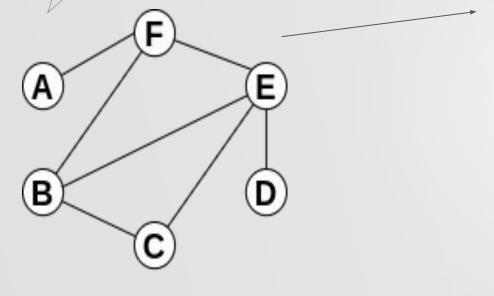






- Goal:
 - Find a good representation of a graph G = (V, E)



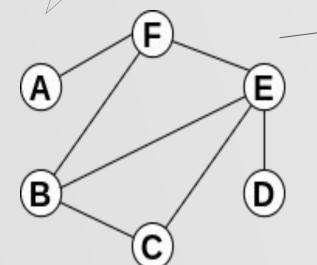
Node embedding:

$$v\mapsto [f_1(v),\ldots,f_d(v)]$$



Goal:

Find a good representation of a graph G = (V, E)

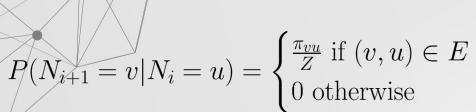


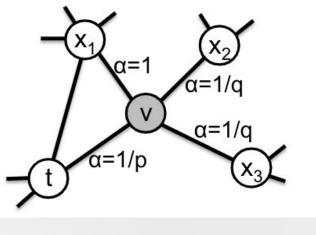
Node embedding:

$$v\mapsto [f_1(v),\ldots,f_d(v)]$$

Strategy:

- Optimize embedding to preserve similarities
- Similarities defined as a "neighborhood" notion
- Use (biased) random walks to define neighborhood







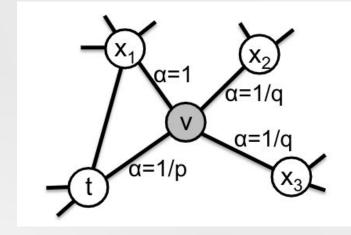
Unbiased random walk

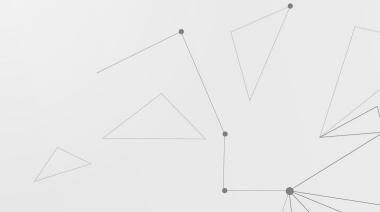
$$P(N_{i+1} = v | N_i = u) = \begin{cases} \frac{\pi_{vu}}{Z} & \text{if } (v, u) \in E \\ 0 & \text{otherwise} \end{cases}$$

$$\pi_{uv}$$

Transition probability, e.g.

$$\pi_{uv}=w_{vu}$$



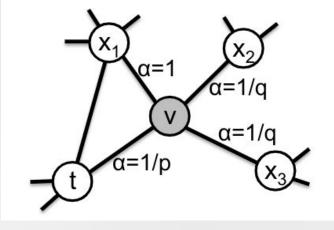


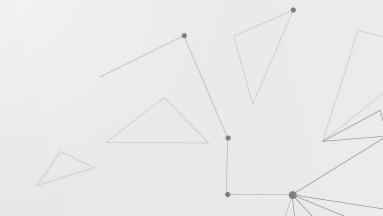
 ${\it Z}$ Normalization factor

Unbiased random walk

$$P(N_{i+1} = v | N_i = u) = \begin{cases} \frac{\pi_{vu}}{Z} & \text{if } (v, u) \in B \\ 0 & \text{otherwise} \end{cases}$$

Transition probability, e.g. $\pi_{uv}=w_{vu}$





Unbiased random walk

Unbiased random walk
$$P(N_{i+1} = v | N_i = u) = \begin{cases} \frac{\pi_{vu}}{Z} \text{ if } (v, u) \in E \\ 0 \text{ otherwise} \end{cases}$$

Transition probability, e.g. $\pi_{uv} = w_{vu}$



Biased random walk

$$\alpha_{pq}(t,x) = \begin{cases} \frac{1}{p} & \text{if } d_{tx} = 0\\ 1 & \text{if } d_{tx} = 1\\ \frac{1}{q} & \text{if } d_{tx} = 2 \end{cases}$$

Biased transition probability

$$\pi_{vx} = lpha_{pq}(t,x) \cdot w_{vx}$$

 $\alpha=1$

Z Normalization factor

Unbiased random walk

$$P(N_{i+1} = v | N_i = u) = \begin{cases} \frac{\pi_{vu}}{Z} & \text{if } (v, u) \in E \\ 0 & \text{otherwise} \end{cases}$$

$$_{\scriptscriptstyle
u}$$
 π_{u}

Transition probability, e.g. $\pi_{uv}=w_{vu}$

Biased random walk

Search bias
$$\alpha_{pq}(t,x) = \begin{cases} \frac{1}{p} \text{ if } d_{tx} = 0 \\ 1 \text{ if } d_{tx} = 1 \end{cases}$$

Biased transition probability

$$\pi_{vx} = lpha_{pq}(t,x) \cdot w_{vx}$$

 $\alpha=1$

Unbiased random walk

$$P(N_{i+1} = v | N_i = u) = \begin{cases} \frac{\pi_{vu}}{Z} & \text{if } (v, u) \in E \\ 0 & \text{otherwise} \end{cases}$$

 π_{uv}

Transition probability, e.g. $\pi_{uv}=w_{vu}$

return parameter p:

- large -> exploration
- small -> backtrack, local

in-out parameter q:

- large -> stay close to t
- small -> exploration

DeepWalk: q=p=1



Z Normalization factor

$$\alpha_{pq}(t,x) = \begin{cases} \frac{1}{p} \text{ if } d_{tx} = 0\\ 1 \text{ if } d_{tx} = 1\\ \frac{1}{q} \text{ if } d_{tx} = 2 \end{cases}$$
 Search bias

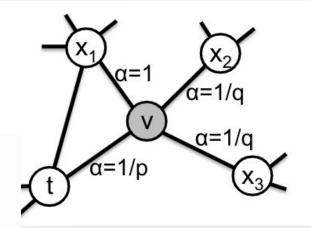
Biased transition probability

$$\pi_{vx} = \alpha_{pq}(t,x) \cdot w_{vx}$$

 $\alpha=1$

CLASS Node2Vec (edge_index, embedding_dim, walk_length, context_size, walks_per_node=1, p=1, q=1, num_negative_samples=1, num_nodes=None, sparse=False) [source]

The Node2Vec model from the "node2vec: Scalable Feature Learning for Networks" paper where random walks of length walk_length are sampled in a given graph, and node embeddings are learned via negative sampling optimization.

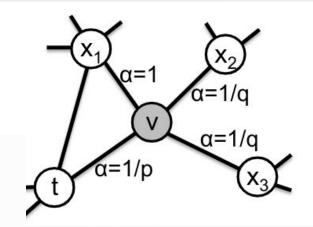


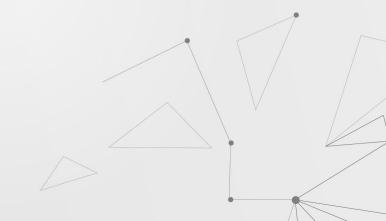


O2 Biased random walks Length

CLASS Node2Vec (edge_index, embedding_dim, walk_length, context_size, walks_per_node=1, p=1, q=1, num_negative_samples=1, num_nodes=None, sparse=False) [source]

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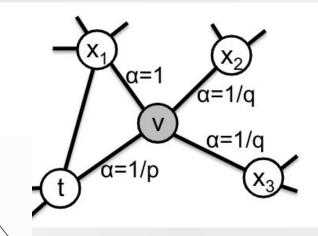


Length

CLASS Node2Vec (edge_index, embedding_dim, walk_length, context_size, walks_per_node=1, p=1, q=1, num_negative_samples=1, num_nodes=None, sparse=False) [source]

The Node2Vec model from the "node2vec: Scalable Feature Learning for Networks" paper where random walks of length walk_length are sampled in a given graph, and node embeddings are learned via negative sampling optimization.

Return parameter



In-out parameter

Length

CLASS Node2Vec (edge_index, embedding_dim, walk_length, context_size, walks_per_node=1, p=1, q=1, num_negative_samples=1, num_nodes=None, sparse=False) //source]

The Node2Vec model from the "node2vec: Scalable Feature Learning for Networks" paper where random walks of length walk_length are sampled in a given graph, and node embeddings are learned via negative sampling optimization.

Return parameter

Length of the RW to extract from a long sample

In-out parameter

 $\alpha = 1$

- Sample **I=6**, **k=3**: {u, s4, s5, s6, s8, s9}
 - 1. **u:** \$4,\$5,\$6 2. **\$4:** \$5,\$6,\$8
 - 3. **\$5**: \$6,\$8,\$9

O2 Biased random walks Length

CLASS Node2Vec (edge_index, embedding_dim, walk_length, context_size, walks_per_node=1, p=1, q=1, num_negative_samples=1, num_nodes=None, sparse=False) //source]

The Node2Vec model from the "node2vec: Scalable Feature Learning for Networks" paper where random walks of length walk_length are sampled in a given graph, and node embeddings are learned via negative sampling optimization.

loader (**kwargs)

Return parameter

Length of the RW to extract from a long sample

In-out parameter

 $\alpha = 1$

- Sample **I=6**, **k=3**: {u, s4, s5, s6, s8, s9}
 - 1. **u:** \$4,\$5,\$6 2. **\$4:** \$5,\$6,\$8
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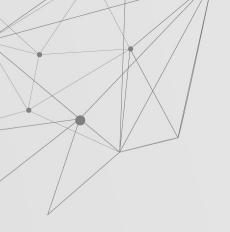




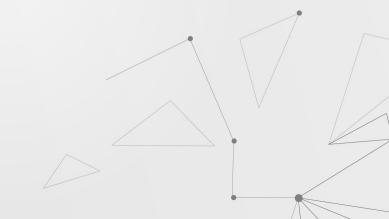
```
Loader over list of nodes
def loader(self, **kwargs):
     return DataLoader(range(self.adj.sparse_size(0));
                            collate_fn=self.sample, **kwargs)
                                                                    def sample(self, batch):
                                                                         if not isinstance(batch, torch.Tensor):
                                                                              batch = torch.tensor(batch)
                                                                         return self.pos_sample(batch), self.neg_sample(batch)
                                                                                                                 A batch of indices
pos_sample(self, batch):
                                                                                           def neg_sample(self, batch):
                                                                                102
batch = batch.repeat(self.walks_per_node)
                                                                                103
                                                                                               batch = batch.repeat(self.walks per node * self.num negative samples)
                                                                                104
rowptr, col, _ = self.adj.csr()
                                                                                               rw = torch.randint(self.adj.sparse_size(0),
                                                                                105
rw = random_walk(rowptr, col, batch, self.walk_length, self.p, self.q)
                                                                                106
                                                                                                                (batch.size(0), self.walk_length))
                                                                                               rw = torch.cat([batch.view(-1, 1), rw], dim=-1)
if not isinstance(rw, torch.Tensor):
                                                                                107
    rw = rw[0]
                                                                                108
                                                                                109
                                                                                               walks = []
walks = []
                                                                                110
num_walks_per_rw = 1 + self.walk_length + 1 - self.context_size
                                                                                               num_walks_per_rw = 1 + self.walk_length + 1 - self.context_size
                                                                                111
for j in range(num walks per rw):
                                                                                112
                                                                                               for j in range(num walks per rw):
    walks.append(rw[:, j:j + self.context size])
                                                                                                   walks.append(rw[:, j:j + self.context size])
                                                                                113
return torch.cat(walks, dim=0)
                                                                                               return torch.cat(walks, dim=0)
                                                                                114
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num_walks_per_rw = 1 + self.walk_length + 1
                                                                                              num_walks_per_rw = 1 + self.walk_length + 1 - self.context_size
                                                                               111
for j in range(num_walks_per_rw):
                                                                               112
                                                                                              for j in range(num walks per rw):
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                             torch.ops.torch_cluster.random_walk
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return torch.cat(walks, dim=0)
                                                                                              return torch.cat(walks, dim=0)
                                                                                114
                                                    Initial nodes
                                                                                                                  A fake RW
                             torch.ops.torch_cluster.random_walk
```



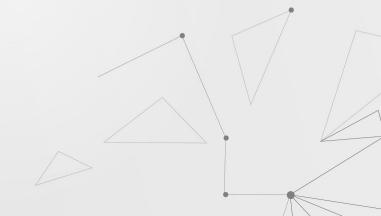
... notebook ...



Definition of the embedding f(v)

```
self.embedding = Embedding(N, embedding_dim, sparse=sparse)
```

torch.nn.Embedding



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torch.nn.Embedding

CLASS torch.nn.Embedding(num_embeddings, embedding_dim, padding_idx=None, max_norm=None, norm_type=2.0, scale_grad_by_freq=False, sparse=False, _weight=None)

[SOURCE]

A simple lookup table that stores embeddings of a fixed dictionary and size.

This module is often used to store word embeddings and retrieve them using indices. The input to the module is a list of indices, and the output is the corresponding word embeddings.



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[SOURCE]

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This module is often used to store word embeddings and retrieve them using indices. The input to the module is a list of indices, and the output is the corresponding word embeddings.

sparse (bool, optional) – If True, gradient w.r.t. weight matrix will be a sparse tensor. See Notes for more
details regarding sparse gradients.



The loss maximizes the probability of a neighborhood given u

$$P_f(v|u) := \frac{\exp(f(v)^T f(u))}{\sum_{w \in V} \exp(f(w)^T f(u))}$$



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Random walk

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Random walk

Expensive

Negative sampling

(deepwalk uses hierarchical softmax)

• Manipulate the loss to $f(v)^T f(u) + \mathbb{E}_{w \sim p} (f(w)^T f(u))$

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Random walk

Expensive

Negative sampling

(deepwalk uses hierarchical softmax)

- Manipulate the loss to $f(v)^T f(u) + \mathbb{E}_{w \sim p}(f(w)^T f(u))$
- Approximate p by defining a positive/negative class

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Random walk

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- Manipulate the loss to $f(v)^T f(u) + \mathbb{E}_{w \sim p} (f(w)^T f(u))$
- Approximate **p** by defining a **positive/negative class**
- Sample +/- by sampling true/fake RW

The loss maximizes the probability of a neighborhood given u

$$v|u\rangle := \frac{\exp(f(v)^T f(u))}{\sum_{w \in V} \exp(f(w)^T f(u))}$$

$$P_f(N_s(u)|u) := \prod_{v \in N_s(u)} P_f(v|u)$$

Random walk

Expensive

Negative sampling

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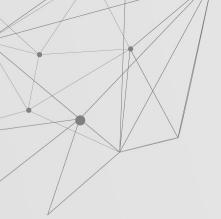
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Details of math: On word embeddings - Part 2: Approximating the Softmax https://ruder.io/word-embeddings-softmax/

CLASS Node2Vec (edge_index, embedding_dim, walk_length, context_size, walks_per_node=1, p=1, q=1, num_negative_samples=1, num_nodes=None, sparse=False) [source]

The Node2 Vec model from the "node2vec: Scalable Feature Learning for Networks" paper where random walks of length walk_length are sampled in a given graph, and node embeddings are learned via negative sampling optimization.

 num_negative_samples (int, optional) – The number of negative samples to use for each positive sample. (default: 1)



```
def loss(self, pos_rw, neg_rw):
    r"""Computes the loss given positive and negative random walks."""
    # Positive loss.
    start, rest = pos rw[:, 0], pos rw[:, 1:].contiquous()
    h_start = self.embedding(start).view(pos_rw.size(0), 1,
                                         self.embedding dim)
    h rest = self.embedding(rest.view(-1)).view(pos rw.size(0), -1,
                                                self.embedding dim)
    out = (h start * h rest).sum(dim=-1).view(-1)
    pos loss = -torch.log(torch.sigmoid(out) + EPS).mean()
    # Negative loss.
    start, rest = neg_rw[:, 0], neg_rw[:, 1:].contiguous()
    h_start = self.embedding(start).view(neg_rw.size(0), 1,
                                         self.embedding dim)
    h_rest = self.embedding(rest.view(-1)).view(neg_rw.size(0), -1,
                                                self.embedding dim)
    out = (h start * h rest).sum(dim=-1).view(-1)
    neg_loss = -torch.log(1 - torch.sigmoid(out) + EPS).mean()
    return pos_loss + neg_loss
```

Divide first node from the rest of the RW

```
def loss(self, pos rw, neg rw):
    r"""Computes the loss given positive and negative random walks."""
    # Positive loss.
   -start, rest = pos rw[:, 0], pos rw[:, 1:].contiquous()
    h_start = self.embedding(start).view(pos_rw.size(0), 1,
                                         self.embedding dim)
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                                                self.embedding dim)
    out = (h start * h rest).sum(dim=-1).view(-1)
    pos loss = -torch.log(torch.sigmoid(out) + EPS).mean()
    # Negative loss.
    start, rest = neg_rw[:, 0], neg_rw[:, 1:].contiguous()
    h_start = self.embedding(start).view(neg_rw.size(0), 1,
                                         self.embedding dim)
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    neg_loss = -torch.log(1 - torch.sigmoid(out) + EPS).mean()
    return pos loss + neg loss
```

Divide first node from the rest of the RW

Compute the embeddings

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def loss(self, pos rw, neg rw):
    r"""Computes the loss given positive and negative random walks."""
    # Positive loss.
    -start, rest = pos rw[:, 0], pos rw[:, 1:].contiquous()
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    # Negative loss.
    start, rest = neg_rw[:, 0], neg_rw[:, 1:].contiguous()
    h_start = self.embedding(start).view(neg_rw.size(0), 1,
                                         self.embedding dim)
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                                                self.embedding dim)
    out = (h start * h rest).sum(dim=-1).view(-1)
    neg_loss = -torch.log(1 - torch.sigmoid(out) + EPS).mean()
    return pos loss + neg loss
```

Divide first node from the rest of the RW

Compute the embeddings

Loss for the **positive class**: true RW

```
def loss(self, pos rw, neg rw):
    r"""Computes the loss given positive and negative random walks."""
    # Positive loss.
   -start, rest = pos rw[:, 0], pos rw[:, 1:].contiquous()
   - h start = self.embedding(start).view(pos rw.size(0), 1,
                                         self.embedding dim)
   h_rest = self.embedding(rest.view(-1)).view(pos_rw.size(0), -1,
                                                self.embedding dim)
    out = (h start * h rest).sum(dim=-1).view(-1)
    pos loss = -torch.log(torch.sigmoid(out) + EPS).mean()
    # Negative loss.
    start, rest = neg_rw[:, 0], neg_rw[:, 1:].contiguous()
    h_start = self.embedding(start).view(neg_rw.size(0), 1,
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                                                self.embedding dim)
    out = (h start * h rest).sum(dim=-1).view(-1)
    neg_loss = -torch.log(1 - torch.sigmoid(out) + EPS).mean()
    return pos loss + neg loss
```

def loss(self, pos rw, neg rw): r"""Computes the loss given positive and negative random walks.""" Divide first node from the rest of the RW # Positive loss. -start, rest = pos rw[:, 0], pos rw[:, 1:].contiquous() - h_start = self.embedding(start).view(pos_rw.size(0), 1, Compute the embeddings self.embedding dim) h rest = self.embedding(rest.view(-1)).view(pos_rw.size(0), -1, self.embedding dim) out = (h start * h rest).sum(dim=-1).view(-1) Loss for the **positive class**: true RW pos loss = -torch.log(torch.sigmoid(out) + EPS).mean() # Negative loss. start, rest = neg_rw[:, 0], neg_rw[:, 1:].contiguous() h_start = self.embedding(start).view(neg_rw.size(0), 1, self.embedding dim) The same, but for the h rest = self.embedding(rest.view(-1)).view(neg rw.size(0), -1, **negative class (fake RW)** self.embedding dim) out = (h_start * h_rest).sum(dim=-1).view(-1) neg_loss = -torch.log(1 - torch.sigmoid(out) + EPS).mean()

return pos loss + neg loss

```
def loss(self, pos rw, neg rw):
                                                                r"""Computes the loss given positive and negative random walks."""
 Divide first node from the rest of the RW
                                                               # Positive loss.
                                                               -start, rest = pos rw[:, 0], pos rw[:, 1:].contiquous()
                                                               - h_start = self.embedding(start).view(pos_rw.size(0), 1,
               Compute the embeddings
                                                                                                     self.embedding dim)
                                                               h rest = self.embedding(rest.view(-1)).view(pos_rw.size(0), -1,
                                                                                                           self.embedding dim)
                                                                out = (h start * h rest).sum(dim=-1).view(-1)
Loss for the positive class: true RW
                                                                pos loss = -torch.log(torch.sigmoid(out) + EPS).mean()
                                                                # Negative loss.
                                                               start, rest = neg_rw[:, 0], neg_rw[:, 1:].contiguous()
                                                               h_start = self.embedding(start).view(neg_rw.size(0), 1,
                                                                                                    self.embedding dim)
      The same, but for the
                                                               h rest = self.embedding(rest.view(-1)).view(neg rw.size(0), -1,
             negative class (fake RW)
                                                                                                           self.embedding dim)
                                                               out = (h_start * h_rest).sum(dim=-1).view(-1)
                                                               neg_loss = -torch.log(1 - torch.sigmoid(out) + EPS).mean()
                                                               return pos loss + neg loss
                         Total loss
```

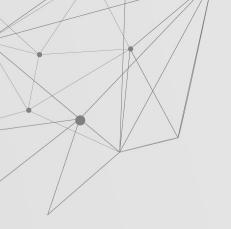
04 node2vec: Full implementation

```
def forward(self, batch=None):
    """Returns the embeddings for the nodes in :obj:`batch`."""
    emb = self.embedding.weight
    return emb if batch is None else emb[batch]
```



1 4 node2vec: Full implementation

```
def forward(self, batch=None):
    """Returns the embeddings for the nodes in :obj:`batch`."""
    emb = self.embedding.weight
    return emb if batch is None else emb[batch]
```



04 node2vec: Full implementation

... notebook ...

