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NUMERICAL TREATMENT OF EDUCATIONAL CHAOS OSCILLATOR

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A mathematical model of a recently suggested chaos oscillator for educational purposes is treated and numerical results are presented. Bifurcation diagrams, phase portraits, power spectra, Lyapunov exponents are simulated. In addition, the Feigenbaum number is estimated.

Keywords: Chaos; electrical circuits; educational aids.

1. Introduction

Electric circuits generating chaotic waveforms are the most convenient training aids accompanying courses on *Nonlinear Dynamics and Chaos* [Moon, 1987; Ott, 1993; Strogatz, 1994; Nicolis, 1995; Alligood *et al.*, 2000; Hilborn, 2001; Sprott, 2001]. From a didactical and technical point of view the educational electric circuit to our opinion should have the following features:

- The architecture of chaos oscillator should be similar to classical second order sine wave oscillator, e.g. LC or Wien-bridge based circuit.
- The oscillator should not be higher than the third order system.
- The circuit should be an autonomous oscillator.
- The active unit should be linear and separated from the nonlinear element(s).
- The nonlinear element should be single and simple, e.g. a general-purpose diode.
- The current–voltage characteristic (or transfer function) of the nonlinear element should be continuous, smooth, unambiguous and monotone function.

- The circuit components should be cheap and commercially available.
- The circuit should be easy to build and tune up in a laboratory.
- The circuit should operate at kHz frequencies to simplify the measuring procedures.
- The mathematical model should be clear and unambiguous.

Commonly for the educational purposes a periodically driven nonlinear oscillator (a nonautonomous circuit) proposed by Linsay [1981] and investigated by Testa *et al.* [1982a] is used [Prusha, 1997; Hilborn, 2001]. The main shortcoming of the Linsay's oscillator is in the fact that the mechanism behind its chaotic behavior is not completely understood. There was a long 20 years lasting and unfinished controversy concerning the mechanism, e.g. [Hunt, 1982; Testa *et al.*, 1982b].

Recently in a short paper [Tamaševičius *et al.*, 2005], we suggested an extremely simple electric circuit meeting all the above-mentioned requirements (Fig. 1).

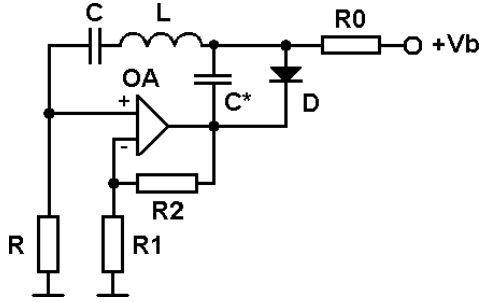


Fig. 1. Circuit diagram of chaotic oscillator.

Later the circuit was studied by means of SPICE simulations [Peters, 2005] and investigated experimentally [Juškevičius *et al.*, 2005]. This circuit is described by a set of three ordinary differential equations [Tamaševičius *et al.*, 2005]:

$$\begin{aligned}\dot{X} &= Y, \\ \dot{Y} &= -X + aY - Z, \\ \varepsilon \dot{Z} &= b + Y - c(\exp Z - 1).\end{aligned}\quad (1)$$

In the present paper we elaborate the mathematical model, and show comprehensive numerical results demonstrating different modes of nonlinear and chaotic oscillations. Students using MATLAB or similar computing tools can easily repeat all the numerical simulations performed in this work. Therefore, in addition to hardware experiments with the circuit in Fig. 1 the developed mathematical model can be suggested as a software part of training aids for students taking courses on *Nonlinear Dynamics and Chaos*.

2. Equations Analysis

Equation (1) has *four* control parameters. Its steady-state solution is the following:

$$X_0 = -Z_0, \quad Y_0 = 0, \quad Z_0 = \ln\left(\frac{b}{c} + 1\right). \quad (2)$$

Let us perform the transformations:

$$X = X_0 + x, \quad Y = Y_0 + y, \quad Z = Z_0 + z. \quad (3)$$

Then after simple algebra Eq. (1) reads as follows:

$$\begin{aligned}\dot{x} &= y, \\ \dot{y} &= -x + ay - z, \\ \varepsilon \dot{z} &= y - (b + c)(\exp z - 1).\end{aligned}\quad (4)$$

We note that parameter c is by *ten* orders smaller than parameter b [Tamaševičius *et al.*, 2005]. Therefore, in Eq. (4) it can be naturally omitted. Moreover, in any case the sum $(b + c)$ can be denoted

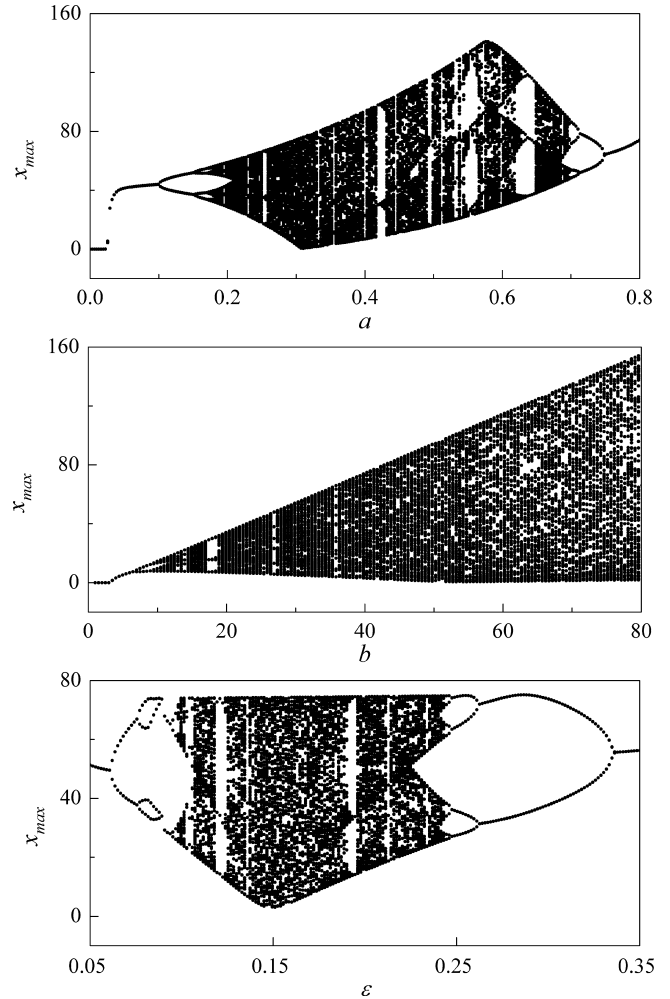


Fig. 2. Local maxima x_{\max} versus different control parameters. Top: x_{\max} versus a at $b = 40$, $\varepsilon = 0.15$; middle: x_{\max} versus b at $a = 0.30$, $\varepsilon = 0.15$; bottom: x_{\max} versus ε at $a = 0.30$, $b = 40$.

as a new parameter b^* . So, we have a compact set of equations with the number of control parameters reduced to only *three*:

$$\begin{aligned}\dot{x} &= y, \\ \dot{y} &= -x + ay - z, \\ \varepsilon \dot{z} &= y - b(\exp z - 1).\end{aligned}\quad (5)$$

3. Bifurcation Diagrams

The dependence of the system behavior on the control parameters is demonstrated in Fig. 2.

4. Phase Portraits

A series of phase portraits presented in Fig. 3 illustrate different oscillatory modes.

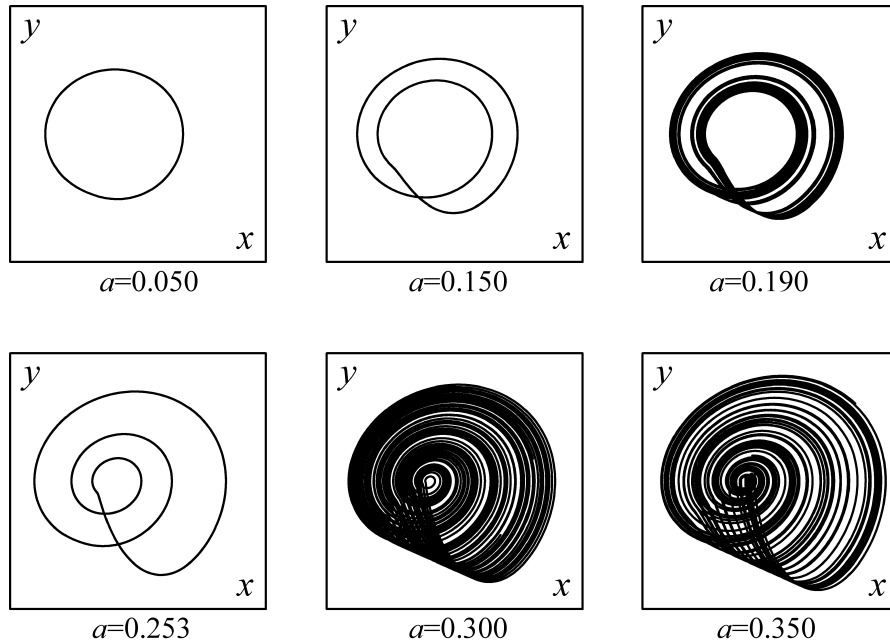


Fig. 3. Phase portraits for different values of control parameter a . Other parameters are fixed at $b = 40$, $\varepsilon = 0.15$.

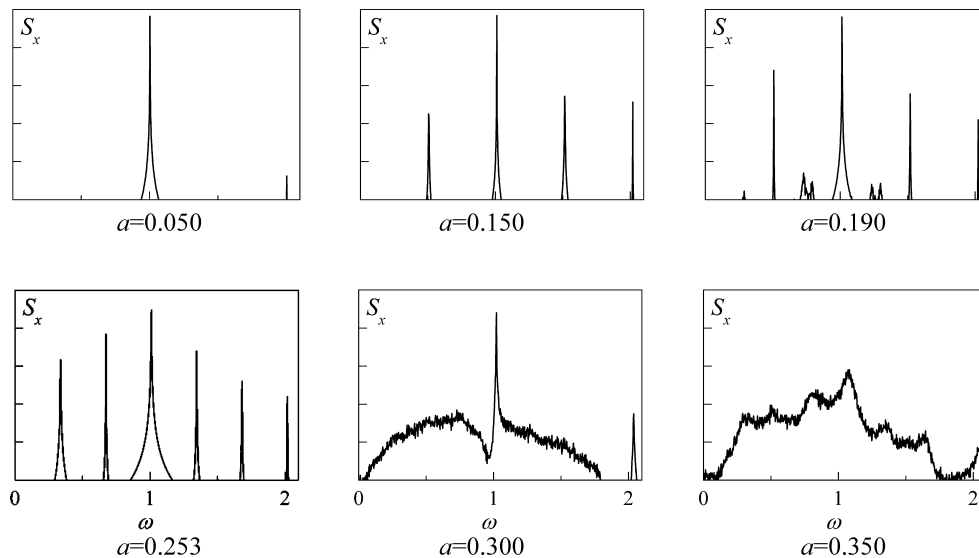


Fig. 4. Power spectra taken from $x(t)$ for different values of control parameter a . Other parameters are fixed at $b = 40$, $\varepsilon = 0.15$. Vertical scale 10 dB/div.

5. Power Spectra

The power spectra shown in Fig. 4 with the increase of control parameter has developed from discrete to continuous format, indicating transition from periodic and chaotic oscillations.

6. Feigenbaum Number and Lyapunov Exponents

In the case of period-doubling route to chaos it is conventional to characterize the bifurcation

sequence by the following ratio:

$$\delta_n = \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}}, \quad (6)$$

where a_n is the control parameter at a critical point of the n th bifurcation. For large n the ratio δ_n tends to a constant value $\delta_F = 4.669\dots$, the so-called Feigenbaum number. One can see from Table 1 that already for $n = 5$ the ratio δ_n is very close to δ_F .

Table 1. Bifurcation parameters.

n	1	2	3	4	5
a_n	0.09920	0.15210	0.17155	0.17670	0.17780
δ_n	—	—	2.72	3.78	4.68

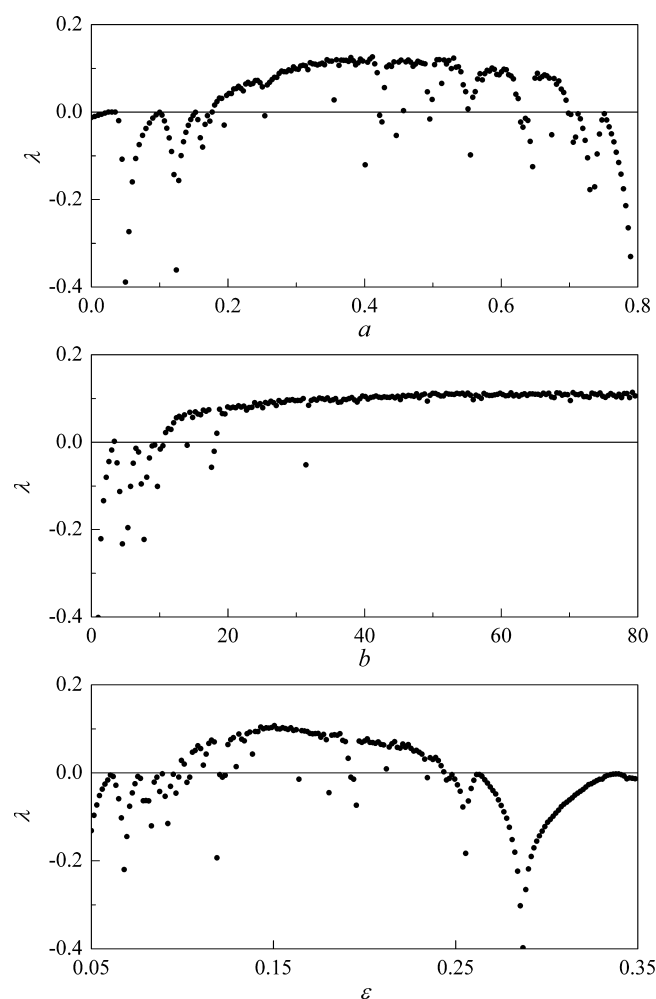


Fig. 5. Leading Lyapunov exponent λ as a function of different control parameters. (Top) $\lambda(a)$ at $b = 40$, $\varepsilon = 0.15$; (middle) $\lambda(b)$ at $a = 0.30$, $\varepsilon = 0.15$; (bottom) $\lambda(\varepsilon)$ at $a = 0.30$, $b = 40$.

Quantitative characterization of chaotic states is presented in Fig. 5 with the Lyapunov exponents. Their positive values do confirm chaotic behavior of the system.

7. Conclusions

A mathematical model describing dynamical behavior of the recently proposed electric circuit

[Tamaševičius et al., 2005] has been considered. Numerical simulations have been performed and a full-scale collection of illustrations including bifurcation diagrams, phase portraits, power spectra, Feigenbaum number and Lyapunov exponents has been presented. Numerical results exhibit all the characteristic features observed experimentally for the hardware prototype [Juškevičius et al., 2005]. Namely, (i) prior to chaotic states the system undergoes period-doubling bifurcations; (ii) narrow periodic windows can be detected in a chaotic domain; (iii) even for relatively small cascade numbers, say $n = 5$ the universal Feigenbaum number $\delta_F = 4.669 \dots$ can be estimated.

Acknowledgment

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