt} &= -\frac{k_y}{k_x} \cdot y' - \frac{k_z}{k_x} \cdot z' \\ \frac{1}{\tau} \cdot \frac{dy'}{dt} &= \frac{k_x}{k_y} \cdot x' + a \cdot y' \\ \frac{1}{\tau} \cdot \frac{dz'}{dt} &= \frac{b}{k_z} + k_x \cdot V_S \cdot \frac{x' \cdot z'}{V_S} - c \cdot z'\end{aligned}$$

These equations can easily get implemented as blocks or as an electronic circuit.

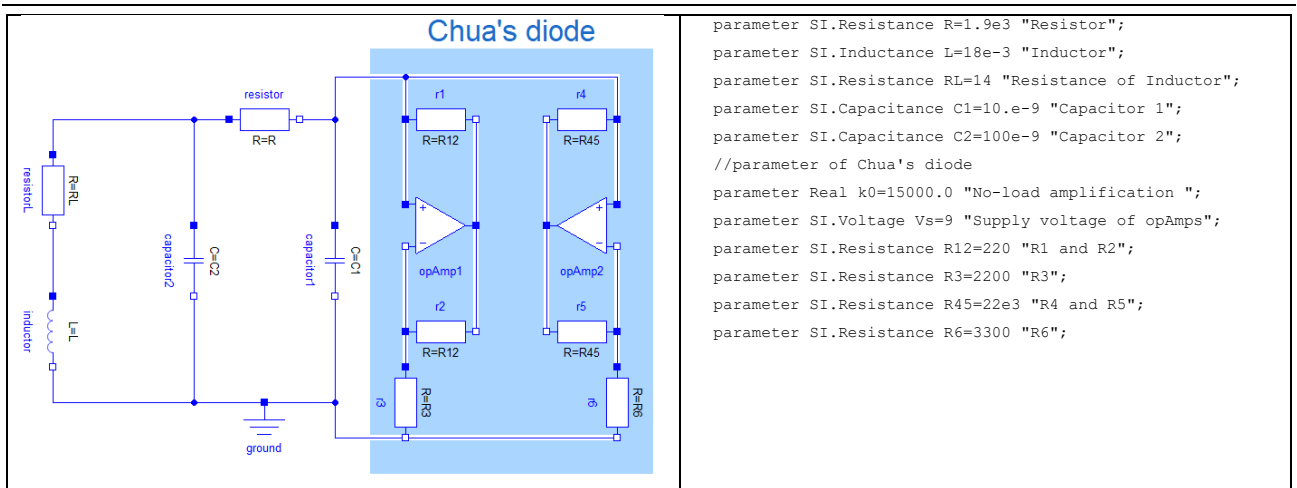
## 5 Chua's Circuit

<https://link.springer.com/book/10.1007/978-3-319-05900-6> (1.1)

<https://nonlinear.eecs.berkeley.edu/chaos/chaos.html#> Working With Chaos Simulation

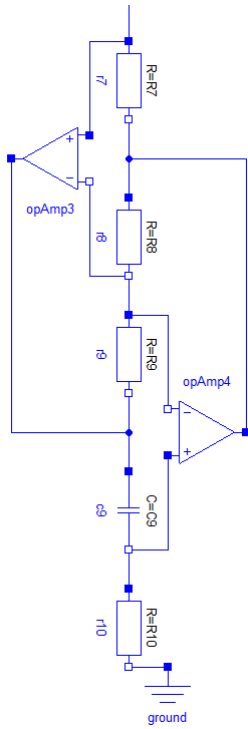
$$\begin{aligned}
 L \cdot \frac{di_L}{dt} &= v_2 - R_L \cdot i_L \\
 C_2 \cdot \frac{dv_2}{dt} &= -i_L - \frac{v_2 - v_1}{R} \\
 C_1 \cdot \frac{dv_1}{dt} &= -i_{NL} + \frac{v_2 - v_1}{R} \\
 -i_{NL}(v_1) &= \begin{cases} -\infty < v_1 < -V_e \rightarrow G_b \cdot (v_1 + V_e) - G_a \cdot V_e \\ -V_e < v_1 < +V_e \rightarrow G_a \cdot v_1 \\ +V_e < v_1 < +\infty \rightarrow G_b \cdot (v_1 - V_e) + G_a \cdot V_e \end{cases} \\
 -\frac{i_{NL}}{v_1} &= \begin{cases} -\infty < v_1 < -V_e \rightarrow G_b - (G_a - G_b) \cdot \frac{V_e}{v_1} \\ -V_e < v_1 < +V_e \rightarrow G_a \\ +V_e < v_1 < +\infty \rightarrow G_b + (G_a - G_b) \cdot \frac{V_e}{v_1} \end{cases} \\
 -\frac{di_{NL}}{dv_1} &= \begin{cases} -\infty < v_1 < -V_e \rightarrow G_b \\ -V_e < v_1 < +V_e \rightarrow G_a \\ +V_e < v_1 < +\infty \rightarrow G_b \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 \tau_L \cdot \dot{v}_{RL} &= v_2 - v_{RL} \\
 \tau_2 \cdot \dot{v}_2 &= +v_1 - v_2 - \frac{R}{R_L} \cdot v_{RL} \\
 \tau_1 \cdot \dot{v}_1 &= -v_1 + v_2 + R \cdot g \cdot v_1 \\
 g(v_1) &= \begin{cases} |v_1| > V_e \rightarrow G_b + (G_a - G_b) \cdot \frac{V_e}{|v_1|} \\ |v_1| < V_e \rightarrow G_a \end{cases}
 \end{aligned}$$





## Chua's Circuit: Inductor Replacement



$$R_7 = 100 \, \Omega$$

$$R_8 = 1 \, k\Omega$$

$$R_9 = 1 \, k\Omega$$

$$R_{10} = 1,8 \, k\Omega$$

$$C_9 = 100 \, nF$$

$$R_7 \cdot i + R_8 \cdot (i + i_{OA4}) = 0$$

$$R_9 \cdot (i + i_{OA4}) + v_{C9} = 0$$

$$i + i_{OA4} + i_{OA3} = i_{C9}$$

$$i_{C9} = C_9 \cdot \frac{dv_{C9}}{dt}$$

$$v = R_{10} \cdot i_{C9}$$

$$(i + i_{OA4}) = -\frac{R_7}{R_8} \cdot i$$

$$v_{C9} = \frac{R_7 \cdot R_9}{R_8} \cdot i$$

$$i_{C9} = C_9 \cdot \frac{R_7 \cdot R_9}{R_8} \cdot \frac{di}{dt}$$

$$v = C_9 \cdot \frac{R_7 \cdot R_9 \cdot R_{10}}{R_8} \cdot \frac{di}{dt}$$

$$L = C_9 \cdot \frac{R_7 \cdot R_9 \cdot R_{10}}{R_8} = 18 \, mH$$

## 6 Chaotic Diode Circuit

[https://www.researchgate.net/publication/309351711\\_A\\_simple\\_chaotic\\_circuit\\_with\\_a\\_light-emitting\\_diode](https://www.researchgate.net/publication/309351711_A_simple_chaotic_circuit_with_a_light-emitting_diode)

$$\begin{aligned}C \cdot \frac{dv_1}{dt} &= -\frac{v_2}{R} \\C \cdot \frac{dv_2}{dt} &= -\frac{v_3}{R} \\C \cdot \frac{dv_3}{dt} &= -\frac{v_1}{R} - \frac{v_3}{R_b} - \frac{v_4}{R} \\ \frac{v_4}{R_a} &= -I_{ds} \cdot \left( e^{\frac{v_2}{nV_t}} - 1 \right)\end{aligned}$$

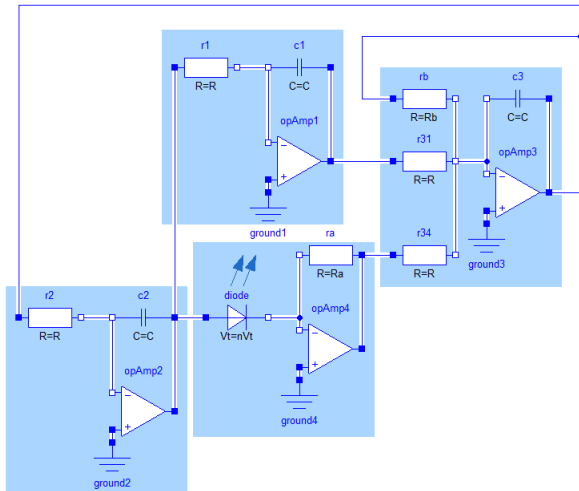
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$$\begin{aligned}\tau &= R \cdot C \\ \tau \cdot \frac{dv_1}{dt} &= -v_2 \\ \tau \cdot \frac{dv_2}{dt} &= -v_3 \\ \tau \cdot \frac{dv_3}{dt} &= -v_1 - \frac{R}{R_b} \cdot v_3 + R_a \cdot I_{ds} \cdot \left( e^{\frac{v_2}{nV_t}} - 1 \right)\end{aligned}$$


---

$$\begin{aligned}a &= \frac{R_a \cdot I_{ds}}{nV_t} \\ b &= \frac{R}{R_b} \\ \tau \cdot \dot{x}_1 &= -x_2 \\ \tau \cdot \dot{x}_2 &= -x_3 \\ \tau \cdot \dot{x}_3 &= -x_1 + a \cdot (e^{x_2} - 1) - b \cdot x_3\end{aligned}$$


---



```
parameter SI.Current Ids=1e-9 "Diode: saturation current";
parameter SI.Voltage nVt=2*26e-3
" n*voltage equ. of temperature";
//time constant and dimensionless parameters
parameter SI.Time Tau=0.1e-3 "Time constant";
parameter Real a=1/5200 "Parameter a = Ra*Ids/nVt";
parameter Real b=1.5 "Parameter b = R/Rb";
//configuration of the opAmps
parameter SI.Capacitance C=10e-9
"Capacitance of {c1, c2, c3}";
parameter SI.Resistance R=Tau/C "Resistance of {r1,r2,r3,r4}";
parameter SI.Resistance Ra=a*nVt/Ids "Resistance of ra";
parameter SI.Resistance Rb=R/b "Resistance of rb";
```

## 7 Chaotic Oscillator

[https://www.researchgate.net/publication/230925506\\_A\\_simple\\_chaotic\\_oscillator\\_for\\_educational\\_purposes](https://www.researchgate.net/publication/230925506_A_simple_chaotic_oscillator_for_educational_purposes)

[https://www.researchgate.net/publication/259216097\\_NUMERICAL\\_TREATMENT\\_OF\\_EDUCATIONAL\\_CHAOS\\_OSCILLATOR](https://www.researchgate.net/publication/259216097_NUMERICAL_TREATMENT_OF_EDUCATIONAL_CHAOS_OSCILLATOR)

$$i_L = C \cdot \frac{dv_C}{dt}$$

$$L \cdot \frac{di_L}{dt} = \left(k - 1 - \frac{R_L}{R}\right) \cdot R \cdot i_L - v_C - v_C^*$$

$$k = 1 + \frac{R_2}{R_1}$$

$$C^* \cdot \frac{dv_{C^*}}{dt} = I_0 + i_L - I_{DS} \cdot \left(e^{\frac{v_{C^*}}{nV_t}} - 1\right)$$

$$I_0 \approx \frac{V_b}{R_0}$$

$$\tau = \sqrt{L \cdot C}$$

$$Z = \sqrt{\frac{L}{C}}$$

$$a = \left(k - 1 - \frac{R_L}{R}\right) \cdot \frac{R}{Z}$$

$$b = \frac{Z \cdot I_0}{nV_t}$$

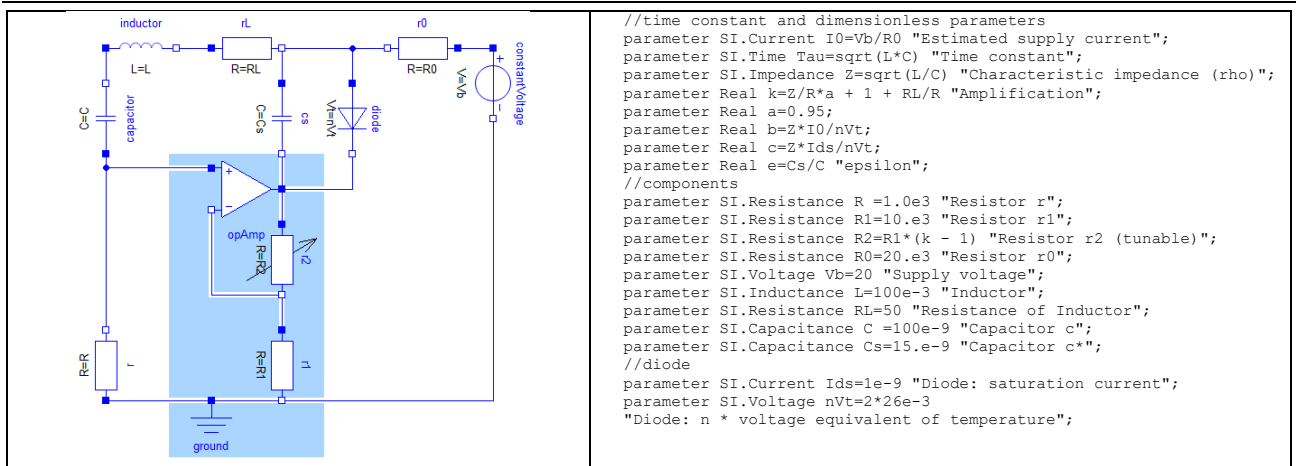
$$c = \frac{Z \cdot I_{DS}}{nV_t}$$

$$e = \frac{C^*}{C}$$

$$\tau \cdot \frac{\dot{v}_C}{nV_t} = \frac{Z \cdot i_L}{nV_t}$$

$$\tau \cdot \frac{Z \cdot i_L}{nV_t} = \left(k - 1 - \frac{R_L}{R}\right) \cdot \frac{R}{Z} \cdot \frac{Z \cdot i_L}{nV_t} - \frac{v_C}{nV_t} - \frac{v_{C^*}}{nV_t}$$

$$\tau \cdot e \cdot \frac{\dot{v}_{C^*}}{nV_t} = \frac{Z \cdot I_0}{nV_t} + \frac{Z \cdot i_L}{nV_t} - \frac{Z \cdot I_{DS}}{nV_t} \cdot \left(e^{\frac{v_{C^*}}{nV_t}} - 1\right)$$



## 8 Colpitts Oscillator

<https://link.springer.com/book/10.1007/978-3-319-05900-6> (1.3)

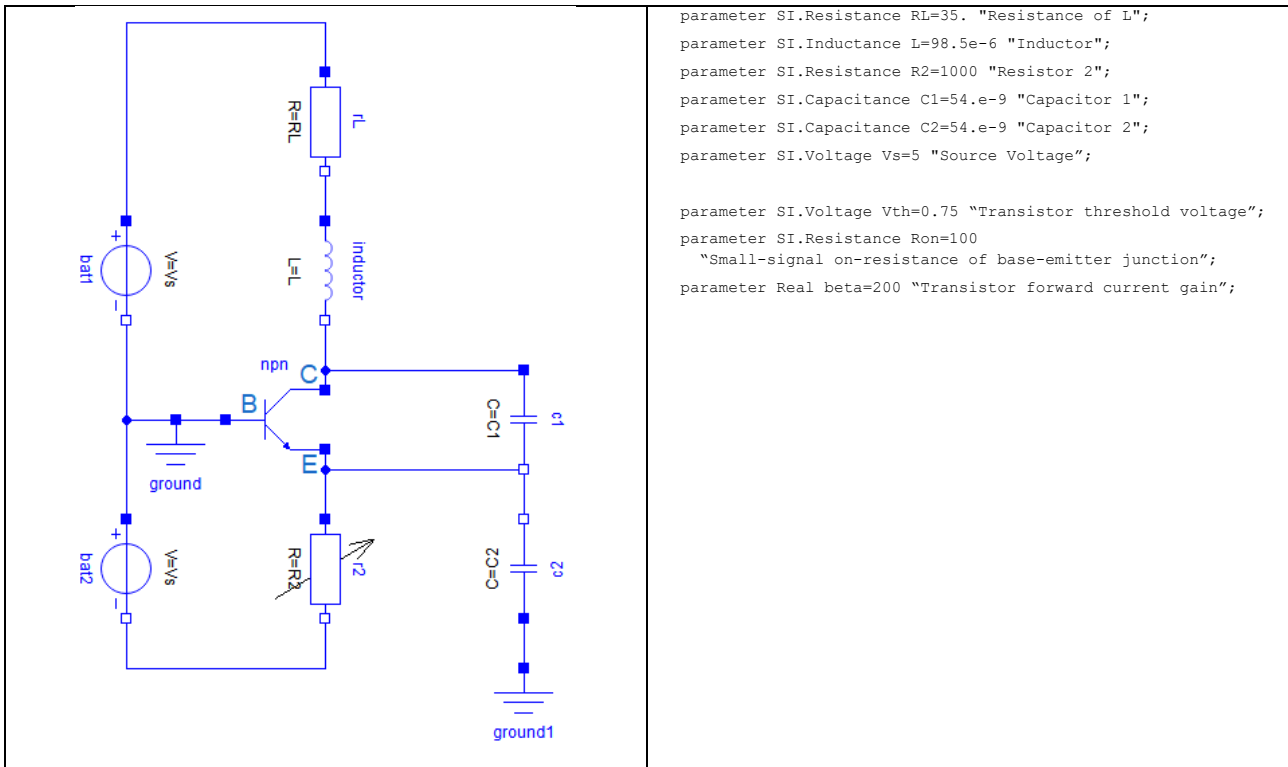
<https://ieeexplore.ieee.org/document/331536>

$$C_1 \cdot \frac{dv_1}{dt} = i_L - \beta \cdot i_B$$

$$C_2 \cdot \frac{dv_2}{dt} = -\frac{V_{s-} + v_2}{R_2} - i_L - i_B$$

$$L \cdot \frac{di_L}{dt} = V_{s+} - v_1 + v_2 - R_L \cdot i_L$$

$$i_B = \begin{cases} v_2 = v_{BE} \leq V_{th} \rightarrow 0 \\ v_2 = v_{BE} > V_{th} \rightarrow \frac{v_2 - V_{th}}{R_{on}} \end{cases}$$



## 9 Shinriki Oscillator

[https://pawp.physik.uni-wuerzburg.de/~slueck/PhyAmSa09/Home\\_files/Examensarbeit\\_Lueck.pdf](https://pawp.physik.uni-wuerzburg.de/~slueck/PhyAmSa09/Home_files/Examensarbeit_Lueck.pdf)

<https://ieeexplore.ieee.org/abstract/document/1456241>

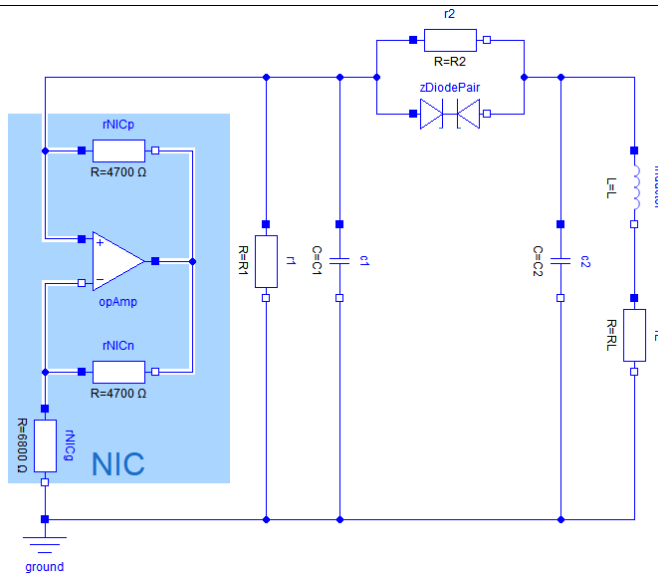
$$i_z = \begin{cases} |v_z| < V_{bt} & 0 \\ |v_z| \geq V_{bt} & \text{sign}(v_z) \cdot [a \cdot (|v_z| - V_{bt}) + b \cdot (|v_z| - V_{bt})^3 + c \cdot (|v_z| - V_{bt})^5] \end{cases}$$

$$C_1 \cdot \frac{dv_1}{dt} = -i_{NIC} - \frac{v_1}{R_1} - i_z$$

$$C_2 \cdot \frac{dv_2}{dt} = i_z - i_L$$

$$v_2 = L \cdot \frac{di_L}{dt} + R_L \cdot i_L$$

$$g_{NIC} = \frac{di_{NIC}}{dv_{NIC}} = \begin{cases} |v_{NIC}| > V_{Lim} & g_+ \\ |v_{NIC}| \leq V_{Lim} & g_- \end{cases}$$



```
parameter SI.Inductance L=320e-3 "Inductor";
parameter SI.Resistance RL=100. "Resistor of L";
parameter SI.Resistance R1=60e3 "Resistor 1";
parameter SI.Resistance R2=20e3 "Resistor 2";
parameter SI.Capacitance C1=10.e-9 "Capacitor 1";
parameter SI.Capacitance C2=100e-9 "Capacitor 2";
```

$$V_{bt} = 3.3 V$$

$$a = 1,0862 \frac{mA}{V}$$

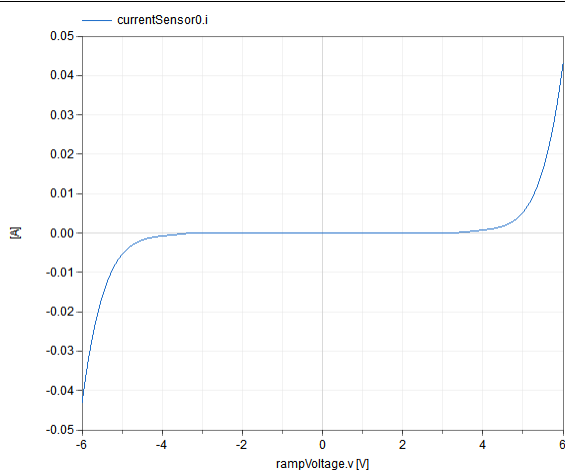
$$b = -0,1615 \frac{mA}{V^3}$$

$$c = 0,3021 \frac{mA}{V^5}$$

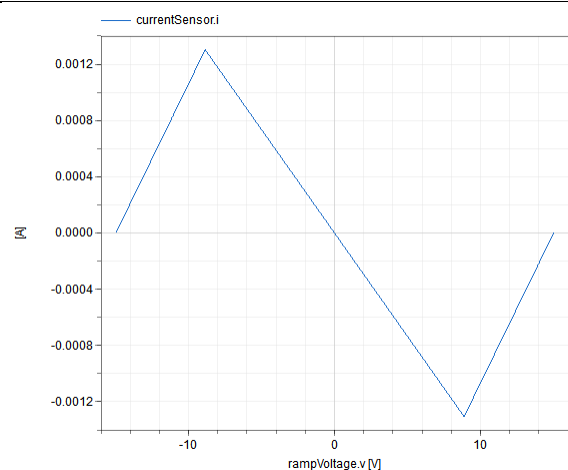
$$V_{Lim} = V_s \cdot \frac{6800}{4700 + 6800}$$

$$g_+ = + \frac{1000}{4700} mS$$

$$g_- = - \frac{1000}{6800} mS$$



Characteristic of the Zener diode pair



Characteristic of the NIC  
(negative impedance converter)

Investigation of the Zener diode pair approximation:

$$i_z = \begin{cases} 0 & |v_z| < V_{bt} \\ \text{sign}(v_z) \cdot [a \cdot (|v_z| - V_{bt}) + b \cdot (|v_z| - V_{bt})^3 + c \cdot (|v_z| - V_{bt})^5] & |v_z| \geq V_{bt} \end{cases}$$

$$\frac{di_z}{dv_z} = \begin{cases} 0 & |v_z| < V_{bt} \\ \text{sign}(v_z) \cdot [a + 3b \cdot (|v_z| - V_{bt})^2 + 5c \cdot (|v_z| - V_{bt})^4] & |v_z| \geq V_{bt} \end{cases}$$

The first derivative  $\frac{di_z}{dv_z}$  is not continuous at  $|v_z| = V_{bt}$ .

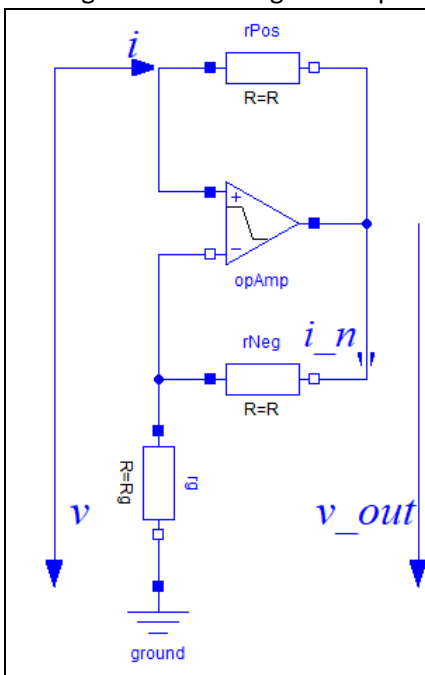
If voltage  $v$  is prescribed, the current  $i$  can be unambiguously determined.

If current  $i$  is prescribed, between  $-V_{bt}$  and  $+V_{bt}$  there is a manifold of solutions for the voltage  $v$ .

For this application, this restriction has no influence.

This restriction could be solved by adaption the approximation, i.e. exchange the horizontal line in the range  $-V_{bt} < v_z < +V_{bt}$  against a characteristic with small constant positive slope and adapt the polynomial approximation to achieve a one times continuously differentiable characteristic.

Investigation of the negative impedance converter (NIC):



As long as the opAmp operates in the linear region:

$$i_n = \frac{v_{out} - v}{R} = \frac{v}{R_g} \rightarrow v_{out} = v \cdot \frac{R + R_g}{R_g}$$

$$i = \frac{v - v_{out}}{R} = -\frac{v}{R_g}$$

$$g_- = \frac{1}{R_g}$$

When the opAmp's output saturates:

$$V_{Lim} = V_s \cdot \frac{R_g}{R + R_g}$$

$$v \geq +V_{Lim}: i = \frac{v - V_{Lim}}{R}$$

$$g_+ = \frac{1}{R}$$

If voltage  $v$  is prescribed, the current  $i$  can be unambiguously determined.

If current  $i$  is prescribed, in the range between the zero crossings the voltage  $v$  has 3 possible solutions.

For this application, this restriction has no influence.

## 10 Jerk Circuit

<https://link.springer.com/book/10.1007/978-3-319-05900-6> (3.1)

<https://sprott.physics.wisc.edu/pubs/paper352.pdf>

The name of the system stems from the third derivative of  $x$ , which – in a mechanical system – is the derivative of acceleration called jerk.

$$\ddot{x} + A \cdot \ddot{x} + f(\dot{x}) + x = 0$$

$$f(\dot{x}) = R \cdot I_S \cdot \left( e^{\frac{\dot{x}}{nV_t}} - 1 \right)$$

$$\dot{x} = y$$

$$\ddot{x} = \dot{y} = z$$

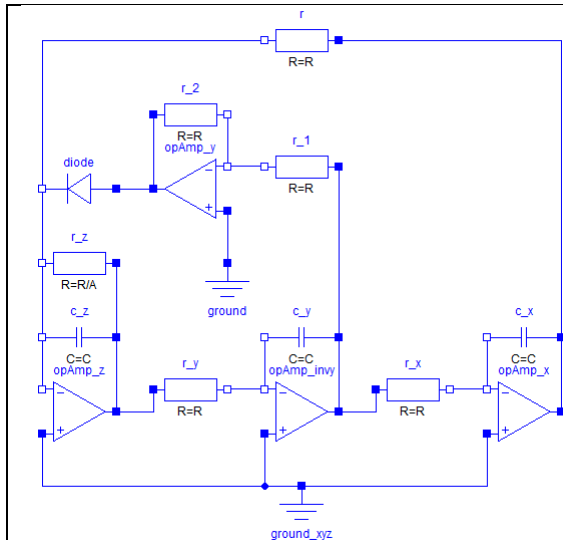
$$\ddot{x} = \dot{y} = \dot{z}$$

$$\dot{z} = -A \cdot z - x - f(y)$$

The values stay pretty inside a practicable range for a normal voltage supply.

Using 4 operational amplifiers and an acceleration factor of 1000, the circuit can be implemented as follows.

The parameter  $A$  influences only the feedback resistor at opAmp\_z:



```
parameter Real A=0.3 "Parameter to be varied";
parameter SI.Resistance R=1e3 "Resistance";
parameter SI.Capacitance C=1e-6 "Capacitance";
parameter SI.Current Ids=1e-12 "Sat.current";
parameter SI.Voltage nVt=26e-3 " voltage equ.";
```

$A=0.3$  for periodic results

$A=1.0$  for chaotic results