LINKÖPING UNIVERSITY
Department of Biomedical Engineering

TBME08 Biomedical Modeling and Simulation
TEN1, Written examination
Saturday the 3rd of June 2017, 14:00-18:00

You are allowed to use **pen, rubber, lecture notes** (submitted with exam), **dictionary** (standard bi-lingual) and a **calculator** (with a cleared memory) when treating the questions.

Write your identification number (no name) on every paper. **Pages without identification number are handled as if they were blank and not taken care of**.

Make sure that you present no more than **one task per page** using **single sided pages**. You are of course allowed to use more than one page per task.

A maximum score of 40 is possible.

Grading	
<19	fail
19–26	3
27–33	4
≥34	5

Results will be available within 10 working days after the exam. ECTS grades will be given based on your total score.

Good luck!

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1. Specify the linearity/nonlinearity and if the systems below are static/dynamic. (2p)

a) $y(t) + (y(t-2))^3 = 2u(t-2)$

b) $\ddot{y}(t) + 2\dot{y}(t) + 7y(t) = u(t)$

c) $y(t+3) = \sin(u(t-1))$

d) $y(t-1) = 0.5\sqrt{u(t+1)}$

2.

a) Calculate the index of the following system. (2p)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 2 \end{bmatrix} x = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} u$$

b) Why is the index interesting when trying to simulate a system? (1p)

c) What is the index of a system described by an ordinary differential equation (ODE)? (1p)

3. Consider the differential equation

$$\dot{y}(t) = -2y(t) + 3$$
$$y(0) = 0$$

Euler's explicit method ($x_{n+1} = x_n + hf(x_n)$) is used to simulate the differential equation with a step size h=0.1 s. The exact solution for the differential equation is given by

$$y(t) = \frac{3}{2}(1 - e^{-2t})$$

Compute the global error at time t=0.2s. (3p)

4. Which of the systems below are ARX, ARMAX, OE or BJ? Motivate your answer (2p).

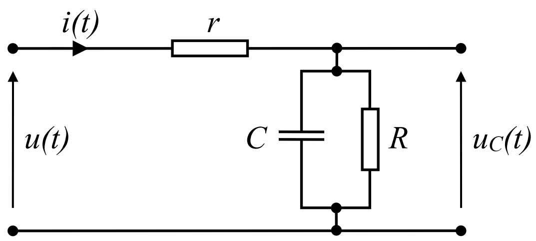
a) $y(t) = \frac{q^{-1}+3q^{-3}}{1+2q^{-1}} u(t) + \frac{1}{1+2q^{-1}} e(t)$

b) $y(t) = \frac{q^{-1}+3q^{-3}}{1+2q^{-1}} u(t) + e(t)$

c) $y(t) = \frac{q^{-1}+3q^{-3}}{1+2q^{-1}} u(t) + \frac{q^{-1}}{1+2q^{-1}} e(t)$

d) $y(t) = \frac{q^{-1}+3q^{-3}}{1+2q^{-1}+q^{-2}} u(t) + \frac{q^{-1}}{1+2q^{-1}} e(t)$

5. A three-element Windkessel model can be used for modelling hemodynamic responses. The electrical equivalent of the model is given by



The system is partly described by

$$i(t) = \frac{u_C(t)}{R} + C\frac{du_C(t)}{dt}$$

$$u(t) = ri + u_C$$

a) Give a state space description of the complete system, where the blood flow *i(t)* is the input and the blood pressure *u(t)* the output. (2p)

b) Estimate the output of the Windkessel model above at $t = 0.2$ s (i.e. $u(0.2)$) using Euler´s explicit method ($x_{n+1} = x_n + hf(x_n)$). (3p)

Simulation parameters:

$r = 0.2$
$R = 2.0$
$C = 0.25$

$i(t) = \begin{cases} 0, & t < 0 \\ 100, & t \geq 0 \end{cases}$

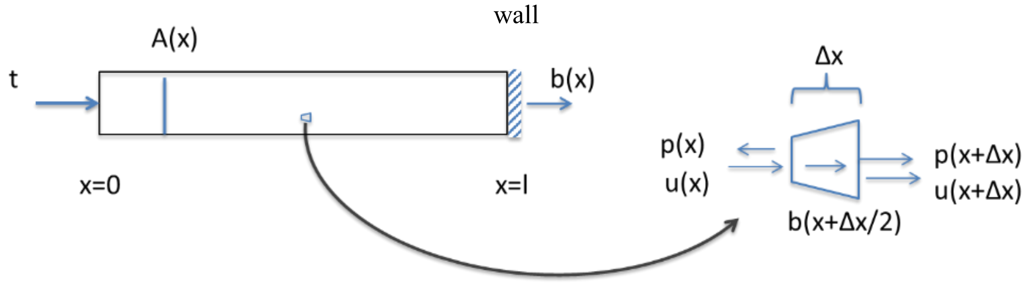
$u_C(t) = 75,\ t \leq 0$

$h = 0.1$ s

6. The equation below is used for modelling and simulation of the heat transfer using finite element methods. Specify the field variable and what the different operators in the equation consider. (2p)

$$\rho c \frac{\partial T}{\partial t} = \nabla(k\nabla T)+\mathrm{Q}$$

7. The strong statement was used for obtaining the governing domain and boundary equations in the following beam using finite element methods for analysing the stress in the beam. Derive the given governing equation using strong form statement for **the domain only**. (4p)



Parameters:
u(x) displacement **(unknown)**
b(x) distributed loading or external force (force/length)
t load/traction
p(x) internal force
σ (x) stress
ε(x) strain
A(x) cross-section
E(x) Young's modulus

Hooke's law:

σ(x) = E(x)ε(x)

Hint: Multiply b($x + \frac{\Delta x}{2}$) by the length of the element to get the force.

8. The Hodgkin-Huxley cell membrane model is given by:

$$I = C_M \frac{dV}{dt} + \bar{g}_K n^4 (V - V_K) + \bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_L (V - V_L)$$

a) What do the parameters *m*, *n* and *h, and* $\bar{g}$ model? (2p)

b) Specify the following properties/types for the above equation. (1p)

- static or dynamic
- linear or non-linear

c) Draw the Hodgkin-Huxley cell membrane electrical circuit model and explain how the Hodgkin-Huxley equation is associated to the circuit. (3p)

9.

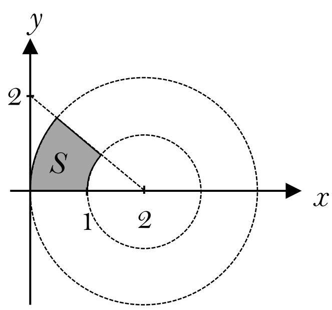
a) Why is it not suitable to use pseudo random number generators in lottery applications? (1p)

b) Random numbers play a central role in stochastic Monte Carlo simulations. Create a random number generator (*rng*), using inversion, that produces random numbers x that are distributed according to the probability density function

$$P(x) = \begin{cases} 2 - 2x, & 0 \le x \le 1 \\ 0, & x < 0 \text{ } or \text{ } x > 1 \end{cases}$$

The *rng* should be based on a random number *U* that is uniformly distributed between 0 and 1. (2p)

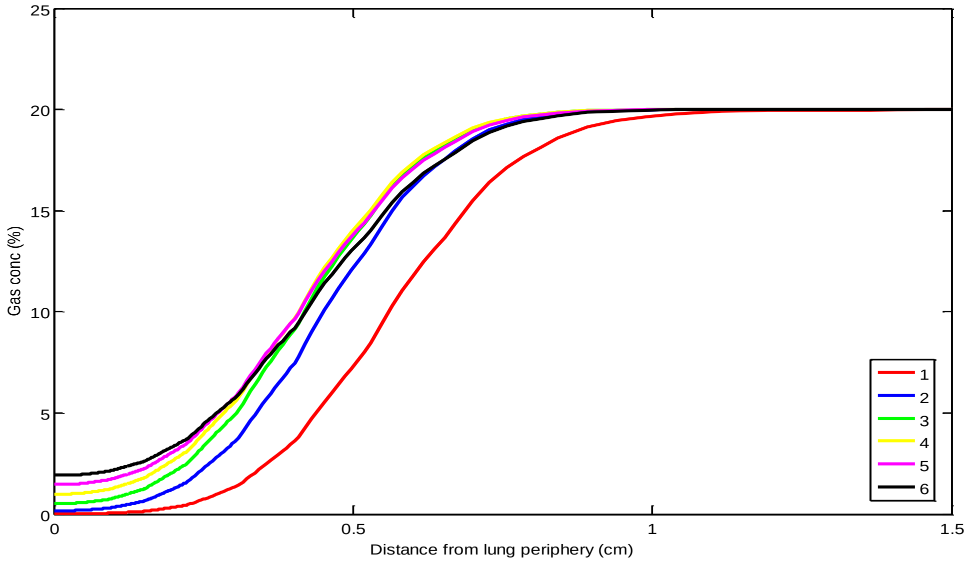
c) Describe an *rng*, that produces uniformly distributed random points [*x*,*y*] on the surface *S*. The *rng* should be based on pairs of random numbers [U_1,U_2] that both are uniformly distributed between 0 and 1. (3p)



10. The 1D (x = distance, t = time) transport equation for the concentration of a gas along a tube with variable cross-sectional area $A(x)$ including alveoli and $a(x)$ without alveoli, can be written (c = concentration, U = convective velocity):

$$\frac{\partial c}{\partial t} = D\,\frac{a}{A}\,\frac{\partial^2 c}{\partial^2 x^2} + \frac{D}{A}\frac{da}{dx}\,\frac{\partial c}{\partial x} - U\,\frac{\partial c}{\partial x}$$

Below is a graph showing the oxygen concentration in the lung periphery obtained from solving the equation for a breathing cycle of 2.5s inspiration and 2.5s expiration. The lung was initially oxygen free and six time points are shown, t = 0.9, 1.1, 1.3, 1.5, 1.7 and 1.9s from start (curves 1 to 6).



a) Describe the effects of convection and diffusion during inspiration, for x in the lung periphery using the graph above. Specifically address gas front, stationary gas front, when and where are convection and diffusion important. (4p)

b) How will curve #3 (t = 1.3 s) change if U is doubled compared to that used in the graph above? Illustrate by drawing $c(x,t)$ for original U and with doubled U. Also motivate and emphasize most important changes in the curve. (2p)