

AI505 – Optimization

Sheet 06, Spring 2025

Exercises with the symbol $+$ are to be done at home before the class. Exercises with the symbol $*$ will be tackled in class. The remaining exercises are left for self training after the exercise class. Some exercises are from the text book and the number is reported. They have the solution at the end of the book.

Exercise 1⁺ (11.1)

Exercise 2⁺ (11.2)

Exercise 3⁺ (11.3)

Suppose we want to solve:

$$\begin{aligned} &\text{minimize } 6x_1 + 5x_2 \\ &\text{subject to } 3x_1 - 2x_2 \geq 5. \end{aligned}$$

How would we translate this problem into a linear program in equality form with the same minimizer?

Exercise 4

Consider the following problem:

$$\begin{aligned} &\text{minimize } 5x_1 + 4x_2 \\ &\text{s.t. } 2x_1 + 3x_2 \leq 5 \\ &\quad 4x_1 + x_2 \leq 11 \end{aligned}$$

Solve the problem numerically implementing the simplex algorithm.

Exercise 5⁺ (11.4)

Exercise 6⁺ (11.5)

Exercise 7^{*}

A pharmaceutical company produces three types of drugs: A, B, and C. These drugs require raw materials, chemical processing time, and packaging units. The goal is to maximize profit, taking into account restrictions due to resource availability and production balance.

Drug A contributes 60 DKK per unit to the profit, drug B contributes 100 DKK per unit, drug C contributes 80 DKK per unit. The consumptions per units of drug of raw materials, chemical processing time, and packaging units are given in Table ?? together with the quantities available. While consumptions of raw material and processing time cannot exceed the amount available, excess of packaging units is allowed. Extra packaging units can be bought but each extra unit reduces profit at 5 DKK per unit while packaging units left can be reused in the next production period and hence contribute positively to the profit with the same amount of DKK per unit. Due to contractual obligations, Drug B must be produced in exactly twice the amount of Drug A.

Formulate the problem in linear programming terms. Write first the *instantiated* model and then the abstract, *general* model separating model from data.

Exercise 8^{*}

Drug	A	B	C	
raw material (Kg)	5	8	6	600
processing time (hours)	3	4	5	400
packaging units	2	3	1	200
profit	60	100	80	-5

Table 1: Data from the pharmaceutical company

Consider the following linear programming problem:

$$\begin{aligned}
 &\text{maximize } Z = 60x_1 + 100x_2 + 80x_3 - 5x_4 \\
 &\text{s.t. } 5x_1 + 8x_2 + 6x_3 \leq 600 \\
 &\quad 3x_1 + 4x_2 + 5x_3 \leq 400 \\
 &\quad 2x_1 + 3x_2 + x_3 = 200 + x_4 \\
 &\quad x_2 = 2x_1 \\
 &\quad x_1, x_2, x_3 \geq 0 \\
 &\quad x_4 \in \mathbb{R}
 \end{aligned}$$

Your tasks:

- Transform the problem in standard form
- Transform the problem in equality form
- Solve the problem numerically with `scipy.optimize.linprog`. Read this [tutorial](#).