Solutions to the TBME08 exam, 20180530

1. a) linear - dynamic
   b) non linear - dynamic
   c) non linear static
   d) linear dynamic

2. a) The system can be written as

$$
\begin{aligned}
&\dot{z}_3 + z_2 = u \\
&z_3 = u \\
&\dot{z}_1 + 2z_1 = u
\end{aligned}
$$

We want to write the equation in explicit form, i.e. $\dot{z}$ as a function of z, input and possibly derivatives of the input. We see that

$$
\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} -2z_1 + u \\ \\ -z_2 + u \end{bmatrix}
$$

The components $\dot{z}_1$ and $\dot{z}_3$ are already given explicitly but not $\dot{z}_2$. This means that the index is larger than 0.

We differentiate the algebraic part

$$\dot{z}_3 = \dot{u}$$

we replace $\dot{z}_3$ with $\dot{z}_3 = -z_2 + u$ resulting in

$$-z_2 + u = \dot{u}$$

we differentiate the equation

$$-\dot{z}_2 + \dot{u} = \ddot{u}$$

Now $\dot{z}$ is expressed explicily

$$
\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} -2z_1 + u \\ \dot{u} - \ddot{u} \\ -z_2 + u \end{bmatrix}
$$

We have differentiated twice in order to express $\dot{z}$ explicity so INDEX = 2.

3. Given the differential equation $\dot{y}(t) = -3y(t)$, the approximated solution with Euler explicit method becomes $y_{n+1} = (1-3h)\,y_n$ that gives at time $t = 0.3$ s $y_3 = 0.343$.

   Exact solution: $y(0.3) = 0.4066$.

   Global error at time $t = 0.3$ s: $E = y(0.3) - y_3 = 0.0636$

   Local error at time $t = 0.3$ s: $e = y(3) - z_{t=0.3} = 0.0224$

   where $z_{t=0.3} = (1 - 3 \cdot 0.1) \cdot y(0.2) = 0.3842$

4. The predictor is: $\widehat{y}(t) = 1.5y(t-1) - 0.7y(t-2) + u(t-1) + 0.5u(t-2)$ that gives $\widehat{y}(5) = 8.8150$.

5. a) The model for the system is

$$\begin{aligned} i(t) &= \frac{1}{R}u_C(t) + C\frac{du_C(t)}{dt} \\ u(t) &= ri(t) + u_C(t) \end{aligned}$$

   It is a first order differential equation so the state has one component

$$x_1(t) = u_C(t)$$

   State-space description becomes

$$\begin{aligned} \dot{x_1}(t) &= -\frac{1}{RC}x_1(t) + \frac{1}{C}i(t) \\ y(t) &= x_1(t) + ri(t) \end{aligned}$$

   where we can say

$$A = -\frac{1}{RC}, \quad B = \frac{1}{C}, \quad C = 1, \quad D = r$$

   b) The simulation of the model with Euler's explicit method gives

$$\begin{aligned} &x_{1,n+1} = x_{1,n} + h\left(-\tfrac{1}{RC}x_{1,n} + \tfrac{1}{C}i_n\right) \\ &y_n = x_{1,n} + ri_n \end{aligned}$$

$$\begin{array}{lll} n=0 & t=0 & x_{1,0} = u_C(0) = 75 \\ n=1 & t=0.2 & x_{1,0.2} = x_{1,0} + h\left(-\tfrac{1}{RC}x_{1,\,0} + \tfrac{1}{C}i_0\right) = 75 + 0.2\left(-\tfrac{1}{RC}75 + \tfrac{1}{C}100\right) = 105 \\ n=2 & t=0.4 & x_{1,0.4} = x_{1,0.2} + h\left(-\tfrac{1}{RC}x_{1,0.2} + \tfrac{1}{C}i_{0.2}\right) = 131 \\ & & y_{0.4} = x_{1,0.4} + 0.2 \cdot 100 = 181 \end{array}$$

6. Model ARX693 should be chosen because it has a better FIT that means that it can reproduce better the behaviour of the system