Exam 2017 solutions- part 2:

6. The equation is a simplified version of bioheat equation. The field variable is temperature (T), the term on the left hand side represents/models the change of temperature vs. time. The first term on the right hand side considers temperature change in 3D space. Note the difference of the operators for 1D and 3D. Q is heat source or heat sink, e.g. blood perfusion that cools the area, the additional heat sources can be represented differently in different models.

7. For finding the governing equation and boundary conditions using the strong form, forces on the two sides of the element must be in equilibrium and the Hooke's law should be satisfied.

$$-p(x)+b(x+\frac{\Delta x}{2})\Delta x+p(x+\Delta x)=0 \quad \text{(Force equilibrium)}$$

After dividing by Δx and taking the limit:

$$\lim_{\Delta x \to 0}\frac{p(x+\Delta x)-p(x)}{\Delta x}+b(x+\frac{\Delta x}{2})=0$$

$$\frac{dp(x)}{dx}+b(x)=0$$

By definition of stress-strain, will give:

$$\sigma(x)=\frac{p(x)}{A(x)}, \qquad \varepsilon(x)=\lim_{\Delta x \to 0}\frac{u(x+\Delta x)-u(x)}{\Delta x}=\frac{du}{dx}$$

$$\sigma(x)=E(x)\varepsilon(x) \quad \text{(Hooke's law)}$$

By replacing the above equations, the domain governing equation, composing the strong form equation, will be obtained.

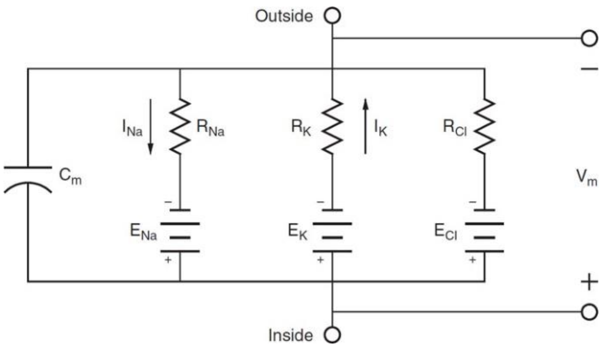
$$\frac{d}{dx}(AE\frac{du}{dx})+b(x)=0, \qquad 0<x<l \qquad \text{Governing equation}$$

$$\left.\begin{array}{ll} \sigma(x)=(E\frac{du}{dx})_{x=0}=t, & x=0 \\ u(x)=\bar{u} & x=l \end{array}\right\} \quad \text{Boundary conditions}$$

8. a) In the HH model, m, n and h model the probabilities of the Na and K channels being open or closed (you should specify in the exam). They are also referred to as state variables since they predict the status of the variable or model for the next time instance. g bar is the maximum possible conductivity of the corresponding ion channels.

b) The equation is non-linear and dynamic.

c) You will not need to calculate Vm from the electrical model in this course, however, you need to be able to associate different terms of the circuit to the mathematical model. The electrical circuit is as below. Details of the comparison is available in the HH lecture slide 27. One can relate each term in the equation to the current in each branch in the electrical circuit where the equation can be obtained by implementing KCL to the circuit, that is, the sum of the capacitive current and the ionic current is equal to zero when the cell is at the resting status. You should also be able to motivate how the increase in the potential and speed of the circuit is related to the Hodgkin Huxley equation.



9. MC is answered in a separate document.