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A simple chaotic oscillator for educational purposes

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Abstract
A novel, very simple chaotic oscillator is described. It is intended for
training laboratories accompanying courses on nonlinear dynamics and chaos
for undergraduate, postgraduate and PhD students. The oscillator consists of
an operational amplifier, an LCR resonance loop, an extra capacitor, a diode
as a nonlinear element and three auxiliary resistors. Chaotic oscillations are
demonstrated both experimentally and numerically.

Electronic oscillators generating chaotic waveforms are the most convenient tools for practical
training of students taking courses on nonlinear dynamics and chaos [1–6]. From a didactical
point of view, the oscillator should not be higher than a third-order system and preferably
autonomous. It should contain a single, simply defined and common nonlinear unit. Smooth,
monotonous and unambiguous nonlinear functions are preferred to piecewise linear, non-
monotonous and ambiguous ones. From a technical point of view the circuit should contain
as few elements as possible. All the devices should be commercially available and cheap.
The circuit should be easy to build and tune up. The oscillator should operate at kilohertz
frequencies to simplify the measuring procedures.

A number of chaotic oscillators have been described in the literature (see, e.g. [7]). A
systematic and didactic way to design a third-order autonomous chaotic oscillator is to start
with a second-order normal sinusoidal oscillator, either an LC or Wien-bridge type. The next
step is to insert an additional energy storage element, say an inductor L^* or capacitor C^*, and
an appropriate nonlinear device, e.g. a diode. In this paper we suggest a novel, very simple,
apparently simplest third-order autonomous chaotic oscillator. The oscillator is shown in
figure 1. It includes a non-inverting amplifier and a series LCR resonance circuit in a positive
feedback loop. In addition, an extra capacitor C^* and a diode D are inserted in the circuit.

In an experimental prototype, the circuit parameters were the following: $L = 100$ mH,
$C = 100$ nF ($\rho = \sqrt{L/C} = 1$ kΩ, fundamental frequency $f^* \approx 1/2\pi\sqrt{LC} \approx 1.6$ kHz),
$C^* = 15$ nF, $R = 1$ kΩ, $R_1 = R_2 = 10$ kΩ (R_2 variable resistor), and $R_0 = 20$ kΩ. The gain of
the amplifier $k = R_2/R_1 + 1$. The variation of R_2 from 0 to 10 kΩ allows one to tune k from
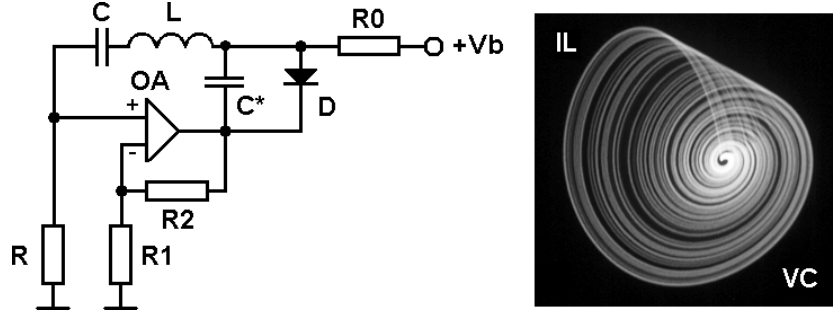
Figure 1. Circuit diagram of the chaotic oscillator (left) and phase portrait (I_L versus V_C) (right). Gain $k \approx 1.4$. The diode is dc-forward-biased via resistor R_0: $V_b = 20$ V ($I_0 \approx 1$ mA).

1 to 2. A common LM741 type operational amplifier was employed and a 1N4148 type general purpose diode was used.

There are three dynamical variables in the oscillator: V_C, I_L and V_{C^*}, i.e. the voltage across the capacitor C, the current through the inductor L, and the voltage across the capacitor C^* respectively. The variable I_L can be taken either as a voltage drop across the resistor R or as the output signal from the amplifier OA. Since the capacitors C and C^* are not grounded the variables V_C and V_{C^*} should be taken by means of an oscilloscope with a differential input. Two-dimensional projections (phase portraits) of the three-dimensional phase trajectory (I_L, V_C, V_{C^*}), specifically (I_L versus V_C), (I_L versus V_{C^*}) and (V_{C^*} versus V_C) can easily be displayed on the screen of an oscilloscope. One of them, namely (I_L versus V_C) is presented in figure 1 (right) to illustrate the chaotic performance of the oscillator.

Applying the Kirchhoff laws one comes to the following set of three ordinary differential equations describing the dynamics of the oscillator:

$$C \, dV_C/dt = I_L, \qquad L \, dI_L/dt = (k-1)RI_L - V_C - V_{C^*}, \qquad C^* \, dV_{C^*}/dt = I_0 + I_L - I_D. \tag{1}$$

Here $I_0 \approx V_b/R_0$ ($R_0 \gg R, \rho$) is the dc bias current and $I_D = f(V_{C^*})$ is the nonlinear current–voltage characteristic of the diode:

$$I_D = I_S(\exp(eV_D/k_B T) - 1), \tag{2}$$

where I_S and V_D is the saturation current and the voltage across the diode, respectively (note that due to parallel connection $V_D = V_{C^*}$); e is the electron charge, k_B is the Boltzmann constant and T is the temperature. By introducing the dimensionless variables and parameters

$$x = \frac{V_C}{V_T}, \qquad y = \frac{\rho I_L}{V_T}, \qquad z = \frac{V_{C^*}}{V_T}, \qquad \theta = \frac{t}{\tau},$$

$$V_T = \frac{k_B T}{e}, \qquad \rho = \sqrt{\frac{L}{C}}, \qquad \tau = \sqrt{LC}, \qquad a = (k-1)\frac{R}{\rho}, \tag{3}$$

$$b = \frac{\rho I_0}{V_T}, \qquad c = \frac{\rho I_S}{V_T}, \qquad \varepsilon = \frac{C^*}{C},$$

equations convenient for analysis and numerical simulation are obtained:

$$\dot{x} = y, \qquad \dot{y} = ay - x - z, \qquad \varepsilon \dot{z} = b + y - c(\exp z - 1). \tag{4}$$

Here the dots above the variables denote their derivatives over θ. For certain sets of the control parameters a, b and ε (normally $c \ll 1$, therefore the oscillations are insensitive to its value)
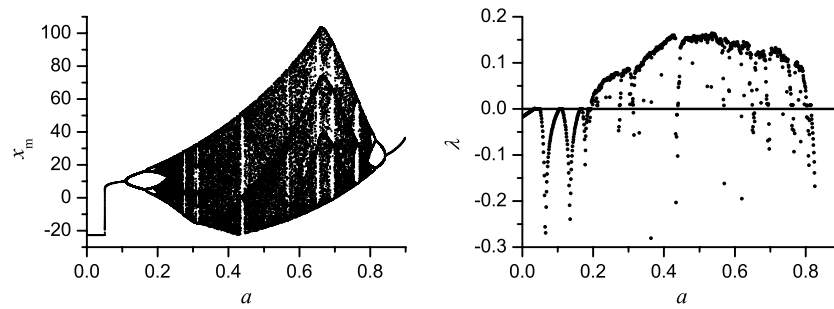
Figure 2. Bifurcation diagram from equation (4), i.e. the local maxima x_m of the variable $x(t)$ versus control parameter a (left) and leading Lyapunov exponent λ as a function of control parameter a (right). Other parameters are fixed at $b = 30$, $\varepsilon = 0.13$ and $c = 4 \times 10^{-9}$.

the system generates chaotic oscillations as illustrated in figure 2 with the bifurcation diagram and Lyapunov exponent. Positive values of the latter confirm the chaotic behaviour of the oscillator. One can see that the oscillator exhibits a universal period-doubling route to chaos, i.e. a scenario typical of many nonlinear systems. There are also narrow periodic windows in the chaotic domain and reverse bifurcations at higher values of the parameter a.

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