

# Algorytmika

## Ćwiczenia 5

Adrian Herda

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### Zadanie 22

Zmienna losowa

$$L \sim \text{Uni}\{0, \dots, n-1\}$$

to znaczy że:

$$\Pr[L = k] = \frac{1}{n}, \text{ dla } 0 \leq k < n.$$

Zmienna losowa

$$X = \max\{L, n - L - 1\}$$

$$X = \begin{cases} L, & \text{jeśli } L > \frac{n-1}{2} \\ n - L - 1, & \text{jeśli } L \leq \frac{n-1}{2} \end{cases}$$

$$X = \begin{cases} k, & \text{jeśli } k > \frac{n-1}{2} \\ n - k - 1, & \text{jeśli } k \leq \frac{n-1}{2} \end{cases}$$

Dla  $n = 2m$  czyli parzystego mamy  $\frac{n-1}{2} = m - \frac{1}{2}$  wtedy:

$$\begin{aligned} \sum_{k=0}^{2m-1} \max\{k, 2m - k - 1\} &= \sum_{k=0}^{m-1} (2m - k - 1) + \sum_{k=m}^{2m-1} k = \\ \frac{m(2m-1+m)}{2} + \frac{m(m+2m-1)}{2} &= \frac{m(3m-1) + m(3m-1)}{2} = 3m^2 - m \end{aligned}$$

A więc wartość oczekiwana:

$$E(X) = \frac{1}{n} \sum_{k=0}^{2m-1} \max\{k, 2m - k - 1\} = \frac{1}{2m} \cdot (3m^2 - m) = \frac{3m-1}{2} = \frac{3}{4}n - \frac{1}{2}$$

Dla  $n = 2m + 1$  czyli parzystego mamy  $\frac{n-1}{2} = \frac{2m}{2} = m$  wtedy:

$$\begin{aligned} \sum_{k=0}^{2m} \max\{k, 2m - k\} &= \sum_{k=0}^m (2m - k) + \sum_{k=m+1}^{2m} k = \\ \frac{m(2m+m)}{2} + \frac{m(m+1+2m)}{2} &= \frac{m}{2} \cdot (6m+1) \end{aligned}$$

A więc wartość oczekiwana:

$$E(X) = \frac{1}{n} \sum_{k=0}^{2m} \max\{k, 2m - k\} = \frac{1}{n} \cdot \frac{m}{2} \cdot (6m+1) = \frac{1}{n} \cdot \frac{n-1}{4} \cdot (3n-2)$$