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AI505 – Optimization

Sheet 02, Spring 2025

Exercises with the symbol $^+$ are to be done at home before the class. Exercises with the symbol * will be tackled in class. The remaining exercises are left for self training after the exercise class. Some exercises are from the text book and the number is reported. They have the solution at the end of the book.

Exercise 1$^+$ (6.1)

What advantage does second-order information provide about the point of convergence that first-order information lacks?

Exercise 2$^+$ (6.2)

When finding roots in one dimension, when would we use Newton's method instead of the bisection method?

Exercise 3* (6.4, 6.9)

Apply Newton's method to $f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T H\mathbf{x}$ starting from $\mathbf{x}_0 = [1, 1]$. What have you observed? Use H as follows:

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1000 \end{bmatrix}$$

Next, apply gradient descent to the same optimization problem by stepping with the unnormalized gradient. Do two steps of the algorithm. What have you observed? Finally, apply the conjugate gradient method. How many steps do you need to converge?
Repeat the exercise for:

$$f(\mathbf{x}) = (x_1 + 1)^2 + (x_2 + 3)^2 + 4.$$

Exercise 4$^+$ (6.5)

Compare Newton's method and the secant method on $f(x) = x^2 + x^4$, with $x_0 = -3$ and $x_0 = -4$. Run each method for 10 iterations. Make two plots:

1. Plot f vs. the iteration for each method.
2. Plot f' vs. x. Overlay the progression of each method, drawing lines from $(x_i, f'(x_i))$ to $(x_{i+1}, 0)$ to $(x_{i+1}, f'(x_{i+1}))$ for each transition.

What can we conclude about this comparison?