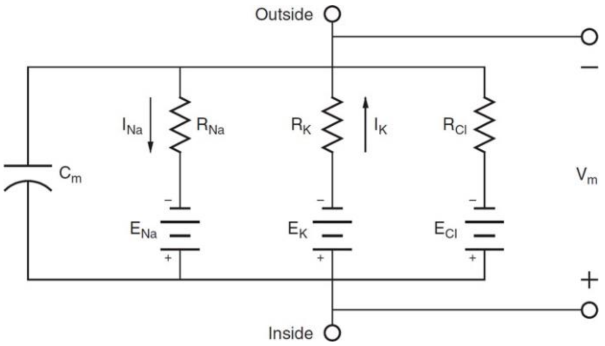
Exam 180530 solutions:

7.

a) You will not need to calculate Vm from the electrical model in this course, however, you need to be able to associate different terms of the circuit to the mathematical model. The electrical circuit is as below. Details of the comparison is available in the HH lecture slide 27. One can relate each term in the equation to the current in each branch in the electrical circuit where the equation can be obtained by implementing KCL to the circuit, that is, the sum of the capacitive current and the ionic current is equal to zero when the cell is at the resting status. You should also be able to motivate how the increase in the potential and speed of the circuit is related to the Hodgkin Huxley equation.



b) The simulations steps are available in detail in the lecture and also in the codes provided for the lab. The level of the detail needed in the answer, will be mentioned in the question, however, it is not expected that you know the equations by heart but you need to explain the main steps for this question.

8. a) the field variable for the electrical models in FEM is voltage with tensor rank 0 since it is a scalar quantity.

b) The computational time can be reduced by the below settings:

- Choosing the minimum required refinement in the mesh or by using mesh refinements only at the corners or geometry with a higher importance in the simulation.

- By using axisymmetric simulation (should be shortly explained), as the geometry in this example is symmetric

9. See slide 38 in the FEM lecture.

$s(x+\frac{\Delta x}{2})\Delta x + q(x)A(x) - q(x+\Delta x)A(x+\Delta x) = 0$ (heat flux equilibrium)

After dividing by Δx and taking the limit:

$$\lim_{\Delta x \to 0} s\left(x+\frac{\Delta x}{2}\right) + \frac{-q(x+\Delta x)A(x+\Delta x)+q(x)A(x)}{\Delta x} = 0$$

$$s - \frac{d(qA)}{dx} = 0$$

$$q = -k\frac{dT}{dx}$$

Strong form

$$\begin{cases} s + \dfrac{d}{dx}(Ak\dfrac{dT}{dx}) = 0, & 0 < x < l \quad \text{Governing equation} \\ -q = k\dfrac{dT}{dx} = \bar{q}, & x = 0 \\ T = \bar{T}, & x = l \end{cases}$$

Boundary conditions

10. MC is answered in a separate document.

11. At x<1 cm convection is negligible, so the third term on right hand side vanishes. Second term is always negative, more in periphery where da/dx grows in importance, accounts for diffusion towards more peripheral parts due to the expanding cross section. First term on right hand side changes sign where $\frac{\partial c}{\partial x}$ is maximal (x = 0.5-0.6 cm). For smaller x, the term is positive and accounts for build-up of gas in periphery; for larger x, the term is negative. Hence, first term on right hand side explains the decrease in gas concentration at x=0.7 and the increase at x=0.3.