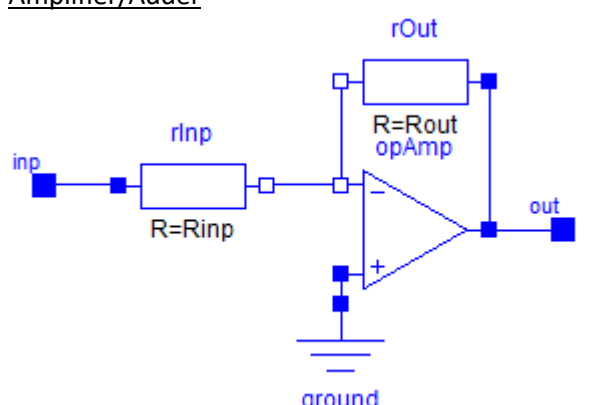
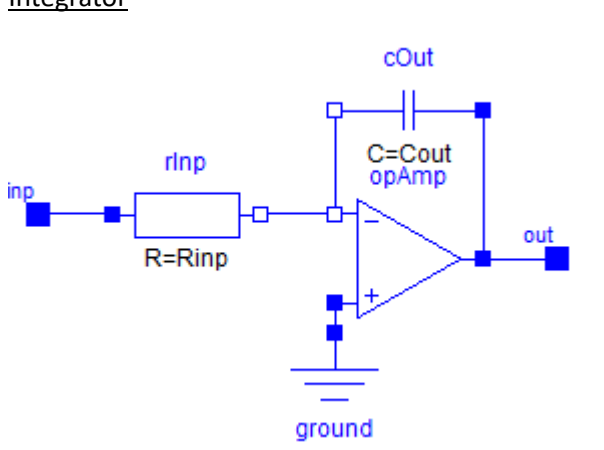
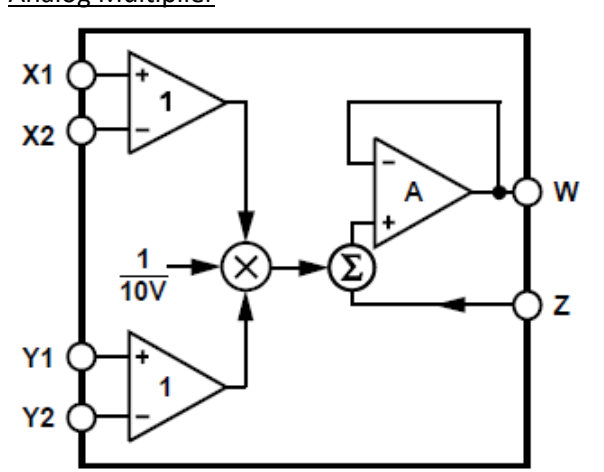


OpAmp-Circuits

<p><u>Amplifier/Adder</u></p> 	$\frac{inp.v}{R_{inp}} + \frac{out.v}{R_{out}} = 0$ $-out.v = k \cdot inp.v$ $k = \frac{R_{out}}{R_{inp}}$ <p>It is possible to add several inputs.</p>
<p><u>Integrator</u></p> 	$\frac{inp.v}{R_{inp}} + C_{out} \cdot \frac{d out.v}{dt} = 0$ $-out.v = v_0 + \frac{1}{\tau} \cdot \int_0^t inp.v \cdot dt$ $\tau = R_{inp} \cdot C_{out}$ <p>It is possible to integrate the sum of several inputs.</p>
<p><u>Analog Multiplier</u></p> 	<p>Functional Block Diagram of AD633</p> <p>Division by 10 V (scaling) inhibits overflow.</p> <p>Additional summing input Z is omitted.</p> <p>Negative inputs of X- and Y-amplifiers are connected to ground.</p> <p>Possible implementations:</p> <ul style="list-style-type: none"> • Gilbert cell • $y = e^{\ln(x_1) + \ln(x_2)}$ • $y = \frac{(x_1 + x_2)^2 - (x_1 - x_2)^2}{4}$

1 Lotka-Volterra

https://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equations

https://en.wikipedia.org/wiki/Competitive_Lotka%E2%80%93Volterra_equations

<https://sprott.physics.wisc.edu/pubs/paper288.pdf>

As a 2-dimensional predator-prey-model no chaos is reported:

$$\begin{aligned}\frac{dx}{dt} &= r_x \cdot x - d_x \cdot x \cdot y \\ \frac{dy}{dt} &= r_x \cdot x \cdot e_y \cdot y - d_y \cdot y\end{aligned}$$

We might interpret x as number of hares (prey) and y as foxes (predator).

r_x is the reproduction rate of hares, d_x the deathrate of hares due to foxes.

e_y is the efficiency in growing foxes from hares, d_y the (natural) deathrate of foxes.

A n -dimensional Lotka-Volterra model is defined for $1 \leq i \leq n$ (n designates the number of species):

$$\frac{dx_i}{dt} = r_i \cdot x_i \cdot \left(1 - \sum_{j=1}^n a_{ij} \cdot x_j \right)$$

The vector $[r]$ describes the reproduction rates of the species whereas the quadratic matrix $[a]$ describes the competition between species.

2 van der Pol Circuit

https://en.wikipedia.org/wiki/Van_der_Pol_oscillator

B. van der Pol and J. van der Mark, „Frequency Demultiplication”, Nature 120 (1927), p. 363-364, ISSN 0028-0836, DOI <https://doi.org/10.1038/120363a0>

Balthasar van der Pol reported 1927 strange phenomena about oscillations in a series resonance circuit containing a vacuum electron triode. Due to the nonlinear characteristic of the triode the autonomous circuit is able to maintain periodic oscillations, and with harmonic excitation it is able to produce chaos.

$$\begin{aligned}
 i &= C \cdot \frac{dv_c}{dt} \\
 L \cdot \frac{di}{dt} + \tilde{R}(i) \cdot i + v_c &= \begin{cases} 0 \\ \hat{V} \cdot \cos(\omega \cdot t - \pi) \end{cases} \\
 \tilde{R}(i) &= R_0 \cdot \left[1 - \frac{1}{3} \cdot \left(\frac{i}{I_0} \right)^2 \right] \\
 L \cdot \frac{d^2 i}{dt^2} + R_0 \cdot \left[1 - \left(\frac{i}{I_0} \right)^2 \right] \cdot \frac{di}{dt} + \frac{1}{C} \cdot i &= \begin{cases} 0 \\ \hat{V} \cdot \omega \cdot \sin(\omega \cdot t) \end{cases} \\
 \omega_0 &= \frac{1}{\sqrt{L \cdot C}} \\
 \mu &= R_0 \cdot \sqrt{\frac{C}{L}} \\
 \tau &= \omega_0 \cdot t \\
 x &= \frac{i}{I_0} \\
 A &= \frac{\omega \cdot C \cdot \hat{V}}{I_0} \\
 w &= \frac{\omega}{\omega_0} \\
 \frac{d^2 x}{d\tau^2} + \mu \cdot [1 - x^2] \cdot \frac{dx}{d\tau} + x &= \begin{cases} 0 \\ A \cdot \sin(w \cdot \tau) \end{cases} \\
 \frac{dx}{d\tau} &= y \\
 \frac{dy}{d\tau} &= -\mu \cdot [1 - x^2] \cdot y - x + \begin{cases} 0 \\ A \cdot \sin(w \cdot \tau) \end{cases}
 \end{aligned}$$

Calculating back from per-unit-parameters:

$$\mu = 0.2, w = 1.15, A = [0..1]$$

and some assumptions:

$$C = 1 \mu F, \omega_0 = 2\pi \cdot 1000 \frac{rad}{s}, I_0 = 0.2 A$$

we obtain physical parameters:

$$\begin{aligned}
 L &= \frac{1}{\omega_0^2 \cdot C} = 25.330 mH \\
 R_0 &= \mu \cdot \sqrt{\frac{L}{C}} = \frac{\mu}{\omega_0 \cdot C} = 31.831 \Omega \\
 \hat{V} &= A \cdot \frac{I_0}{\omega \cdot C} = [0..27.679] V
 \end{aligned}$$

Investigating correlation between physical and scaled values:

$$x = \frac{i}{I_0}$$

$$y = \frac{dx}{d\tau} = \frac{1}{\omega_0 \cdot I_0} \cdot \frac{di}{dt}$$

$$i = C \cdot \frac{dv_C}{dt} \rightarrow v_C = v_{C0} + \frac{1}{C} \cdot \int_{t_0}^t i \cdot dt'$$

$$z = v_C \cdot \frac{\omega_0 \cdot C}{I_0}$$

Investigating the nonlinear resistance:

$$v_R = R_0 \cdot I_0 \cdot \left[\left(\frac{i}{I_0} \right) - \frac{1}{3} \cdot \left(\frac{i}{I_0} \right)^3 \right]$$

$$\frac{v_R}{i} = R_0 \cdot \left[1 - \frac{1}{3} \cdot \left(\frac{i}{I_0} \right)^2 \right]$$

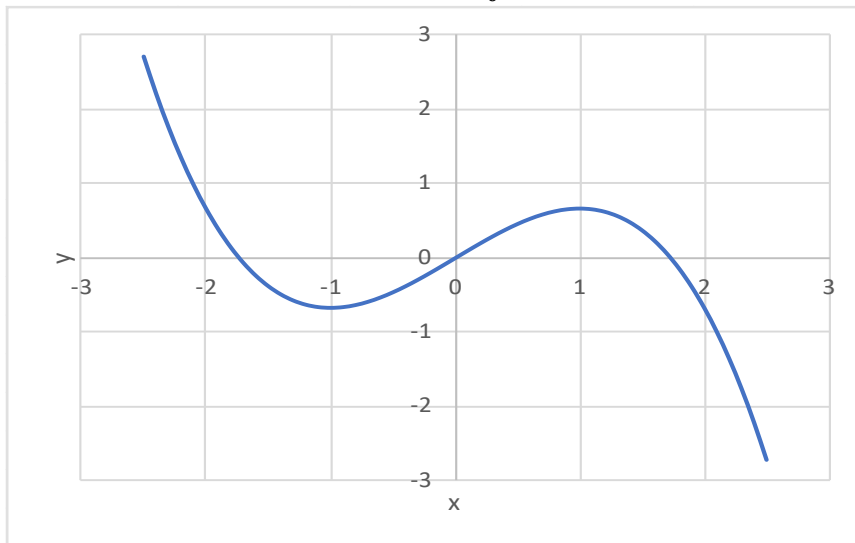
$$\frac{dv_R}{di} = R_0 \cdot \left[1 - \left(\frac{i}{I_0} \right)^2 \right]$$

$$y = x - \frac{x^3}{3}$$

$$\frac{y}{x} = 1 - \frac{x^2}{3}$$

$$\frac{dy}{dx} = 1 - x^2$$

Extrema are found at $x = \pm 1 \leftrightarrow i = \pm I_0$.



3 Lorenz System

http://en.wikipedia.org/wiki/Lorenz_attractor

Developed 1963 by Edward Lorenz to model atmospheric convection.

x is proportional to the rate of convection, y to the horizontal temperature variation and z to the vertical temperature variation. σ depicts the Prandtl number, ρ the Rayleigh number and β the physical dimensions.

The original parameters were: $\sigma = 10, \rho = 28, \beta = \frac{8}{3}$

$$\begin{aligned}\frac{dx}{dt} &= \sigma \cdot (y - x) \\ \frac{dy}{dt} &= x \cdot (\rho - z) - y \\ \frac{dz}{dt} &= x \cdot y - \beta \cdot z\end{aligned}$$

$\beta = \frac{1}{3}$ leads to a periodic solution.

For an implementation as an electronic circuit, the equations have to be scaled to keep the variables within the desired range. This can be compared with calculating per-unit values by dividing by reference values:

$$\begin{aligned}x' &= \frac{x}{k_x} \\ y' &= \frac{y}{k_y} \\ z' &= \frac{z}{k_z} \\ t' &= \frac{t}{\tau}\end{aligned}$$

We also have to take into account that the analog multiplier divides by E_R to avoid overflow of the output.

After that, none of the computing block should encounter an overflow.

This leads to the following set of equations:

$$\begin{aligned}\frac{1}{\tau} \cdot \frac{dx'}{dt'} &= -\sigma \cdot x' + \sigma \cdot \frac{k_y}{k_x} \cdot y' \\ \frac{1}{\tau} \cdot \frac{dy'}{dt'} &= \rho \cdot \frac{k_x}{k_y} \cdot x' - y' - \frac{k_x \cdot k_z \cdot V_S}{k_y} \cdot \frac{x' \cdot z'}{V_S} \\ \frac{1}{\tau} \cdot \frac{dz'}{dt'} &= \frac{k_x \cdot k_y \cdot V_S}{k_z} \cdot \frac{x' \cdot y'}{V_S} - \beta \cdot z'\end{aligned}$$

These equations can easily get implemented as blocks or as an electronic circuit.

4 Roessler System

https://en.wikipedia.org/wiki/R%C3%B6ssler_attractor

A simple system of 3 ordinary nonlinear differential equations to study chaos without physical background.

$$\begin{aligned}\frac{dx}{dt} &= -y - z \\ \frac{dy}{dt} &= x + a \cdot y \\ \frac{dz}{dt} &= b + (x - c) \cdot z\end{aligned}$$

$a = 0.2$, $b = 0.2$ and $c = 1$ give periodic results. Changing $c = 5.7$ reveals chaotic results.

For an implementation as an electronic circuit, the equations have to be scaled to keep the variables within the desired range. This can be compared with calculating per-unit values by dividing by reference values:

$$\begin{aligned}x' &= \frac{x}{k_x} \\ y' &= \frac{y}{k_y} \\ z' &= \frac{z}{k_z} \\ t' &= \frac{t}{\tau}\end{aligned}$$

We also have to take into account that the analog multiplier divides by E_R to avoid overflow of the output. After that, none of the computing block should encounter an overflow.

This leads to the following set of equations:

$$\begin{aligned}\frac{1}{\tau} \cdot \frac{dx'}{dt} &= -\frac{k_y}{k_x} \cdot y' - \frac{k_z}{k_x} \cdot z' \\ \frac{1}{\tau} \cdot \frac{dy'}{dt} &= \frac{k_x}{k_y} \cdot x' + a \cdot y' \\ \frac{1}{\tau} \cdot \frac{dz'}{dt} &= \frac{b}{k_z} + k_x \cdot V_S \cdot \frac{x' \cdot z'}{V_S} - c \cdot z'\end{aligned}$$

These equations can easily get implemented as blocks or as an electronic circuit.

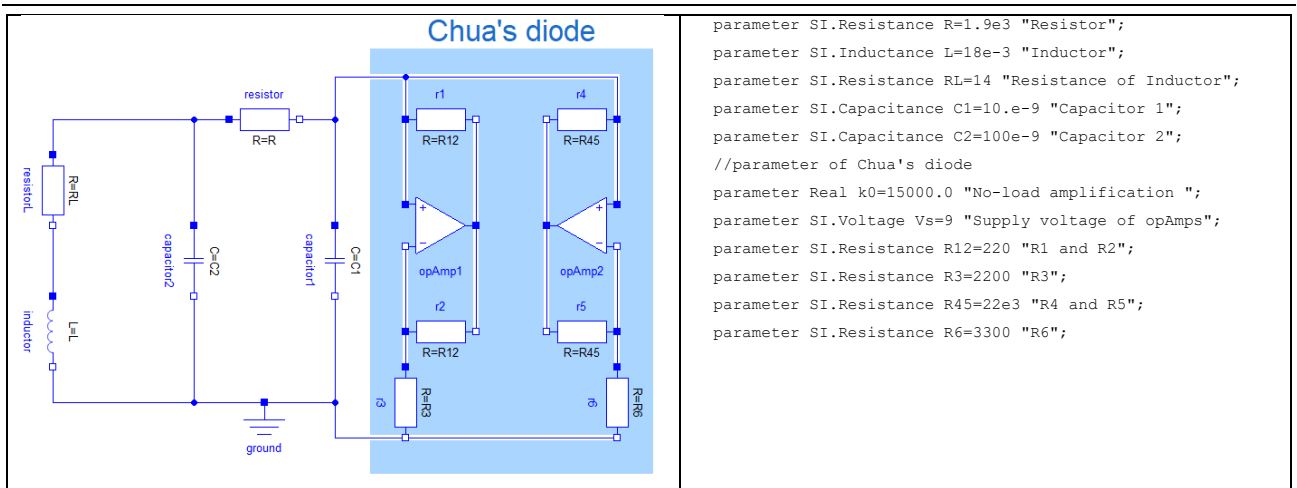
5 Chua's Circuit

<https://link.springer.com/book/10.1007/978-3-319-05900-6> (1.1)

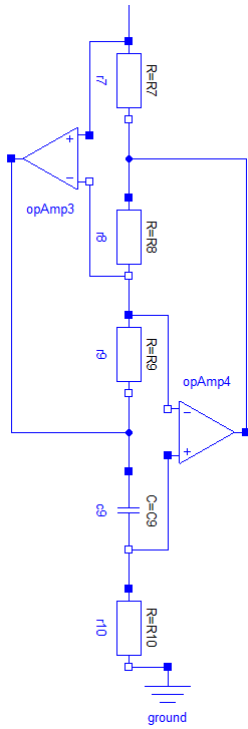
<https://nonlinear.eecs.berkeley.edu/chaos/chaos.html#> Working With Chaos Simulation

$$\begin{aligned}
 L \cdot \frac{di_L}{dt} &= v_2 - R_L \cdot i_L \\
 C_2 \cdot \frac{dv_2}{dt} &= -i_L - \frac{v_2 - v_1}{R} \\
 C_1 \cdot \frac{dv_1}{dt} &= -i_{NL} + \frac{v_2 - v_1}{R} \\
 -i_{NL}(v_1) &= \begin{cases} -\infty < v_1 < -V_e \rightarrow G_b \cdot (v_1 + V_e) - G_a \cdot V_e \\ -V_e < v_1 < +V_e \rightarrow G_a \cdot v_1 \\ +V_e < v_1 < +\infty \rightarrow G_b \cdot (v_1 - V_e) + G_a \cdot V_e \end{cases} \\
 -\frac{i_{NL}}{v_1} &= \begin{cases} -\infty < v_1 < -V_e \rightarrow G_b - (G_a - G_b) \cdot \frac{V_e}{v_1} \\ -V_e < v_1 < +V_e \rightarrow G_a \\ +V_e < v_1 < +\infty \rightarrow G_b + (G_a - G_b) \cdot \frac{V_e}{v_1} \end{cases} \\
 -\frac{di_{NL}}{dv_1} &= \begin{cases} -\infty < v_1 < -V_e \rightarrow G_b \\ -V_e < v_1 < +V_e \rightarrow G_a \\ +V_e < v_1 < +\infty \rightarrow G_b \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 \tau_L \cdot \dot{v}_{RL} &= v_2 - v_{RL} \\
 \tau_2 \cdot \dot{v}_2 &= +v_1 - v_2 - \frac{R}{R_L} \cdot v_{RL} \\
 \tau_1 \cdot \dot{v}_1 &= -v_1 + v_2 + R \cdot g \cdot v_1 \\
 g(v_1) &= \begin{cases} |v_1| > V_e \rightarrow G_b + (G_a - G_b) \cdot \frac{V_e}{|v_1|} \\ |v_1| < V_e \rightarrow G_a \end{cases}
 \end{aligned}$$



Chua's Circuit: Inductor Replacement



$$R_7 = 100 \, \Omega$$

$$R_8 = 1 \, k\Omega$$

$$R_9 = 1 \, k\Omega$$

$$R_{10} = 1,8 \, k\Omega$$

$$C_9 = 100 \, nF$$

$$R_7 \cdot i + R_8 \cdot (i + i_{OA4}) = 0$$

$$R_9 \cdot (i + i_{OA4}) + v_{C9} = 0$$

$$i + i_{OA4} + i_{OA3} = i_{C9}$$

$$i_{C9} = C_9 \cdot \frac{dv_{C9}}{dt}$$

$$v = R_{10} \cdot i_{C9}$$

$$(i + i_{OA4}) = -\frac{R_7}{R_8} \cdot i$$

$$v_{C9} = \frac{R_7 \cdot R_9}{R_8} \cdot i$$

$$i_{C9} = C_9 \cdot \frac{R_7 \cdot R_9}{R_8} \cdot \frac{di}{dt}$$

$$v = C_9 \cdot \frac{R_7 \cdot R_9 \cdot R_{10}}{R_8} \cdot \frac{di}{dt}$$

$$L = C_9 \cdot \frac{R_7 \cdot R_9 \cdot R_{10}}{R_8} = 18 \, mH$$

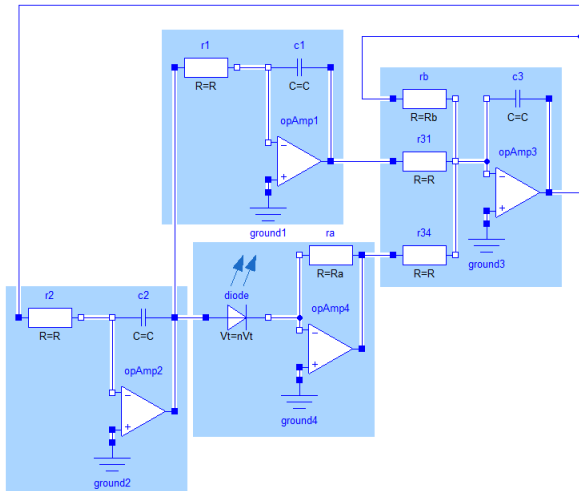
6 Chaotic Diode Circuit

https://www.researchgate.net/publication/309351711_A_simple_chaotic_circuit_with_a_light-emitting_diode

$$\begin{aligned}C \cdot \frac{dv_1}{dt} &= -\frac{v_2}{R} \\C \cdot \frac{dv_2}{dt} &= -\frac{v_3}{R} \\C \cdot \frac{dv_3}{dt} &= -\frac{v_1}{R} - \frac{v_3}{R_b} - \frac{v_4}{R} \\ \frac{v_4}{R_a} &= -I_{ds} \cdot \left(e^{\frac{v_2}{nV_t}} - 1 \right)\end{aligned}$$

$$\begin{aligned}\tau &= R \cdot C \\ \tau \cdot \frac{dv_1}{dt} &= -v_2 \\ \tau \cdot \frac{dv_2}{dt} &= -v_3 \\ \tau \cdot \frac{dv_3}{dt} &= -v_1 - \frac{R}{R_b} \cdot v_3 + R_a \cdot I_{ds} \cdot \left(e^{\frac{v_2}{nV_t}} - 1 \right)\end{aligned}$$

$$\begin{aligned}a &= \frac{R_a \cdot I_{ds}}{nV_t} \\ b &= \frac{R}{R_b} \\ \tau \cdot \dot{x}_1 &= -x_2 \\ \tau \cdot \dot{x}_2 &= -x_3 \\ \tau \cdot \dot{x}_3 &= -x_1 + a \cdot (e^{x_2} - 1) - b \cdot x_3\end{aligned}$$



```
parameter SI.Current Ids=1e-9 "Diode: saturation current";
parameter SI.Voltage nVt=2*26e-3
" n*voltage equ. of temperature";
//time constant and dimensionless parameters
parameter SI.Time Tau=0.1e-3 "Time constant";
parameter Real a=1/5200 "Parameter a = Ra*Ids/nVt";
parameter Real b=1.5 "Parameter b = R/Rb";
//configuration of the opAmps
parameter SI.Capacitance C=10e-9
"Capacitance of {c1, c2, c3}";
parameter SI.Resistance R=Tau/C "Resistance of {r1,r2,r3,r4}";
parameter SI.Resistance Ra=a*nVt/Ids "Resistance of ra";
parameter SI.Resistance Rb=R/b "Resistance of rb";
```

7 Chaotic Oscillator

https://www.researchgate.net/publication/230925506_A_simple_chaotic_oscillator_for_educational_purposes

https://www.researchgate.net/publication/259216097_NUMERICAL_TREATMENT_OF_EDUCATIONAL_CHAOS_OSCILLATOR

$$i_L = C \cdot \frac{dv_C}{dt}$$

$$L \cdot \frac{di_L}{dt} = \left(k - 1 - \frac{R_L}{R}\right) \cdot R \cdot i_L - v_C - v_C^*$$

$$k = 1 + \frac{R_2}{R_1}$$

$$C^* \cdot \frac{dv_{C^*}}{dt} = I_0 + i_L - I_{DS} \cdot \left(e^{\frac{v_{C^*}}{nV_t}} - 1\right)$$

$$I_0 \approx \frac{V_b}{R_0}$$

$$\tau = \sqrt{L \cdot C}$$

$$Z = \sqrt{\frac{L}{C}}$$

$$a = \left(k - 1 - \frac{R_L}{R}\right) \cdot \frac{R}{Z}$$

$$b = \frac{Z \cdot I_0}{nV_t}$$

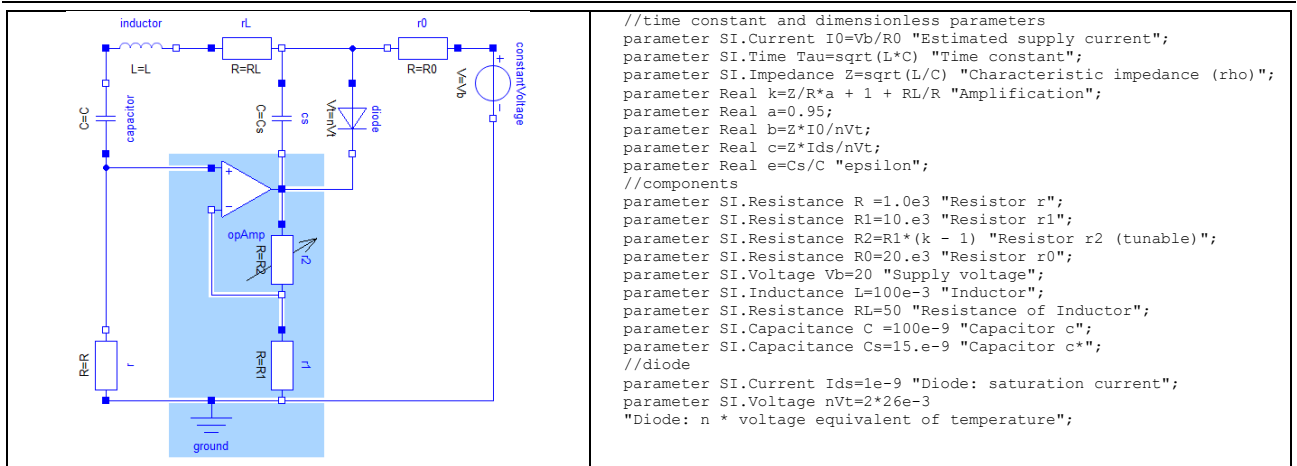
$$c = \frac{Z \cdot I_{DS}}{nV_t}$$

$$e = \frac{C^*}{C}$$

$$\tau \cdot \frac{\dot{v}_C}{nV_t} = \frac{Z \cdot i_L}{nV_t}$$

$$\tau \cdot \frac{Z \cdot i_L}{nV_t} = \left(k - 1 - \frac{R_L}{R}\right) \cdot \frac{R}{Z} \cdot \frac{Z \cdot i_L}{nV_t} - \frac{v_C}{nV_t} - \frac{v_{C^*}}{nV_t}$$

$$\tau \cdot e \cdot \frac{\dot{v}_{C^*}}{nV_t} = \frac{Z \cdot I_0}{nV_t} + \frac{Z \cdot i_L}{nV_t} - \frac{Z \cdot I_{DS}}{nV_t} \cdot \left(e^{\frac{v_{C^*}}{nV_t}} - 1\right)$$



8 Colpitts Oscillator

<https://link.springer.com/book/10.1007/978-3-319-05900-6> (1.3)

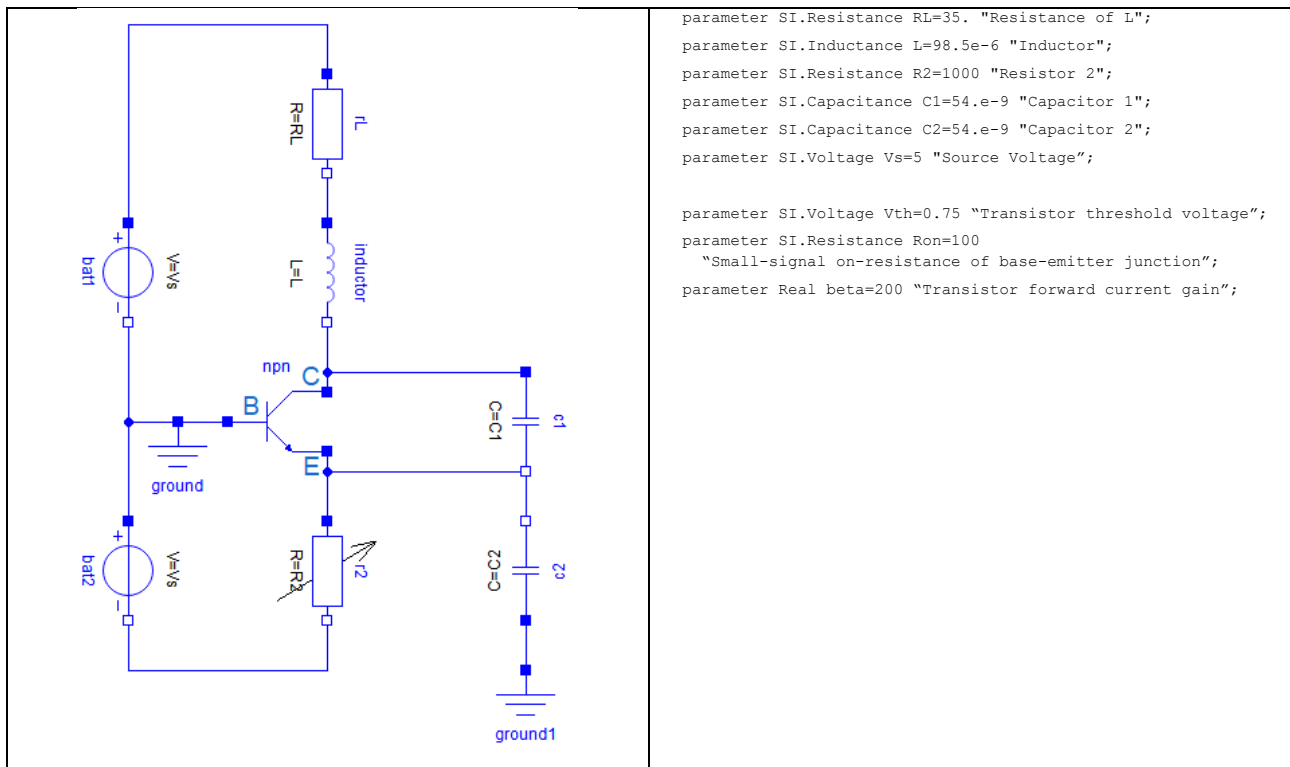
<https://ieeexplore.ieee.org/document/331536>

$$C_1 \cdot \frac{dv_1}{dt} = i_L - \beta \cdot i_B$$

$$C_2 \cdot \frac{dv_2}{dt} = -\frac{V_{s-} + v_2}{R_2} - i_L - i_B$$

$$L \cdot \frac{di_L}{dt} = V_{s+} - v_1 + v_2 - R_L \cdot i_L$$

$$i_B = \begin{cases} v_2 = v_{BE} \leq V_{th} \rightarrow 0 \\ v_2 = v_{BE} > V_{th} \rightarrow \frac{v_2 - V_{th}}{R_{on}} \end{cases}$$



9 Shinriki Oscillator

https://paw.n.physik.uni-wuerzburg.de/~slueck/PhyAmSa09/Home_files/Examensarbeit_Lueck.pdf

<https://ieeexplore.ieee.org/abstract/document/1456241>

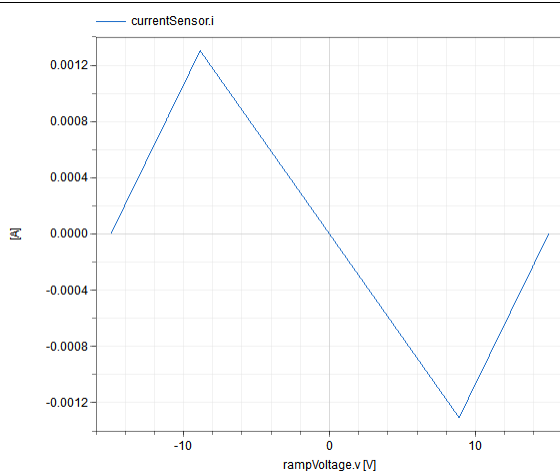
$$i_z = \begin{cases} |v_z| < V_{bt} & 0 \\ |v_z| \geq V_{bt} & \text{sign}(v_z) \cdot [a \cdot (|v_z| - V_{bt}) + b \cdot (|v_z| - V_{bt})^3 + c \cdot (|v_z| - V_{bt})^5] \end{cases}$$

$$C_1 \cdot \frac{dv_1}{dt} = -i_{NIC} - \frac{v_1}{R_1} - i_z$$

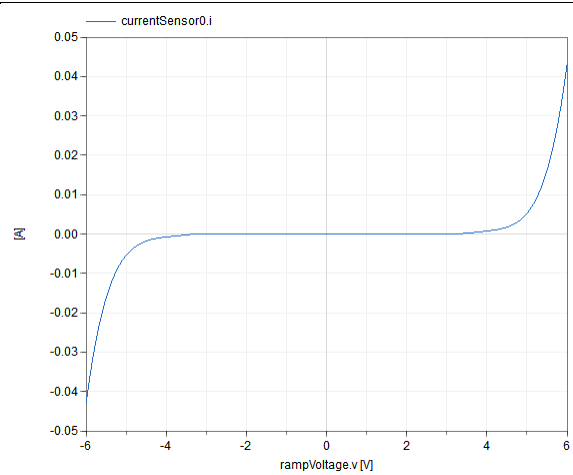
$$C_2 \cdot \frac{dv_2}{dt} = i_z - i_L$$

$$v_2 = L \cdot \frac{di_L}{dt} + R_L \cdot i_L$$

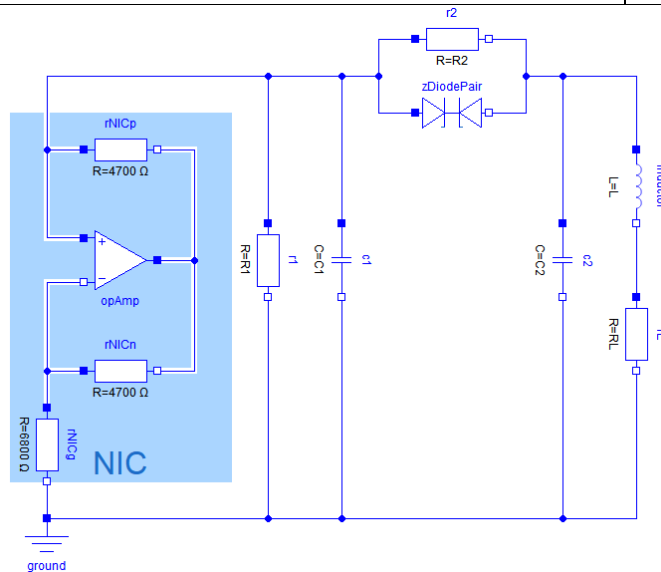
$$g_{NIC} = \frac{di_{NIC}}{dv_{NIC}} = \begin{cases} |v_{NIC}| > V_{Lim} & g_+ \\ |v_{NIC}| \leq V_{Lim} & g_- \end{cases}$$



Characteristic of negative impedance converter



Characteristic of Zener diode pair



```
parameter SI.Inductance L=320e-3 "Inductor";
parameter SI.Resistance RL=100. "Resistor of L";
parameter SI.Resistance R1=60e3 "Resistor 1";
parameter SI.Resistance R2=20e3 "Resistor 2";
parameter SI.Capacitance C1=10.e-9 "Capacitor 1";
parameter SI.Capacitance C2=100e-9 "Capacitor 2";
```

$$a = 1,0862 \frac{mA}{V}$$

$$b = -0,1615 \frac{mA}{V^3}$$

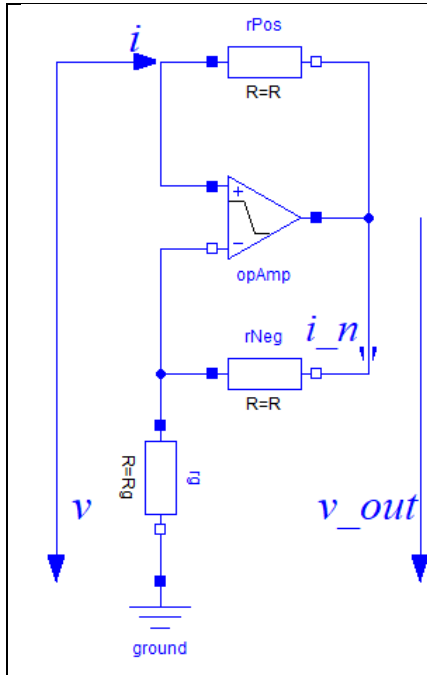
$$c = 0,3021 \frac{mA}{V^5}$$

$$V_{Lim} = V_s \cdot \frac{6800}{4700 + 6800}$$

$$g_+ = + \frac{1000}{4700} mS$$

$$g_- = - \frac{1000}{6800} mS$$

Investigation of the negative impedance converter (NIC):



As long as the opAmp operates in the linear region:

$$i_n = \frac{v_{out} - v}{R} = \frac{v}{R_g} \rightarrow v_{out} = v \cdot \frac{R + R_g}{R_g}$$

$$i = \frac{v - v_{out}}{R} = -\frac{v}{R_g}$$

$$g_- = \frac{1}{R_g}$$

When the opAmp's output saturates:

$$V_{Lim} = V_s \cdot \frac{R_g}{R + R_g}$$

$$v \geq +V_{Lim}: i = \frac{v - V_{Lim}}{R}$$

$$g_+ = \frac{1}{R}$$

10 Jerk Circuit

<https://link.springer.com/book/10.1007/978-3-319-05900-6> (3.1)

<https://sprott.physics.wisc.edu/pubs/paper352.pdf>

The name of the system stems from the third derivative of x , which – in a mechanical system – is the derivative of acceleration called jerk.

$$\ddot{x} + A \cdot \dot{x} + f(\dot{x}) + x = 0$$

$$f(\dot{x}) = R \cdot I_S \cdot \left(e^{\frac{\dot{x}}{nV_t}} - 1 \right)$$

$$\dot{x} = y$$

$$\ddot{x} = \dot{y} = z$$

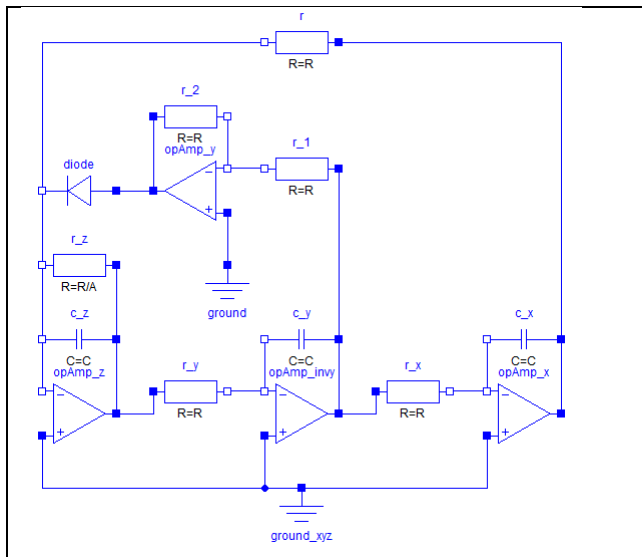
$$\ddot{x} = \ddot{y} = \dot{z}$$

$$\dot{z} = -A \cdot z - x - f(y)$$

The values stay pretty inside a practicable range for a normal voltage supply.

Using 4 operational amplifiers and an acceleration factor of 1000, the circuit can be implemented as follows.

The parameter A influences only the feedback resistor at opAmp_z:



```
parameter Real A=0.3 "Parameter to be varied";
parameter SI.Resistance R=1e3 "Resistance";
parameter SI.Capacitance C=1e-6 "Capacitance";
parameter SI.Current Ids=1e-12 "Sat.current";
parameter SI.Voltage nVt=26e-3 " voltage equ.";
```

A=0.3 for periodic results

A=1.0 for chaotic results