LINKÖPING UNIVERSITY
Department of Biomedical Engineering

TBME08 Biomedical Modeling and Simulation
TEN1, Written examination
Wednesday 30th of May 2018, 14:00-18:00

You are allowed to use **pen, rubber, lecture notes** (submitted with exam), **dictionary** (standard bi-lingual) and a **calculator** (with a cleared memory) when treating the questions.

Write your identification number (no name) on every paper. **Pages without identification number are handled as if they were blank and not taken care of**.

Make sure that you present no more than **one task per page** using **single sided pages**. You are of course allowed to use more than one page per task.

The answers may be written in either **English** or **Swedish**.

A maximum score of 40 is possible.

Grading	
< 20	fail
20–26	3
27–33	4
≥ 34	5

Results will be available within 10 working days after the exam. ECTS grades will be given based on your total score.

Good luck!

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1. Specify the linearity/nonlinearity and if the systems below are static/dynamic. (2p)

a) $y(t+1) + y(t) = u(t-1) + 3u(t-2)$

b) $y(t) = u^2(t-1)$

c) $y(t) = \cos(u(t))$

d) $\dot{y}(t) + 3y(t) = 0.4u(t)$

2. Calculate the index of the following system. (2p)

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \dot{z} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix} z = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$

3. Consider the differential equation

$$\dot{y}(t) = -3y(t)$$
$$y(0) = 1$$

Euler's explicit method ($x_{n+1} = x_n + hf(x_n)$) is used to simulate the differential equation with a step size h=0.1 s. The exact solution for the differential equation is given by

$$y(t) = y_0 e^{-3t}$$

where $y_0 = y(0)$. Compute the local and global error at time t=0.3s. Is the simulation of the system with the given numerical method numerically stable? (3p)

4. Consider the model

$$y(t) + a_1 y(t-1) + a_2 y(t-2) = b_1 u(t-1) + b_2 u(t-2) + e(t)$$

with the parameters identified as

$$a_1 = -1.5, \quad a_2 = 0.7 \quad b_1 = 1.0, \quad b_2 = 0.5$$

By making a step-response experiment, i.e.

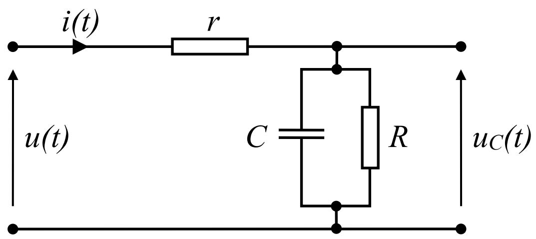
$$\begin{cases} u(t) = 0 & t < 0 \\ y(t) = 0 & t \leq 0 \\ u(t) = 1 & t \geq 0 \end{cases}$$

the output sequence obtained is (see next page)

$$y(1) = 1.0$$
$$y(2) = 3.0$$
$$y(3) = 5.3$$
$$y(4) = 7.35$$

Which value will $\hat{y}(5)$ have? (2p)

5. A three-element Windkessel model can be used for modelling hemodynamic responses. The electrical equivalent of the model is given by



The system is partly described by

$$i(t) = \frac{u_C(t)}{R} + C\,\frac{du_C(t)}{dt}$$

$$u(t) = ri + u_C$$

a) Give a state space description of the complete system, where the blood flow *i(t)* is the input and the blood pressure *u(t)* the output. (2p)

b) Estimate the output of the Windkessel model above at t = 0.4 s (i.e. $u(0.4)$) using Euler´s explicit method ($x_{n+1} = x_n + hf(x_n)$). (3p)

Simulation parameters:

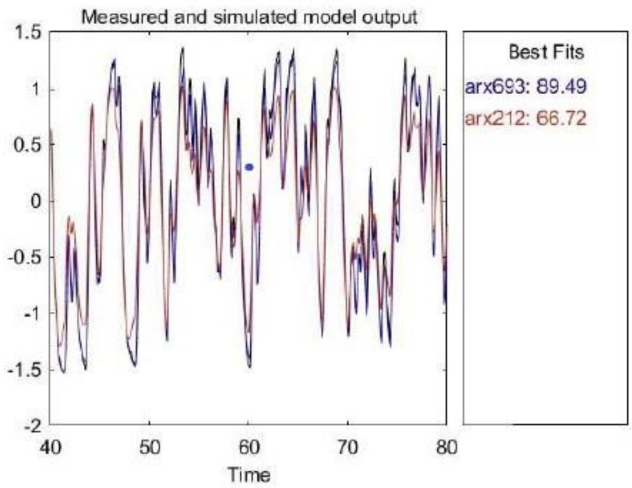
$r = 0.5$
$R = 3.0$
$C = 0.5$

$i(t) = \begin{cases} 0, & t < 0 \\ 100, & t \geq 0 \end{cases}$

$u_C(t) = 75,\ t \leq 0$

$h = 0.2$ s

6. You have estimated two ARX models: arx212 and arx693. Fig. below shows the output signals of the models (model output) with the corresponding FIT. Which model should you choose? Why? (2p).



7. The Hodgkin-Huxley cell membrane model is given by:

$$I = C_M \frac{dV}{dt} + \bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_L (V - V_L)$$

a) Draw the Hodgkin-Huxley cell membrane electrical circuit model and explain how the Hodgkin-Huxley equation is equivalent to the circuit. (3p)

b) How can the Hodgkin Huxley model be simulated using numerical methods? (3p)

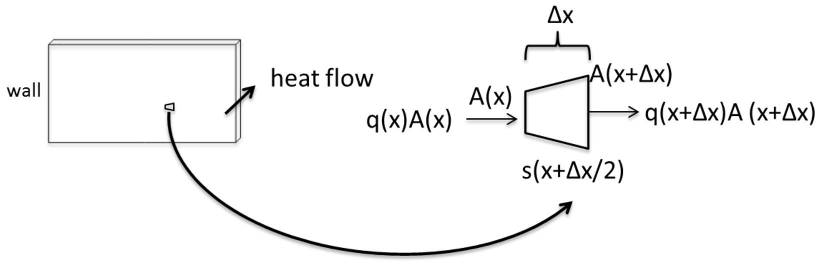
8. The electrical field in a cylinder of tissue is to be simulated using COMSOL muliphysics or an equivalent software. The electrical field is applied by a cone shaped electrode in the middle of the cylinder. (3p)

a) What is the field variable in the simulation and what tensor rank does it have?

b) Name two factors or techniques that can be used to reduce the computational time of the simulation.

9. The strong statement was used for obtaining the governing domain and boundary equations in the following beam using finite element methods for analysing the heat flow in the beam. Derive the given governing equation using strong form statement for **the domain only**. (3p)

Note: The description of the parameters and the strong form statement is given in the lecture notes attached to the exam.



10.

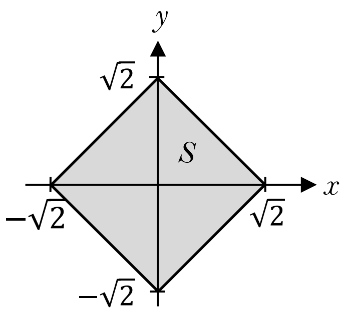
a) What is the main difference between a *true rng* (random number generator) and a *pseudo rng*? (1p)

b) Random numbers play a central role in stochastic Monte Carlo simulations. Create an *rng*, using **inversion**, that produces random numbers x that are distributed according to the probability density function

$$P(x) = \begin{cases} \cos(x \cdot \pi) + 1, & 0 \leq x \leq 1 \\ 0, & x < 0 \text{ } or \text{ } x > 1 \end{cases}$$

The *rng* should be based on a random number U that is uniformly distributed between 0 and 1. (3p)

c) Describe an *rng*, that produces uniformly distributed random points [x,y] on the surface S. The *rng* should be based on pairs of random numbers [U_1,U_2] that both are uniformly distributed between 0 and 1. (2p)



11. The 1D (x = distance, t = time) transport equation for the concentration of a gas along a tube with variable cross-sectional area $A(x)$ including alveoli and $a(x)$ without alveoli, can be written (c = concentration, U = convective velocity):

$$\frac{\partial c}{\partial t} = D\frac{a}{A}\frac{\partial^2 c}{\partial^2 x^2} + \frac{D}{A}\frac{da}{dx}\frac{\partial c}{\partial x} - U\frac{\partial c}{\partial x}$$

Below is a graph showing the oxygen concentration in the lung periphery obtained from solving the transport equation in a realistic lung geometry with a breathing cycle of inspiration 2.5s and expiration 2.5s. The lung was initially oxygen free. The time point is t=0.9s from start. The dotted blue curve is for a diffusion constant D=0.225 cm^2/s (that of oxygen) and the solid red line for D=0.

Explain why diffusion causes this effect and specifically: why for x=0.3 the dotted line has higher $c(x,t)$ than the solid line, and for x=0.7 the dotted line has lower $c(x,t)$ than the solid line. The explanation should be related to the terms in the transport equation. (6p)