Department of Mathematics and Computer Science
University of Southern Denmark, Odense

April 28, 2025
Marco Chiarandini

AI505 – Optimization

Sheet 09, Spring 2025

Exercises with the symbol $^+$ are to be done at home before the class. Exercises with the symbol * will be tackled in class. The remaining exercises are left for self training after the exercise class. Some exercises are from the text book and the number is reported. They have the solution at the end of the book.

Exercise 1$^+$ (19.2)

A Boolean satisfiability problem, often abbreviated SAT, requires de- termining whether a Boolean design exists that causes a Boolean-valued objective function to output true. SAT problems were the first to be proven to belong to the difficult class of NP-complete problems.21 This means that SAT is at least as difficult as all other problems whose solutions can be verified in polynomial time. Consider the Boolean objective function:

$$f(\boldsymbol{x}) = x_1 \wedge (x_2 \vee \neg x_3) \wedge (\neg x_1 \vee \neg x_2)$$

Formulate the problem as an integer linear program. Can any Boolean satisfiability problem be formulated as an integer linear program? Solve the problem with scipy.

Exercise 2*

Consider $\max\{\boldsymbol{c}^T\boldsymbol{x} \mid A\boldsymbol{x} = \boldsymbol{b}, \boldsymbol{x} \in \mathbb{Z}^n\}$. Sometimes the solution of the linear relaxation is already integral. Can you find a sufficient condition for the matrix A for that to happen?

Exercise 3*

Consider the following problem:

$$\begin{aligned} &\min x_1 + x_2 \\ \text{subject to: } &\frac{2}{3}x_1 + \frac{1}{2}x_2 \geq 1 \\ &x_1, x_2 \geq 0 \\ &x_1, x_2 \in \mathbb{Z} \end{aligned}$$

Derive a Chvatal-Gomory cut.

Exercise 4

Apply the linear programming branch and bound algorithm to the following instance of the 0-1 knapsack problem: values $v = [9, 4, 2, 3, 5, 3]$, weights $w = [7, 8, 4, 5, 9, 4]$ and capacity $W = 20$.

Exercise 5

Consider the problem of selecting students for a swimming medley relay team. We have a table showing times for each swimming style of five students:
We need to choose a student for each of the four swimming styles such that the total relay time is minimized. Formulate the problem as a MILP and solve it in Python.

Student	backstroke	breaststroke	butterfly	freestyle
A	43.5	47.1	48.4	38.2
B	45.5	42.1	49.6	36.8
C	43.4	39.1	42.1	43.2
D	46.5	44.1	44.5	41.2
E	46.3	47.8	50.4	37.2