Algorytmika
Ćwiczenia 5

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Zadanie 22

Zmienna losowa

$$L \sim \mathrm{Uni}\{0, \cdots, n-1\}$$

to znaczy że:

$$\Pr[L = k] = \frac{1}{n},\ \text{dla } 0 \le k < n.$$

Zmienna losowa

$$X = \max\{L, n - L - 1\}$$

$$X = \begin{cases} L, & \text{jeśli } L > \frac{n-1}{2} \\ n - L - 1, & \text{jeśli } L \le \frac{n-1}{2} \end{cases}$$

$$X = \begin{cases} k, & \text{jeśli } k > \frac{n-1}{2} \\ n - k - 1, & \text{jeśli } k \le \frac{n-1}{2} \end{cases}$$

Dla $n = 2m$ czyli parzystego mamy $\frac{n-1}{2} = m - \frac{1}{2}$ wtedy:

$$\sum_{k=0}^{2m-1} \max\{k, 2m - k - 1\} = \sum_{k=0}^{m-1} (2m - k - 1) + \sum_{k=m}^{2m-1} k =$$

$$\frac{m(2m - 1 + m)}{2} + \frac{m(m + 2m - 1)}{2} = \frac{m(3m-1) + m(3m-1)}{2} = 3m^2 - m$$

A więc wartość oczekiwana:

$$E(X) = \frac{1}{n} \sum_{k=0}^{2m-1} \max\{k, 2m - k - 1\} = \frac{1}{2m} \cdot (3m^2 - m) = \frac{3m-1}{2} = \frac{3}{4}n - \frac{1}{2}$$

Dla $n = 2m + 1$ czyli parzystego mamy $\frac{n-1}{2} = \frac{2m}{2} = m$ wtedy:

$$\sum_{k=0}^{2m} \max\{k, 2m - k\} = \sum_{k=0}^{m} (2m - k) + \sum_{k=m+1}^{2m} k =$$

$$\frac{(m+1)(2m+m)}{2} + \frac{m(m + 1 + 2m)}{2} = \frac{3m^2 + 3m + 3m^2 + m}{2} = \frac{6m^2 + 4m}{2} = 3m^2 + 2m$$

A więc wartość oczekiwana:

$$E(X) = \frac{1}{n} \sum_{k=0}^{2m} \max\{k, 2m - k\} = \frac{m(3m+2)}{n} = \frac{(n-1) \cdot \left(3 \cdot \left(\frac{n-1}{2}\right) + 2\right)}{2n} =$$

$$\frac{(n-1) \cdot (3n+1)}{4n} = \frac{3n^2 - 2n - 1}{4n}$$